

FOUR-BODY SCALE IN UNIVERSAL FEW-BOSON SYSTEMS.

מנואל פאבון סבאסטואן מלך יוהנס קירשר בצלאל בזק
B. Bazak¹ J. Kirscher² S. König³ M. Pavón Valderrama⁴
N. Barnea¹ U. van Kolck^{5,6}
וביראירה ון כולק ניר ברנע

¹The Racah Institute of Physics, The Hebrew University

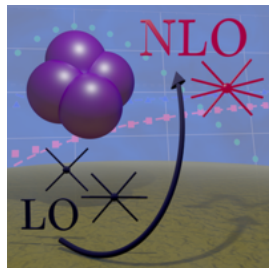
²Department of Physics and Astronomy, The University of Manchester

³Institut für Kernphysik, Technische Universität Darmstadt

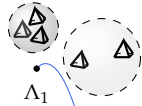
⁴Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University

⁵Institut de Physique Nucléaire, Université Paris-Saclay

⁶Department of Physics, University of Arizona



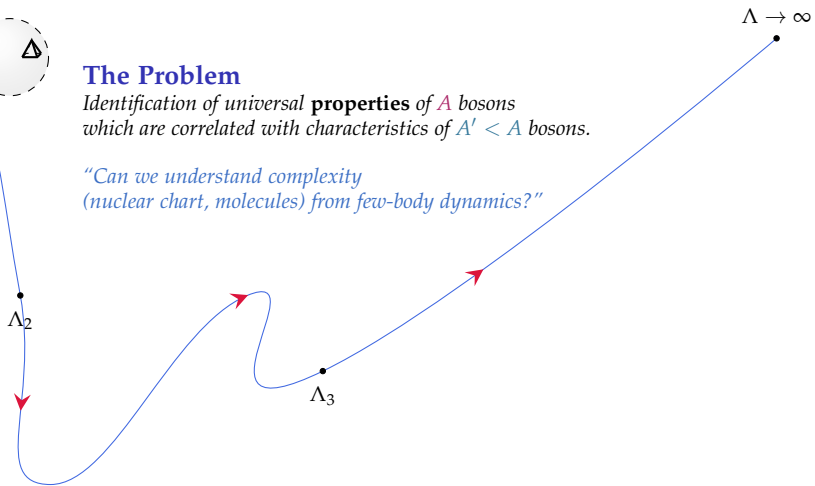
Phys. Rev. Lett. **122**, 143001 [↗](#)

