Low-SUSY exact D-instanton corrections from SFT

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Based on: 2108.04265, 2110.06949 w/ Sergei Alexandrov, Ashoke Sen

2204.02981 w/ Sergei Alexandrov, Atakan Hilmi Firat, Manki Kim, Ashoke Sen

D-instantons

In $\mathbb{R}^{1,3} \times \mathcal{M}$ D-instantons are D-branes wrapping a cycle in $\mathcal{M}$ at a point in $\mathbb{R}^{1,3}$

(Dirichlet b.c.s in all $\mathbb{R}^{1,3}$ directions)

String Field Theory		Quantum Field Theory
D-branes	~	solitons
D-instantons	~	instantons

D-instanton corrections to effective action

D-instantons give non-perturbative contributions $\propto e^{-c/g_s}$ to string amplitudes + effective action (Green + Gutperle '97) (Green '94, '77)

For example in 10-d IIB theory (in Einstein frame)

$$S = S_{sugra} + \frac{1}{\alpha'}\int d^{10}x \left[A\,\zeta(3)\,\tau_2^{3/2} + B\,\tau_2^{-1/2} + C\,e^{2\pi i \tau} \right] R^4 + \cdots$$

$\tau = \langle C^{(0)} \rangle + i e^{\langle\phi\rangle}$

A : tree-level higher-derivative correction — Gross + Witten '86

B : one-loop correction — Green + Schwarz '82

C : leading D-instanton correction — Green + Gutperle '97

A, B can be found exactly by string worldsheet calculation

C cannot

D-instanton effects

$\mathcal{C}$ involves divergent integrals over zero modes.

Regularising these requires going off-shell.

This cannot be done in worldsheet CFT: need SFT

Previously, dualities used to determine $\mathcal{C}$

1) IIB in 10d from S-duality + susy — Green, Gutperle '97

2) Type II on CY_3 from mirror symmetry + susy, S duality — Robles-Llana, Rocek '06; Saueressig, Theis, Vandoren; Alexandrov, Pioline '08; Saueressig, Vandoren; Alexandrov, Banerjee '15

D-instanton effects from SFT

Recent progress comes from first-principle computation of leading D-instanton effects with divergences understood from String Field theory

1) IIB string in 10d — Sen; Agmon, Balthazar, Cho, Rodriguez, Yin

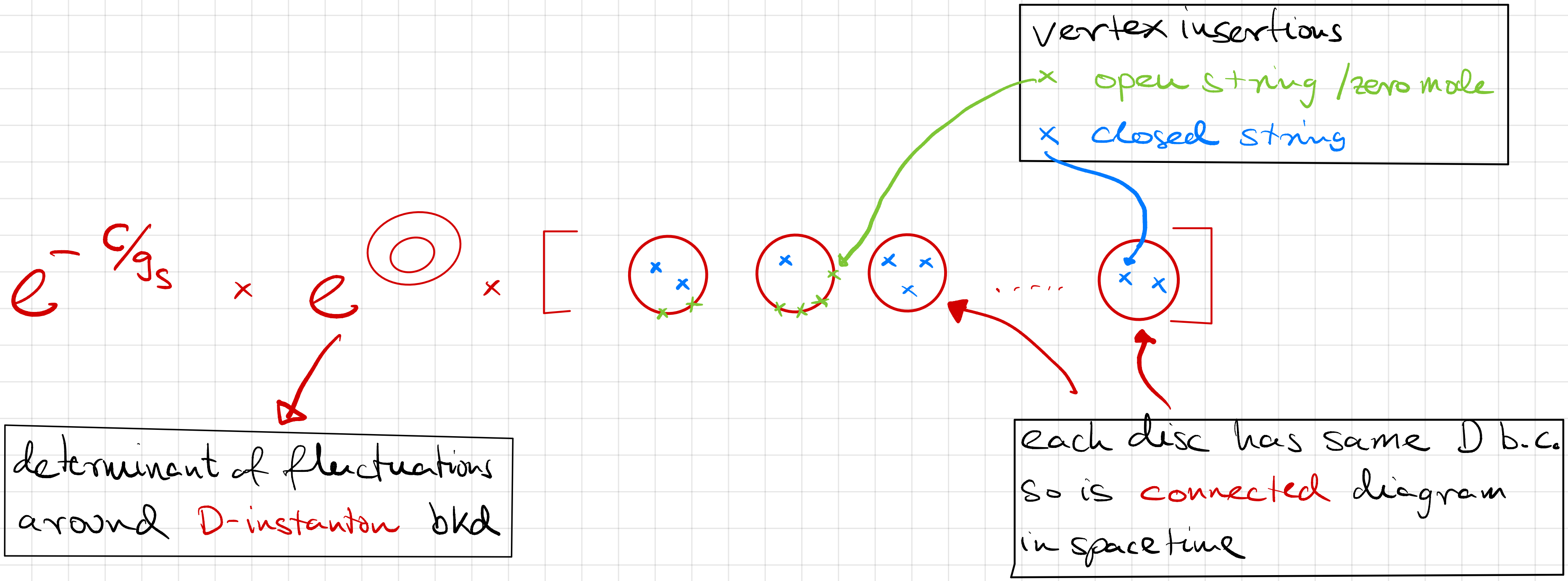
2) Type II on CY ($\mathcal{N}=2$ susy) — Alexandrov, Sen, BS

