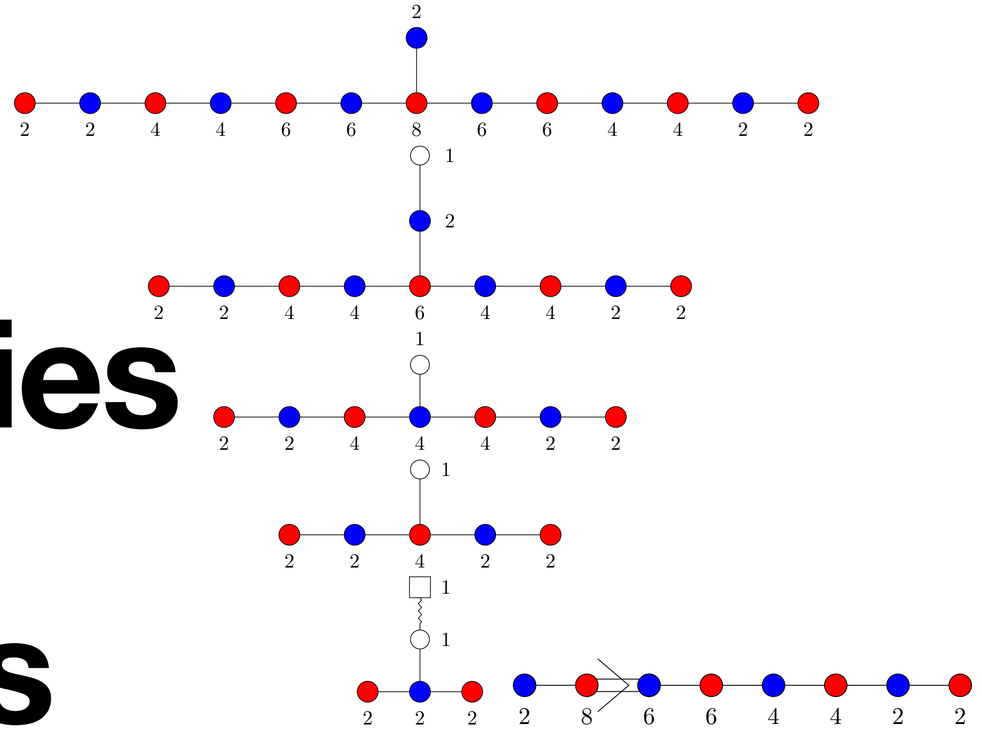
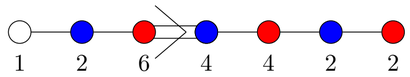
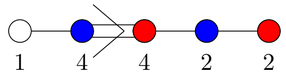
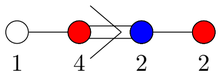
Symplectic singularities Phase diagrams and Magnetic Quivers

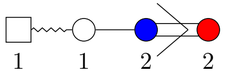
Status Update on theories with 8 supercharges 2023











Amihay Hanany

Over the past 10 years we faced an impressive progress in the understanding of theories with 8 supercharges

Hypermultiplet moduli space

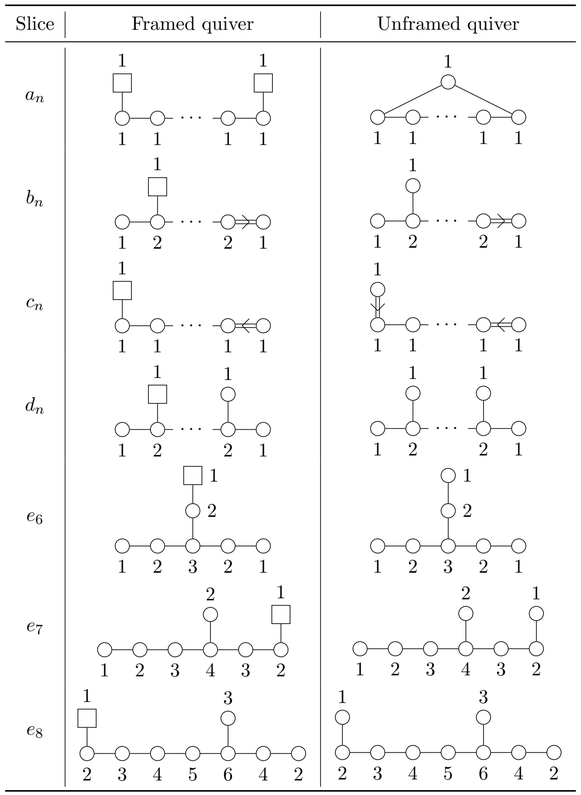
Theoretical tools

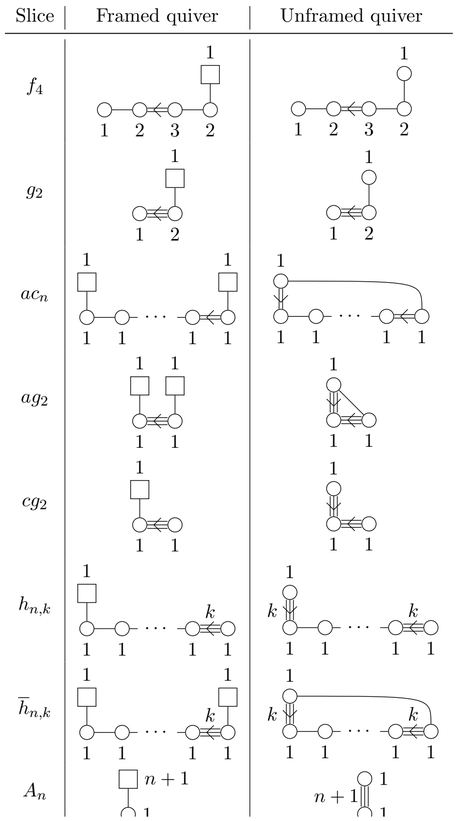
- Applied to a host of gauge theories — mostly quivers
- May be living on brane systems
- 3d $\mathcal{N} = 4$ Coulomb branches — Monopole Formula
- Phase (Hasse) Diagrams — Quiver subtraction vs Higgs mechanism
- Magnetic Quivers — not possible without branes systems!

