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Theoretical study of deeply Virtual Compton Scattering of ^{4}He

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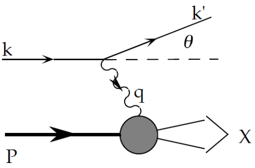
Introduction

Some history

Inclusive DIS process $A(e,e')X \implies$ *Parton distribution functions (PDFs)*



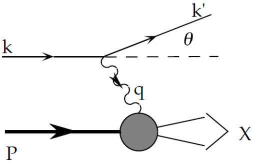
$$\frac{d^2\sigma}{d\theta d\nu} \propto F_2^N(x) = \sum_q e_q^2 x f_q^N(x)$$

x is the longitudinal momentum fraction for a quark q in a nucleon N

Consider the ratio

$$\boldsymbol{R(x) = \frac{F_2^A(x)}{F_2^d(x)}}$$

Inclusive DIS process $A(e, e')X \implies$ *Parton distribution functions (PDFs)*

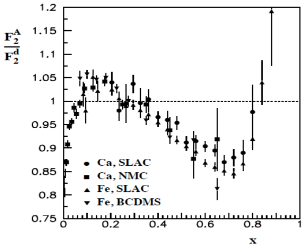


$$\frac{d^2\sigma}{d\theta d\nu} \propto F_2^N(x) = \sum_q e_q^2 x f_q^N(x)$$

x is the longitudinal momentum fraction for a quark q in a nucleon N

Consider the ratio

$$R(x) = \frac{F_2^A(x)}{F_2^d(x)} \longrightarrow \text{EMC (Cern (1983))} \longrightarrow R(x) \neq 1$$



$$x = \frac{Q^2}{2M_A\nu} \to x \in [0; \frac{M_A}{M} \approx A]$$

- $x \leq 0.2$: "Shadowing region"
- $0.3 \leq x \leq 0.7$: "EMC region"
- $0.8 \leq x \leq 1$: "Fermi motion region"

What is the right way to explain the EMC trend?

- ***Elastic scattering*** $\longrightarrow$ **Form factors**
  $F(Q^2) \longrightarrow$ no inner parton structure

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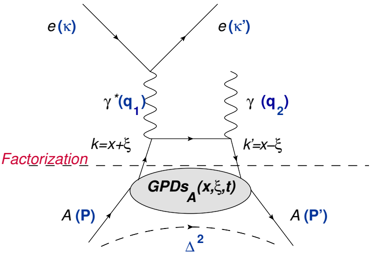
  Longitudinal momentum space

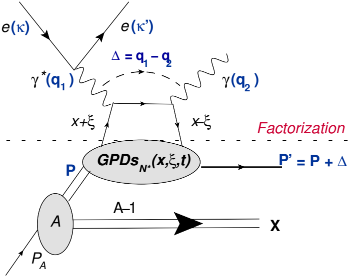
- **????** $\longrightarrow \mathcal{F}_q(x, Q^2, ??..) \longrightarrow$ *Transverse plane*

We can do a *tomography* of nuclei in coordinate space.

Exclusive processes: DVCS off nuclei in handbag approximation

Two different *channel* for DVCS off nuclei: **coherent and incoherent**





- Factorization property($\Delta^2 \ll Q^2$):
  - **HARD PART** $\Longrightarrow$ **perturbative QED & QCD**
  - **SOFT PART** $\Longrightarrow$ **non-perturbative QCD** $\longrightarrow$ **Generalized Parton Distributions**
- GPDs depend on :
  - $\Delta^2 = (P' - P)^2 = (q_1 - q_2)^2$
  - $\xi = -\frac{\Delta^+}{2\bar{P}^+}$
  - $x = \frac{\bar{k}^+}{\bar{P}^+}$
  - $Q^2 = -(\kappa - \kappa')^2$
- $x \leq \xi$: GPDs describe **antiquarks**; $-\xi \leq x \leq \xi$: GPDs describe $q\bar{q}$ **pairs**; $x \geq \xi$: GPDs describe **quarks**