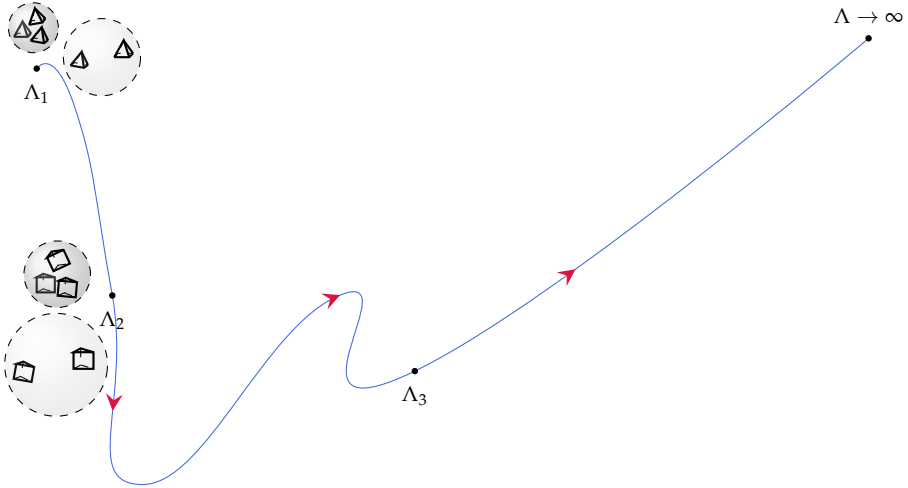


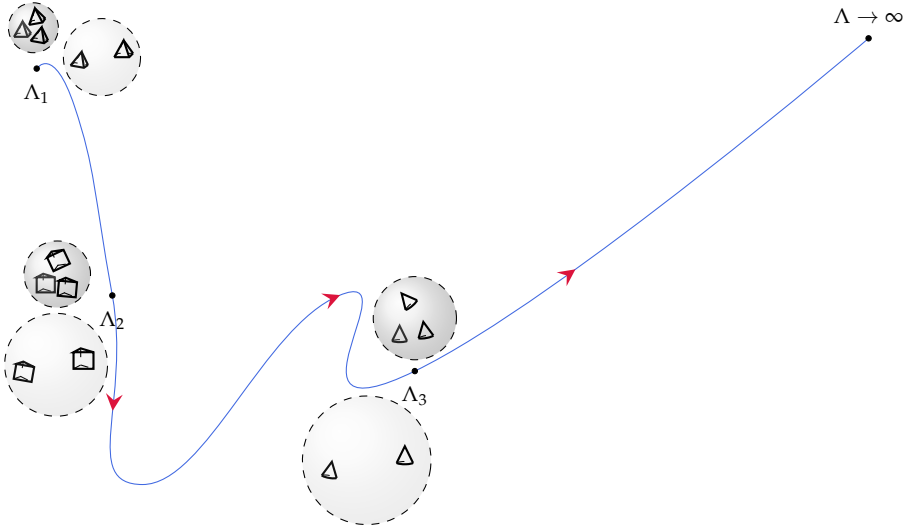
The Problem

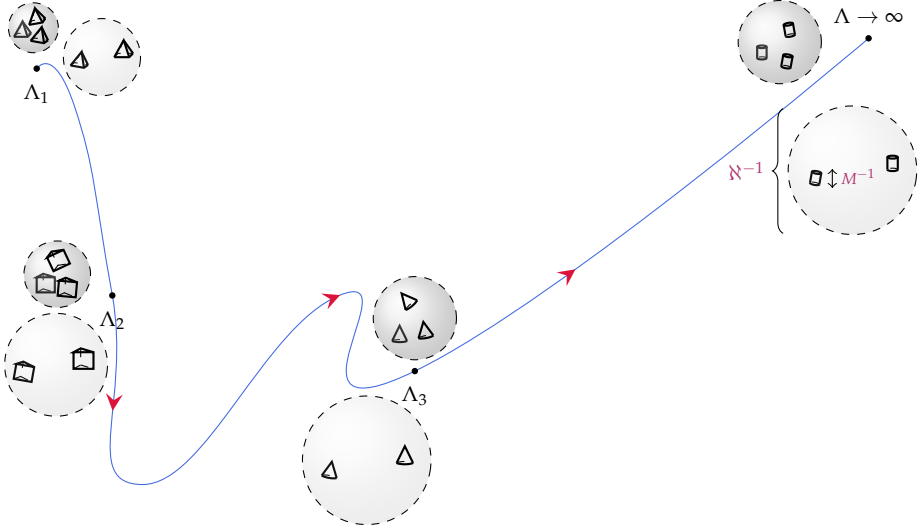
Identification of universal **properties** of A bosons
which are correlated with characteristics of $A' < A$ bosons.

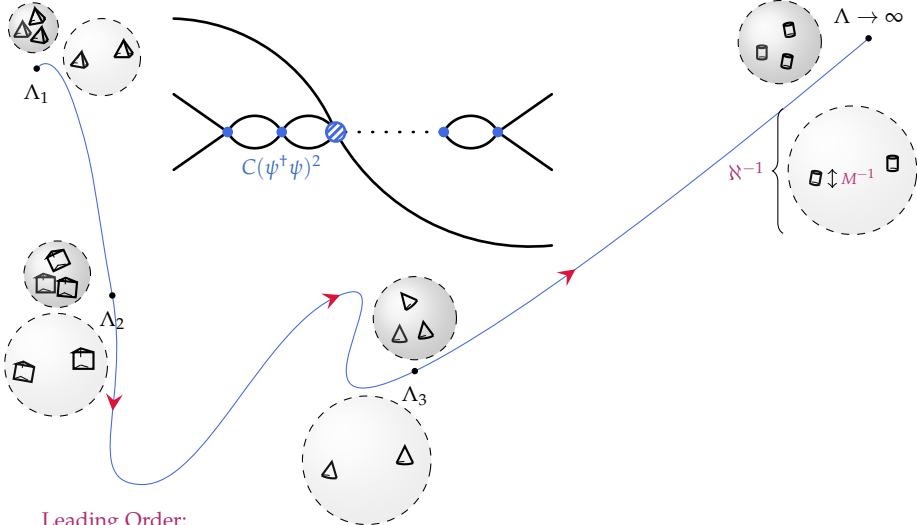
*“Can we understand complexity
(nuclear chart, molecules) from few-body dynamics?”*







