Time-dependent exploration of the triple-alpha reaction

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The Project

Can we learn about the triple-alpha reaction from time-dependent mean-field (TDHF) studies?

Open questions include:

- Can this single-particle-based method produce cluster states?
- What are the right degrees of freedom to study
- How do we extract the relevant observables from TDHF

Methodology

Our calculations use time-dependent Hartree-Fock (TDHF) [8] with the Skyrme interaction [9] with an unmodified version of the Sky3D code [10, 11]. We use the SLy4d interaction [12] which was fitted with no centre of mass correction as ideal for TDHF calculations.

A ground state alpha particle is calculated in static Hartree-Fock to a well-converged solution in a 16×16×16 fm coordinate space box with 1 fm grid spacing in each Cartesian direction. Time-dependent calculations for ^{4}He+^{4}He collisions are performed by placing two identical ^{4}He ground states separated by a given amount and initialised with instantaneous boost vectors at $t = 0$ which are calculated from a user-defined impact parameter and centre of mass energy of the collision, accounting for the Coulomb tracjectory of the reacting ^{4}He nuclei as they come from infinity.

At later times during the ^{4}He+^{4}He collision, the wave functions of the combined ^{8}Be* nucleus are saved to be used as a starting point for a further TDHF calculation in a larger box. For each specific calculation, the particular parameters used are given as the results are presented in the next section.

To initiate a two alpha particle collision, two ^{4}He ground states were placed in a coordinate grid box with dimension 20×16×20 fm with centres at $(0,0,-4)$ fm and $(0,0,4)$ fm. The alphas were given intial boosts to send them traveling towards each other with impact parameter $b = 0$ fm and centre of mass energy $E_{CM} = 1.0$ MeV. The nuclei fuse and remain fused for the duration of the TDHF calculation. The quadrupole moment of the matter distribution, defined as

$$Q = \sqrt{\frac{5}{16\pi}} \int d^3r\rho(r)(2z^2 - x^2 - y^2), \qquad (1)$$

where ρ is the total nucleon density in the entire multinucles system, is shown in Figure 1. Also shown is the Fourier power spectrum of the time signal of the quadrupole oscilliations. The time-series for the transformation is sampled starting at 800 fm/c = 2.64 zs, for 2048 data points, which is up to $\simeq$ 29.7 zs. The positions of the peaks are rather insensitive to the sampling window. Two states are apparent in the spectrum: One at 8.5 MeV and the other at 14.7 MeV. Both shows some fragmentation, presumably due to artificial discretisation in the coordinate space box [13].

Two-alpha reaction

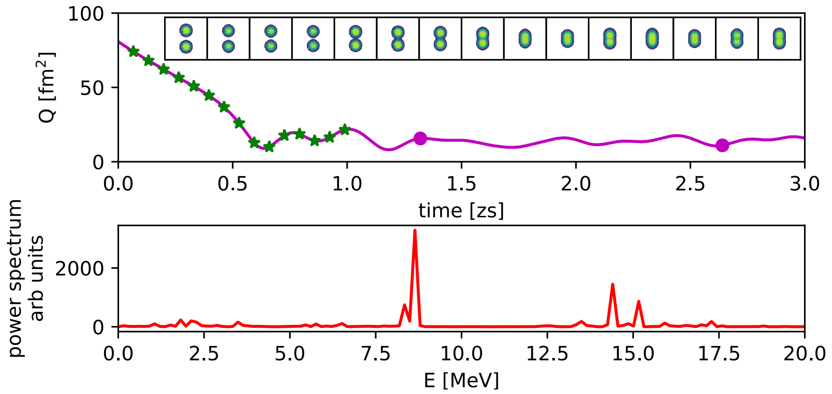


Figure 1: Upper panel shows the quadrupole moment of matter distribution of two colliding alpha particles. Two solid circles show the snapshots after 2000 and 4000 iterations when the wave functions of the ^{8}Be* are taken to be used as starting points in the triple alpha calculations. Starred points correspond to the density snapshots in the inset frames. The lower panel shows the power spectrum of the oscillations of the compound ^{8}Be nucleus.

