Electroweak processes in few-nucleon systems

A. Baroni[a] & M. Viviani[b]

[a] LANL, Los Alamos, NM (USA)
[b]INFN, Sezione di Pisa &
Department of Physics, University of Pisa
Pisa (Italy)

24th European Conference on Few-Body Problems in Physics

September 1-6, 2019
Guildford, Surrey (UK)

Outline

Collaborators

- R. Schiavilla *Jefferson Lab. & ODU, Norfolk (VA, USA)*
- M. Piarulli & S. Pastore *GWU, St.Louis (MS, USA)*
- L. Girlanda *University of Salento & INFN-Lecce, Lecce (Italy)*
- A. Kievsky & L.E. Marcucci - *INFN-Pisa & Pisa University, Pisa (Italy)*
- A. Lovato, B. Wiringa, & S. Pieper, *ANL, Argonne (IL, USA)*

EFT approach

Chiral symmetry

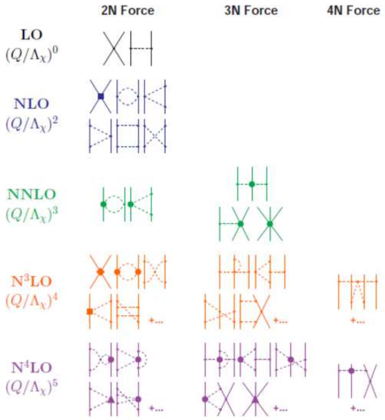
- $\mathcal{L}_{QCD}$ (almost) invariant under $SU(2)$ “isospin” and “axial” rotations since m_u, m_d “small”
- Invariance also for *locals* rotations: Introducing external vector and axial currents
- Non-linear realization of the chiral symmetry for nucleons and nuclei [Weinberg, 1968, 1990], [CCWZ, 1969], [Gasser & Leutwyler, 1984], ...
- $\rightarrow \chi$EFT $=$ Lagrangian written in terms of nucleonic and pionic d.o.f. – automatic inclusion of external vector and axial currents
- Infinite number of unknown coupling constants (low energy constants – LECs)
- All quantities can be written as powers of Q/Λ_χ, $Q \sim$ small momenta or the pion mass, $\Lambda_\chi \sim 1$ GeV

Potentials and currents from χEFT

- To obtain effective operators acting only on nucleons d.o.f.:
  - S-matrix: for a given process $NN \rightarrow NN$ define V so that (on-shell) $\langle NN|T_{\rm EFT}|NN\rangle \equiv \langle NN|T_V|NN\rangle$
  - Unitary transformation: find U in order to decouple $|NN\rangle$ Hilbert space from $|NN\pi\rangle$, etc.
- Alternative: Lattice χEFT [Lee *et al.*, 2010]

NN & 3N interaction

For more information see for example [Epelbaum, 2010], [Machleidt & Entem, 2011]



NN interaction

- Jülich N4LO [Epelbaum, Krebs, & Meissner, 2014], [Reinert, Krebs, & Epelbaum, 2017]
- Idaho N4LO [Entem, Machleidt, & Nosyk, 2017]

LEC's fitted to the NN database or πN database

3N interaction

- N2LO [Epelbaum *et al*, 2002]
- 3N force at N3LO & N4LO [Krebs *et al.*, 2012-2013]

– At N2LO there are two LECS c_D and c_E: fitted to some 3N epxt. data (see later)
– At N3LO no new parameters
– At N4LO 10 new LECs [Girlanda *et al., 2011, 2019*]

Ingredients to study EW transitions

- Initial/final wave functions + transition operators (currents & charges)
- For transition to continuum: scattering wave functions
- Inclusive & semi-inclusive reactions: integral techniques (Lorentz integral tranform – Laplace integral transform)

Example

Transition $|\alpha\rangle + \gamma \to |\beta\rangle$

$$\langle\beta|H_{e.m.}|\alpha;\boldsymbol{q}\lambda\rangle = \langle\Psi_\beta|\mathcal{K}_1|\Psi_\alpha\rangle \qquad \mathcal{K}_1 = \frac{-e}{\sqrt{2\omega\Omega}}\int d\boldsymbol{x}\; e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\,\widehat{\epsilon}_{\boldsymbol{q}\lambda}\cdot\hat{\boldsymbol{J}}(\boldsymbol{x})$$

- $\mathcal{K}_1$ acts only on the nucleons' d.o.f.
- $|\alpha\rangle$, $|\beta\rangle$ initial & final nuclear states, Ψ_α, Ψ_β corresponding w.f.
- $\boldsymbol{q}$, ω, $\hat{\epsilon}_{\boldsymbol{q}\lambda}$ = momentum, energy, polarization of the absorbed photon
- for virtual photons, one needs also the m.e. of $\hat{\boldsymbol{q}}\cdot\widehat{\boldsymbol{J}}$ and ρ

$$J^\mu(\boldsymbol{q}) = \int d\boldsymbol{x}\; e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\,\widehat{J}^\mu(\boldsymbol{x}) \qquad \mu = 0,1,2,3$$

Current conservation (CC) $\nabla \cdot \widehat{\boldsymbol{J}}(\boldsymbol{x}) = -i[H, \rho(\boldsymbol{x})]$

- Strict interplay between H, $\widehat{\boldsymbol{J}}$ and $\widehat{\rho}$ [Buchmann *et al*, 1985], [Riska, 1989], [Schiavilla *et al*, 1990]

$$\widehat{\rho}(\boldsymbol{x}) = \sum_{i=1}^{A} \frac{1+\tau_z(i)}{2}\delta(\boldsymbol{r}_i - \boldsymbol{x}) \quad H = \sum_i T_i + \sum_{ij} V_{ij} + \cdots \rightarrow \boldsymbol{J}(x) = \sum_i \boldsymbol{J}_i^{(1)} + \sum_{ij} \boldsymbol{J}_{ij}^{(2)}(x) + \cdots$$

- $\boldsymbol{J}_{ij}^{(2)}(x)$ = meson exchange currents
- Old approach: $\boldsymbol{J}$ constructed not consistently with V; CC verified "by hand" [Marcucci *et al.*, 2005]
- EFT approach: H and J^μ derived from the same Lagrangian [Park *et al*, 1993], [Kolling *et al*, 2009], [Pastore *et al*, 2009], [Schiavilla *et al.*, 2018]
- However:
  - different cutoff used to regularize the short-range parts of V and $\boldsymbol{J}$
  - different orders of chiral expansions of V and $\boldsymbol{J}$
- In the present work: same regulators for V and $\boldsymbol{J}$
- See also [Krebs *et al.*, 2019], for a more systematic approach

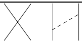
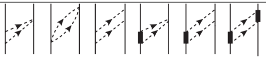
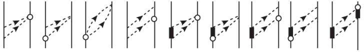
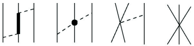
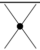
Weak transitions

- Vector current: CVC hypothesis: V^μ derived from J^μ_{EM}
- Axial current: PCAC (conservation in the limit $m_\pi \longrightarrow 0$) [Park *et al*, 2003], [Baroni *et al.*, 2015–2016], [Krebs *et al.*, 2016]

EFT potentials with Δ–particle d.o.f.