GPDs in a nutshell (i)

GPDs are introduced considering the *light cone correlator*:

$$F^A_{S,S'} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P'S'|\bar{\psi}\Big(-\frac{z^-}{2}\Big)\gamma^+\psi\Big(\frac{z^-}{2}\Big)|PS\rangle$$

$$= \frac{1}{2P^+}\Big[H^A_q(x,\xi,t)\bar{u}(P',S')\gamma^+u(P,S) + E^A_q(x,\xi,t)\bar{u}(P',S')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(P,S)\Big] + ..$$

→ For a target of spin S, the number of GPDs is **$(2S+1)^2$**

Form factor

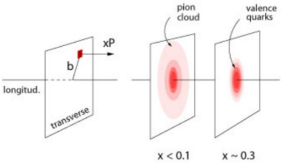
$$\sum_N \int_{-1}^{1} dx \sum_{q=u,d} e_q H^A_q(x,\xi,t) = F^A_1(t)$$

PDFs (when $P = P'$, i.e $t = \xi = 0$)

$$H^A_q(x,0,0) = q^A_q(x)$$

Probabilistic interpretation in *impact parameter space*

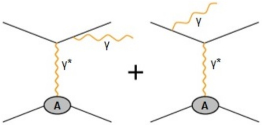
$$\rho^q(x,\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\vec{\Delta}_\perp} H^q(x,0,\Delta^2_\perp)$$



GPDs in a nutshell (ii)

- At JLab kinematics, ***Bethe Heitler*** process interferes with DVCS enhancing this latter. For this reason, it is convenient to measure **asimmetries**, ie.
  $A_{LU} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$

$$\sigma \propto T_{BH}^2 + T_{DVCS}^2 + \mathcal{I}_{BH-DVCS}$$



that can be expressed in terms of

- **Charge Form Factor**

$$T_{BH} \propto F_i(\Delta^2)$$

- **Compton Form Factor** ($\propto$ GPDs)

$$T_{DVCS} \propto \mathcal{H}(\xi, \Delta^2) = \int dx \frac{H_q^A(x, \xi, \Delta^2)}{x \pm \xi + i\epsilon} = \Re e\mathcal{H}(\xi, \Delta^2) + i\Im m\mathcal{H}(\xi, \Delta^2)$$

- for a **nuclear target**, it is difficult to disentangle coherent and incoherent channels because of the large energy gap between the photons and the slow-recoiling systems which requires different detectors

Why is ^{4}He a golden nucleus?

- ^{4}He is a typical few body system and it is theoretically well known
- exact and realistic calculations are difficult BUT possible
- $J^{\pi}_{^4He} = 0^+$ e $I_{^4He} = 0 \implies$ only one chiral-even GPD at LO
- CLAS and ALERT collaboration are carrying on an experimental program at JLab using ^{4}He target

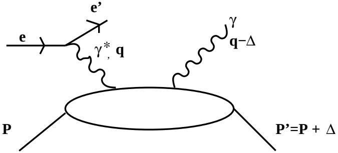
> Coherent (**PRL 119, 202004 (2017)**) and incoherent (**PRL 123, 032502 (2019)**) DVCS off ^{4}He has been measured at the Jefferson Laboratory!

- good perspectives at the forthcoming EIC

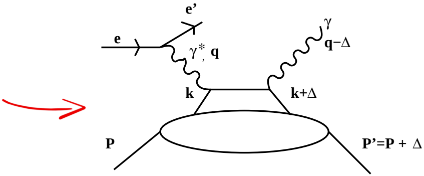
> Our point is to obtain models necessary to distinguish effects due to “conventional” or to “exotic” nuclear structure in order to proper interpret the data.

