Response functions and cross sections for inclusive neutrino scattering off ^{2}H, ^{3}H and ^{3}He

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Why neutrinos?

Why neutrino scattering is important?

- probe for weak interaction and electroweak unification
- knowledge about neutrino scattering off nuclei is a prerequisite for investigations of neutrino masses and their oscillations

Theoretical approach

- **semi-phenomenological**: AV18 NN potential (or other semi-phenomenological ones) and related current operators
- potentials and currents derived from **chiral effective field theory** (χ**EFT**): problems with regularizations

Our framework

Ingredients of our work

- the **momentum-space approach** allows us to easily work with neutrino reactions off the deuteron and trinucleons: ^{3}He and ^{3}H
- currently we use the **semi-phenomenological AV18** NN potential
- we employ only the one **single nucleon current** operator
- we treat the hadronic arm in the **non-relativistic** manner

What else?

Some remarks

- χEFT potentials are provided in momentum space and will be easily incorporated in our future calculations
- relativistic effects are supposed to be of no great importance for the considered neutrino energies
- possibility to **avoid partial wave representation** of nuclear states by using the so-called "three dimensional" calculations

Studied reactions

A=2

$$\nu_e\ (\bar{\nu}_e) + {}^2\mathrm{H} \rightarrow\ \nu_e\ (\bar{\nu}_e) + {}^2\mathrm{H}$$
$$\nu_e\ (\bar{\nu}_e) + {}^2\mathrm{H} \rightarrow\ \nu_e\ (\bar{\nu}_e) + n + p$$
$$\nu_e + {}^2\mathrm{H} \rightarrow\ e^- + p + p$$
$$\bar{\nu}_e + {}^2\mathrm{H} \rightarrow\ e^+ + n + n$$

A=3

Neutral current (**NC**) breakup reactions

$$\nu_e\ (\bar{\nu}_e) + {}^3\mathrm{H}\ \rightarrow\ \nu_e\ (\bar{\nu}_e) + n + {}^2\mathrm{H}\ \ (+\ n + n +\ p)$$
$$\nu_e\ (\bar{\nu}_e) + {}^3\mathrm{He}\ \rightarrow\ \nu_e\ (\bar{\nu}_e) + p + {}^2\mathrm{H}\ \ (+\ p + p + \mathrm{n})$$

Charged current (**CC**) breakup reactions

$$\bar{\nu}_e + {}^3\mathrm{H}\ \rightarrow\ e^+ + n + n + n$$
$$\bar{\nu}_e + {}^3\mathrm{He}\ \rightarrow\ e^+ + n + {}^2\mathrm{H}\ \ (+\ n + n + p)$$

Kinematics

Energy-momentum conservation

$$E + M_{ini} = \sqrt{\mathbf{k}'^2 + m_\ell^2} + \sum_i \sqrt{\mathbf{p_i}^2 + M_i^2}$$

$$\mathbf{k} = \mathbf{k}' + \sum_i \mathbf{p_i}$$

To evaluate the total cross section we need to know the boundaries for $E' = \sqrt{\mathbf{k}'^2 + m_\ell^2} \Rightarrow$

E'_{max} : $\quad S_{nuc} = (\sum_i p_i^\mu)^2 = (k^\mu - k'^\mu)^2 = (\sum_i M_i)^2 \equiv M_{tot}^2$

Condition on E'_{max}

$$E'_{max} : \qquad [E_{ini} - E'(\mathbf{k}')]^2 - (\mathbf{k} - \mathbf{k}')^2 = M_{tot}^2$$

This can be solved **analytically**.

Kinematics: non-relativistic approach

Assuming all hadronic masses are much bigger than their momenta, we can write

Energy momentum conservation - non relativistic

$$E + M_{ini} = \sqrt{\mathbf{k}'^2 + m_\ell^2} + \sum_i M_i + \frac{\mathbf{Q}^2}{2\,M_{tot}} + E_{CM}$$

$$\mathbf{k} = \mathbf{k}' + \sum_i \mathbf{p_i} \qquad \Rightarrow \qquad \mathbf{Q} = \sum_i \mathbf{p_i}$$

Where $\mathbf{Q} = \mathbf{k} - \mathbf{k}'$ and E_{CM} is the hadronic kinetic energy in the hadronic center of mass.

