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Defence and Security Accelerator

University of Strathclyde Engineering

Multi-Static Radar for Manoeuvre Detection

Simão da Graça Marto*
Massimiliano Vasile
Sebastian Diaz Riofrio
Christos Ilioudis
Carmine Clemente
*simao.da-graca-oliveira-marto@strath.ac.uk

Space Situation Awareness (SSA)

- The ever-increasing number of space missions and potential catastrophic effects of a collision has made SSA a topic of great interest over the last years.
- Monitoring of targets such as ballistic missiles and spy satellites makes SSA also critical to national defense.

In this presentation

- Metrics used for manoeuvre detection
- Assessment of the quality of these metrics
- Application to test cases

Manoeuvre detection

- Determine, from noisy observations of the state, if a manoeuvre was performed,

- Using a metric of the form $G(x_0, x_f)$ which indicates whether a measure was performed. Such a metric is itself a random variable.

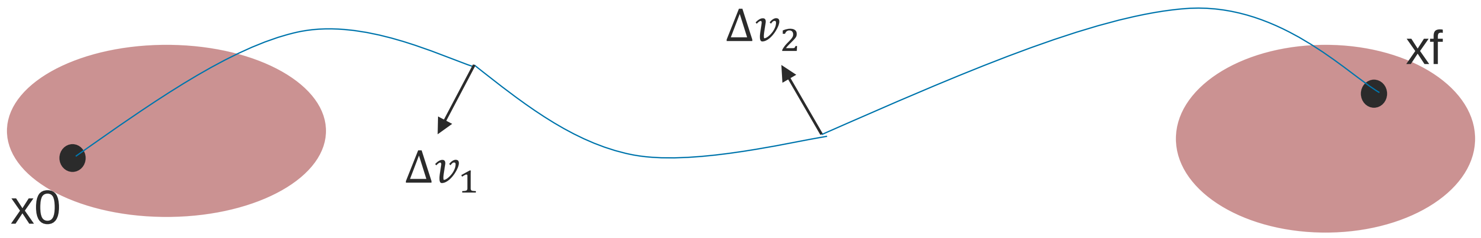
Optimal Control Based Manoeuvre Detection

- If one assumes any manoeuvre that might happen is optimal with respect to delta-v, a simple metric would be

$$G(x_0, x_f) = \min_u \int \|u(t)\| dt$$
$$s.t.: \ x(t_0, u) = x_0$$
$$x(t_f, u) = x_f$$

Impulse Sequence

- Approximating manoeuvres as a sequence of instantaneous impulses, the total delta-v for such a manoeuvre can be used as a metric
- Optimised using Majorization-Minimization



$$G_{\Delta v} = \sum_{k} \Delta v_k$$

Smooth Cost Function

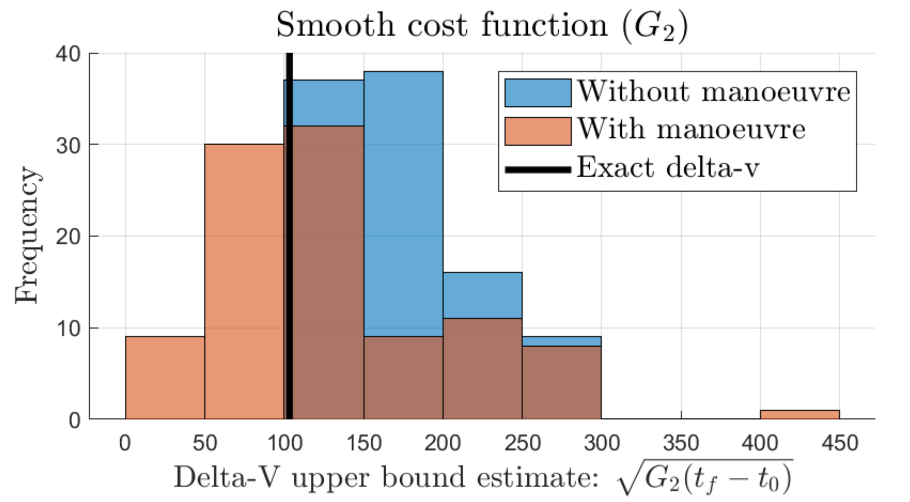
- $G_2 = \min \int \|u(t)\|^2 dt$
- To account for uncertainty in this metric, the optimization is over confidence sets Ω.

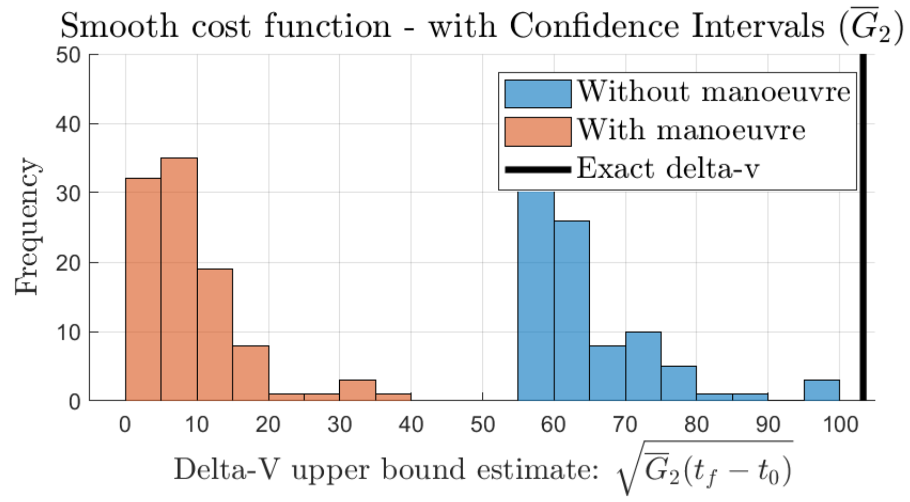
$$\overline{G}_2(\mathbf{x}_0, \mathbf{x}_f) = \min_{u} \int_{t_0}^{t_f} \|u(t)\|^2 \mathrm{d}t$$

$$s.t.\ \mathbf{x}(t_0) \in \Omega(\mathbf{y}_0)$$

$$\mathbf{x}(t_f) \in \Omega(\mathbf{y}_f)\ .$$

$$\Omega(y) = \left\{ \mathbf{x} : (y - h(\mathbf{x}))^T R^{-1} (y - h(\mathbf{x})) < F_{\chi_k^2}^{(-1)}(p) \right\}$$





