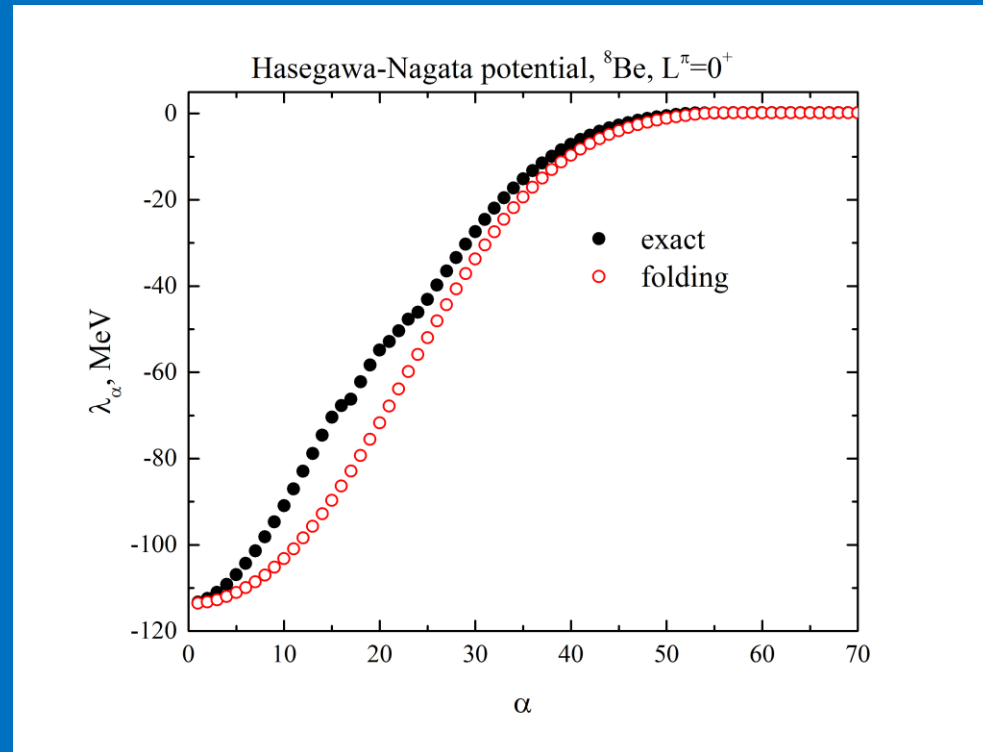
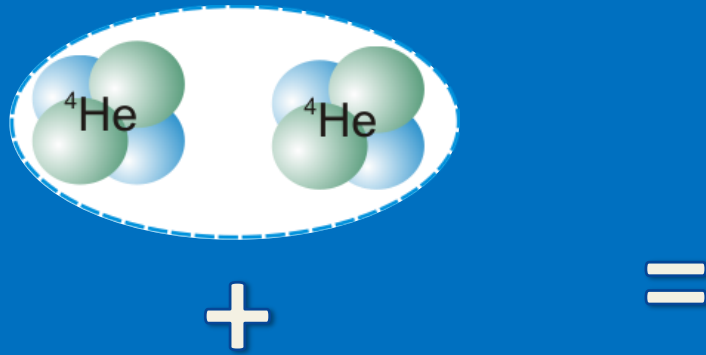


Influence of the Pauli principle on two-cluster potential energy



Yuliya Lashko, Viktor Vasilevsky and **Gennady Filippov**

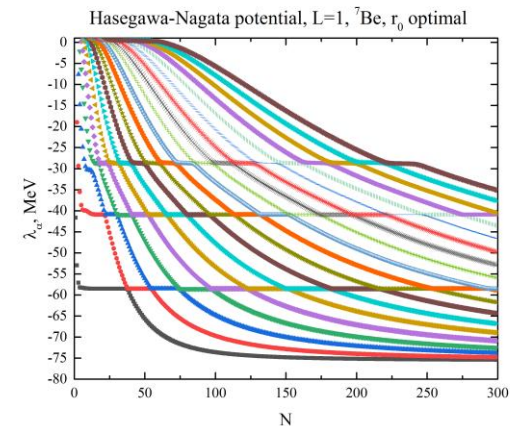
*Bogolyubov Institute for Theoretical Physics
Kyiv, Ukraine*



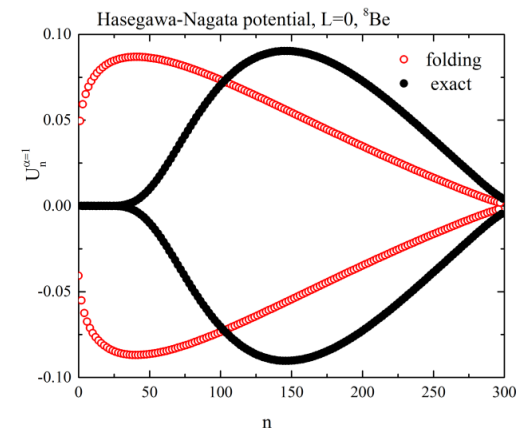
This presentation studies effects of the antisymmetrization on potential energy of two interacting nuclear clusters

1. Potential energy matrix $\hat{V}_N(r, \tilde{r}) = \sum_{n,m=0}^{N-1} \Phi_{nL}(r, b) \langle n | \hat{V} | m \rangle \Phi_{mL}(\tilde{r}, b)$

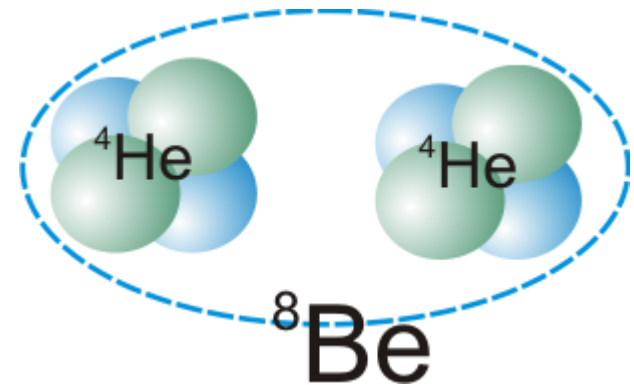
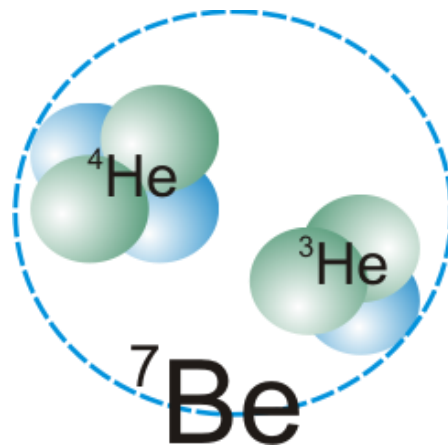
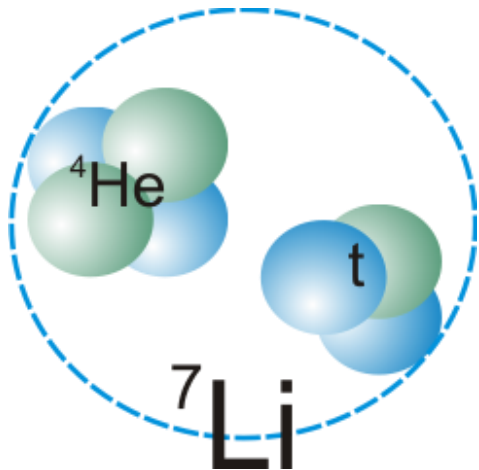
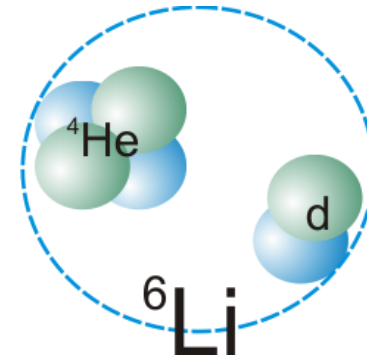
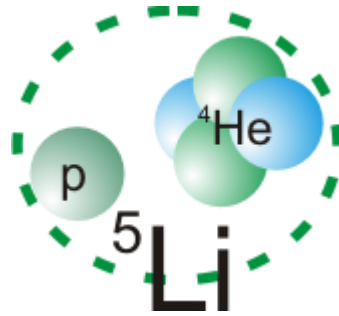
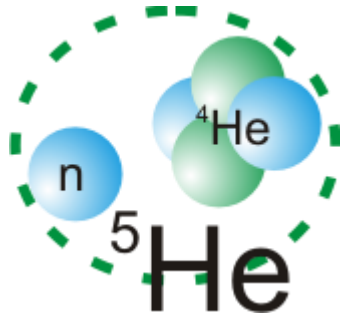
2. **Eigenvalues** of potential energy operator: folding vs exact potential



3. **Eigenfunctions** of potential energy operator : folding vs exact potential



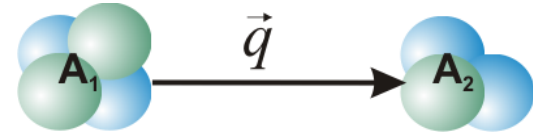
We analyse two-cluster potential energy for the lightest nuclei of p-shell with a pronounced two-cluster structure



We explore a microscopic two-cluster model of light nuclei

Intrinsic cluster w.f.

$$\Psi_{LM} = \hat{\mathcal{A}} \{ [\psi_1(A_1, s_1, b) \psi_2(A_2, s_2, b)]_S f_L(q) Y_{LM}(\hat{\mathbf{q}}) \}$$



Antisymmetrization operator

W.f. of relative motion

Algebraic version of Resonating Group Method: $\Psi_{LM} = \sum_{n=0}^{\infty} C_{nL} |n, L, M\rangle_A$

number of quanta

Cluster oscillator functions $|n, L, M\rangle_A = \hat{\mathcal{A}} \{ \psi_1(A_1) \psi_2(A_2) \Phi_{nL}(q, b) Y_{LM}(\hat{\mathbf{q}}) \}$

A set of linear equations is solved to find expansion coefficients C_{mL} :

$$\sum_{m=0}^{\infty} \left\{ \langle n, L | \hat{H} | m, L \rangle_A - E \cdot \Lambda_m \delta_{n,m} \right\} C_{mL} = 0$$

Eigenvalues of the norm kernel

$$\langle n, L | \tilde{n}, L \rangle_A = \Lambda_n \delta_{n,\tilde{n}}$$

Inter-cluster wave function in r - and p -space can be expanded into a set of h.o. functions with the same coefficients

$$f_L(q) = \sum_{n=0}^{\infty} C_n \Phi_{nL}(q, b) \quad f_L(p) = \sum_{n=0}^{\infty} C_n \Phi_{nL}(p, b)$$

Oscillator functions:

$$\Phi_{nL}(r, b) = (-1)^n \mathcal{N}_{nL} b^{-3/2} \rho^L e^{-\frac{1}{2}\rho^2} L_n^{L+1/2}(\rho^2), \quad \rho = \frac{r}{b}$$

$$\Phi_{nL}(p, b) = \mathcal{N}_{nL} b^{3/2} \rho^L e^{-\frac{1}{2}\rho^2} L_n^{L+1/2}(\rho^2), \quad \rho = pb,$$

where b is the oscillator length, and

$$\mathcal{N}_{nL} = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+3/2)}}$$

Neglecting Pauli principle gives folding approximation

$$\Psi_{LM}^{(F)} = [\psi_1(A_1, s_1, b) \psi_2(A_2, s_2, b)]_S \boxed{f_L^{(F)}(q)} Y_{LM}(\hat{\mathbf{q}})$$

solution of the two-body Schrödinger equation

$$\left\{ \hat{T}_q + V^{(F)}(q) + E^{(th)} - E \right\} f_L^{(F)}(q) = 0$$

with folding potential

$$\begin{aligned} V^{(F)}(q) &= \int d\tau_1 d\tau_2 |\psi_1(A_1)|^2 |\psi_2(A_2)|^2 \sum_{i \in A_1} \sum_{j \in A_2} \hat{V}(\mathbf{r}_{ij}) \\ &= \int d\mathbf{r}_1 d\mathbf{r}_2 \boxed{\rho_1(\mathbf{r}_1) \rho_2(\mathbf{r}_2)} \hat{V}(\mathbf{r}_{12}) \end{aligned}$$

single particle local densities of the clusters

The idea is to compare the exact and folding two-cluster potentials via separable representation of the potentials

$$V_L^{(E)}(\tilde{\mathbf{q}}, \mathbf{q}) \quad \longleftrightarrow \quad \left\| \left\langle nL \left| \hat{V} \right| mL \right\rangle \right\|_N \quad V^{(F)}(q)$$

Exact two-cluster potential is a nonlocal operator

$$V_L^{(E)}(\tilde{\mathbf{q}}, q) = \left\langle \hat{\mathcal{P}}_L(\tilde{\mathbf{q}}) \left[\hat{V} \right] \hat{\mathcal{P}}_L(q) \right\rangle \quad \boxed{\hat{V} = \sum_{i < j} \hat{V}(\mathbf{r}_{ij})}$$

$$\hat{\mathcal{P}}_L(q) = \hat{\mathcal{A}} \{ [\psi_1(A_1, s_1, b) \psi_2(A_2, s_2, b)]_S \delta(r - q) Y_{LM}(\hat{\mathbf{r}}) \}$$

The potential energy matrix of dimension $N \times N$ approximates the exact potential as follows:

$$\widehat{V}_N(r, \tilde{r}) = \sum_{n,m=0}^{N-1} \Phi_{nL}(r, b) \langle nL | \widehat{V} | mL \rangle \Phi_{mL}(\tilde{r}, b),$$

$$\widehat{V}_N(p, \tilde{p}) = \sum_{n,m=0}^{N-1} \Phi_{nL}(p, b) \langle nL | \widehat{V} | mL \rangle \Phi_{mL}(\tilde{p}, b),$$

which in limiting case $N \rightarrow \infty$ coincides with the original exact form:

$$\lim_{N \rightarrow \infty} \widehat{V}_N(r, \tilde{r}) = \delta(r - \tilde{r}) \widehat{V}(r)$$

$$\lim_{N \rightarrow \infty} \widehat{V}_N(p, \tilde{p}) = \widehat{V}(p, \tilde{p})$$

Potential energy matrix can be reduced to a diagonal form

$$\left\| \left\langle nL \left| \widehat{V} \right| mL \right\rangle \right\|_N = \left\| U \right\|^{-1} \left\| \begin{array}{ccc} \lambda_1 & & \\ & \lambda_2 & \\ & & \dots \\ & & & \lambda_N \end{array} \right\| \left\| U \right\|$$

