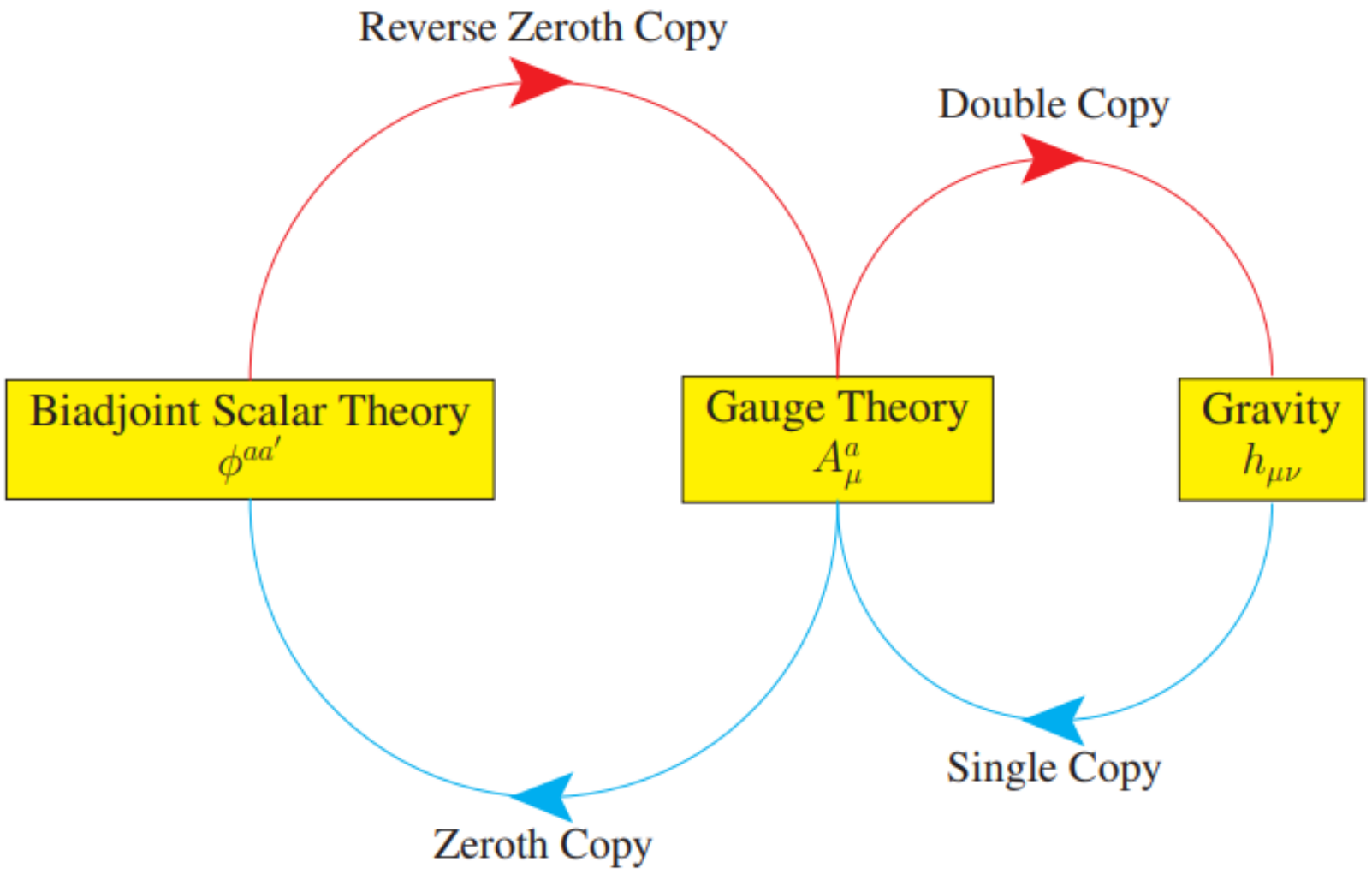

Reverse Zeroth Copy
Double Copy
Biadjoint Scalar Theory
$\phi^{aa'}$
Gauge Theory
A^a_μ
Gravity
$h_{\mu\nu}$
Single Copy
Zeroth Copy

Representation Theory for
Categorical Symmetries

based on 2304.03789, 2305.17165

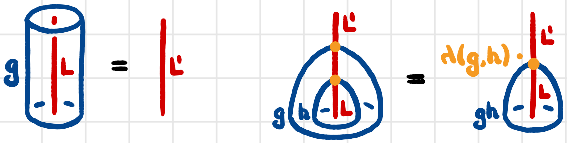
Thomas Bartsch, M. Bullimore, A. Grigoletto

Durham University

QFT with finite group symmetry G:

↳ **local** ops. in representations of G

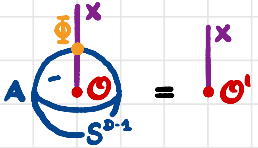
↳ **line** ops. in **2**-representations of G



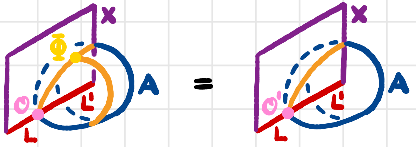
↳ **(n−1)-dim.** ops. in **n**-representations of G[1]

QFT with higher fusion category symmetry C:

↳ twisted sector **local** ops. in representations of the tube category TC[2]



↳ twisted sector **line** ops. in **2**-representations of the tube **2**-category T_2C



↳ twisted sector **(n−1)-dim.** ops. in **n**-representations of the tube **n**-category T_nC

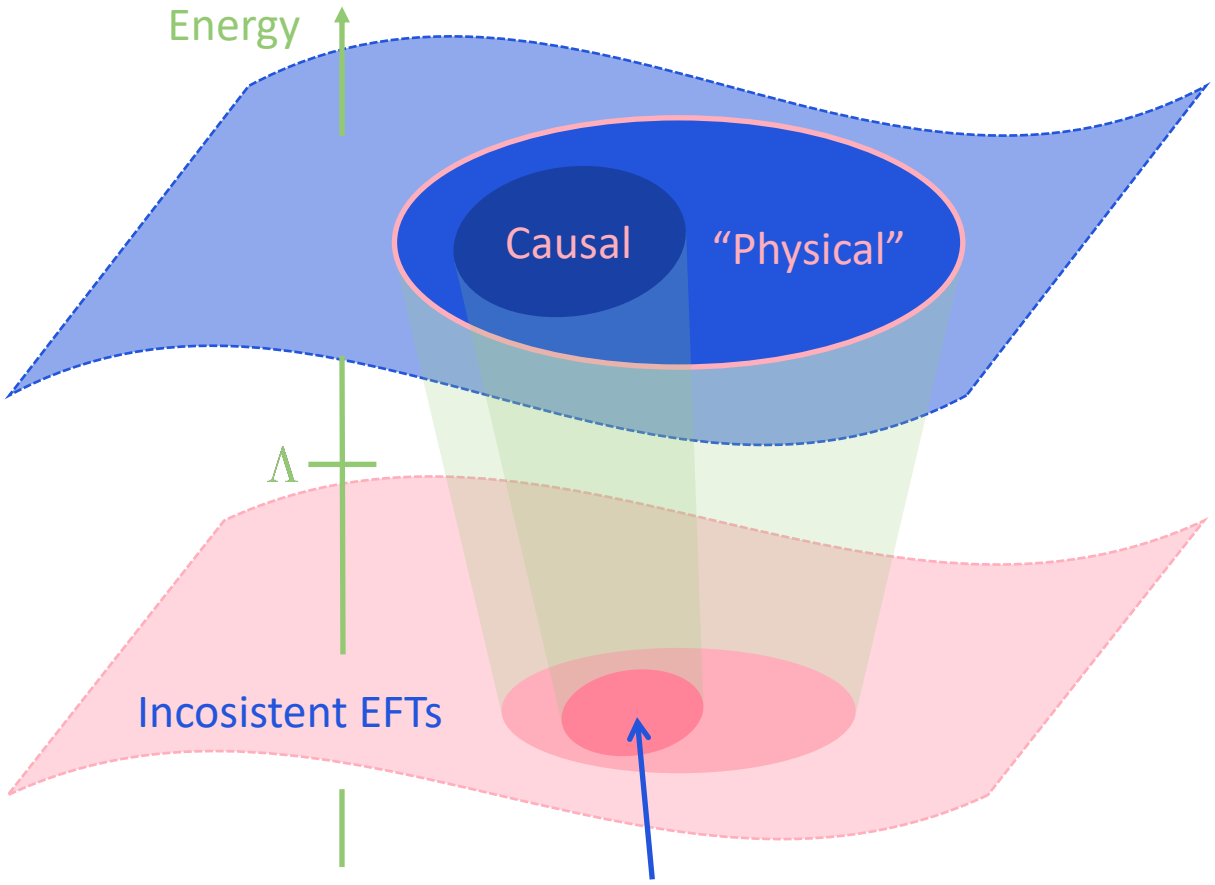
[1] [TB, Bullimore, Grigoletto], [Bhardwaj, Schäfer-Nameki]

[2] [Lin, Okada, Seifnashri, Tachikawa]

Integrated Correlators Beyond Localisation

- We are interested in $\mathcal{N} = 4$ SYM with gauge group $SU(N)$. This is dual to string theory in $AdS_5 \times S^5$.
- Integrated correlators are four-point correlation functions $\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle$, where $\mathcal{O}_p \sim \mathrm{Tr}(\phi^p)$, that have had their spacetime dependence integrated out with a certain measure.
- The integrated correlators of $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_p \mathcal{O}_p \rangle$ can be computed by supersymmetric localisation - deforming the partition function of $\mathcal{N} = 2^*$ SYM. These are exact in N and τ.
- We are now looking at the integrated correlators of $\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle$ in the large N limit. The unintegrated correlators can be computed using a hidden symmetry, and we have found a formula that reproduces all the integrated results.
- We can use this to fix coefficients in the α' expansion of the Mellin amplitude of $\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle$.