Likelihood based

- Similarly to hypothesis testing in statistics, where the null hypothesis is that no manoeuvre occurred.
- Simplest option is to use the Mahalanobis distance on final state under the assumption that no manoeuvre occurs [2]:

$$G_{MD} = (\overline{\mathbf{x}}_f - \hat{\mathbf{x}}_f)^T (\Sigma_p + \Sigma_y)^{-1} (\overline{\mathbf{x}}_f - \hat{\mathbf{x}}_f)$$

- An alternative, proposed in this work, is to take the following minimization:

$$\overline{G}_{\mathrm{MD}}(\mathbf{y}_0, \mathbf{y}_f) = \min_{\mathbf{x}_0} \; (\mathbf{y}_0 - \mathbf{h}(\mathbf{x}_0))^T R^{-1} (\mathbf{y}_0 - \mathbf{h}(\mathbf{x}_0)) + (\mathbf{y}_f - \mathbf{h}(\mathbf{x}_f))^T R^{-1} (\mathbf{y}_f - \mathbf{h}(\mathbf{x}_f))$$

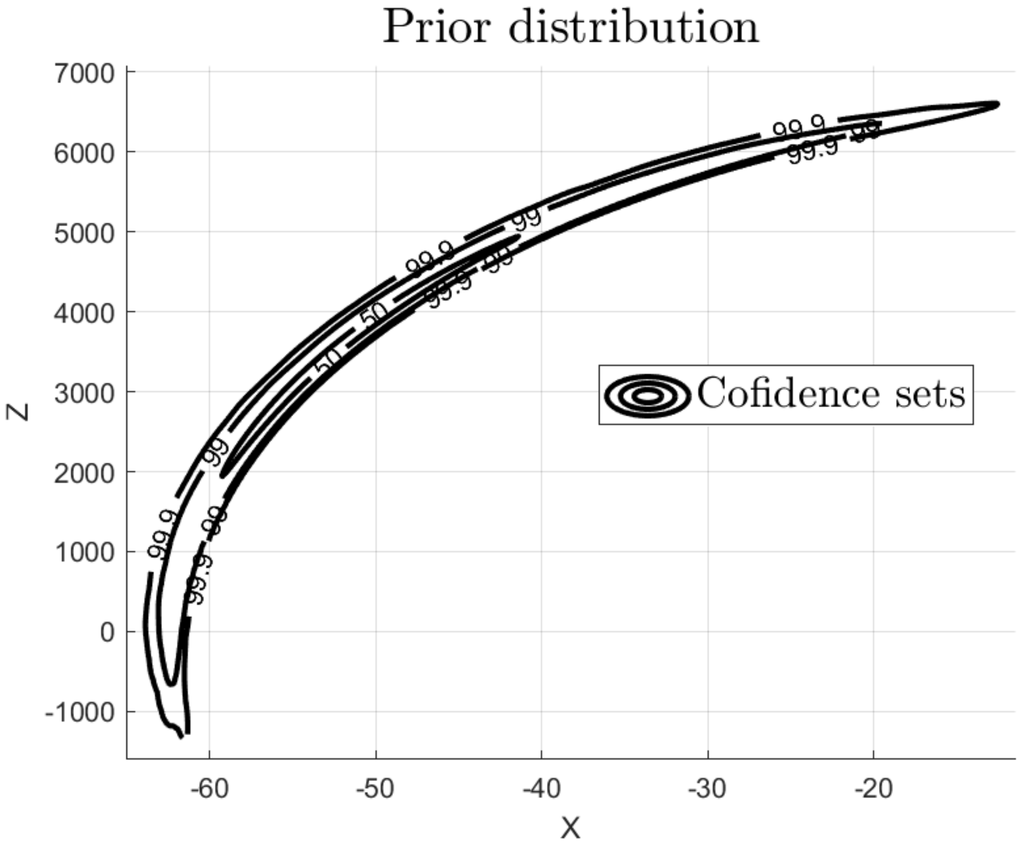
$$s.t. \; \mathbf{x}_f = F(\mathbf{x}_0) \; .$$

[2] - J. M. Montilla, J. C. Sanchez, R. Vazquez, J. Galan-Vioque, J. R. Benayas, and J. Siminski, ‘Manoeuvre detection in Low Earth Orbit with Radar Data’

Likelihood based – Gaussian Mixture

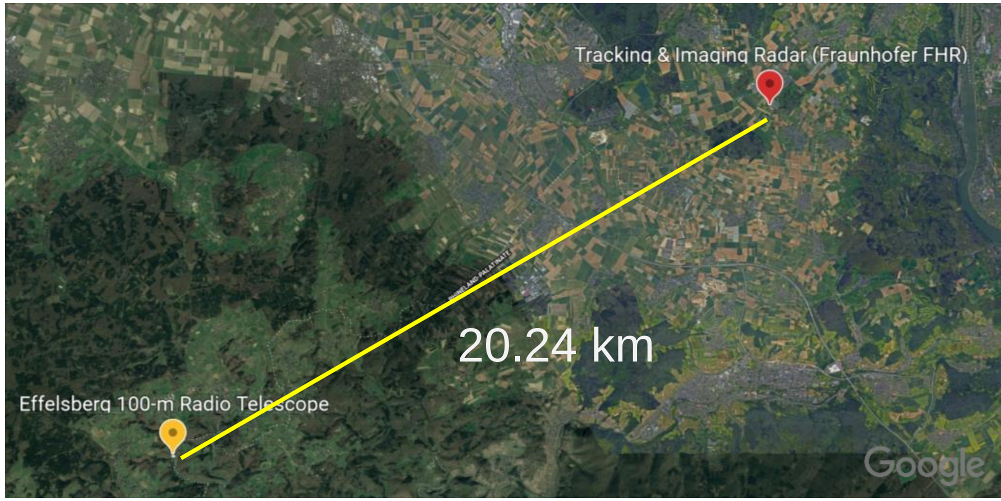
- An alternative approach is to model the uncertainty as a Gaussian Mixture model: $p(\boldsymbol{x}) = \sum_k w_k N(\boldsymbol{x}, \boldsymbol{\mu}_k, \Sigma_k)$
- Each Gaussian is propagated with an Unscented Transform, and the weight update is given by:
- $$\overline{w}_k^{t+1} = \frac{w_k^t \exp\left(-\frac{1}{2}\boldsymbol{r}^T \Sigma_k^y \boldsymbol{r}\right)}{\left(2\pi \det(\Sigma_k^y)\right)^{n/2}}$$
- The normalization factor is the metric:

