Fun with strong-field scattering

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FPUK 2023

17 August 2023

work with Bogna, Bu, Casali, Cristofoli, Ilderton, Klisch, MacLeod, Mason, Nekovar, Sharma, Tourkine & Zhu

Scattering amplitudes 101

How do we usually compute *scattering amplitudes*?

Perturbation theory (few exceptions)

- QFT: turn Feynman diagram crank
- String theory: correlators in free worldsheet CFT at fixed genus

Additional (almost implicit) assumption:

- expanding around a *trivial* field configuration

Strong-field scattering

Suppose we consider scattering in a *non-trivial* (asymp. flat) field configuration:

- Background a fixed solution to classical (non-linear) equations of motion
- Treated *non-perturbatively* $\leftrightarrow$ 'strong' background field
- $\Rightarrow$ use background field theory [Furry, DeWitt, 't Hooft, Abbott,...]
- Scattering quantum perturbations on strong background encodes back-reaction/depletion effects

Strong-field QFT describes **many** interesting scenarios:

- Non-linear regime of QED: Schwinger pair-production, beam-induced emission, vacuum birefringence
- High-energy regime of QCD: heavy ion collisions, colour-glass condensate, Balitsky-JIMWLK and BFKL evolution equations
- Non-linear effects in GR: pair-production by horizons, self-force expansion, radiation-reaction and memory effects

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...not to mention anything with a cosmological constant: 'scattering' in (A)dS

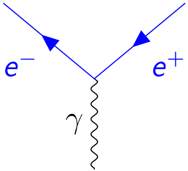
(I'll only focus on asymptotically flat scenarios)

However, strong-field scattering is a **hard** problem

- Background-coupled Feynman rules a nightmare, String worldsheet CFT not free
- Functional d.o.f. in background $\Rightarrow$ no rational functions
- Non-pert. effects: e.g., no Huygens' principle $\leftrightarrow$ tails
- S-matrix may not exist as a unitary operator

Example:

$\gamma \longrightarrow e^- \, e^+$ in QED



In a trivial EM background,
this is process vanishes at tree-level

But non-vanishing
in strong EM background fields!
$\Rightarrow$
non-linear Breit-Wheeler pair production

[Breit-Wheeler, Reiss, Narozhny-Nikishov-Ritus, Ritus]

For instance, in an *impulsive plane wave* background

$$e\,A = -\delta(x^-)\,c_\perp\,x^\perp\,\mathrm{d}x^- \,, \qquad \mathrm{d}s^2 = 2\,\mathrm{d}x^-\,\mathrm{d}x^+ - (\mathrm{d}x^\perp)^2$$

differential probability is [Ilderton]

$$\mathbb{P}_{\mathrm{NBW}} = \frac{\alpha}{3\pi} + \frac{4\,\alpha}{3\pi}\,\frac{c_0^2 - 1}{c_0\,\sqrt{c_0^2+4}}\,\tanh^{-1}\left(\frac{c_0}{\sqrt{c_0^2+4}}\right)$$

for $c_0 := |c_\perp|/m_e$

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All-orders in background electric field $c_\perp$!

Broad interest

Non-pert. backgrounds induce new physics!

Similar processes (non-linear Compton scattering, photon helicity flip, trident pair production)

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Strong field scattering has been studied for a *long* time since 1930s [Sauter, Volkov, Furry,...]

State-of-the-art

Despite study for ~ 100 years, precision frontiers of strong-field QFT are low:

- QED in plane wave background $\to$ 4-point tree [Baier-Katkov-Strakhovenko, Ritus,...] , 2-point 1-loop [Toll, Ritus]
- QCD in plane wave background $\to$ 4-point tree [TA-Casali-Mason-Nekovar] , 2-point 1-loop [TA-Ilderton]
- GR in plane wave background $\to$ 3-point tree [TA-Casali-Mason-Nekovar]

Roughly NLO/N^{2}LO precision around background

Stark contrast

...with N$^\infty$LO information in a trivial background:

all-multiplicity tree- and loop-level formulae for gluon/graviton scattering

[Parke-Taylor, Witten, Roiban-Spradlin-Volovich, Hodges, Cachazo-Skinner, Cachazo-He-Yuan,

TA-Casali-Skinner, Geyer-Mason-Monteiro-Tourkine,...]

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A countably **infinite** precision gap in even the simplest strong backgrounds!

Note:

High-multiplicity scattering in strong backgrounds a serious problem!

- more external states $\Rightarrow$ more powers of small coupling
- but also more insertions of background-dressed wavefunctions and propagators
- background insertions can compensate powers of coupling

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High mult. can dominate low mult. in a strong background

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Of course not!