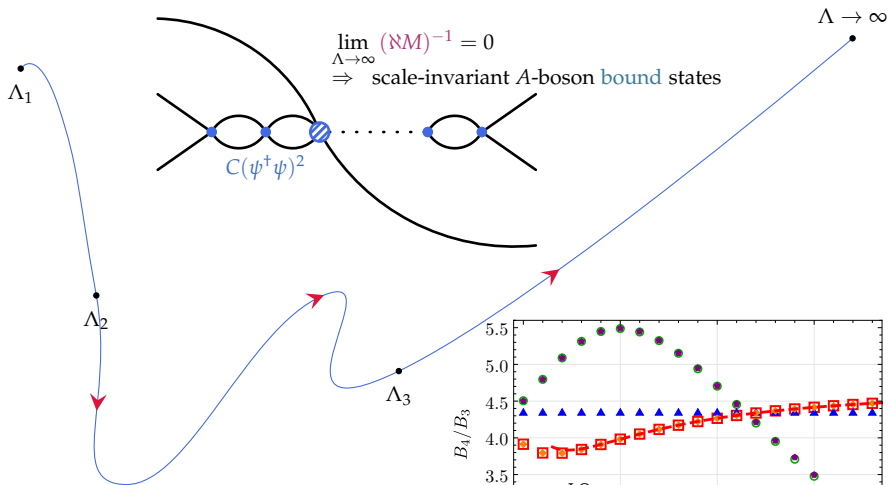




Leading Order:

$$\mathcal{P}(A+n \text{ 2-/3-body collisions}) = \mathcal{P}(A \text{ 2-/3-body collisions}) \quad \forall |n| \leq A$$

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger\psi)^2 - D(\psi^\dagger\psi)^3$$



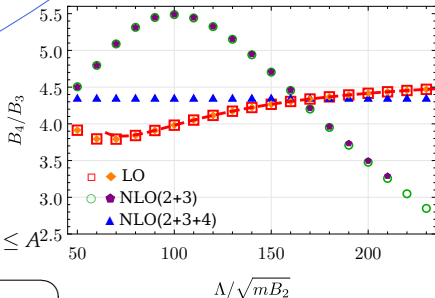
Leading Order:

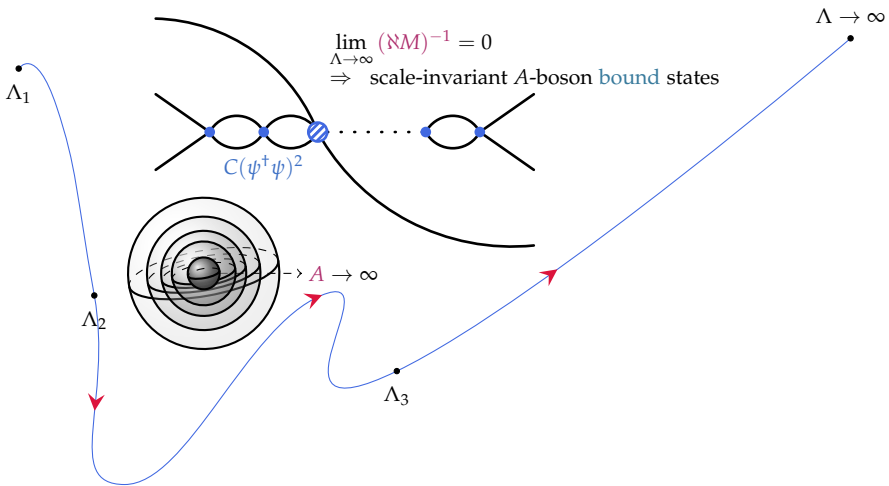
$$\mathcal{P}(A + n \text{ }^{2\text{-}/3\text{-body} \text{ collisions}}) = \mathcal{P}(A \text{ }^{2\text{-}/3\text{-body} \text{ collisions}}) \quad \forall |n| \leq A^{2.5}$$

