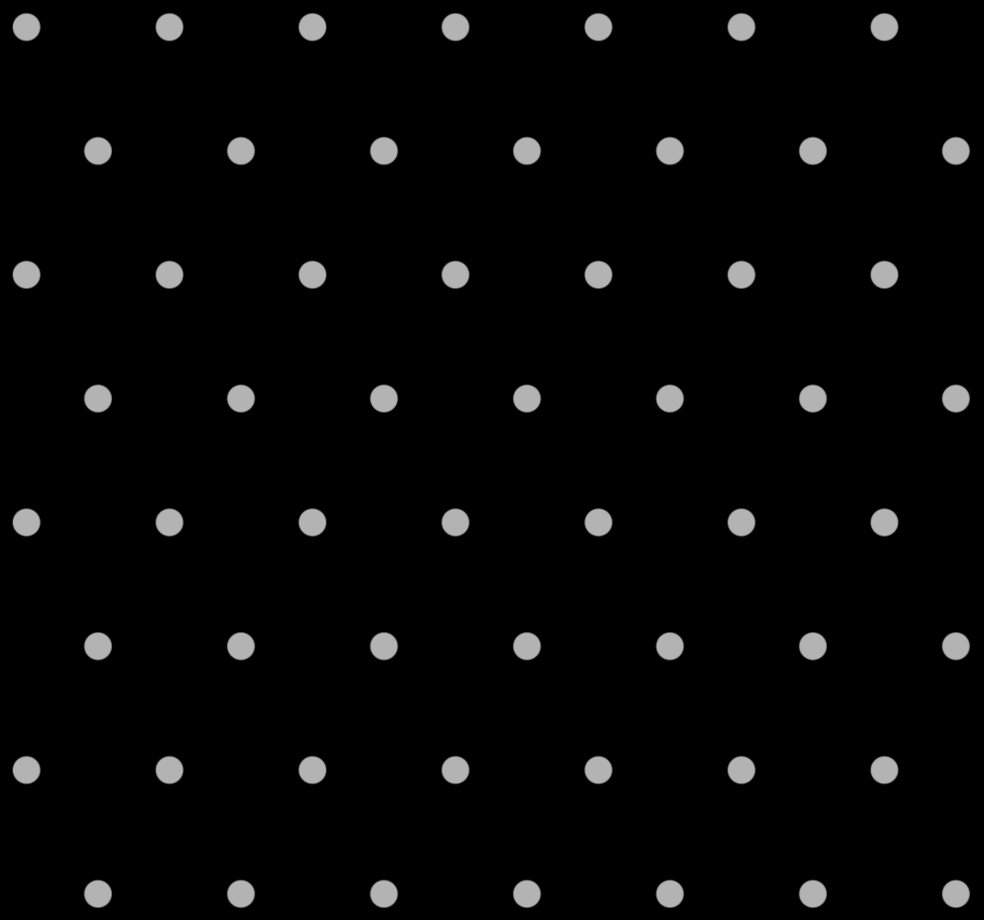
PHASES OF AN ULTRACOLD GAS:

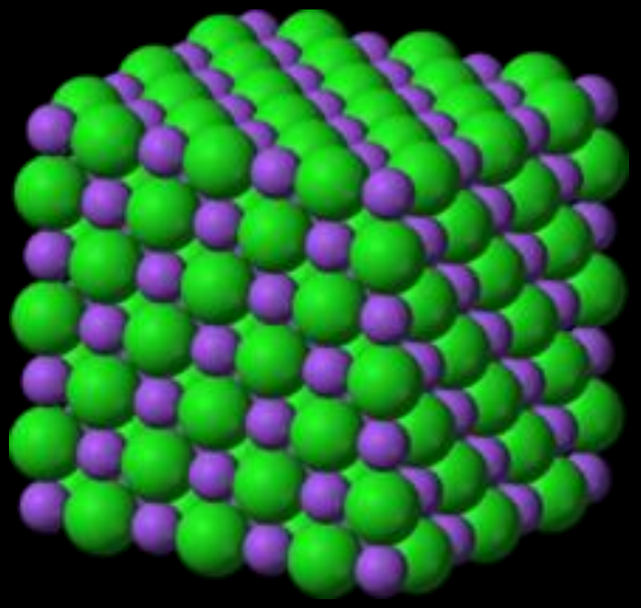
BOSONS IN A QUASICRYSTALLINE LATTICE

Dean Johnstone

HERIOT WATT UNIVERSITY

CRYSTALS





- Periodic arrangement of atoms on a discrete set of points.

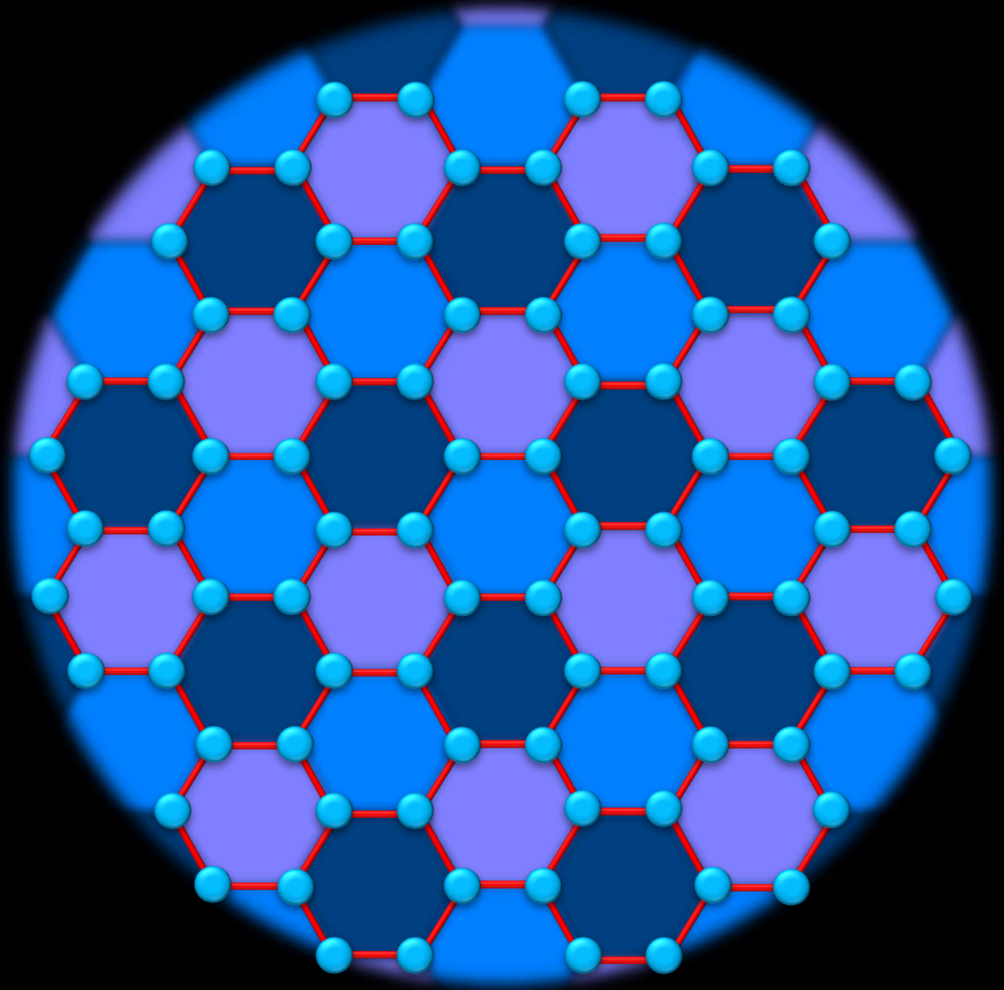
PERIODIC TILING

- Vertex model of a Euclidean plane tiling can seen as an effective tight-binding model.