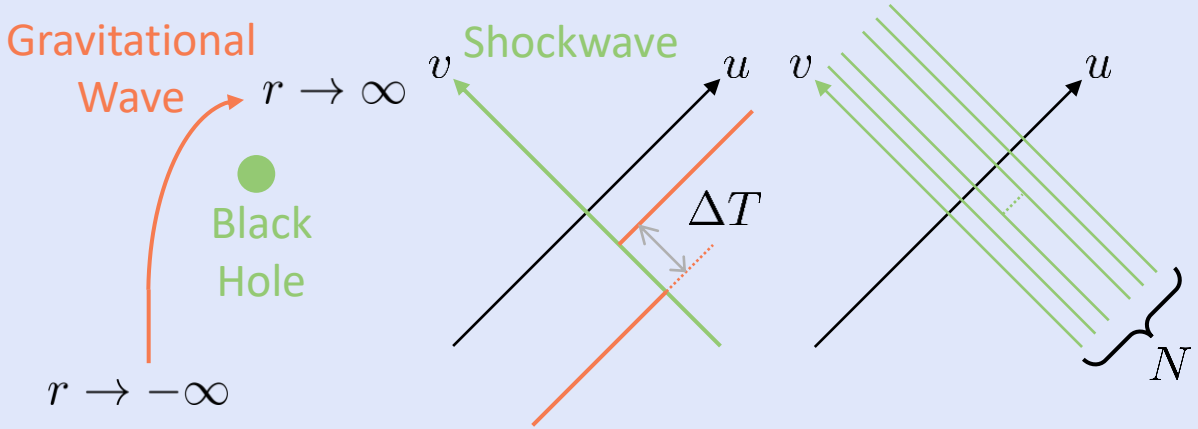
How can causality be used to identify "consistent" EFTs of gravity?



Infrared causality:

$$\Delta T^{\rm EFT} \gtrsim -T_{\rm min}$$

Some Examples…



$$\to \mathcal{L}_{\rm EFT} = \frac{1}{2}R + \frac{c_{\rm GB}}{\Lambda^2}R^2_{\rm GB} + \dots$$

Careful about when a) computation can be done and b) EFT can be trusted

$$\left|\Delta T^{\rm EFT}\right| / T_{\rm min} \ll |c_{\rm GB}|$$

→ Infrared causal when $|c_{\rm GB}| \lesssim 1$.
→ Cannot be sharpened with sequence of shocks.
→ Consistent with S-matrix/positivity bounds.

Calvin Y.-R. Chen

Imperial College London

Fixed points of 3d fermionic field theories

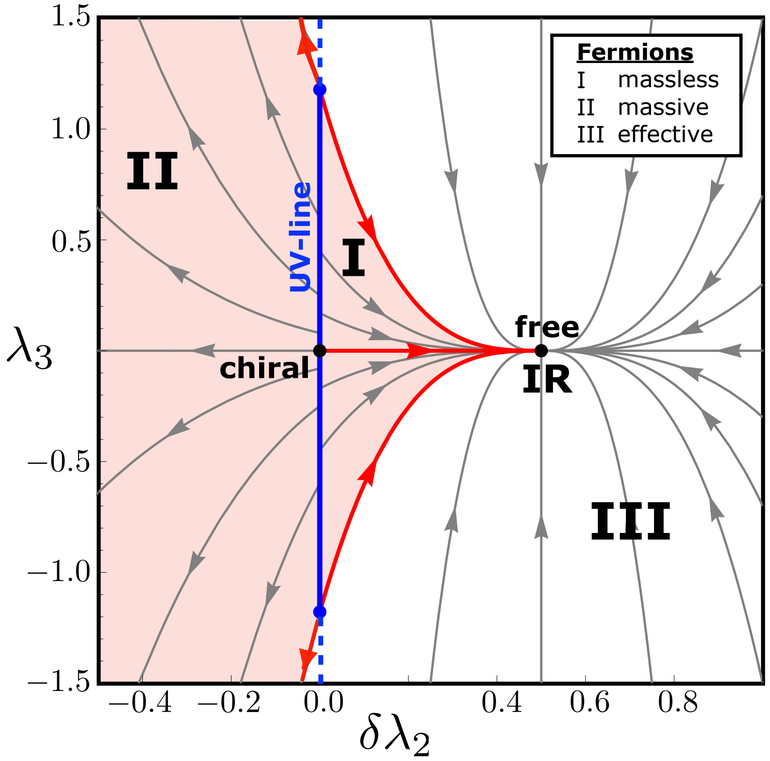
Charlie Cresswell-Hogg
University of Sussex

Now with fundamental six-fermion interactions!

$$S = \int \mathrm{d}^3x \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + M \bar{\psi}_a \psi_a + \tfrac{1}{2} G (\bar{\psi}_a \psi_a)^2 + \tfrac{1}{3!} H (\bar{\psi}_a \psi_a)^3 \right\}$$

Featuring:

- A line of fixed points
- Massless fermions without chiral symmetry
- CFTs with spontaneously broken scale invariance
- Boson-fermion duality



University of Sussex

Higher-form symmetry, chiral MHD & holography

arXiv:2205.03619 [hep-th] with Ruth Gregory and Nabil Iqbal

Arpit Das (3 CPT Maths, Durham University)

Objective

Finite temperature physics of a chiral MHD plasma: System with an axial $U(1)_A$ current j_A that is afflicted by an ABJ anomaly, where the corresponding vector $U(1)_V$ current j_V has been coupled to dynamical electromagnetism.

Weakly-coupled Physics

Lagrangian: describes a massless Dirac fermion ψ coupled to dynamical electromagnetism with photon a (with $f = da$),

$$S[a,\psi] = \int d^4x \left(-\frac{1}{4e^2} f^2 + \overline{\psi}\left(\gamma^\mu \partial_\mu - i\gamma^\mu a_\mu \psi\right)\right)$$

Aim: Construct and study a holographic model possessing the symmetries of the above action $S[a,\psi]$.

E & M: Symmetries and Anomaly

$U(1)_V^{(0)}$ Vector Symmetry: $\partial_\mu j_V^\mu = 0$, where $j_V^\mu = \overline{\psi}\,\gamma^\mu\,\psi$ (current for vector phase rotations, $\psi \to e^{i\alpha}\psi$)

$U(1)_A^{(0)}$ Axial Symmetry: $\partial_\mu j_A^\mu = 0$, where $j_A^\mu = \overline{\psi}\,\gamma^\mu\gamma^5\,\psi$ (current for axial phase rotations, $\psi \to e^{i\alpha\gamma^5}\psi$)

$U(1)^{(1)}$ Magnetic Symmetry: Conservation of magnetic flux (Bianchi Identity),

$$\partial_\mu J^{\mu\nu} = 0, \qquad J^{\mu\nu} := \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} f_{\rho\sigma}$$

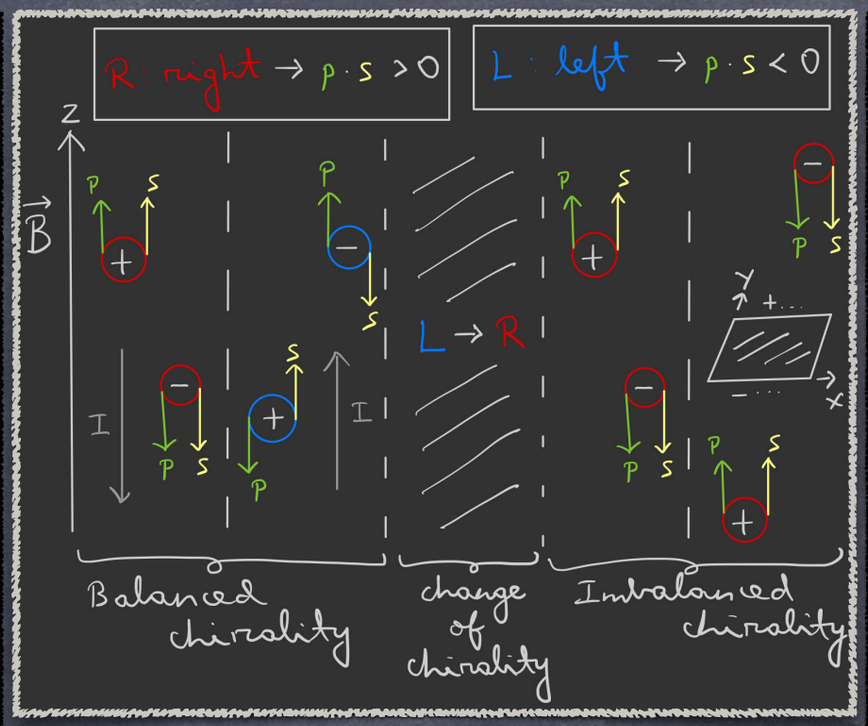
ABJ anomaly: @ QM level, the axial symmetry is broken to,

$$\partial_\mu j_A^\mu = k\,\epsilon_{\mu\nu\rho\sigma} J^{\mu\nu} J^{\rho\sigma}$$

$$\Rightarrow \quad \dot{\rho}_A \sim \vec{E}\cdot\vec{B} \quad \xrightarrow[\text{MHD}]{} \quad \mu_A = e^{-\Gamma_A}\mu_A$$

implying, (external) parallel electric and magnetic fields can change the axial charge density (ρ_A).