Orthogonal matrix

Eigenvectors of the potential energy matrix define new eigenfunctions

$$\phi_\alpha(r, b) = \sum_{n=0}^{N-1} U_n^\alpha \Phi_n(r, b),$$
$$\sum_{n=0}^{N-1} |U_n^\alpha|^2 = 1$$
$$\phi_\alpha(p, b) = \sum_{n=0}^{N-1} U_n^\alpha \Phi_n(p, b).$$

Oscillator basis realizes specific form of separable potentials

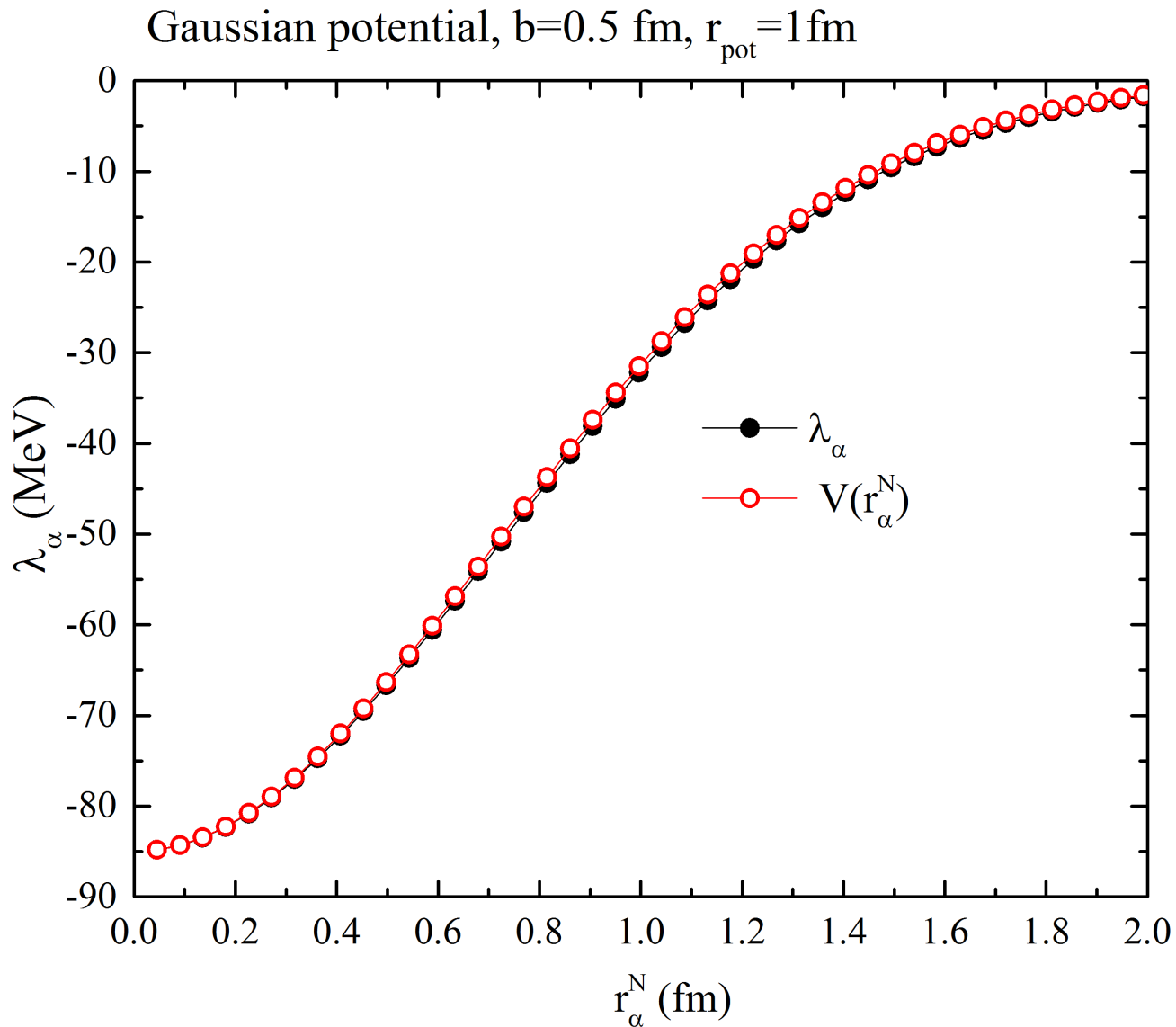
$$V_N(x, \tilde{x}) = \sum_{\alpha=1}^N \lambda_{\alpha} \phi_{\alpha}(x, b) \phi_{\alpha}(\tilde{x}, b)$$

$x = r$ OR $x = p.$

- ✓ The eigenvalues λ_{α} coincide with potential energy $V(r)$ at some discrete points r_{α}

$$\lambda_{\alpha} = V(r_{\alpha})$$

Eigenvalues perfectly coincide with the original potential energy in discrete points for oscillator length $b=r_{\text{pot}}/2$



Eigenfunctions of the folding potential operator are universal

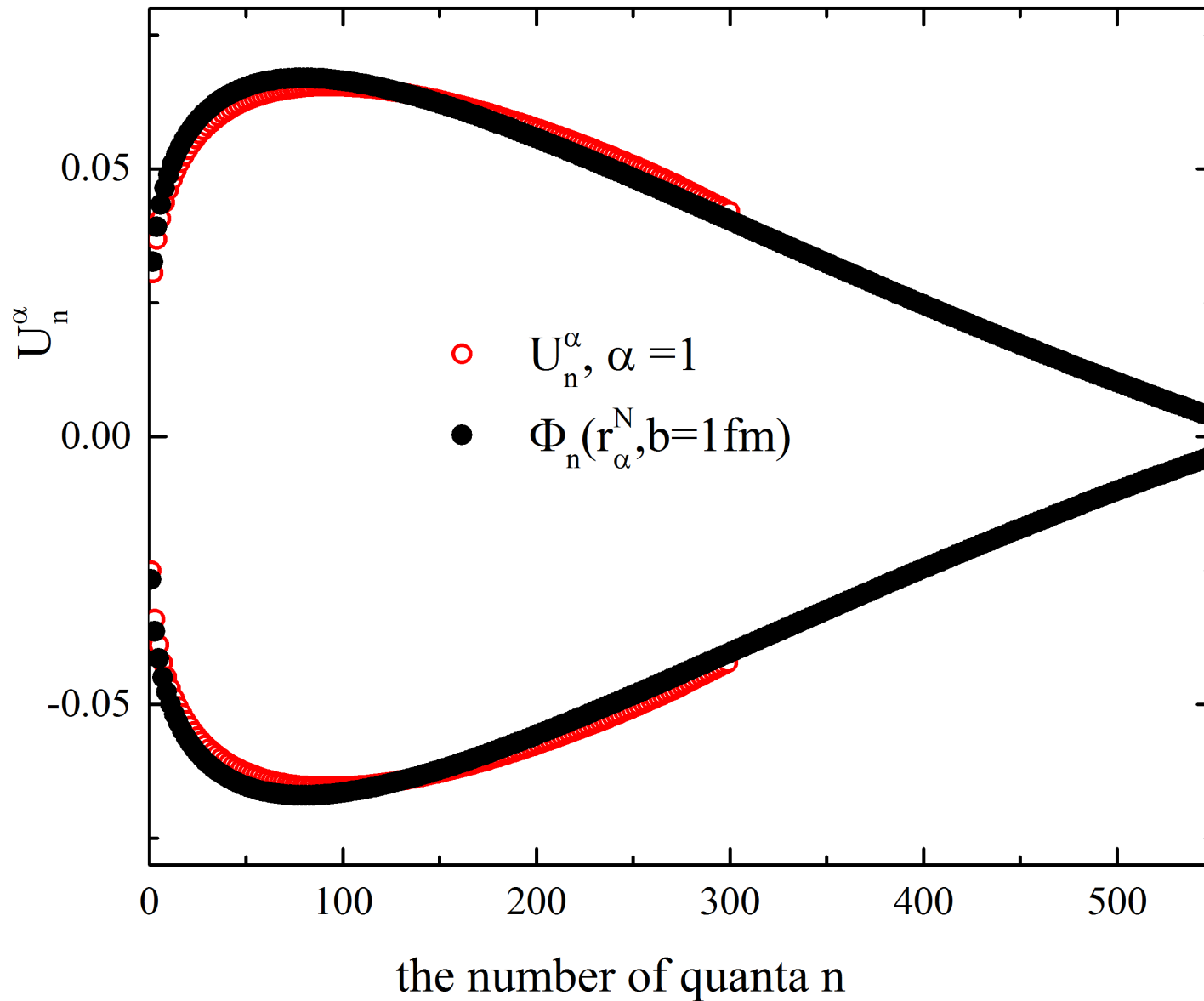
- ✓ Eigenfunctions of the folding potential energy operator in the momentum representation are Bessel functions

$$\phi_{\alpha}(p, b) = \mathcal{N}_{\alpha} \sqrt{\frac{2}{\pi}} j_L(pr_{\alpha})$$

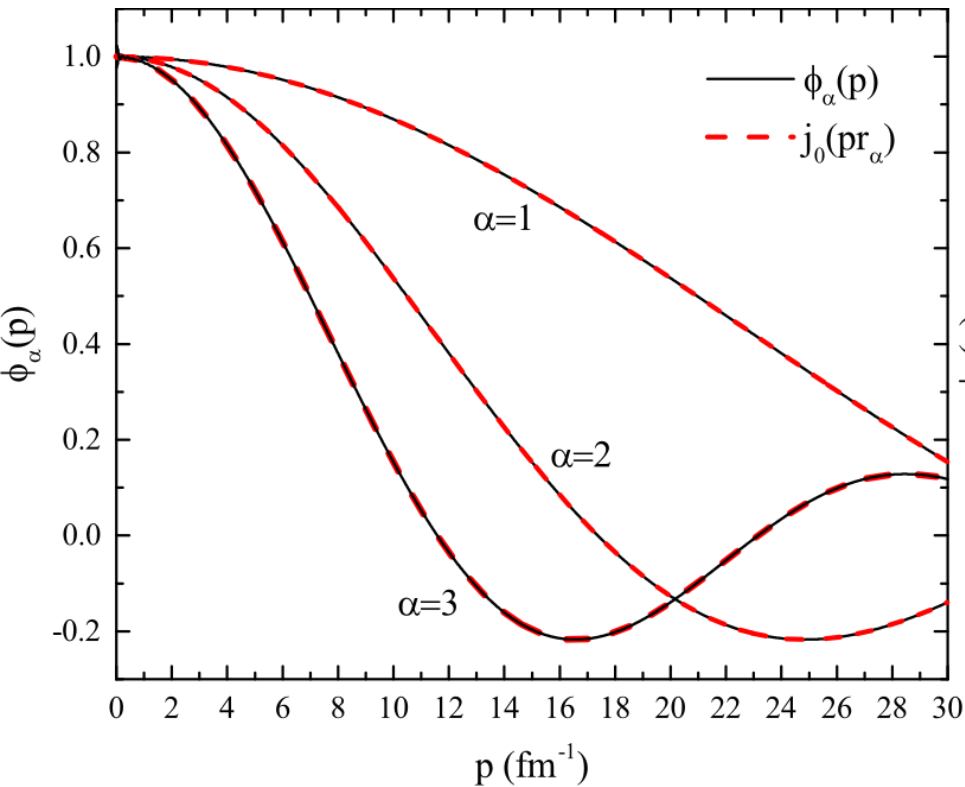
- ✓ Eigenfunctions of the folding potential energy operator in discrete representation are harmonic oscillator functions

$$U_n^{\alpha} = \langle \Phi_{nL}(p, b) | \mathcal{N}_{\alpha} \sqrt{\frac{2}{\pi}} j_L(pr_{\alpha}) \rangle = \mathcal{N}_{\alpha} \Phi_{nL}(r_{\alpha}, b)$$

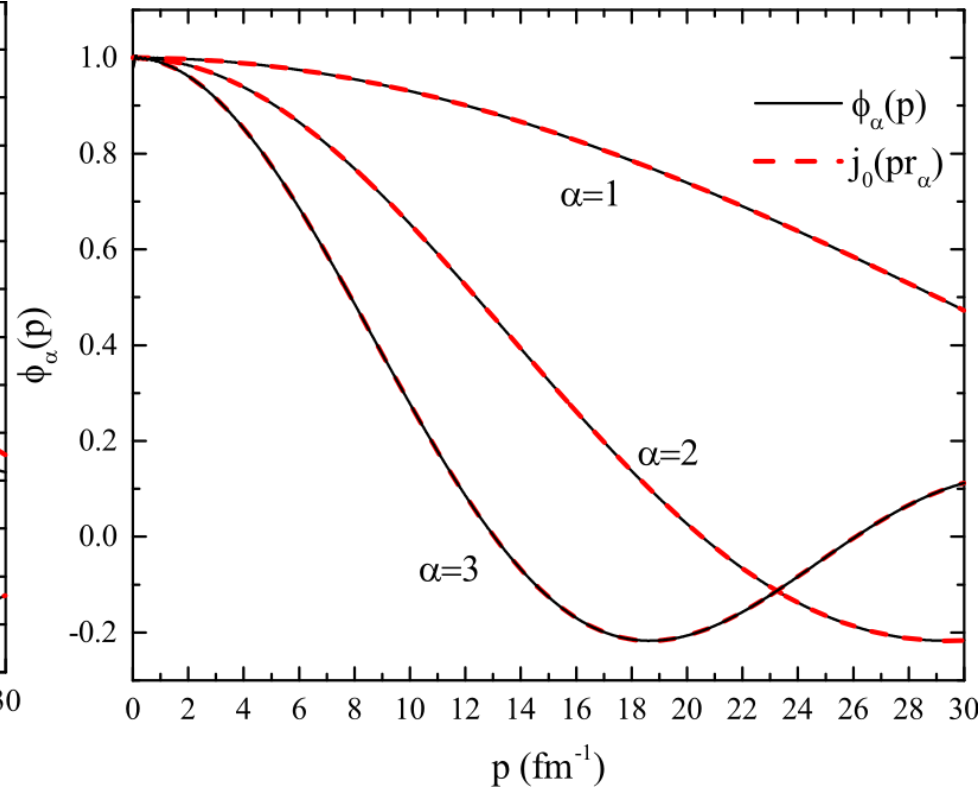
Eigenfunctions U_α^N of folding potentials in discrete representation are harmonic oscillator functions $\Phi_n(r_\alpha^N, b)$



Eigenfunctions $\phi_\alpha(p)$ of the folding potential operator in the momentum representation are Bessel functions $j_0(pr_\alpha)$