3) Type I etc on CY ($\mathcal{N}=1$ susy) — Sergei Alexandrov, Atakan Hilmi Firat, Manki Kim, Ashoke Sen, BS

Leading order D-instanton effects

String amplitudes $\propto g_s^{-\chi}$ (χ Euler # of worldsheet)

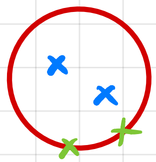
Leading (max χ) contribution in given D-instanton sector



At higher order many more wsheet topologies possible

Leading order D-instanton effects

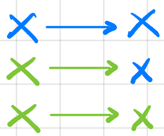
Calculation in two parts. For D2 wrapping $L_\gamma \hookrightarrow CY_3$ (Lag. 3 cycle)

1) $\prod_i \Big[$  $\Big]_i \equiv \prod_i a_i$

CFT$_2$ computation w/ D b.c.s

<u>do not</u> impose momentum conservation (yet)

<u>unphysical divergences</u> from vertex operators (in some pictures) close to each other

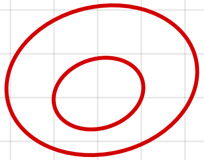


In D-<u>brane</u> disc diagrams these are total derivatives removed by analytic continuation

Not possible for D-instantons but modern SFT methods allow to separate picture-changing from vertex ops & avoid

Leading order D-instanton effects

Calculation in two parts

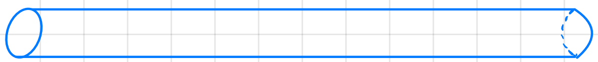
2) e  $\equiv e^{\mathcal{A}}$

$\mathcal{A}$ partition fn for spectrum of open strings w/ end(s) on D-instanton

$$\mathcal{A} = \int_0^\infty \frac{dt}{2t} \, \mathrm{Tr}\left[e^{-2\pi t L_0} (-1)^{F_s} \right]$$

possible divergences

$t \to 0$



$\ell = \frac{1}{t} \to \infty$

No closed string tachyons ✓

(massless closed strings no-force condition $\Rightarrow$ no divergence)

$t \to \infty$ IR divergence from massless open strings

Divergence & SFT

Sen showed how to regulate divergences in $\mathcal{A}$

with enough susy massive bosons/fermions cancel

Divergences come from zero modes $\mathcal{A}_{zm} \sim$ "$\frac{O_f \, O_{gh}}{O_b}$"

Zero modes can be regulated with $L_0 \to L_0 + h$ (separation along some direction)