Changes in ρ_A leads to chirality imbalance, resulting in generation of an electric current parallel to the external magnetic field: CME



Holographic Dictionary:

QG in D+1 dim	Holographic Duality	CFT in D dim
Extra dim.	⟷	Energy probe
$G_N, \alpha' \to 0$ (IR)	⟷	$N, \lambda \to \infty$ (UV)
Gauge fields	⟷	Currents
Metric fluctuat^{n}s	⟷	Stress-tensor

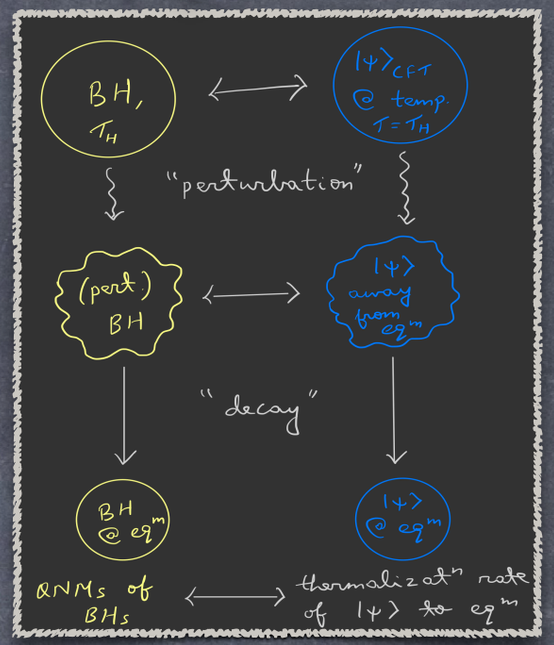
$$\longrightarrow \quad E_1 \leftrightarrow j_A^\mu, \qquad B_2 \leftrightarrow J^{\mu\nu}$$

Holographic bulk action:

$$S_5 = \int_{\mathcal{M}^5} \left|dE_1\right|^2 + \left|dB_2\right|^2 + k\,E_1 \wedge \star dB_2 \wedge \star dB_2 + \cdots$$

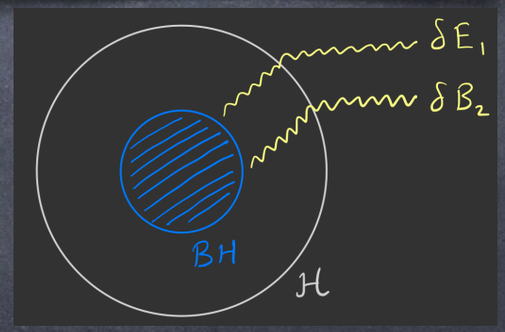
$$J^{\mu\nu} = \frac{\delta S_5}{\delta B^{\mu\nu}(\infty)}, \qquad j_A^\mu = \frac{\delta S_5}{\delta E^\mu(\infty)}$$

QNM captures damping:

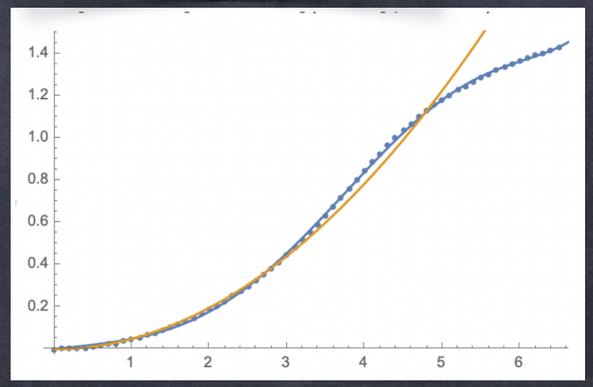


implying, $\Gamma_A \leftrightarrow \omega_{qnm}$

Decaying E & B modes



Result: $\Gamma_A(b) \sim b^2$ (for small b)

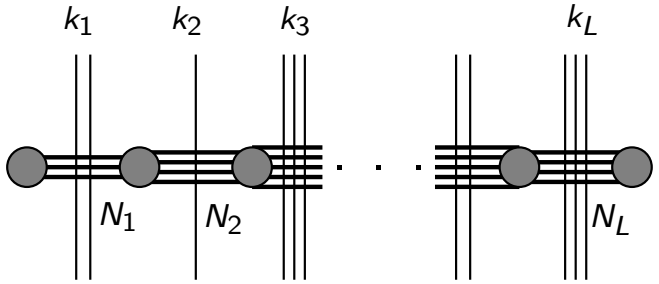


Wilson loops for 5d and 3d conformal linear quivers

Ali Fatemiabhari (Swansea University) with Carlos Nunez

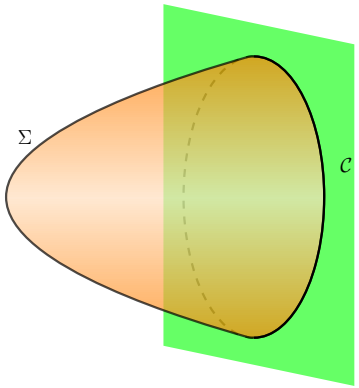
- Classification of a set of Type II backgrounds with AdS_{d+1} factors being holographic duals to SCFTs in d dimensions

- Study conformal and supersymmetric linear quiver field theories with $\mathcal{N}=1$ SUSY in 5d and $\mathcal{N}=4$ in 3d



- Brane embeddings dual to Wilson loop operator insertions in the corresponding gauge theories:

$$W_{\mathbb{R}} = \mathrm{Tr}_{\mathbb{R}}\, \mathcal{P} e^{i \oint \left(A_\mu \dot{x}^\mu + \sigma_3 \sqrt{-\dot{x}^2}\right) d\tau}.$$



Based on Nucl.Phys.B 989 (2023) 116125 [arXiv:2209.07536]

The Celestial Chiral Algebra of SDGR on Eguchi-Hanson

Top-down celestial holography through **twisted holography**.

[Costello, Paquette, Sharma '23], [Costello, Paquette '22], [Costello, Gaiotto '18], [Costello, Li '16] ...

$$\boxed{\text{QG in } 4d,\ \Lambda = 0 \text{ spacetime}} \xleftrightarrow[\text{Holography}]{\text{Celestial}} \boxed{\textbf{CCFT on } \mathbb{CP}^1_{\text{celestial}}}$$

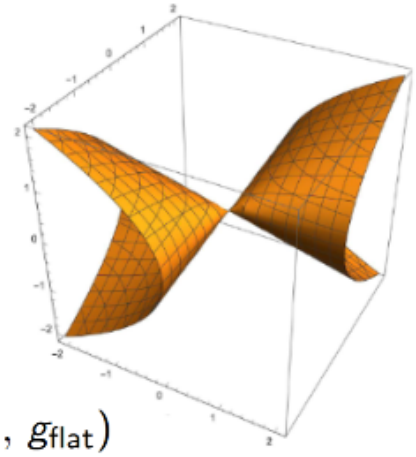
[Strominger '17] ...

Can we get self-dual **Einstein** - rather than conformal - gravity in the bulk?

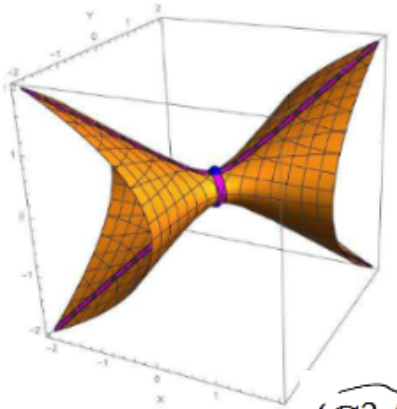
How far we got:

Tree-level dictionary involving self-dual gravity on Eguchi-Hanson space.

