Study of 3- and 4-neutron systems using the hyperspherical method

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Outline

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3. Time-delay
4. Results for 3*n*
5. Results for 4*n*
6. Conclusions & Outlook

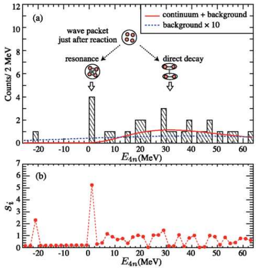
In collaboration with

- C. Greene *Purdue Un., West Lafayette (USA)*

Adiabatic HH method

Experiment:

- Candidate $4n$ resonance found by [Kisamori *et al.*, 2016] in the reaction ^{4}He(^{8}He^{8}Be)
- $E_{4n} = 0.83 \pm 0.75 stat \pm 1.25 syst$ MeV
- see also [Bertulani & Zelevinsky, Nature News & Views, 2016]



Theory:

- [Hiyama *et al.*, 2016]: [FY+GEM] **NO!** *"...a remarkable actractive 3N force would be required..."*
- [Shirokov *et al.*, 2016]: [HORSE] **YES!**, *"4n resonance at E = 0.8 Mev, Γ = 1.4 MeV"*
- [Gandolfi *et al.*, 2017]: [QMC] **YES!** *"... 3n resonance exists below the 4n resonance ..."*
- [Fossez *et al.*, 2017]: [NCGSM] **YES!** *"...energy compatible with the expt. ..."*
- [Deltuva, 2018]: [FY] **NO!** *"...absence of an observable 4n resonance."* (low-energy final state interactions?)
- [Deltuva & Lazauskas, 2019]: **NO!** they point out "... shortcomings in the work by Gandolfi et al. ..."

Adiabatic HH method

Separate the hyper-radial (slow) motion from the hyper-angular (fast) motion of the particles
obtain effective potentials $U(\rho)$ of the hyperradius
from them it is possible to have indications if bound or quasi-bound states are possible

$$H = -\frac{\hbar^2}{m}\left[\frac{\partial^2}{d\rho^2} + \frac{D-1}{\rho}\frac{\partial}{d\rho} - \frac{L^2(\Omega)}{\rho^2}\right] + V(\rho,\Omega)$$

- $D = 3(A-1)$ hyperradius $\rho = \frac{2}{A}\sum_{i<j}(\mathbf{r}_i - \mathbf{r}_j)^2$
- Hyperangular momentum operator $L^2(\Omega)$
- HH functions are the eigenfunctions $L^2(\Omega)Y_{[K]}(\Omega) = K(K+D-2)Y_{[K]}(\Omega)$

$$\Psi(\rho,\Omega) = \sum_\ell \frac{f_\ell(\rho)}{\rho^{\frac{D-1}{2}}}\Phi_\ell(\rho,\Omega)$$

$$\left\{-\frac{\hbar^2}{m}\left[\frac{L^2(\Omega)}{\rho^2} + \frac{(D-1)(D-3)}{4\rho^2}\right] + V(\rho,\Omega)\right\}\Phi_\ell(\rho,\Omega) = U_\ell(\rho)\Phi_\ell(\rho,\Omega)$$

Application to study $3n \to 3n$ and $4n \to 4n$

- No $2n$, $3n$ bound states $\to$ all the $U_\ell \to 1/\rho^2$
- For $\rho \to \infty$ $\Phi_\ell \sim Y_{[K]}(\Omega)$
- $U_\ell \to \frac{\hbar^2}{m}\mathcal{L}(\mathcal{L}+1)/\rho^2 \qquad \mathcal{L} = K + (D-3)/2$

$3n$ system

J^π	$(3/2)^-$
K_{min}	1
$\mathcal{L}$	5/2
$\mathcal{L}(\mathcal{L}+1)$	35/4

$4n$ system

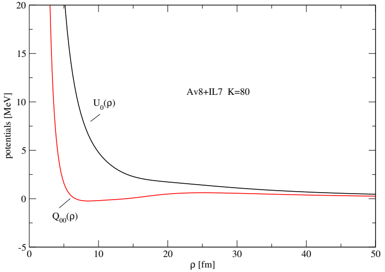
J^π	0^+
K_{min}	2
$\mathcal{L}$	5
$\mathcal{L}(\mathcal{L}+1)$	30