Path integral for $\mathcal{A}^h_{z.m.}$ matched to $SFT_{z.m.}$ in Siegel gauge

Siegel gauge singular as $h \to 0$

instead work w/ gauge-inv path integral, integrate out ghosts

$$\mathcal{A}_{z.m} = N \int \prod_\mu dx^\mu \prod_\alpha d\chi_\alpha$$

N ⟶ Determined exactly comparing SFT & worldsheet amplitudes

D-instanton effective action

D-instantons correct hypermultiplet metric in $\mathcal{N}=2$ theories

$$S_{hm} = -\frac{1}{2}\int d^4x \; G_{ij}(\varphi)\,\partial_\mu \varphi^i \partial^\mu \varphi^j$$

sum D-instanton cycles weighted by D-inst action T_γ

$$G_{ij} = g_{ij} + \sum_\gamma e^{-T_\gamma}\left(h^{(\gamma)}_{ij} + \dots\right)$$

$$\varphi^i = \phi^i + \lambda^i$$

expansion in quantum fluctuations

Leading - g_s λ^4 term in $S_{h.m}$ when ∂_m, ∂_n act on T_γ

$T_\gamma \sim 1/g_s$

$$\sim -\frac{1}{4}\int d^4x \; e^{-T_\gamma}\,\partial_m T_\gamma\,\partial_n T_\gamma\, h^{(\gamma)}_{ij}\,\lambda^m \lambda^n \partial_\mu \lambda^i \partial_\nu \lambda^j$$

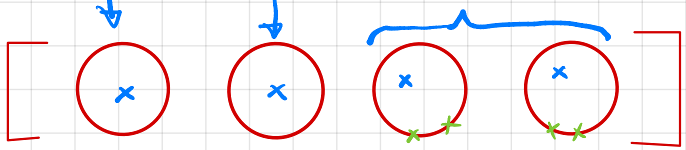
corresponding amplitude is

$$\mathcal{A} = (2\pi)^4\,\delta^{(4)}(\Sigma p_i)\left[\epsilon_1^m \epsilon_2^n \epsilon_3^i \epsilon_4^j\,\partial_m T_\gamma\,\partial_n T_\gamma\, h^{(\gamma)}_{ij}\, e^{-T_\gamma}\; p_3\cdot p_4 + \text{perm}\right]$$

D-instanton effective action

$$\mathcal{A} = (2\pi)^4\, \delta^{(4)}\!\left(\Sigma p_i\right)\, e^{-T_\gamma} \left[\underbrace{\epsilon_1^m \partial_m T_\gamma}\; \underbrace{\epsilon_2^n \partial_n T_\gamma}\; \underbrace{\epsilon_3^i \epsilon_4^j h_{ij}^{(\gamma)}\; p_3 \cdot p_4} \;+\; \text{perm} \right]$$

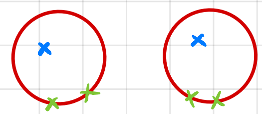
$$e^{-C/g_s} \times e^{\circledcirc} \times$$



correction to metric extracted from

(Do not impose momentum conservation)

$$p_3 \cdot p_4\, h_{ij}^{(\gamma)} \sim \underbrace{\int \Pi\, d\chi^{\delta}\, d\chi^{\dot{\delta}}}$$



Integral over fermionic zero modes from $e^{\circledcirc}$

Field redefinition

$$\varphi^m \longrightarrow \varphi^m + e^{-T_\gamma}\xi^m$$

$$d\varphi^m \longrightarrow d\varphi^m - e^{-T_\gamma}\xi^m\, dT_\gamma + \dots$$

Terms proportional to dT_γ cannot be compared

Disc diagrams

integration contour ABOVE real line

$0 \to z$ OPE gives $d\,T_\gamma$ corrections which we ignore

e.g.

$$a_B = i \pi \kappa T_2\, \chi^\alpha \hat{\chi}^\beta \int_{-\infty}^{\infty} dz \left\langle V_B(i)\; c\, e^{-\phi/2} S_\alpha(0)\, e^{-\phi/2} S_\beta(z) \right\rangle$$

$-1 \quad + \quad -\tfrac{1}{2} \quad + \quad -\tfrac{1}{2} \quad = -2$ picture

where

$$V_B = 2\, b_{\mu\nu}\, c \bar{c} \left(\partial X^\mu + i\, p\cdot\psi\, \psi^\mu \right) e^{-\bar{\phi}}\, \bar{\psi}^\nu$$

10-d spinors

$$\chi^\alpha = \eta \otimes x^\alpha \qquad \hat{\chi}^\alpha = \bar{\eta} \otimes x^{\dot{\alpha}}$$

