Keeping matter in the loop in 3D Quantum Gravity

FPUK Meeting

Durham, August 2023

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DAMTP

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$$\langle Z_{scalar}[M] \rangle_{grav} = \int \left(Dg_{\mu\nu}\right)_M e^{-I_{EH}[g_{\mu\nu}]} Z_{scalar}[g_{\mu\nu}]$$

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- It is not just another way to compute one-loop determinants.
  We will be able to quantify quantum corrections to metric fluctuations.

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Based on arxiv:2302.12281+2304.02668
with Ioana Coman, Jackson Fliss and Claire Zukowski

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Outline

- Chern-Simons theory
- Wilson spool: construction
- Testing the Wilson spool: one-loop determinants
- Quantum Wilson spool: G_N corrections

Chern-Simons Theory

Synergy with three-dimensional gravity

In 2+1 dimensions, we have the **luxury** of casting general relativity in terms of:

[Acucharro & Townsend; Witten]

Einstein-Hilbert: Metric, curvature → Local variables. Spacetime is explicit.

OR

Chern-Simons: Gauge connections → Gauge Theory. Topological nature is explicit.

How to interpret Chern-Simons theory as a theory of gravity?

$$S_{CS}[\mathrm{A}] = \frac{k}{4\pi}\int_M Tr\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$$

It is not just a matter of actions and equations of motion.
Other important **INPUTS** are:

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It is not just a matter of actions and equations of motion.
Other important **INPUTS** are:

1. Gauge Group:
Organization of the massless modes.
Determine the surrounding.

$A \in SL(2,\mathbb{R})\times SL(2,\mathbb{R})$: AdS_3 Lorentzian Gravity

$A \in SU(2)\times SU(2)$: dS_3 Euclidean Gravity

2. Boundary Conditions:
Setup the AdS/CFT dictionary.
Regular spacetime metric.

$$A - A_{AdS} = O(1)$$

$$g_{\mu\nu} \sim Tr(A_L - A_R)^2$$

Next, we would like to add matter fields

Einstein-Hilbert: Metric, curvature

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Chern-Simons: Gauge connections

This has been an open problem.
How to introduce fields coupled to $A_{L,R}$ while keeping gravity topological?

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OR

Chern-Simons: Gauge connections

$$\log(Z_{scalar}[g_{\mu\nu}]) = \frac{1}{4} \mathbb{W}_j[A_L, A_R]$$

$$\langle \mathbb{W}_j \rangle_{grav} = \int DA_{L/R} e^{ik_L S[A_L] + ik_R S[A_R]} \mathbb{W}_j[A_L, A_R]$$

Wilson Spool

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dS_3 Quantum Gravity

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Focus mainly on massive scalar fields coupled to dS_3 gravity. **Why?**

- We can use the full power of SU(2) Chern-Simons theory.

  [Carlip 1992; AC, Lashkari, Maloney 2011; Anninos, Denef, Law, Sun 2022]

- Make predictions for G_N corrections without the aid of holography.

- Interesting non-standard representations of SU(2).

  [AC, Sabella-Garnier, Zukowski, 2019]

dS_3 Quantum Gravity

- Gauge group: $SU(2)\times SU(2)$ leads to dS_3 Euclidean Gravity
- Action: $-ik_L S_{CS}[A_L] - ik_R S_{CS}[A_R] = I_{EH}[g_{\mu\nu}] - i\delta\, I_{GCS}[g_{\mu\nu}]$
- Couplings: $k_L = \delta + i\,\frac{\ell}{4G_N} \longrightarrow r_L = k_L + 2$

  $k_R = \delta - i\,\frac{\ell}{4G_N} \longrightarrow r_R = k_R + 2$
- Dictionary: $A_L = i\left(\omega^a + \frac{e^a}{\ell}\right)L_a$

  $A_R = i\left(\omega^a - \frac{e^a}{\ell}\right)\bar{L}_a$

dS_3 Quantum Gravity

Background S^3 connections

$$a_L = i\,L_1 d\rho + i(\sin\rho\, L_2 - \cos\rho\;\; L_3)(d\varphi - d\tau)$$
$$a_R = -i\bar{L}_1 d\rho - i(\sin\rho\, \bar{L}_2 + \cos\rho\, \bar{L}_3)(d\varphi + d\tau)$$