Project

- NN/3N potentials and charge/currents derived form χEFT with contributions of the Δ–particle d.o.f.
- $\rightarrow$ χEFT with nucleons, pions & Δ's: See, for example, [Bernard, Kaiser, & Meissner, 1995], [Hemmert, Holstein, & Kambor, 1998], [Krebs, Epelbaum, & Meissner, 2007]
- Further "condition": *NN* & 3*N* forces local in *r*-space [Piarulli *et al.*, 2014-2016]
- Another local *r*-space potential derived from χEFT [Gezerlis *et al.*, 2014], [Lynn *et al.*, 2017]

Case	NN	3N
LO Q^0		
NLO Q^2		
N2LO Q^3		
N3LO Q^4		—

Fit & Regularization

Regularization in r-space

- Diagrams with pion exchanges $\sim \frac{1}{r^n}$ [Epelbaum *et al.*, 2004], [Valderrama *et al.*, 2008]
  - Regulated in r space with the function

$$C_{R_L}(r) = 1 - \frac{1}{(r/R_L)^6\, e^{(r-R_L)/a_L} + 1}$$

- Contact terms
  - Using Fierz transformation to eliminate ∇^2 terms
  - Neglected some terms $\sim \frac{d^2}{dr^2}$ giving small contributions
  - at the end: L^2 and $(L \cdot S)^2$ operators
  - Short-range part regularized with

$$\delta(\mathbf{r}) \longrightarrow C_{R_S}(r) = \frac{1}{\pi^{3/2} R_S^3} e^{-(r/R_S)^2}$$

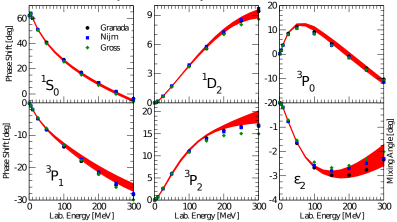
Fits & χ^2

- $\sim$ 25 LECs fitted to the NN database
- Interactions *I*: Fit of data up to 125 MeV
- Interactions *II*: Fit of data up to 200 MeV
- Choice $(R_L, R_S) = (0.8, 1.2)$ fm: models *a*
- Choice $(R_L, R_S) = (0.7, 1.0)$ fm: models *b*

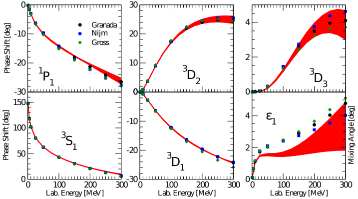
model	$E_{\rm Lab}$ (MeV)	N_{pp+np}	χ^2
Ia	0–125	2668	1.05
Ib	0–125	2665	1.07
IIa	0–200	3698	1.37
IIb	0–200	3695	1.37

NN phase-shifts and deuteron radial components

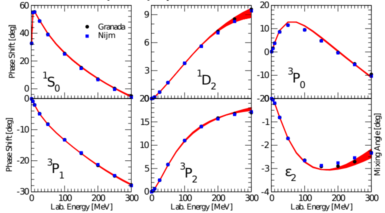
$n-p$ $T=1$ phase-shifts



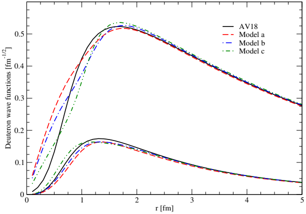
$n-p$ $T=0$ phase-shifts



$p-p$ phase-shifts



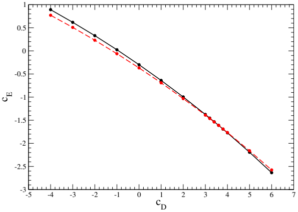
Deuteron radial components



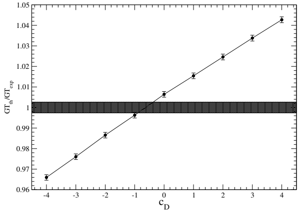
3N force – choice of c_D & c_E

- For $A > 2$ inclusion of the 3NF at N2LO [Epelbaum *et al*, 2002] + Fujita-Miyazawa
- Two LECs to be fixed: c_D & c_E
- Method 1: $B(^3\mathrm{H})$ and $a^{(2)}_{n-d}$ Ia, Ib, IIa, IIb
- Method 2: $B(^3\mathrm{H})$ and Gamow-Teller (GT) matrix element Ia$*$, Ib$*$, IIa$*$, IIb$*$
- Bound– and continuum..states $A = 3$ calculations performed using the HH technique [Kievsky *et al.*, 2008]

Fit of $B(^3\mathrm{H})$



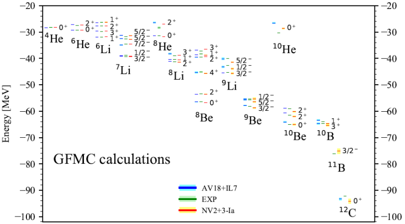
Fit of the GT matrix element of trutiom beta decay

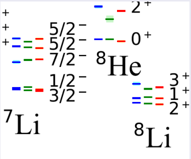


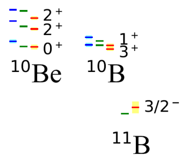
	Ia$*$	Ib$*$	IIa$*$	IIb$*$
c_D	-0.635	-4.71	-0.61	-5.25
c_E	-0.09	0.55	-0.35	0.05

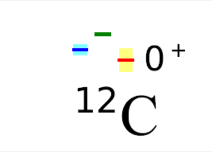
GFMC spectrum of light nuclei

NN+3N interaction: Ia [Piarulli *et al.*, 2018]









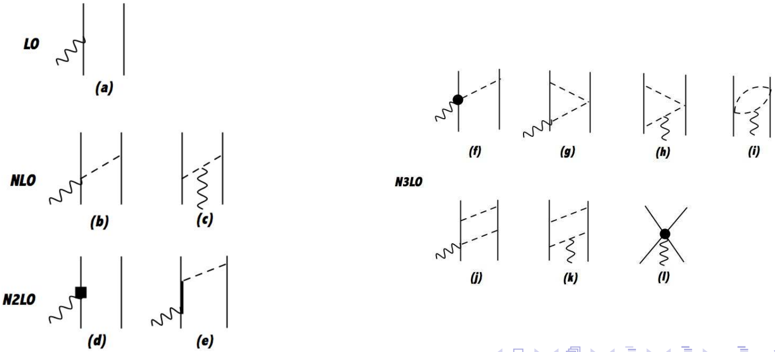
Nice reproduction of the energy levels – 3N force fitted using only $A = 3$ data!!!
but too soft neutron matter EOS – studies with Ia*–IIb* in progress