$$\mathrm{G}_{\mathrm{GMM}} = \sum_k \overline{w}_k^{t+1}$$



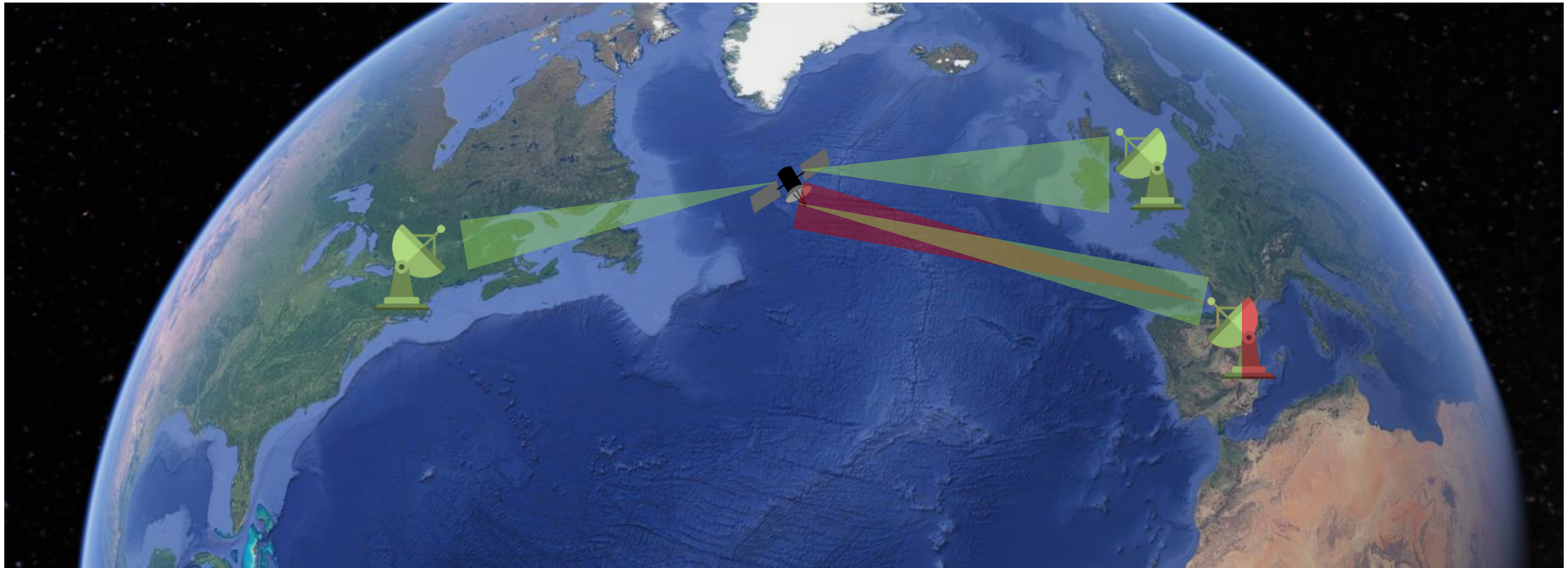
Conventional SSA

- SSA is primarily achieved using Radar in monostatic configuration;
- Radio telescopes can be used as passive receivers located in bistatic configuration;
- Due to the target relative distance, such bistatic configurations can be considered pseudo-monostatic;



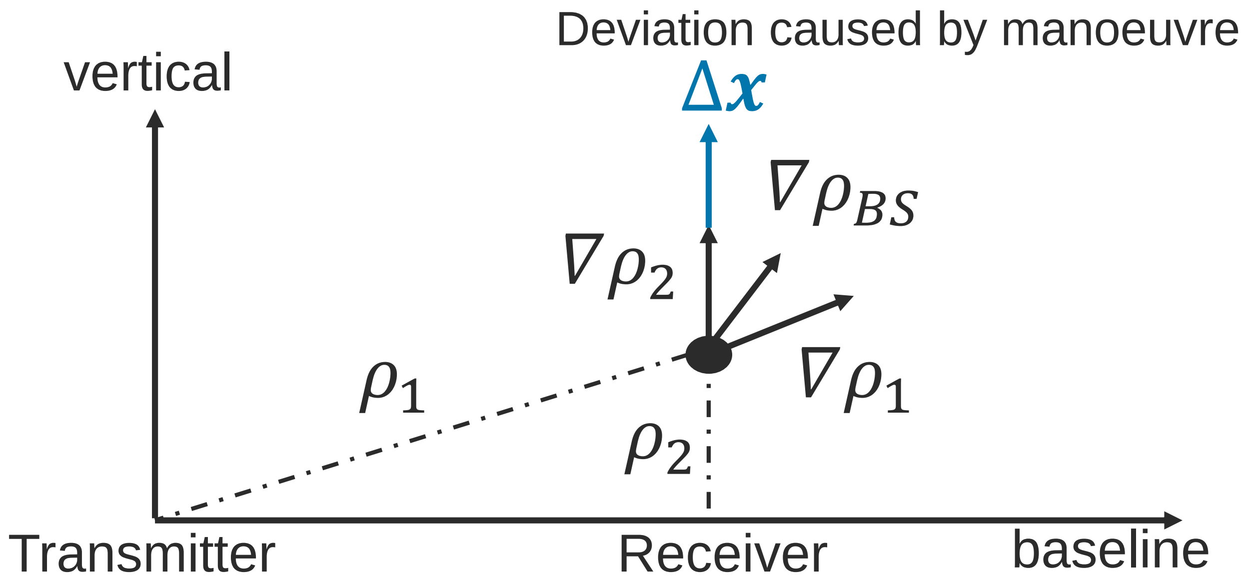
Long Baseline Bistatic Radar

- Remote telescopes can be used in combination with a monostatic Radar to form **distributed** bistatic pairs;
- Developed within the NATO SET-293 “RF SENSING FOR SPACE SITUATIONAL AWARENESS”



