# Correlation analysis and statistical uncertainty of three-nucleon scattering observables

**Yu. Volkotrub**, R. Skibiński, J. Golak, K. Topolnicki, H. Witała
*for the LENPIC Collaboration*

M.Smoluchowski Institute of Physics, Jagiellonian University

LENPIC

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## Introduction

- Nucleon-deuteron scattering
- Theoretical uncertainties in few-nucleon sector

## Tools

- Forces: OPE-Gaussian, chiral SMS
- The Faddeev approach to $3N$ scattering

## Results

- Propagation of potential uncertainties to the elastic nucleon-deuteron scattering $3N$ observables up to $E = 200$ MeV.
- Angular dependence of correlation coefficients between various three-nucleon observables

## Summary

# Why to study nucleon-deuteron scattering?

$N + d \rightarrow N + d$
$N + d \rightarrow N + N + N$

- Because this relatively simple reaction beyond $2N$ system makes the demanding test of two-nucleon force models (which are usually fitted to all $2N$ data).
- Exact theoretical methods and numerical solutions are available (including three-nucleon force, Coulomb interaction, relativity and etc.).
- Many observables are sensitive to various terms of interaction and it gives deeper insight to structure of nuclear interactions.
- Understanding of nuclear force is the basics of nuclear physics and can be investigated in this way.

## Theoretical uncertainties in few-nucleon sector

### Uncertainty quantification for nuclear interactions:

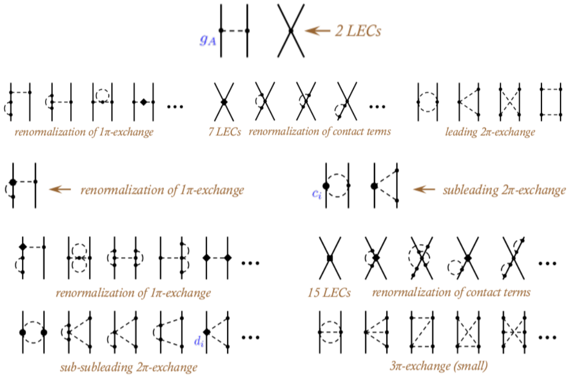
- An estimation by comparing predictions based on various models of nuclear interactions
    - The spread of predictions obtained by numerous models like AV18, CD-Bonn, chiral models, ...
- Application of a covariance matrix of $2N$ potential parameters to estimate uncertainty
    - the statistical uncertainties from an error propagation of potential parameters uncertainties to various nuclear observables
- Utilizing power-counting arguments to estimate the systematic uncertainties
    - truncation errors for $\chi$EFT's
- Bayesian: can fit the above methods into this framework
- Theoretical methods introduce their own uncertainties (small in the Faddeev approach for *Nd* scattering) and suffer from finite computational accuracy

# Two-nucleon forces from $\chi$EFT

From E. Epelbaum's lecture at the summer school of "Strong interaction in the nuclear medium: new trends"

Chiral expansion of the $2N$ force: $V_{2N} = V_{2N}^{(LO)} + V_{2N}^{(NLO)} + V_{2N}^{(N^2LO)} + V_{2N}^{(N^3LO)} + V_{2N}^{(N^4LO)} + \dots$

- LO:
- NLO:
- $N^2$LO:
- $N^3$LO:




+ isospin-breaking corrections...

van Kolck et al. '93,'96; Friar et al. '99,'03,'04; Niskanen '02; Kaiser '06; E.E. et al. '04,'05,'07; ...

- The newest and the best model "SMS" is chiral $N^4$LO potential with semilocal regularization in momentum space, the SMS $N^4$LO+ (2018, Bochum (LENPIC)) ⟵ 27 LECs

  P. Reinert presented this model at the session yesterday.

  P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54, 86 (2018).