- Useful to look at $U_\ell(\rho)$ compared to $\frac{\hbar^2}{m}\mathcal{L}(\mathcal{L}+1)/\rho^2$
- Without V, $\Phi_0 = Y_{[K_{min}]}$ and $U_0 = \frac{\hbar^2}{m}\mathcal{L}(\mathcal{L}+1)/\rho^2$
- $\mathcal{L} = K_{min} + (D-3)/2$
- This term is the kinetic energy contribution

The problem reduces to the solution of a set of ordinary coupled differential equations

$$-\frac{\hbar^2}{m} f''_\ell(\rho) + \sum_{\ell'} \left[P_{\ell\ell'}(\rho) \frac{d}{d\rho} + Q_{\ell\ell'}(\rho) \right] f_{\ell'}(\rho) + U_\ell(\rho) f_\ell(\rho) = E f_\ell(\rho)$$

$$P_{\ell\ell'}(\rho) = 2 \left\langle \Phi_\ell(\rho,\Omega) \frac{\partial}{\partial\rho} \Phi_{\ell'}(\rho,\Omega) \right\rangle_\Omega \qquad Q_{\ell\ell'}(\rho) = \left\langle \Phi_\ell(\rho,\Omega) \frac{\partial^2}{\partial\rho^2} \Phi_{\ell'}(\rho,\Omega) \right\rangle_\Omega$$



- One adiabatic approximation ($P_{00} = 0$)
- For bound-states: rigorous upperbound to the energy of the system, usually very close to the correct energy

$$-\frac{\hbar^2}{m} f''_0(\rho) + \left[Q_{00}(\rho) + U_0(\rho) \right] f_0(\rho) = E f_0(\rho)$$

Boundary conditions

- For bound states $f_\ell(\rho) \to 0$
- For scattering states $f_\ell(\rho) \to \delta_{\ell_0 \ell} e^{-iQ\rho} - \mathcal{S}_{\ell_0 \ell} e^{+iQ\rho}$
- $\hbar^2 Q^2/m = E \qquad \mathcal{S} = \text{S-matrix}$

Calculation of Φ_ℓ & U_ℓ

- Expansion in HH functions $\Phi_\ell = \sum_{[K]} a_{[K],\ell}(\rho) Y_{[K]}(\Omega)$
- HH functions included up to $K = K_{max} \qquad K_{max} > 100$ needed
- Very difficult to obtain Φ_ℓ & U_ℓ for medium-large ρ
- Mainly used in atomic and molecular physics

$$P_{\ell\ell'} = 2\sum_K a_{K,\ell}(\rho) a'_{K,\ell}(\rho) \qquad Q_{\ell\ell'} = \sum_K a_{K,\ell}(\rho) a''_{K,\ell}(\rho)$$

$$\langle \Phi_\ell(\rho,\Omega) \Phi_{\ell'}(\rho,\Omega) \rangle_\Omega = \delta_{\ell\ell'}$$

$$P_{\ell'\ell} = -P_{\ell\ell'} \qquad Q_{\ell'\ell} = +Q_{\ell\ell'}$$

Time-delay

Time-delay

- Time-delay in scattering $\tau = 2d\delta(E)/dE$ [Wigner, 1955]
- $\delta(E) =$ phase-shift
- Generalized for multichannel scattering [Smith, 1960]

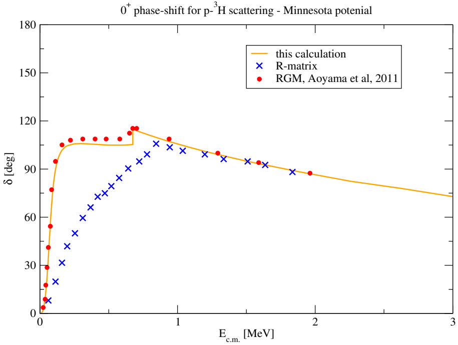
$$T_D = iS\frac{dS^\dagger}{dE} \qquad eigenvalues \rightarrow \tau_i$$