Status Report

Symplectic singularities

- This talk reviews the current understanding we have of moduli spaces for theories with 8 supercharges
- The focus is on
- Higgs branches in 3, 4, 5, 6 dimensions
- Coulomb branch in 3 dimensions

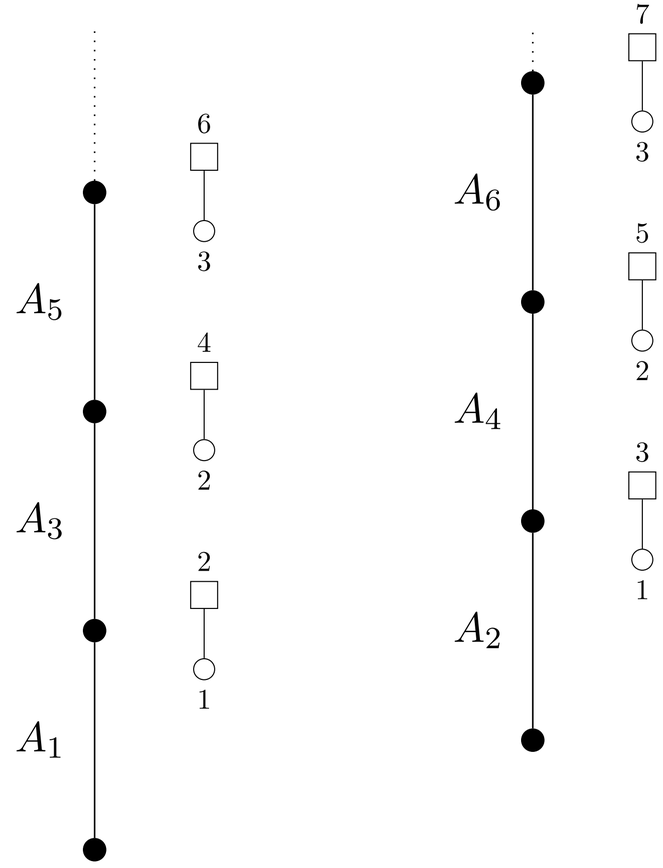




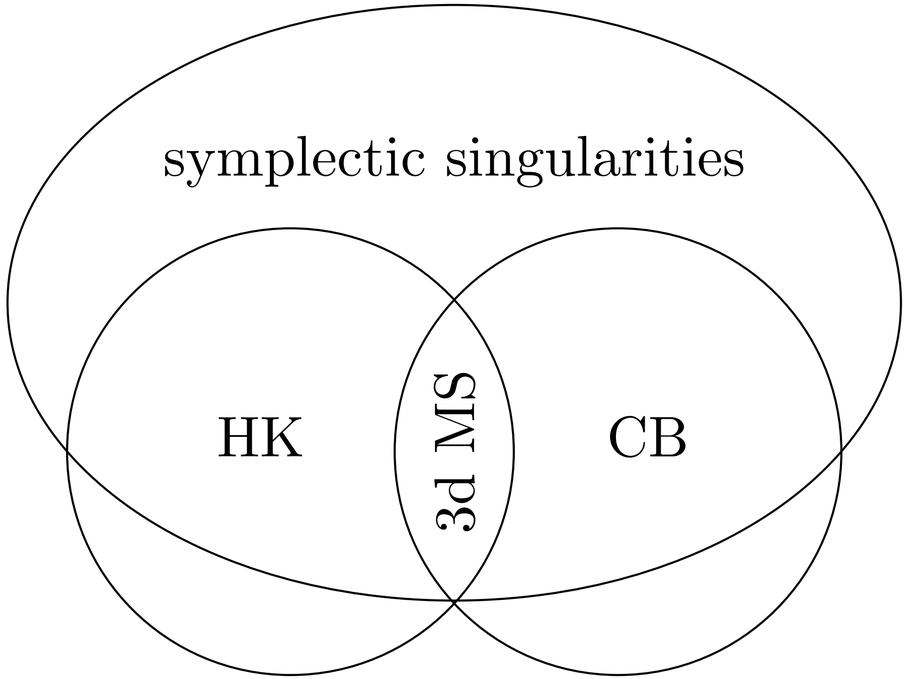
Symplectic Singularities

alternative names

- HyperKähler cones
- HyperKähler singularities
- holomorphic symplectic varieties
- The last name really represents the approach we take:
- supersymmetry; extended