[Bittleston, SH, Skinner '23], [Bu, SH, Skinner '22]

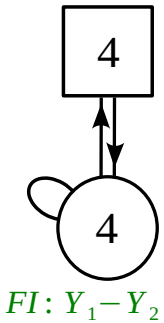


$(\mathbb{C}^2/\mathbb{Z}_2\,,\, g_{\text{flat}})$

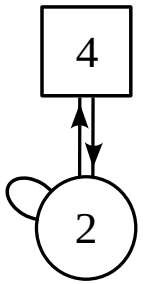


$(\widetilde{\mathbb{C}^2/\mathbb{Z}_2}\,,\, g_{\text{EH}})$

Probing Bad Theories with the Dualisation Algorithm

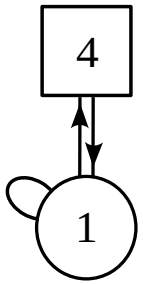


$$= \delta\left(Y_1 - Y_2\right) \times$$



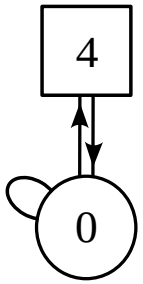
$$\times \left(singlets\right)$$

$$+ \delta\left(Y_1 - Y_2 \pm (\cdots)\right) \times$$



$$\times \left(singlets\right)$$

$$+$$



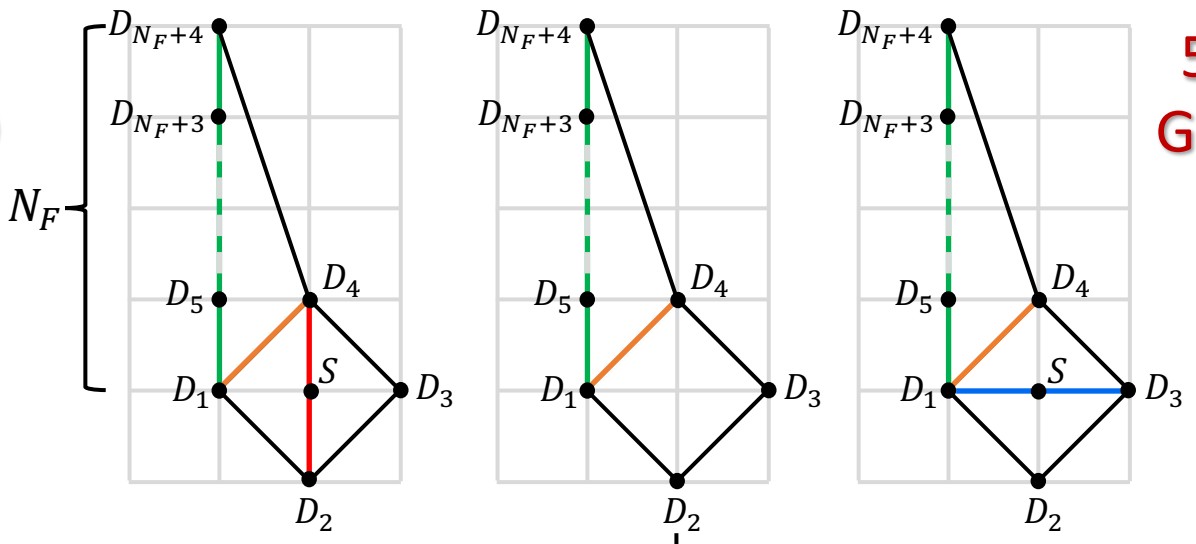
$$\times \left(singlets\right)$$

G_2 Manifolds from 4d $\mathcal{N} = 1$ Theories – Evyatar Sabag

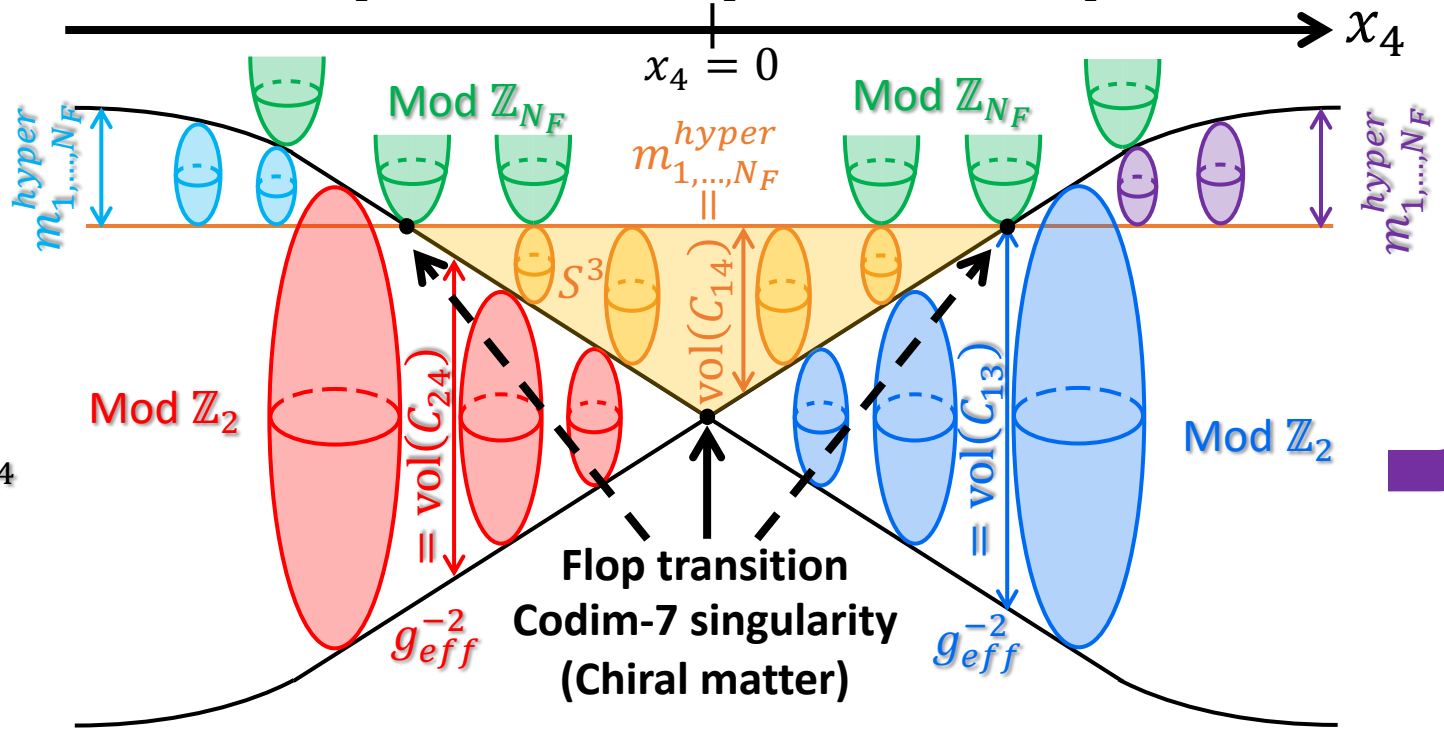
5d $\mathcal{N} = 1$ E_{N_F+1} Seiberg theory (partially resolved) toric diagrams

Low energy 5d $SU(2) + N_F F$ gauge theory

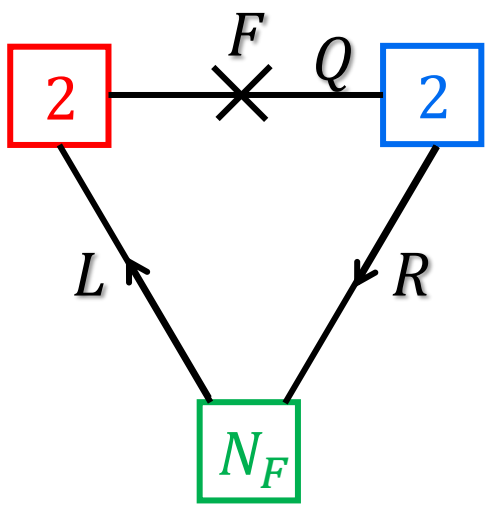
Fibered over x_4



5d $\mathcal{N} = 1$ Gaiotto-Kim Domain wall



4d $\mathcal{N} = 1$ quiver

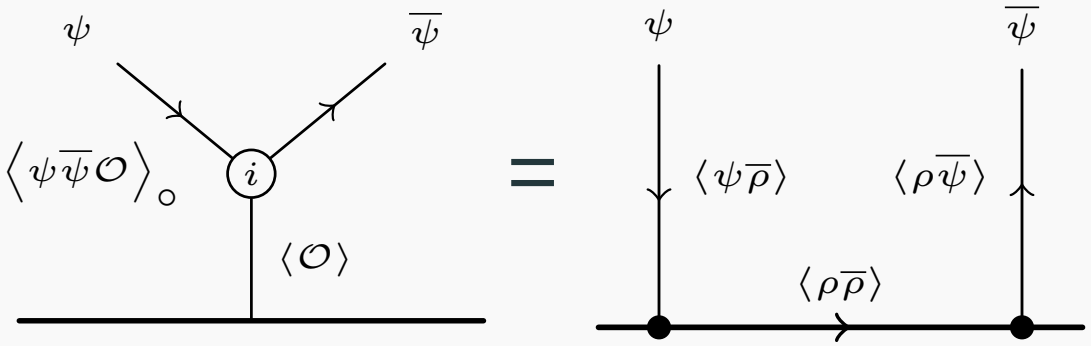


$$W = FQ^2 + LQR$$

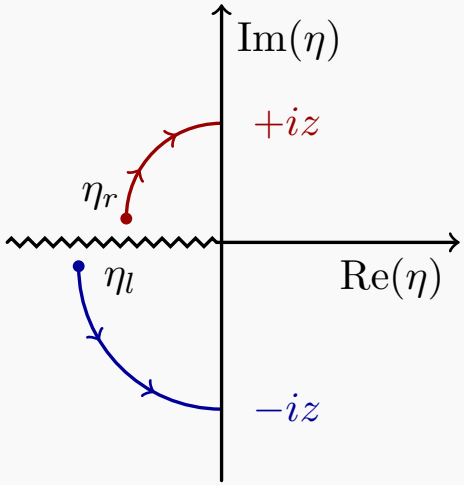
Evyatar Sabag - Oxford
A. Braun, ES, M. Sacchi, S. Schäfer-Nameki (2023)