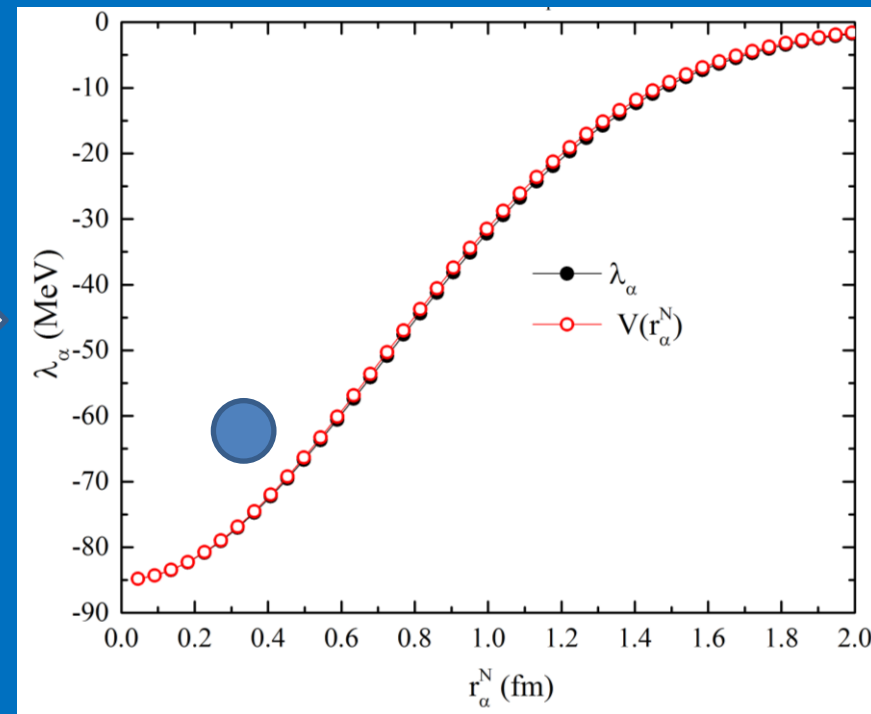
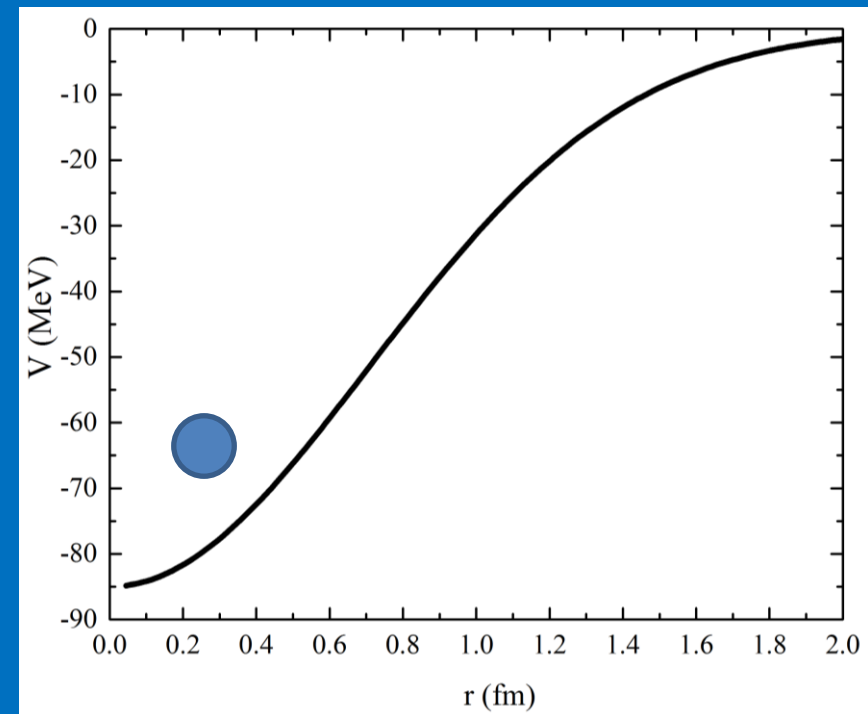


Gaussian potential, $b=r_{\text{pot}}$



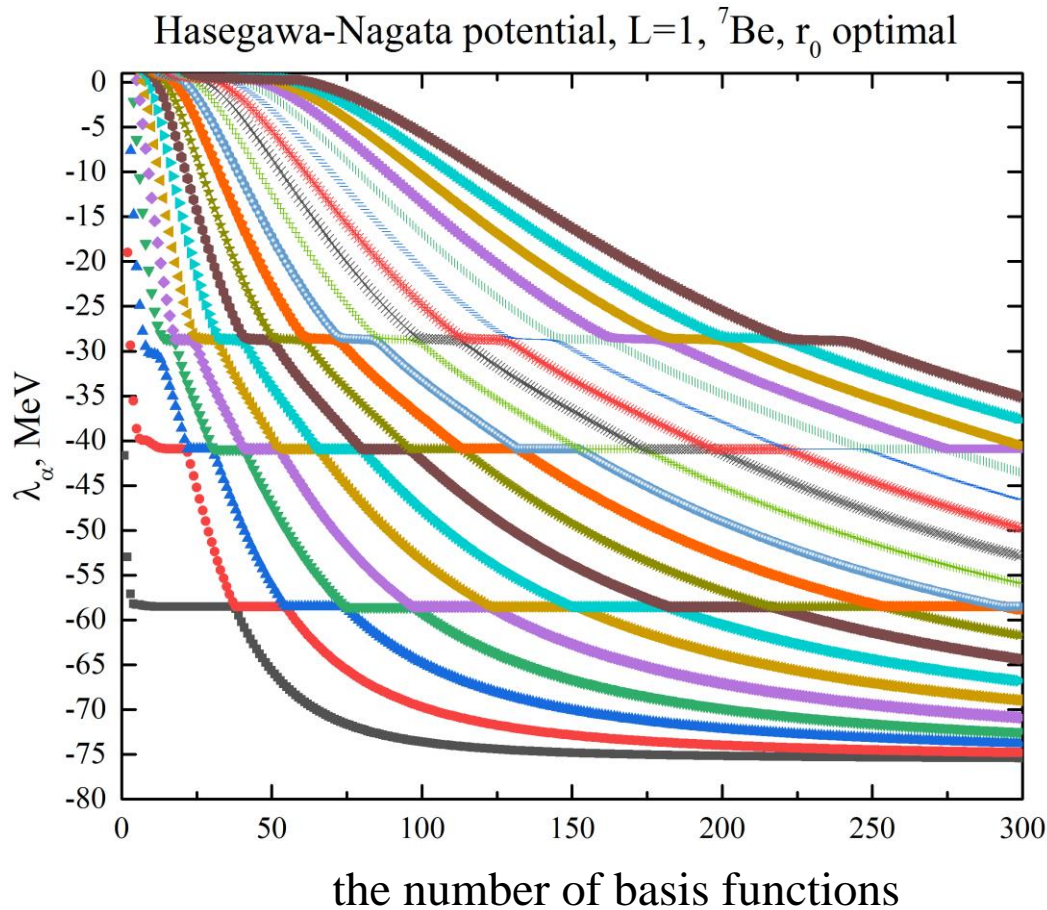
Yukawa potential, $b=r_{\text{pot}}$

Eigenvalues of the potential energy operator reproduce the behavior of the potential in some discrete points



A limited number of eigenstates of a short-range potential energy matrix gives nonzero contribution to the S-matrix

2. Eigenvalues of potential energy operator: folding vs exact potential



We employ three nucleon-nucleon potentials (+Coulomb) which have been often used in cluster models

Input parameters

NN potential

- Minnesota potential (MP)
- Volkov potential (VP)
- Hasegawa-Nagata potential (MHNP)

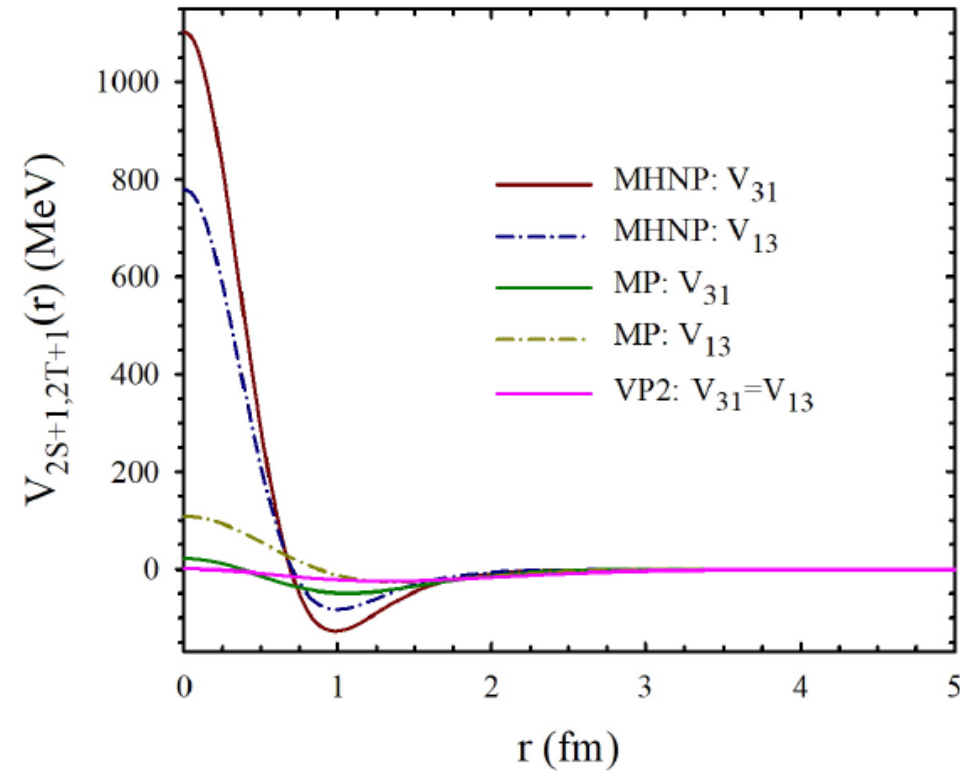
Basis

300 Oscillator functions

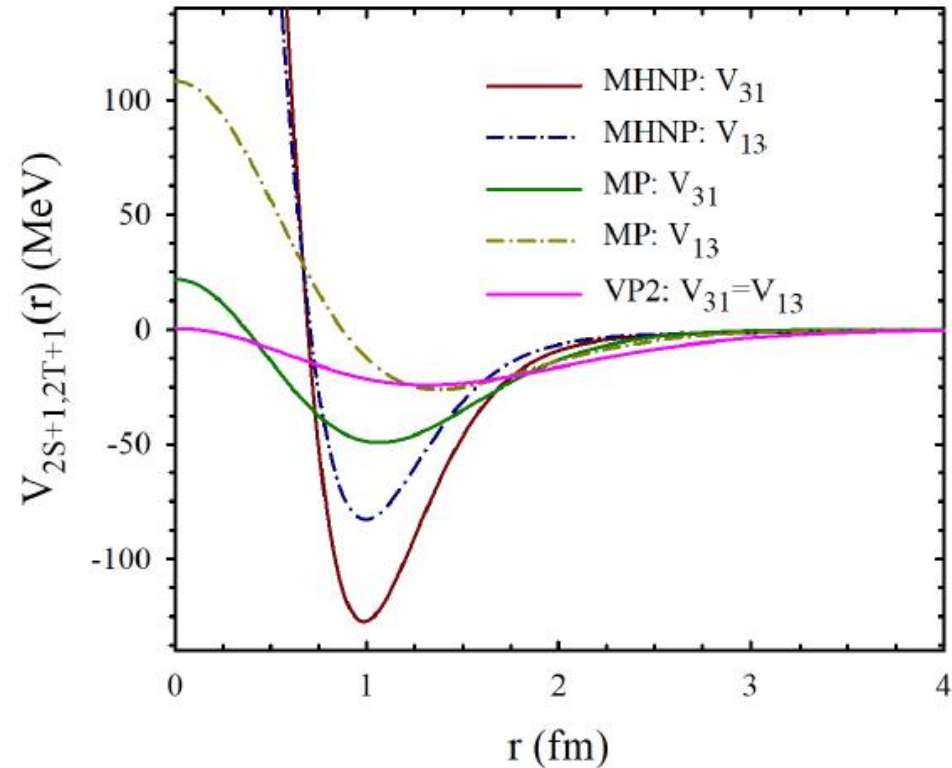
Adjustable parameters

Oscillator length b : is adjusted to minimize energy of the two-cluster threshold

The larger is the core in the nucleon-nucleon potential, the deeper is its attractive part