Effect of bistatic radar on accuracy

- Each receiver measures the bistatic range: $\rho_{BS} = \rho_1 + \rho_2$
- Each range measurement reduces the variance in the state estimate along the direction of its gradient
- Each additional receiver increases the chances that the deviation caused by the manoeuvre will be detected

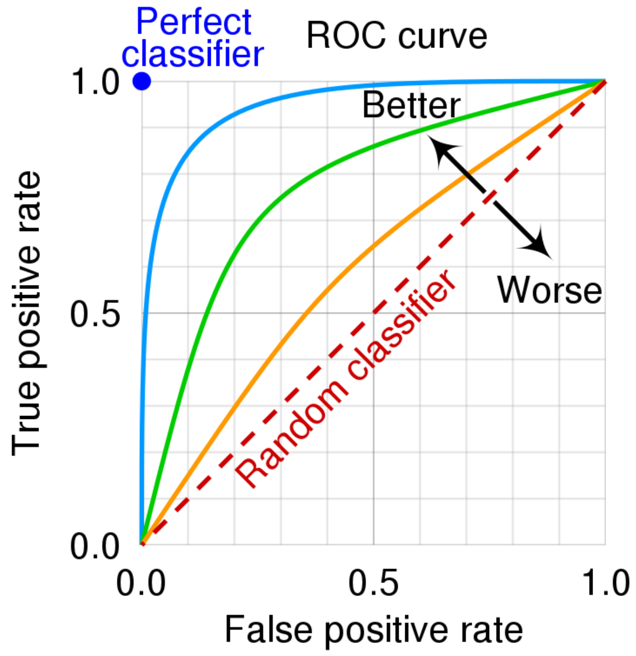


Quality of a metric

- A binary classifier's accuracy is assessed by its Receiver Operator Characteristic (ROC) Curve, which plots the true positive rate against the false positive rate.
- The area under this curve (AUC), corresponds to $P(G_{\Delta v} > G_0)$
- Making a quadratic-Gaussian approximation to G the AUC is estimated with $q\{G\}$

$$G(\Delta\mathbf{x}) \approx \Delta\mathbf{x}^T \hat{A}_\Delta \Delta\mathbf{x}$$

$$q\{G\} = \Delta\mathbf{x}^T \frac{\hat{A}_\Delta}{\sqrt{\mathrm{tr}(APAP) + \delta\mathbf{x}^T APA\delta\mathbf{x}}} \Delta\mathbf{x}$$



Source: CMG Lee and MartinThoma, Wikimedia Commons.

Test case 1

Tx: Millstone Hill Radar

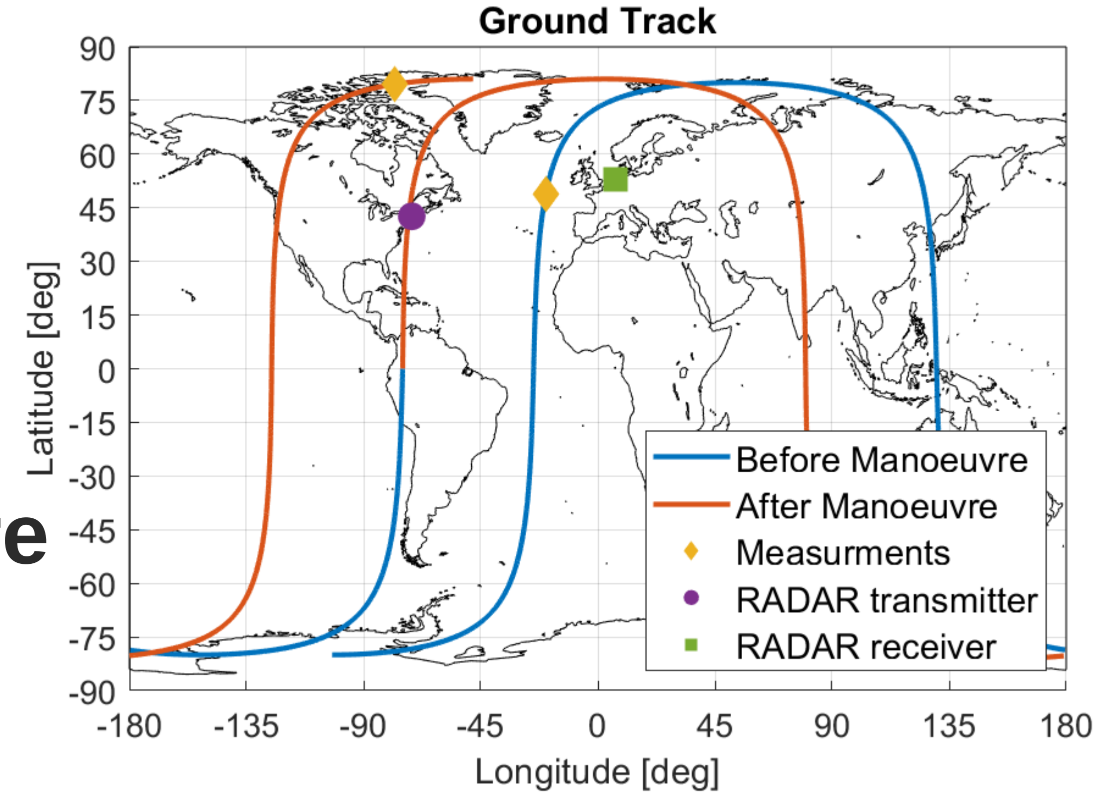
- Millstone Hill Steerable Antenna (MISA)
- 23 meter radius full steerable antenna
- Location: MIT Haystack Observatory, Massachusetts, USA