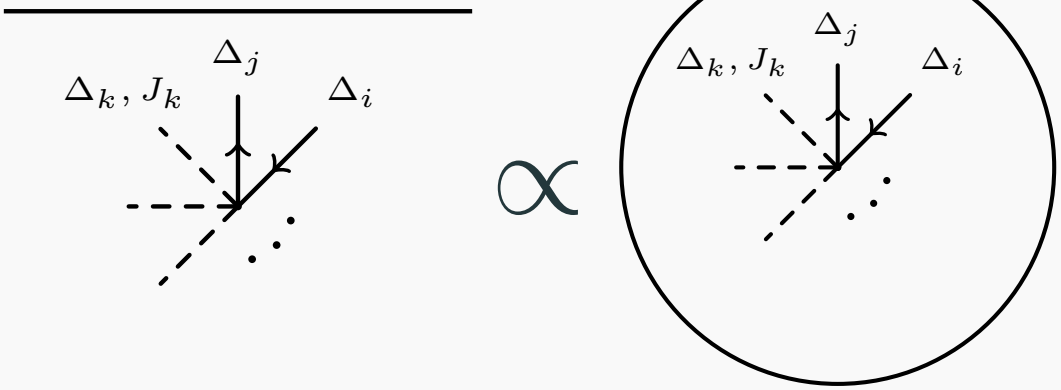
FPUK 2023

Bootstrapping Fermions With Defects



Wick Rotations & de-Sitter Space

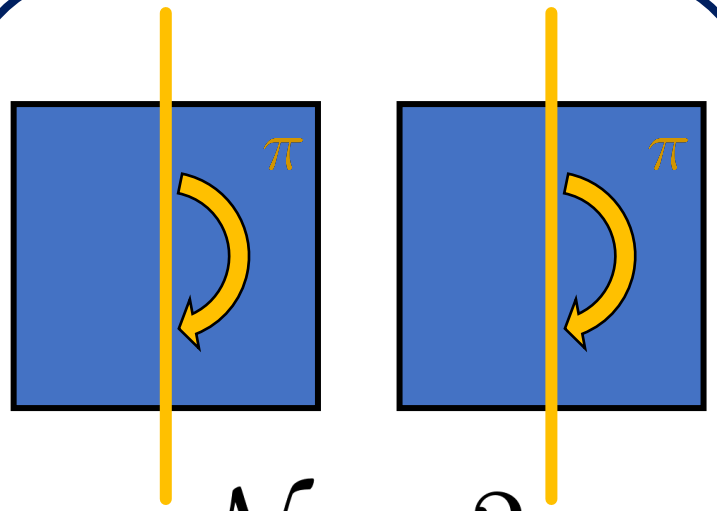




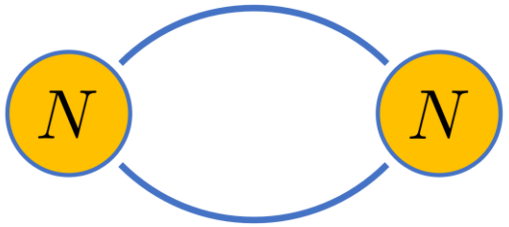
More on Orbifolds of $AdS_5 \times S^5$

Torben Skrzypek

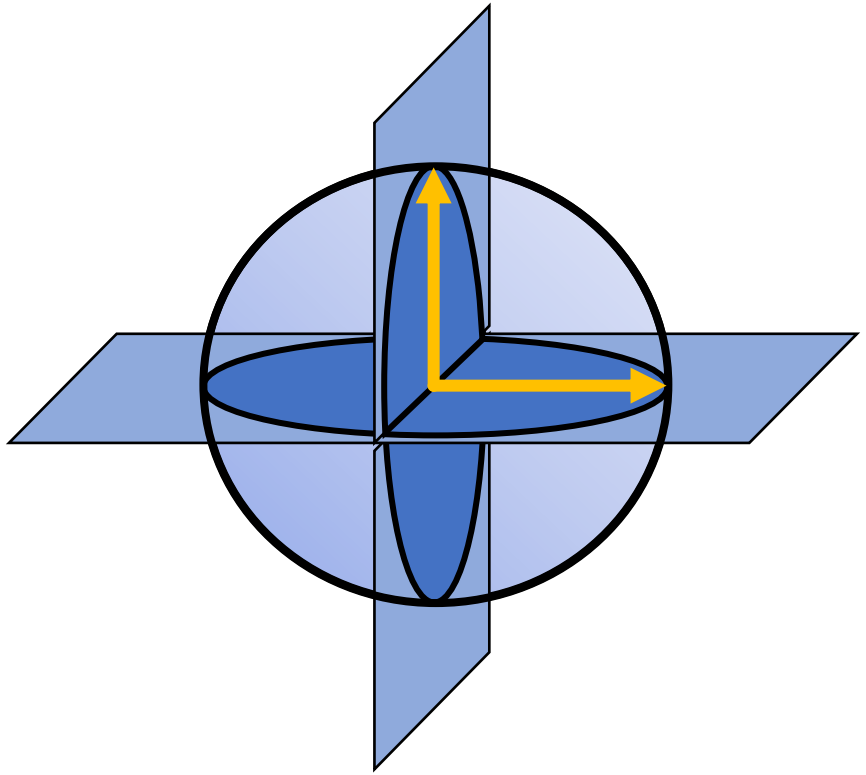
ICL

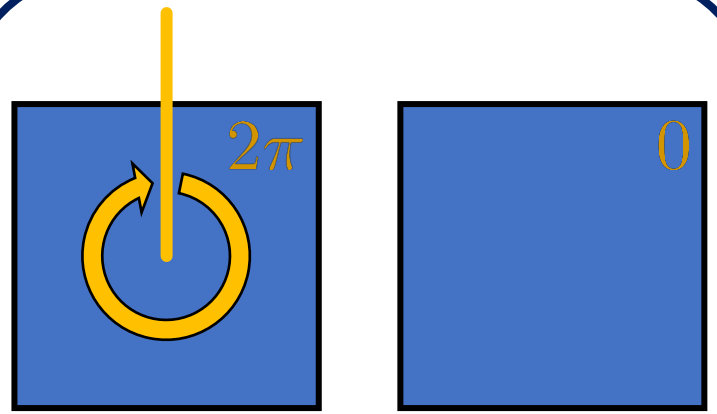


$\mathcal{N} = 2$



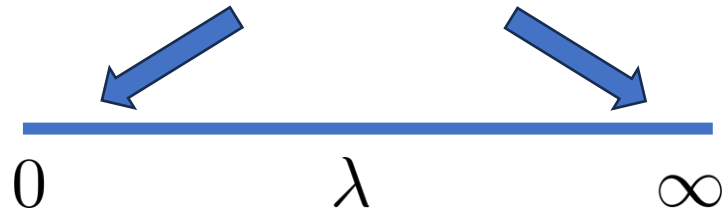
Stringy corrections?





$\mathcal{N} = 0$

Who's afraid of Tachyons?