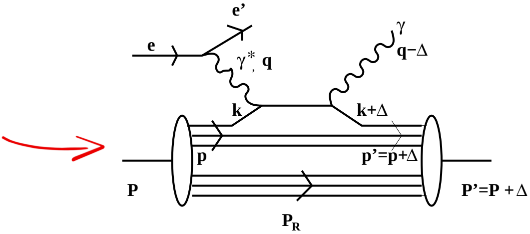
Coherent DVCS off ^{4}He

Coherent DVCS channel



Handbag approximation





Impulse approximation (IA)

Our formalism for the nuclear GPD (S. F., S.Scopetta, M. Viviani, PRC 98 (2018) 015203)

A convolution formula for the GPD H_q can be obtained in terms of:

- GPDs of the inner nucleons

$$H_q^{^4He}(x,\xi,\Delta^2) = \sum_N \int_{|x|}^1 \frac{dz}{z} h_N^{^4He}(z,\xi,\Delta^2) \, H_q^N\left(\frac{x}{\zeta},\frac{\xi}{\zeta},\Delta^2\right)$$

- light-cone momentum distribution

$$h_N^{^4He}(z,\Delta^2,\xi) = \int dE \int d\vec{p}\, P_N^{^4He}(\vec{p},\vec{p}+\vec{\Delta},E)\,\delta\left(z-\frac{\bar{p}^+}{\bar{P}^+}\right)$$

$$= \frac{M_A}{M}\int dE \int_{p_{min}}^{\infty} dp \int_0^{2\pi} d\phi\, p\, \tilde{M}\, P_N^{^4He}(\vec{p},\vec{p}+\vec{\Delta},E)$$

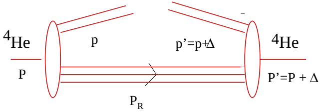
where $\xi_A = \frac{M_A}{M}\xi$, $\tilde{z} = z+\xi_A$, $\tilde{M} = \frac{M}{M_A}\left(M_A+\frac{\Delta^+}{\sqrt{2}}\right), p_{min} = f(z,\xi_A,E)$

As an input, one needs the non-diagonal spectral function and the nucleonic GPDs (we used the Goloskokov-Kroll model (EPJA 47 212 (2014)).

$$P_N^{^4He}(\vec{p}, \vec{p} + \vec{\Delta}, E) = \rho(E) \sum_{\alpha\, \sigma_N} \langle P + \Delta | -p\, E\, \alpha, p + \Delta\, \sigma_N \rangle$$
$$\langle p\, \sigma_N, -p\, E\, \alpha | P \rangle$$

2-body channel

- $\langle ^4$ He $|p, {}^3$H $\rangle$;
- $\langle ^4$ He $|n, {}^3$He $\rangle$;



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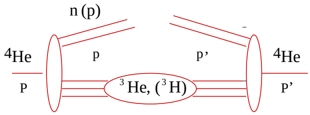
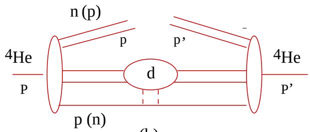
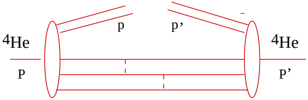


(a)



(b)

+



(c)

3-body channel

- $\langle ^4$ He $|p, d\ n\rangle$;
- $\langle ^4$ He $|n, d\ p\ \rangle$;

4-body channel

- $\langle ^4$ He $|n, p\ n\ p\rangle$;
- $\langle ^4$ He $|p, n\ p\ n\ \rangle$.