EM transitions

EM charge & current from χEFT

[Park *et al*, 1993], [Kolling *et al*, 2009], [Pastore *et al*, 2009]
Including the Δ d.o.f. [Schiavilla *et al.*, 2018]

Diagrams with the inclusion of the Δ d.o.f. up to N2LO



Determination of the LECs

3 new LECs d_1^S, d_2^S, d_1^V

- d_1^S, d_2^S multiply isoscalar operators
- d_1^V multiplies an isovector operator
- Fitted to μ_d, $\mu_{^3\mathrm{H}}$, and $\mu_{^3\mathrm{He}}$

	Ia*	Ib*	IIa*	IIb*
d_1^S	-0.00999	-0.02511	-0.01170	-0.04955
d_2^S	-0.06571	-0.02384	-0.04714	-0.07947
d_1^V	-0.05120	-0.03509	-0.05128	-0.03880

Deuteron magnetic moment expt. 0.8574 n.m.

	Ia*	Ib*	IIa*	IIb*
LO	0.8498	0.8485	0.8501	0.8501
N2LO(RC)	-0.0062	-0.0061	-0.0065	-0.0072
N3LO	0.0137	0.0151	0.0138	0.0145
TOT	0.8573	0.8575	0.8574	0.8574

Isoscalar magnetic moment expt. 0.4257 n.m.

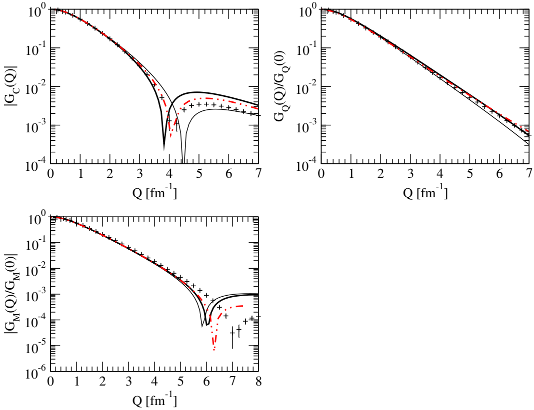
	Ia*	Ib*	IIa*	IIb*
LO	0.4089	0.4075	0.4091	0.4089
NLO	0.0015	0.0020	0.0012	0.0018
N2LO	-0.0062	-0.0043	-0.0052	-0.0071
N3LO	0.0229	0.0215	0.0218	0.0231
TOT	0.4271	0.4267	0.4269	0.4267

Isovector magnetic moment expt -2.553 n.m.

	Ia*	Ib*	IIa*	IIb*
LO	-2.1823	-2.1755	-2.1815	-2.1787
NLO	-0.1967	-0.2257	-0.1967	-0.2255
N2LO	-0.0388	-0.0657	-0.0395	-0.0617
N3LO	-0.1355	-0.0864	-0.1354	-0.0872
TOT	-2.5533	-2.5533	-2.5531	-2.5531

Deuteron form factors

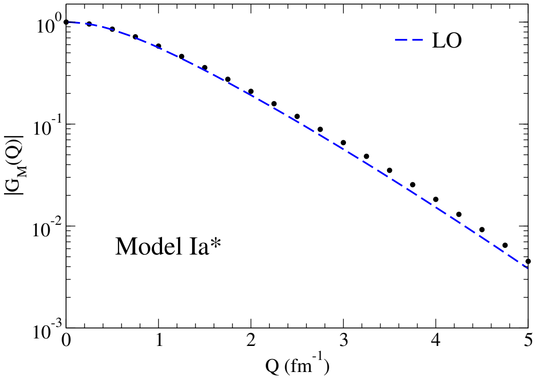
Conventional approach [Marcucci *et al.*, 2015]
AV18/UIX interaction + meson exchange current [Marcucci *et al.*, 2005]



- Thin black line: IA results
- Thick black line: IA+MEC (FULL) results
- Dash-double-dotted red line: approximate inclusion of relativistic effects [Friar, 1975]
- Experimental data: see the review paper [Marcucci *et al.*, 2015]

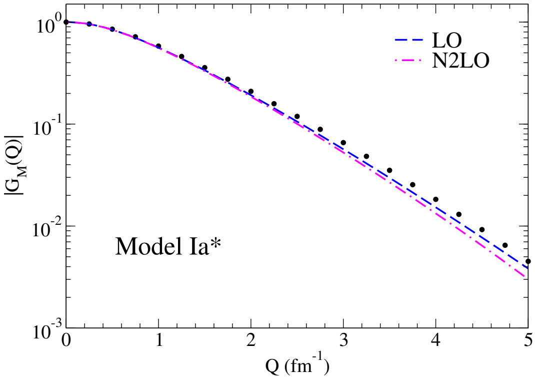
Deuteron magnetic form factor (1)

Dependence on the current orders (cumulative)
Interaction Ia∗



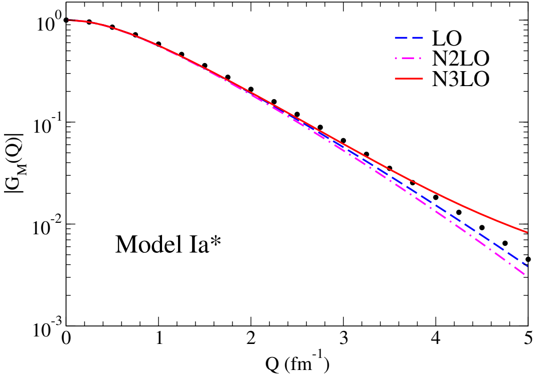
Deuteron magnetic form factor (1)

Dependence on the current orders (cumulative)
Interaction Ia$*$



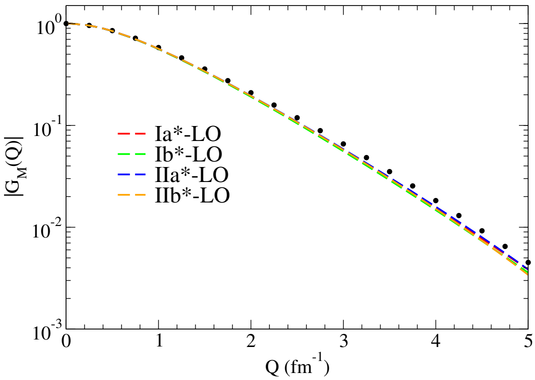
Deuteron magnetic form factor (1)