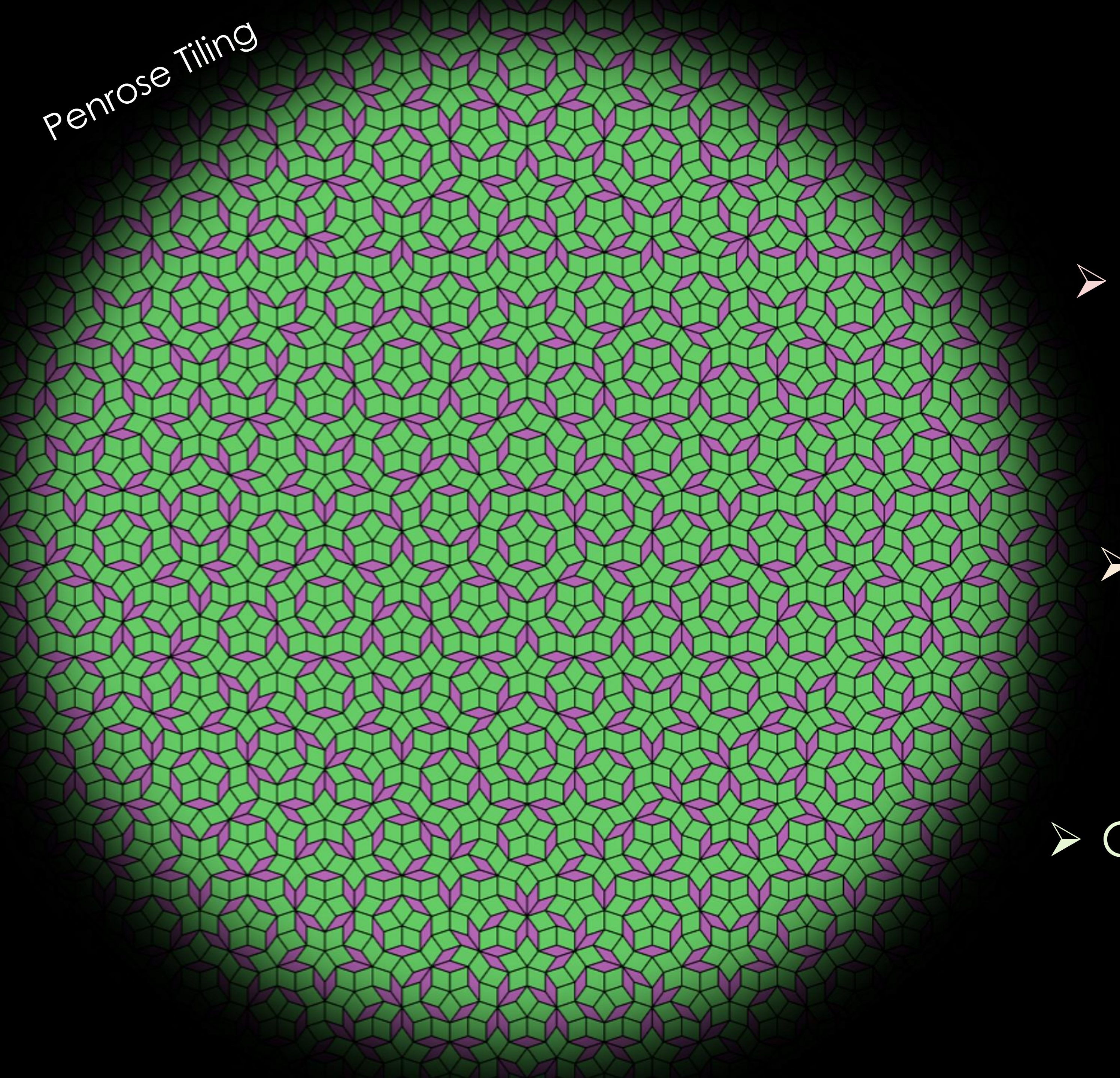
- **Edges** correspond to **bonds**.

- Intersection of **edges/vertices** correspond to **lattice sites**.

PERIODIC TILING



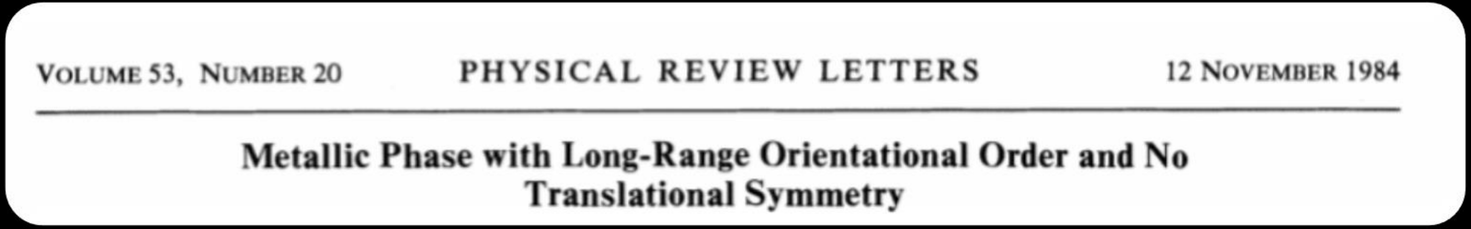
- Vertex model of a Euclidean plane tiling can seen as an effective tight-binding model.
- **Edges** correspond to **bonds**.
- Intersection of **edges/vertices** correspond to **lattice sites**.



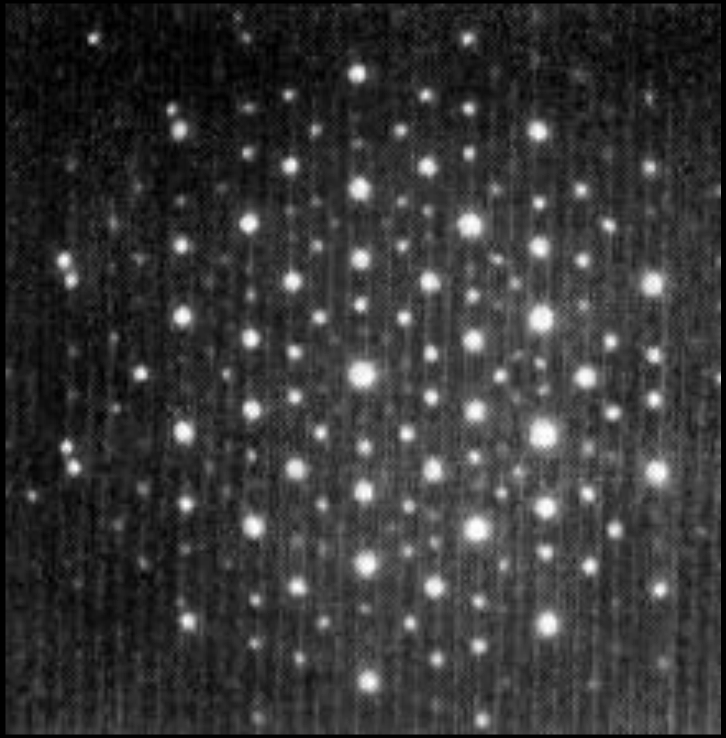
APERIODIC TILING

- No translational invariance, but still possess long-range order.
- Self-similar pattern that fills all of space continuously.
- Can have rotational symmetries outwith ***Crystallographic Restriction Theorem***.

DISCOVERY OF QUASICRYSTALS

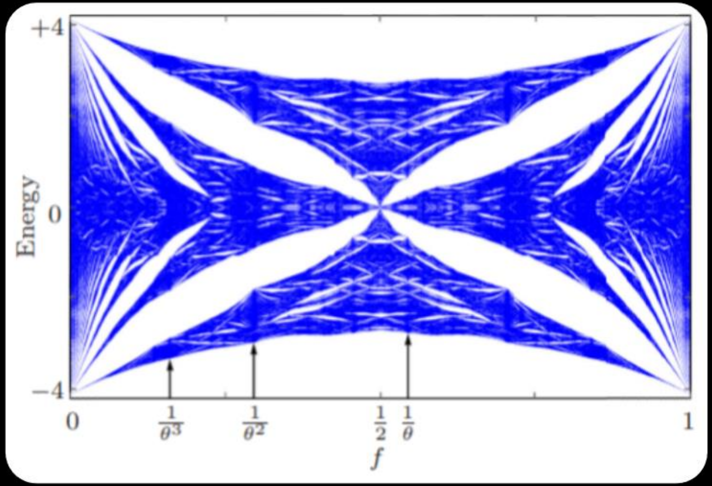

VOLUME 53, NUMBER 20 PHYSICAL REVIEW LETTERS 12 NOVEMBER 1984

Metallic Phase with Long-Range Orientational Order and No Translational Symmetry



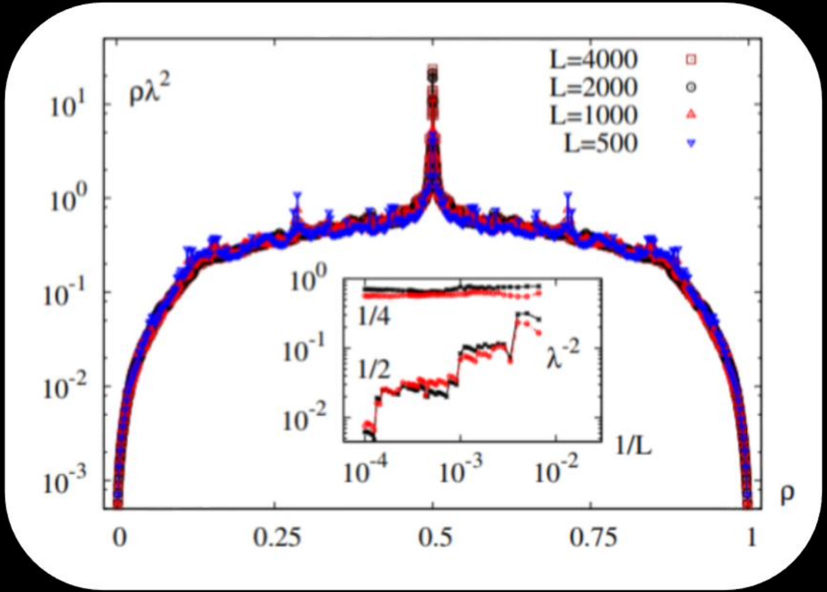
Dan Shechtman

"Hofstadter Butterfly of a Quasicrystal"



