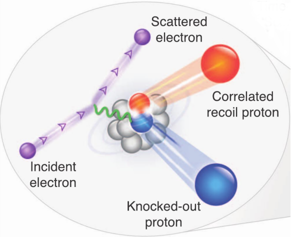

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Nuclear Short Range Correlations and Universality

Nir Barnea

24th European Conference on Few-Body Problems in Physics
University of Surrey, UK, 3 September 2019

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Introduction

Short Range Correlations in a many-body system

Heavy Fermions

The Mara river, Kenya (2016).

Short Range Correlations in a many-body system

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The short range wave function - Universality

We start with the 2-body Schrodinger ...

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(\boldsymbol{r})\right]\psi = E\psi$$

Vanishing distance, $r \longrightarrow 0$

- The energy becomes negligible $E \ll \hbar^2/mr^2$
- The w.f. ψ assumes an asymptotic **energy independent** form φ

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi(\boldsymbol{r}) = 0 \qquad\qquad r\varphi(r) = 0|_{r=0}$$

- φ **is a universal function** (in a limited sense - V dependent)

The short range wave function - Factorization

The 2-body system

$$\psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi(\boldsymbol{r})$$

The A-body system

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) \xrightarrow[r_{12} \to 0]{} \varphi(\boldsymbol{r}_{12}) A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_N)$$

The 2-body system

$$\psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi(\boldsymbol{r})$$

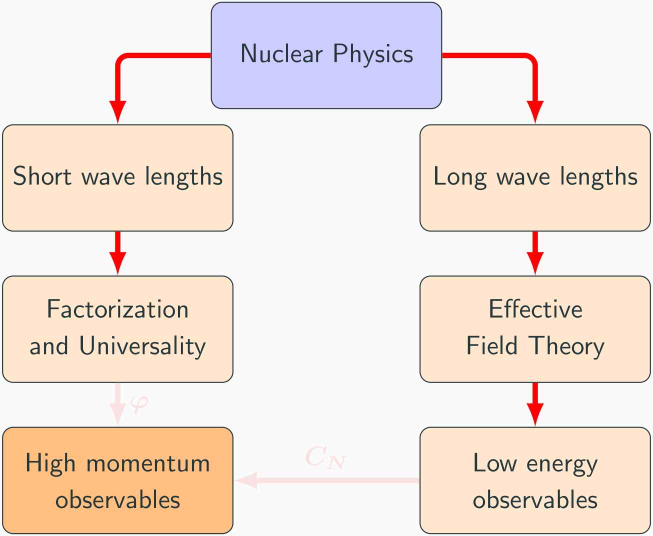
$$O_{12} \approx \delta(\boldsymbol{r}_{12}) \Longrightarrow \langle\psi|O_{12}|\psi\rangle \approx C_2 \langle\varphi|O_{12}|\varphi\rangle$$

The A-body system

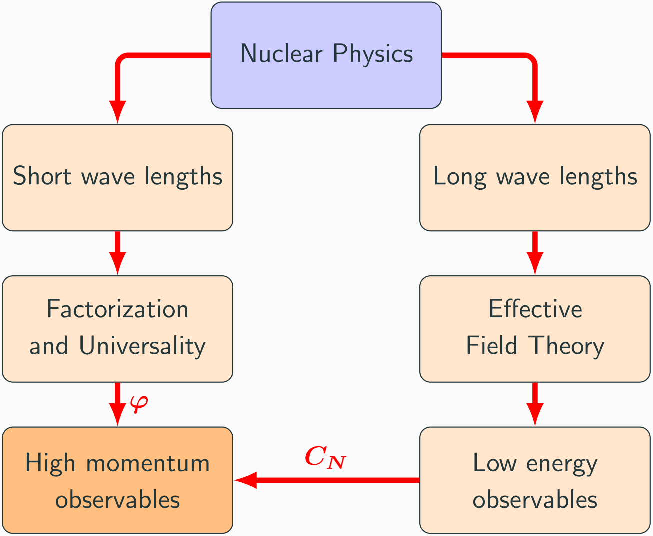
$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) \xrightarrow[r_{12} \to 0]{} \varphi(\boldsymbol{r}_{12}) A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_N)$$

$$\langle\Psi| \sum O_{ij} |\Psi\rangle \approx \underbrace{\tfrac{N(N-1)}{2} \langle A|A\rangle}_{C_N} \langle\varphi|O_{12}|\varphi\rangle$$