- Time-delay matrix T_D related to the “density of resonant states” [Aymar, Greene, & Luc-Koenig, 1996]
- Resonances $=$ where τ_i have maxima

Example

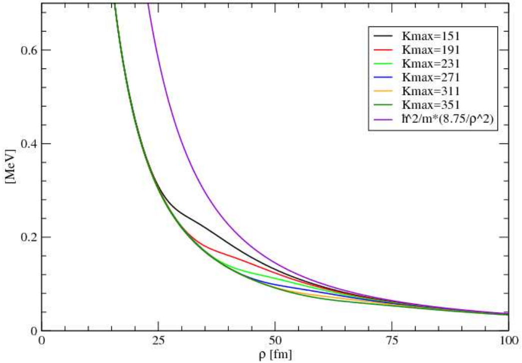
Results for $p - {}^3\mathrm{H}$ scattering

- Test case: Minnesota potential
- 0^+ phase-shift $\delta(E)$ calculated using the Kohn variational principle + HH expansion
- E_r = maximum of $\delta'(E)$, $\Gamma = 2/\delta'(E_R)$: $E_r = 0.07$ MeV, $\Gamma = 0.07$ MeV
- Poles of a Padé approximation of the S-matrix [Rakityansky, Sofianos, & Elander (2011)]: $E_r = 0.07$ MeV, $\Gamma = 0.07$ MeV
- [Aoyama, 2016] (complex scaling): $E_r = 0.07$ MeV $\Gamma = 0.06$ MeV



Trineutron (1)

$J = 3/2^-$ AV18+Urbana interaction
Lowest eigenpotential – convergence with K_{max}



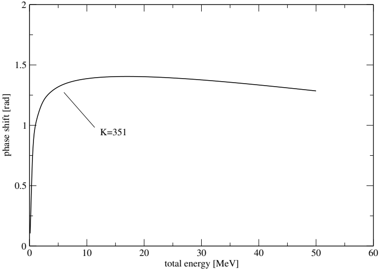
Trineutron (2)

$J = 3/2^-$ AV18+Urbana interaction

phase-shift at $E = 2$ MeV
Preliminary – Only the lowest AHH taken into account

K_{max}	δ [rad]
151	1.158
191	1.200
231	1.223
271	1.241
311	1.252
351	1.259

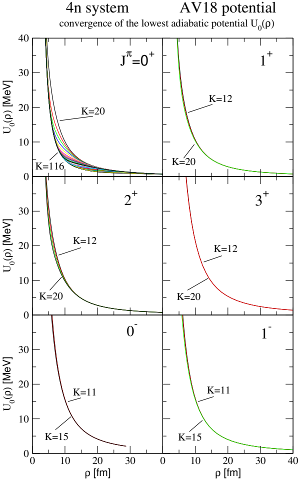
Preliminary – Only the lowest AHH taken into account



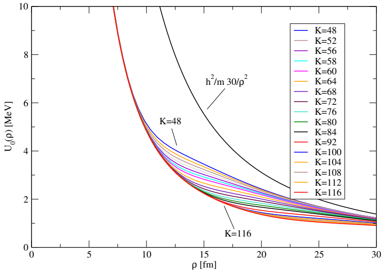
From the time-delay: $E_r = 0.1$ MeV $\Gamma \approx 0.1$ MeV
Preliminary

Tetraneutron (1)

Lowest potential for different waves
preliminary

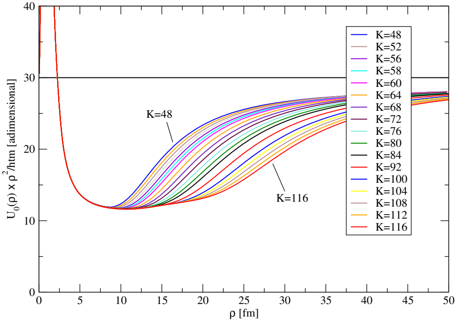


0^+ Lowest eigenpotential – convergence
$K \equiv K_{max}$
AV18 potential preliminary