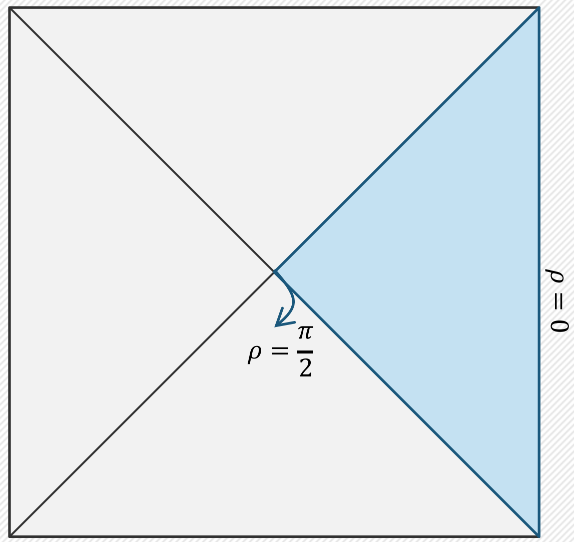
Holonomies

$$P\exp\oint_\gamma\; a_{L/R} \sim e^{2\pi i\, L_3 h_{L/R}}$$

$$h_L = 1$$
$$h_R = -1$$

Geometry: Static Patch

$$ds^2 = \ell^2(\cos^2\rho\;\, d\tau^2 + \sin^2\rho\;\, d\varphi^2 + d\rho^2)$$



AdS_3 Quantum Gravity

Background BTZ connections

$$a_L = L_0 d\rho + (e^{\rho} L_+ - e^{-\rho}\frac{M\ell + J}{2k} L_-)(dt + d\varphi)$$

$$a_R = -\bar{L}_0 d\rho - (e^{\rho} \bar{L}_- - e^{-\rho}\frac{M\ell - J}{2k} \bar{L}_+)(dt - d\varphi)$$

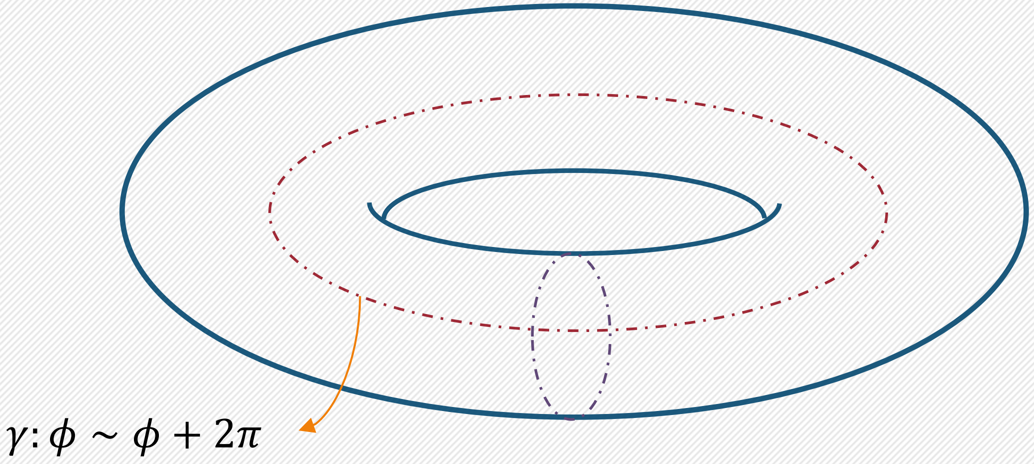
Holonomies

$$P \exp \oint_{\gamma} a_{L/R} \sim e^{2\pi i\, L_3 h_{L/R}}$$

$$h_L = \tau$$
$$h_R = \bar{\tau}$$

$$\tau = 2i\sqrt{\frac{M\ell + J}{2k}}$$

Euclidean BTZ



Wilson Spool

Construction

Wilson lines

The metric encodes distances: geodesic distances.
What replaces geodesic length in a Chern-Simons theory?

$$W_R(C_{ij}) = \left\langle i \middle| P \exp \int_{C_{ij}} A \middle| j \right\rangle$$

Wilson line encodes the dynamics of a massive point particle.
Natural replacement of geodesic equation.

[Witten 1989; Carlip 1989; Ammon, AC, & Iqbal 2013; AC, Sabella-Garnier, Zukowski, 2019]

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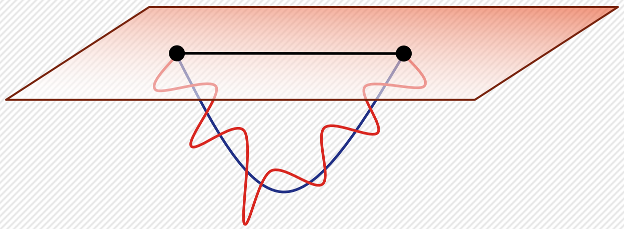

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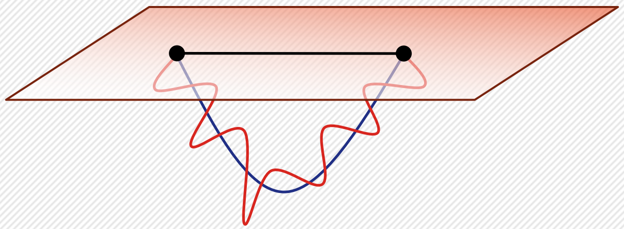
Geodesic length