Our model for the spectral function (S. F., S.Scopetta, M. Viviani, PRC 98 (2018) 015203)

$$
\begin{aligned}
P_N^{^4He}(\vec{p},\vec{p}+\vec{\Delta},E) &= n_0(\vec{p},\vec{p}+\vec{\Delta})\delta(E) + P_1(\vec{p},\vec{p}+\vec{\Delta},E) \\
&= n_0(|\vec{p}|,|\vec{p}+\vec{\Delta}|,\cos\theta_{\vec{p},\vec{p}+\vec{\Delta}})\delta(E) + P_1(|\vec{p}|,|\vec{p}+\vec{\Delta}|,\cos\theta_{\vec{p},\vec{p}+\vec{\Delta}},E) \\
&\simeq a_0(|\vec{p}|)a_0(|\vec{p}+\vec{\Delta}|)\delta(E) + \sqrt{n_1(|\vec{p}|)n_1(|\vec{p}+\vec{\Delta}|)}\,\delta(E-\bar{E})
\end{aligned}
$$

where

- the total momentum distribution is $n(p)$

$$n_1(|\vec{p}|) = n(|\vec{p}|) - n_0(|\vec{p}|)$$

- $n_0(k)$ is the momentum distribution of the recoiling system in the ground state

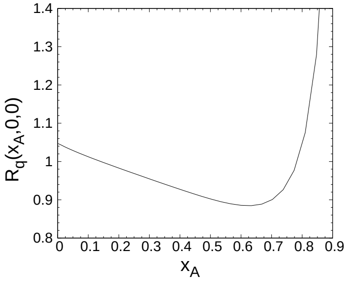
$$n_0(|\vec{p}|) = |a_0(|\vec{p}|)|^2$$

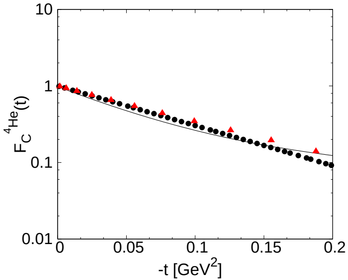
  with

$$a_0(|\vec{p}|) =< \Phi_3(1,2,3)\chi_4\eta_4|j_0(|\vec{p}|R_{123,4})\Phi_4(1,2,3,4) > \quad .$$

- $\boldsymbol{n(p)}$ has been evaluated for the 4-body and 3-body systems within the Av18 NN interaction + UIX three-body forces
- $\bar{E}$ is the average excitation energy of the recoiling system (model of diagonal s.f. by **M. Viviani et al., PRC 67(2003) 034003**).

Some checks for our model





- **EMC-like effect**

$$R_q(x,0,0) = \frac{H_q^A(x_A,0,0)}{2(H_q^p(x_A,0,0)+H_q^n(x_A,0,0))}$$

✓ Good EMC-like behavior;

- **Charge form factor**

$$F_C^{^4He}(\Delta^2) = \frac{1}{2}\sum_q e_q \int_0^1 dx H_q^{^4He}(x,\xi,\Delta^2)$$

Data (•) from **PRC 160, 4 (1987)**, theoretical one-body calculation (▲) by **Marcucci et al., PRC 58, 3069 (1998)**.

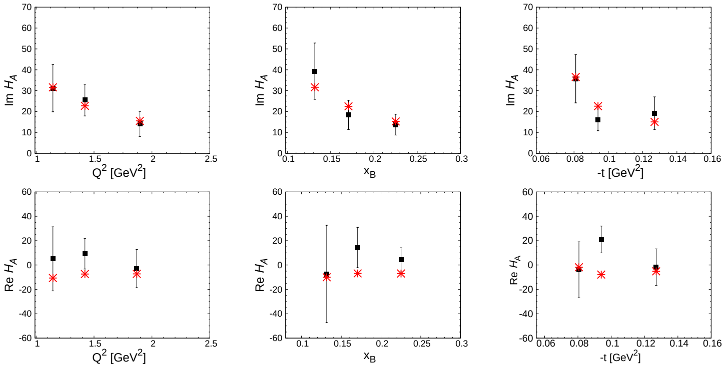
✓ Good agreement with the experimental data.