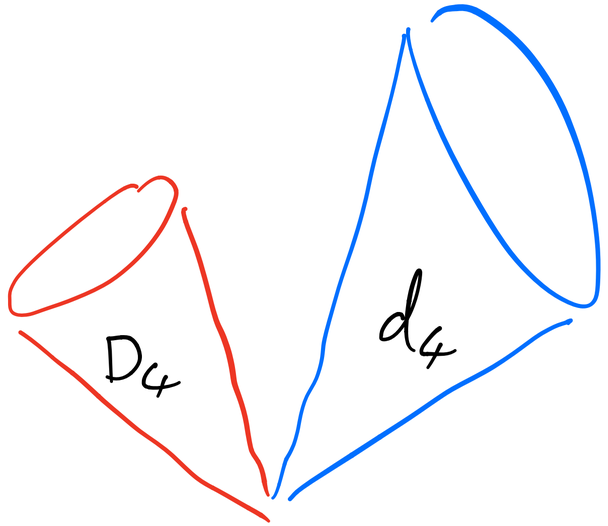
Symplectic singularities & Physics

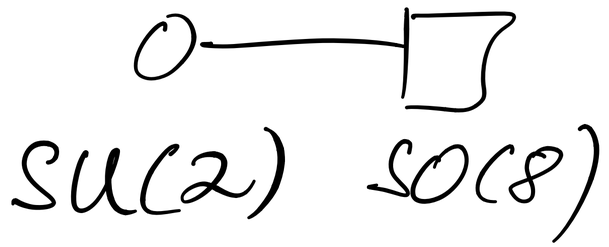


Gauge theories

symplectic singularities

- These moduli spaces are relevant to gauge theories as they show as Higgs branches
- 3d Coulomb branches
- hence physically relevant

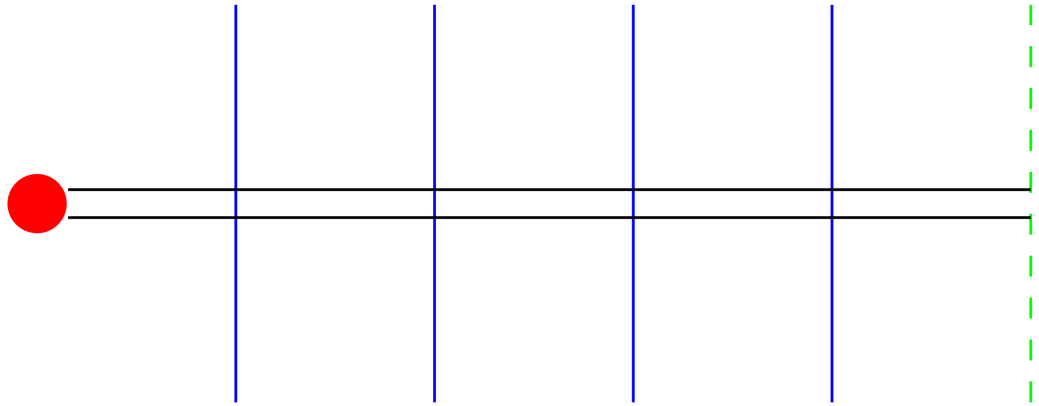




Brane systems

symplectic singularities

- These singularities naturally arise on brane systems
- most notably through the Affine Grassmanian
- hence physically relevant



String theory constructions

symplectic singularities

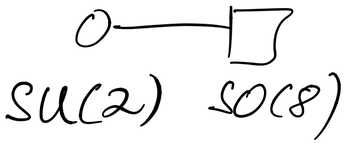
- Other constructions in string theory with 8 supercharges like
- F theory
- M theory on singular spaces
- similar
- While these share the low energy theory with the brane systems, their Higgs branches remains a challenge for future work

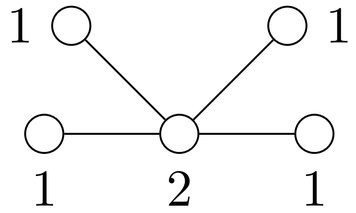
Lagrangian theories

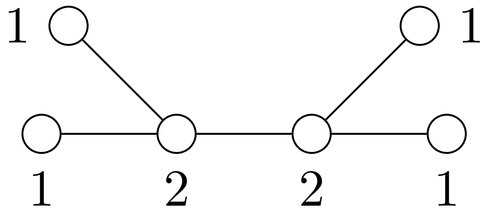
HKLR 1987

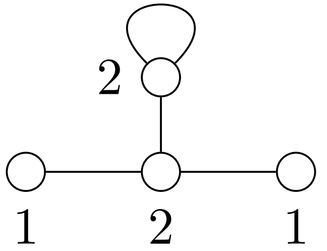
- Higgs branches HK quotient — classical physics, good at weak coupling
- It is important to note as opposed to (wrong) standard lore:
- Higgs branches are not classically exact