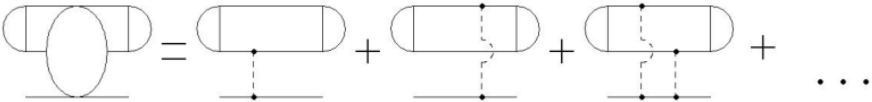
# The OPE-Gaussian potential

R. Navarro Pérez, J. E. Amaro, and E. Ruiz Arriola, Phys. Rev. C 89 (2014) 064006

- The OPE-Gaussian interaction can be decomposed as
$$V(\vec{r}) = V_{short}(\vec{r})\theta(r_c - r) + V_{long}(\vec{r})\theta(r - r_c)$$
where $r_c$ = 3 fm.
- The long range part has two parts: OPE part and electromagnetic corrections
$$V_{long}(\vec{r}) = V_{OPE}(\vec{r}) + V_{em}(\vec{r})$$
- The short range part is
$$V_{short}(\vec{r}) = \sum_{n=1}^{18} \hat{O}_n \left[ \sum_{i=1}^{4} V_{i,n} F_i(r) \right], \; F_i(r) = e^{-r^2/(2a_i^2)}$$
where $\hat{O}_n$ are the same operators as in the AV18 + three additional operators; $V_{i,n}$ and $a_i$ are unknown coefficients to be determined from NN data, $F_i$ are radial Gaussian functions.
- Authors prepared and used “3$\sigma$ self-consistent database” to fix free parameters.
- **Finally, they obtained values of all 42 free parameters and their uncertainties (statistically well defined standard deviations and correlation coefficients).**
- The OPE-Gaussian force can be seen as a remastered the AV18 interaction.

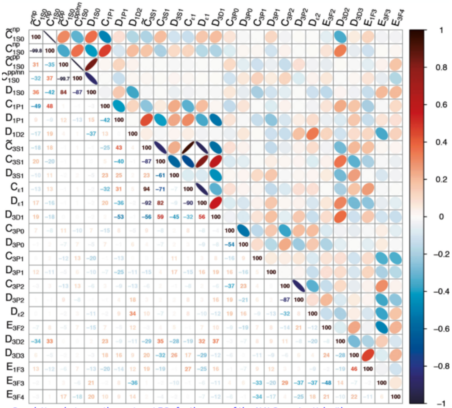
# Formalism for $2N$ and $3N$ scattering

- $2N$ bound state: Schrödinger equation,
- $2N$ scattering state: Lippmann-Schwinger equation for the $t$-matrix (interaction + free propagation)
  $t(E) = VG_0(E)V + VG_0(E)VG_0(E)V + \ldots$
  $G_0(E) \equiv \lim_{\epsilon \to 0^+} \frac{1}{E - H_0 + i\epsilon}$
- $3N$: Faddeev equation:
  $T = tP\phi + (1 + tG_0)V_{123}^{(1)}(1 + P)\phi + tPG_0T + (1 + tG_0)V_{123}^{(1)}(1 + P)G_0T$
- In the presented work we neglect the $3N$ interactions and apply only the two-body force, which enters the Faddeev equation via the $t$-matrix operator
  $T = tP\phi + tPG_0T$
  and in this case, we have the transition amplitude $U = PG_0^{-1} + PT$
  which can be represented given this diagram



# How to estimate statistical uncertainties?

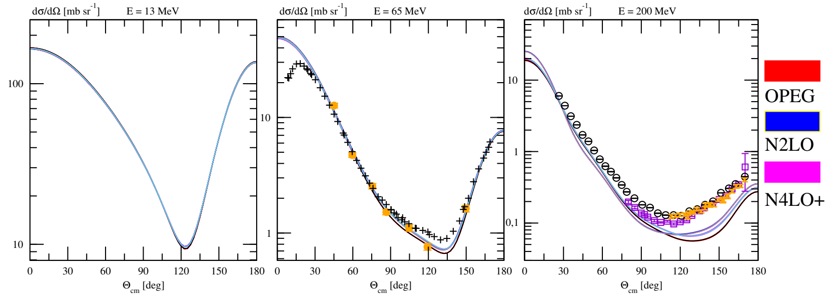
- Statistical uncertainties – here: uncertainties of 3*N* observables arising from uncertainties of 2*N* force parameters.
- Knowing 2*N* force parameters and their correlation matrix we sample many (50) sets of potential parameters.
- For each set we solve Faddeev equation and compute 3*N* observables.
- Thus for each observable (at given energy and scattering angle) we have 50+1 predictions.
- Basing on these predictions we estimate the uncertainty of given 3*N* observable. This can be done in various ways, which in practice leads to similar results. We use $\Delta_{68\%}$ – a difference between maximal and minimal value which are taken over 34 (68% of 50) predictions based on different sets of the 2*N* potential parameters.




Correlations between the various LECs for the case of the N4LO+ potential with cutoff dependence Λ = 450 MeV. The lower triangle gives the correlation coefficients in percent.
P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54, 86(2018).