Today

Try to convince you that:

- strong-field scattering an important theoretical challenge
  - where many ‘standard’ methods break down
- all-multiplicity results *are* possible
  - chiral backgrounds w/ functional dof
  - remarkably simple results
- teach something about *radiative* structures in QFT
  - collinear splitting and chiral algebras

Basics

What exactly do we mean by a strong-field amplitude?

Denote fields by $\mathcal{F}$, classical action $S[\mathcal{F}]$

- let Φ be exact solution to e.o.m.s – the *background*.
- evaluate action on $S[\Phi + \phi]$, discard all terms less than $O(\phi^2)$

$\rightarrow$ obtain background field action $S[\Phi; \phi]$

[DeWitt, 't Hooft, Boulware, Abbott]

governs fluctuations ϕ on background Φ

Tree-level strong-field amplitudes: $\{\phi_1, \ldots, \phi_n\}$ solutions to free, background-coupled eqs with appropriate bndry conds. Define:

$$\varphi_0^{[n]} := \sum_{i=1}^{n} \varepsilon_i \, \phi_i$$

$\varphi_k^{[n]}$ non-linear recursive solution at $O(\mathrm{g}^k)$

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Strong-field, *n*-point tree amplitude:

$$\mathcal{M}_n^{(0)} := \left. \frac{\delta^n S\left[\Phi; \varphi^{[n]}_{\max\{0,n-3\}}\right]}{\delta\varepsilon_1 \cdots \delta\varepsilon_n} \right|_{\varepsilon_1 = \cdots = \varepsilon_n = 0}$$

Upshot

Strong-field amps = multi-linear piece of background field action

[Schwinger, Boulware-Brown, Arafeva-Faddeev-Slavnov, Abbott-Grisaru-Schaefer, Jevicki-Lee, Rosly-Selivanov,...]

- 'perturbiner' definition extremely robust
- coincides w/ S-matrix when it exists
- when it doesn't, still encodes expected dynamical content of scattering [TA-Nakach-Tseytlin, Ilderton-Lindved, Kim-Kraus-Monten-Myers]
- higher loops: use ℓ-loop effective action [Costello]

What does it mean to compute a strong-field amp?

In general, amplitudes look like:

$$\mathcal{M}_n^{(0)} = \int \underbrace{\mathrm{d}\mu_n}_{\text{measure}} \; \overbrace{\mathcal{I}_n^{(0)}}^{\text{integrand}} \; \underbrace{\mathcal{V}_n}_{\text{wavefunctions}}$$

- trivial background: integrals give rational function + momentum conservation
- general strong fields: *cannot* perform integrals analytically
- 'compute strong-field amp' $\rightsquigarrow$ determine $\mathrm{d}\mu_n$, $\mathcal{I}_n^{(0)}$, $\mathcal{V}_n$ analytically

Example:

Scalar QED: photon emission in plane wave
('non-linear Compton scattering')

$$A^{\mathrm{PW}} = -a_\perp(x^-)\, x^\perp\, \mathrm{d}x^- \,, \qquad a_\infty := \int_{-\infty}^{+\infty} \mathrm{d}x^-\, a_\perp(x^-)$$

$$\mathcal{M}_3^{(0)}(p \to p' + k) = e\, \delta^3_{+,\perp}(p' + k - p + e\, a_\infty) \int\limits_{-\infty}^{+\infty} \mathrm{d}x^-$$

$$\times\; \epsilon(k) \cdot P(x^-) \exp\left[\mathrm{i} \int^{x^-} \mathrm{d}s\, \frac{k \cdot P(s)}{(p-k)_+} \right] ,$$

for $P_\mu := p_\mu - e\, \delta^\perp_\mu\, a_\perp + \frac{\delta^-_\mu}{2p_+}(2ep \cdot a - e^2\, a^2)$

All-order physics

Non-perturbative background $\rightarrow$ *infinite* order in coupling when expanded

Even at low precision/multiplicity!

Theoretical data

This simple fact underpins many interesting theoretical applications of strong-field scattering:

- Particle-sourced backgrounds $\leftrightarrow$ eikonal resummation [’t Hooft, Amati-Ciafaloni-Veneziano, Kabat-Ortiz, TA-Cristofoli-Tourkine]
  - ▷ Constrains exact solutions (e.g., ultrarel. Kerr [TA-Cristofoli-Tourkine])
  - ▷ Higher-mult. $\rightarrow$ eikonal/particle-beam + emission [Lodone-Rychkov, TA-Ilderton-MacLeod]

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- Building blocks for self-force expansion [Ilderton-Torgrimsson, TA-Cristofoli-Ilderton, TA-Cristofoli-Ilderton-Klisch]
  - ▷ Probe + emission = self-force waveform [TA-Cristofoli-Ilderton-Klisch]

Basic question:

Can we compute high-multiplicity scattering amplitudes in (any) strong field QFT?

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YES!