Coherent DVCS: a comparison with EG6 data, $\mathcal{H}_A$ (S. F., S.Scopetta, M. Viviani, PRC 98 (2018) 015203)

Our results (**red stars**) compared with experimental results (**black squares**)

$$\Im m \mathcal{H}_A(\xi, t) = \sum_{q=u,d,s} e_q^2 (H_q^A(\xi, \xi, \Delta^2) - H_q^A(-\xi, \xi, \Delta^2))$$

$$\Re e \mathcal{H}_A(\xi, t) = \mathrm{Pr} \sum_{q=u,d,s} e_q^2 \int_0^1 \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right) (H_q^A(x, \xi, t) - H_q^A(-x, \xi, t))$$



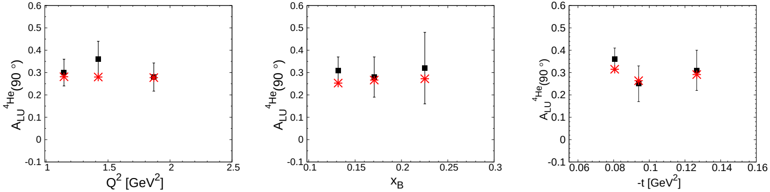
Coherent DVCS: a comparison with EG6 data, A_{LU} (S. F., S.Scopetta, M. Viviani, PRC 98 (2018) 015203)

Beam spin asimmetry as a function of azimuthal angle $\phi = 90^o$

$$A_{LU}(\phi) = \frac{\alpha_0(\phi)\,\Im m(\mathcal{H}_A)}{\alpha_1(\phi) + \alpha_2(\phi)\,\Re e(\mathcal{H}_A) + \alpha_3(\phi)\left(\Re e(\mathcal{H}_A)^2 + \Im m(\mathcal{H}_A)^2\right)}.$$

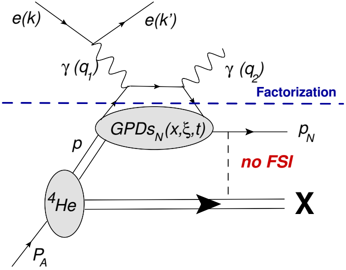
where $\alpha_i(\phi)$ are kinematical coefficients from **A. V. Belitsky et al., Phys. Rev. D 79, 014017 (2009)**.

Results of our approach VS **EG6 data**



From left to right, the quantity is shown in the experimental Q^2, x_B and $-t$ bins, respectively.

Incoherent DVCS off ^{4}He



The **beam spin asimmetry** (observable) is:

$$A_{LU} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

where
$\pm$refers to positive(negative) beam polarizations.

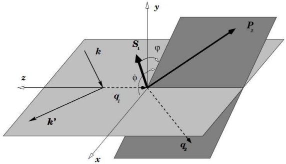
Fundamental starting points for our **Impulse Approximation** approach are:

- kinematical **off shellness**:
$$p_0 = M_A - \sqrt{M_{A-1}^{2*} + \vec{p}^2} \simeq M_N - E - T_{rec} \implies p^2 \neq m^2$$
- general expression for **cross section**
$$(d\sigma^\pm)_{INC} = (2\pi)^4 \frac{1}{2P_A \cdot k} \sum_N \sum_X |\mathcal{A}^\pm|^2 \delta^4(P_A + k - k' - p_X - p_N - q_2) LIPS$$

where $LIPS = d\tilde{p}_X d\tilde{k}' d\tilde{q}_2 d\tilde{p}_N$

Incoherent DVCS: work in progress (i)

In a frame where the target nucleus is at rest, the cross section and its azimuthal dependence are expressed in terms of a ***convolution formula*** between:



- the **diagonal** spectral function $P_N^{^4He}$ of the inner nucleons

$$d\sigma^{\lambda, {}^4He}_{Incoh} = \sum_N \frac{p \cdot k}{P_A \cdot k} \frac{M_A}{p_0} \sum_E \int d\bar{p}\; P_N^{^4He}(\vec{p}, E) \quad d\sigma^{\lambda, N}_{COH}$$