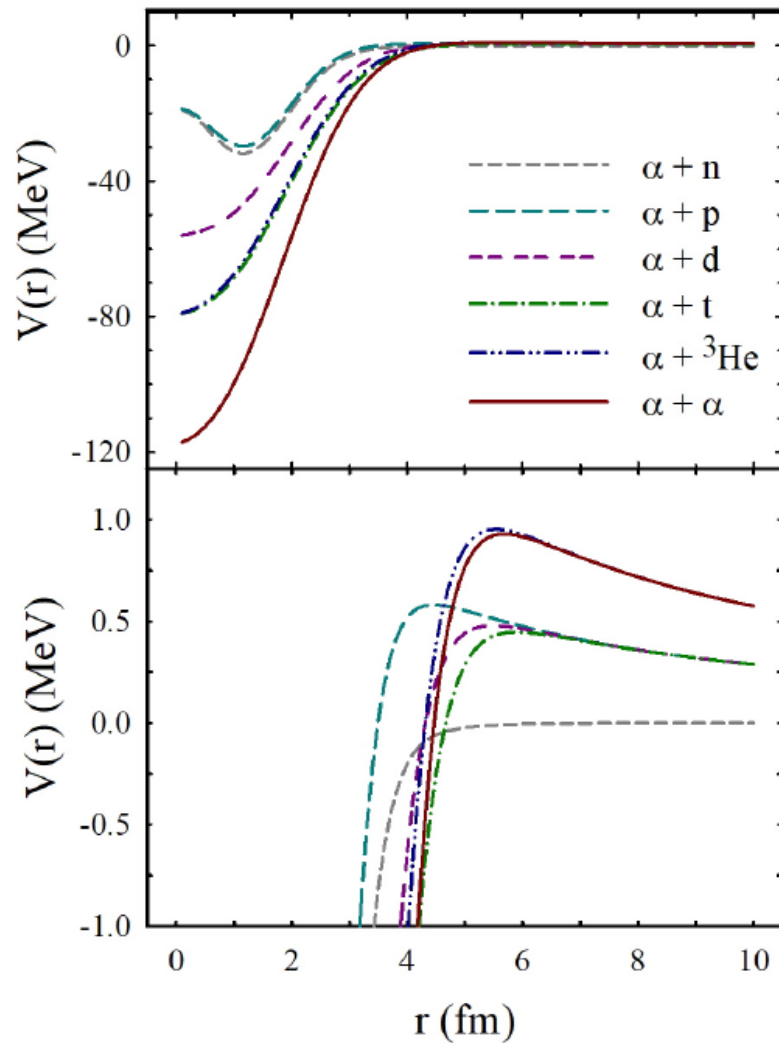


Even components of the potentials as a function of distance between nucleons

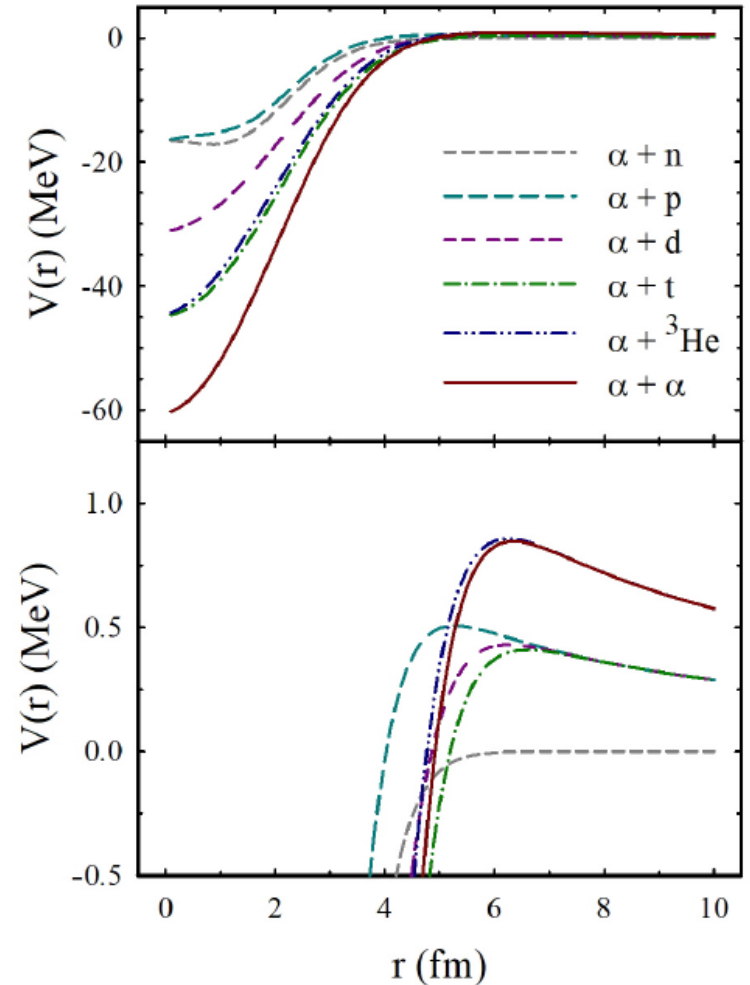


Attractive part of the even components of the potentials

Despite of the huge core in MHNP nucleon-nucleon potential, there is no such a core in cluster-cluster folding potential

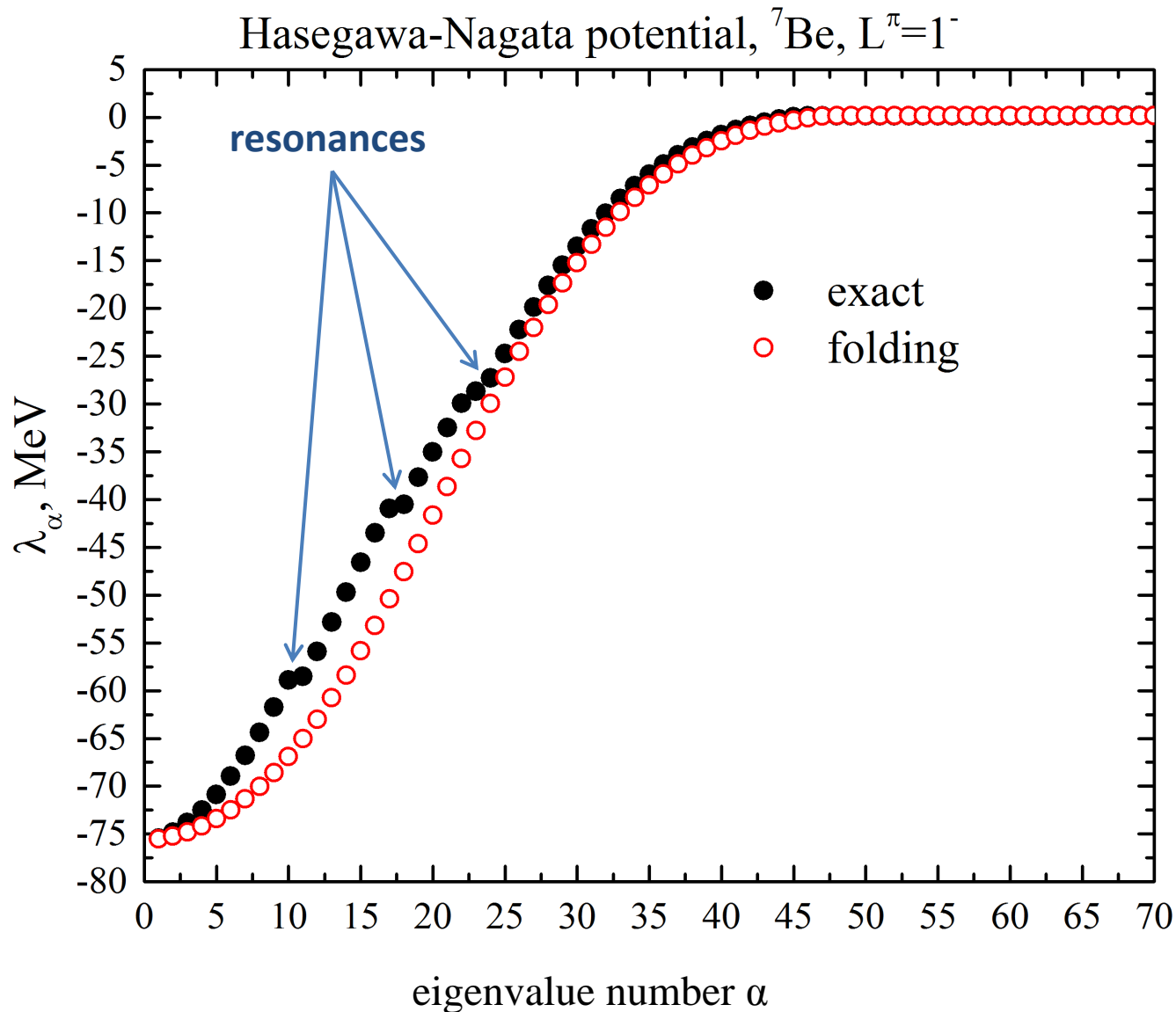


Folding potentials generated by the MHNP

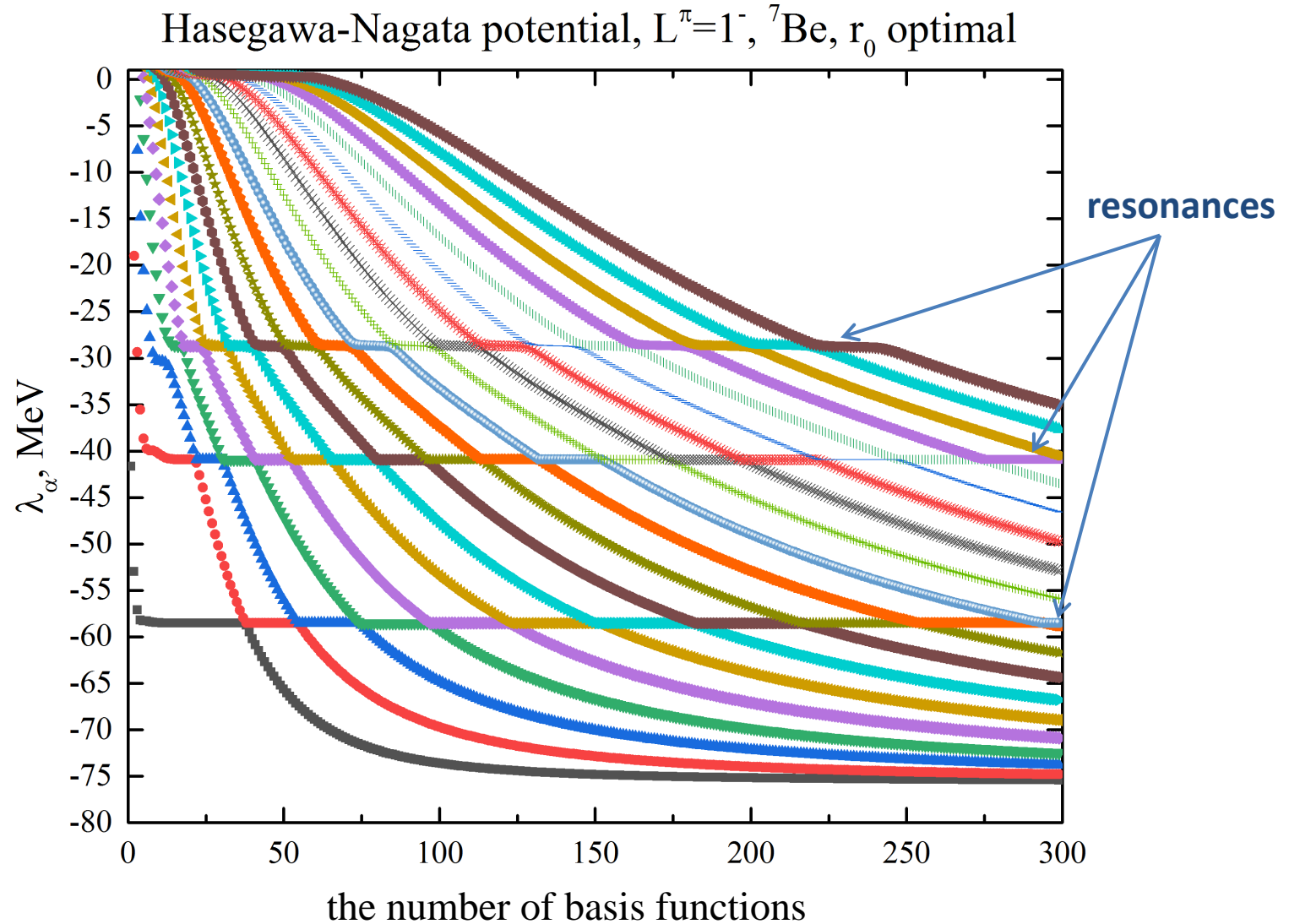


Folding potentials generated by the VP

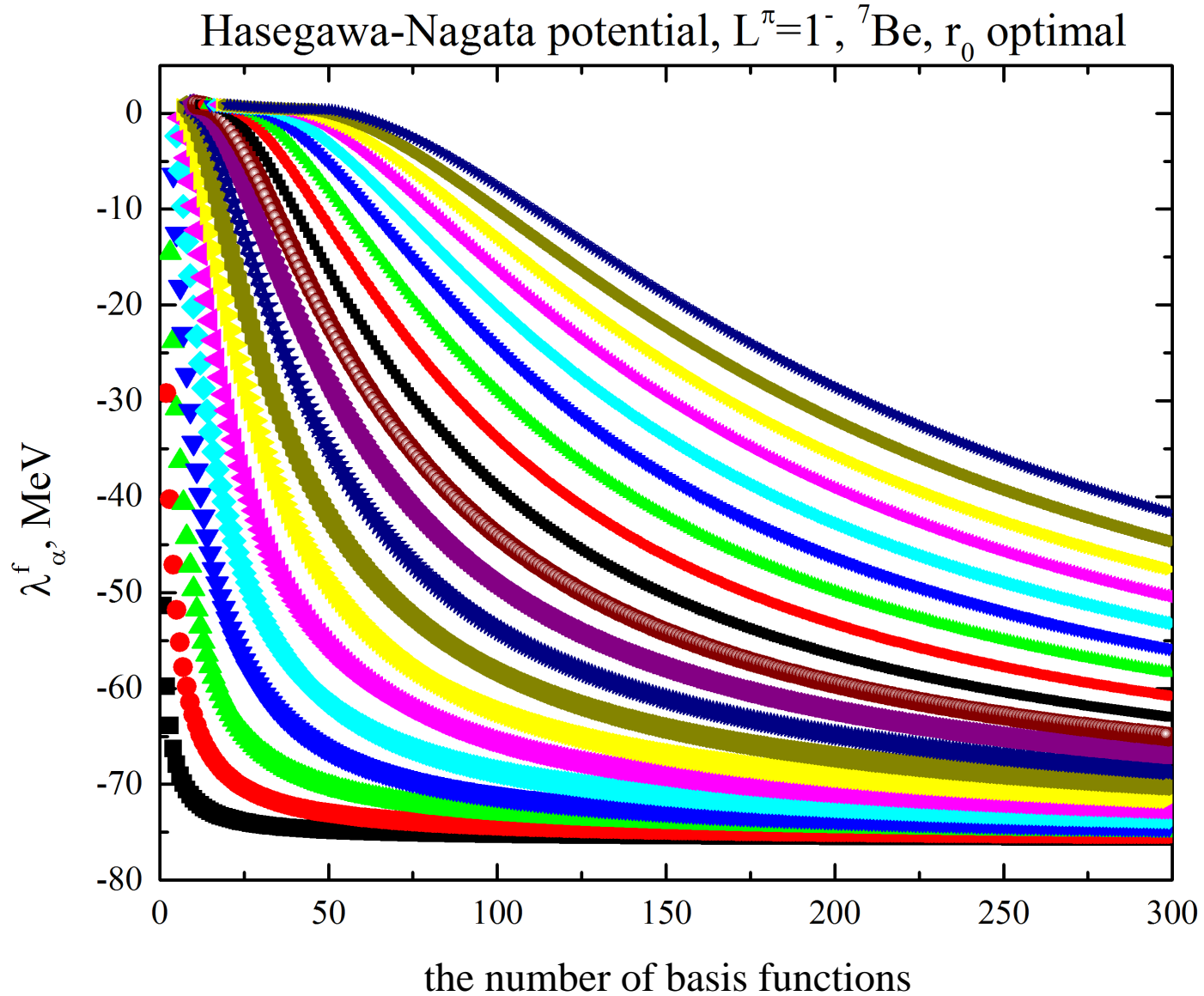
Eigenvalues of the exact and folding potential energy matrix are quite close to each other