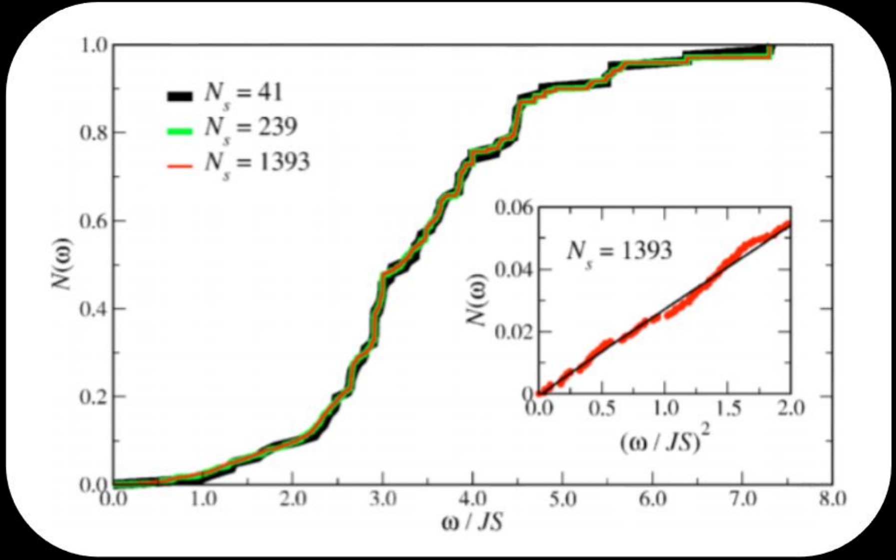
Phys. Rev. B 94, 205437 (2016)

"Conduction in quasi-periodic and quasi-random lattices: Fibonacci, Riemann, and Anderson models"



Phys. Rev. B 94, 214204 (2016)

"Quantum Fluctuations and Excitations in Antiferromagnetic Quasicrystals"



Phys. Rev. B 71, 104427 (2005)

PHYSICAL REVIEW LETTERS **122,** 110404 (2019)

Editors' Suggestion | Featured in Physics

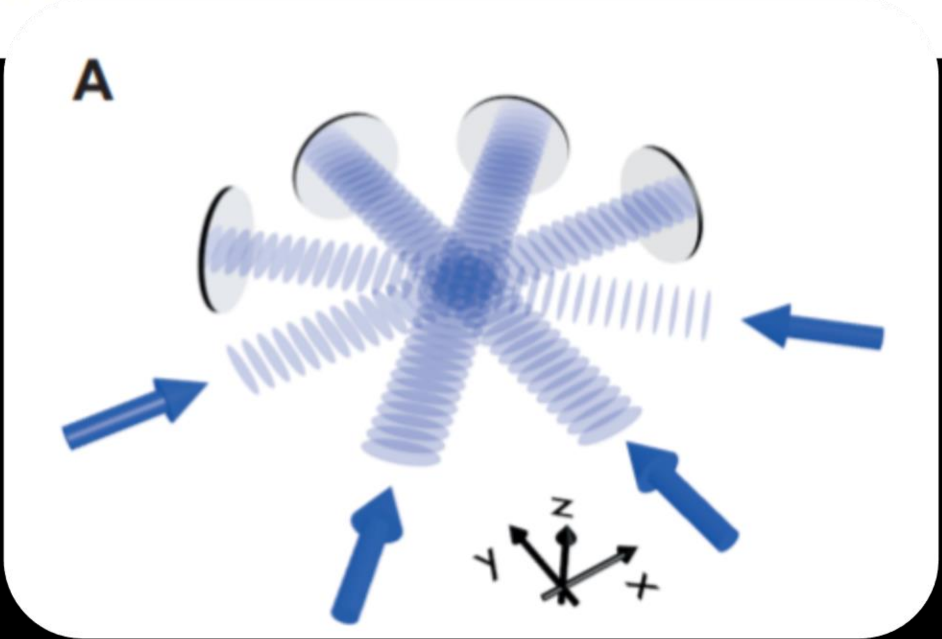
Matter-Wave Diffraction from a Quasicrystalline Optical Lattice

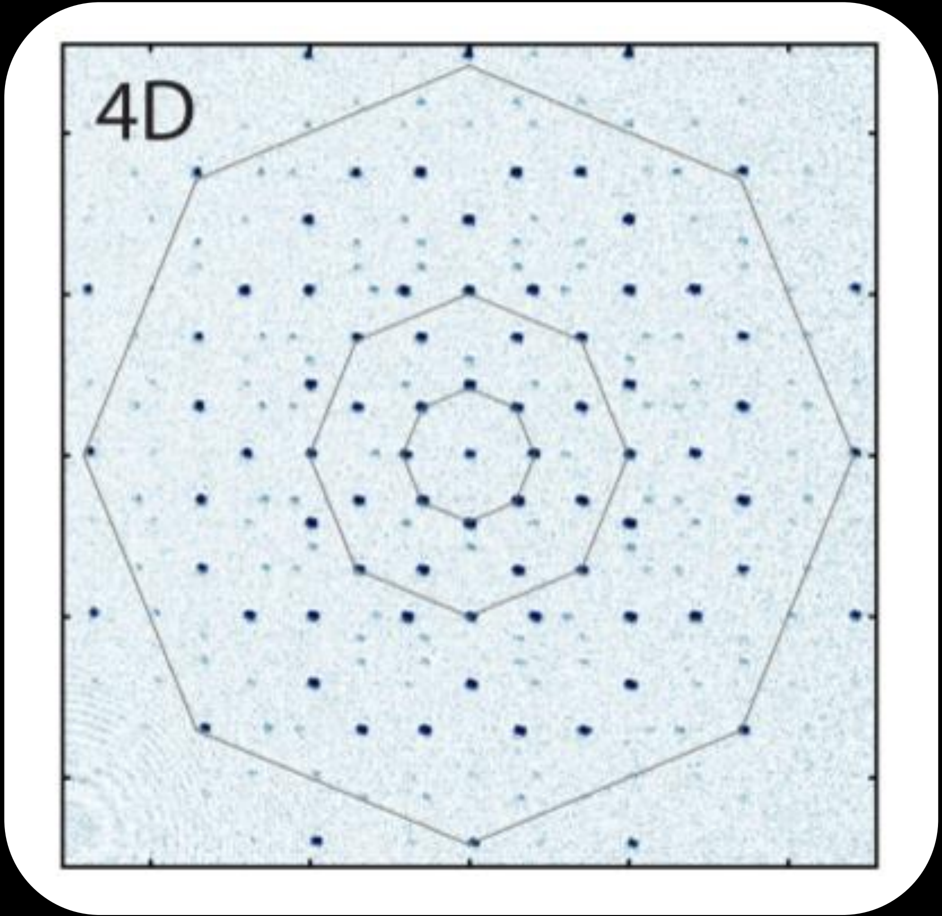
Konrad Viebahn, Matteo Sbroscia, Edward Carter, Jr-Chiun Yu, and Ulrich Schneider*
Cavendish Laboratory, University of Cambridge, J. J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom

(Received 26 November 2018; published 20 March 2019)