Today:

Gluon scattering in self-dual radiative Yang-Mills fields

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But can also do:

- gravitons in self-dual radiative spacetimes [TA-Mason-Sharma]
- YM form factors in self-dual radiative gauge fields [Bogna-Mason]
- gluons in self-dual dyons [TA-Bogna-Mason-Sharma to appear]
- gravitons in self-dual Taub-NUT [TA-Bogna-Mason-Sharma to appear]

What is a SD radiative gauge field? [van der Burg, Newman, Goldberg]

1. Asymptotically flat solution to Yang-Mills equations in Minkowski space $\mathbb{M}$
2. Uniquely determined by characteristic data at $\mathscr{I}^+$
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Functional dof: $\tilde{\mathcal{A}}(u, z, \bar{z})$

- spin weight $+1$, conformal weight -1
- otherwise unconstrained (modulo regularity)

$$F = \partial_u \tilde{\mathcal{A}}\, \mathrm{d}u \wedge \mathrm{d}\bar{z} + O(r^{-1})$$

On one hand, a simplified setting...

- self-dual/chiral background

...on the other hand

- still has unconstrained, functional dof
- totally intractable with conventional methods
- encodes backreaction/beam depletion effects [Dinu-Ilderton]
- high-precision on chiral background $\Rightarrow$ non-chiral backgrounds

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So how do we proceed?

Twistor theory

From physical data on $\mathbb{M}$ to geometric data on $\mathbb{PT} \subset \mathbb{CP}^3$

$$x^{\alpha\dot{\alpha}} = \frac{1}{\sqrt{2}} \begin{pmatrix} x^0 + x^3 & x^1 - \mathrm{i}x^2 \\ x^1 + \mathrm{i}x^2 & x^0 - x^3 \end{pmatrix} \qquad \text{coords on } \mathbb{M}$$

$Z^A = (\mu^{\dot{\alpha}}, \lambda_\alpha)$ homogeneous coords on $\mathbb{CP}^3$

$$\mathbb{PT} = \{Z^A \in \mathbb{CP}^3 \,|\, \lambda_\alpha \neq 0\}$$

Related by *incidence relations*

$$\mu^{\dot{\alpha}} = x^{\alpha\dot{\alpha}}\, \lambda_\alpha$$

$$x \in \mathbb{M} \leftrightarrow X \cong \mathbb{CP}^1 \hookrightarrow \mathbb{PT}, \text{ linear \& holomorphic}$$

So what?

First key fact:

Theorem (Ward 1977)

$\exists$ a 1:1-corresp between SD Yang-Mills fields on $\mathbb{M}$ and holomorphic vector bundles $E \to \mathbb{PT}$ (+ technical conditions)

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In real money: E equipped with partial connection $\bar{D} : \Omega^{p,q}(\mathbb{PT}, E) \to \Omega^{p,q+1}(\mathbb{PT}, E)$ obeying $\bar{D}^2 = 0$

Locally, $\bar{D} = \bar{\partial} + \mathsf{a}\,, \quad \mathsf{a} \in \Omega^{0,1}(\mathbb{PT}, \mathrm{End}\,E)\,, \qquad \bar{\partial}\mathsf{a} + [\mathsf{a}, \mathsf{a}] = 0$

So what?

Second key fact:

Theorem (Ward-Wells 1991)

$\exists$ an isomorphism between:
i.) helicity ± 1 gluons coupled to SD background gauge field, and
ii.) $H^{0,1}_{\bar{D}}(\mathbb{PT}, \mathcal{O}(\pm 2 - 2) \otimes \mathrm{End}\, E)$

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In real money: SD background-coupled gluon wavefunctions represented on $\mathbb{PT}$ by

$$a_{\pm}(Z) \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(\pm 2 - 2) \otimes \mathrm{End}\, E) \ : \quad \bar{D} a_{\pm} = 0, \ a_{\pm} \neq \bar{D} f$$

Upshot

Twistor theory provides:

1. a description of SD gauge fields manifesting integrability
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Further simplification for SD radiative backgrounds:

- $\bar{D} = \bar{\partial} + \tilde{\mathcal{A}}(\mu^{\dot\alpha}\bar{\lambda}_{\dot\alpha}, \lambda, \bar{\lambda})\, \bar{\lambda}^{\dot\alpha}\, \mathrm{d}\bar{\lambda}_{\dot\alpha}$ [Newman, Sparling]
- trivial twistor reps for $a_{\pm}$ still in cohomology [TA-Mason-Sharma]

Ingredients

For SD radiative backgrounds, $E \to \mathbb{PT}$ admits **holomorphic trivialization** on lines X:

$$\exists\, \mathrm{H}(x,\lambda) : E|_X \to \mathbb{C}^N \;\text{ s.t. }\quad \mathrm{H}^{-1}\bar{D}|_X \mathrm{H} = \bar{\partial}|_X$$

$$\text{Encodes SD rad. field: } \mathrm{H}^{-1}\,\lambda^{\alpha}\partial_{\alpha\dot{\alpha}}\mathrm{H} = \lambda^{\alpha}\,A_{\alpha\dot{\alpha}}(x)$$