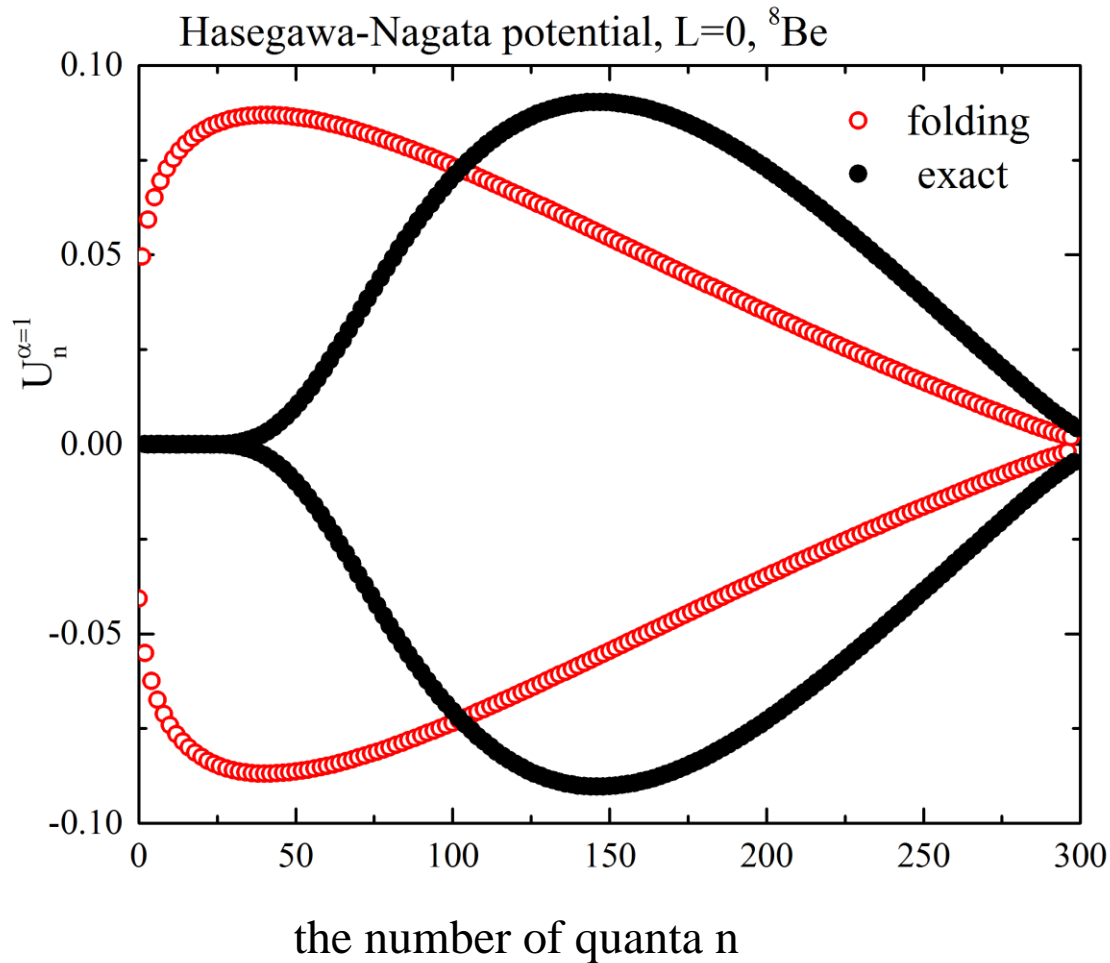
Dependence of eigenvalues of the exact potential on the number of functions exhibits resonance behavior



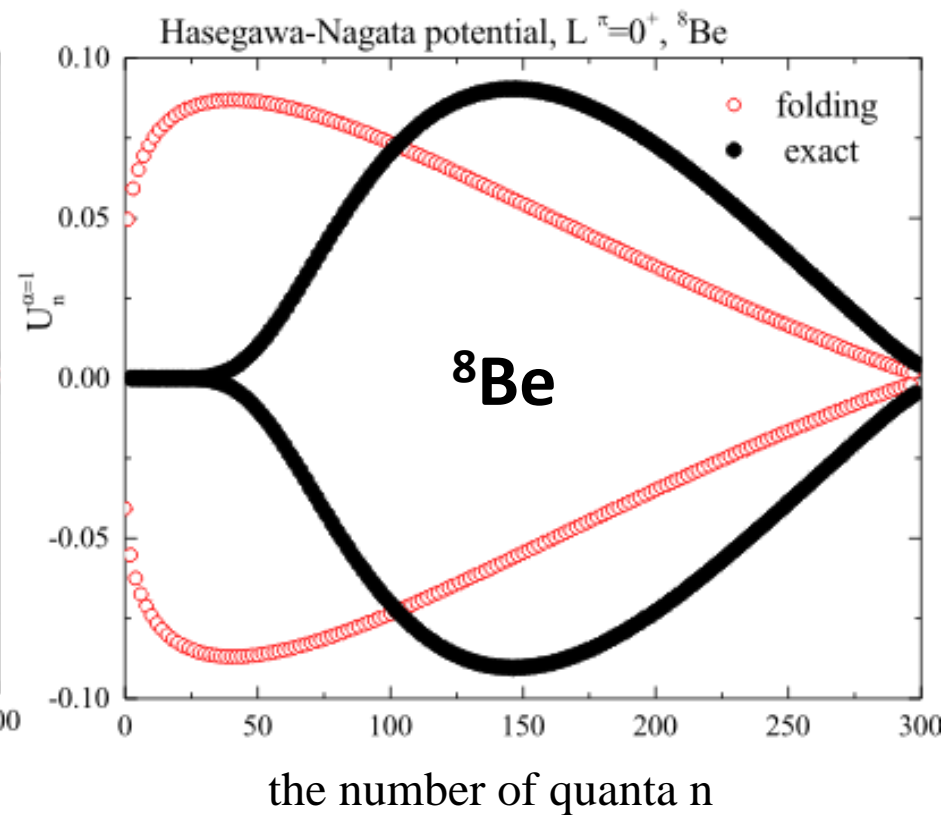
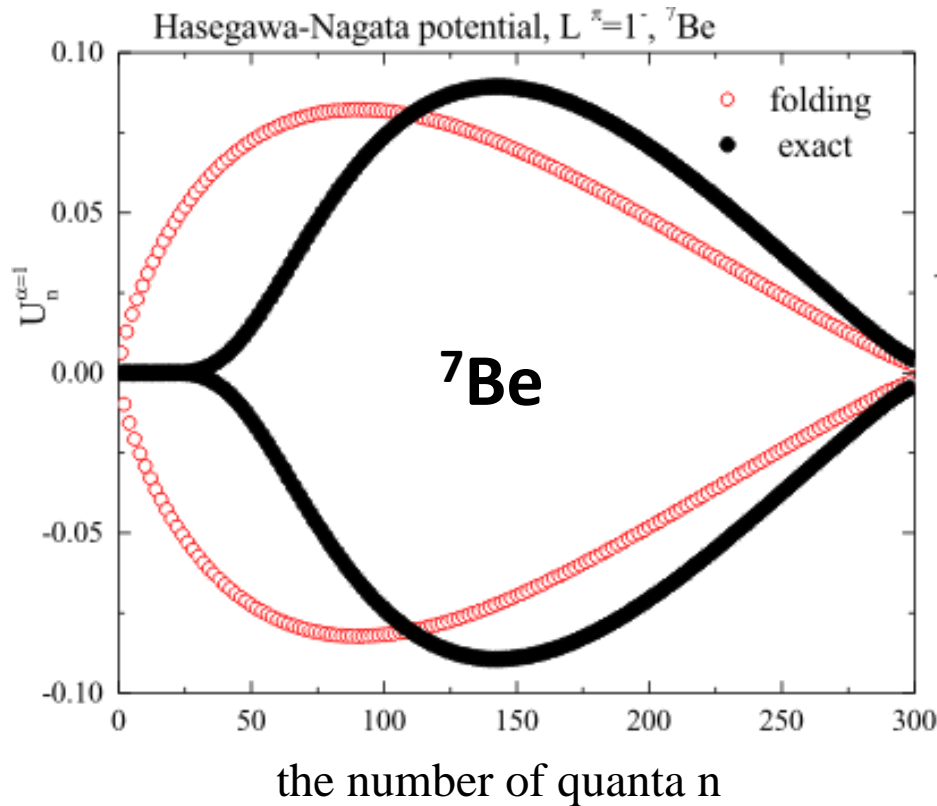
Contrary, none of the eigenstates of the folding potential has a resonance behavior



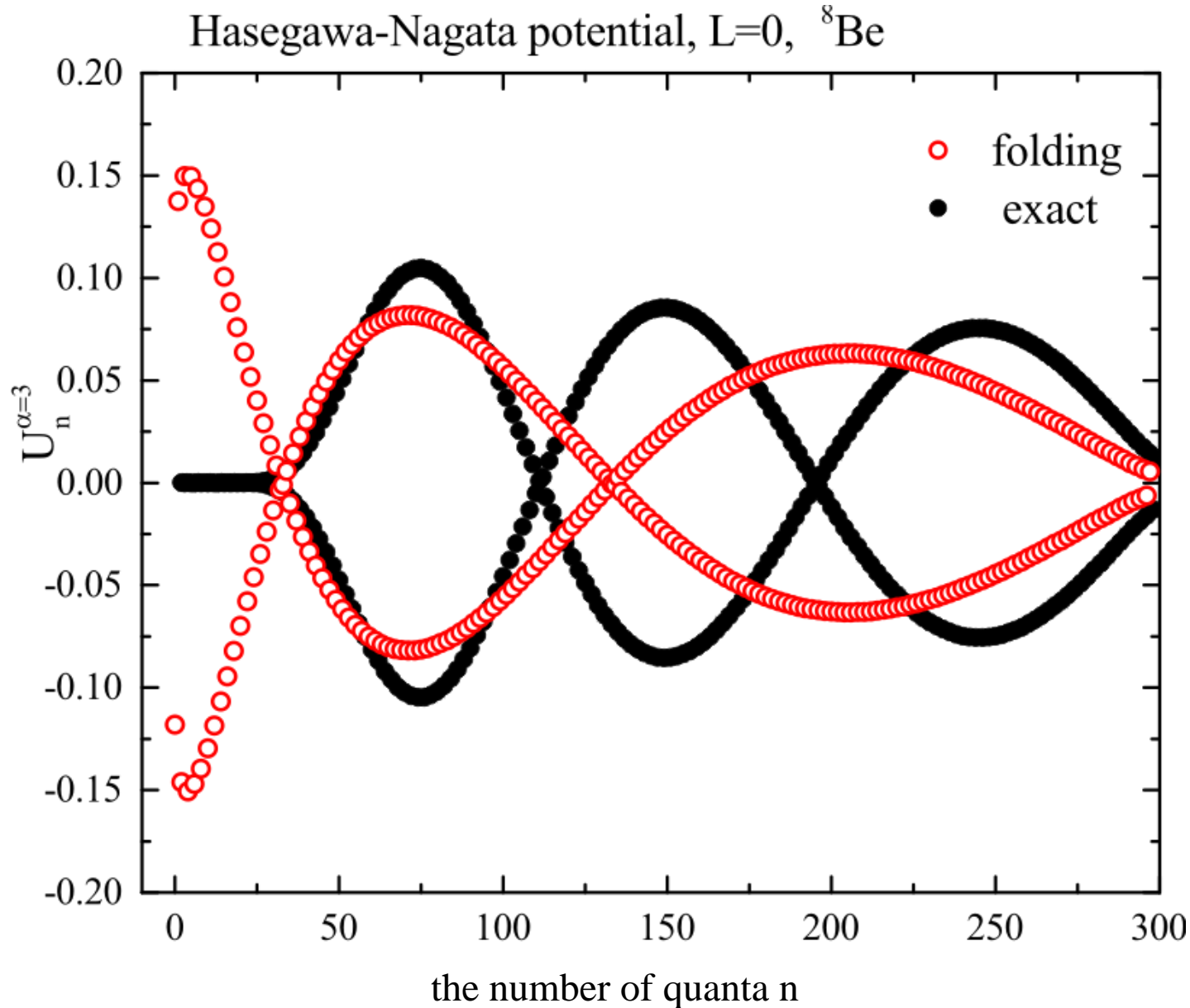
3. **Eigenfunctions** of potential energy operator : folding vs exact potential



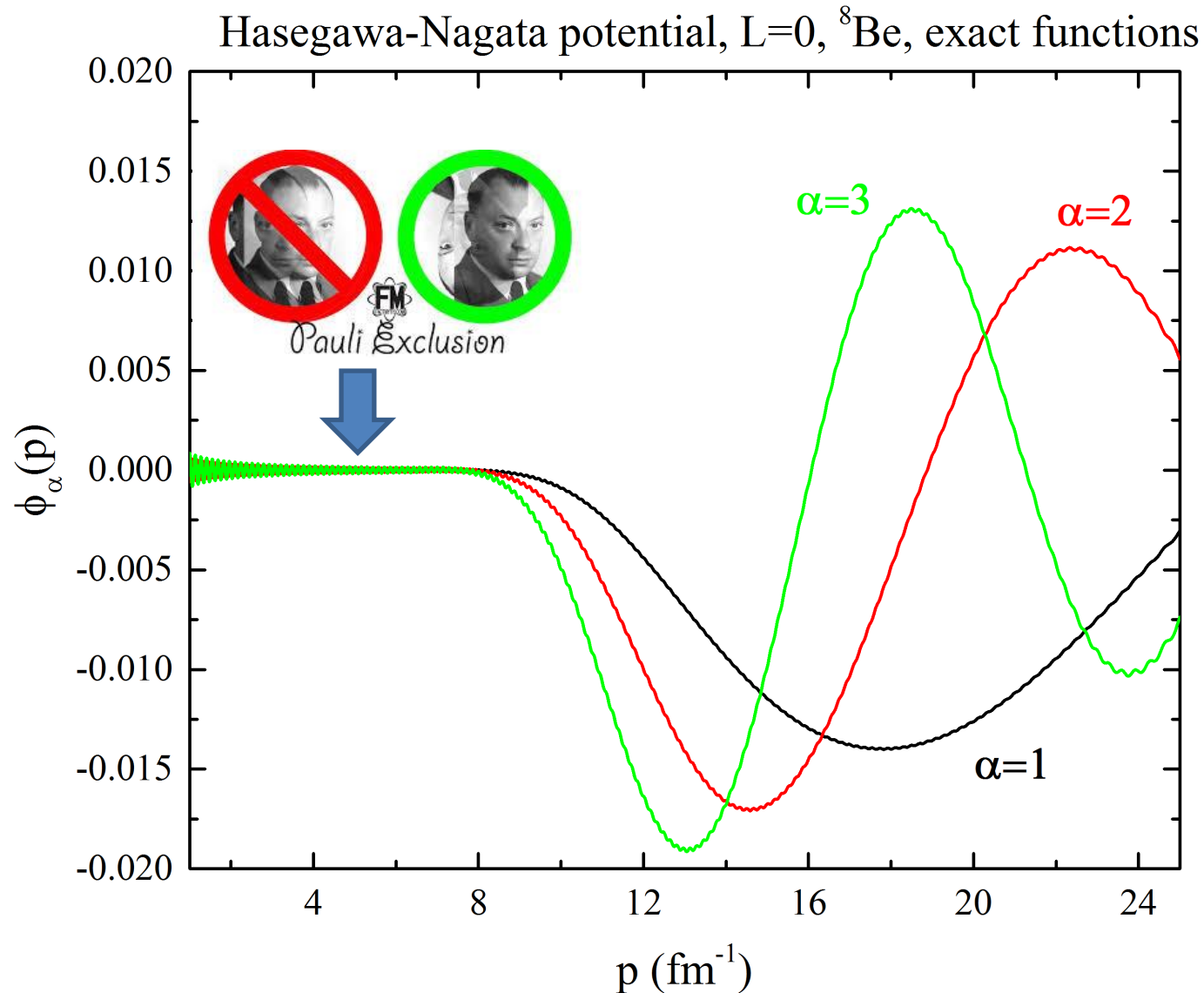
The eigenfunctions of the exact potential corresponding to non-resonance values of α is suppressed at the range $n < 50$