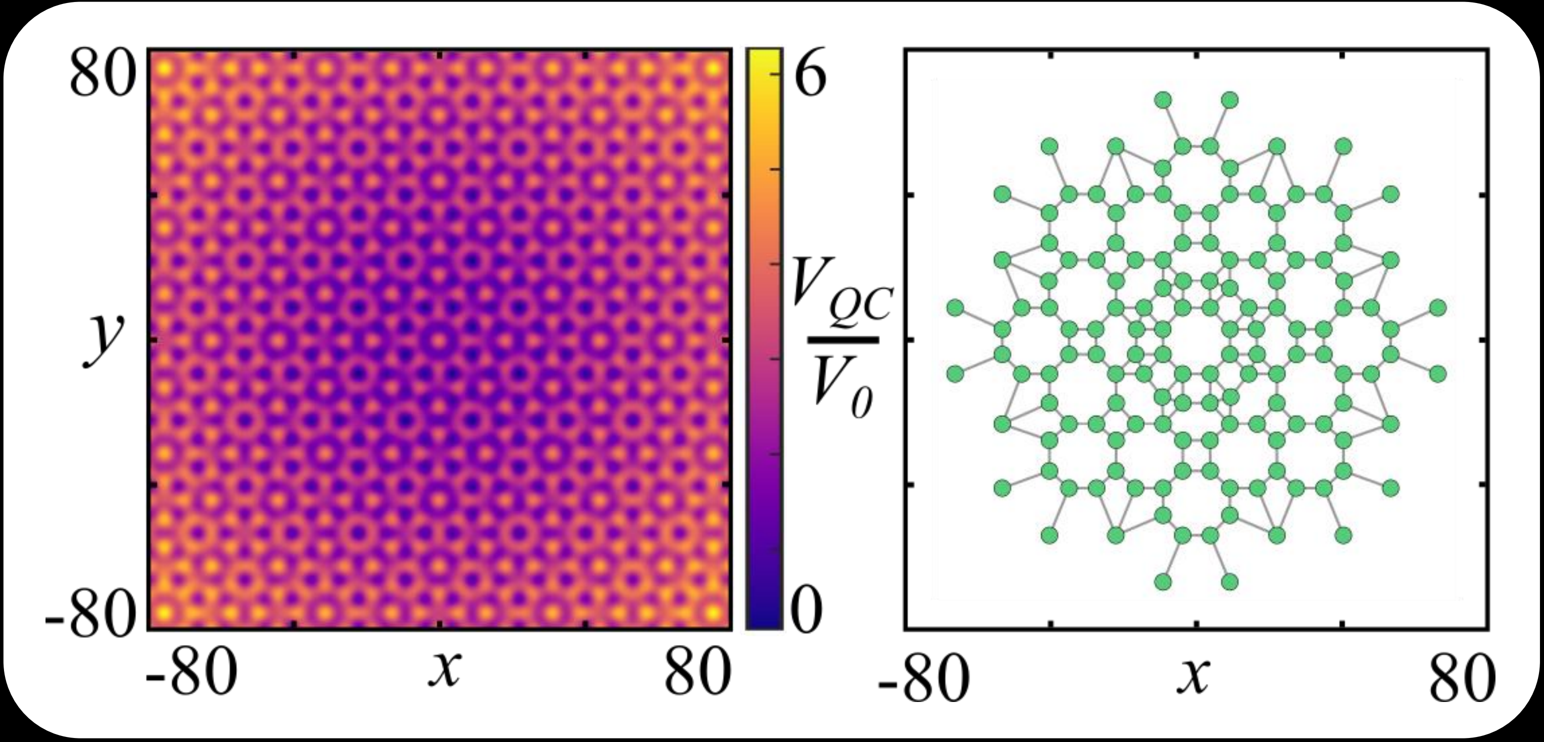
Quasicrystals are long-range ordered and yet nonperiodic. This interplay results in a wealth of intriguing physical phenomena, such as the inheritance of topological properties from higher dimensions, and the presence of nontrivial structure on all scales. Here, we report on the first experimental demonstration of an eightfold rotationally symmetric optical lattice, realizing a two-dimensional quasicrystalline potential for ultracold atoms. Using matter-wave diffraction we observe the self-similarity of this quasicrystalline structure, in close analogy to the very first discovery of quasicrystals using electron diffraction. The diffraction dynamics on short timescales constitutes a continuous-time quantum walk on a homogeneous four-dimensional tight-binding lattice. These measurements pave the way for quantum simulations in fractal structures and higher dimensions.

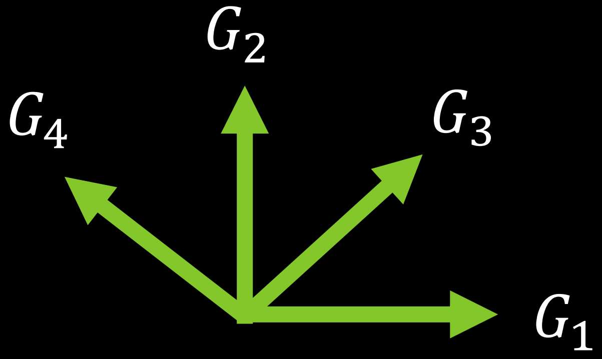
DOI: 10.1103/PhysRevLett.122.110404





QUASICRYSTALLINE POTENTIAL



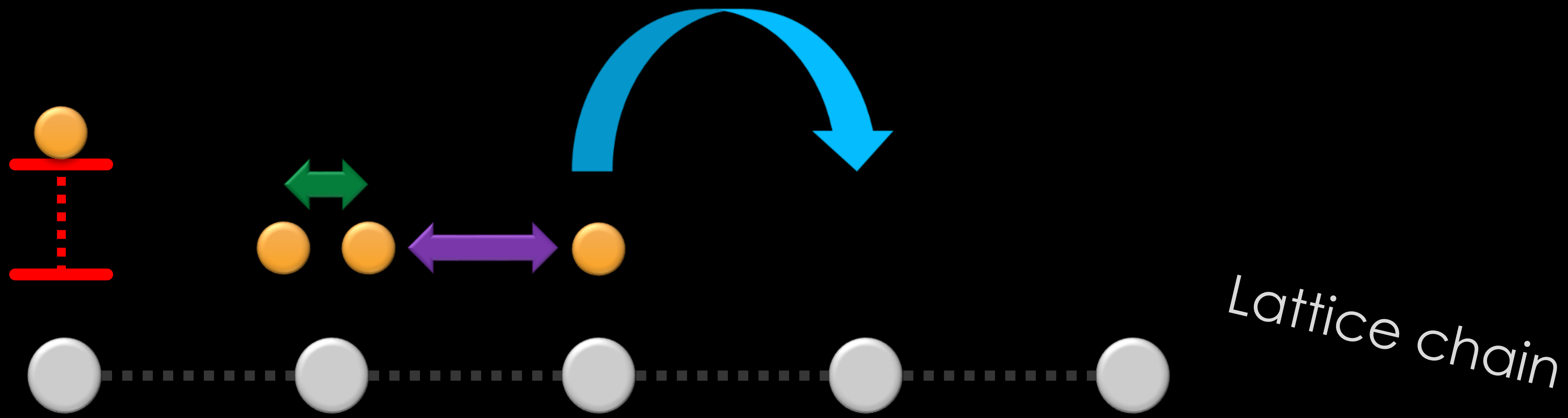


$$V_0 \sum_{i=1}^{4} \cos^2\left(\frac{\boldsymbol{G}_i \, . \, \boldsymbol{r}}{2}\right)$$

Lattice Potential

TIGHT-BINDING MODEL

$$\hat{H} = \sum_{i} (\varepsilon_i - \mu)\hat{n}_i + \sum_{i} \frac{U_i}{2}\hat{n}_i(\hat{n}_i - 1) - \sum_{<i,j>} J_{ij}\hat{b}_i^{\dagger}\hat{b}_j + \sum_{<i,j>} V_{ij}\hat{n}_i\hat{n}_j$$



TIGHT-BINDING MODEL

$$\hat{H} = \sum_i (\varepsilon_i - \mu)\hat{n}_i + \sum_i \frac{U_i}{2}\hat{n}_i(\hat{n}_i - 1) - \sum_{<i,j>} J_{ij}\hat{b}_i^\dagger \hat{b}_j + \sum_{<i,j>} V_{ij}\hat{n}_i\hat{n}_j$$

Homogenous Lattice

$$\hat{H} = (\varepsilon - \mu)\sum_i \hat{n}_i + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - J\sum_{<i,j>} \hat{b}_i^\dagger \hat{b}_j + V\sum_{<i,j>} \hat{n}_i\hat{n}_j$$