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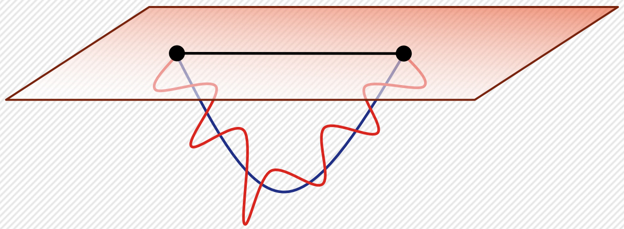
Casimir $c_2 = -\frac{m^2}{4\Lambda}$



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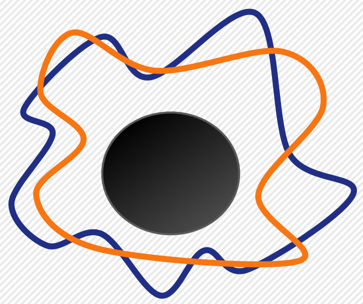
$$W_R(C) = Tr_R\left(P \exp \oint_C A\right) = \int DU \exp(-S(U,A)_C)$$



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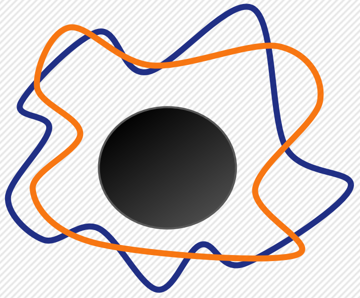
Infinite dimensional representation of G.
Encodes quantum numbers of the particle.

Path integral of a single particle state.



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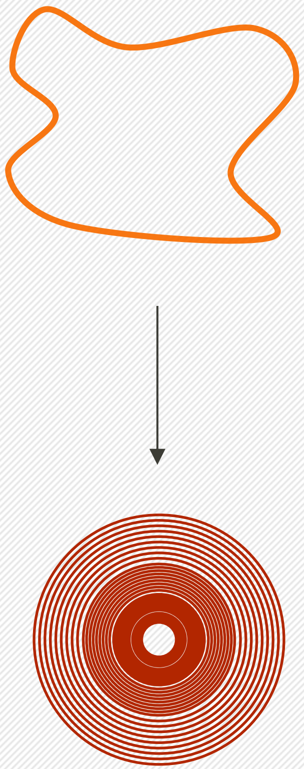
We want to capture fields. How to get fields from single particles states?



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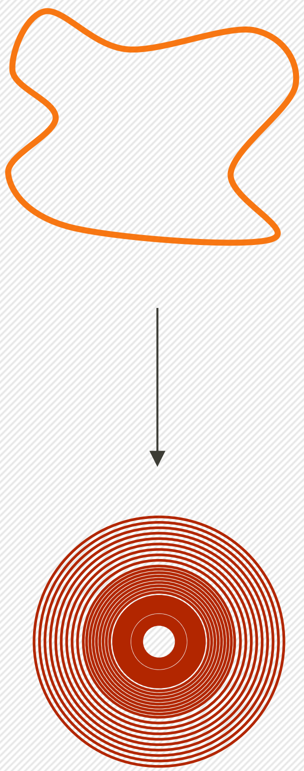
Our proposal: to spool

$$\mathbb{W}_j[A_L, A_R] = i \int_{\mathcal{C}} \frac{d\alpha}{\alpha} \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} \mathrm{Tr}_j\left(P e^{\frac{\alpha}{2\pi}\oint A_L}\right) \mathrm{Tr}_j\left(P e^{-\frac{\alpha}{2\pi}\oint A_R}\right)$$



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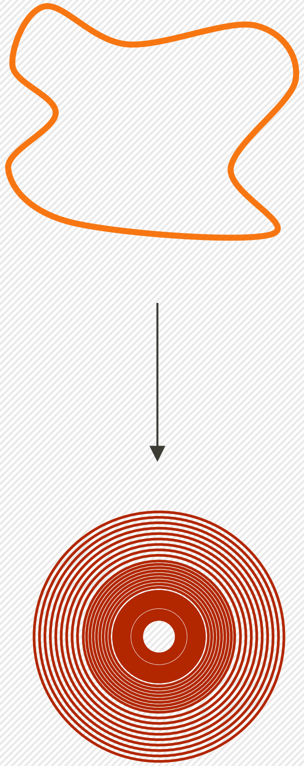
$\sim \log\det(-\nabla^2 + m^2\ell^2)$ Why?