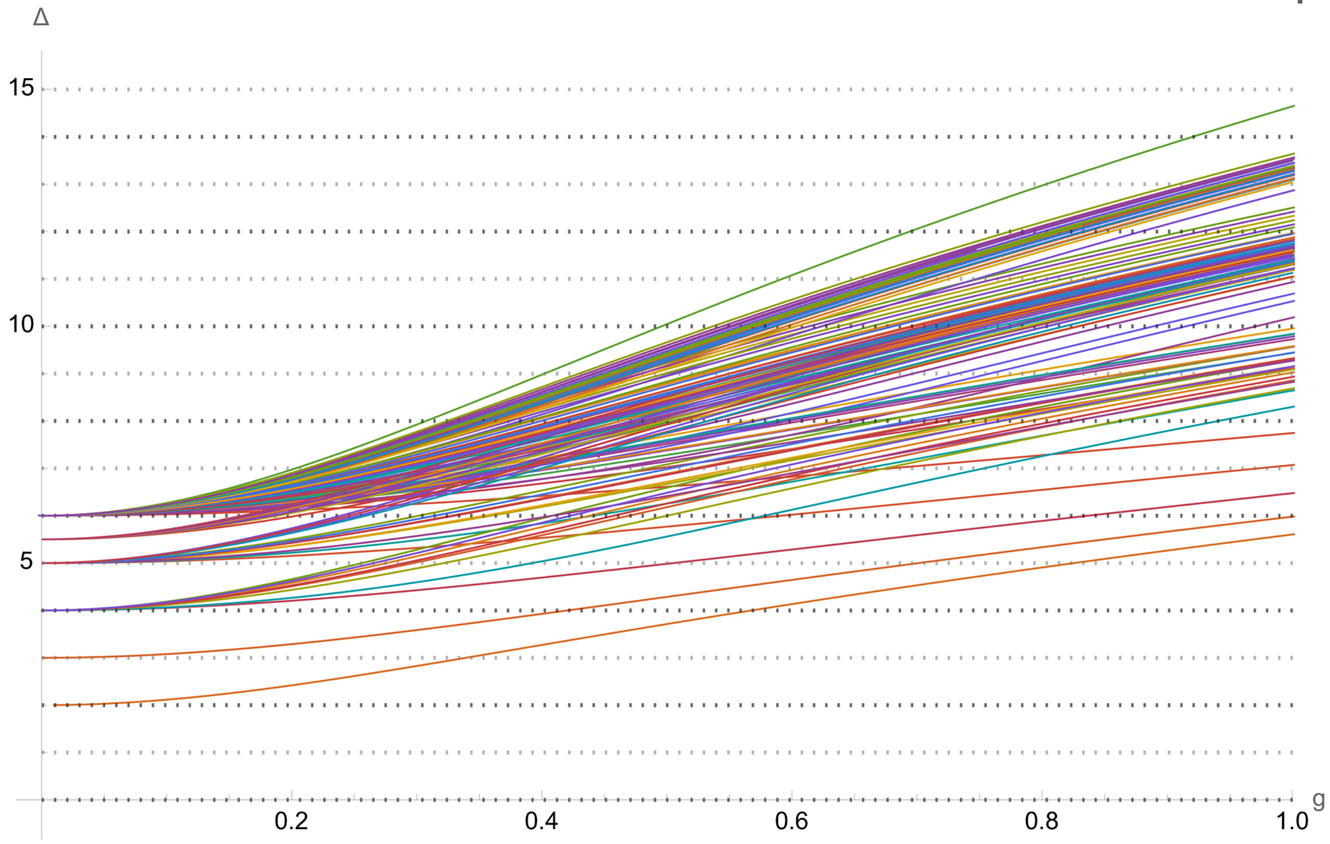


QSC Solver for Excited States

Nikolay Gromov, Árpád Hegedűs,
Julius Julius, **Nika Sokolova**

based on
arXiv: 2306.12379

We are focused on 4d $\mathcal{N} = 4$ SYM theory The spectrum is in-principle known from integrability. What about the structure constants? We can use integrability + conformal bootstrap (bootstrability), but for that we need to know the spectrum very well.



Quantum Spectral Curve Solver allows us to follow the spectrum for many excited states (all states up to bare dimension < 7) from weak to strong coupling. QSC Solver is available on GitHub (github.com/julius-julius/qsc).

We can reach the strong coupling and fit the data. We can find the subleading order for many states, and also can assign states to the Kaluza-Klein towers.

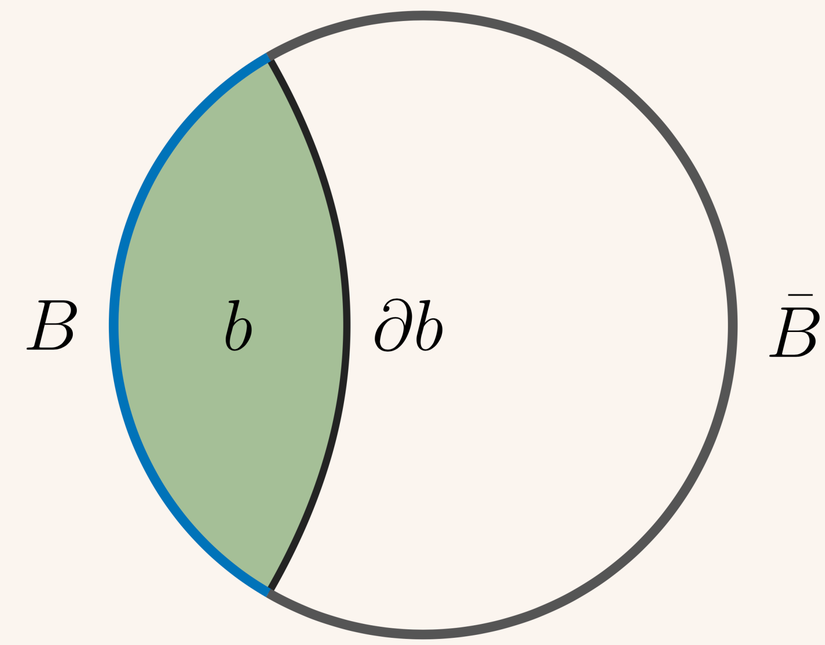
Breaking the degeneracies at strong coupling, it is possible to unmix the dispersive sum rules [Alday, Hansen, Silva, 22' 23'] and find some of the structure constants.

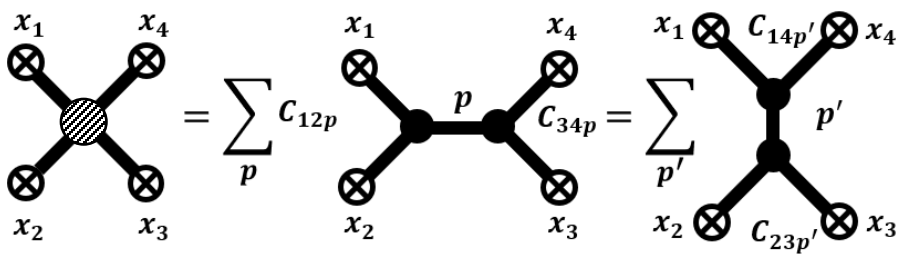
Information recovery in JT gravity

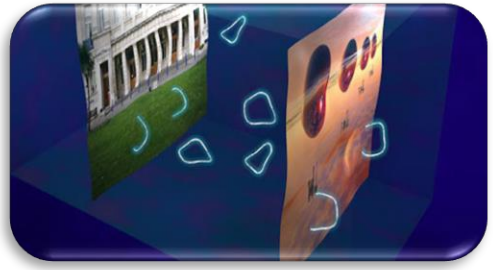
Neil Talwar, Swansea University

- QES $\rightsquigarrow$ EWR (info in b is encoded in B)
- Information recovery in JT + RTN
  - Hayden-Preskill decoding criterion with refinements
  - Backreaction and state-dependence
  - Information recovery in the late radiation
  - Petz recovery map

$$S(B) = \frac{A(\partial b)}{4G} + S_{\text{bulk}}(b)$$





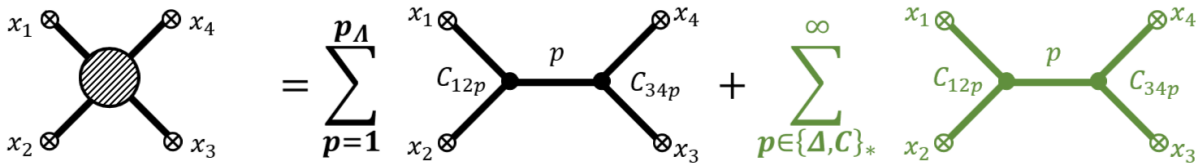


New conformal bootstrap strategies with applications in 1*d*

Mitchell Woolley
Supervisor: Dr. Costis Papageorgakis

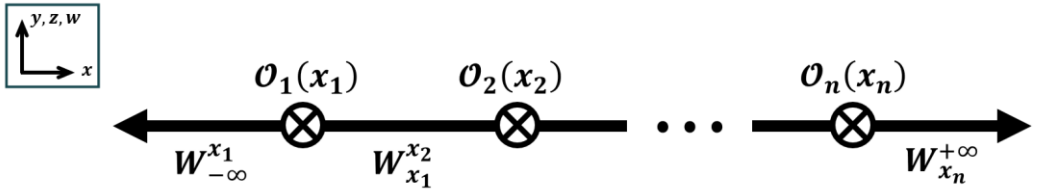
Based on [2306.15730] with V. Niarchos, C. Papageorgakis, P. Richmond, A. G. Stapleton

- Improved truncation of conformal block expansions



- Cheap optimization algorithms that *approximately solve* crossing equations for CFT data

- Application to 1*d* Wilson line defect SCFT in 4*d* $\mathcal{N}$=4 SYM and much more



Integrated correlators in $\mathcal{N}=4$ and modular properties

- Integrated correlator of four 1/2-BPS superconformal primary with dimension p

$$\mathcal{C}_{G_N}(\tau,\bar{\tau}) = d\mu\, \langle \mathcal{O}_2(x_1,Y_1)\,\mathcal{O}_2(x_2,Y_2)\,\mathcal{O}_p(x_3,Y_3)\,\mathcal{O}_p(x_4,Y_4)\rangle\ .$$

- Proved the previously conjectured recursion relation arcoss N for all gauge groups. And later also extends to general charge $p>2$.

$$\Delta_\tau \mathcal{C}_{SU(N)} - 4c_{SU(N)}\Big[\mathcal{C}_{SU(N+1)} - 2\,\mathcal{C}_{SU(N)} + \mathcal{C}_{SU(N-1)}(\tau,\bar{\tau})\Big] \\ -(N+1)\,\mathcal{C}_{SU(N-1)} + (N-1)\,\mathcal{C}_{SU(N+1)} = 0\,,$$

- large-N (and similarly large-charge expansions), with modular functions

$$\mathcal{C}_{SU(N)} = \frac{N^2}{4} + \sum_{r=0}^{\infty} \underbrace{N^{\frac{1}{2}-r} E(1/2+\dots)}_{\mathcal{R}^4, d^4\mathcal{R}^4, d^6\mathcal{R}^4, \dots} \pm i \sum_{r=0}^{\infty} \underbrace{N^{2-\frac{r}{2}} D_N(\dots)}_{(p,q)-\text{string}}\ .$$

- 10d hidden symmetry leads to magical cancellation of multi-zetas! This predicts expression at all-loops in planar limit for $\langle \mathcal{O}_p\mathcal{O}_q\mathcal{O}_r\mathcal{O}_s\rangle$ beyond localisation.