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3$$

$$\Psi = 12 \cdot \mathbf{Y} + 6 \cdot \mathbf{H} \xrightarrow{\text{id.}} \mathbf{Y} + \mathbf{H} \quad (\text{S. König, accurate calibration});$$

$$\Psi = \sum_n c_n \cdot e^{-\eta^\dagger A_n \eta} \quad (\text{B. Bazak, } A > 4).$$





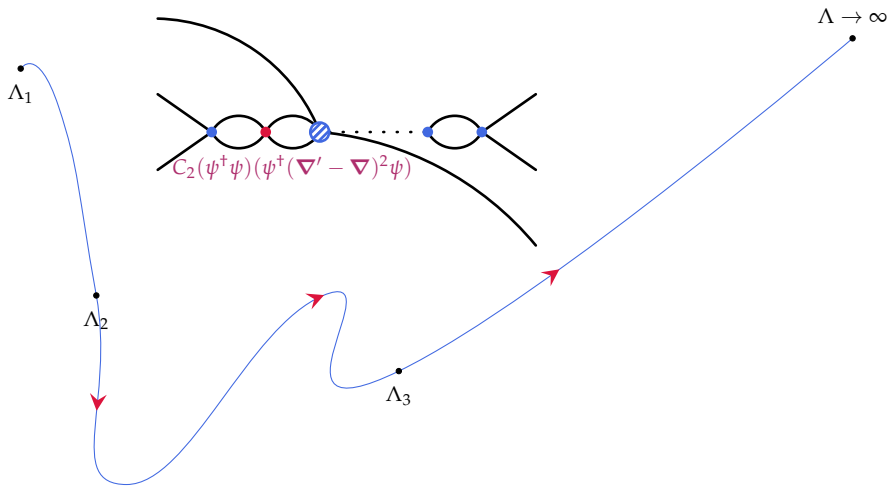
Leading Order:

$$\mathcal{P}(A+n \text{ }^{2\text{-}/3\text{-body} \text{ collisions}}) = \mathcal{P}(A \text{ }^{2\text{-}/3\text{-body} \text{ collisions}}) \quad \forall |n| \leq A$$

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3$$

J. von Stecher, PRL 107, 200402 (2011); M. Gattobigio, A. Kievsky, and M. Viviani, PRA 84, 052503 (2011);

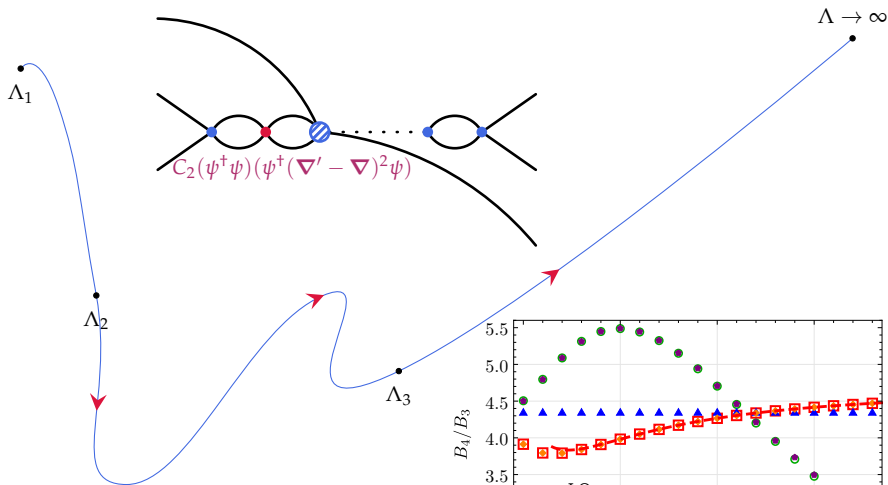
B. Bazak, M. Elyahu, and U. van Kolck, PRA 94, 052502 (2016).