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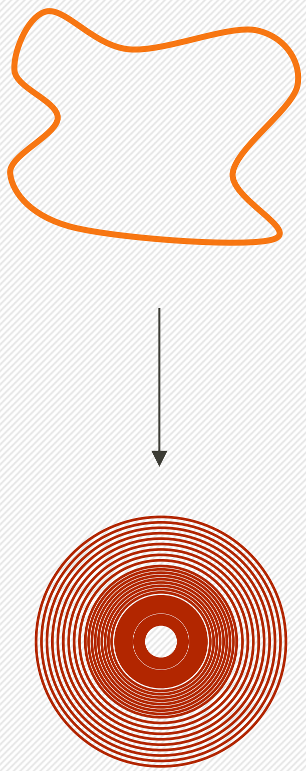
Connections: Capture the geometry



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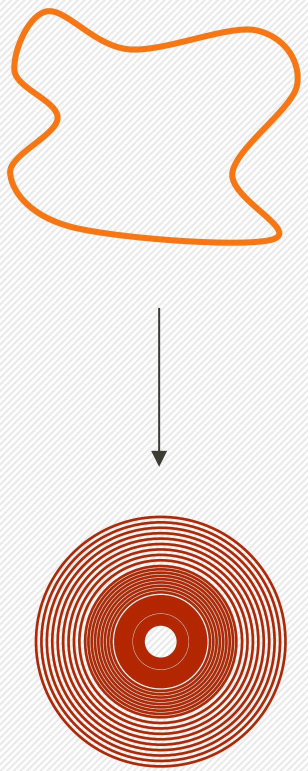
Representation: carries the mass, single particle info.



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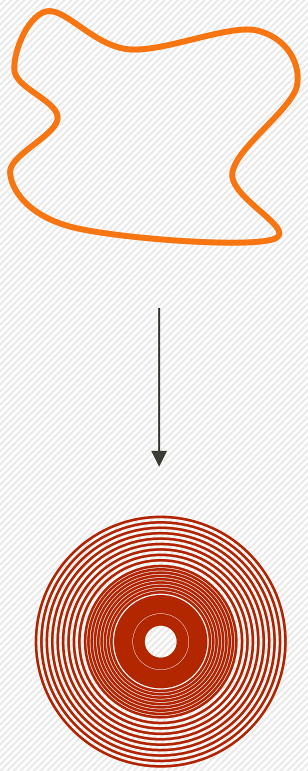
Measure and contour serve two purposes:

- Regulate UV divergences
- Poles that $\mathcal{C}$ will wrap make the Wilson loop wind arbitrarily many times.



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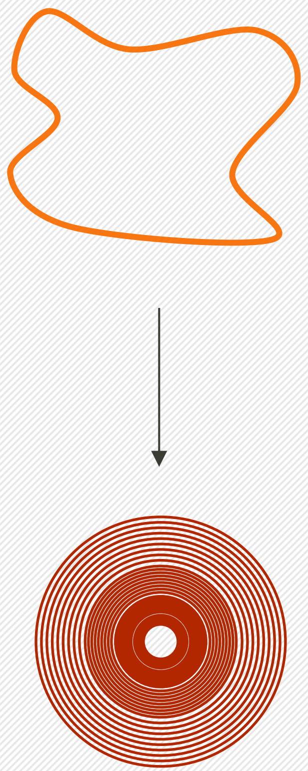
$$\text{“=”} \quad \sum_n \frac{1}{n} \text{``}\mathrm{Tr}_j\left(Pe^{\frac{n}{2\pi}\oint A}\right)\text{''}$$

Caution! Just for intuitive purposes.



Representations of su(2)

$$\mathbb{W}_j[A_L, A_R] = i \int_{\mathcal{C}} \frac{d\alpha}{\alpha} \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} \underbrace{\mathrm{Tr}_j}\left(P e^{\frac{\alpha}{2\pi}\oint A_L}\right) \underbrace{\mathrm{Tr}_j}\left(P e^{-\frac{\alpha}{2\pi}\oint A_R}\right)$$

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But unitary (standard) representations of su(2) have $j = 0,1,2, \ldots$ and positive Casimir!!!!

Non-Standard Representations of su(2)

Complementary-type

$$L_3^\dagger = L_3$$
$$L_\pm^\dagger = -L_\mp$$

$$j = -\frac{1}{2}(1+\nu),$$
$$\nu \in (-1,1)$$

$$m^2\ell^2 = 1-\nu^2$$

$$\chi_j(z) = \mathrm{Tr_j}\left(e^{2\pi i z L_3}\right) = \frac{e^{2\pi i\, z\nu}}{2i\sin\pi z}$$

Principal-type

$$L_3^\dagger = \mathcal{S} L_3 \mathcal{S}$$
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$$\mathcal{S}: j \to \bar{j} = -1-j$$

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Important:

- The norm of states is positive.
- They differ from so(3,1) reps.
- Only considering spin zero.