Higgs branches are NOT classically exact





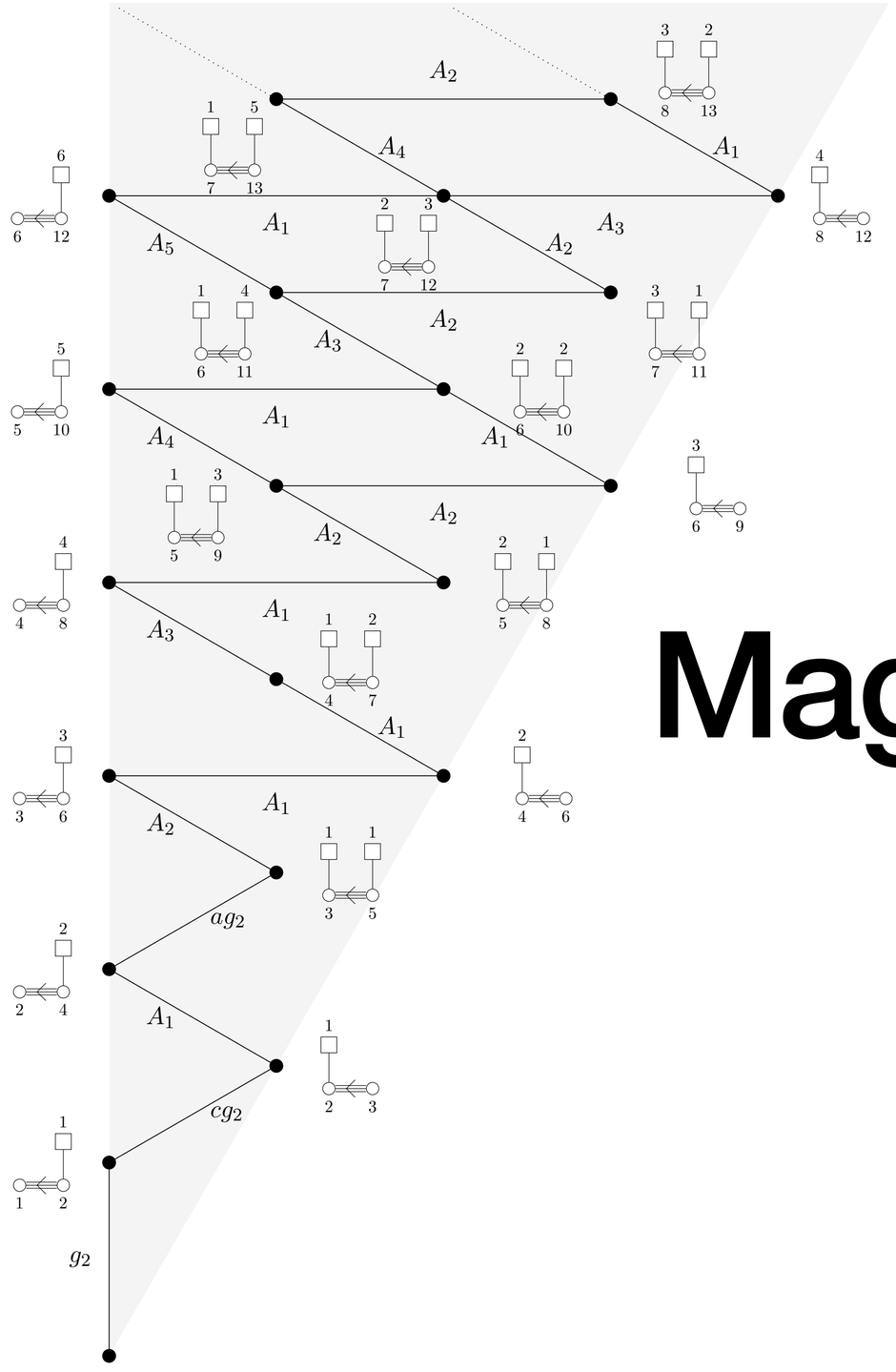




Higgs Branch

changes at special points with extra massless states

- stays fixed along the Coulomb branch
- stays fixed along continuous variation of gauge coupling
- until we get extra massless states
- At such points they become very different from the classical result
- So how do evaluate the new Higgs branch?



Magnetic Quivers

Lagrangian theories

Coulomb branch

- Lagrangians are very important for construction of 3d Coulomb branches
- Many quantum corrections
- But under control
- as space of dressed monopole operators
- The monopole formula (CHZ 2013)

Characterization of moduli spaces

physical quantities — ordered by ease of computation

- Dimension (quaternionic)
- Global symmetry
- Phase (Hasse) diagram
- Representation content of the chiral ring (Hilbert Series)
- Chiral ring - generators and relations

The Monopole Formula

CHZ 2013

- Computes the Hilbert Series for the Coulomb branch of a 3d gauge theory with 8 supercharges
- opened a way to evaluate Coulomb branch on an industrial scale
- Convinced mathematicians to study such constructions
- Opened the way to evaluate Higgs branches away from the perturbative point

Magnetic Quiver

Brane system

- Computes the Higgs branch of some theory in 3,4,5,6 dimensions
- especially when the Higgs branch is not a HK quotient
- Extends the notion of 3d mirror symmetry
- Gives an important window to physics away from the weakly coupled point
- This is a major change in our understanding of such theories

Brane system

Magnetic Quivers

- Given a theory with some interesting physics at strong coupling
- Embed using brane systems
- Compute the magnetic quiver
- Extract the new physics from the quiver
- Using the monopole formula or the phase diagram

Magnetic Quivers

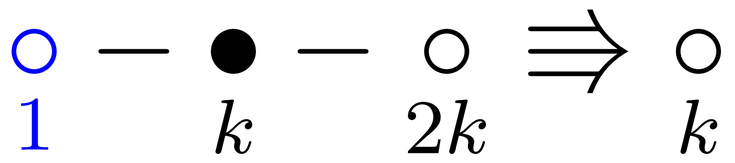
physics at strong coupling

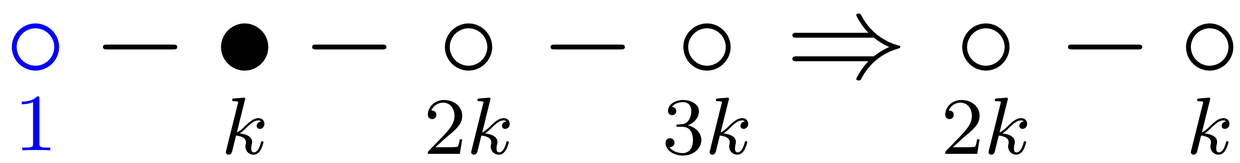
- 6d — physics of tensionless strings
- 5d — physics of massless gauge instantons
- 4d — physics of Argyres Douglas points
- 3d and above — physics of moduli spaces with multiple cones
- moduli space of instantons

Moduli space of instantons

Coulomb branch

- The ADHM quiver is a HK quotient for a classical gauge group
- Problem — no such quotient for exceptional gauge groups
- The affine Dynkin diagram is a Coulomb branch construction for any gauge group
- exceptional
- non simply laced