To evaluate E'_{max} we impose $E_{CM} = 0$ and solve it **numerically**.

The transition matrix element

The squared matrix element for the transition can be written as

$$|M_{fi}|^2 = |\mathrm{L}_\alpha \mathrm{N}^\alpha|^2 = L_{\mu\nu} N^{\mu\nu}$$

where, if we **do not measure** the particles' spin

$$\begin{aligned} L^{\mu\nu} &= \quad 2\, k'_\alpha\, k_\beta\, \mathrm{Tr}[(\gamma^\alpha \pm m_\ell)\, \gamma^\mu\, \gamma^\beta\, \gamma^\nu (1+\gamma^5)] \\ &= 8\,(k^\mu\, k^{\nu\prime} + k^\nu\, k^{\mu\prime} - (k \cdot k')\, g^{\mu\nu} \mp i\, \epsilon^{\alpha\beta\mu\nu} k_\alpha k_\beta{}') \end{aligned}$$

and

$$N^{\mu\nu} = \sum_{M_i\, M_f} (N^\mu)^*\, N^\nu,$$

$$N^\mu = \langle \Psi_f\, \mathbf{P_f}\, M_f |\, j^\mu_W\, | \Psi_i\, \mathbf{P_i}\, M_i \rangle$$

The 1-body current operator

The $j^{\alpha}(k)$ is the **single nucleon current** has the form

The one body weak current matrix element for the k-*th* particle

$$j^{\mu}(k) = \bar{u}_{\mathbf{p}'_k}^{\lambda'_k}\left[F_1\,\gamma^{\mu} + \frac{i}{2M}\,F_2\,\sigma^{\mu\nu}q_{\nu} + F_A\gamma^{\mu}\gamma_5 + F_P\frac{q^{\mu}}{M}\gamma_5\right]u_{\mathbf{p}_k}^{\lambda_k}$$

where the nucleon **form factors** $F_i\,(Q^2)$ are prepared separately for the neutron and the proton.

These form-factors are **different** for **CC** and **NC** induced reactions. The **CC** current operator contains also the isospin $\tau^{\pm}$ operator.

R-functions

We define

R-functions

$$R_{ij} = \sum_{M_i\, M_f} \int df\, \delta(E_{CM} - E_f) \langle \Psi_f^{(-)} | j_{3N}^i | \Psi \rangle \left(\langle \Psi_f^{(-)} | j_{3N}^j | \Psi \rangle \right)^*$$

There are the so-called R-functions which can be evaluated for inclusive cross section.

Next we decide to work in the reference frame where $\mathbf{Q} \parallel \hat{\mathbf{z}}$ and we switch to **spherical coordinates**.

The differential cross section

The only **components** that **survive** with these choices are

$\Rightarrow R_{PP}$, R_{MM}, R_{ZZ},R_{Z0} and R_{00}.

The **cross section** can be written as

The differential cross section

$$\frac{d^3\sigma}{dE'\, d^2\hat{\Omega}} = \frac{G_F^2 \cos^2\theta_C}{(2\pi)^2} F(Z,E') \frac{k'}{8E} \left|M_{if}\right|^2$$

This red terms are set to 1 for **NC** reactions.

where now

Transition matrix element

$$\left|M_{if}\right|^2 = V_{PP}\, R_{PP} + V_{MM}\, R_{MM} + V_{ZZ}\, R_{ZZ} + V_{Z0}\, R_{Z0} + V_{00}\, R_{00}$$

CC reactions

As we have seen the cross section contains a function to treat the **Coulomb distortion** of the emerging charged lepton wave function.