Gravitational, Electromagnetic and Quantum Interaction: From String to Cloud Theory

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M. B. Al-Fadhli, University of Lincoln, UK.

inspirehep.net/literature/2663665 & inspirehep.net/literature/2661450

Motivation

The Planck Legacy 2018 release has confirmed the existence of an enhanced lensing amplitude in the cosmic microwave background power spectra, which prefers a positively curved early Universe (early curved bulk) with a confidence level of over 99% [1,2].

Besides, while the Sun induces curvature in a likely flat spacetime background, Earth encounters a curved background (curved bulk) due to the Sun presence, where its induced curvature is influenced by the bulk curvature in addition to its energy density and flux.

Aims

This study aims to formulate interaction field equations that

I. consider the background curvature, signified as the curvature of 4D conformal bulk, and its impact on celestial objects that are regarded as 4D relativistic cloud-worlds.

II. reflect the impact of gravity, expressed by the curvature of 4D conformal bulk, on quantum fields that are regarded as 4D relativistic quantum clouds.

I. Electromagnetism and Gravitation

To incorporate the bulk curvature and its feasible evolution over the conformal time, a modulus of spacetime deformation, E_D, in terms of energy density is introduced. The modulus can be expressed in terms of the resistance of the bulk or in terms of the field strength of the bulk, $\mathcal{F}_{\lambda\rho}$, by using the Lagrangian formulation of energy density existing in the bulk as a manifestation of vacuum energy density as follows

$$E_D = \frac{T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}}{R_{\mu\nu}/\mathcal{R}} = \frac{-\mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}}{4\mu_0}$$

Because of the bulk's constant modulus under the condition of constant vacuum energy density and by considering its expansion, a dual action concerning the conservation of energy on global (bulk) and local (cloud world) scales can be formulated as

$$S = \int_B \left[\frac{-\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}}{4\mu_0}\right]\sqrt{-\tilde{g}} \int_C \left[\frac{R_{\mu\nu}g^{\mu\nu}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}} + \frac{L_{\mu\nu}g^{\mu\nu}}{\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}}\right]\sqrt{-g}\, d^4\rho\, d^4\sigma$$

where R is the Ricci scalar denoting a localized curvature, which is induced in the bulk by a celestial object that is regarded as a 4D relativistic cloud-world of metric g_{uv} and Lagrangian density L while $\mathcal{R}$ is the scalar curvature of the 4D conformal bulk of metric $\tilde{g}_{\mu\nu}$ and Lagrangian density $\mathcal{L}$. The global-local action should hold for any variation as

$$\delta S = \int_B \begin{bmatrix} \frac{-\mathcal{F}_{\lambda\rho}\mathcal{F}^{\rho}_{\gamma}\delta\tilde{g}^{\lambda\gamma}}{2\mu_0} \\ -\frac{\tilde{g}^{\lambda\gamma}\delta\mathcal{F}_{\lambda\rho}\mathcal{F}^{\rho}_{\gamma}}{2\mu_0} + \\ \mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}\frac{\tilde{g}_{\mu\nu}\delta\tilde{g}^{\mu\nu}}{8\mu_0} \end{bmatrix}\sqrt{-\tilde{g}} \int_C \begin{bmatrix} \frac{R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}}{\mathcal{R}} - \frac{\mathcal{R}_{\mu\nu}\delta\tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu}\delta\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R \\ -\frac{g_{\mu\nu}R\delta g^{\mu\nu}}{2\mathcal{R}} - \frac{g_{\mu\nu}L\delta g^{\mu\nu}}{2\mathcal{L}} \\ +\frac{L_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta L_{\mu\nu}}{\mathcal{L}} - \frac{\mathcal{L}_{\mu\nu}\delta\tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu}\delta\mathcal{L}_{\mu\nu}}{\mathcal{L}^2}L \end{bmatrix}\sqrt{-g}\,\varphi^2\, d^8\alpha$$

The cloud-world's boundary term can be transformed based on Stokes' theorem as

$$\int_C \left[\frac{g^{\mu\nu}\delta R_{\mu\nu}}{\mathcal{R}}\right]\sqrt{-g}\, d^4\rho = \int_C \left[\frac{1}{\mathcal{R}}\nabla_\rho\left(g^{\mu\nu}\delta\Gamma^{\rho}_{\nu\mu} - g^{\mu\rho}\delta\Gamma^{\sigma}_{\sigma\mu}\right)\right]\sqrt{-g}\, d^4\rho = \epsilon\int_{\partial C}\left[\frac{K}{\mathcal{R}}\right]\sqrt{|q|}\, d^3\varrho$$

where the Ricci tensor variation is expressed in terms of the Palatini identity. The whole transformed boundary action, S_b, is

$$S_b = \int_{\partial B}\epsilon\left[\frac{f_\lambda}{2\mu_0}\right]\sqrt{-\tilde{p}}\left(\epsilon\int_{\partial C}\left(\left[\frac{K}{\mathcal{R}}\right] + \left[\frac{l}{\mathcal{L}}\right]\right)\sqrt{|q|} - \epsilon\int_{\partial C}\left(\left[\frac{R\mathcal{K}}{\mathcal{R}^2}\right] + \left[\frac{L\ell}{\mathcal{L}^2}\right]\right)\sqrt{|p|}\right) d^3\varrho\, d^3\varsigma$$

where f_λ is the 4D Lorentz force density; K and $\mathcal{K}$ are the traces of the cloud-world and the bulk extrinsic curvatures; l and ℓ are the extrinsic traces of Lagrangian density on their boundaries; q and p are the determinants of their induced metrics, respectively.

II. Electromagnetism, Gravitation and Quantum

Regarding the wave-particle duality and analogous to the constant bulk modulus, the curvature of the bulk, including both that which is conformal, $\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}$, and induced by a celestial object, $R_{\mu\nu}g^{\mu\nu}$, can be considered constant regarding quantum fields. Accordingly, the action can be extended in terms of quantum fields as

$$S = \int_B \left[\frac{-\mathcal{F}_{\lambda\rho}\tilde{g}^{\lambda\gamma}\mathcal{F}_{\gamma\alpha}\tilde{g}^{\rho\alpha}}{4\mu_0}\right]\sqrt{-\tilde{g}}\int_C\left[\frac{R_{\mu\nu}g^{\mu\nu}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}}\right]\sqrt{-g}\int_Q\left[\frac{p_\mu p_\nu q^{\mu\nu}}{\pi_\mu\pi_\nu g^{\mu\nu}} + \frac{L_{\alpha\beta}q^{\alpha\lambda}L_{\lambda\gamma}q^{\beta\gamma}}{2\chi_0\mathcal{L}_{\mu\nu}g^{\mu\nu}}\right]\sqrt{-q}\,\vartheta^2 d^{12}\sigma$$

where $L_{\alpha\beta}L^{\alpha\beta}/2\chi_0$ and $p_\mu p^\nu$ are the Lagrangian and four momenta of two entangled quantum fields, respectively. The action should hold for any variation as

$$\delta S = \int_B \begin{bmatrix} \frac{-\mathcal{F}_{\lambda\rho}\mathcal{F}^{\rho}_{\gamma}\delta\tilde{g}^{\lambda\gamma}}{2\mu_0} \\ -\frac{\tilde{g}^{\lambda\gamma}\delta\mathcal{F}_{\lambda\rho}\mathcal{F}^{\rho}_{\gamma}}{2\mu_0} + \\ \mathcal{F}_{\lambda\rho}\mathcal{F}^{\lambda\rho}\frac{\tilde{g}_{\mu\nu}\delta\tilde{g}^{\mu\nu}}{8\mu_0} \end{bmatrix}\sqrt{-\tilde{g}}\int_C \begin{bmatrix} \frac{R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}}{\mathcal{R}} \\ -\frac{\mathcal{R}_{\mu\nu}\,\delta\tilde{g}^{\mu\nu} + \tilde{g}^{\mu\nu}\delta\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R \\ -\frac{g_{\mu\nu}\delta g^{\mu\nu}}{2\mathcal{R}}R \end{bmatrix}$$

$$\int_Q \begin{bmatrix} \frac{p_\mu p_\nu\delta q^{\mu\nu} + q^{\mu\nu}\delta(p_\mu p_\nu)}{\pi_\mu\pi^\nu} - p_\mu p^\nu\frac{(\pi_\mu\pi_\nu)\delta g^{\mu\nu} + g^{\mu\nu}\delta(\pi_\mu\pi_\nu)}{(\pi_\mu\pi^\nu)^2} - \frac{p_\mu p^\nu}{\pi_\mu\pi^\nu}\frac{q_{\mu\nu}\delta q^{\mu\nu}}{2} + \\ \frac{L_{\alpha\gamma}L^{\gamma}_{\beta}\delta q^{\alpha\lambda} + q^{\alpha\lambda}L_{\alpha\beta}\delta L^{\gamma}_{\beta}}{\chi_0\mathcal{L}} - L_{\alpha\beta}L^{\alpha\beta}\frac{\mathcal{L}_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta\mathcal{L}_{\mu\nu}}{\chi_0\mathcal{L}^2} - L_{\alpha\beta}L^{\alpha\beta}\frac{q_{\mu\nu}\delta q^{\mu\nu}}{4\chi_0\mathcal{L}} \end{bmatrix}\sqrt{-q}\,\vartheta^2 d^{12}\sigma$$