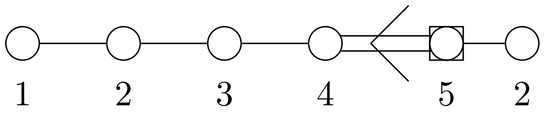


Non simply laced quivers

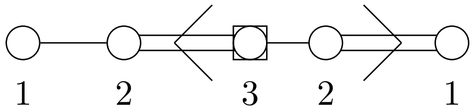
No known Lagrangian or path integral

- A small modification of the monopole formula
- A whole new set of moduli spaces
- A window to exotic moduli spaces
- like rank 1 4d theories

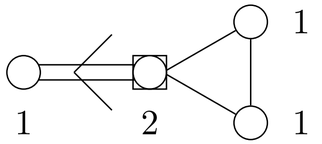
C_5



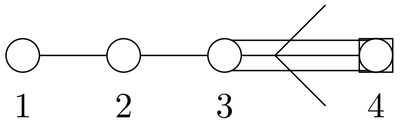
$C_3 \times A_1$



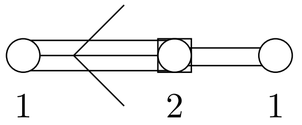
$C_2 \times U_1$



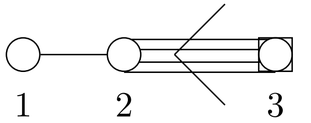
A_3



$A_1 \times U_1$



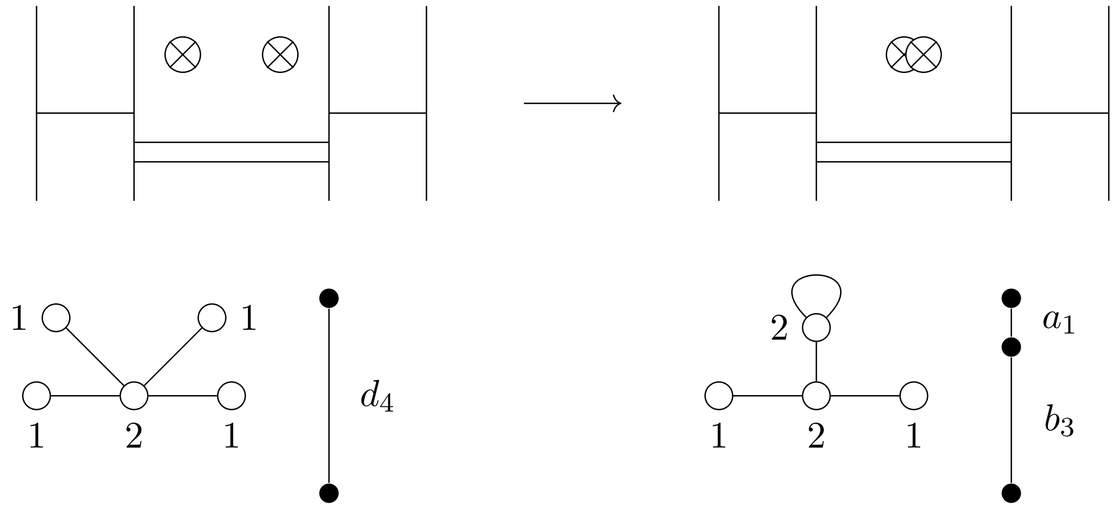
A_2



Magnetic quiver in 6 dimensions

Physics of tensionless strings

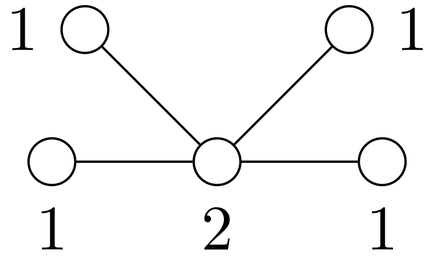
- BPS strings become tensionless at special points in the moduli space
- like the origin of the Tensor Branch
- What appears to be infinitely many massless states gets replaced by an effective description using a new Higgs branch, given by a magnetic quiver

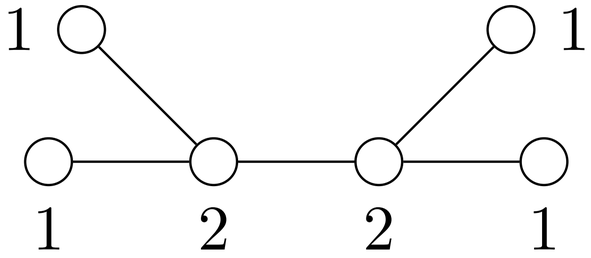


Magnetic Quivers in 5 dimensions

massless gauge instantons

- massless gauge instantons are known as giving new physics in 5 dimensions
- lead to increase in Higgs branch dimension
- lead to global symmetry enhancement
- dramatically change the chiral ring of the theory
- all this is captured precisely by magnetic quivers





Magnetic Quivers in 4 dimensions

New physics at Argyres Douglas (AD) points

- It is known that mutually non local states become massless at AD points
- Hence the need for new degrees of freedom
- Nicely captured by magnetic quivers

