Creases, corners and caustics: non-smooth structures on horizons

Harvey Reall

DAMTP, Cambridge University

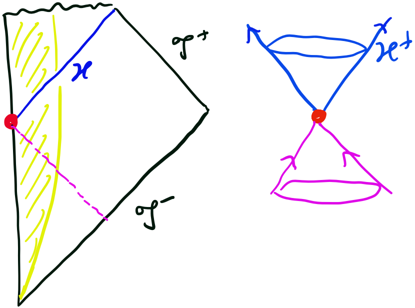
Maxime Gadioux and HSR, 2303.15512

Introduction

Every point of an event horizon $\mathcal{H}$ belongs to a null geodesic that lies within $\mathcal{H}$. These geodesics are the *generators* of $\mathcal{H}$.

A generator cannot have a future endpoint, i.e., it cannot leave $\mathcal{H}$ to the future.

Generators can have *past* endpoints:



Horizon non-smoothness

We assume that spacetime is smooth.

Theorem: $\mathcal{H}$ is an achronal continuous hypersurface.
(Achronal: no two points of $\mathcal{H}$ are timelike separated.)

$\mathcal{H}$ is *not smooth* except in very special cases e.g. a time-independent black hole.

What is the nature of the non-smoothness of $\mathcal{H}$?

There exist examples of spacetimes for which $\mathcal{H}$ is non-differentiable on a dense set (Chrusciel & Galloway 96)

Theorem (Beem & Krolak 97):

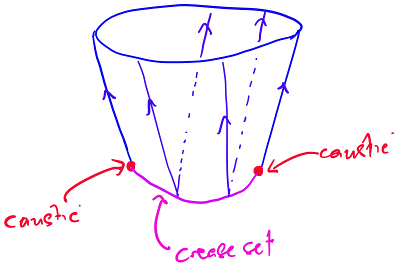
- $\mathcal{H}$ is differentiable at p iff p lies on *exactly one* generator
- A point lying on more than one generator is an endpoint (converse untrue)

Let $\mathcal{H}_{\rm end}$ be the set of past endpoints of horizon generators.

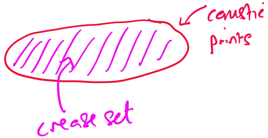
In explicit examples of gravitational collapse or black hole mergers, $\mathcal{H}_{\rm end}$ consists of a 2d spacelike *crease set* where *pairs* of generators enter $\mathcal{H}$, together with its boundary, which is a line of *caustic points* (where "infinitesimally nearby generators intersect")

(Hughes *et al* 94, Shapiro *et al* 95, Lehner *et al* 99, Husa & Winicour '99, Hamerly & Chen 10, Cohen *et al* 11, Emparan & Martinez 16, Bohn *et al* 16, Emparan *et al* 17)

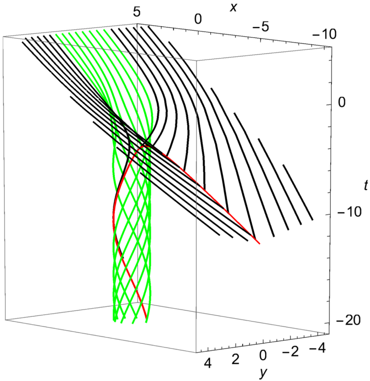
In 2+1 dimensions:

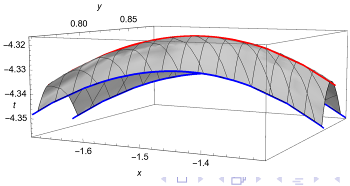


Asymmetric gravitational collapse in 3+1 dimensions:



Non-axisymmetric black hole merger (Emparan *et al* 17):





What features of these spacetimes lead to this simple structure for $\mathcal{H}_{\rm end}$?

What other structures are possible?

Assumptions

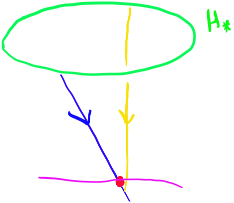
- Spacetime is *globally hyperbolic*
- $\mathcal{H}$ is *smooth at late time*: there exists a Cauchy surface Σ to the future of $\mathcal{H}_{\rm end}$ such that $H_\star \equiv \Sigma \cap \mathcal{H}$ is smooth

(No assumptions about equations of motion.)

We show that $\mathcal{H}_{\rm end}$ is the past *null cut locus* of $H_\star$.