Dependence on the current orders (cumulative)
Interaction Ia∗



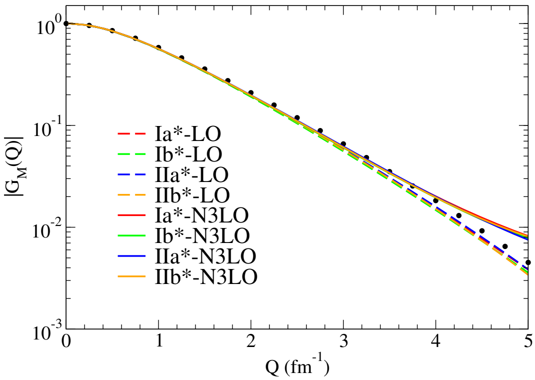
Deuteron magnetic form factor (2)

Dependence on the NN interaction

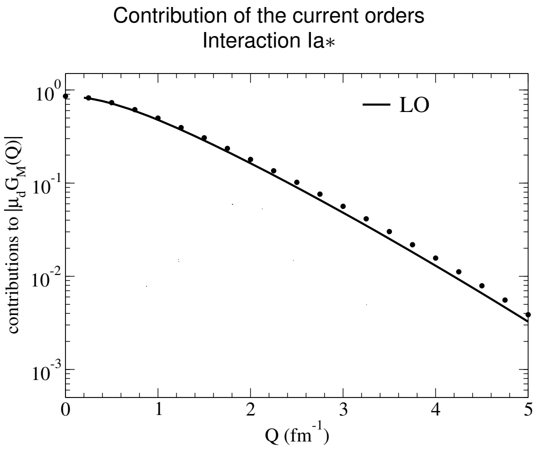


Deuteron magnetic form factor (2)

Dependence on the NN interaction

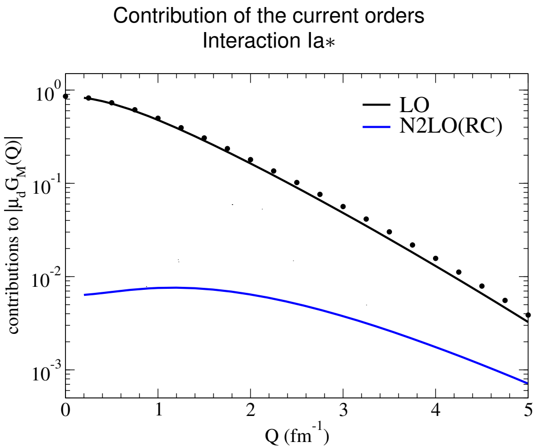


Deuteron magnetic form factor (3)



- Problems with some N3LO terms
- "NM"= non minimal $\sim d_1^S$

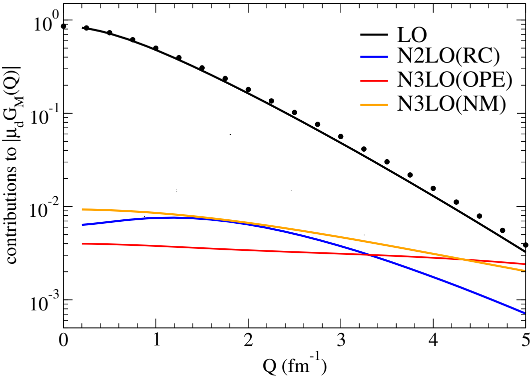
Deuteron magnetic form factor (3)



- Problems with some N3LO terms
- "NM"= non minimal $\sim d_1^S$

Deuteron magnetic form factor (3)

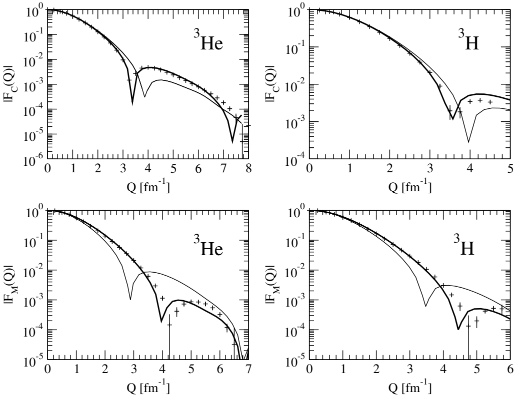
Contribution of the current orders
Interaction Ia$*$



- Problems with some N3LO terms
- "NM"= non minimal $\sim d_1^S$

Trinucleon form factors

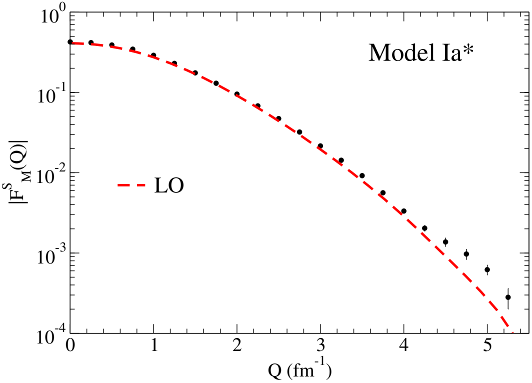
Conventional approach [Marcucci *et al.*, 2015]
AV18/UIX interaction + meson exchange current [Marcucci *et al.*, 2005]



- thin black line: IA results
- Thick black line: IA+MEC (FULL) results
- Experimental data: see the review paper [Marcucci *et al.*, 2015]

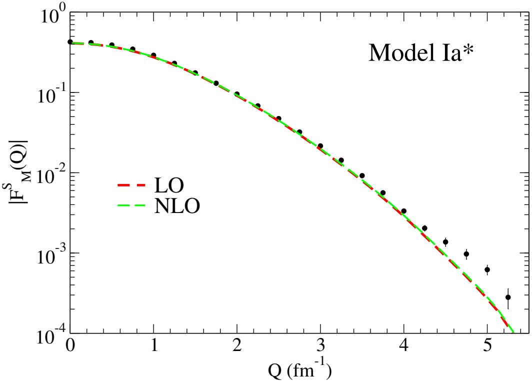
Trinucleon magnetic form factors (1)

Calculation with the new interactions/currents
Isoscalar magnetic form factor



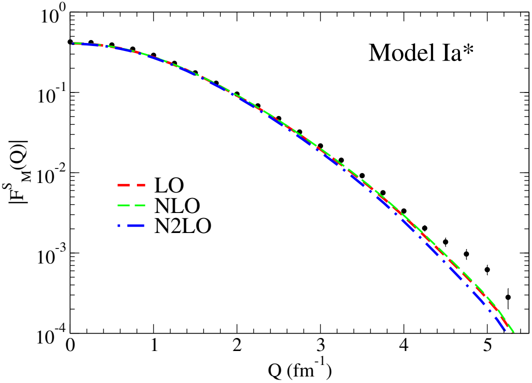
Trinucleon magnetic form factors (1)