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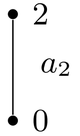
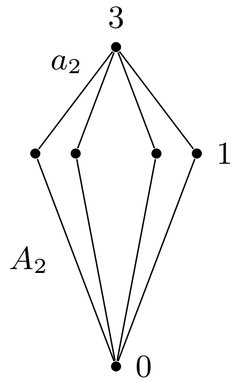
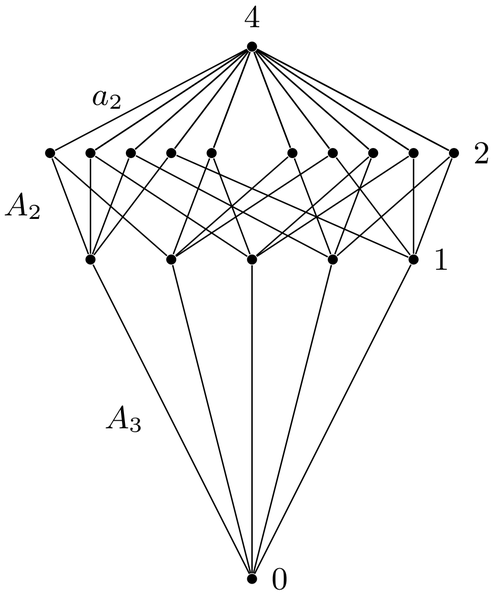
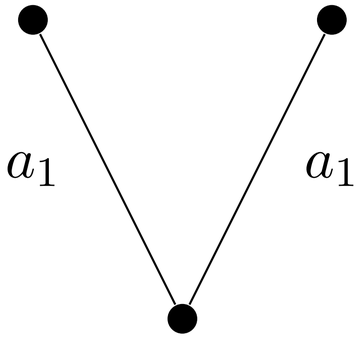
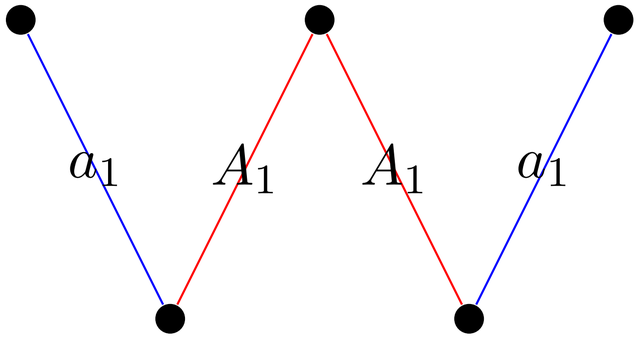
$\mathbb{H}$			

Magnetic Quivers in dimensions higher than 2

Moduli spaces which are unions of multiple cones

- Some gauge theories have Higgs branches which are unions of 2 cones
- Of particular note are the family of very even nilpotent orbits
- This can be generalized to include multiple cones
- There are families of quivers with multiple cones
- Their moduli spaces are given by Magnetic Quivers



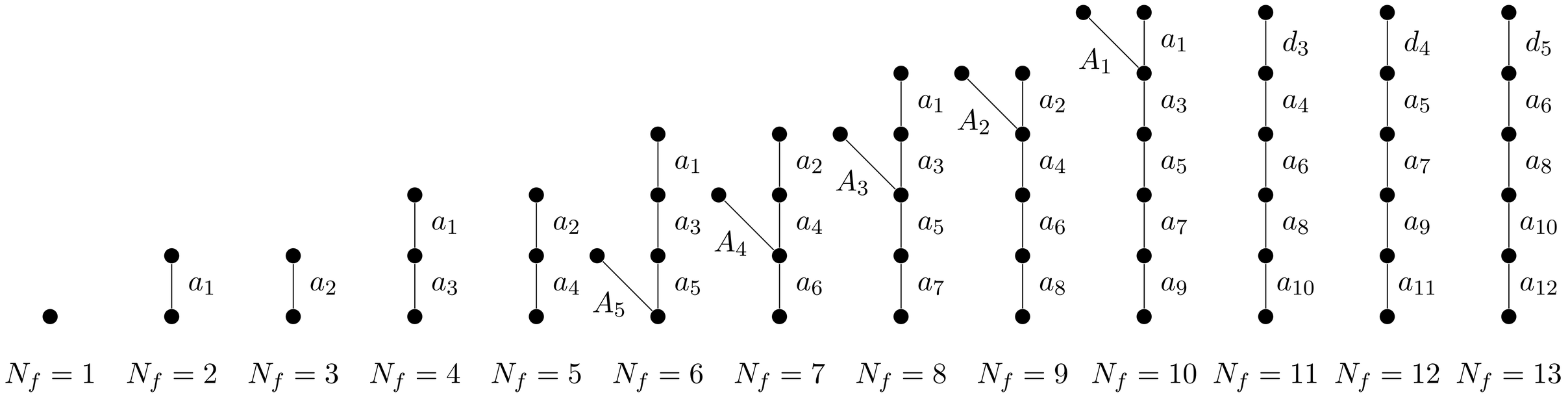


Physical Phenomena

4d — multiple cones

SQCD SU(6) with flavors

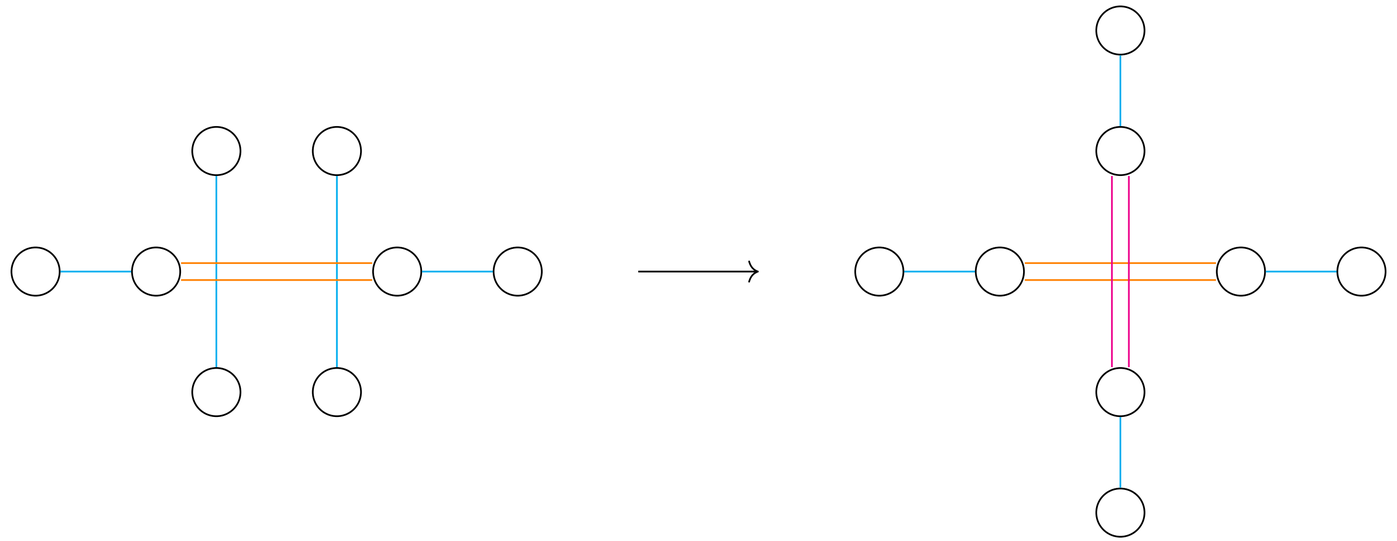
- a union of the mesonic and baryonic branches