# Propagation of statistical errors with chiral forces

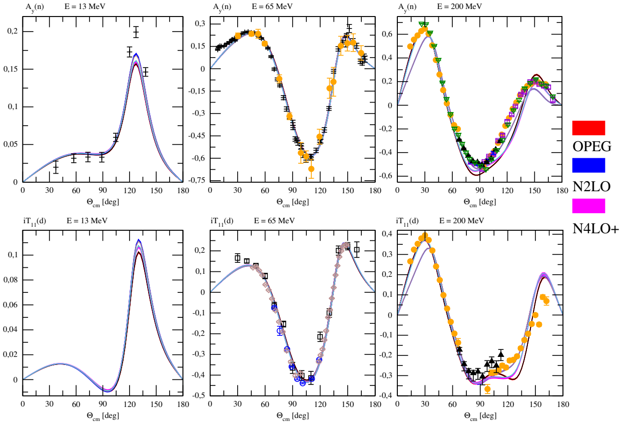
- The OPE-Gaussian and the new SMS potentials allow us to study propagation of uncertainties to 3*N* system.




- Statistical errors for the chiral SMS force are of similar magnitude as the ones for the OPE-Gaussian.
- Similar magnitudes at $N^2$LO and $N^4$LO+.

# Propagation of statistical errors with chiral forces

$\vec{n} + d \to n + d$




$n + \vec{d} \to n + d$

$A_y(n)$ — the neutron vector analyzing power
$iT_{11}(d)$ — the deuteron tensor analyzing power

- Statistical errors for the chiral SMS forces are of similar magnitude as the ones for the OPE-Gaussian.

# Correlations among observables in few-nucleon systems

GOAL:

- explore correlations between 2*N* and 3*N* observables
- could impact on future methods of fixing free parameters of the 2*N* and N-body potentials, especially the case of correlations in a 3*N* system should deliver information on possible restrictions on data sets used during fitting the 3*N* potential parameters

Examples

- Analysis of correlations among the 3*N* observables.
- Angular dependence of correlation coefficients for pairs of 3*N* observables. Correlation coefficient for a given pair of 3*N* observables can depends on:
  - a scattering energy;
  - a scattering angle;
  - a model of *NN* interaction;
  - an order of chiral expansion

# Correlation coefficients between choosen 3$N$ observables

$E = 13\ MeV, \theta = 45^\circ$

**OPEG**

| | $d\sigma/d\theta$ | $A_y(n)$ | $iT_{11}(d)$ | $T_{20}$ | $T_{21}$ | $T_{22}$ |
|---|---|---|---|---|---|---|
| $d\sigma/d\theta$ | 1. | −0.31 | −0.24 | 0.084 | −0.043 | −0.11 |
| $A_y(n)$ | −0.31 | 1. | 0.89 | 0.37 | −0.35 | −0.33 |
| $iT_{11}(d)$ | −0.24 | 0.89 | 1. | 0.6 | −0.57 | −0.52 |
| $T_{20}$ | 0.084 | 0.37 | 0.6 | 1. | −0.99 | −0.84 |
| $T_{21}$ | −0.043 | −0.35 | −0.57 | −0.99 | 1. | 0.8 |
| $T_{22}$ | −0.11 | −0.33 | −0.52 | −0.84 | 0.8 | 1. |

**N$^2$LO**

| | $d\sigma/d\theta$ | $A_y(n)$ | $iT_{11}(d)$ | $T_{20}$ | $T_{21}$ | $T_{22}$ |
|---|---|---|---|---|---|---|
| $d\sigma/d\theta$ | 1. | −0.46 | −0.44 | −0.13 | 0.2 | 0.11 |
| $A_y(n)$ | −0.46 | 1. | 0.99 | 0.9 | −0.92 | −0.89 |
| $iT_{11}(d)$ | −0.44 | 0.99 | 1. | 0.88 | −0.89 | −0.87 |
| $T_{20}$ | −0.13 | 0.9 | 0.88 | 1. | −1. | −0.98 |
| $T_{21}$ | 0.2 | −0.92 | −0.89 | −1. | 1. | 0.98 |
| $T_{22}$ | 0.11 | −0.89 | −0.87 | −0.98 | 0.98 | 1. |

**N$^4$LO**

| | $d\sigma/d\theta$ | $A_y(n)$ | $iT_{11}(d)$ | $T_{20}$ | $T_{21}$ | $T_{22}$ |
|---|---|---|---|---|---|---|
| $d\sigma/d\theta$ | 1. | 0.096 | 0.25 | 0.62 | −0.58 | −0.56 |
| $A_y(n)$ | 0.096 | 1. | 0.94 | 0.68 | −0.68 | −0.74 |
| $iT_{11}(d)$ | 0.25 | 0.94 | 1. | 0.8 | −0.8 | −0.78 |
| $T_{20}$ | 0.62 | 0.68 | 0.8 | 1. | −1. | −0.93 |
| $T_{21}$ | −0.58 | −0.68 | −0.8 | −1. | 1. | 0.91 |
| $T_{22}$ | −0.56 | −0.74 | −0.78 | −0.93 | 0.91 | 1. |