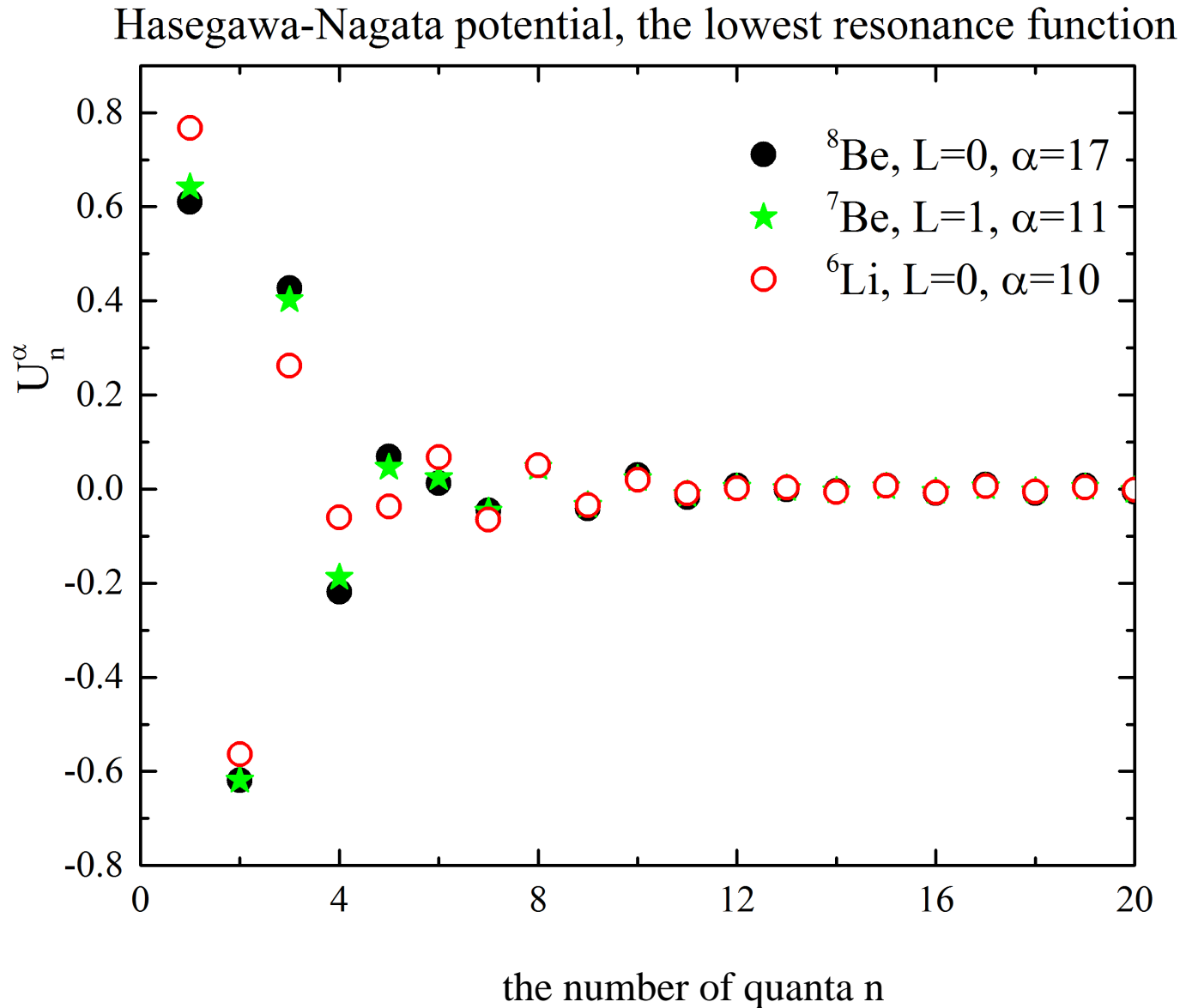
Eigenfunctions of folding potential has a maximum at lower number of quanta n than eigenfunctions of exact potential



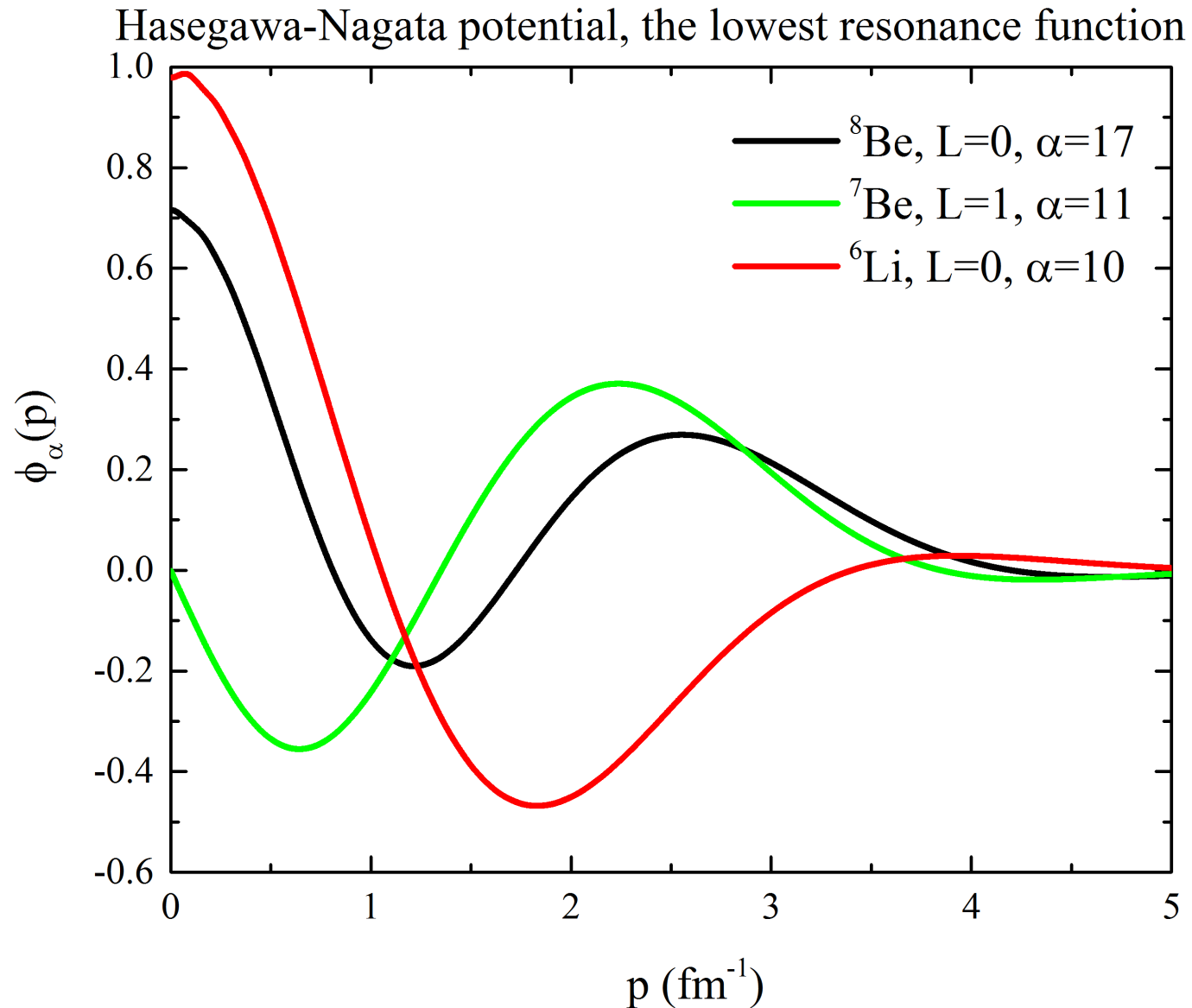
Non-resonance eigenfunctions of the exact potential in p-representation are suppressed in the range $p < 10 \text{ fm}^{-1}$



Eigenfunctions of resonance states of the exact potential are localized at low values of oscillator quanta $n < 10$

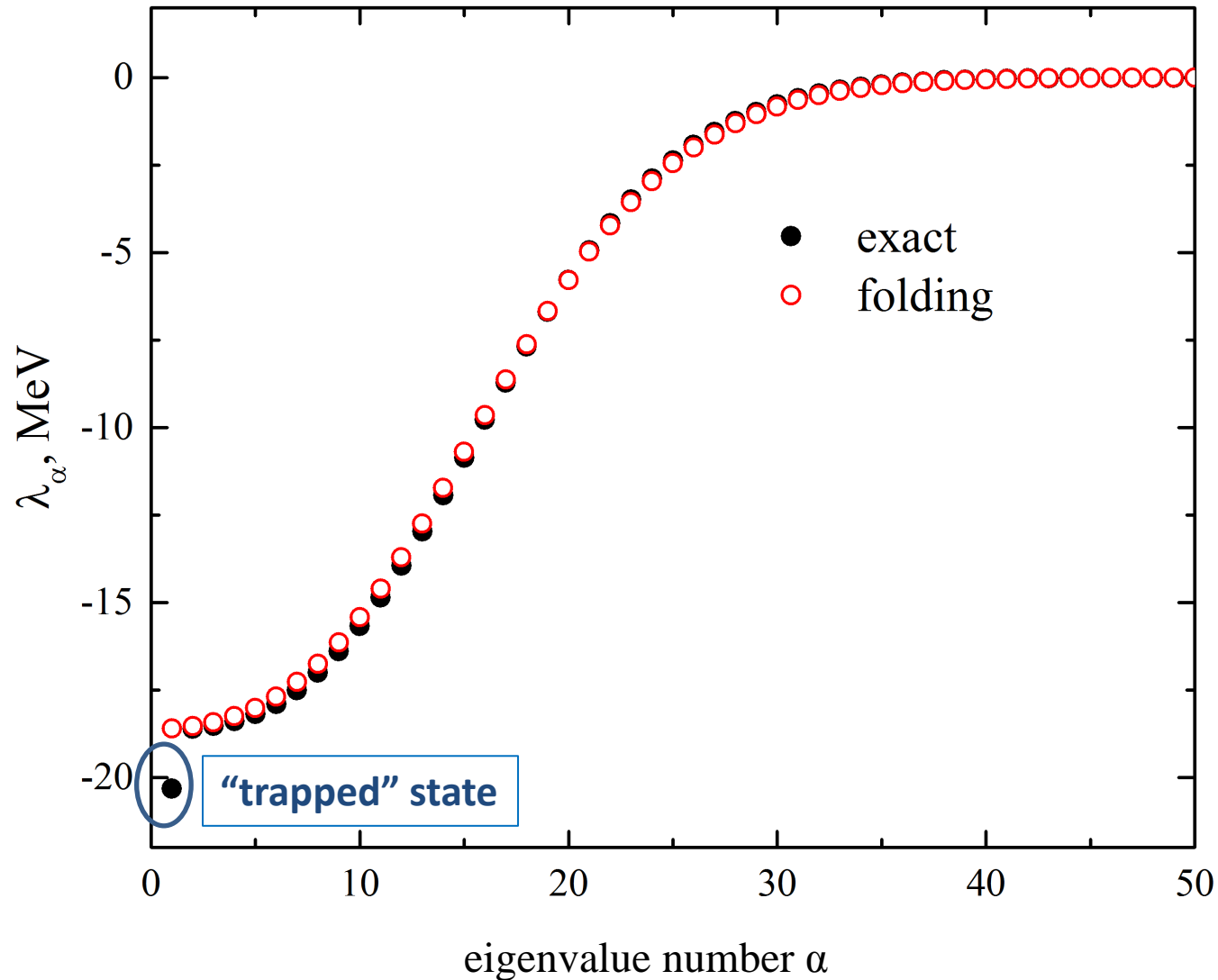


Eigenfunctions of resonance states of the exact potential in p-representation are localized at low values of p