Three-alpha dynamics

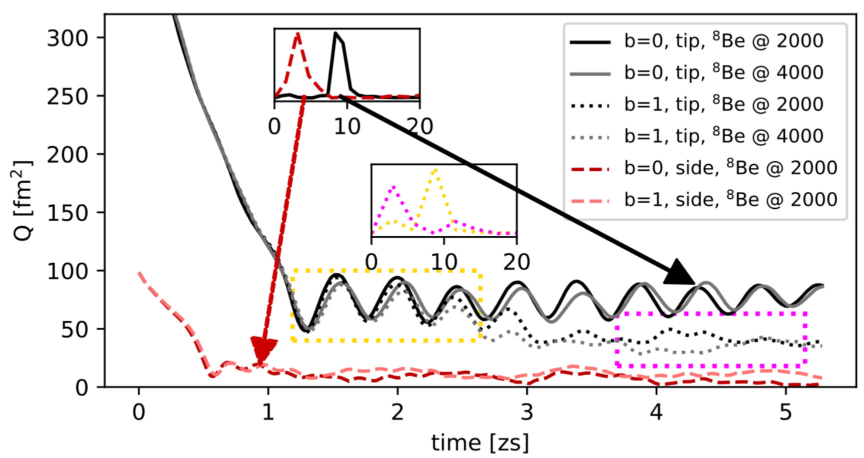
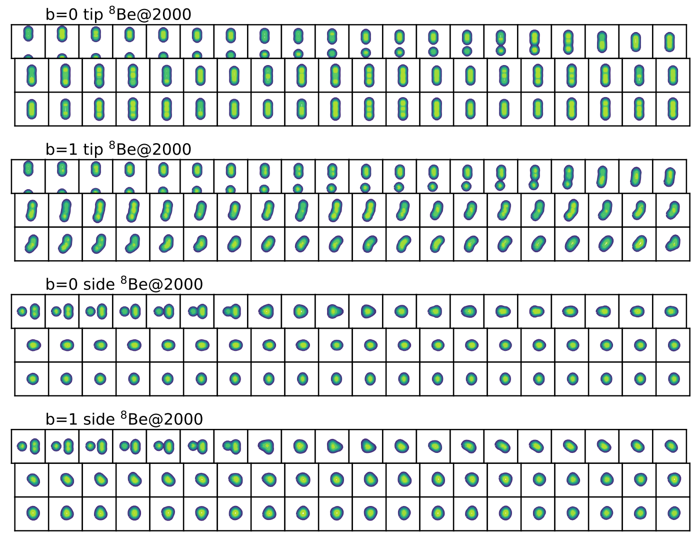


Figure 2: Quadrupole moment of the matter density during collisions of ^{4}He with compound ^{8}Be* nucleus with initial conditions as per the legend and discussed in the text. The two insets show the Fourier power spectrum of parts of the quadropole moment: The upper inset panel shows the spectrum of the resonance created in a tip collision (solid black) and a side collision (dashed red) both at b=0 and using the 2000 configuration of ^{8}Be*. The lower panel shows two spectra from the b=1 tip 2000 configuration with the yellow (lighter) dotted line from the time signal as shown in the yellow (lighter) dotted box, and the pink (darker) dotted line showing the spectrum from the later time signal in the pink (darker) dotted box.

snapshots



Results & discussion

The parameter space for triple alpha (^{8}Be*+^{4}He) reactions is much larger than for ^{4}He+^{4}He: The ^{8}Be* is not spherical, so there will be dependence on the initial orientation of the reacting nuclei. The ^{8}Be* nucleus is not in a stationary state, so there may be dependence upon the exact configuration of the ^{8}Be* at the moment of impact. Here, we make a study of these extra parameters, but concede that a much fuller study is needed for a complete picture.

Two different starting configurations are used for the ^{8}Be* nucleus, as indicated by the solid circles on the line in Figure 1. These are somewhat arbitrarily chosen and lablled configurations 2000 and 4000 (because of the number of iterations in the 2-α TDHF calcuation), though we note that one of the configurations is near a maximum in the value of Q while the other is near a mininum. Starting orientations are limited to the two extremes of impinging along the long or short axes of the ^{8}Be, labelled "tip" and "side" collisions respectively, and with impact parameters selected between $b = 0$ fm and $b = 1$ fm only. The centre of mass collision energy is fixed at $E_{cm} = 2.0$ MeV.

Figure 2 shows a summary of the results for simulations up to 5 zs. Figure 3 shows some details of the evolution of the density leading to the results of Figure 2. The b=0 tip configurations lead to a rather stable large-amplitude oscillation which remain in a chain state. Side configuraitons lead to more compact states with smaller-amplitude oscillations in which triangular configurations appear.

Mixing of the mean-field TDHF configurations via a Fourier spectrum analysis gives an estimate for the energies of the excited states of ^{12}C involved. The upper inset panel in Figure 2 shows the power spectrum from the chain state oscillatons at around 9 MeV, and from the triangular oscillations at around 4 MeV. The lower inset panel shows that in a b=1 tip collision, the nucleus initially oscillates in the 9 MeV chain state before quickly (∼2 zs) decaying to the 4 MeV triangular state. This interpretation is seen in the snapshots of the time-dependent density in Figure 3, and qualitatively agrees with a previous study [4].

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