GROUND STATES

$\mu = 0$

Fixed ***N*** atoms for ***L*** sites

$$D_f = \frac{(N+L-1)!}{N!\,(L-1)!}$$

$\mu \neq 0$

Max ***z*** atoms for ***L*** sites

$$D_g = (z+1)^L$$

$\{N, L, z\} = 10$

$$D_f \sim 10^5$$

$$D_g \sim 10^{10}$$

GUTZWILLER MEAN-FIELD

$$|\Psi\rangle = \prod_{i}^{L} \sum_{n=0}^{z} f_n^{(i)} \, |n_i\rangle$$

$$\hat{b}_i^\dagger \hat{b}_j \rightarrow \langle \hat{b}_i^\dagger \rangle \hat{b}_j + \langle b_j \rangle \hat{b}_i^\dagger - \langle \hat{b}_i^\dagger \rangle \langle b_j \rangle$$

$$\hat{n}_i \hat{n}_j \rightarrow \langle \hat{n}_i \rangle \hat{n}_j + \langle \hat{n}_j \rangle \hat{n}_i - \langle \hat{n}_i \rangle \langle \hat{n}_j \rangle$$

$$\hat{H} \rightarrow \sum_{i}^{L} \hat{H}_i$$

$$D_g = z + 1$$

OBSERVABLES AND PHASES

Transport Order Parameter

$$\varphi_i = \langle b_i \rangle = \sum_{n=0}^{z} \sqrt{n}\, f_n^{(i)} f_{n-1}^{*(i)}$$

$$\bar{\varphi} = \max(\varphi) - \min(\varphi)$$

Density Order Parameter

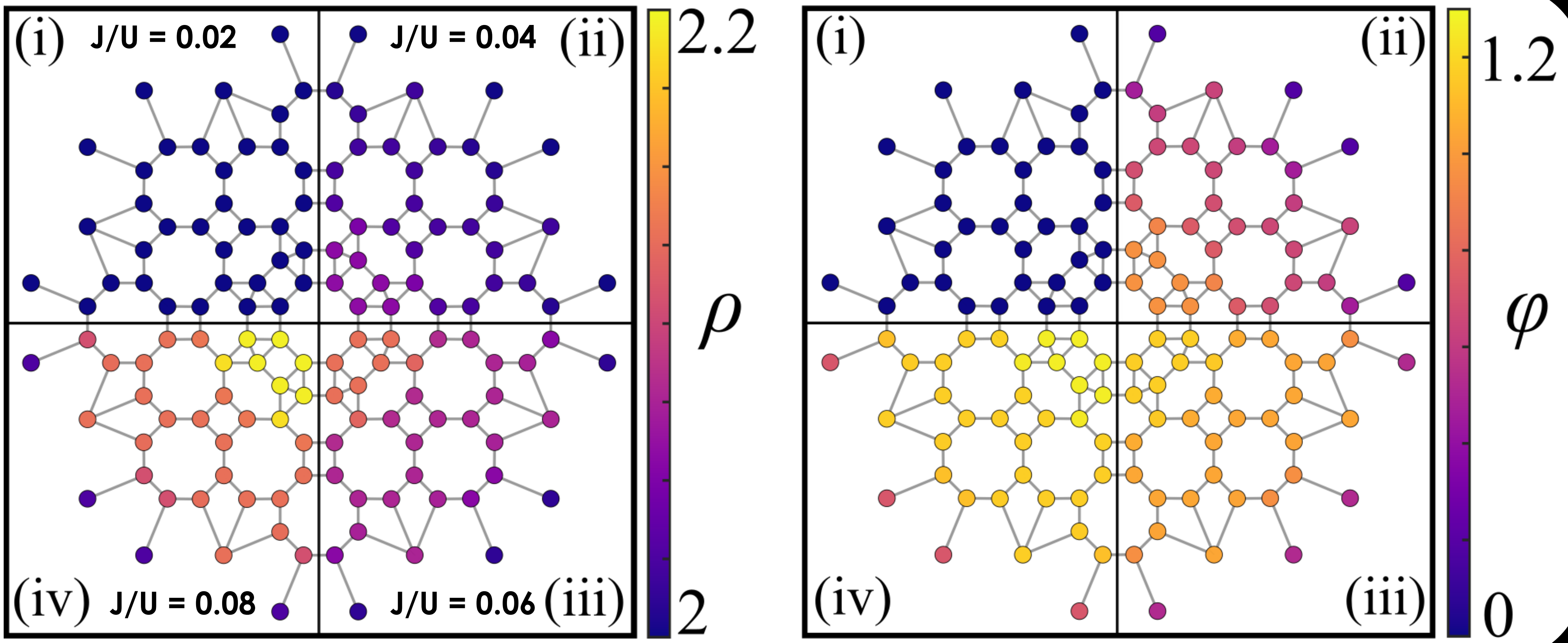
$$\rho_i = \langle n_i \rangle = \sum_{n=0}^{z} n \left| f_n^{(i)} \right|^2$$

$$\bar{\rho} = \max(\rho) - \min(\rho)$$

OBSERVABLES AND PHASES

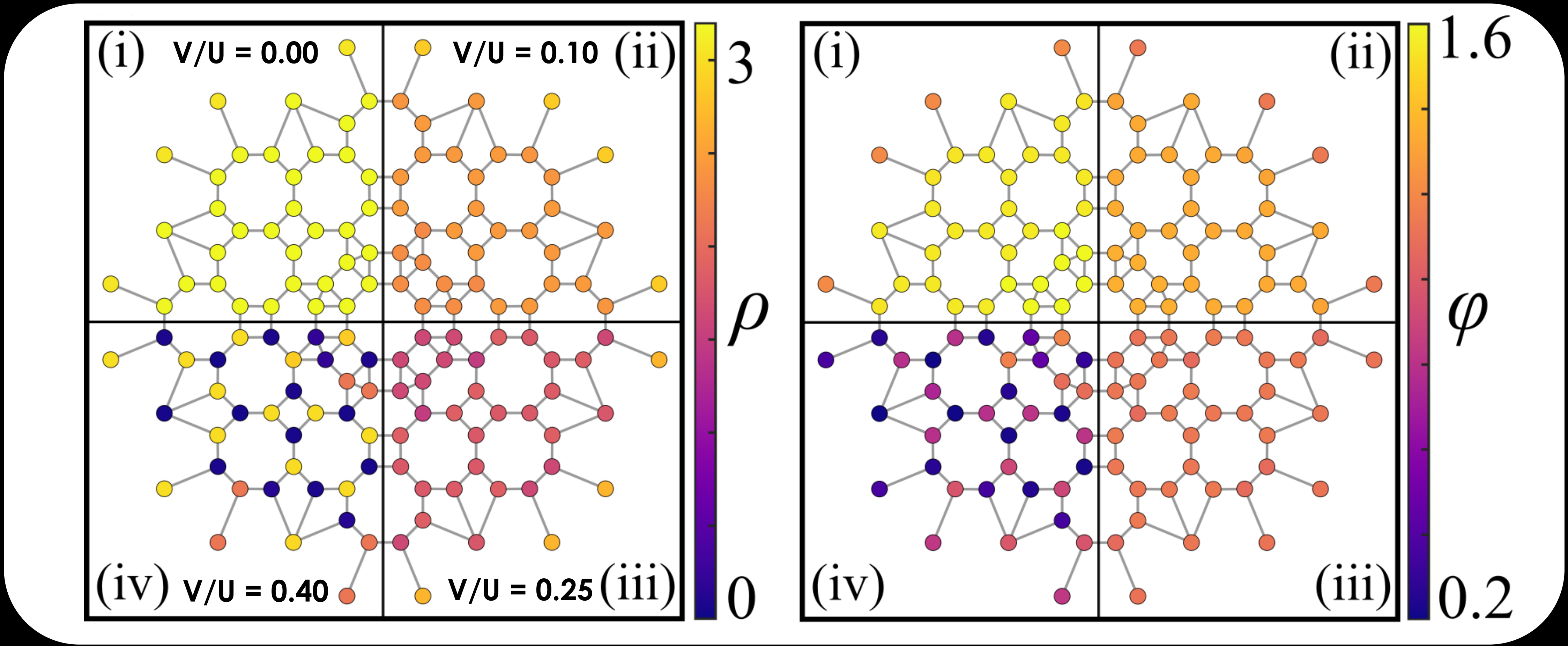
Phase	$\bar{\varphi}$	$\bar{\rho}$
Mott Insulator (MI)	≈ 0	≈ 0
Superfluid (SF)	> 0	≈ 0
Density Wave (DW)	≈ 0	> 0
Supersolid (SS)	> 0	> 0