Null cut locus

A null geodesic emitted orthogonally to $H_\star$ cannot be deformed to a timelike curve from $H_\star$ *locally*. A *null cut point* is the first point along a such a null geodesic beyond which it can be deformed into a timelike curve. The *null cut locus* of $H_\star$ is the set of all null cut points.



In Riemannian geometry a cut locus can be very complicated (e.g. fractal). But it can be decomposed into parts with simpler structure (Itoh & Tanaka 1998). We obtained a Lorentzian analogue of this decomposition.

A point in a null cut locus lying on exactly one generator must be a caustic point (Beem & Ehrlich 81, Kemp 84, Kupeli 85). So we can classify points of $\mathcal{H}_{\rm end}$ as follows:

- caustic points
- non-caustic points
  - "normal crease points": lie on exactly 2 generators
  - points on ≥ 3 generators

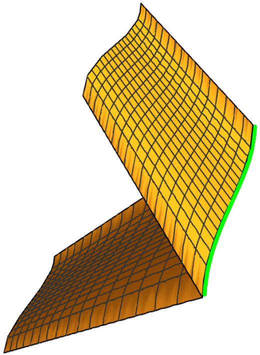
We prove:

- (a) Normal crease points form a 2d spacelike *crease submanifold*
- (b) All other points form a set of (Hausdorff) dimension ≤ 1

Creases

Normal crease points form a 2d spacelike *crease submanifold*.

If a Cauchy surface Σ intersects this submanifold then the horizon cross-section $\Sigma \cap \mathcal{H}$ possesses a crease:



Perestroikas

Let τ be a time function and Σ_τ denote a Cauchy surface of constant τ

$\Sigma_\tau \cap \mathcal{H}$ is the "horizon at time τ". This will have some arrangement of creases, caustics etc

As τ varies, this arrangement may undergo a qualitative change at a critical value of τ. We call this a *perestroika* (restructuring).

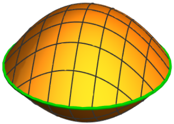
A crease perestroika occurs at a time τ for which Σ_τ is tangent to the crease submanifold.

Near the point of tangency, $\mathcal{H}$ is (part of) the union of two intersecting null hypersurfaces. By introducing Riemannian normal coordinates around this point we can determine the exact local behaviour of $\mathcal{H}$.

There are three qualitatively different possibilities. Shift τ so that perestroika occurs at $\tau = 0$.

Flying saucer

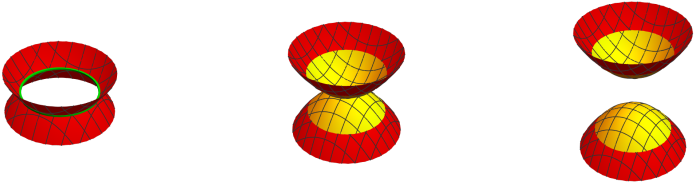
This perestroika describes the nucleation of a component of $\mathcal{H}$ in generic gravitational collapse



Length of crease and angle at crease scale as $\sqrt{\tau}$, area scales as τ

Collapse of hole in horizon

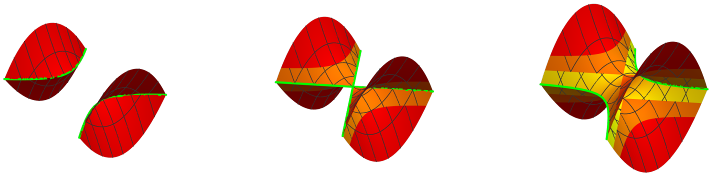
In examples of gravitational collapse or a black hole merger, some choices of time function give a brief period where horizon has toroidal topology (Hughes *et al* 94, Siino 97, Cohen *et al* 11, Bohn *et al* 16). The "hole in the torus" collapses superluminally. The collapse is described by a perestroika:



Length of crease and angle at crease scale as $\sqrt{-\tau}$.

Black hole merger

This perestroika describes the merger of two (locally) disconnected sections of horizon e.g. two merging black holes.