The Fermi function

$$F(Z,E') = 2\,(\gamma+1)\,[2k']^{2\gamma-2}\left|\frac{\Gamma(\gamma - iy)}{\Gamma(1+2\gamma)}\right|^2$$

where

$$\gamma = \sqrt{1 - Z\alpha} \qquad\qquad y = Z\,\alpha\frac{E'}{k'}$$

The evaluation of the response functions R_{jk}

As we saw, the response functions are defined as

$$R_{jk} = \sum_{m_i,\, m_f} \int df\, \delta(E - E')\, \langle \Psi_f^{(-)} | j^j | \Psi \rangle\, \langle \Psi | j^k | \Psi_f^{(-)} \rangle$$

These depend **only** on $(Q,\ E_{CM})$!
$\rightarrow$ we calculate these quantities on a grid in the (Q, E_{CM}) plane, and then use the grid for any incoming neutrino energy.

Grid properties

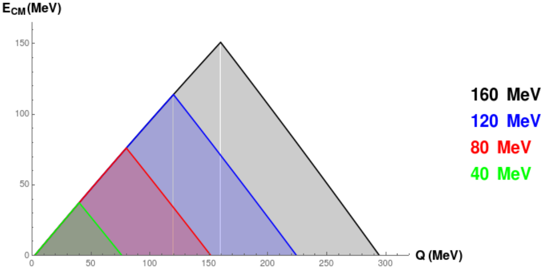
- E_{CM} energies up to pion threshold production ($\sim$150 MeV)
- need to set kinematics boundaries
- denser grid around the axes origin (wait for more info...)

Physical constraints

The allowed physical regions are the ones for which

- $E_{CM} > 0$
- $|\cos\theta'| \leq 1$

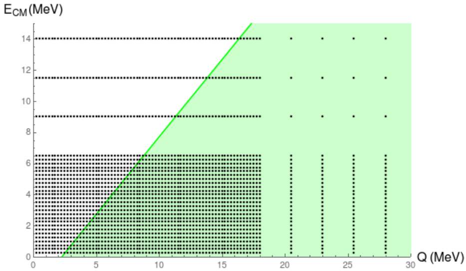
The possible maximal region under non-relativistic treatment is the **black** one in the figure, and it contains all subregions for smaller initial energies



The grid

The grid used to interpolate the R_{jk} functions

- is denser for small initial energies due to the behavior of these functions
- has different steps for momentum and energy
- is regular in order to use the interpolation method



Numerical approach

→We have to evaluate the R_{jk} functions,

- we need more than 10,000 points on the Q-E_{CM} grid to have convergent results
- the computational cost of the grid depends dramatically on the reaction studied
- a single executable running on one core may take **more than one month** to produce the grid for trinucleon reactions

Is there another way?

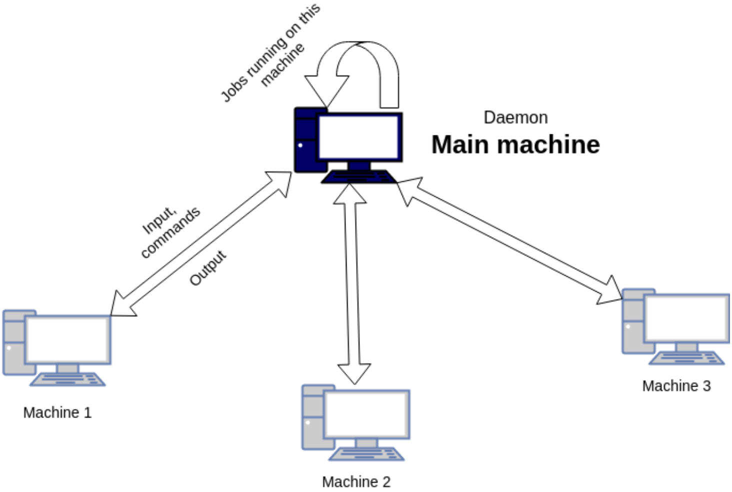
Parallel network computation

Our group has developed a script to run multiple processes on many machines at the same time.