MI-SF



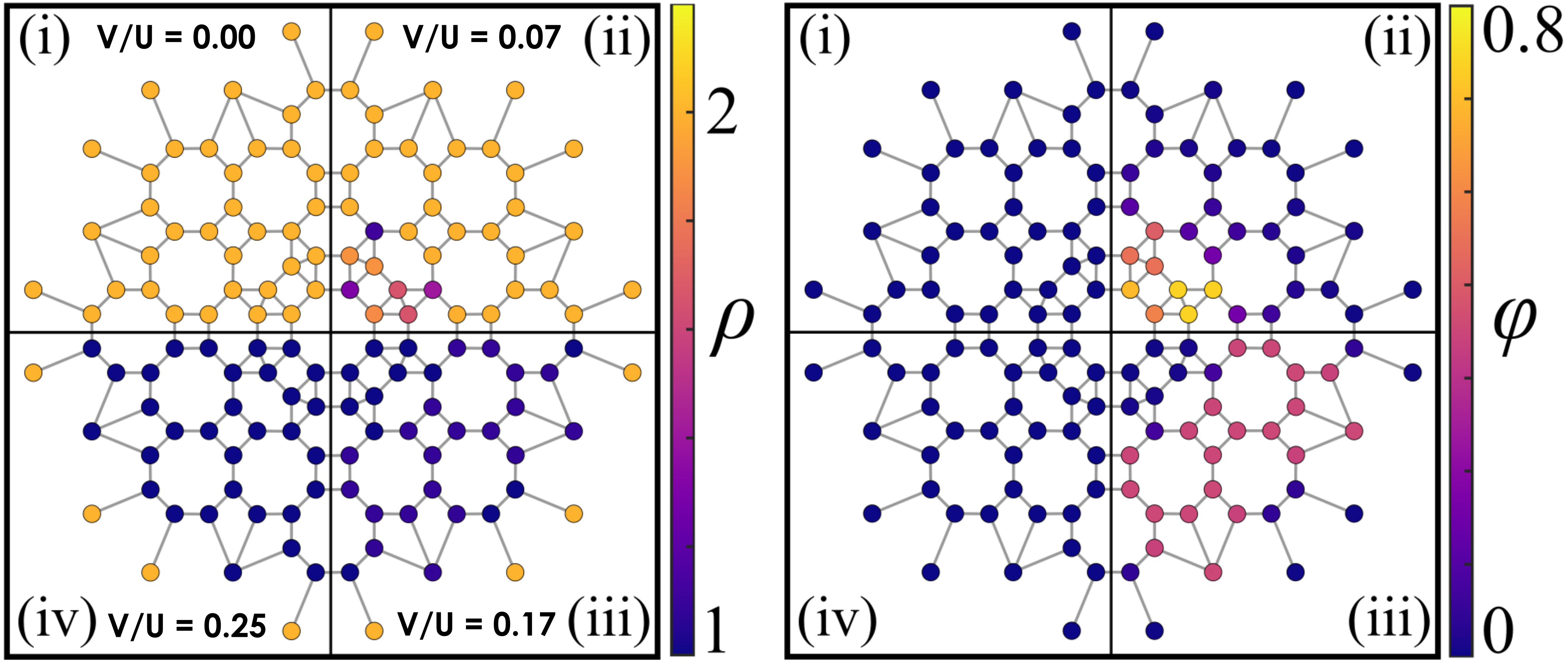
$\mu/U = 1.5$

SF-SS



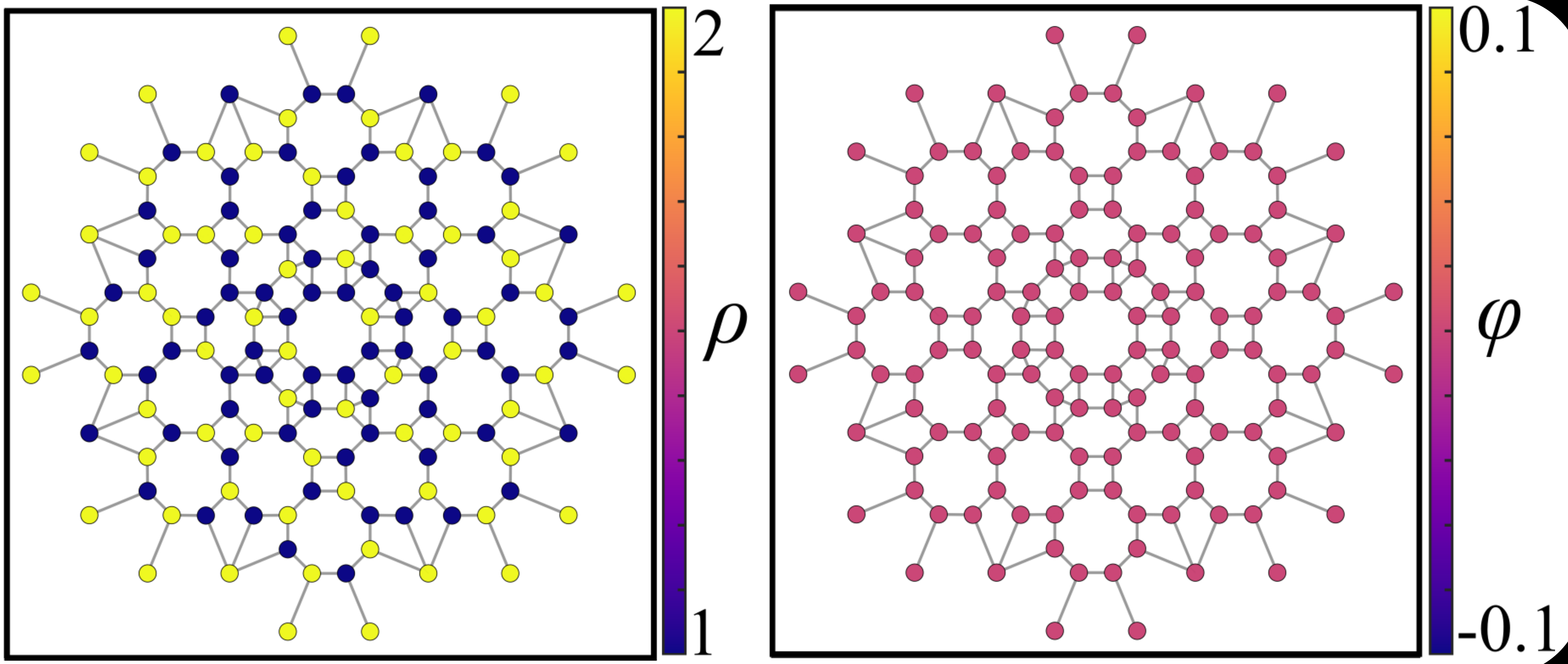
$\mu/U = 2.5$, $J/U = 0.1$

MI-DW



μ/U = 1.5, J/U = 0.01

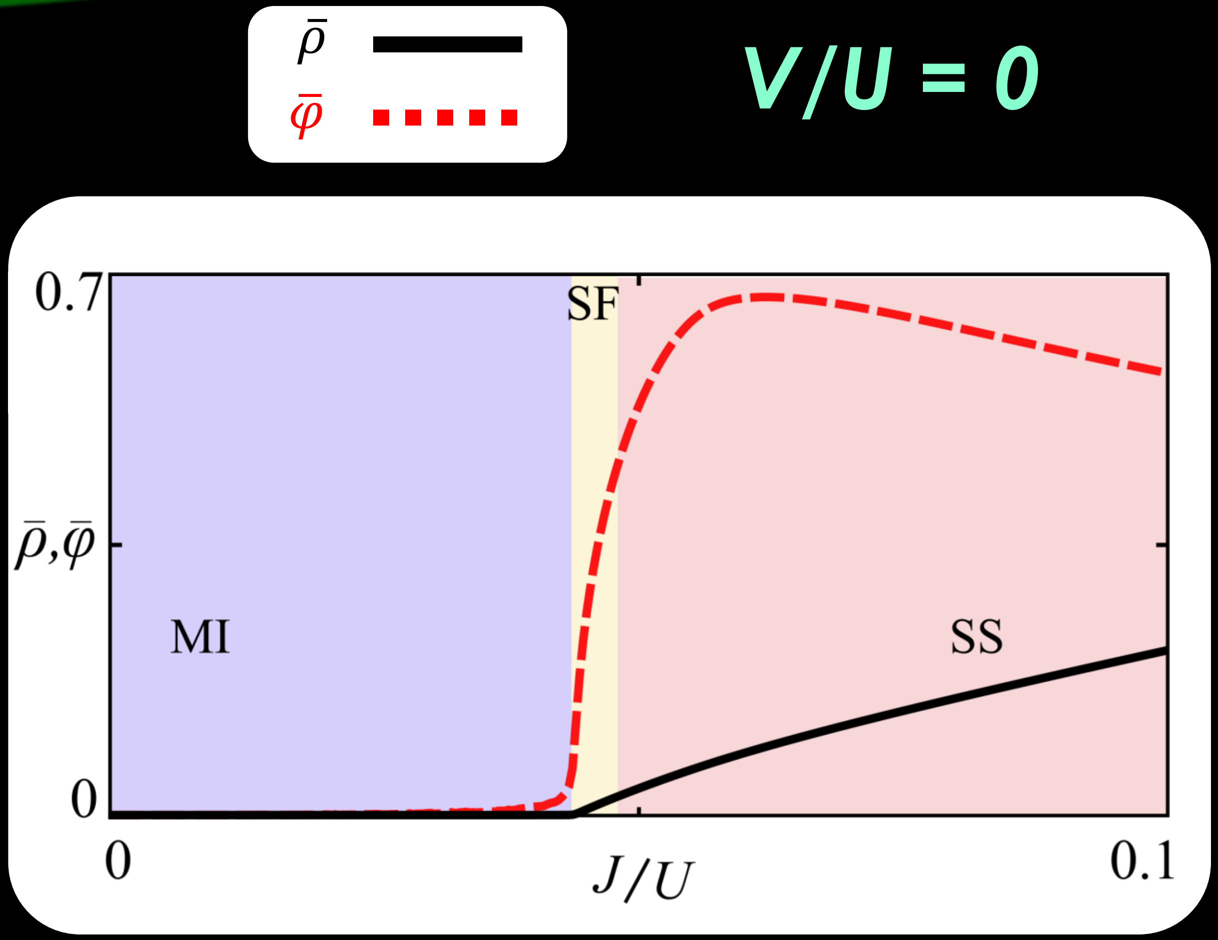
SYMMETRY BREAKING



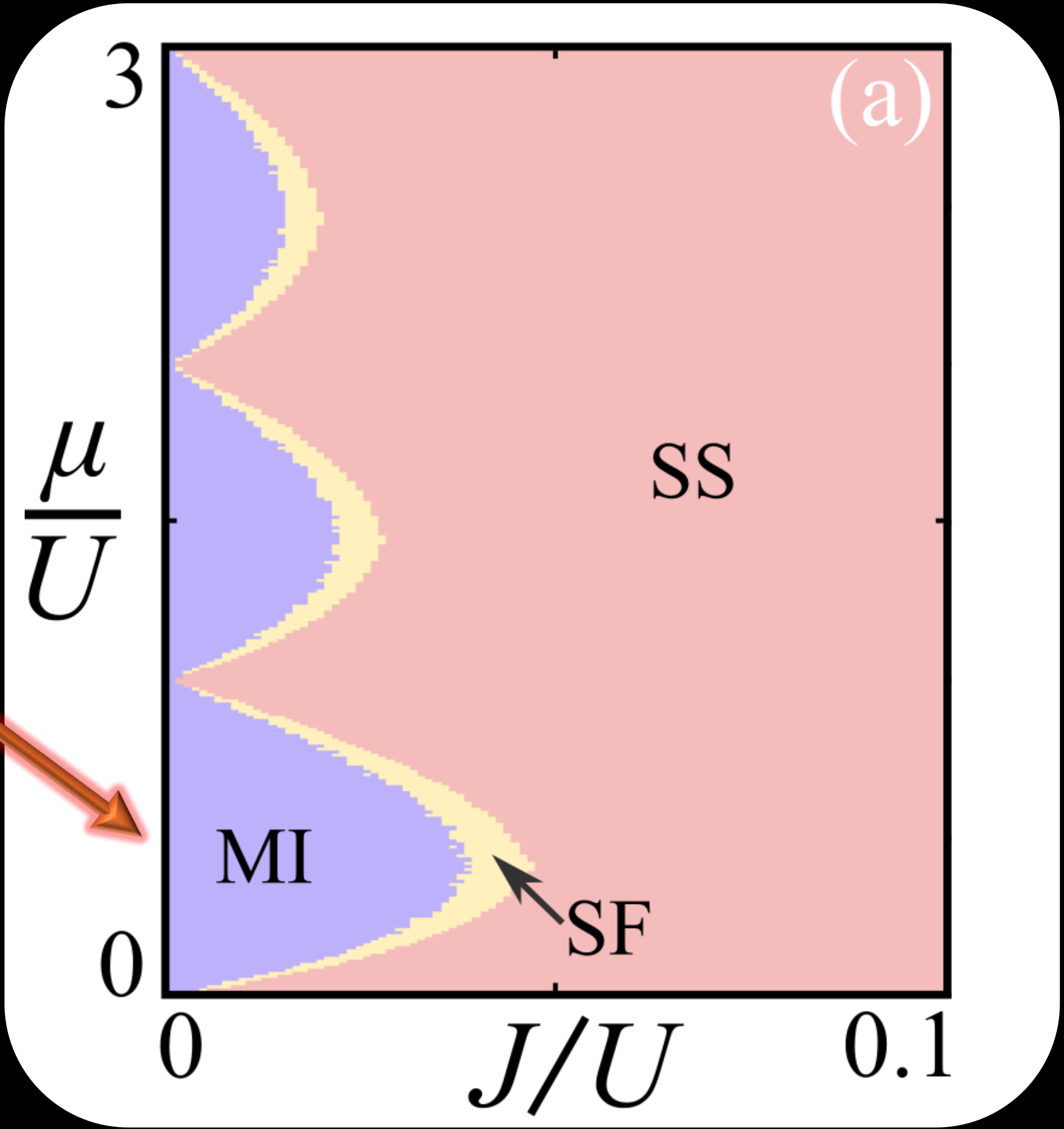
μ/U = 1.5, J/U = 0.01, V/U = 0.1

PHASE DIAGRAMS

$V/U = 0$

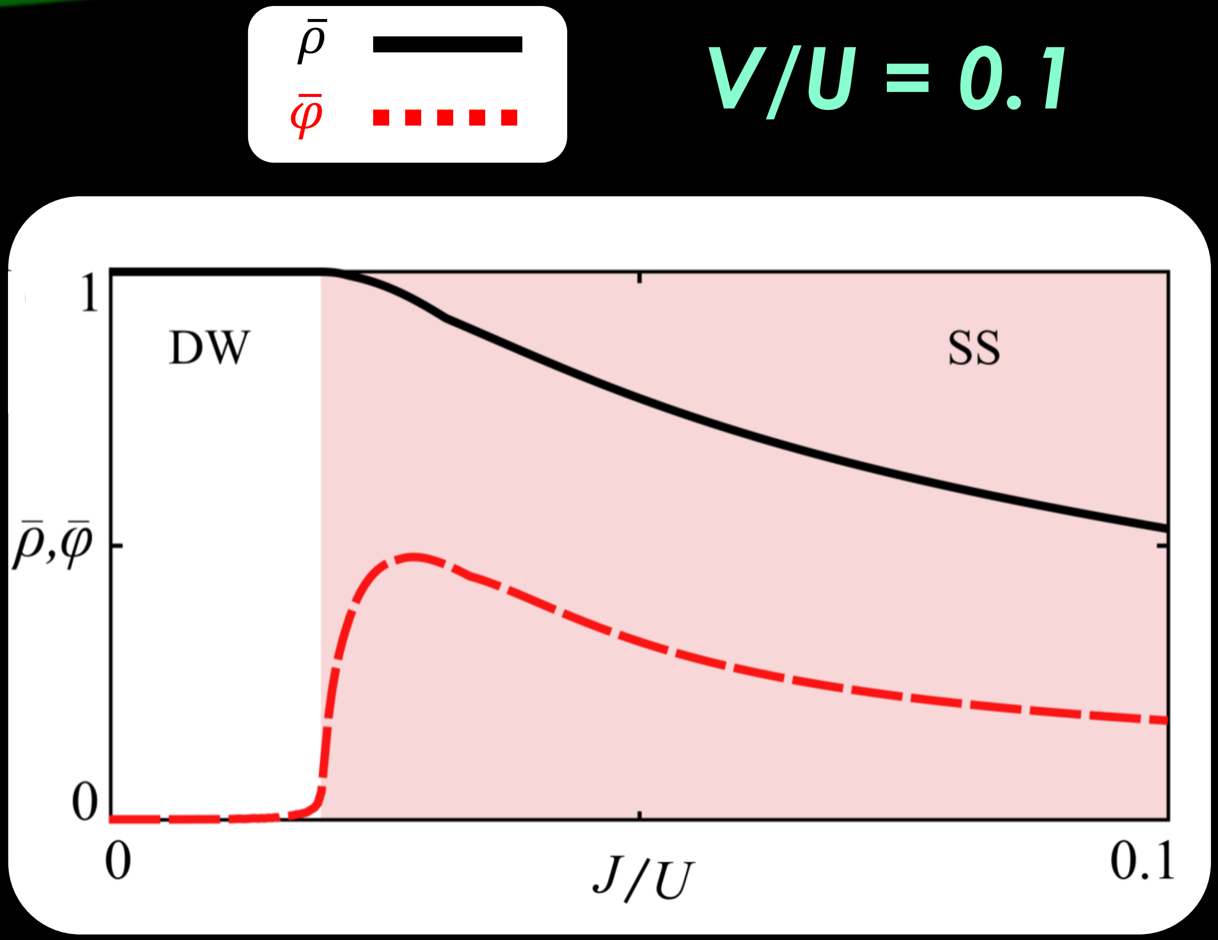


$\mu/U = 0.5$