Calculation with the new interactions/currents
Isoscalar magnetic form factor



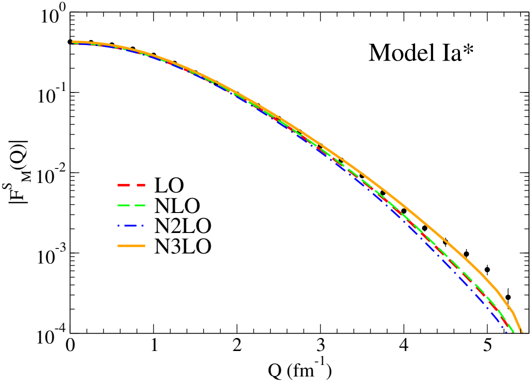
Trinucleon magnetic form factors (1)

Calculation with the new interactions/currents
Isoscalar magnetic form factor



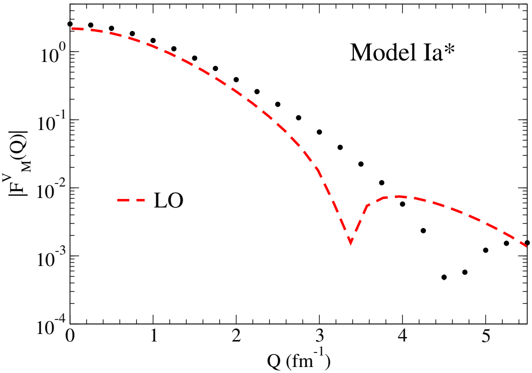
Trinucleon magnetic form factors (1)

Calculation with the new interactions/currents
Isoscalar magnetic form factor



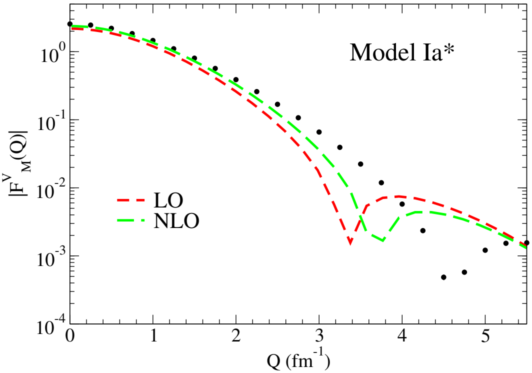
Trinucleon magnetic form factors (2)

Calculation with the new interactions/currents
Isovector magnetic form factor



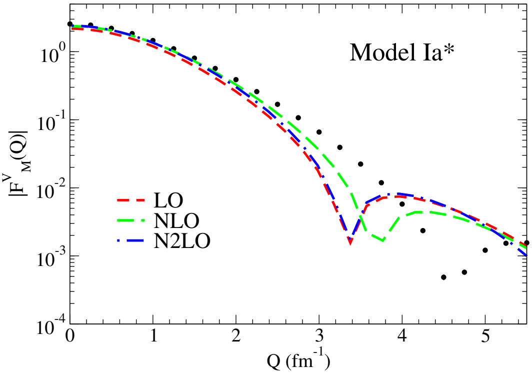
Trinucleon magnetic form factors (2)

Calculation with the new interactions/currents
Isovector magnetic form factor



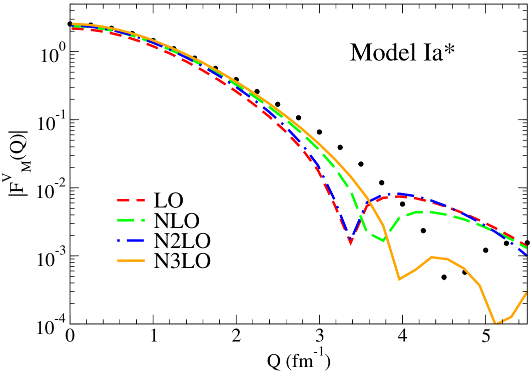
Trinucleon magnetic form factors (2)

Calculation with the new interactions/currents
Isovector magnetic form factor



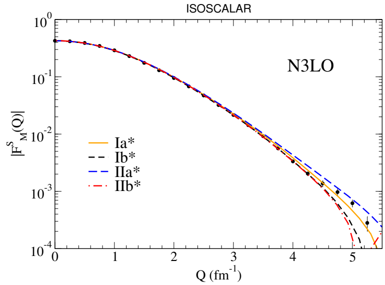
Trinucleon magnetic form factors (2)

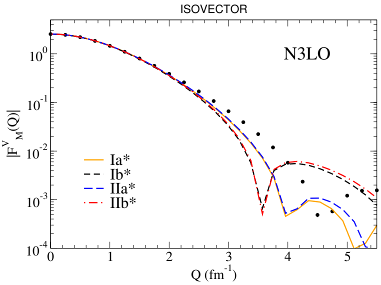
Calculation with the new interactions/currents
Isovector magnetic form factor



Trinucleon magnetic form factors (3)

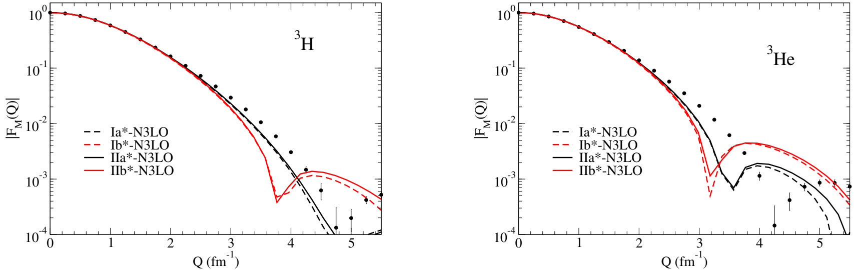
Calculation with the new interactions/currents
Dependence on the interaction





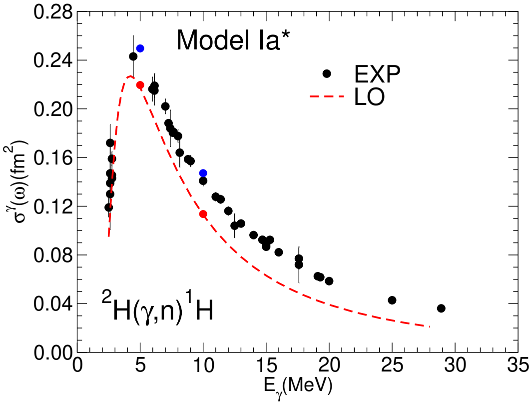
Trinucleon magnetic form factors (4)