**N$^4$LO+**

| | $d\sigma/d\theta$ | $A_y(n)$ | $iT_{11}(d)$ | $T_{20}$ | $T_{21}$ | $T_{22}$ |
|---|---|---|---|---|---|---|
| $d\sigma/d\theta$ | 1. | −0.62 | −0.8 | 0.2 | 0.099 | −0.48 |
| $A_y(n)$ | −0.62 | 1. | 0.92 | 0.52 | −0.69 | −0.3 |
| $iT_{11}(d)$ | −0.8 | 0.92 | 1. | 0.35 | −0.6 | −0.031 |
| $T_{20}$ | 0.2 | 0.52 | 0.35 | 1. | −0.95 | −0.89 |
| $T_{21}$ | 0.099 | −0.69 | −0.6 | −0.95 | 1. | 0.73 |
| $T_{22}$ | −0.48 | −0.3 | −0.031 | −0.89 | 0.73 | 1. |

$E = 65\ MeV, \theta = 45^\circ$

**OPEG**

| | $d\sigma/d\theta$ | $A_y(n)$ | $iT_{11}(d)$ | $T_{20}$ | $T_{21}$ | $T_{22}$ |
|---|---|---|---|---|---|---|
| $d\sigma/d\theta$ | 1. | −0.43 | −0.48 | 0.6 | −0.75 | 0.67 |
| $A_y(n)$ | −0.43 | 1. | 0.98 | −0.19 | 0.19 | −0.34 |
| $iT_{11}(d)$ | −0.48 | 0.98 | 1. | −0.22 | 0.21 | −0.4 |
| $T_{20}$ | 0.6 | −0.19 | −0.22 | 1. | −0.95 | 0.17 |
| $T_{21}$ | −0.75 | 0.19 | 0.21 | −0.95 | 1. | −0.27 |
| $T_{22}$ | 0.67 | −0.34 | −0.4 | 0.17 | −0.27 | 1. |

**N$^2$LO**

| | $d\sigma/d\theta$ | $A_y(n)$ | $iT_{11}(d)$ | $T_{20}$ | $T_{21}$ | $T_{22}$ |
|---|---|---|---|---|---|---|
| $d\sigma/d\theta$ | 1. | −0.23 | −0.19 | 0.55 | −0.61 | −0.3 |
| $A_y(n)$ | −0.23 | 1. | 0.99 | 0.62 | −0.57 | −0.63 |
| $iT_{11}(d)$ | −0.19 | 0.99 | 1. | 0.66 | −0.62 | −0.62 |
| $T_{20}$ | 0.55 | 0.62 | 0.66 | 1. | −0.99 | −0.78 |
| $T_{21}$ | −0.61 | −0.57 | −0.62 | −0.99 | 1. | 0.77 |
| $T_{22}$ | −0.3 | −0.63 | −0.62 | −0.78 | 0.77 | 1. |

**N$^4$LO**

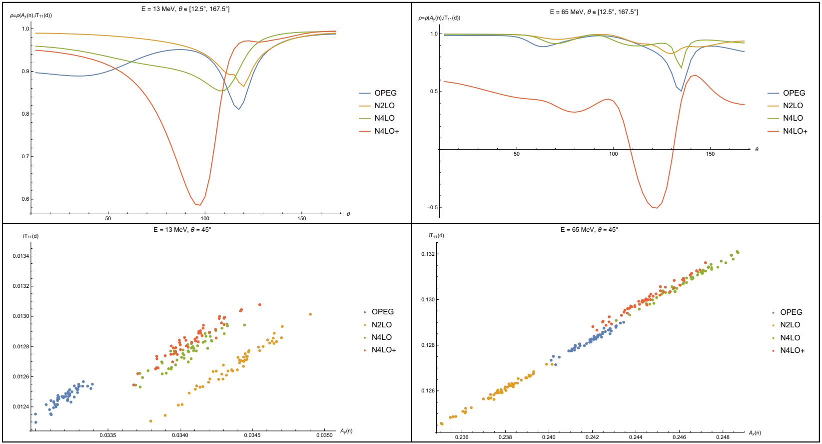
| | $d\sigma/d\theta$ | $A_y(n)$ | $iT_{11}(d)$ | $T_{20}$ | $T_{21}$ | $T_{22}$ |
|---|---|---|---|---|---|---|
| $d\sigma/d\theta$ | 1. | −0.48 | −0.49 | 0.39 | −0.52 | 0.4 |
| $A_y(n)$ | −0.48 | 1. | 0.99 | 0.11 | −0.0078 | −0.56 |
| $iT_{11}(d)$ | −0.49 | 0.99 | 1. | 0.17 | −0.068 | −0.61 |
| $T_{20}$ | 0.39 | 0.11 | 0.17 | 1. | −0.98 | −0.54 |
| $T_{21}$ | −0.52 | −0.0078 | −0.068 | −0.98 | 1. | 0.45 |
| $T_{22}$ | 0.4 | −0.56 | −0.61 | −0.54 | 0.45 | 1. |