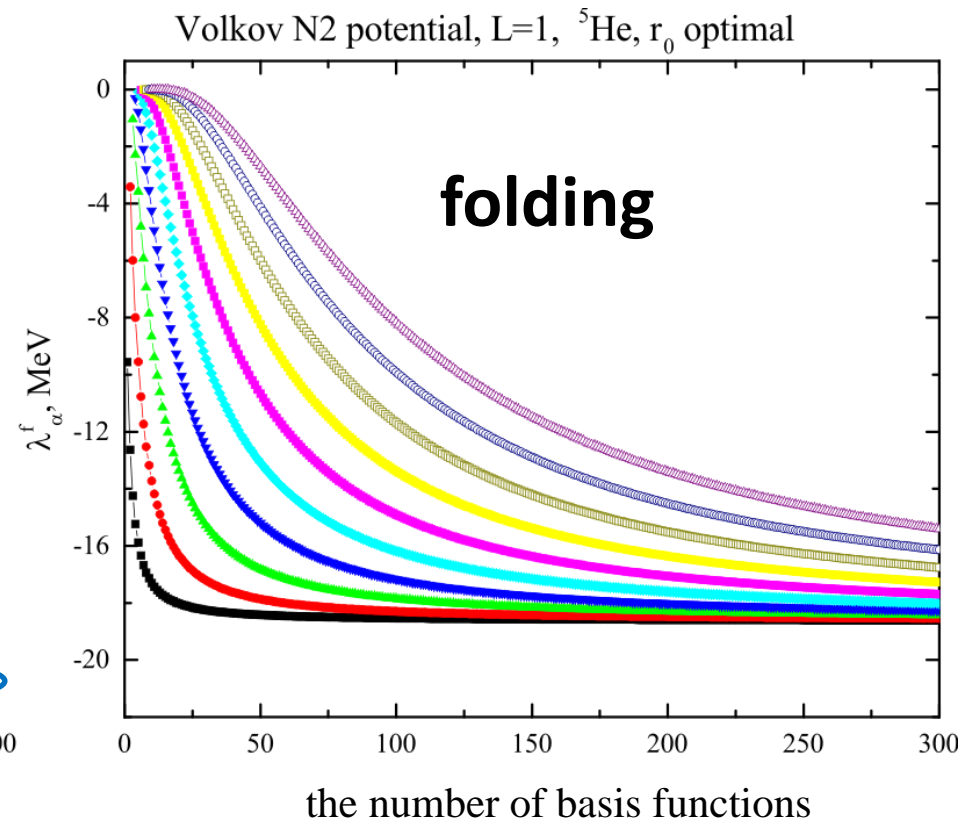
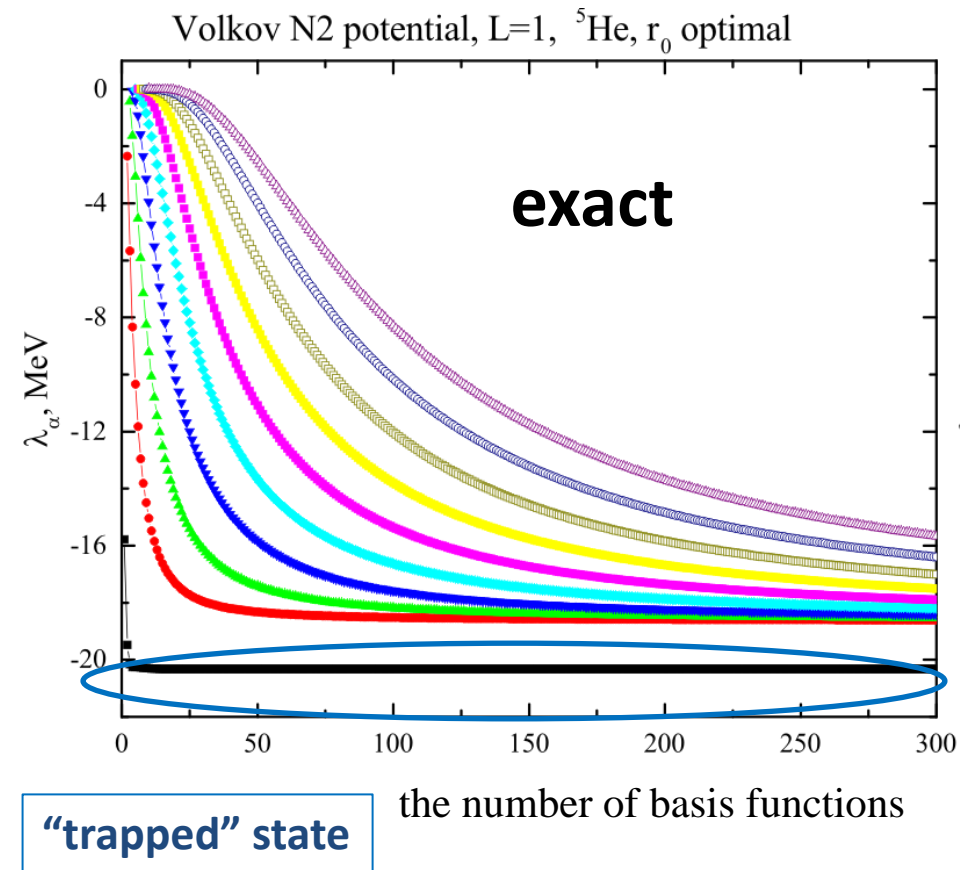


The eigenvalues of the exact Volkov N2 potential differ from those of the folding potential at a single point $\alpha=1$

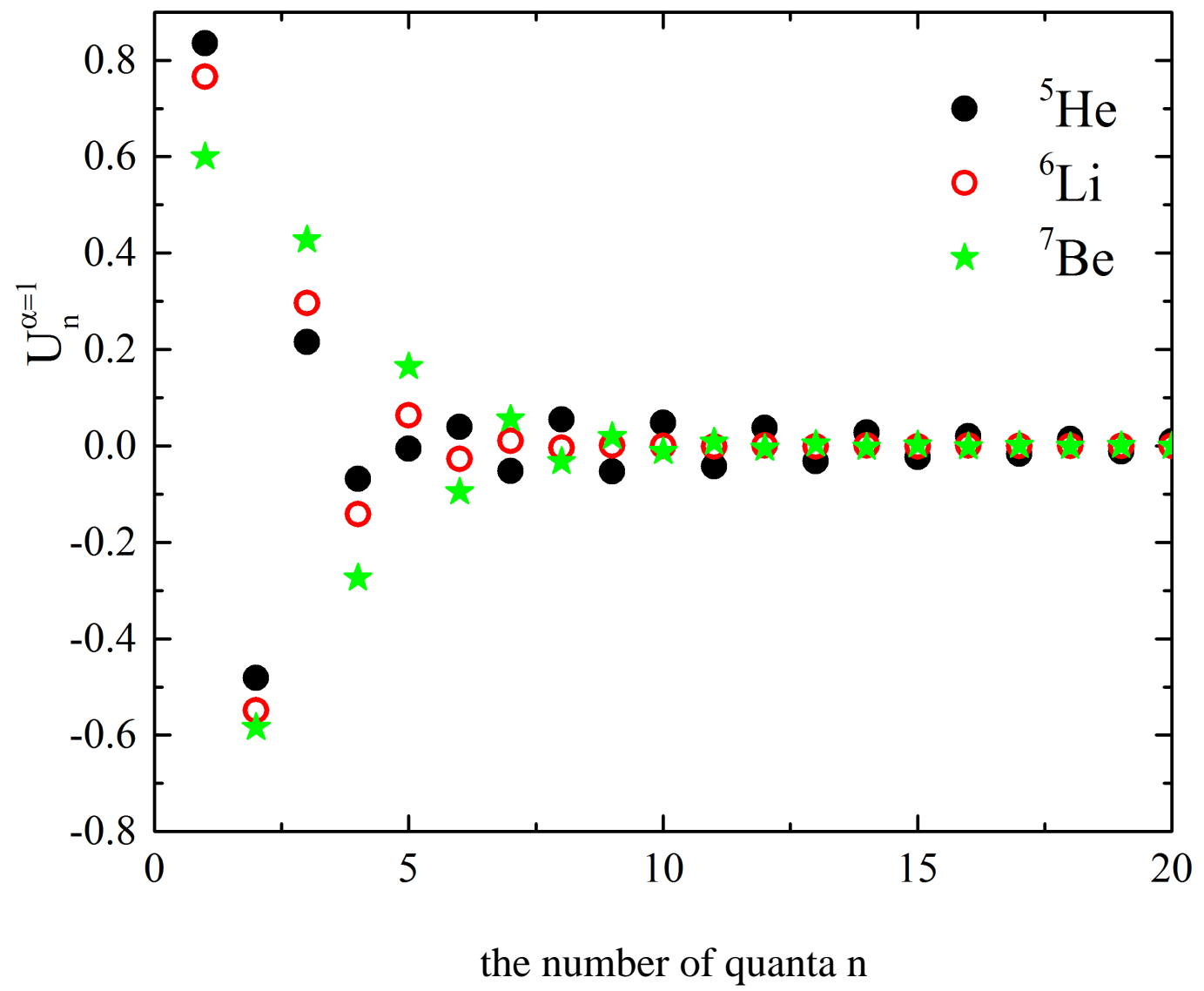
Volkov potential N2, $L=1$, ${}^5\text{He}$, r_0 optimal



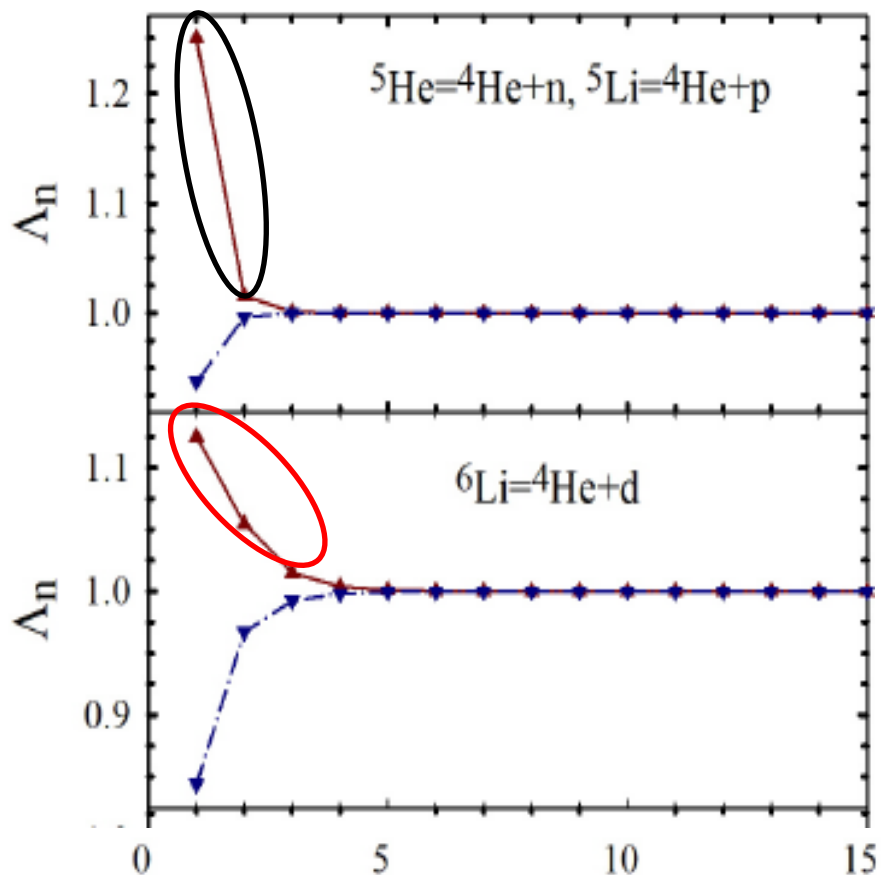
Dependence of eigenvalues of the exact Volkov N2 potential on the number of basis functions shows the existence of a “trapped” state at $\alpha=1$



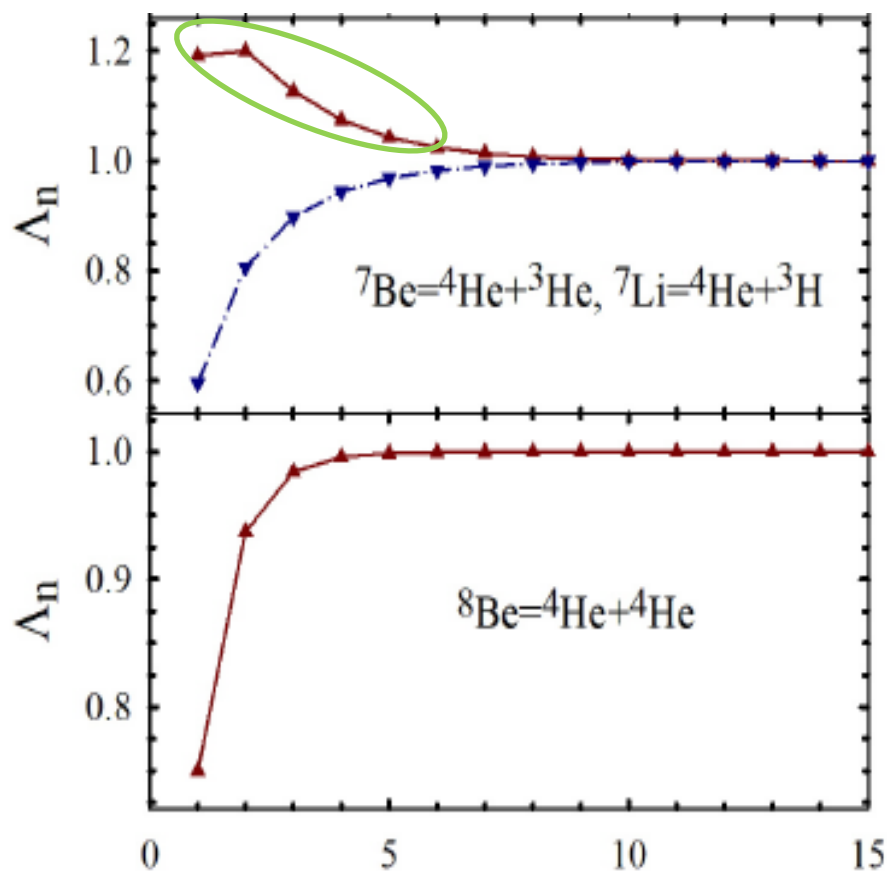
Eigenfunction of a “trapped” state in Volkov N2 potential is similar to the resonances in Hasegawa-Nagata potential



A “trapped” state in Volkov N2 potential appears only for the cluster configurations characterized by eigenvalues of the norm kernel $\Lambda_n > 1$