Calculation with the new interactions/currents
^{3}H & ^{3}He form factors



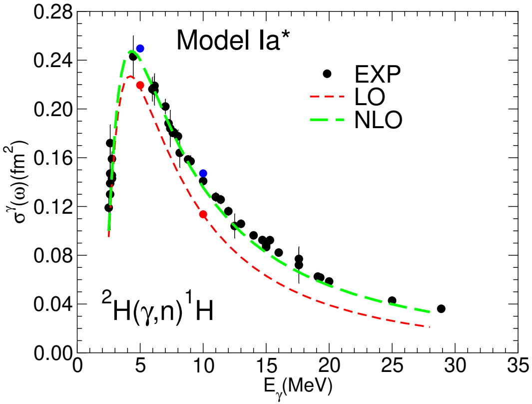
Phodisintegration of the deuteron (1)

Calculation with the new interactions/currents
Interaction Ia$*$



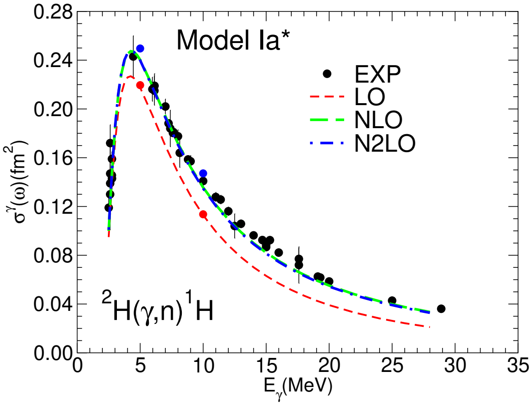
Phodisintegration of the deuteron (1)

Calculation with the new interactions/currents
Interaction Ia∗



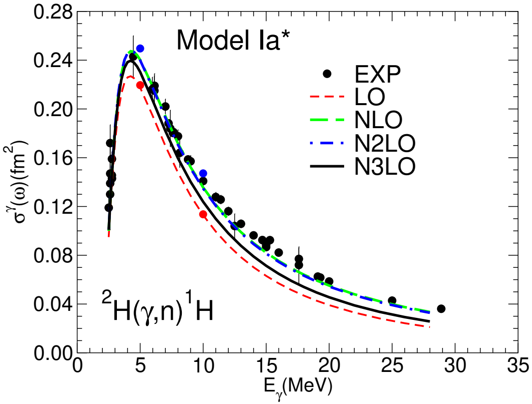
Phodisintegration of the deuteron (1)

Calculation with the new interactions/currents
Interaction Ia∗



Phodisintegration of the deuteron (1)

Calculation with the new interactions/currents
Interaction Ia$*$



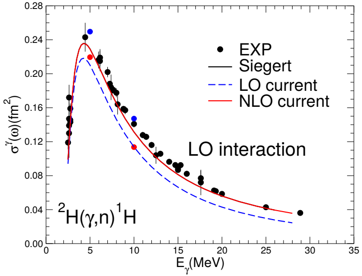
Phodisintegration of the deuteron (2)

Multipole expansion of the EM current operator

$$T^E_{J,M} = -i\frac{q^{J-1}}{(2J+1)!!}\sqrt{\frac{J+1}{J}}\int d^3x\left[\nabla\cdot \boldsymbol{J}_C(\boldsymbol{x}) + \frac{q^2}{J+1}\nabla\cdot\Big(\boldsymbol{x}\times\boldsymbol{\mu}(\boldsymbol{x})\Big)\right]x^J Y_{JM}(\hat{\boldsymbol{x}})$$

Current conservation (CC)

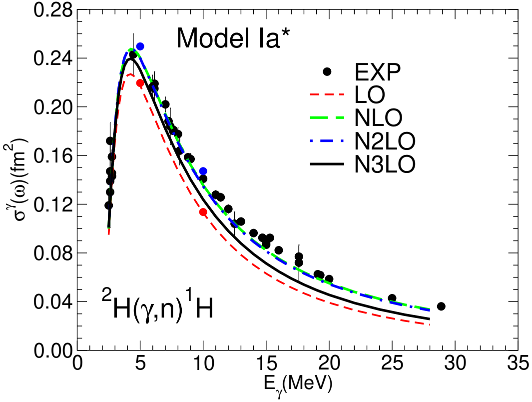
$$\nabla\cdot \boldsymbol{J}(x) + i\Big[H_0,\rho(\boldsymbol{x})\Big] = 0 \qquad \langle J_f,M_f|\Big[H_0,\rho(\boldsymbol{x})\Big]|J_i,M_i\rangle = (E_f-E_i)\langle J_f,M_f|\rho(\boldsymbol{x})|J_i,M_i\rangle$$



- This observable is dominated by the E_1 multipole
- V at LO (one-pion exchange + LO contact terms)
- $\boldsymbol{J}$ at LO+NLO (single nucleon current + one-pion exchange diagrams)
- In this case CC is exactly verified
- However, if V and J do not verify CC, E_ℓ not well calculated
- [Siegert, 1937]: compute E_1 forcing CC

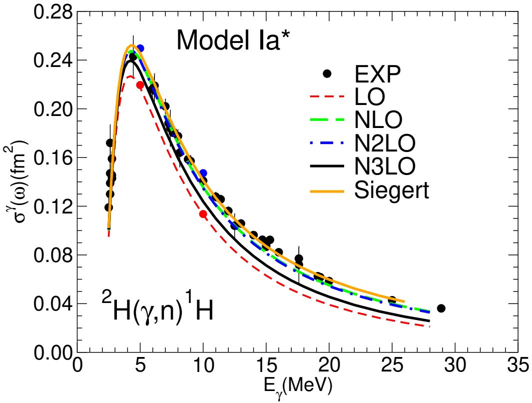
Phodisintegration of the deuteron (3)

Calculation with the new interactions/currents
Interaction Ia∗ Using Siegert

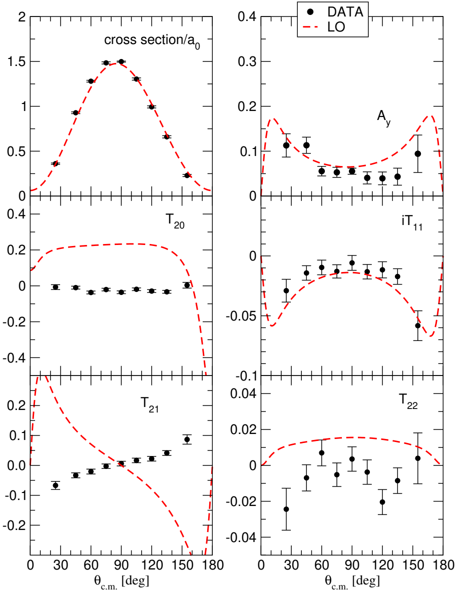


Phodisintegration of the deuteron (3)

