Computation of Proper Elements. Reconnecting Groups of Simulated Space Debris. ⋆

Tudor Vartolomei[1], Alessandra Celletti[2], and Giuseppe Pucacco[3]

[1] Department of Mathematics, University of Rome Tor Vergata
Via della Ricerca Scientifica 1, 00133 Rome, Italy
vartolom@mat.uniroma2.it
[2] Department of Mathematics, University of Rome Tor Vergata
Via della Ricerca Scientifica 1, 00133 Rome, Italy
celletti@mat.uniroma2.it
[3] Department of Physics, University of Rome Tor Vergata
Via della Ricerca Scientifica 1, 00133 Rome, Italy
Giuseppe.Pucacco@roma2.infn.it

Abstract. Proper orbital elements (or proper elements) are quasi-integrals of motion, namely quantities that are stable for long periods of time. The proper elements are obtained from the mean elements after a canonical change of coordinates that averages a Hamiltonian function w.r.t. the semi-short and long-periodic variables. We show that proper elements can be used to identify groups of simulated space debris associated to the same break-up event. The proposed method allows us to track the evolution of families of space debris.

Keywords: Proper elements · Space debris · Normal form · Simulator.

1 Introduction

The implemented procedure for reconnecting groups of space debris is composed by several ingredients. We develop a Hamiltonian model that describes the dynamics of space debris around the Earth, implement a Lie series normalization procedure to compute the proper elements associated to semi-major axis, eccentricity and inclination, and create a software, which reproduces the break-up model Evolve 4.0 provided by NASA, used for generating the synthetic families of space debris obtained after a simulated collision or explosion.

2 Method used

The procedure used to obtain the results is summarized by the following steps:

1. Create a Hamiltonian model that includes the perturbation due to the Earth (as a non-spherical shape), Moon and Sun, Solar radiation pressure.

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2. Implement a Lie series normalization procedure to determine the canonical transformation from the mean elements to the proper elements.
3. Use the break-up simulator to create synthetic space debris groups. The software returns the initial orbital elements of each fragment generated after a break-up event.
4. Propagate the mean elements of each space debris from the groups and save the data at some certain time over the period $(t_0, t_1, \ldots, T)$.
5. With the generating function obtained at step 2 and the evolution of the mean elements obtained at step 4, we compute the corresponding proper elements at time $(t_0, t_1, \ldots, T)$.
6. Implement statistical and visual methods to compare the evolution of the mean and proper elements.

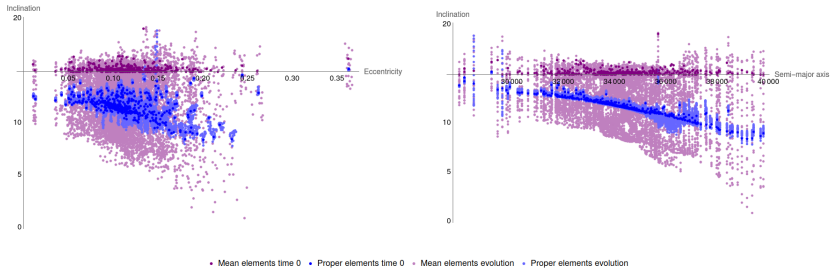

Fig. 1. The comparison between variation of mean elements (purple and light purple dots) and proper elements (blue and light blue dots) for the fragments generated by a collision between a spacecraft ($\{a, e, i, M, \omega, \Omega\} = \{34300\,km, 0.1, 15°, 55°, 34°, 26°\}$) of 1000 kg and a projectile of 6 kg at a relative velocity of 5500 m/s.

3 Main results

An experiment performed following the steps above is presented in Figure 1. It describes the comparison between the evolution of the mean elements of each fragment (purple and light purple dots) and the proper elements of each fragment (blue and light blue dots). The plot highlights the stability of the proper elements over 200 years, while the evolution of the fragments in the mean elements has larger variations.

References

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