- the DVCS cross section off a bound nucleon

Differentiating with respect to the experimental variables, one gets

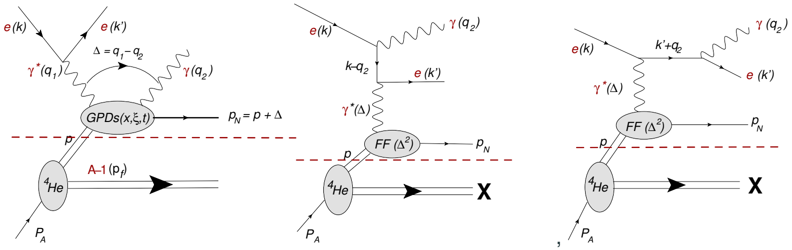
$$\frac{d\sigma^{\lambda, {}^4He}_{Incoh}}{dx_B dQ^2 d\Delta^2 d\phi} \propto K \sum_N \int dE \int d\vec{p}\, P_N^{^4He}(\vec{p}, E) |\mathcal{A}^{N,\lambda}_{Coh}(p, p_N, K)|^2 g(p, p_N, K)$$

where

- K includes different combination of kinematical variables: $Q^2, \phi, x_B, y, \Delta^2$
- $g(p, p_N, K)$ arises from the integration of LIPS and accounts for the flux factor

Incoherent DVCS: work in progress (ii)

Schematically $d\sigma^{\pm} \propto \int d\vec{p} \int dE P_N^{^4He}(\vec{p}, E) |A_{Coh}^{N,\pm}(p, p_N, kin)|^2$ with $|\mathcal{A}_{Coh}^{\pm}|^2 = \mathcal{T}_{BH}^2 + \mathcal{T}_{DVCS}^2 + \mathcal{I}_{DVCS-BH}^{\pm}$.

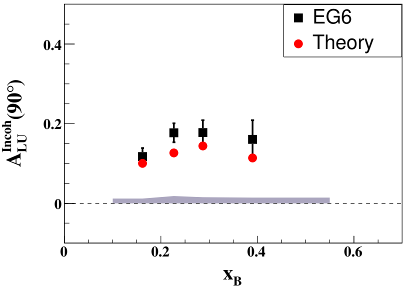


The observable we computed is :

$$A_{LU}^{Incoh} \equiv A_{LU}^{p,^4He} \propto \frac{\int d\vec{p} \int dE P_p^{^4He}(\vec{p}, E) \mathcal{I}_{DVCS-BH}^{\pm}(p, p_N, K)}{\int d\vec{p} \int dE P_p^{^4He}(\vec{p}, E) |\mathcal{T}_{BH}(p, p_N, K)|^2}$$

- our expression for $|\mathcal{T}_{BH}(p, p_N, K)|^2 = c_0^{BH} + c_1^{BH}\cos(\phi) + c_2^{BH}\cos(2\phi)$ is a generalization for a moving bound nucleon of results by **Muller et al.(2002)**
- $\mathcal{I}_{DVCS-BH}^{\pm} \implies s_1^{\mathcal{I}}(p, p_N, K)\sin(\phi)$ accounts for beam polarization and contains CFF, i.e $\Im m\, \mathcal{H}(\xi_{N^*}, \Delta^2, Q^2)$.

- For nucleon GPD H_q^N, again, we used **GK model (2013)** evaluated for $\xi_{N*} = \frac{Q^2}{(p+p_N)(q_1+q_2)} \neq \frac{x_B}{2-x_B} = \xi$
- For the diagonal spectral function $P_p^{^4He}(\vec{p}, E)$ we use an Av18-based model (**M. Viviani et al., PRC 67, 034003 (2003)**) update of **Ciofi et al., PRC 53 1689 (1996)** . Realistic energy dependence only in the ground part.
- Our results are compared with the experimental data from EG6 collaboration at JLab (**M. Hattawy et al., PRL 123, 032502 (2019)**). Each point, at a given x_B, corresponds to an *almost definitive* experimental analysis.



PRELIMINARY!!