Rx: Westerbork radar

- Westerbork Synthesis Radio Telescope (WSRT)
- 25 meter radius antenna
- Location: Westerbork, Netherlands

Inclination Change Manoeuvre

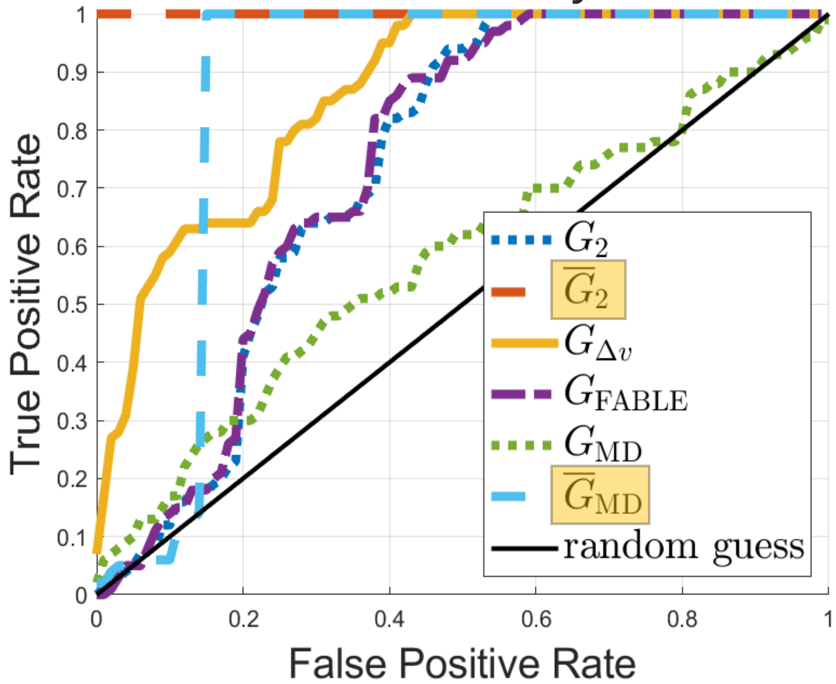
- 1 degree inclination change
- delta-v: ~100m/s