The system

- has a **central daemon** on the main running machine which **controls and sends** the inputs for the jobs to be done
- each job produces only the values of the R_{jk} functions for a given Q-E_{CM} configuration (input) and can be run on any machine
- all the **results** (output) from the jobs are checked by the daemon and stored **on the main machine**
- the theoretical number of the parallel jobs if all the machines are working on the same code is simply the number of cores per machine summed for all the machines

A simple diagram



Pratical example

Imagine our grid consists in 1000 points, and each points takes approximately 2 hours.
For one single executable (1 core) it would take

$$\text{time} = 2000 \text{ hours} \sim 83 \text{ d}$$

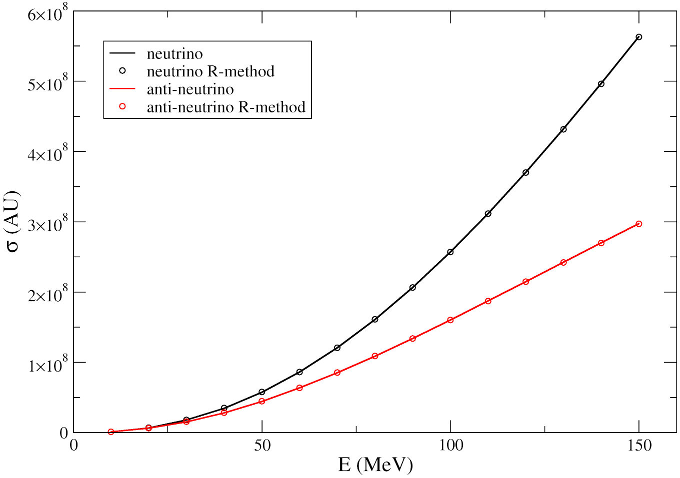
Now imagine we have 4 machines which can work with 6 cores each and have approximately the same calculation speed

$$\#\text{parallel jobs} = 6\times 4 = 24 \qquad \text{time} = 83/24 \sim 3 \text{ d } 11 \text{ h}$$

Comparison of the two methods for $\nu_e + {}^2\mathrm{H} \rightarrow \nu_e + p + n$

$E(MeV)$	$d\sigma_{PWD}$	$d\sigma_{PWT}$	% err	$d\sigma_{RD}$	$d\sigma_{RT}$	% err
10	0.543	0.536	1.365	1.09	1.08	0.864
20	4.67	4.64	0.614	6.84	6.79	0.664
30	14.	13.9	0.373	18.	17.9	0.54
40	29.2	29.1	0.257	34.8	34.6	0.422
50	50.5	50.4	0.199	57.5	57.3	0.252
60	78.1	78.	0.136	86.1	86.	0.12
70	112.	112.	0.074	121.	121.	0.065
80	152.	152.	0.08	161.	161.	0.04
90	197.	197.	0.055	206.	206.	0.032
100	248.	247.	0.055	257.	257.	0.057
110	303.	303.	0.009	311.	312.	0.123
120	362.	362.	0.05	370.	370.	0.12
130	424.	424.	0.023	432.	432.	0.151
140	489.	489.	0.014	496.	497.	0.145
150	557.	557.	0.024	563.	564.	0.12