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Representations of sl(2)

Lowest/Highest weight

$$L_0^\dagger = L_0$$
$$L_\pm^\dagger = L_\mp$$

$$c_2 = j(j-1)$$
$$j > 0$$

$$L_0|j,0\rangle = j|j,0\rangle,\ L_+|j,0\rangle = 0$$

$$\chi_j(z) = \mathrm{Tr_j}\left(e^{2\pi i z L_0}\right) = \frac{e^{2\pi i\, zj}}{(1-e^{2\pi i z})}$$

Mass and spin: $sl(2)_L \times sl(2)_R$

$$j_L = \frac{\Delta + s}{2}, \qquad j_R = \frac{\Delta - s}{2}$$

$$\Delta(\Delta - 2) = m^2 \ell^2$$

Testing the Wilson Spool

One-loop determinants

One-loop determinants

Does the Wilson spool reproduce the one-loop determinant on S^3?

$$\log(Z_{scalar}[S^3]) = \log\det(-\nabla^2 + m^2\ell^2)^{-\frac{1}{2}}$$
$$\stackrel{?}{=} \frac{1}{4}\mathbb{W}_j[a_L, a_R]$$

Collect appropriate data according to definition

$$\mathbb{W}_j[A_L, A_R] = i \int_{\mathcal{C}} \frac{d\alpha}{\alpha} \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} \mathrm{Tr}_j(Pe^{\frac{\alpha}{2\pi}\oint A_L})\mathrm{Tr}_j(Pe^{-\frac{\alpha}{2\pi}\oint A_R})$$

Characters

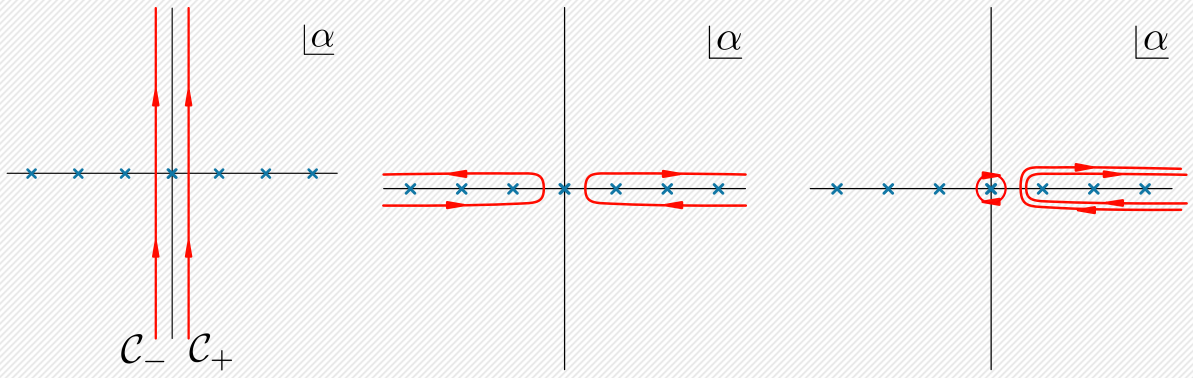
$$\chi_j(z) = \mathrm{Tr_j}\left(e^{2\pi i z L_3}\right) = \frac{e^{\pi i\, z(2j+1)}}{2i \sin \pi z}$$

Holonomies

$$P \exp \oint_\gamma \ a_{L/R} \sim e^{2\pi i\, L_3 h_{L/R}}$$

$$h_L = 1$$
$$h_R = -1$$

Contour: $\mathcal{C} = \mathcal{C}_+ \cup \mathcal{C}_-$

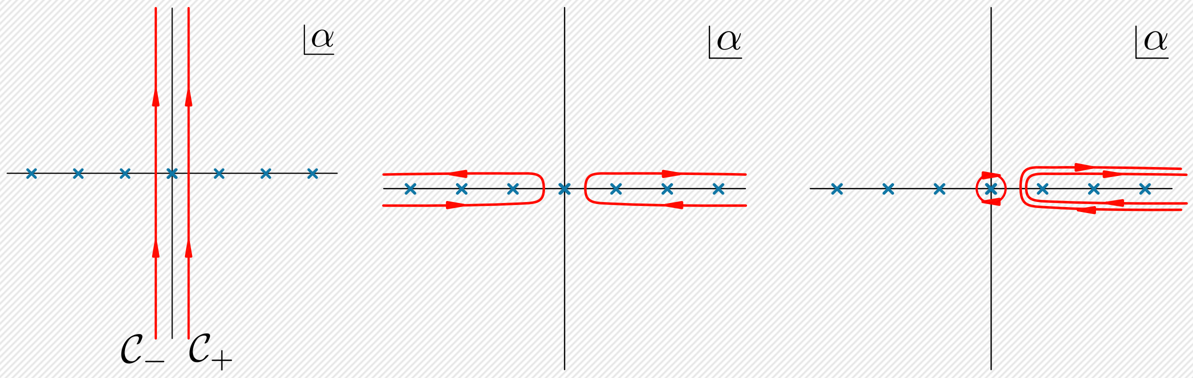


Collect appropriate data according to definition

$$\frac{1}{4}\mathbb{W}_j[A_L,A_R] = \frac{i}{4}\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\,\mathrm{Tr}_j(Pe^{\frac{\alpha}{2\pi}\oint A_L})\mathrm{Tr}_j(Pe^{-\frac{\alpha}{2\pi}\oint A_R})$$

$$= -\frac{i}{16}\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\frac{\alpha}{2}}{\sin^3\frac{\alpha}{2}}\,e^{i(2j+1)\alpha}$$