**N$^4$LO+**

| | $d\sigma/d\theta$ | $A_y(n)$ | $iT_{11}(d)$ | $T_{20}$ | $T_{21}$ | $T_{22}$ |
|---|---|---|---|---|---|---|
| $d\sigma/d\theta$ | 1. | −0.45 | −0.48 | 0.25 | −0.037 | 0.37 |
| $A_y(n)$ | −0.45 | 1. | 0.99 | 0.22 | −0.25 | −0.63 |
| $iT_{11}(d)$ | −0.48 | 0.99 | 1. | 0.28 | −0.31 | −0.67 |
| $T_{20}$ | 0.25 | 0.22 | 0.28 | 1. | −0.95 | −0.43 |
| $T_{21}$ | −0.037 | −0.25 | −0.31 | −0.95 | 1. | 0.37 |
| $T_{22}$ | 0.37 | −0.63 | −0.67 | −0.43 | 0.37 | 1. |

A few pairs of 3$N$ observables which are strongly correlated/uncorrelated independently on the nuclear model and scattering energy exist! They are, e.g., $A_y$ vs i$T_{11}$; $T_{20}$ vs $T_{21}$; $T_{20}$ vs $T_{22}$ .

# Angular dependence of correlation coefficients between $A_y(n)$ and $iT_{11}(d)$ at $E = 13$ and 65 MeV




Predictions obtained with the chiral $N^4LO+$ SMS force differs from remain results.

# Summary

## Part I

- The dominant theoretical uncertainties arise from using various models of the *NN* interaction.
- The statistical errors are small.
- In general, the theoretical uncertainties remain smaller than the experimental ones.

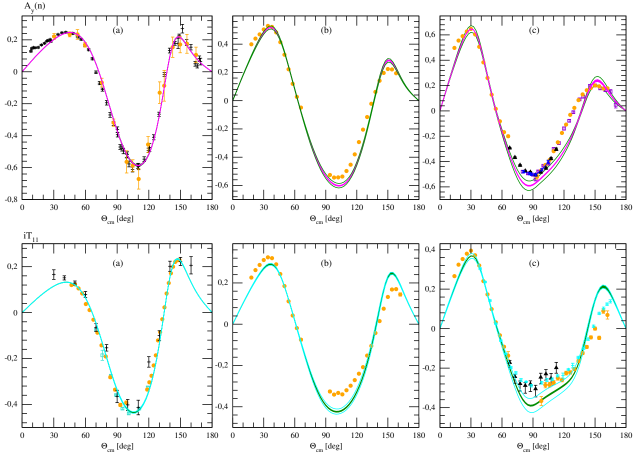
More discussion about theoretical uncertainties in
R.Skibiński, Yu. Volkotrub. et al., Phys. Rev. C98, 014001 (2018).

## Part II

- We have investigated correlations between various three-nucleon observables.
- We have found pairs of 3*N* observables which are strongly correlated or remain uncorrelated independently on the model of nuclear forces and reaction energy.
- We expect the reason is probably as a sensitivity of various observables to different partial waves contributions to the scattering amplitude.
- This research is ongoing.

Thank you for your kind attention!

# Statistical & Systematic (truncation) Errors



# Correlation coefficients between 2N observables with various chiral forces

| | dσ/dθ | P | R | $R'$ | A | $A'$ | $C_{NN}$ | D |
|---|---|---|---|---|---|---|---|---|
| dσ/dθ | 1. | 0.18 | 0.79 | 0.48 | -0.77 | 0.48 | 0.5 | 0.84 |
| P | 0.18 | 1. | -0.14 | -0.41 | 0.061 | -0.49 | 0.47 | -0.088 |
| R | 0.79 | -0.14 | 1. | 0.87 | -0.99 | 0.88 | 0.029 | 0.99 |
| $R'$ | 0.48 | -0.41 | 0.87 | 1. | -0.88 | 0.99 | -0.42 | 0.82 |
| A | -0.77 | 0.061 | -0.99 | -0.88 | 1. | -0.87 | 0.04 | -0.98 |
| $A'$ | 0.48 | -0.49 | 0.88 | 0.99 | -0.87 | 1. | -0.37 | 0.83 |
| $C_{NN}$ | 0.5 | 0.47 | 0.029 | -0.42 | 0.04 | -0.37 | 1. | 0.13 |
| D | 0.84 | -0.088 | 0.99 | 0.82 | -0.98 | 0.83 | 0.13 | 1. |