Inferred from gauge theory, the quantum cloud's boundary term can be transformed as

$$\int_Q\left[\frac{q^{\mu\nu}\delta(p_\mu p_\nu)}{\pi_\mu\pi^\nu}\right]\sqrt{-q}\, d^4\alpha = \int_{\partial Q}\left[\epsilon\frac{J_\lambda(A_\rho + B_\rho)e^{\lambda\gamma}J_\alpha(A_\gamma + B_\gamma)e^{\rho\alpha}}{\pi_\mu\pi^\nu}\right]\sqrt{-e}\, d^3\varsigma$$

where J_λ is the four-current flux from the boundary, ∂Q, while A_ρ and B_ρ are the generated and external four potentials, respectively. By labelling $A_\rho + B_\rho$ as A_ρ, the variation is

$$\int_{\partial Q}\epsilon\left[\frac{J_\lambda A_\rho J^\rho A_\gamma\delta e^{\lambda\gamma}}{\pi_\mu\pi^\nu} - \frac{1}{2}\frac{J_\lambda A_\rho J^\rho A^\lambda}{\pi_\mu\pi^\nu}\left(e_{\mu\nu}\delta e^{\mu\nu} - 2e^{\lambda\gamma}\frac{\delta J_\lambda A_\rho J^\rho A_\gamma}{J_\lambda A_\rho J^\rho A^\lambda} + 2\frac{\delta\pi_\mu\pi^\nu}{\pi_\mu\pi^\nu}\right)\right]\sqrt{-e}\, d^3\varsigma$$

As $e^{\lambda\gamma} = -e_{\lambda\gamma}\delta e^{\lambda\gamma}/\delta e_{\lambda\gamma}$, the last two terms can be treated as

$$\int_{\partial Q}\left[e_{\lambda\gamma}\frac{-\delta(J_\lambda A_\rho J^\rho A_\gamma)e_{\lambda\gamma}}{(J_\lambda A_\rho J^\rho A_\gamma)\delta e_{\lambda\gamma}}\right]\delta e^{\lambda\gamma}\sqrt{-e}\, d^3\varsigma = \int_{\partial Q}\left[e_{\lambda\gamma}\frac{\delta ln(J_\lambda A_\rho J^\rho A_\gamma)}{\delta ln e_{\lambda\gamma}}\right]\delta e^{\lambda\gamma}\sqrt{-e}\, d^3\varsigma$$

where $\emptyset^2 = \delta ln(J_\lambda A_\rho J^\rho A_\gamma)/\delta ln e_{\lambda\gamma}$ resembles a normalized Ricci flow that can be stated as a conformal function based on Weyl's transformation reflecting boundary's evolution.

I. Findings: E-G Interaction Field Equations

By applying the principle of stationary action, the interaction field equations in Ref [3] are

$$\frac{R_{\mu\nu}}{\mathcal{R}} - \frac{1}{2}\frac{R}{\mathcal{R}}g_{\mu\nu} - \frac{R\mathcal{R}_{\mu\nu}}{\mathcal{R}^2} + \frac{R\left(\mathcal{K}_{\mu\nu} - \frac{1}{2}\mathcal{K}\hat{p}_{\mu\nu}\right) - \mathcal{R}\left(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}\right)}{\mathcal{R}^2} = \frac{\hat{T}_{\mu\nu}}{\mathcal{T}_{\mu\nu}}$$

These field equations reveal that the cloud-world's induced curvature over the bulk's conformal (background) curvature is equal to the ratio of the cloud-world's energy density and flux to the bulk's vacuum energy density and flux through the expanding/contracting Universe. The field equations can be simplified to

$$R_{\mu\nu} - \frac{1}{2}R(g_{\mu\nu} + 2\tilde{g}_{\mu\nu}) - \left(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}\right) + \frac{R}{\mathcal{R}}\left(\mathcal{K}_{\mu\nu} - \frac{1}{2}\mathcal{K}\hat{p}_{\mu\nu}\right) = \frac{8\pi G_{\mathcal{R}}}{c^4}\hat{T}_{\mu\nu}$$

The field equations reflect the eight-dimensional DoF as stated in the dual-action, where $\mathcal{R}_{\mu\nu}/\mathcal{R} = \mathcal{R}_{\mu\nu}/\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu} = \tilde{g}_{\mu\nu}$. The bulk boundary term seems to resemble external fields exerted by vacuum energy. The solved equations for a galaxy supermassive core reveal a dual forced vortex model where the early curved bulk could cause the high orbital speeds:

$$ds^2 = \left(1 - \frac{r_s}{r} - \frac{\tilde{r}_p}{\tilde{r}}\right)\left(-c^2dt^2 + S^2\left(\frac{dr^2}{1 + \frac{r_s^2}{r^2} - 2\frac{r_s}{r}} + \frac{r^2d\theta^2 + r^2sin^2\theta d\phi^2}{1 - \frac{r_s}{r} - \frac{\tilde{r}_p}{\tilde{r}}}\right)\right)$$

This metric reduces to the Schwarzschild metric in a flat background ($\tilde{r} \to \infty$) [3]. Finally, the interaction field equations with implicit bulk boundary term are

$$R_{\mu\nu} - \frac{1}{2}R\hat{g}_{\mu\nu} - \left(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}\right) = \frac{8\pi G_{\mathcal{R}}}{c^4}\hat{T}_{\mu\nu}$$

where $\hat{g}_{\mu\nu} = g_{\mu\nu} + 2\tilde{g}_{\mu\nu} - 2\bar{\tilde{g}}_{\mu\nu}$.

II. Findings: E-G-Q Interaction Field Equations

By applying the principle of stationary action, the interaction field equations in Ref [3] are

$$p_\mu - \frac{1}{2}p^\nu(q_{\mu\nu} + 2\tilde{q}_{\mu\nu}) - \left(J^\mu A_\mu - \frac{1}{2}J^\mu A^\nu\zeta_{\mu\nu}\right) + \frac{p^\nu}{\pi^\nu}\left(\mathcal{J}^\mu\mathcal{A}_\mu - \frac{1}{2}\mathcal{J}^\mu\mathcal{A}^\nu\varsigma_{\mu\nu}\right) = \frac{\hbar G_{\mathcal{R}}}{2c^2g_R}\mathcal{T}_\mu$$

Applying the momentum p_μ and stress-energy (gravitational) $\mathcal{T}_\mu$ operators give

$$i\hbar\gamma^\mu\partial_\mu\psi - \frac{1}{2}i\hbar\gamma^\mu\partial^\nu(q_{\mu\nu} + 2\tilde{q}_{\mu\nu})\psi - \left(J^\mu A_\mu - \frac{1}{2}J^\mu A^\nu\zeta_{\mu\nu}\right)\psi + \left(\mathcal{J}^\mu\mathcal{A}_\mu - \frac{1}{2}\mathcal{J}^\mu\mathcal{A}^\nu\varsigma_{\mu\nu}\right)i\hbar\frac{\gamma^\mu\partial_\mu\psi}{\pi^\nu} = \frac{1}{2}\frac{\hbar}{x^\mu}R\partial_R\psi$$

where the quantum cloud's boundary term seems to resemble the Isospin including both the charge and hypercharge while the bulk's boundary term seems to resemble the Higgs mechanism that is governed by the spin-spin coupling between conventional and vacuum energy fields. For an electron in a flat background, these equations reduce to

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = e\gamma^\mu A_\mu\psi$$

which represents the Dirac equation and the interaction with the electromagnetic field.

References

[1] Di Valentino, E. (2020) Planck evidence for a closed Universe and a possible crisis for cosmology. *Nat. Astron.* 4 196–203.

[2] Handley, W. (2021) Curvature tension: Evidence for a closed universe. *Phys. Rev. D.* 103, 041301.

[3] Al Fadhli, M. (2023) Gravitational, Electromagnetic and Quantum Interaction: From String to Cloud Theory. *MDPI Phys. Sci. Forum* 7 (1), 55.