Short and Long



Short and Long



Theoretical develpoments in nuclear physics

- **Levinger - Photoabsorption**

  J. S. Levinger, Phys. Rev. 84, 43 (1951).

- **Amado, Woloshyn - Momentum Distribution**

  R. D. Amado, Phys. Rev. C 14, 1264 (1976).

  R. D. Amado and R. M. Woloshyn, Phys. Lett. B 62, 253 (1976).

- **Zabolitsky - Coupled Cluster**

  J. G. Zabolitsky and W. Ey, Phys. Lett. B 76, 527 (1978).

- **Frankfurt, Strikman - Factorization**

  L. Frankfurt, and M. Strikman, Phys. Rep. **160**, 235 (1988).

- **Bogner, Roscher - Factorization**

  S. K. Bogner and D. Roscher, Phys. Rev. C **86**, 064304 (2012).

- **Ciofi degli Atti - Electron scattering**

  C. Ciofi degli Atti, Phys. Rep. 590, 1 (2015).

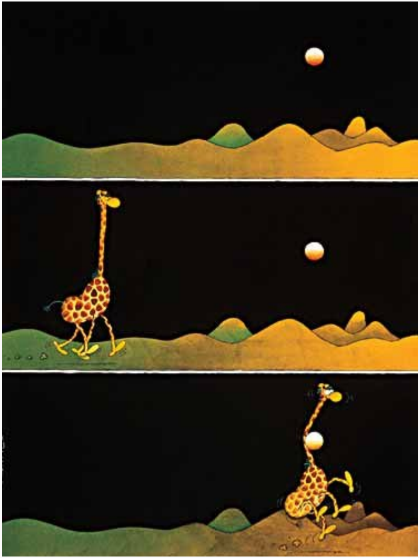
Factorization

and

Universality

in

Nuclear-Physics



The Generalized Contact Formalism

Observations

- Nuclei are dense object
- The interaction range is significant
- There could be different interaction channels - not only s-wave
- Model dependent asymptotic form

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

φ_α are the **zero-energy 2-body nuclear WFs**

- Consequently in general we don't expect to see Tan's universal results to hold

The Nuclear Contact(s)

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp,\ nn,\ pn\}$$

- For each pair there are different channels

$$\alpha = (s, \ell)jm$$

- For each pair we define the contact matrix

$$C_{ij}^{\alpha\beta} \equiv N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

using the normalization $\int_{k_F} \frac{d\boldsymbol{k}}{(2\pi)^3} |\tilde{\varphi}_\alpha(\boldsymbol{k})|^2 = 1$

- For $\ell = 0$ we need consider **4** contacts

$$\{C_{pp}^{S=0}, C_{nn}^{S=0}, C_{np}^{S=0}, C_{np}^{S=1}\}$$

- Adding isospin symmetry the number of contacts is **2**,

$$C_{np}^{S=1} \qquad C_{np}^{S=0} = C_{pp}^{S=0} = C_{nn}^{S=0}$$

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The nuclear contact relations/applications

- The quasi-deuteron model - photoabsorption cross-section
- The 1-body and 2-body momentum distributions
- The nuclear spectral function
- Electron scattering
- Symmetry energy
- Two-body density
- ...

Momentum Distributions

Momentum distributions

1-body neutron and proton momentum distributions

$$n_n(\boldsymbol{k}), \;\; n_p(\boldsymbol{k})$$

2-body nn, np, pp momentum distributions

$$F_{nn}(\boldsymbol{k}), \;\; F_{pn}(\boldsymbol{k}), \;\; F_{pp}(\boldsymbol{k})$$

Momentum distributions

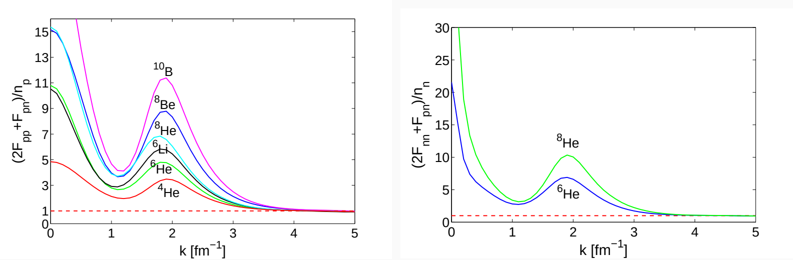
The **asymptotic** relations between the 1-body and 2-body momentum distributions **reads**

$$\boxed{n_p(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{pp}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})}$$

$$\boxed{n_n(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{nn}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})}$$

These are **model independent** relations, that hold regardless of the specific form of nuclear interaction or the asymptotic functions φ_α.