Next-to-leading Order (**3 bodies, 3 constraints**):

Two long-range constraints, one short-range constraint.

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger\psi)^2 - D(\psi^\dagger\psi)^3$$

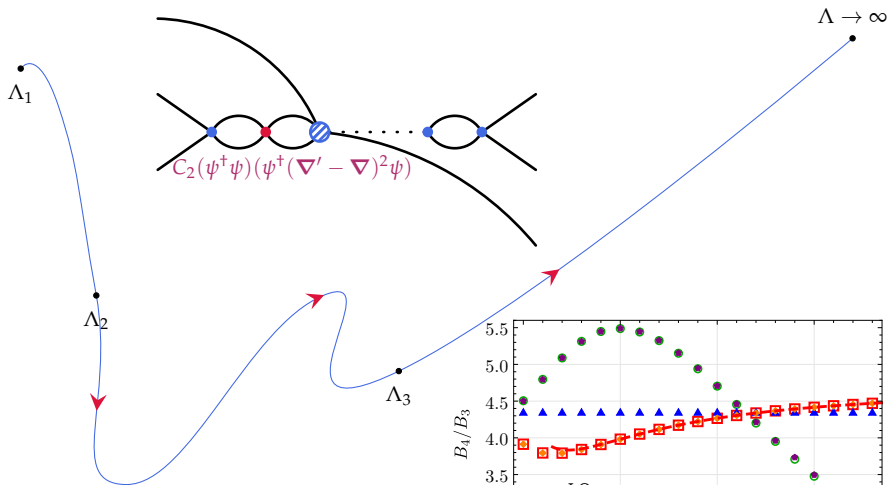


Next-to-leading Order (3 bodies, 3 constraints):

Two long-range constraints, one short-range constraint.

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger\psi)^2 - D(\psi^\dagger\psi)^3$$

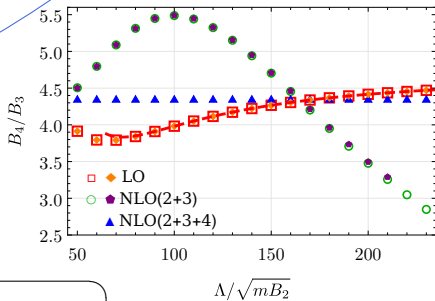
$$\Lambda/\sqrt{mB_2}$$

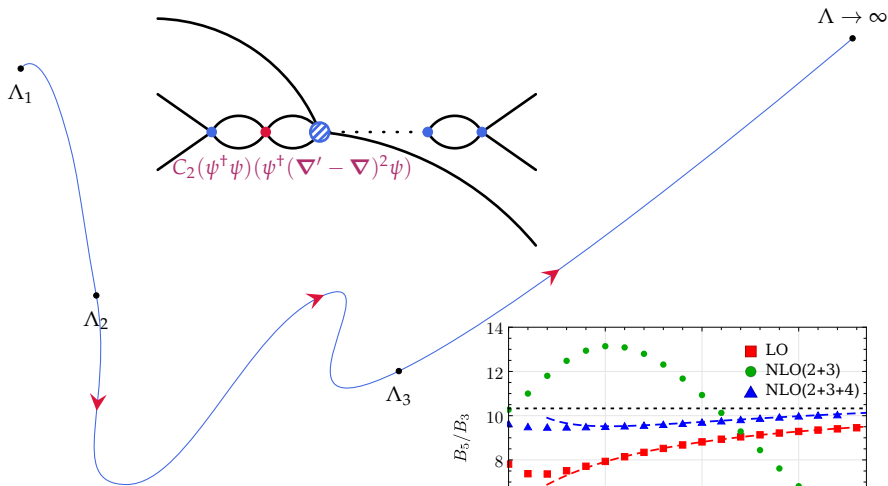


(3 bodies, 3 constraints):

\Rightarrow four-body details are resolved!