| | dσ/dθ | P | R | $R'$ | A | $A'$ | $C_{NN}$ | D |
|---|---|---|---|---|---|---|---|---|
| dσ/dθ | 1. | -0.15 | 0.61 | 0.59 | -0.6 | 0.59 | -0.2 | 0.63 |
| P | -0.15 | 1. | -0.23 | -0.53 | 0.15 | -0.61 | 0.54 | -0.19 |
| R | 0.61 | -0.23 | 1. | 0.85 | -0.98 | 0.84 | -0.32 | 1. |
| $R'$ | 0.59 | -0.53 | 0.85 | 1. | -0.85 | 0.99 | -0.71 | 0.81 |
| A | -0.6 | 0.15 | -0.98 | -0.85 | 1. | -0.82 | 0.38 | -0.97 |
| $A'$ | 0.59 | -0.61 | 0.84 | 0.99 | -0.82 | 1. | -0.67 | 0.81 |
| $C_{NN}$ | -0.2 | 0.54 | -0.32 | -0.71 | 0.38 | -0.67 | 1. | -0.24 |
| D | 0.63 | -0.19 | 1. | 0.81 | -0.97 | 0.81 | -0.24 | 1. |

| | dσ/dθ | P | R | $R'$ | A | $A'$ | $C_{NN}$ | D |
|---|---|---|---|---|---|---|---|---|
| dσ/dθ | 1. | -0.096 | 0.95 | 0.87 | -0.94 | 0.88 | -0.57 | 0.95 |
| P | -0.096 | 1. | -0.13 | -0.29 | 0.093 | -0.32 | 0.36 | -0.11 |
| R | 0.95 | -0.13 | 1. | 0.95 | -1. | 0.96 | -0.66 | 1. |
| $R'$ | 0.87 | -0.29 | 0.95 | 1. | -0.95 | 1. | -0.83 | 0.94 |
| A | -0.94 | 0.093 | -1. | -0.95 | 1. | -0.95 | 0.67 | -0.99 |
| $A'$ | 0.88 | -0.32 | 0.96 | 1. | -0.95 | 1. | -0.81 | 0.94 |
| $C_{NN}$ | -0.57 | 0.36 | -0.66 | -0.83 | 0.67 | -0.81 | 1. | -0.62 |
| D | 0.95 | -0.11 | 1. | 0.94 | -0.99 | 0.94 | -0.62 | 1. |

| | dσ/dθ | P | R | $R'$ | A | $A'$ | $C_{NN}$ | D |
|---|---|---|---|---|---|---|---|---|
| dσ/dθ | 1. | -0.25 | 0.93 | 0.87 | -0.93 | 0.86 | -0.66 | 0.94 |
| P | -0.25 | 1. | -0.35 | -0.5 | 0.29 | -0.54 | 0.53 | -0.33 |
| R | 0.93 | -0.35 | 1. | 0.96 | -0.99 | 0.95 | -0.76 | 1. |
| $R'$ | 0.87 | -0.5 | 0.96 | 1. | -0.95 | 1. | -0.88 | 0.94 |
| A | -0.93 | 0.29 | -0.99 | -0.95 | 1. | -0.94 | 0.75 | -0.99 |
| $A'$ | 0.86 | -0.54 | 0.95 | 1. | -0.94 | 1. | -0.88 | 0.94 |
| $C_{NN}$ | -0.66 | 0.53 | -0.76 | -0.88 | 0.75 | -0.88 | 1. | -0.72 |
| D | 0.94 | -0.33 | 1. | 0.94 | -0.99 | 0.94 | -0.72 | 1. |

$E = 10\ MeV, \theta = 45^\circ$

| | dσ/dθ | P | R | $R'$ | A | $A'$ | $C_{NN}$ | D |
|---|---|---|---|---|---|---|---|---|
| dσ/dθ | 1. | 0.22 | -0.5 | -0.42 | 0.34 | -0.5 | 0.18 | -0.54 |
| P | 0.22 | 1. | -0.49 | -0.32 | -0.031 | -0.71 | 0.57 | -0.28 |
| R | -0.5 | -0.49 | 1. | 0.82 | -0.79 | 0.9 | -0.56 | 0.96 |
| $R'$ | -0.42 | -0.32 | 0.82 | 1. | -0.9 | 0.88 | -0.84 | 0.7 |
| A | 0.34 | -0.031 | -0.79 | -0.9 | 1. | -0.65 | 0.55 | -0.78 |
| $A'$ | -0.5 | -0.71 | 0.9 | 0.88 | -0.65 | 1. | -0.82 | 0.74 |
| $C_{NN}$ | 0.18 | 0.57 | -0.56 | -0.84 | 0.55 | -0.82 | 1. | -0.31 |
| D | -0.54 | -0.28 | 0.96 | 0.7 | -0.78 | 0.74 | -0.31 | 1. |