Numerical verification of the momentum relations



VMC calculations of light nuclei

R. B. Wiringa, *et al.*, PRC **89**, 024305 (2014)

- Series of 1-body, 2-body momentum distributions .
- The data is available for $2 \leq A \leq 10$ and $A = 12, 16, 40$.
- The calculations were done with the VMC method.
- Potential - AV18+UX.

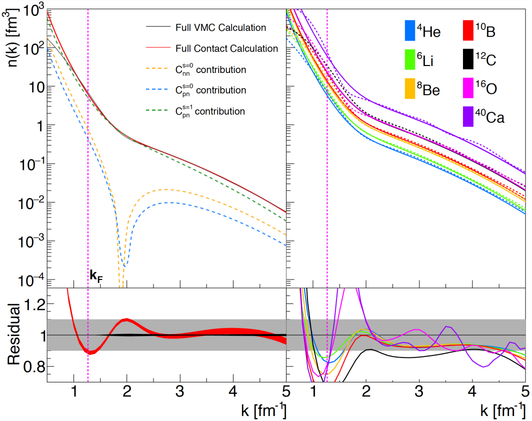
The momentum relations holds for $4\ \mathrm{fm}^{-1} \leq k \leq 5\ \mathrm{fm}^{-1}$

Further numerical verifications

The resulting **asymptotic** 1-body momentum distribution

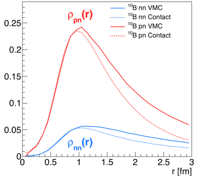
$$n_n(\boldsymbol{k}) \longrightarrow C_{np}^{s=0}|\tilde{\varphi}_{np}^{s=0}(\boldsymbol{k})|^2 + C_{np}^{s=1}|\tilde{\varphi}_{np}^{s=1}(\boldsymbol{k})|^2 + 2C_{nn}^{s=0}|\tilde{\varphi}_{nn}^{s=0}(\boldsymbol{k})|^2$$

Comparing with the VMC data:

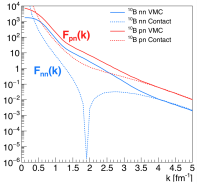


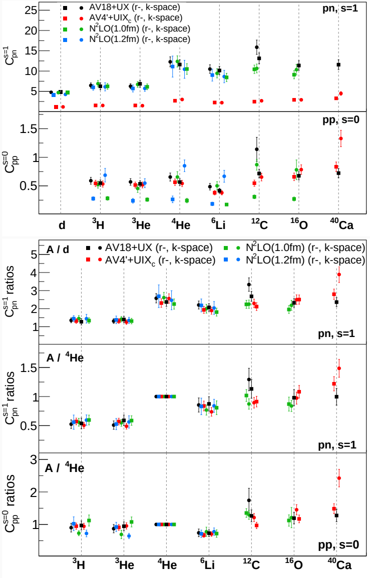
Surprisingly, the agreement holds for $k_F \leq k \leq 6\ \mathrm{fm}^{-1}$

Contacts from 2-body densities



Contacts from 2-body momentum distributions





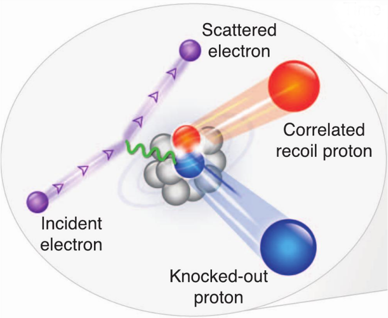
Short Range Experiments

Electron scattering

The Bjorken scaling parameter - x_B

$$x_B = \frac{Q^2}{2m\omega}$$

Where the virtual photon carries $(\boldsymbol{q}, \omega)$ and $Q^2 = -q^2$



- Kinematical considerations:

  For $n-1 < x_B \leq n$ the **"active subsystem"** must include n-nucleons.

- The cross-section ratio σ_A/σ_D is (almost) flat for $1.4 < x_B \leq 2$.

Kinematical considerations

Devide the nucleus into n **active** nucleons and $A-n$ **spectators**

A momentum q is absorbed by the **active** nucleons.