4d spinors

$\eta, \bar{\eta}$ $\mathcal{N}=2$ covariantly constant spinors

Disc diagrams

$$a_\beta = b_{\mu\nu} \int_{-\infty}^{\infty} dz \left\langle c\bar{c}\left(\bar\partial X^\mu + i p\cdot\psi\psi^\mu\right) e^{-\bar\phi}\bar\psi^\nu e^{ipX}(i)\, c\, e^{-\phi/2} S_\alpha(0)\, e^{-\phi/2} S_\beta(z) \right\rangle$$

$\bar\partial X^\mu$ only possible contraction w/ $e^{ip\cdot X}$

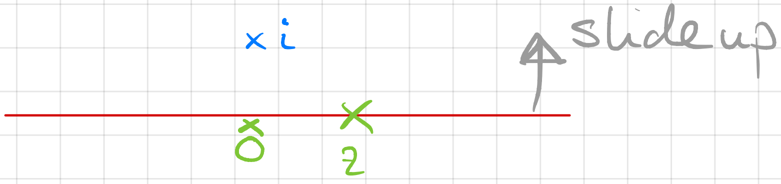
but $p^\mu b_{\mu\nu} = 0$ (mass-shell condition) so drop

Dirichlet bc for anti-holomorphic dim 0 operator

$$\bar{c}\, e^{-\bar\phi}\bar\psi^\nu(i) = -c\, e^{-\phi}\psi^\nu(-i)$$

$$a_\beta = -b_{\mu\nu} \int_{-\infty}^{\infty} dz \left\langle i c\, p\cdot\psi\psi^\mu(i)\, c e^{-\phi}\psi^\nu(-i)\, c\, e^{-\phi/2} S_\alpha(0)\, e^{-\phi/2} S_\beta(z) \right\rangle$$

Disc diagrams



$\otimes\!\!\times \sim \frac{1}{z-i}$ pick up pole

$\mathring{\delta}$

$$a_\beta = -b_{\mu\nu}\int_{-\infty}^{\infty} dz \left\langle i c\, \underbrace{p\cdot\psi\psi^\mu}_{\text{Lorentz generator}}(i)\; c e^{-\phi}\psi^\nu(-i)\; c\, e^{-\phi/2}S_\alpha(0)\, e^{-\phi/2}S_\beta(z)\right\rangle$$

$$= -b_{\mu\nu}\frac{i\pi}{2}\left\langle i c\, p_\delta \left(\Gamma^{\delta\mu}\right)_\beta{}^\gamma e^{-\phi/2}S_\gamma(i)\, c e^{-\phi}\psi^\nu(-i)\; c\, e^{-\phi/2}S_\alpha(0)\right\rangle$$

3pt fn can be easily computed. when dust settles

$$\boxed{a_\beta\, d\sigma = \frac{\pi\kappa^2}{4V} J_\gamma{}^R\, d\sigma}$$

$dB^{\mu\nu\rho} = -\frac{\kappa}{4V}\epsilon^{\mu\nu\rho\tau}\partial_\tau\sigma$

$J_\gamma^R = T_2 V_\gamma$ $\quad$ V: CY_3 volume

Disc diagrams

We can compute all other disc diagrams

e.g.

$$a_\Lambda d\xi^\Lambda + a^\Lambda d\tilde{\xi}_\Lambda = -2\pi^3 d\Theta_\gamma + \dots$$

other contributions cut for time here but known

where

$$\Theta_\gamma = \int_{L_\gamma} C = q_\Lambda \xi^\Lambda - p^\Lambda \tilde{\xi}_\Lambda$$

$A^\Lambda \cap B_\Sigma = \delta^\Lambda_\Sigma$

$\Lambda = 0, \dots h^{2,1}(CY_3)$ labels symplectic basis of A^Λ, B_Λ 3-cycles

$\xi^\Lambda, \tilde{\xi}_\Lambda$: RR fields from RR 3 forms wrapping symplectic 3-cycles