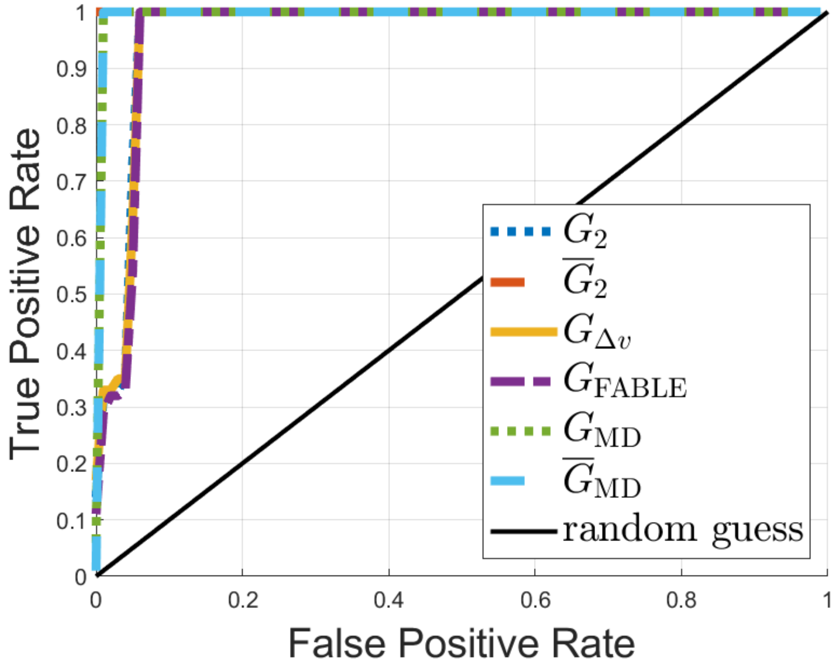
Test case 1 – MEO inclination change

- $\sigma_\rho = 100m,\ \sigma_\alpha = \sigma_\beta = 1'$

Monostatic data only

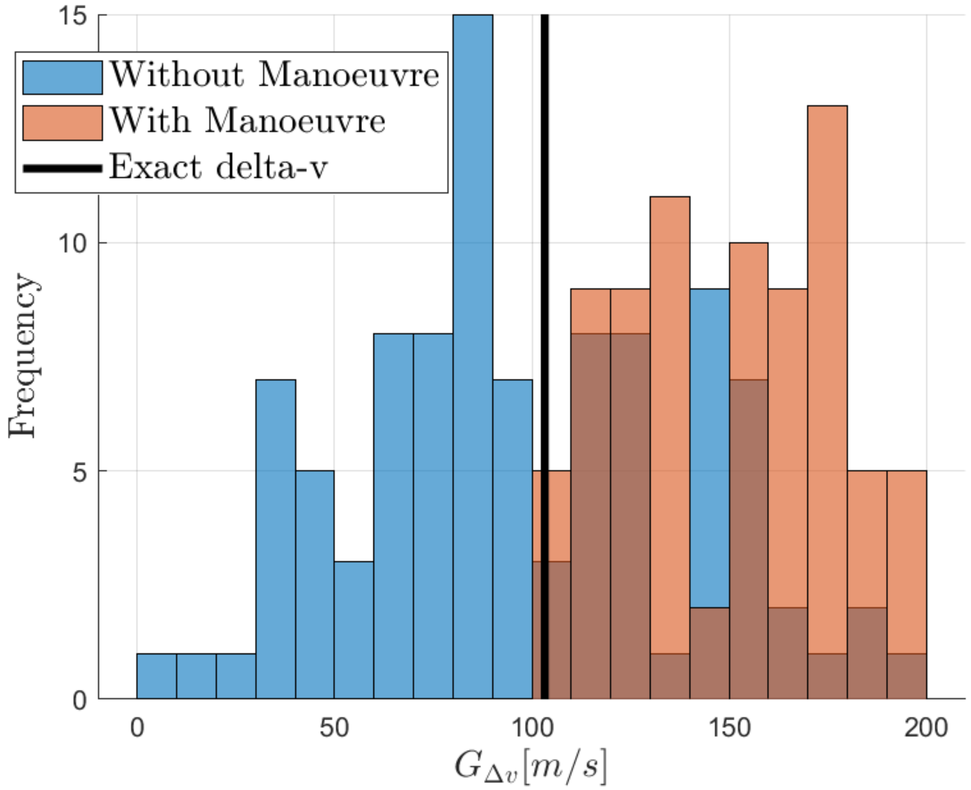


Monostatic + Bistatic data



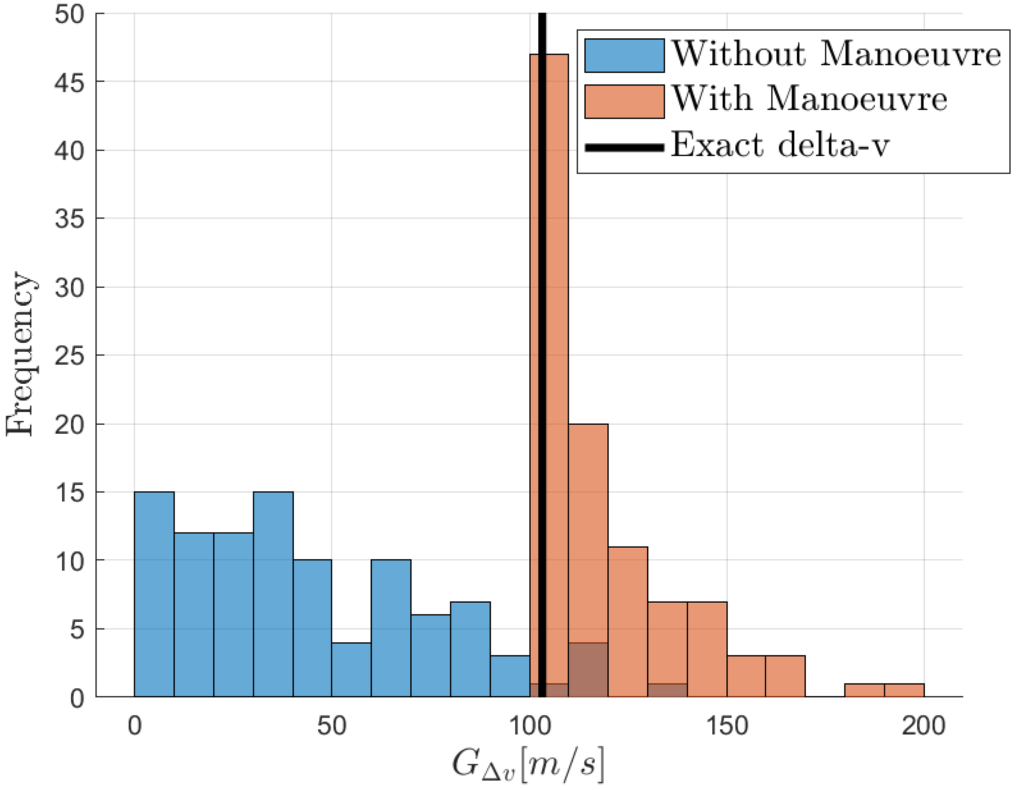
Test case 1 – MEO inclination change

Delta-v estimates with monostatic observations



RMSE = 73.8m/s

Delta-v estimates with bistatic observations



RMSE = 24.7m/s

Test case 1 – quality of metric results

- Which metric is best depends on the manoeuvre being performed, among other factors. Making a small manoeuvre approximation, we can write our quality of metric as a quadratic form:

$$\tilde{q}\{G\} = \frac{\Delta\mathbf{x}^T \hat{A} \Delta\mathbf{x}}{\sqrt{\mathrm{tr}\left(APAP\right)}} = \delta\mathbf{x}_f^T A_q\{G\} \delta\mathbf{x}_f$$

- By taking the eigenvalues of $A_q\{\overline{G}_{MD}\} - A_q\{G_2\}$, we can find manoeuvres for which G_2 becomes a better metric than $\overline{G}_{MD}$

$$\begin{bmatrix} 4.98 \times 10^9 \\ 1.36 \times 10^8 \\ 8.43 \times 10^6 \\ 1.12 \times 10^7 \\ 1.80 \times 10^3 \\ -6.16 \times 10^3 \end{bmatrix}$$

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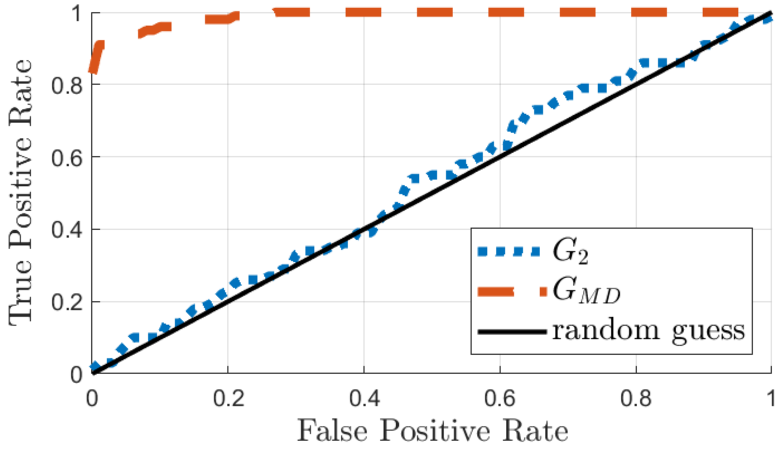
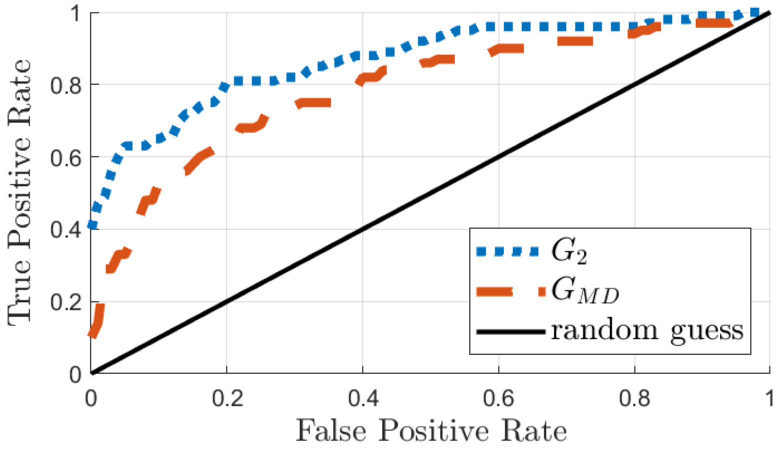
Test case 1 – quality of metric results