Calculation with the new interactions/currents
Interaction Ia$*$ Using Siegert



Proton-deuteron capture



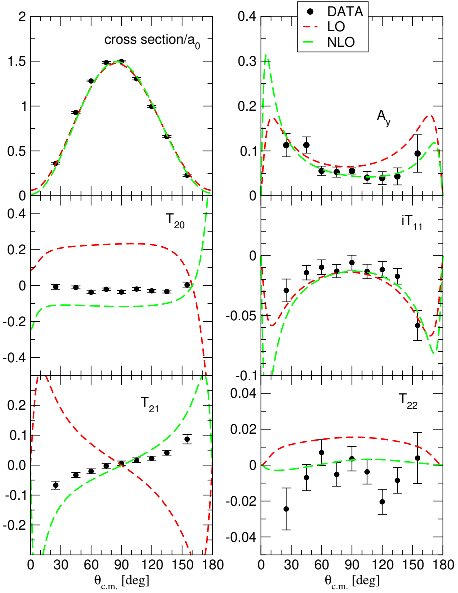
PRELIMINARY
AV18 + new EM currents
$E_{cm} = 2$ MeV

Data: [Smith & Knutson, 1999]

Big effects from
$\langle \Psi_{^3\mathrm{He}} | j_{EM} | \Psi_{pd}^{L=1,S=3/2,J} \rangle$

Calculations with the new interactions in progress

Proton-deuteron capture



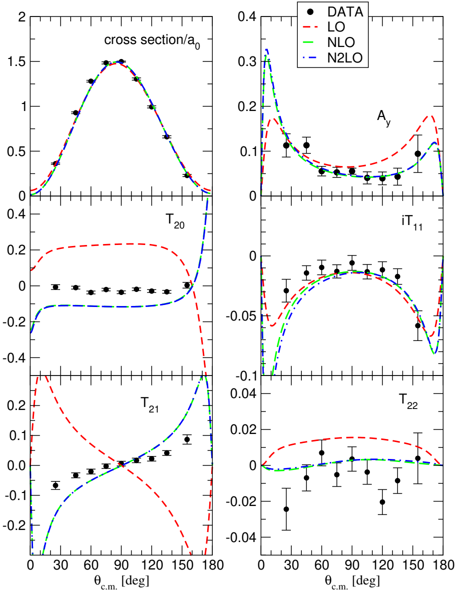
PRELIMINARY
AV18 + new EM currents
$E_{cm} = 2$ MeV

Data: [Smith & Knutson, 1999]

Big effects from
$\langle \Psi_{^3\mathrm{He}} | j_{EM} | \Psi_{pd}^{L=1,S=3/2,J} \rangle$

Calculations with the new interactions in progress

Proton-deuteron capture



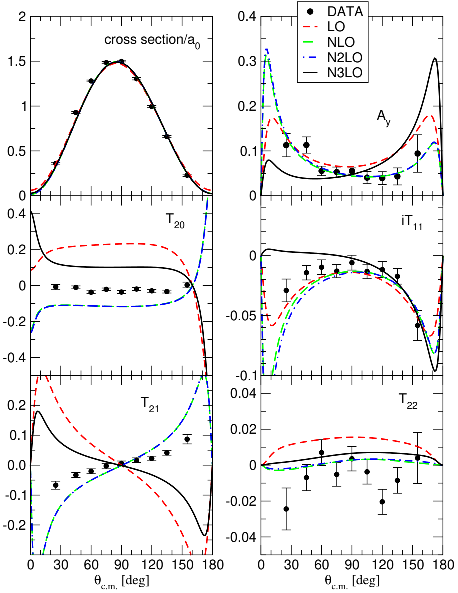
PRELIMINARY
AV18 + new EM currents
$E_{cm} = 2$ MeV

Data: [Smith & Knutson, 1999]

Big effects from
$\langle \Psi_{^3\mathrm{He}} | j_{EM} | \Psi_{pd}^{L=1,S=3/2,J} \rangle$

Calculations with the new interactions in progress

Proton-deuteron capture



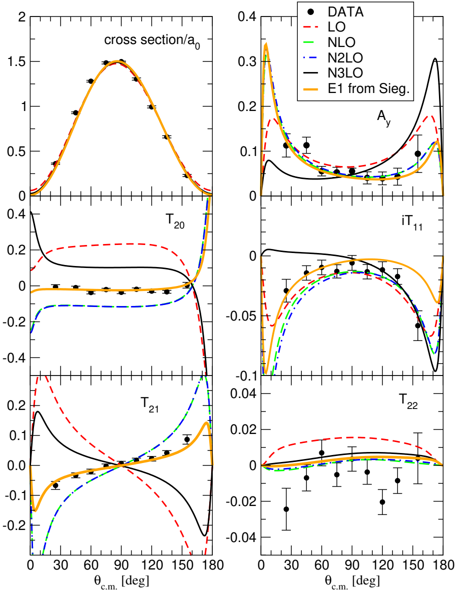
PRELIMINARY
AV18 + new EM currents
$E_{cm} = 2$ MeV

Data: [Smith & Knutson, 1999]

Big effects from
$\langle \Psi_{^3\text{He}} | j_{EM} | \Psi_{pd}^{L=1,S=3/2,J} \rangle$

Calculations with the new interactions in progress

Proton-deuteron capture



PRELIMINARY
AV18 + new EM currents
$E_{cm} = 2$ MeV

Data: [Smith & Knutson, 1999]

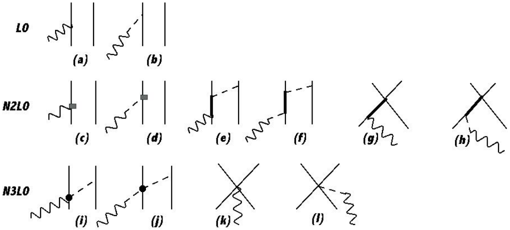
Big effects from
$\langle \Psi_{^3\mathrm{He}} | j_{EM} | \Psi_{pd}^{L=1,S=3/2,J} \rangle$

Calculations with the new interactions in progress

Weak current

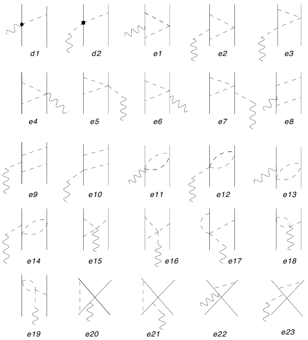
Axial current from χEFT

[Park *et al*, 2003], [Baroni *et al*, 2015], [Krebs *et al*, 2016]
Including the Δ d.o.f.: [Baroni *et al.*, 2018]