The D2 instanton wraps the 3 cycle

$$L_\gamma = q_\Lambda A^\Lambda - p^\Lambda B_\Lambda$$

Comparison with $\mathcal{N}=2$ predictions

$$a_\sigma d\sigma + a_\Lambda d\zeta^\Lambda + a^\Lambda d\tilde{\zeta}_\Lambda + i\frac{dr}{2r}$$

← dilaton + complex structure

$$= \pi^2 T_\gamma^R \left[\frac{\varkappa^2}{4\pi V} d\sigma - 2\pi^3 d\Theta_\gamma + \dots \right]$$

Since we are dropping dT_γ terms we can use

$$0 = dT_\gamma = T_\gamma^R \left(\frac{1}{2}\frac{dr}{r} + \dots \right) + 2\pi i \, d\Theta_\gamma$$

to eliminate $d\Theta_\gamma$ to find metric

$$ds^2_{inst} = \sum_\gamma 2\pi e^\phi g_0 \Omega_\gamma \left(\sum_{k=1}^\infty k^{-1/2} e^{-kT_\gamma} \right) \left(\sum a_m dq^m \right)^2 + O(dT_\gamma)$$

which matches perfectly duality-based predictions.

D-instanton superpotentials in $\mathcal{N}=1$ theories

We consider an orientifold of Type IIB on CY_3

$$\begin{pmatrix} X^\mu, \psi^\mu \\ \mu \in \mathbb{R}^{1,3} \end{pmatrix} \oplus (b, c, \beta, \gamma) \oplus \begin{pmatrix} \mathcal{N}=(2,2) \text{ SCFT} \\ \text{w/ spectral flow} \end{pmatrix}$$

S. Odake '89

projected by

$$\Omega : \quad \Phi(z,\bar{z}) \longrightarrow (-1)^h \, \bar{\Phi}(-\bar{z}, -z)$$

to get $\mathcal{N}=1$ susy. Backgrounds with no flux

D. b.c.s on $\mathbb{R}^{1,3}$ bc in $\mathcal{N}=(2,2)$ preserves $J + \bar{J}$ current

Today we consider $O(1)$ instantons only. These have only universal zero modes:

4 bosonic z.m.s $\quad x^\mu$

2 fermionic z.ms $\quad \chi_\alpha$

D-instanton superpotentials in $\mathcal{N}=1$ theories

In unoriented theory

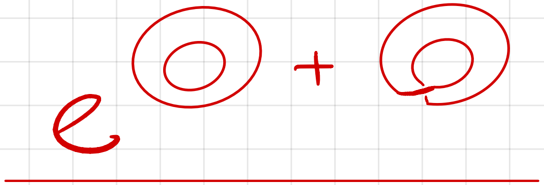
Möbius strip

$\int \frac{dt}{2t} \mathrm{Tr}\left(\frac{\Omega}{2} \ldots\right)$

one boundary of annulus can be on tadpole-canceling brane

As before $t \to 0$ not divergent because no tachyons and tadpoles cancel

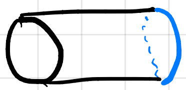
For $t \to \infty$ possible IR problems. Since susy is low we do not have equal # of massless bosons & fermions

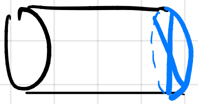


we split $\int_0^\infty \frac{dt}{2t} = \int_0^\delta \frac{dt}{2t} + \int_\delta^\infty \frac{dt}{2t}$

"closed" "open"

"closed string part" $t \to 0$ ie $\ell \to \infty$

$$\int_0^\delta \frac{dt}{2t} Z_A(t) =$$  $+$  $$= \langle B| \int_{1/\delta}^\infty d\ell\, e^{-\pi\ell(L_0+\bar{L}_0)} \left(|B\rangle + 2|B\rangle\right)$$

$$+ \int_0^\delta \frac{dt}{2t} Z_\mu(t) =$$  $$= 2\langle B| \int_{1/4\delta}^\infty d\ell\, e^{-\pi\ell(L_0+\bar{L}_0)} |C\rangle$$

divergences come from $\ell \to \infty$ & cancel between Z_A & Z_μ

& cancel as in $N=2$ for Z_A

Manifestly finite splitting $\epsilon \to 0$ regulator

$$\langle B| \int_{1/\delta}^{1/\epsilon} d\ell \, e^{-\pi \ell (L_0 + \bar{L}_0)} |B\rangle \quad + 2 \langle B| \int_{1/\delta}^{1/\epsilon} d\ell \, e^{-\pi \ell (L_0 + \bar{L}_0)} \left(|B\rangle + |C\rangle \right)$$

$$+ 2 \langle B| \int_{1/4\delta}^{1/\delta} d\ell \, e^{-\pi \ell (L_0 + \bar{L}_0)} |C\rangle$$