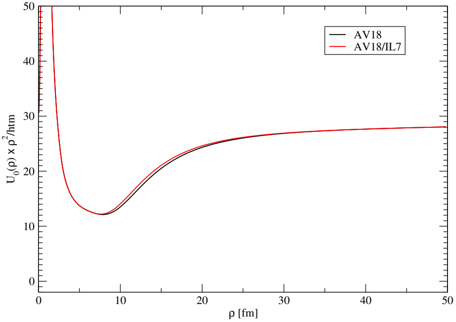


Tetraneutron (2)

Lowest potential rescaled with ρ^2
AV18 potential preliminary



Lowest potential including a 3N interaction
$K = 50$ preliminary



Tetraneutron (3)

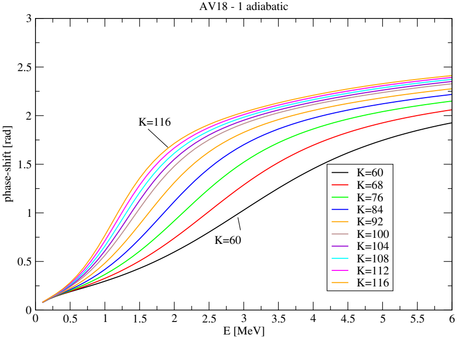
Phase-shift with only the lowest eigenpotential

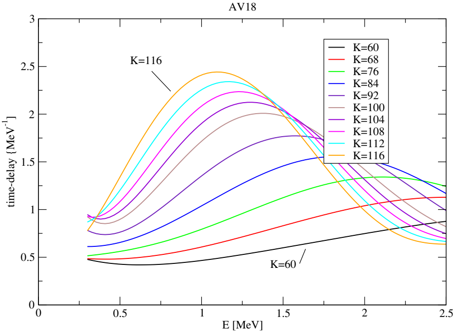
AV18 0^+ state $\mathcal{L} = 5$

$$f_0(\rho) \to j_5(Q\rho) + \tan\delta_0(E)(-y_5(Q\rho))$$

Phase-shift $\delta_0(E)$

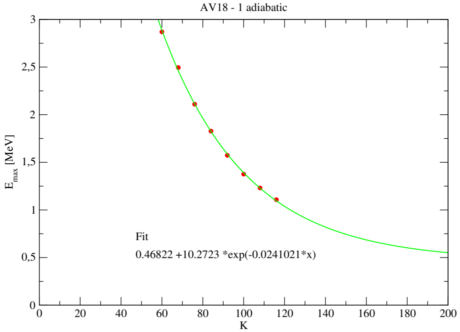
Time-delay $= 2d\delta/dE$





Tetraneutron (4)

Position of the maxima



- Energy of the time-delay "peak"
- Extrapolation

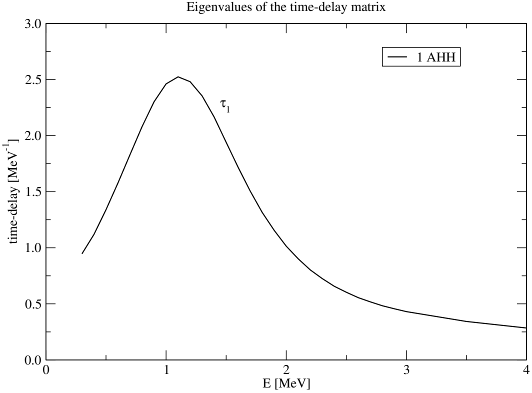
K_{max}	maximum of τ [MeV]
60	2.87
68	2.49
76	2.11
84	1.83
92	1.57
100	1.38
108	1.23
116	1.10
Extp.	0.5

Tetraneutron (5)

Contribution of other AHH - solution of the coupled equations

$$-\frac{\hbar^2}{m} f''_\ell(\rho) + \sum_{\ell'} \left[P_{\ell\ell'}(\rho) \frac{d}{d\rho} + Q_{\ell\ell'}(\rho) \right] f_{\ell'}(\rho) + U_\ell(\rho) f_\ell(\rho) = E f_\ell(\rho) \qquad T_D = iS \frac{dS^\dagger}{dE}$$