Single nucleon current diagram NLO $\sim$ LEC d_R

N4LO diagrams



Tritium beta decay

$$d_R = -\frac{m_\pi}{4\, g_A\, \Lambda_\chi}\, c_D + \frac{m_\pi}{3}\left(c_3 + c_3^\Delta + 2\, c_4 + 2\, c_4^\Delta\right) + \frac{m_\pi}{6\, m}$$

[Gardestig and Phillips, 2006], [Gazit *et al.*, 2009], [Schiavilla, 2018]
Fit of $B(^3\mathrm{H})$ and the GT matrix element in tritium β decay
Ia/b* & IIa/b* chiral Hamiltonians [Baroni *et al.*, 2018]

	Ia*	Ib*	IIa*	IIb*
c_D	-0.635	-4.71	-0.61	-5.25
c_E	-0.09	0.55	-0.35	0.05
LO	0.9272	0.9247	0.9261	0.9263
N2LO	0.0345	0.0517	0.0345	0.0515
N3LO	-0.0108	-0.0261	-0.0102	-0.0272
TOT	0.9509	0.9503	0.9504	0.9506

Experimental value 0.9511 ± 0.0013 (see [Baroni *et al.*, 2016])
Range of c_D and c_E values allowed by the experimental error on $\mathrm{GT}_{\mathrm{exp}}$

	Ia*	Ib*	IIa*	IIb*
c_D	$(-0.89, -0.38)$	$(-4.99, -4.42)$	$(-0.89, -0.33)$	$(-5.56, -4.94)$
c_E	$(-0.01, -0.17)$	$(+0.70, +0.40)$	$(-0.25, -0.45)$	$(+0.23, -0.13)$

N4LO contribution

Contributions obtained with the [Baroni *et al.*, 2016] and [Krebs *et al.*, 2017] formulations of the N4LO axial current

	Ia*	Ib*	IIa*	IIb*
N4LO(B)	-0.0672	-0.0732	-0.0671	-0.0716
N4LO(K)	-0.0364	-0.0540	-0.0365	-0.0543
B-K(OPE)	0.0141	0.0196	0.0142	0.0201
B-K(TPE)	0.0018	0.0024	0.0018	0.0025
B-K(CT)	-0.0467	-0.0412	-0.0466	-0.0399

- OPE = N4LO loop corrections to the OPE axial current
- TPE = N4LO genuine new TPE contributions
- CT = N4LO contact contributions induced by the regularization scheme in configuration space we have adopted
- Contributions at N4LO found to be relatively large and of opposite sign than those at LO
- Difference between the two formalisms at present not clarified

Quantum computation & Nuclear Physics

A. Roggero [INT, Seattle, WA (USA)] & A. Baroni `arXiv:1905:08383`
see also [Roggero & Carlson, 2018], [Dumitrescu *et al.*, 2018]

Quantum computers

- Example: 1 qubit = spin of an electron
  - spin up = $|1\rangle$, spin down = $|0\rangle$
  - but are also possible all the superposition states $\alpha|1\rangle + \beta|0\rangle$...
- Gates: perform simple operation on the qubit
  - X,Y,Z gates = Pauli matrix operations
  - Hadamard gate

$$\mathrm{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \sim \mathrm{X} + \mathrm{Z}$$

- "Ancillary" qubit = qubit in a definite known state

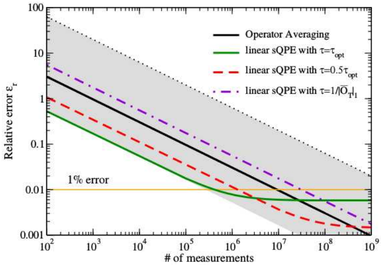
arXiv:1905:08383

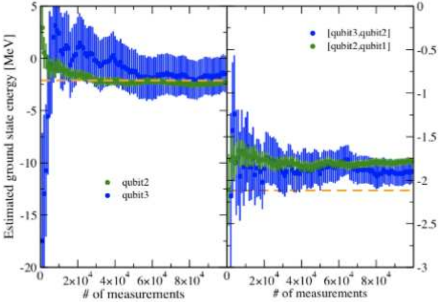
- Deuteron $\Psi = \alpha|S\rangle + \beta|D\rangle$
- The deuteron can be described by a single qubit!
- Using a "normal" computer, compute the matrix elements

$$\begin{bmatrix} \langle S|H|S\rangle & \langle S|H|D\rangle \\ \langle D|H|S\rangle & \langle D|H|D\rangle \end{bmatrix} = aI + bX + cZ$$

  - Potential used: Argonne V6
  - Quantum gates $\rightarrow H\Psi$
  - Run on an IBM simulator
  - Problems: number of iterations, noise, ...

A. Roggero [INT, Seattle, WA (USA)] & A. Baroni `arXiv:1905:08383`





Conclusions & perspectives

Conclusions

- Ongoing effort to construct interactions & EW currents from χEFT but local in configuration space
- Structure and EW processes in light nuclei using the HH & GFMC techniques
- Presented results for
  - Magnetic moments and FFs up to $A = 3$
  - Photo-disintegration of the deuteron
  - GT matrix element of tritium β decay
- Still something to be clarified . . .
  - Better quantification of "theoretical error"
  - Benchmark calculations

Perspectives

- Work in progress
  - FF of $^4\mathrm{He}$
  - $p-d$ & $d-d$ radiative capture at BBN energies (LUNA experiment)
  - $p-p$ & $p-{}^3\mathrm{He}$ astrophysical factors
  - ^{6}He beta decay
  - . . .
- Further advances:
  - Test of the interactions in $p-{}^3\mathrm{He}$ elastic scattering
  - Inclusion of the N4LO contact interaction in the 3N force
- Quantum computing of scattering processes $n-p$, $n-d$, etc.

Conclusions & perspectives

Conclusions

- Ongoing effort to construct interactions & EW currents from χEFT but local in configuration space
- Structure and EW processes in light nuclei using the HH & GFMC techniques
- Presented results for
  - Magnetic moments and FFs up to $A = 3$
  - Photo-disintegration of the deuteron
  - GT matrix element of tritium β decay
- Still something to be clarified . . .
  - Better quantification of “theoretical error”
  - Benchmark calculations

Perspectives

- Work in progress
  - FF of $^4\mathrm{He}$
  - $p - d$ & $d - d$ radiative capture at BBN energies (LUNA experiment)
  - $p - p$ & $p - {}^3\mathrm{He}$ astrophysical factors
  - ^{6}He beta decay
  - . . .
- Further advances:
  - Test of the interactions in $p - {}^3\mathrm{He}$ elastic scattering
  - Inclusion of the N4LO contact interaction in the 3N force
- Quantum computing of scattering processes $n - p$, $n - d$, etc.

Thank you for your attention!