The invariant mass (recall $x_B = Q^2/2m\omega$)

$$
\begin{aligned}
M_n^2 &= (p_A - p_{A-n} + q)^2 \\
&= (p_A - p_{A-n})^2 + q^2 + 2q\cdot(p_A - p_{A-n}) \\
&\approx (M_A - M_{A-n})^2 - Q^2 + 2\omega(M_A - M_{A-n}) \\
&= (M_A - M_{A-n})^2 + Q^2\left[\frac{M_A - M_{A-n}}{x_B m} - 1\right]
\end{aligned}
$$

The original nucleus is stable

$$M_n^2 \geq (M_A - M_{A-n})^2 \Longrightarrow Q^2\left[\frac{M_A - M_{A-n}}{x_B m} - 1\right] \geq 0$$

It follows that

$$\boxed{\frac{M_A - M_{A-n}}{m} \geq x_B \Longrightarrow n \geq [x_B] + 1}$$

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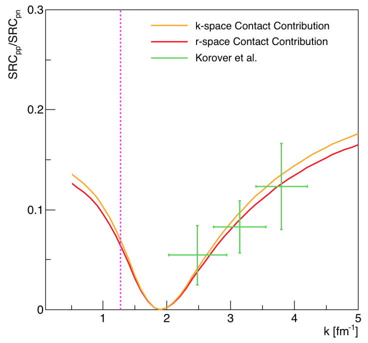
Two-body knockout reactions - naive analysis

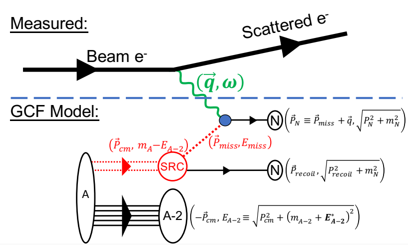
The ratio of short range pp and np pairs is given by

$$\frac{SRC_{pp}(k)}{SRC_{pn}(k)} = \frac{F_{pp}(k)}{F_{pn}(k)} = \frac{C_{pp}^{s=0}|\tilde{\varphi}_{pp(nn)}^{s=0}(k)|^2}{C_{pn}^{s=0}|\tilde{\varphi}_{pn}^{s=0}(k)|^2 + C_{pn}^{s=1}|\tilde{\varphi}_{pn}^{s=1}(k)|^2}$$

^{4}He

Korover et al., Phys. Rev. Lett. 113, 022501 (2014)

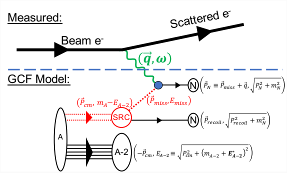




Plane-wave impulse approximation

$$\frac{d^4\sigma}{d\Omega_{k'}\, d\epsilon'_k\, d\Omega_{p'_1}\, d\epsilon'_1} = p'_1 \epsilon'_1 \sigma_{eN} S^N(\boldsymbol{p}_1, \epsilon_1)$$

- $(\boldsymbol{k}', \epsilon'_k)$ - electron final.
- $(\boldsymbol{p}'_1, \epsilon'_1)$ - knocked-out nucleon final.
- $(\boldsymbol{p}_1, \epsilon_1) = (\boldsymbol{p}_{miss}, E_{miss})$ - knocked-out nucleon initial.
- σ_{eN} - off-shell electron-nucleon cross section.
- $S^N(\boldsymbol{p}_1, \epsilon_1)$ - the spectral function.

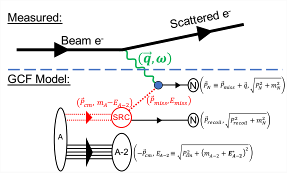


The spectral function

$$S(\boldsymbol{p}_1, \epsilon_1) \equiv \bar{\sum_i} \sum_{s_1, f} \delta(\epsilon_1 + E_f^{A-1} - E_i^A) \left| \langle \Psi_f^{A-1} | a_{\boldsymbol{p}_1, s_1} | \Psi_i^A \rangle \right|^2$$