$\tau < 0$ $\tau = 0$ $\tau > 0$

Angle at creases scales as $\sqrt{|\tau|}$

Crease contribution to black hole entropy

Old idea: some/all of black hole entropy is entanglement entropy of quantum fields across horizon (Bombelli *et al* 86, Srednicki 93, Susskind & Uglum 94). Flat space entanglement entropy exhibits novel features in the presence of a crease (Casini & Huerta 06, Hirata & Takayanagi 06, Klebanov *et al* 12, Myers & Singh 12)

Suggests that a crease might contribute to black hole entropy as

$$\frac{1}{\sqrt{G\hbar}} \int_{\text{crease}} F(\Omega) dl$$

where Ω is angle at crease and $F < 0$ with $F \propto 1/\Omega$ as $\Omega \to 0$. Subleading compared to Bekenstein-Hawking entropy $A/4G\hbar$

Consider "hole in the horizon" perestroika: this term remains finite and non-zero as $\tau \to 0-$, so discontinuous at $\tau = 0$. Consistent with second law as discontinuity positive

Genericity/stability

Which features of $\mathcal{H}_{\mathrm{end}}$ are stable under small perturbations?

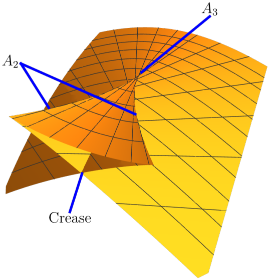
e.g. spherically symmetric collapse: $\mathcal{H}_{\mathrm{end}}$ is a single (caustic) point. This structure for $\mathcal{H}_{\mathrm{end}}$ is unstable: if perturbed then a crease submanifold is present.

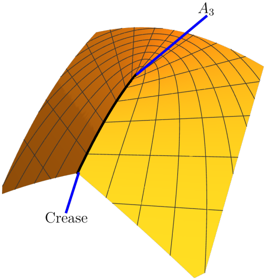
Siino & Koike 04: classification of points of $\mathcal{H}_{\mathrm{end}}$ assuming a particular notion of genericity

Classification reveals that generic caustic points are "of type A_3"

Generic caustic point: A_3

A_3 caustic points form spacelike lines. A horizon cross-section generically has isolated A_3 caustic points. If we extend generators beyond their past endpoints we obtain the swallowtail:

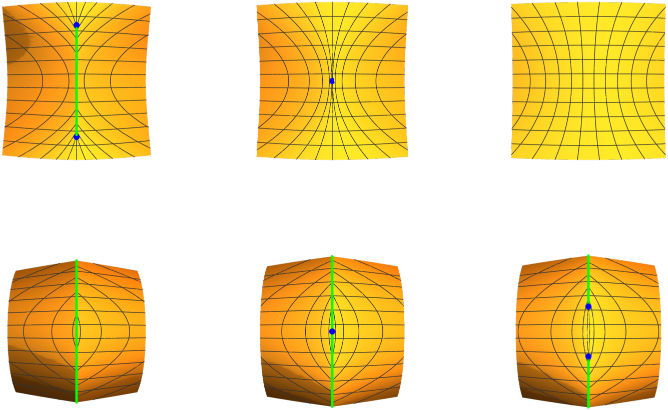




Why can't an A_2 caustic occur on $\mathcal{H}$? Would violate achronality!

A_3 perestroikas

Occur when Σ_τ is tangent to A_3 line.



$\tau < 0$ $\tau = 0$ $\tau > 0$

Gauss-Bonnet term in entropy

A "Gauss-Bonnet" term in gravitational action is topological in 4d but contributes to black hole entropy (Jacobson & Myers 93, Iyer & Wald 94)

$$S_{\rm GB} = \gamma \int_H d^2x \sqrt{\mu} R[\mu]$$

On smooth horizon $S_{\rm GB} = 4\pi\gamma\chi$ where χ is Euler number of H.

For non-smooth horizon, "regulate" $S_{\rm GB}$, defining via a limit of smooth surfaces to obtain same result. $S_{\rm GB}$ is discontinuous in black hole formation or merger, so only $\gamma = 0$ is consistent with 2nd law (Sarkar & Wall 11)

But: does $S_{\rm GB}$ actually need regulating? No: integral is well-defined for creases, corners and A_3 caustics. No longer topological, continuous in black hole formation/merger.

Still find $\gamma = 0$ if no "higher order" terms in entropy but γ unconstrained if such (EFT) terms are present.

Things I didn't have time to tell you about

Corners, corner perestroika

Behaviour of some higher derivative terms in entropy

Bousso entropy bound

I discussed some of these in the Extreme Universe Colloquium (YouTube)

Maxime Gadioux and HSR, 2303.15512