Comparison graph



Results for break-up processes on ^{3}H and ^{3}He

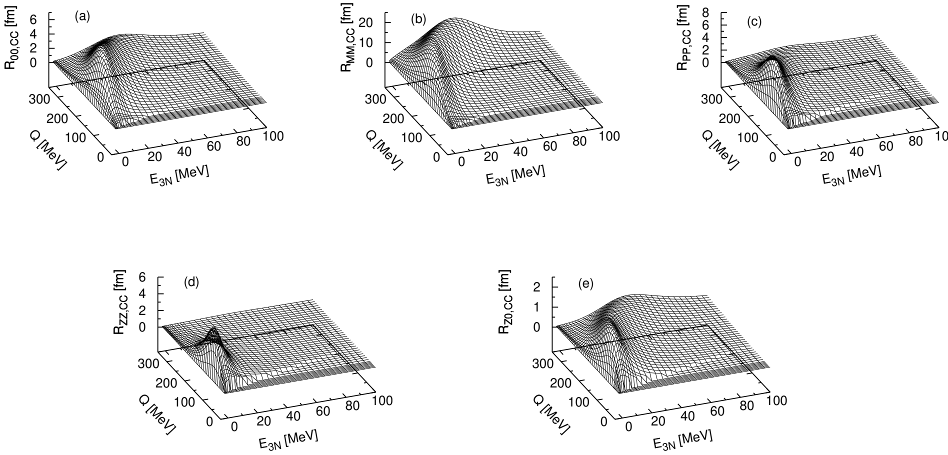
Reactions studied

- **CC** reactions on
  - ^{3}He
  - ^{3}H
- **NC** reactions on
  - ^{3}He
  - ^{3}H

For these reactions

- $\simeq$ **2000 points** on (Q, E_{CM}) grid
- 4 reactions evaluated $\Rightarrow$ **4 grids**
- NC reactions have been studied both for neutrino and antineutrino, while for the CC reactions we restricted ourselves to the antineutrino case

R-functions for $\bar{\nu}_e$ CC breakup on ^{3}He



Cross section for breakup reactions on ^{3}He

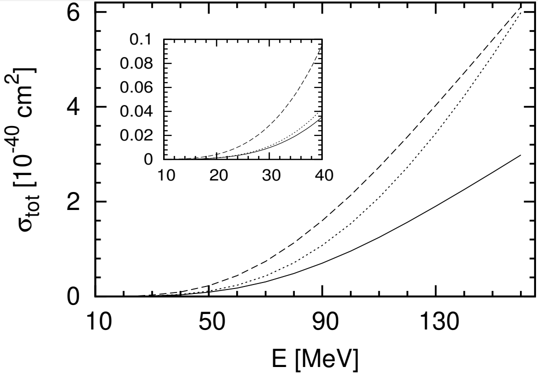


Figure: Cross section for three inclusive (anti)neutrino reactions with ^{3}He: CC electron antineutrino disintegration of $^3\mathrm{He}$ (dashed line), NC electron antineutrino disintegration of $^3\mathrm{He}$ (solid line), NC electron neutrino disintegration of $^3\mathrm{He}$ (dotted line).

Cross section for breakup reactions on ^{3}H

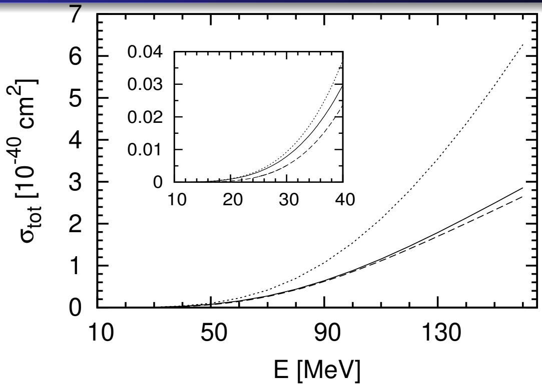


Figure: Cross section for three inclusive (anti)neutrino reactions with ^{3}H: CC electron antineutrino disintegration of $^3\mathrm{H}$ (dashed line), NC electron antineutrino disintegration of $^3\mathrm{H}$ (solid line), NC electron neutrino disintegration of $^3\mathrm{H}$ (dotted line).

Conclusions

- We have calculated the **response function** and the **cross section** for the breakup processes on ^{2}H, ^{3}H and ^{3}He.
- the response function method is **economical** in terms of computation time and allows us to study both ν and $\bar{\nu}$ processes for neutral current reactions **without substantial costs**
- the **agreement** of this method with the traditional integration one is **very good**, it depends on the grid density

Future improvements

This work is a part of the *LENPIC* project and

- aims at calculations with **heavier nuclei**
- we plan to **improve it** using different **potentials** and the corresponding consistent **current operators**

Thank you!