In open-channel variables

$$\int_0^\delta \frac{dt}{2t} \left(Z_A + Z_\mu \right) \rightsquigarrow \lim_{\epsilon \to 0} \int_\epsilon^\delta \frac{dt}{2t} Z_A(t) + \int_{\epsilon/4}^\delta \frac{dt}{2t} Z_M(t)$$

"open string part"

$$\int_\delta^\infty \frac{dt}{2t}$$

As before regulate by separation $L_0 \to L_0 + h$

$$Z_A + Z_M = \mathrm{Tr}(\ldots) = \sum_{r \in NS} s_r e^{-2\pi t h_r} + \frac{1}{2} \sum_{a \in R} \tilde{s}_a e^{-2\pi t h_{\tilde{a}}}$$

$s_r, s_a = \pm 1$
Grass mannparts

with $h_r, h_{\tilde{a}} > 0$

Split all modes into 2 groups

$$\sum \longrightarrow \sum{}' + \sum{}''$$

$\sum'$ contains all modes which become massless as $h \to 0$ and a finite number of other modes such that

$$\sum_{r \in NS}{}' s_r + \frac{1}{2} \sum_{a \in R}{}' \tilde{s}_a = 0$$

$\sum''$ all other modes

$$\sum_{r\in NS}{}' s_r + \frac{1}{2}\sum_{a\in R}{}' \tilde{s}_a = 0$$ allows us to perform integral

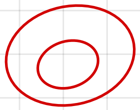
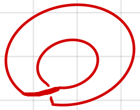
$$\int_\delta^\infty \frac{dt}{2t}\, Z_A' + Z_M' = -\frac{1}{2}\sum_{r\in NS}{}' s_r \log h_r - \frac{1}{4}\sum_{a\in R}{}' \tilde{s}_a \log \tilde{h}_a$$

So $$e^{\int_\delta^\infty \frac{dt}{2t} Z_A + Z_M} = \underbrace{e^{\int_\delta^\infty \frac{dt}{2t} Z_A'' + Z_M''}}_{\text{finite}} \underbrace{\prod_{r\in NS}{}' h_r^{-s_r/2} \prod_{a\in R}{}' h_a^{-\tilde{s}_a/4}}_{\text{re-interpret as path integral of quadratic part of gauge-fixed SFT}}$$

$$= e^{\int_\delta^\infty \frac{dt}{2t} Z_A'' + Z_M''}\, N \int \prod_r{}' d\xi_r \prod_a{}' d\xi_a\, e^{S_{NS} + S_R}$$

The last expression makes sense as we send $h \to 0$

Regulated measure

$e^{\,}$  $+$ 

$$= \frac{1}{2}\lim_{\epsilon\to 0} \exp\left[\int_{\epsilon}^{\delta}\frac{dt}{2t}\, Z_A + \int_{\epsilon/4}^{\delta}\frac{2t}{2t}\, Z_\mu\right] \exp\left[\int_{\delta}^{\infty}\frac{dt}{2t}\, Z_A'' + Z_\mu''\right]$$

$$\times N \int \prod{}' d\xi_r \prod{}' d\xi_a \, e^{S_{NS}+S_R}$$

$$= \frac{1}{2}\lim_{\epsilon\to 0} \exp\left[\int_{\epsilon}^{\delta}\frac{dt}{2t}\, Z_A + \int_{\epsilon/4}^{\delta}\frac{2t}{2t}\, Z_\mu\right]$$