Contour



Collect appropriate data according to definition

$$\frac{1}{4}\mathbb{W}_j[A_L, A_R] = \frac{i}{4}\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\,\mathrm{Tr}_j(Pe^{\frac{\alpha}{2\pi}\oint A_L})\mathrm{Tr}_j(Pe^{-\frac{\alpha}{2\pi}\oint A_R})$$

$$= -\frac{1}{16}\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\frac{\alpha}{2}}{\sin^3\frac{\alpha}{2}}\,e^{i(2j+1)\alpha}$$

$$= i\frac{\pi}{6}(2j+1)^3 - \frac{1}{4\pi^2}Li_3\left(e^{2\pi i(2j+1)}\right) + \mathrm{i}\frac{(2j+1)}{2\pi}Li_2\left(e^{2\pi i(2j+1)}\right)$$
$$-\frac{(2j+1)^2}{2}Li_1(e^{2\pi i(2j+1)})$$

$$j = -\frac{1}{2} + \frac{1}{2}\sqrt{1-m^2\ell^2}$$

Collect appropriate data according to definition

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$$j = -\frac{1}{2} + \frac{1}{2}\sqrt{1-m^2\ell^2}$$

Exact agreement with finite contributions to the scalar one-loop determinant!

$$\log(Z_{scalar}[S^3]) = \log\det(-\nabla^2 + m^2\ell^2)^{-\frac{1}{2}} = \frac{1}{4}\mathbb{W}_j[a_L,a_R]$$

Comments

- Construction of the Wilson spool: for massive scalars we have a derivation of $\mathbb{W}_j[A_L, A_R]$.

$$\det(-\nabla^2 + m^2\ell^2)^{-1} = \prod_{\substack{n\in\mathbb{Z} \\ \lambda_R,\lambda_L}} (n - \lambda_L h_L + \lambda_R h_R)(n + \lambda_L h_L - \lambda_R h_R)$$

  Wilson Spool is an adaptation of QNM method for 1-loop determinants [Denef-Hartnoll-Sachdev] to the Chern-Simons formulation.

- More general backgrounds: due to the construction of the spool we expect it to work (but needs to be checked).
- Benefit: connections are off-shell! We are integrating out matter fields.

Wilson Spool in AdS_3

$$\mathbb{W}_{j_L,j_R}[A_L, A_R] = i\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\,\mathrm{Tr}_{j_L}(Pe^{\frac{\alpha}{2\pi}\oint A_L})\mathrm{Tr}_{j_R}(Pe^{-\frac{\alpha}{2\pi}\oint A_R})$$

Characters

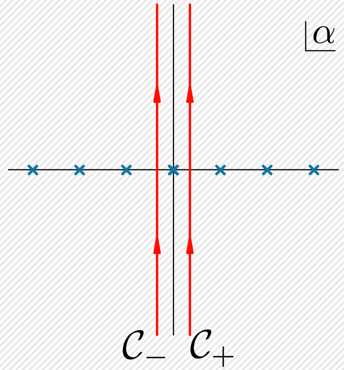
$$\chi_j(z) = \mathrm{Tr_j}\left(e^{2\pi i z L_3}\right) = \frac{e^{2\pi i\, zj}}{(1 - e^{2\pi i z})}$$

Holonomies

$$P\exp\oint_\gamma a_{L/R} \sim e^{2\pi i\, L_3 h_{L/R}}$$

$$h_L = \tau$$
$$h_R = \bar{\tau}$$

Contour: $\mathcal{C} = 2\,\mathcal{C}_+$



Collect appropriate data according to definition

$$\frac{1}{4}\mathbb{W}_{j_L,j_R}[A_L,A_R] = i\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\,\mathrm{Tr}_{j_L}(Pe^{\frac{\alpha}{2\pi}\oint A_L})\mathrm{Tr}_{j_R}(Pe^{-\frac{\alpha}{2\pi}\oint A_R})$$

$$= -\sum_{\pm}\sum_{l,\bar{l}=0}^{\infty}\log\left(1-q^{\frac{1}{2}(\Delta\pm s)+l}\,\bar{q}^{\frac{1}{2}(\Delta\mp s)+\bar{l}}\right)$$

Exact agreement with finite contributions to the scalar one-loop determinant!