Assumptions

- Factorization - $|\Psi_i^A\rangle \xrightarrow[p_{12}\to\infty]{} \sum_\alpha \tilde{\varphi}_{12}^\alpha(\boldsymbol{p}_{12}) \tilde{A}_{12}^\alpha(\boldsymbol{P}_{12}, \ldots)$.
- Plane wave - $|a^\dagger_{\boldsymbol{p}_1, s_1} \Psi_f^{A-1}\rangle \longrightarrow \mathcal{N} \sum_\beta |\boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2\rangle \tilde{A}_{12}^\beta(\boldsymbol{P}_{12}, \ldots)$
- $E_f^{A-1} = E_{A-2} + \epsilon_2$
- Variation in E_{A-2} can be ignored $\longrightarrow E^*_{A-2}$.
- $\langle \tilde{A}_{ab}^\alpha(\boldsymbol{P}) | \tilde{A}_{ab}^\alpha(\boldsymbol{P}) \rangle = C_{ab}^\alpha n_{ab}^\alpha(\boldsymbol{P})$,
- n_{ab}^α is approximately a gaussian.



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In the high momentum limit

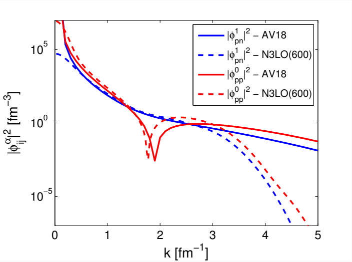
$$S^p(\boldsymbol{p}_1, \epsilon_1) \longrightarrow \sum_\alpha C_{pn}^\alpha S_{pn}^\alpha(\boldsymbol{p}_1, \epsilon_1) + 2 \sum_\beta C_{pp}^\beta S_{pp}^\beta(\boldsymbol{p}_1, \epsilon_1)$$

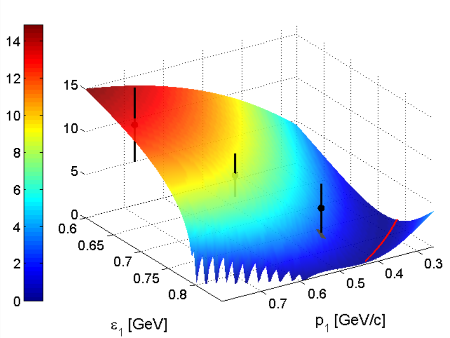
where

$$S_{ab}^\alpha = \frac{1}{4\pi} \int \frac{d\boldsymbol{p}_2}{(2\pi)^3} \delta(E(\boldsymbol{p}_2)) \left| \tilde{\varphi}_{ab}^\alpha\left(\frac{\boldsymbol{p}_1 - \boldsymbol{p}_2}{2}\right) \right|^2 n_{ab}^\alpha(\boldsymbol{p}_1 + \boldsymbol{p}_2)$$

Two-body knockout reactions - revisited

The ratio of short range pp and np pairs
A clear depends on both initial momentum and energy



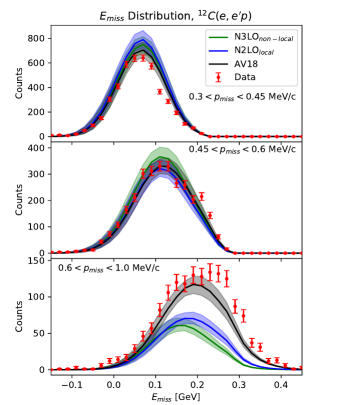


^{4}**He**

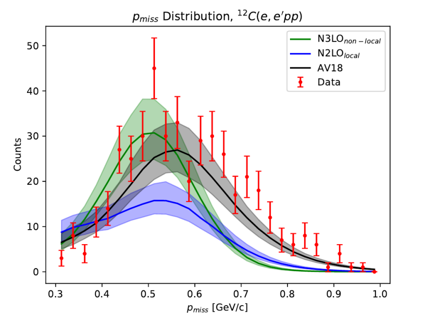
Korover et al., Phys. Rev. Lett. 113, 022501 (2014)

Experiments at $1.4 < x_B \leq 2$

Reanalysis by A. Schmidt *et al.* (CLAS Collaboration), Science **346**, 614 (2014)



No Free Parameters



Contacts taken from **ab-initio** calculations

σ_{CM} taken from previous **experiments**.

E^*_{A-2} is modified in the range $(0, 30)$MeV.

Conclusions

Summary and Conclusions

Factorization and universality and nuclear physics

- The contact formalism can be generalized and applied to NP.
- Many relations can be derived.
- CC theory provides the “missing link” between the contact formalism and the underlying many-body physics.
- **Surprisingly, it seems to be working...**

Thank you !