While we are still playing with our model testing numerical stability dependence on exp. kinematics etc, for the moment **our model reproduces the experimental data trend!**

Our straightforward and workable approaches to DVCS off ^{4}He seem suitable for planning new measurements and interpreting the present data.

Formal development of a theoretical formula for the only **GPD** describing the ^{4}He with an overall good agreement with data.

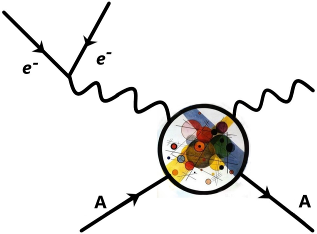
- Realistic AV18 + UIX momentum dependence
- Dependence on E, angles and Δ in the s.f is modeled and not yet realistic

Explicit calculation of the **beam spin asymmetry of a bound moving nucleon**

Evaluation of the incoherent channel considering Final State Interaction

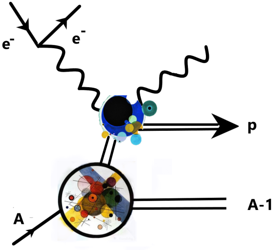
A full realistic evaluation of the spectral function, both diagonal and off-diagonal

Relativistic description of both channels in a Light Front scenario



Thank you ...

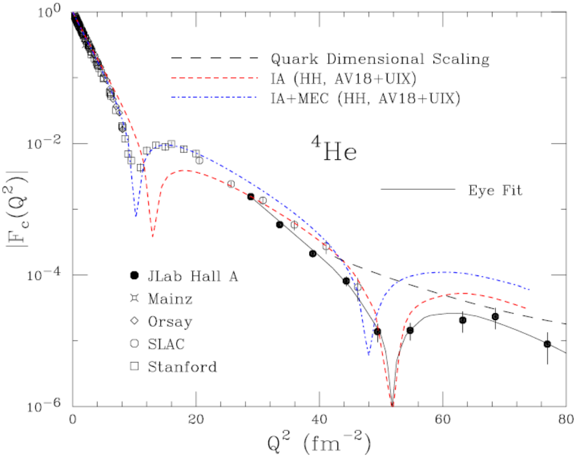
Questions?



Backup slides

Form factor of the ^{4} He at high Q^2

Red dashed line: One body part of the form factor from a direct integration of the diagonal momentum distribution of the ^{4}He within Av18+UIX calculation (**Phys. Rev. Lett. 112, 132503**)

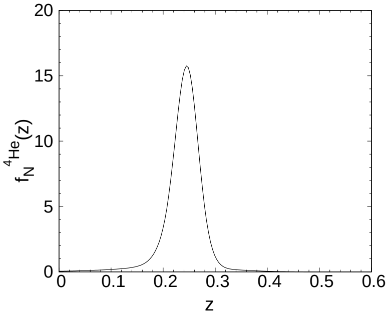


Light cone momentum distribution

Forward limit

$$h_N^{4He}(z,0,0) = {\color{blue} f_N^{4He}(z)} = \int dE \int d\vec{p}\, P_N(\vec{p},E)\,\delta\left(z - \frac{\sqrt{2}p^+}{M}\right).$$

It reproduces in the forward limit the correct IA result for the nuclear PDF



$z \in [x:1]$

$f(z)$ is strongly picked around $z \simeq 0.25$

EMC effect with our model for the off diagonal spectral function

$$R(x_A) = \frac{F_2^{^4He}(x_A)}{2F^d_{2\,l.f}(x_A)}$$

where

$$F_2^A(x) = \sum_q e_q^2 x H_q^{^4He}(x,0,0)$$

$$F^d_{2\;l.f.}(x) = \int_x^{M_d/M} dz \int d\vec{k}\; n_d(|\vec{k}|)\delta\left(z - \frac{M_d}{M}\frac{k_z + \sqrt{k^2+M^2}}{2\sqrt{k^2+M^2}}\right)F_2^N\left(\frac{x}{z}\right)$$