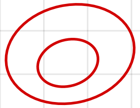
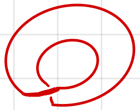
$$\times \exp\left[\int_{\delta}^{\infty}\frac{dt}{2t}\, Z_A + Z_\mu - 3 - \sum_{i=1}^{n} w_i e^{-2\pi y_i t}\right]$$

$$\times \int \prod_{\mu=0}^{3}\frac{d\xi_\mu}{\sqrt{2\pi}} \prod_{a=1}^{2} d\chi_a \prod y_i^{-w_i/2}$$

with a bit of work one can check that this expression is independent of y_i as $\delta\to 0$ so can set $y_i = h$

Regulated measure

The final expression is

$e^{\left(\right.}$  $+$  $^{\left.\right)}$ $$= \frac{1}{2}\int \prod_{\mu=0}^{3} \frac{d\xi_\mu}{\sqrt{2\pi}} \prod_{a=1}^{2} dx_a \; K_0$$

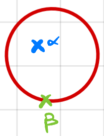
where

$$K_0 = h^{3/2} \lim_{\epsilon\to 0}\lim_{\epsilon'\to 0} \exp\left[\int_{\epsilon}^{1/\epsilon'} \frac{dt}{2t} Z_A + \int_{\epsilon/4}^{1/\epsilon'} \frac{dt}{2t} Z_M + \int_{0}^{1/\epsilon} \frac{dt}{2t}\left(-3 + e^{-2\pi h t}\right)\right]$$

Superpotential

Simplest correction comes to mass term for fermionic superpartners of Kähler moduli

Here disc diagrams are simpler to compute



$$= -2\pi i \, \epsilon_{\alpha\beta} \frac{\partial T_\gamma}{\partial \phi_m}(\phi_0)$$

ϕ_m superpartner of X acts as supercurrent

ϕ_0 bkd values of moduli

Effective action that reproduces this amplitude is (with precise normalization)

$$-\frac{\kappa_4^3}{(2\pi)^4 g_0^2} \int d^4x \sqrt{-G}\, e^{-T_\gamma(\phi_0)} \epsilon_{\alpha\beta} \zeta_m^\alpha \zeta_n^\beta K_0 \frac{\partial T_\gamma}{\partial \phi_m} \frac{\partial T_\gamma}{\partial \phi_n}$$

Superpotential

This term $\sim -\frac{1}{2} e^{K/2} \nabla_I \nabla_J W \, \epsilon_{\alpha\beta} \Psi^{I\alpha} \Psi^{J\beta}$

$$|W_\gamma| e^{K/2} = \frac{\kappa_4^3}{16\pi^2} \mathrm{Re}\left(T_\gamma(\phi_0)\right) K_0 \left| e^{-T_\gamma(\phi_0)} \right|$$

lhs unambiguous under Kähler tr.
$K \to K + f + \bar{f}$
$W_\gamma \to e^{-f} W_\gamma$

we have shown that this superpotential is holomorphic

This is done by relating instanton one-loop amplitudes to threshold corrections in spacefilling branes.

Viewing the D-instanton as an gauge instanton in spacefilling brane its action is controled by gauge coupling of s.f.b.

So 1-loop corrections to D-instanton action (like in K_0) related to 1-loop corrections to gauge coupling of s.f.b.

Superpotential

These relations had been shown in toroidal orientifolds

Akerblom, Blumenhagen, Lüst, Plauschinn, Schmidt-Sommerfeld 07
Abel, Goodsell 07

We derive them in complete generality for any CY_3 orientifold using $\mathcal{N}=(2,2)$ formulation

This allows us to find the Kaplunovsky–Louis threshold formula with which the superpotential is

$$W_\gamma = 2^{-11/2}\,\pi^{-7/2}\,e^{-8\pi^2 f^1(\varphi)}\,e^{-T_\gamma(\varphi+v(\varphi,\bar{\varphi}))}$$

$f^1(\varphi)$ is some holomorphic fn of moduli

$\vec{\varphi} + \vec{v}(\varphi,\varphi)$ are quantum-corrected holomorphic coordinates

Outlook

- Computing D-instanton corrections from 1^{st}-principle SFT with <u>exact</u> measure normalization with $\mathcal{N}=2,1$ susy <u>fully under control</u>
- Consider explicit solvable examples like toroidal orbifolds or Gepner models & perturb
- Analyse cases with non-universal zero modes
- higher order corrections Xi et al.