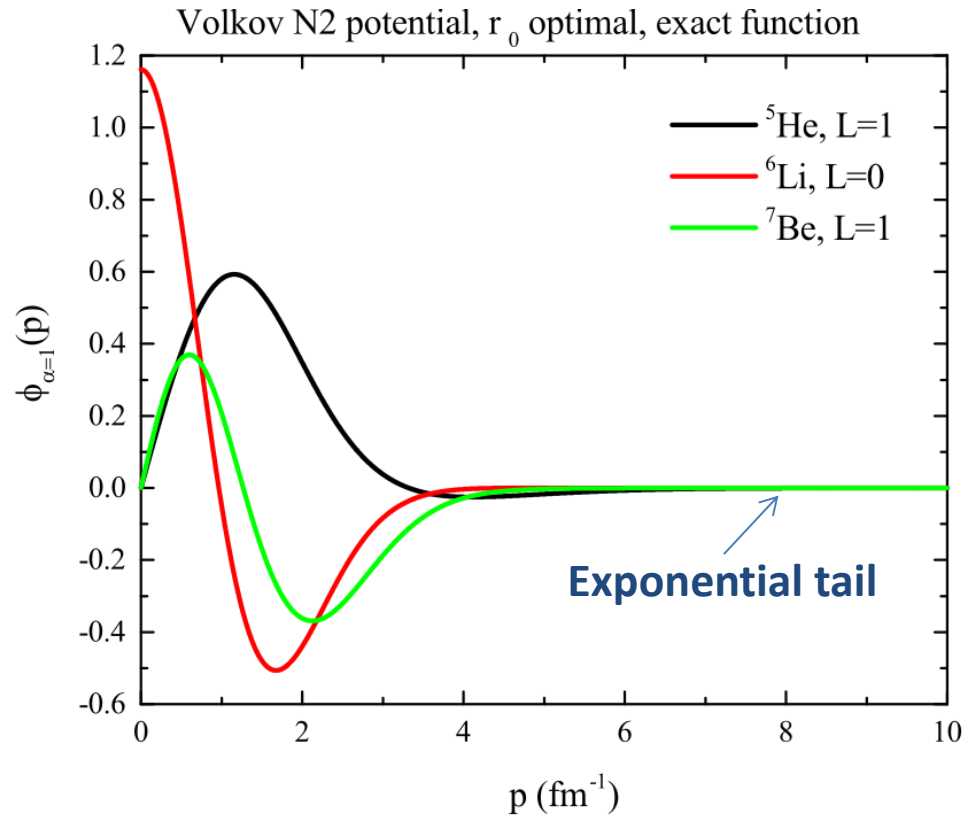
the number of quanta n



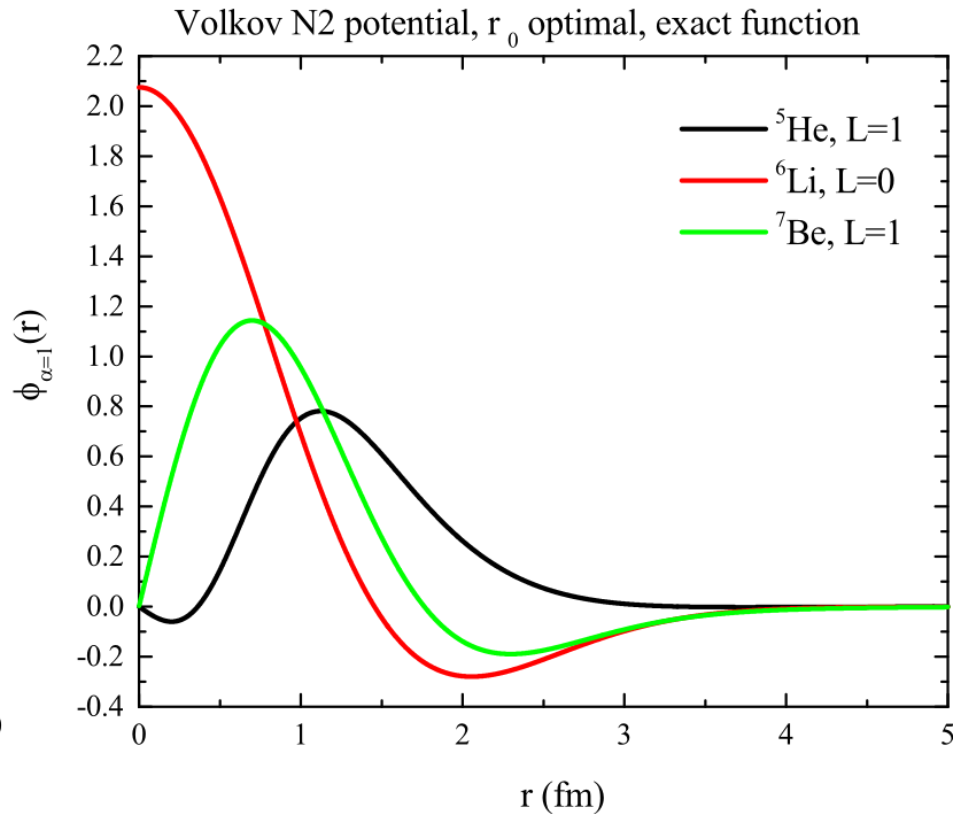
the number of quanta n

Eigenvalues of the norm kernel for lightest p-shell nuclei

With increasing momentum eigenfunctions of a “trapped” state in p-representation decrease exponentially

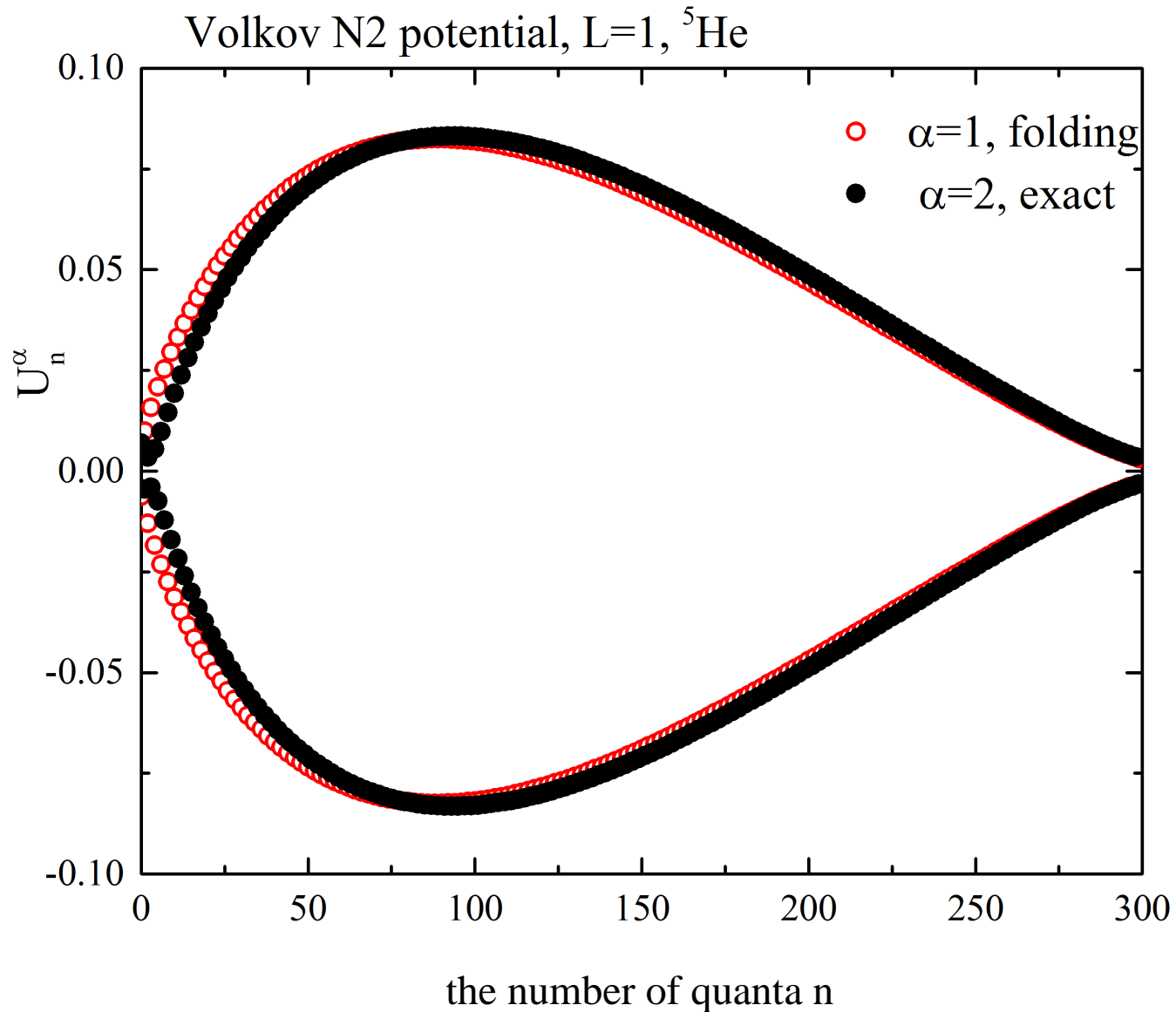


Momentum representation



Coordinate representation

All other eigenfunctions of exact Volkov potential are quite close to those of the folding potential

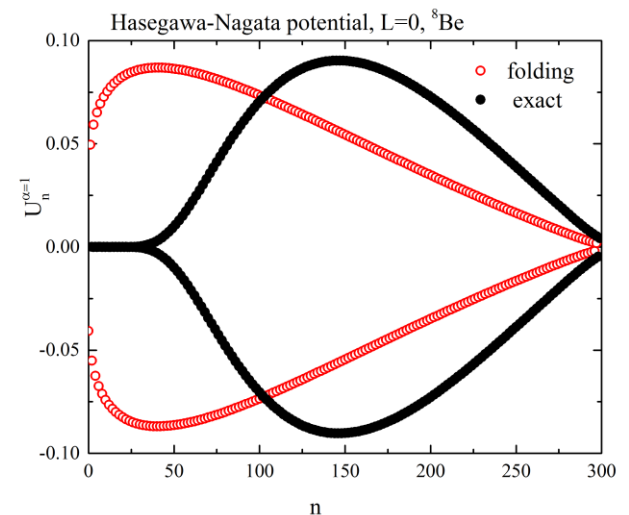
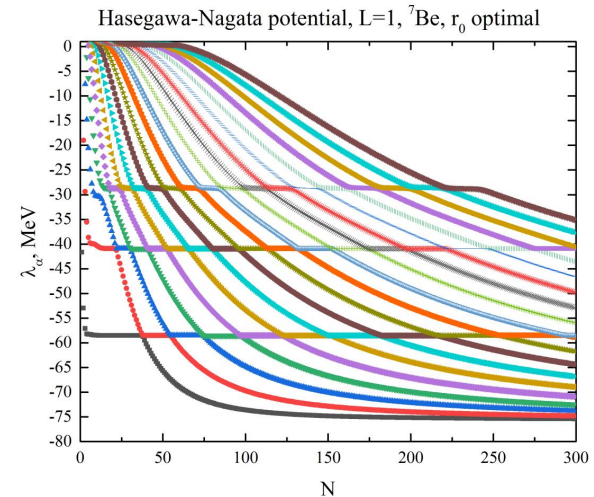


Diagonalization of the potential energy matrix proposes a self-consistent way of reducing a nonlocal potential of two-cluster systems to the local form

1. Dependence of **eigenvalues** of the exact potential on the number of functions exhibits resonance behavior

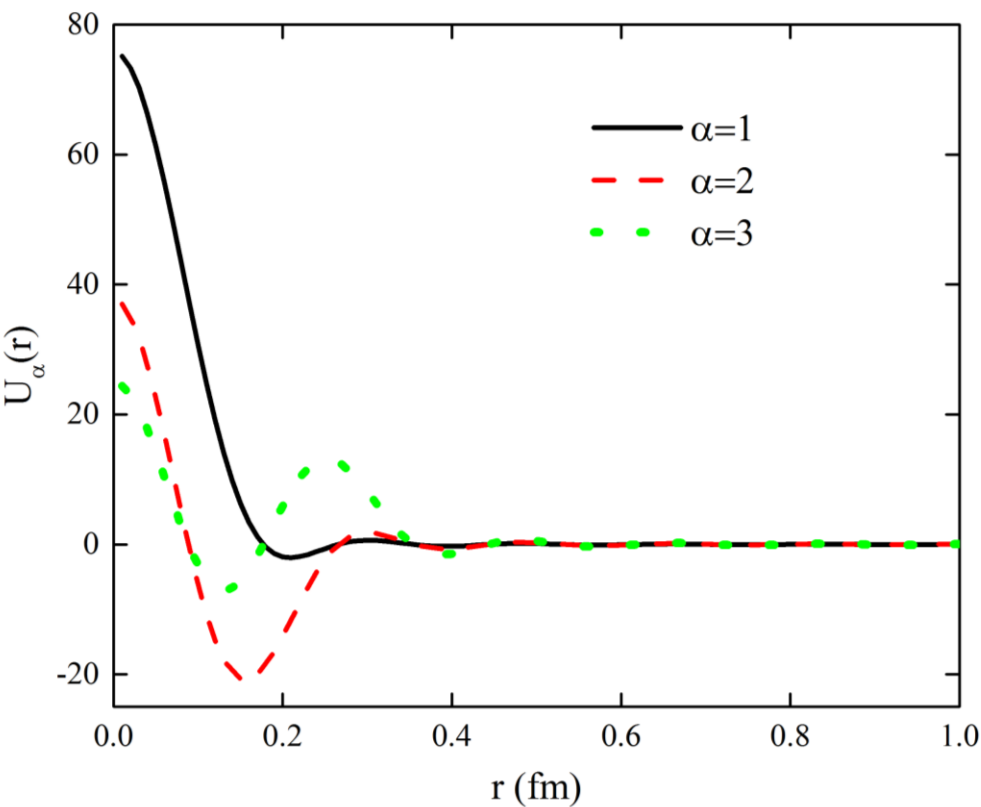


2. **Eigenfunctions** of the exact potential essentially differ from those of folding potential

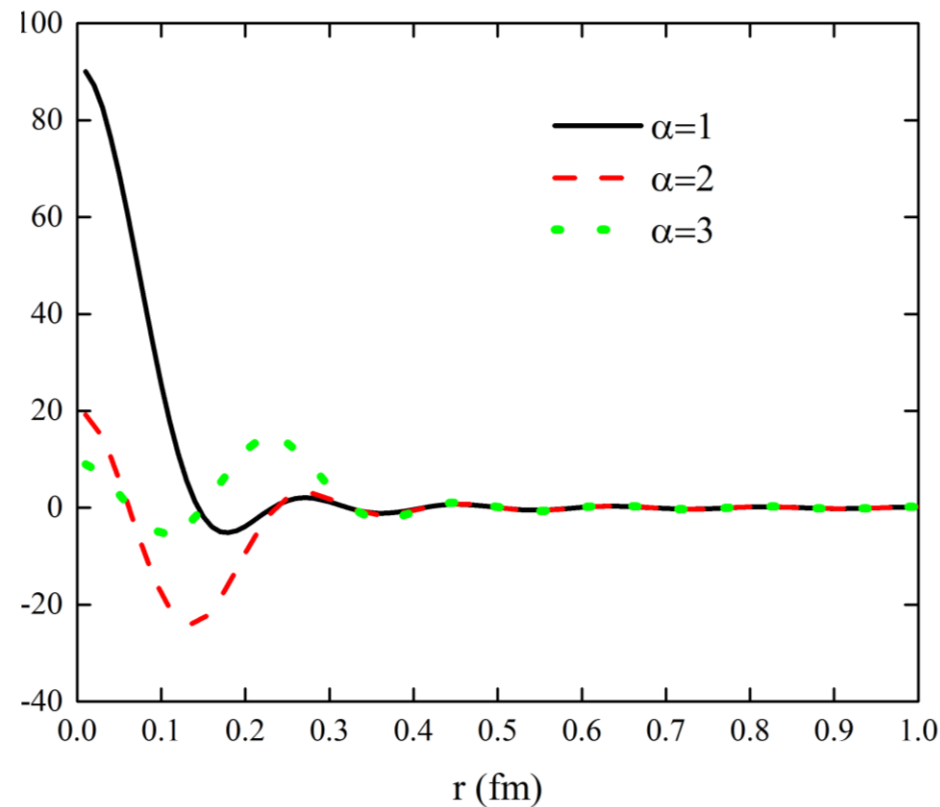


Slides for answering questions

The smaller is the eigenvalue number α , the more the corresponding eigenfunction $U_\alpha(r)$ resembles δ -function



Gaussian potential, $b=r_{\text{pot}}$



Yukawa potential, $b=r_{\text{pot}}$

Discrete coordinates r_α define the location of the α -th zero of the eigenfunction in harmonic oscillator representation

$$U_{N+1}^\alpha = \mathcal{N}_\alpha \Phi_{N+1,L}(r_\alpha, b) = 0$$

They are the zeros of the Laguerre polynomial $L_N^{L+1/2}(\rho)$

For $\rho = r_\alpha/b \ll R_N$ $R_N = \sqrt{4N + 2L + 3}$

$$r_\alpha = \frac{\pi b \left(\alpha + \frac{1}{2}L \right)}{R_N}$$