External gluons characterized by helicity, and:

- asymptotic null momenta $k_{\alpha\dot{\alpha}} = \kappa_{\alpha}\,\tilde{\kappa}_{\dot{\alpha}}$
- colour vector T^{a}

Result

MHV amplitude: gluons r, s negative helicity, all others positive helicity [TA-Mason-Sharma]

$$\frac{\langle r\, s\rangle^4}{\langle 1\, 2\rangle \langle 2\, 3\rangle \cdots \langle (n-1)\, n\rangle \langle n\, 1\rangle} \times \int \mathrm{d}^4 x \, \mathrm{tr}\left(\prod_{i=1}^{n} \mathsf{H}^{-1}(x, \kappa_i)\, \mathsf{T}^{\mathsf{a}_i}\, \mathsf{H}(x, \kappa_i)\, \mathrm{e}^{\mathrm{i}\, k_i \cdot x} \right)$$

where $\langle i\, j\rangle := \epsilon^{\alpha\beta}\, \kappa_{i\,\beta}\, \kappa_{j\,\alpha}$

There are **many** surprising things about this formula:

- *All-multiplicity* result – arbitrarily many positive helicity gluons
- *Much* simpler than naïve expectations: only one spacetime integral!
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How do we know it's right?

- Can be derived directly from the Yang-Mills action!
- Passes all other sanity tests: trivial background, perturbative limit, background gauge invariance

Also...

- Natural conjectures for full tree-level S-matrix (all N^kMHV amps), passing all sanity tests [TA-Mason-Sharma]
- Similar methods work for SD dyon backgrounds

[TA-Bogna-Mason-Sharma]

$$A = \mathsf{c}\left(\frac{\mathrm{d}t}{r} + \frac{z\,\mathrm{d}\bar{z} - \bar{z}\,\mathrm{d}z}{1+|z|^2}\right), \qquad \text{MHV amp:}$$

$$\delta\left(\sum_{i=1}^{n} k_i^0\right) \frac{\langle r\,s\rangle^4}{\langle 1\,2\rangle \cdots \langle n\,1\rangle} \int \mathrm{d}^3\vec{x} \prod_{i=1}^{n} \frac{\langle \iota\, i\rangle^{e_i}}{\langle i\, o\rangle^{e_i}}\, \mathrm{e}^{\mathrm{i}\,\vec{k}_i\cdot\vec{x}}$$

where $\iota^\alpha = (1, z)$, $o_\alpha = (1, 0)$, and $[\mathsf{c}, \mathsf{T}^{\mathsf{a}_i}] = e_i\,\mathsf{T}^{\mathsf{a}_i}$

Collinear limits

Can learn some surprising things from these formulae:

In holomorphic collinear limit $\langle i\, j\rangle \to 0$ [TA-Bu-Zhu to appear]

$$\mathcal{M}_n(\ldots, i^{\mathrm{a}}, j^{\mathrm{b}}, \ldots) \to \frac{f^{\mathrm{abc}}}{\langle i\, j\rangle} \mathcal{M}_{n-1}(\ldots, P^{\mathrm{c}}, \ldots)$$

The *same* holo. collinear splitting as a trivial background!

[Altarelli-Parisi, Birthwright-Glover-Khoze-Marquard]

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Same story for gravitons on SD rad. spacetime

Chiral algebra

Basis for SD perturbations on a SD rad. background forms a *chiral algebra* [TA-Bu-Zhu to appear]

$$\left\{ S^{p\,\mathrm{a}}_{m,r} \,|\, 2p-2 \in \mathbb{Z}_{\geq 0},\; |m| \leq p-1,\; r \in \mathbb{Z} \right\}$$

subject to:

$$[S^{p,\mathrm{a}}_{m,r},\, S^{q,\mathrm{b}}_{n,s}] = f^{\mathrm{abc}}\, S^{p+q-1,\mathrm{c}}_{m+n,r+s}$$

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The same chiral symmetry algebra, $\mathcal{L}\mathfrak{g}[\mathbb{C}^2]$ ('$\mathcal{S}$-algebra'), as trivial background

[Guevara-Himwich-Pate-Strominger, Strominger]

This is surprising, because:

in non-radiative SD backgrounds, splitting functions and chiral algebras *are* deformed

[Bittleston-Heuveline-Skinner]

A conjecture

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more specifically:

Chiral radiation fields on *any* background do not deform the chiral IR physics (splitting functions, celestial OPEs, celestial chiral algebras)

Summary

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