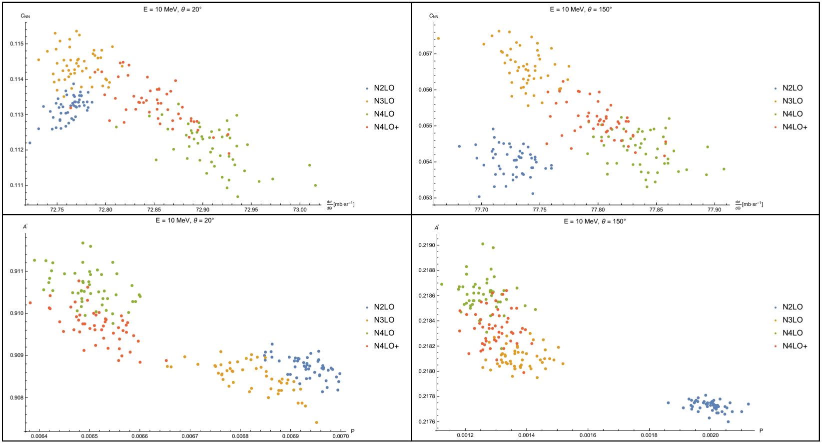
| | dσ/dθ | P | R | $R'$ | A | $A'$ | $C_{NN}$ | D |
|---|---|---|---|---|---|---|---|---|
| dσ/dθ | 1. | 0.22 | -0.24 | -0.31 | 0.27 | -0.25 | 0.21 | -0.22 |
| P | 0.22 | 1. | -0.4 | -0.25 | -0.18 | -0.7 | 0.55 | -0.17 |
| R | -0.24 | -0.4 | 1. | 0.72 | -0.71 | 0.81 | -0.53 | 0.95 |
| $R'$ | -0.31 | -0.25 | 0.72 | 1. | -0.82 | 0.82 | -0.87 | 0.59 |
| A | 0.27 | -0.18 | -0.71 | -0.82 | 1. | -0.46 | 0.47 | -0.73 |
| $A'$ | -0.25 | -0.7 | 0.81 | 0.82 | -0.46 | 1. | -0.86 | 0.62 |
| $C_{NN}$ | 0.21 | 0.55 | -0.53 | -0.87 | 0.47 | -0.86 | 1. | -0.28 |
| D | -0.22 | -0.17 | 0.95 | 0.59 | -0.73 | 0.62 | -0.28 | 1. |

| | dσ/dθ | P | R | $R'$ | A | $A'$ | $C_{NN}$ | D |
|---|---|---|---|---|---|---|---|---|
| dσ/dθ | 1. | -0.14 | 0.9 | 0.83 | -0.84 | 0.85 | -0.71 | 0.89 |
| P | -0.14 | 1. | -0.29 | -0.25 | 0.037 | -0.46 | 0.48 | -0.18 |
| R | 0.9 | -0.29 | 1. | 0.93 | -0.94 | 0.95 | -0.79 | 0.99 |
| $R'$ | 0.83 | -0.25 | 0.93 | 1. | -0.94 | 0.95 | -0.92 | 0.88 |
| A | -0.84 | 0.037 | -0.94 | -0.94 | 1. | -0.86 | 0.75 | -0.94 |
| $A'$ | 0.85 | -0.46 | 0.95 | 0.95 | -0.86 | 1. | -0.92 | 0.89 |
| $C_{NN}$ | -0.71 | 0.48 | -0.79 | -0.92 | 0.75 | -0.92 | 1. | -0.68 |
| D | 0.89 | -0.18 | 0.99 | 0.88 | -0.94 | 0.89 | -0.68 | 1. |