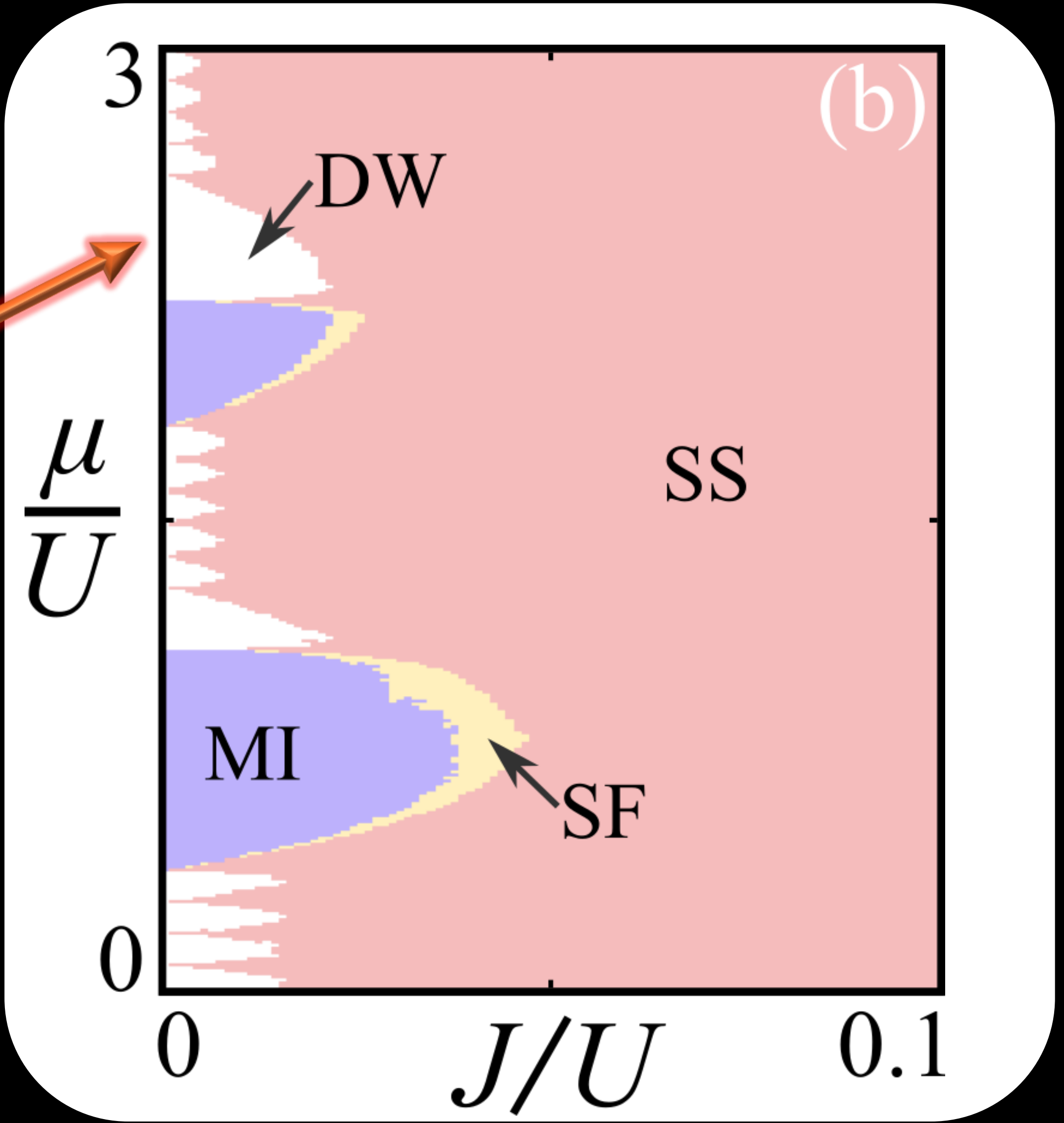


PHASE DIAGRAMS

V/U = 0.1



$\mu/U = 2.4$

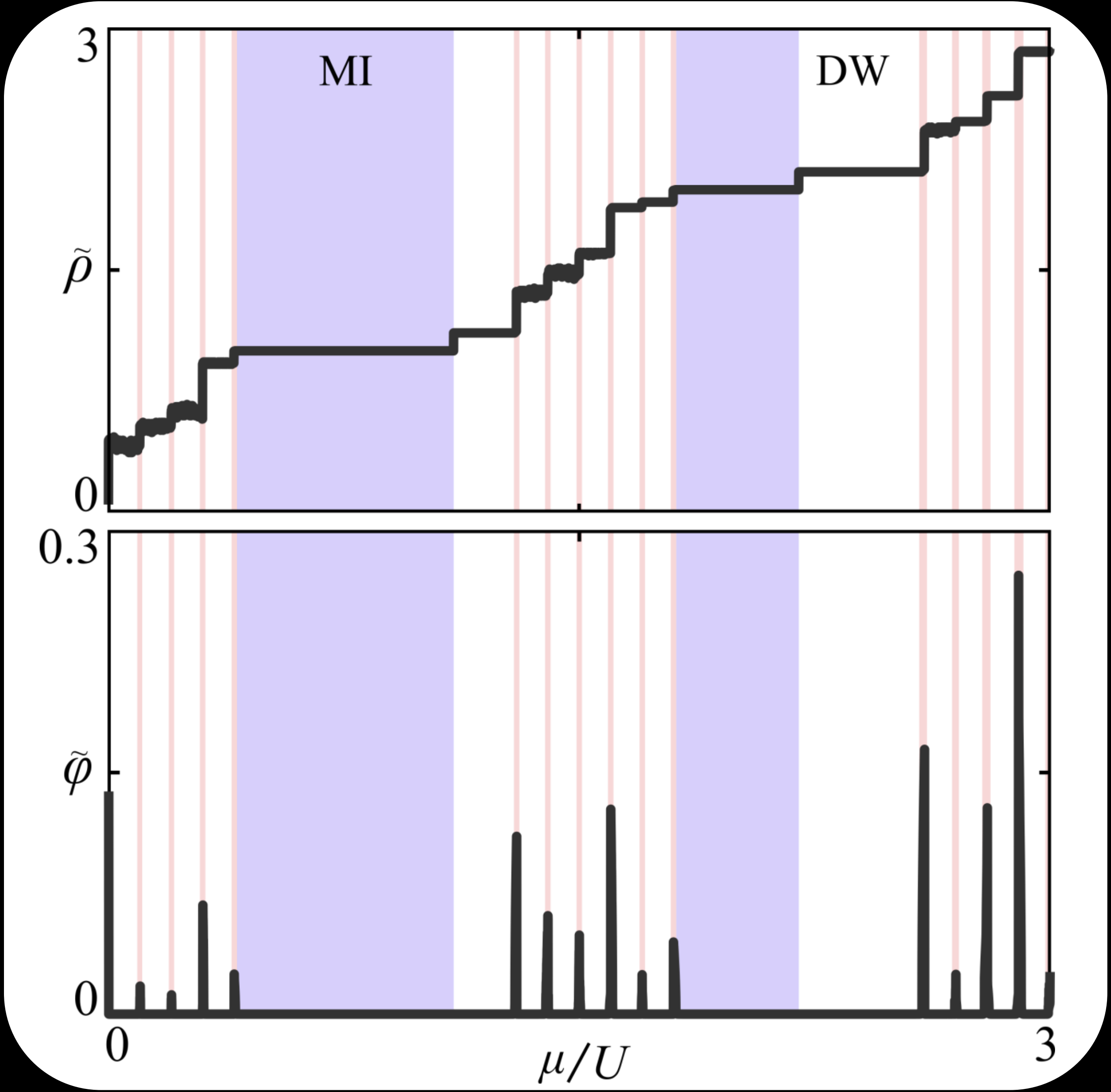


J/U = 0.001

$\tilde{\rho}$ & $\tilde{\varphi}$

Average order parameter across entire lattice

- Fractionalised set of DW states between each MI plateau.



CONCLUSIONS

- Can observe interesting phenomena in quasicrystals without invoking disorder.
- Fluctuations in local coordinate number plays an important role.
- Rotational symmetries of Quasicrystal can be broken.

|QOCA⟩

Phase transitions of an ultracold gas in a quasicrystalline potential

Dean Johnstone,* Patrik Öhberg, and Callum W. Duncan†
SUPA, Institute of Photonics and Quantum Sciences,
Heriot-Watt University, Edinburgh, EH14 4AS, UK

https://arxiv.org/abs/1904.12870

Prof. Patrik Öhberg

Callum W. Duncan

HERIOT WATT UNIVERSITY

CM-CDT

SUPA