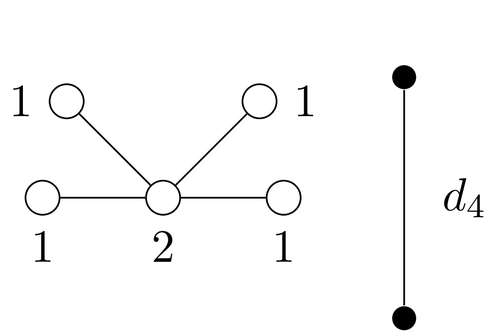


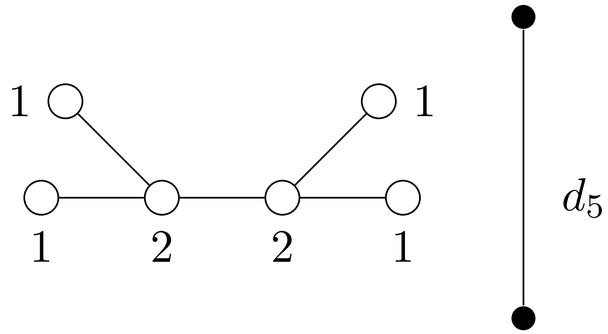
5d — massless instantons

SU(2) with 4 flavors

- finite / infinite coupling



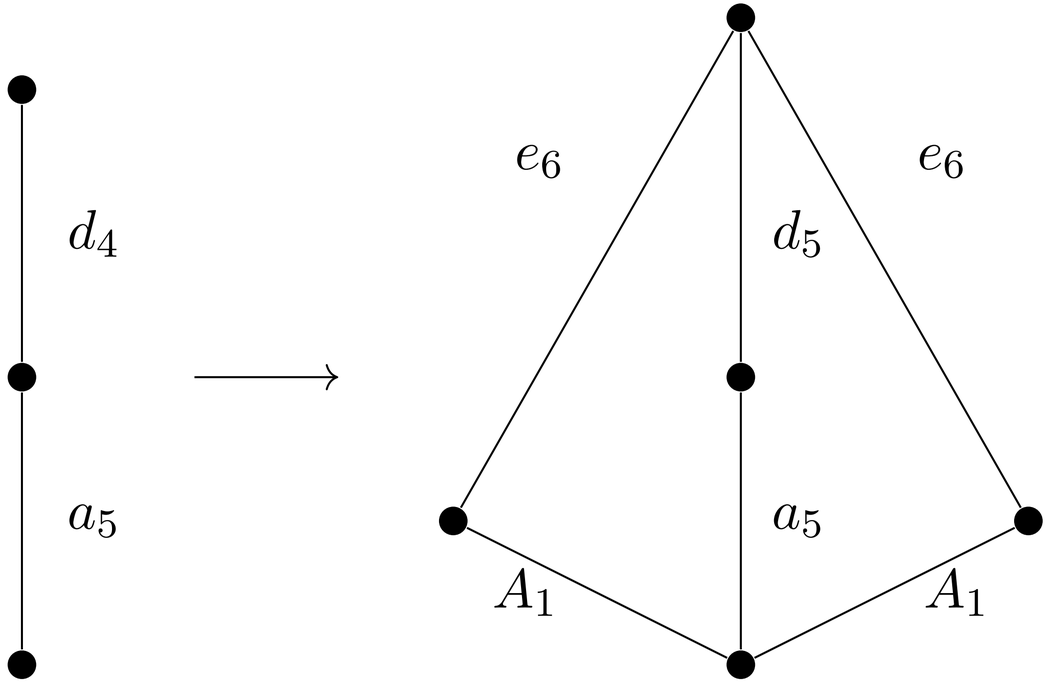




5d — massless gauge instantons

$SU(3)_0$ with 6 flavors

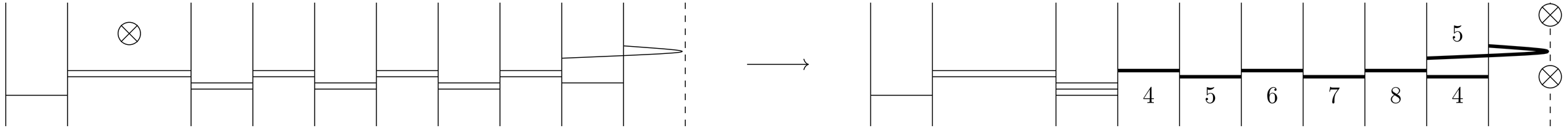
- Phase diagram — finite / infinite coupling