- By taking the eigenvalues of $A_q\{\overline{G}_{MD}\} - A_q\{G_2\}$, we find manoeuvres for which G_2 becomes a better metric than $\bar{G}_{MD}$. (Note: $A_q\{G_{MD}\} = A_q\{\overline{G}_{MD}\}$)
- The proposed quantity aims to be a more general description of how good a metric is at manoeuvre detection

ROC curves for inclination change manoeuvre



ROC curves for manoeuvre defined by eigenvector with negative eigenvalue



Eigenvalues

$$\begin{bmatrix} 4.98 \times 10^9 \\ 1.36 \times 10^8 \\ 8.43 \times 10^6 \\ 1.12 \times 10^7 \\ 1.80 \times 10^3 \\ -6.16 \times 10^3 \end{bmatrix}$$

Test case 2 – Cosmos 2542/2543

We take the Cosmos 2542/2543 satellites shadowing of the American KH-11 satellite.

Tx: Tracking and Imaging Radar (TIRA)

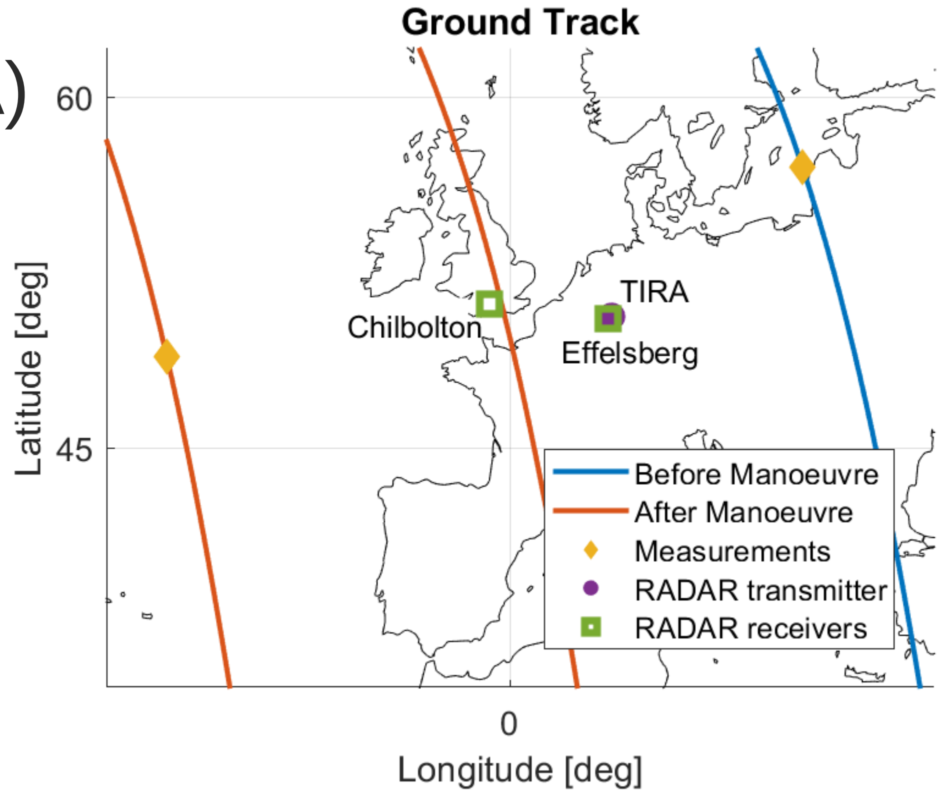
- Location: Wachtberg, Germany

Rx1: Effelsberg Radio Telescope

- Location: Effelsberg, Germany

Rx2: Observatory

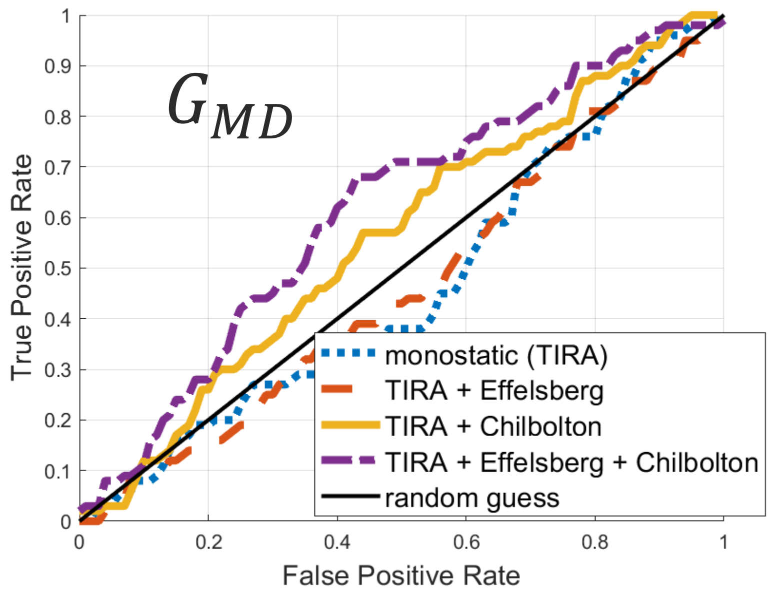
- Location: Chilbolton, England, United Kingdom