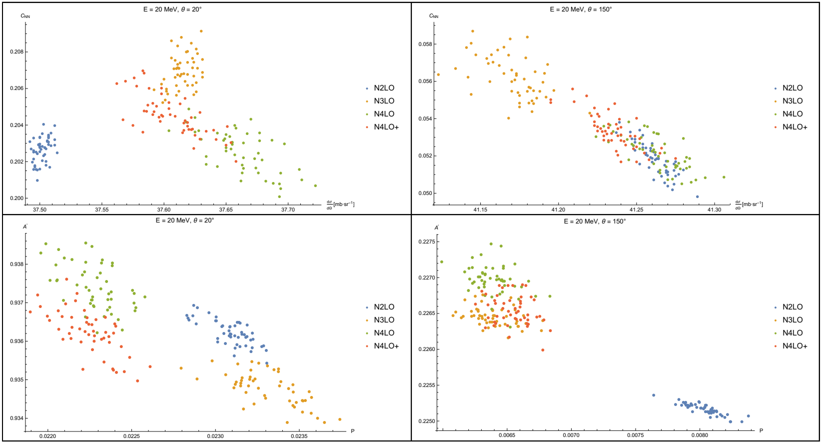
| | dσ/dθ | P | R | $R'$ | A | $A'$ | $C_{NN}$ | D |
|---|---|---|---|---|---|---|---|---|
| dσ/dθ | 1. | -0.34 | 0.84 | 0.77 | -0.71 | 0.81 | -0.76 | 0.82 |
| P | -0.34 | 1. | -0.53 | -0.4 | 0.14 | -0.66 | 0.62 | -0.42 |
| R | 0.84 | -0.53 | 1. | 0.93 | -0.88 | 0.96 | -0.87 | 0.99 |
| $R'$ | 0.77 | -0.4 | 0.93 | 1. | -0.93 | 0.94 | -0.93 | 0.91 |
| A | -0.71 | 0.14 | -0.88 | -0.93 | 1. | -0.78 | 0.74 | -0.91 |
| $A'$ | 0.81 | -0.66 | 0.96 | 0.94 | -0.78 | 1. | -0.96 | 0.9 |
| $C_{NN}$ | -0.76 | 0.62 | -0.87 | -0.93 | 0.74 | -0.96 | 1. | -0.78 |
| D | 0.82 | -0.42 | 0.99 | 0.91 | -0.91 | 0.9 | -0.78 | 1. |

$E = 20\ MeV, \theta = 45^\circ$

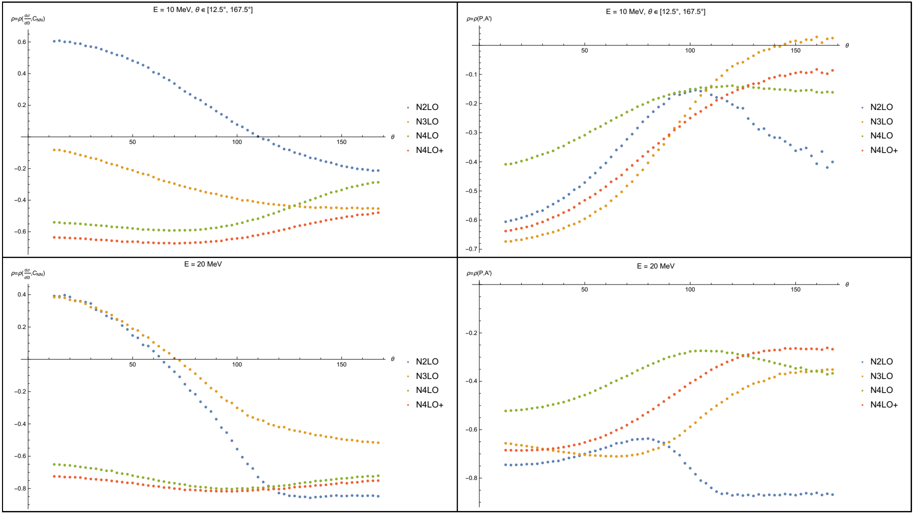
# Correlation table of correlation coefficients between 2N observables with various chiral forces at $E = 10$ MeV



# Correlation coefficients between 2N observables with various chiral forces at $E = 20$ MeV



# Angular dependence of correlation coefficients between 2N observables with various chiral forces



# Nuclear forces from $\chi$EFT - regularization

- Chiral forces (2N,3N, ...) require regularization to avoid divergences in the Lippmann-Schwinger equation and in $\pi - \pi$ loops.
  Various solutions have been proposed. They are:
- **Nonlocal regularization**
  (convenient for applications but introduces unwanted artifacts in a long-range part of interaction)
  in momentum space $V(p^{'}, p) \rightarrow V(p^{'}, p) f(p^{'}, p)$ with
  $f(p^{'}, p) \equiv exp\left(-\left(\frac{p^{'}}{\Lambda}\right)^{2n} - \left(\frac{p}{\Lambda}\right)^{2n}\right)$ where $\Lambda = 450 - 550$ MeV and $n = 2, 3, 4, \ldots$