τ_i, $i = 1, \ldots$ eigenvalues of T_D AV18 potential - $K_{max} = 116$



Contribution of other AHH - solution of the coupled equations

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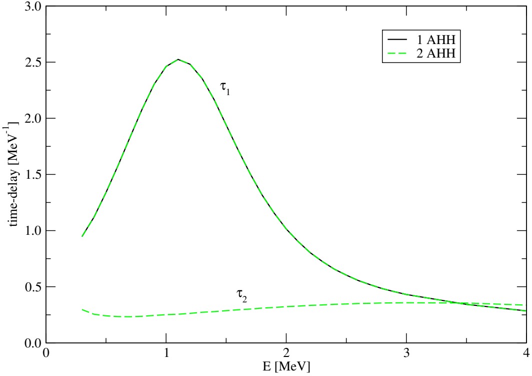
$\tau_i, i = 1, \ldots$ eigenvalues of T_D AV18 potential - $K_{max} = 116$

Eigenvalues of the time-delay matrix



Tetraneutron (5)

Contribution of other AHH - solution of the coupled equations

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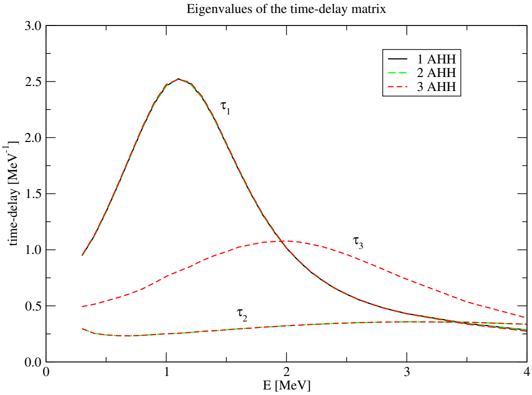

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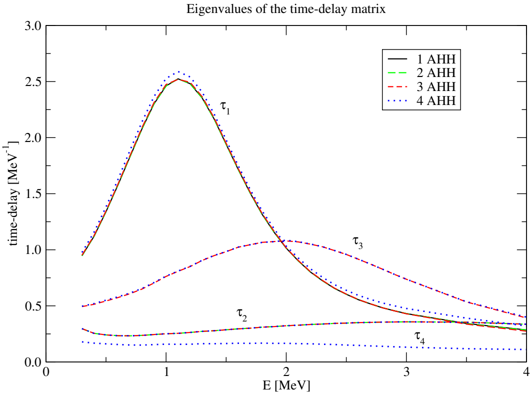
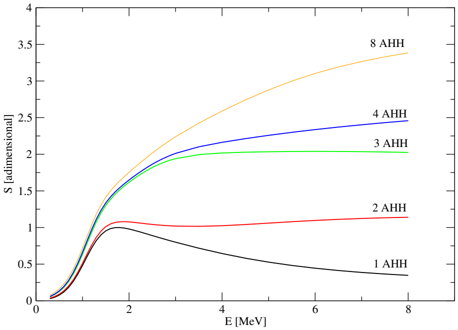


A-body $\rightarrow$ A- body scattering cross section

[Metha *et al.*, 2009] ($D = 3(A-1)$)

$$\Psi(\rho,\Omega) = \sum_{\ell=0}^{N} \frac{f_\ell(\rho)}{\rho^{\frac{D-1}{2}}} \Phi_\ell(\rho,\Omega) \qquad \sigma_{el} = \left(\frac{2\pi}{k}\right)^{D-1} \frac{\Gamma(D/2)}{2\pi^{D/2}} \underbrace{\sum_{\ell,\ell'=0}^{N} |S_{\ell\ell'} - \delta_{\ell,\ell'}|^2}_{S}$$

Tetraneutron - AV18 - Preliminary



Conclusions & Outlook

Conclusions

- Preliminary results for AV18
- There is a time-delay "peak" at $E_R \approx 0.5$ MeV
- Convergence with K_{max} still to be achieved
- Superposition of resonances - no "peak" in the $4n \to 4n$ cross section
- Small effects of the 3N force (important region $\rho \approx 10 - 30$ fm)

Perspectives

- Study of different $NN/3N$ potentials
- Study of transition matrix elements like $\langle 3n|\tau^-|^3\mathrm{H}\rangle$ or $\langle 4n|\tau^-\tau^-|^4\mathrm{He}\rangle$