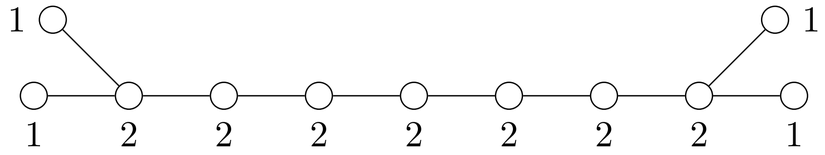


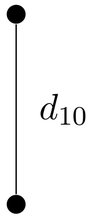
6d — small instanton transition

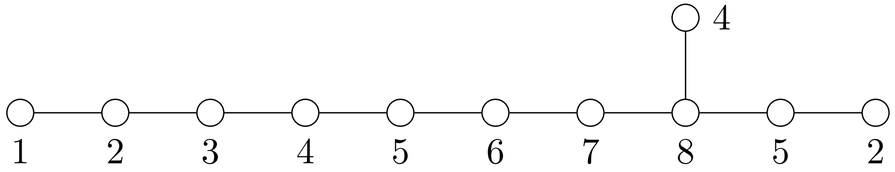
SU(2) with 10 flavors

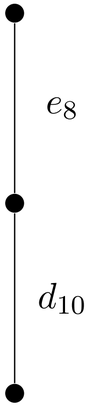
- The Classical Higgs branch — minimal nilpotent orbit of SO(20)
- The moduli space of 1 SO(20) instanton on $\mathbb{C}^2$







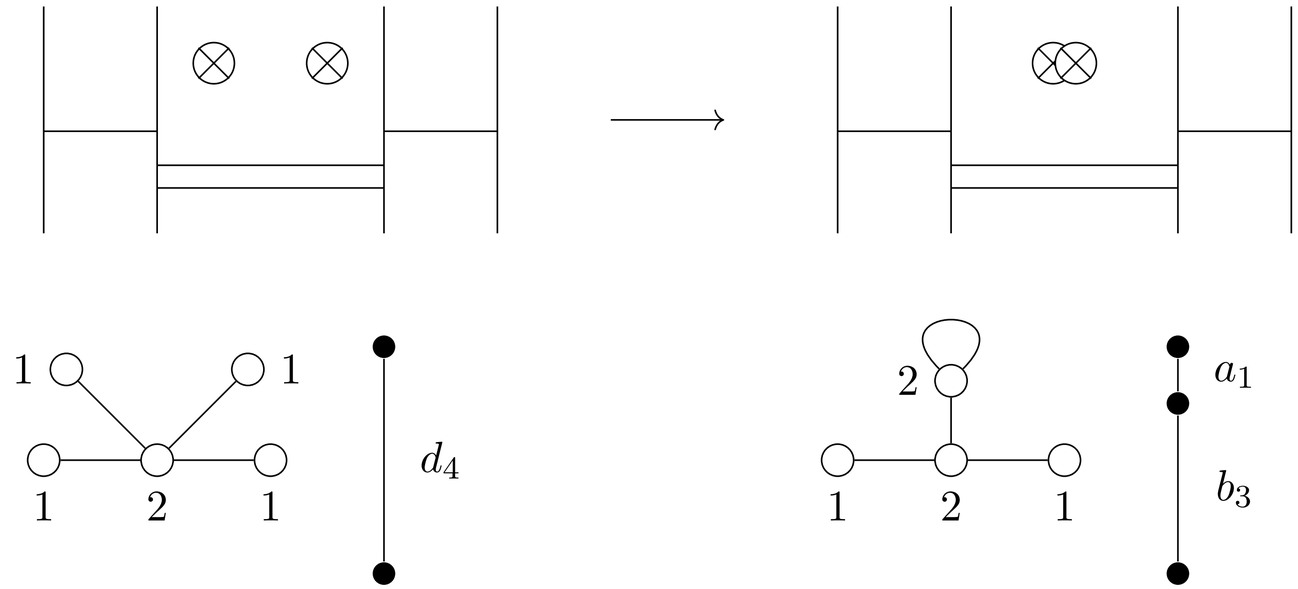




6d — tensionless strings and discrete gauging

SU(2) with 4 flavors

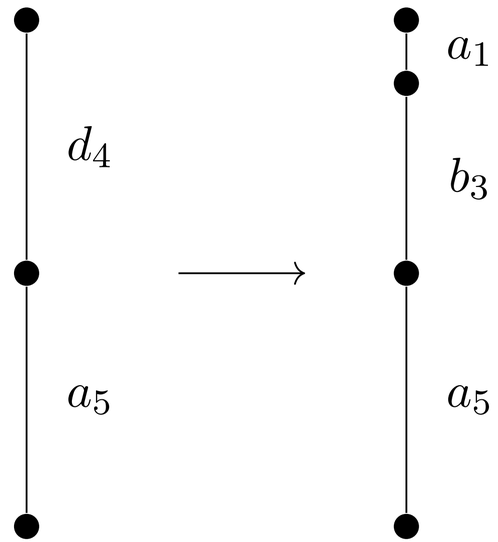
- When n M5 branes coincide on an A-type singularity an S_n group is gauged
- There is symmetry reduction for the A_1, but not for higher values



6d — tensionless strings and S_2 gauging

SU(3) with 6 flavors

- Phase diagram — finite / infinite coupling



Summary

Changing the way we think

- Magnetic Quivers — encodes all data needed to understand strongly coupled moduli spaces
- Phase (Hasse) diagrams — changes the way we analyze symplectic singularities
- Brane systems — very instrumental in getting this progress
- Monopole formula — opened the window to all recent achievements

Thank you !