$$\exp\left(\frac{1}{4}\mathbb{W}_{j_L,j_R}[A_L,A_R]\right) = \prod_{l,\bar{l}=0}^{\infty}\frac{1}{\left(1-q^{\frac{1}{2}(\Delta+s)+l}\,\bar{q}^{\frac{1}{2}(\Delta-s)+\bar{l}}\right)\left(1-q^{\frac{1}{2}(\Delta-s)+l}\,\bar{q}^{\frac{1}{2}(\Delta+s)+\bar{l}}\right)}$$

$$= \log\det(-\nabla_s^2+m^2\ell^2)^{-1}$$

Quantum Wilson spool in dS_3

G_N corrections

The quantum proposal is

Einstein-Hilbert: Metric, curvature

$$Z_{scalar}[g_{\mu\nu}] = \int D\phi \; e^{iS_{matter}[\phi, g_{\mu\nu}]}$$

$$\langle Z_{scalar}[M] \rangle_{grav} = \int (Dg_{\mu\nu})_M \; e^{-I_{EH}[g_{\mu\nu}]} Z_{scalar}[g_{\mu\nu}]$$

OR

Chern-Simons: Gauge connections

$$\log(Z_{scalar}[g_{\mu\nu}]) = \frac{1}{4} \mathbb{W}_j[A_L, A_R]$$

$$\langle \mathbb{W}_j \rangle_{grav} = \int DA_{L/R} e^{ik_L S[A_L] + ik_R S[A_R]} \mathbb{W}_j[A_L, A_R]$$

The next challenge is to quantify gravitational path integrals.

$$\left\langle \mathbb{W}_j[S^3] \right\rangle_{grav} = \int DA_{L/R} e^{ik_L S[A_L] + ik_R S[A_R]} \mathbb{W}_j[A_L, A_R]$$

$$\mathcal{Z}_{grav}[S^3] = \int DA_{L/R} e^{ik_L S[A_L] + ik_R S[A_R]}$$

- Consider fixed topology, still all order in perturbation theory in G_N.
- We need to adapt exact results in Chern-Simons theory:
    - Level is complex
    - Background connection is not trivial
- Assure that exact results are compatible with the non-standard representations

Partition function

There are two things to keep in mind:

- Level is complex: $k = \delta - is$
- Background connection is not trivial: $P \exp \oint_\gamma a_{L/R} \sim e^{2\pi i\, L_3 h_{L/R}}$

We adapted exact methods to incorporate these tweaks:

- Abelianisation [Blau-Thompson]
- Supersymmetric Localization [Kapustin-Willet-Yaakov]

$$\mathcal{Z}_{grav}[S^3] = e^{ir_L S_{CS}[a_L] + i\, r_R S_{CS}[a_R]} \int d\sigma_L d\sigma_R e^{\frac{i\pi}{2} r_L \sigma_L^2} e^{\frac{i\pi}{2} r_R \sigma_R^2} \sin^2(\pi\, (\sigma_L + h_L)) \sin^2(\pi\, (\sigma_R + h_R))$$

with $r_{L/R} = 2 + k_{L/R}$

Partition function

$$\mathcal{Z}_{grav}[S^3] = e^{ir_L S_{CS}[a_L]+i\, r_R S_{CS}[a_R]} \int d\sigma_L d\sigma_R e^{\frac{i\pi}{2} r_L \sigma_L^2}\, e^{\frac{i\pi}{2} r_R \sigma_R^2} \sin^2(\pi\, (\sigma_L + h_L)) \sin^2(\pi\, (\sigma_R + h_R))$$

$$= \int \left(D g_{\mu\nu}\right)_{S^3} e^{-I_{EH}[g_{\mu\nu}] + i\delta\, I_{GCS}[g_{\mu\nu}]}$$

with $r_{L/R} = 2 + k_{L/R}$

Partition function

$$\mathcal{Z}_{grav}[S^3] = e^{ir_L S_{CS}[a_L]+i\, r_R S_{CS}[a_R]} \int d\sigma_L d\sigma_R e^{\frac{i\pi}{2} r_L \sigma_L^2}\, e^{\frac{i\pi}{2} r_R \sigma_R^2} \sin^2(\pi\,(\sigma_L + h_L)) \sin^2(\pi\,(\sigma_R + h_R))$$

with $r_{L/R} = 2 + k_{L/R}$

$$= i e^{-\frac{i\pi}{r_L} - \frac{i\pi}{r_R}} e^{-i\pi r_L + i\pi r_R} \frac{2}{\sqrt{r_L r_R}} \sin\left(\frac{\pi}{r_L}\right) \sin\left(\frac{\pi}{r_R}\right)$$

$$= \frac{8G_N}{i\ell} \exp\left(\frac{\pi\ell}{2G_N}\right) \sinh^2(4\pi \frac{G_N}{\ell})$$

with $r_{L/R} = \pm i \frac{\ell}{4G_N}$

Wilson loop

Care is also needed for exact methods used to evaluate a Wilson loop, since

- Level is complex: $k = \delta - i\, s$
- Background connection is not trivial: $P \exp \oint_\gamma a \sim e^{2\pi i\, L_3 h}$
- Non-standard representations of SU(2)!