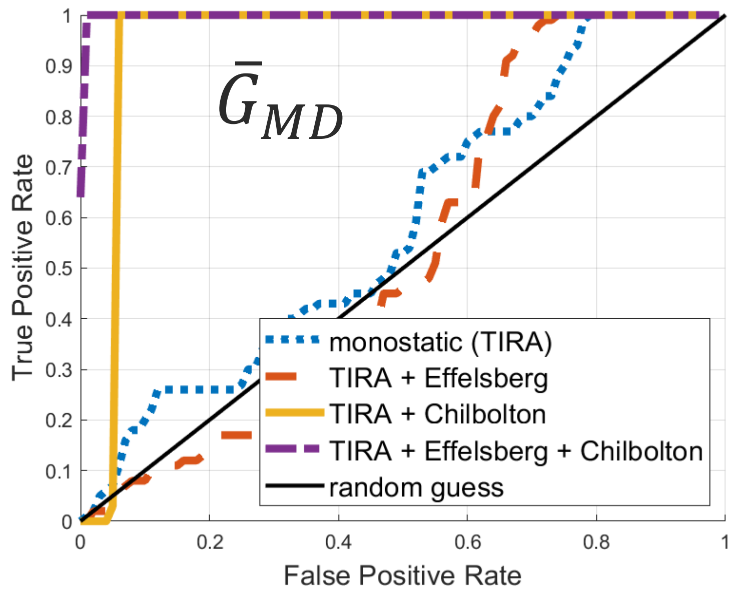


Test case 2 – Cosmos 2542/2543

We take the Cosmos 2542/2543 satellites shadowing of the American KH-11 satellite.

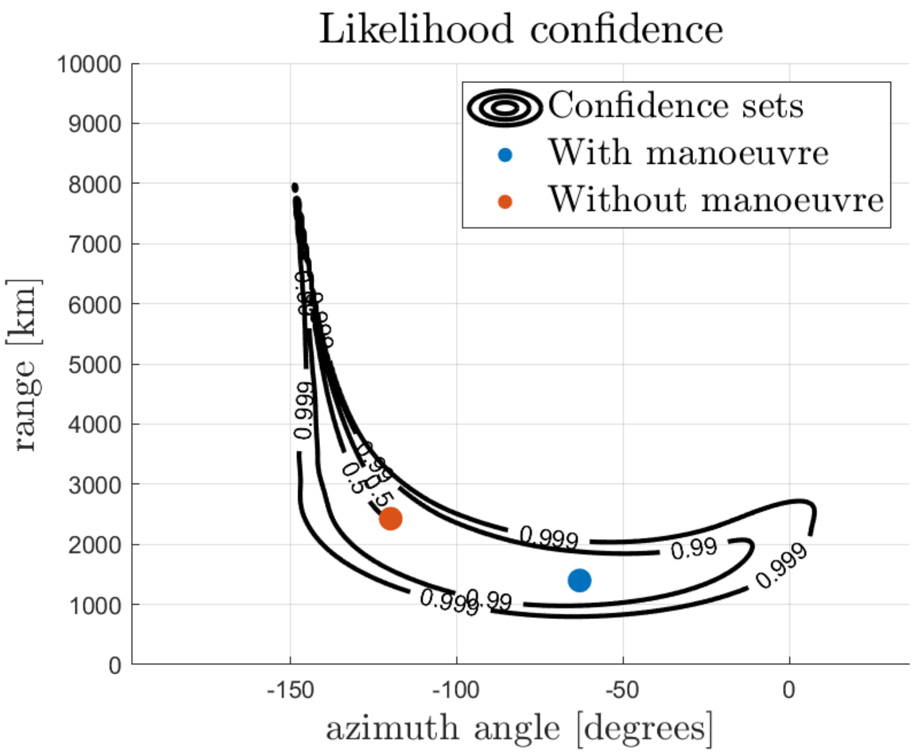
The error in angular observations is increased to 0.5º

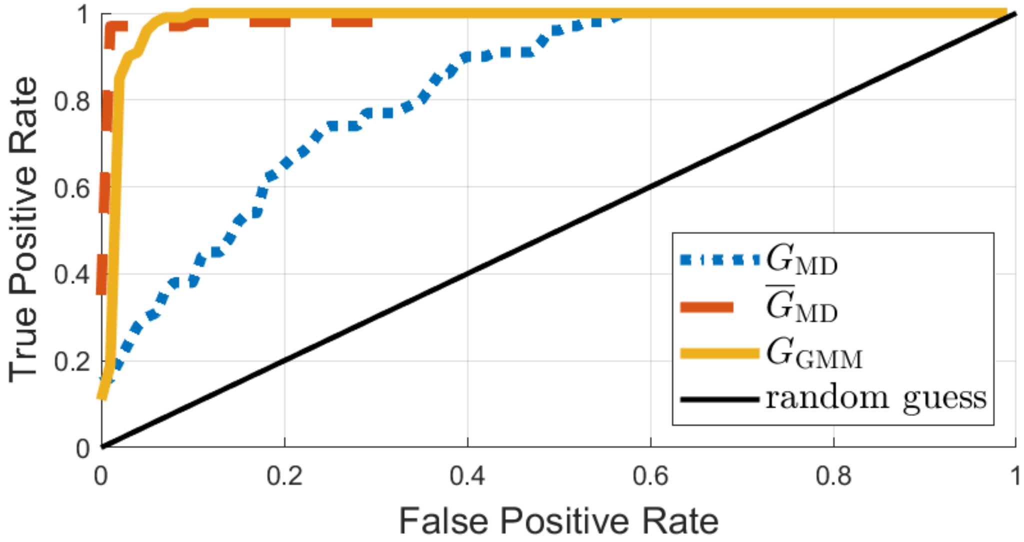




Test case 2 – GMM likelihood

Propagating the uncertainty with a Gaussian Mixture Model leads to a better representation of uncertainty.





Conclusions

- The addition of bistatic radar adds a measurements from a different line of sight, improving manoeuvre detection capabilities
- Proposed new manoeuvre detection metrics which produced good results
- Introduced a quality of metric measure
- Results confirm expectation that larger baselines lead to better manoeuvre detection accuracy

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