Adapted exact methods to incorporate these tweaks:

$$\langle W_j[S^3] \rangle_{SU(2)} = e^{ir\, S_{CS}[a]} \int d\sigma\, e^{\frac{i\pi}{2} r \sigma^2} \sin^2\big(\pi(\sigma + h)\big) \chi_j\, (\sigma + h)$$

Where the character of the non-standard rep is $\chi_j(z) = \dfrac{e^{\pi i\, z(2j+1)}}{2i \sin \pi z}$

Wilson loop

Care is also needed for exact methods used to evaluate a Wilson loop, since

- Level is complex: $k = \delta - i\, s$
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$$\langle W_j[S^3]\rangle_{SU(2)} = e^{ir\, S_{CS}[a]} \int d\sigma\, e^{\frac{i\pi}{2} r\sigma^2} \sin^2\big(\pi(\sigma+h)\big)\chi_j\,(\sigma+h)$$

$$= \frac{1}{2} e^{irS_{CS}[a]} e^{2\pi i\, h\, j}\, e^{i\phi - \frac{2\pi i}{r} c_j} \sqrt{\frac{2}{r}}\, \sin(\frac{\pi(2j+1)}{r})$$

Wilson spool

Combining these results, the quantum Wilson spool is

$$\left\langle \mathbb{W}_j[S^3] \right\rangle_{grav} = \int DA_{L/R} e^{ik_L S[A_L] + ik_R S[A_R]} \mathbb{W}_j[A_L, A_R]$$

$$= i\, e^{ir_L\, S_{CS}[a_L] + i\, r_R S_{CS}[a_R]} \int d\sigma_L d\sigma_R e^{\frac{i\pi}{2} r_L \sigma_L^2}\, e^{\frac{i\pi}{2} r_R \sigma_R^2} \sin^2(\pi\, \sigma_L) \sin^2(\pi\, \sigma_R)$$

$$\times \int_{\mathcal{C}} \frac{d\alpha}{\alpha} \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\, \chi_j\left(\frac{\alpha}{2\pi}(\sigma_L + h_L)\right) \chi_j\left(\frac{\alpha}{2\pi}(\sigma_R + h_R)\right)$$

Wilson spool

Massive scalar fields coupled to dS_3 quantum gravity

$$\langle \log Z_{scalar}[S^3] \rangle_{grav} = \frac{1}{4} \left\langle \mathbb{W}_j[S^3] \right\rangle_{grav} = \frac{i}{4}\, e^{i r_L\, S_{CS}[a_L] + i\, r_R S_{CS}[a_R]} \int d\sigma_L d\sigma_R e^{\frac{i\pi}{2} r_L \sigma_L^2}\, e^{\frac{i\pi}{2} r_R \sigma_R^2} \sin^2(\pi\, \sigma_L) \sin^2(\pi\, \sigma_R)$$

$$\times \int_{\mathcal{C}} \frac{d\alpha}{\alpha} \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\, \chi_j\left(\frac{\alpha}{2\pi}(\sigma_L + h_L)\right) \chi_j\left(\frac{\alpha}{2\pi}(\sigma_R + h_R)\right)$$

$$\frac{\langle \log Z_{scalar}[S^3] \rangle_{grav}}{\mathcal{Z}_{grav}[S^3]} = \log Z_{scalar}[S^3] + \sum_{m=1}^{\infty} \left(\frac{G_N}{\ell}\right)^{2m} (\log Z)_{2m}$$

What do we do with this?

Wilson spool

Massive scalar fields coupled to dS_3 quantum gravity

$$\frac{\langle \log Z_{scalar}[S^3]\rangle_{grav}}{\mathcal{Z}_{grav}[S^3]} = \log Z_{scalar}[S^3] + \sum_{m=1}^{\infty} \left(\frac{G_N}{\ell}\right)^{2m} (\log Z)_{2m}$$

Mass renormalization

$$m_R^2 \ell^2 = m^2 \ell^2 + \frac{96}{5} m^4 \ell^4 e^{-2\pi|m\ell|} \left(\frac{G_N}{\ell}\right)^2 + \ldots$$

Large mass limit (for simplicity)

Concrete predictive statement about how dynamical gravity renormalizes QFT

Conclusions

We have introduced a new object: the Wilson spool.

- Allows us to incorporate matter fields in the Chern-Simons formulation of 3D gravity.

- Tested at $G_N \to 0$, where the Wilson spool reproduces the one-loop determinant of massive scalar fields.

$$\log(Z_{scalar}[M]) = \log\det(-\nabla^2 + m^2\ell^2)^{-\frac{1}{2}}$$
$$= \frac{1}{4}\mathbb{W}_j[a_L, a_R]$$

- We can also make predictions for quantum corrections, without the aid of holography.

Massive higher spin fields

Sum over topologies

Wilson lines, open spools

Quantum corrections in AdS_3

Edge Modes

$\langle \log Z_{scalar} \rangle$ versus $\log \langle Z_{scalar} \rangle$

Massive higher spin fields

Sum over topologies

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Thank you!