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3 - F(\psi^\dagger \psi)^4$$



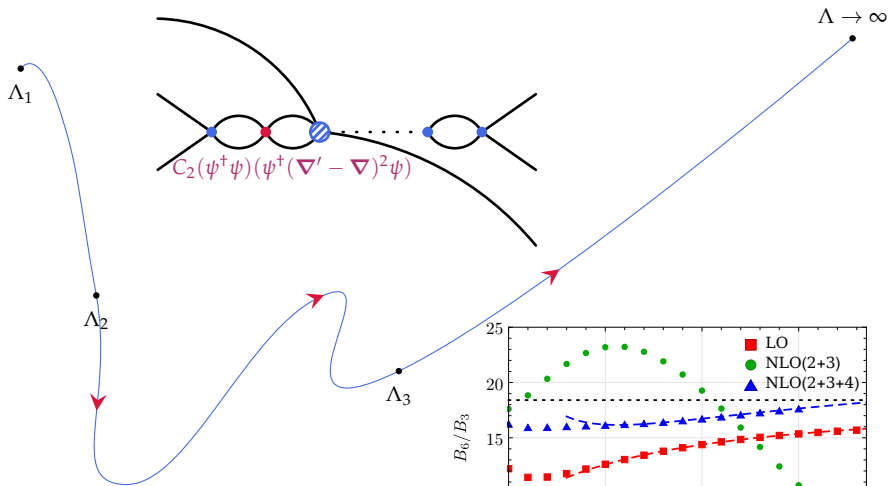


(3 bodies, 3 constraints):

\Rightarrow four-body details are resolved!

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3 - F(\psi^\dagger \psi)^4$$

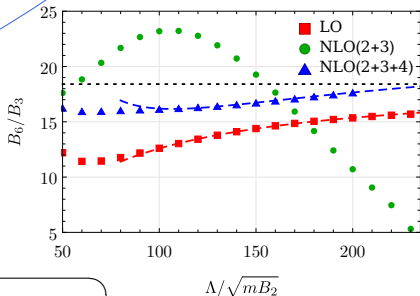
$\Lambda / \sqrt{mB_2}$



(3 bodies, 3 constraints):

\Rightarrow four-body details are resolved!

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3 - F(\psi^\dagger \psi)^4$$



Λ_1

Conjecture

As soon as the $A - 1$ boson system is constrained by more than A parameters, the A -boson system is sensitive to $(A - 1)$ -unobservable interaction details.

 Λ_2 Λ_3 $\Lambda \rightarrow \infty$

$$\lim_{|r_{nm}| \rightarrow 0} \left\{ \text{Diagram of a central sphere with concentric shells and a path} \right\} = \frac{u(\mathbf{r}_{12}, \dots, \mathbf{r}_{A-1,A})}{\prod_{i < j} |\mathbf{r}_{ij}|} \quad \forall n, m$$

and with $0 < |u| < \infty$ for any $|r_{nm}| \rightarrow 0$ (no finite polynomial)

$\Lambda \rightarrow \infty$



Little Fugue in G Minor

Johann Sebastian Bach 1685 - 1750

Piano

4

4/4

2

4

2

3

4

2

3

4

Detailed description: This block shows the first system of the piano part of the Little Fugue in G Minor. It consists of two staves: a treble clef staff and a bass clef staff. The key signature has two flats (B-flat and E-flat), and the time signature is 4/4. The treble staff has a whole rest followed by a quarter note D. The bass staff has a quarter note D, followed by a dotted quarter note A, an eighth note G, a quarter note B-flat, an eighth note A, and a quarter note G. Fingering numbers are provided below the notes: 4 for the first D in the bass, 2 for the D in the treble, 2 for the A, 3 for the G, 4 for the B-flat, 2 for the A, and 4 for the final G.

Pno.

7

4/4

2

5

2

5

3

2

3

4

3

Detailed description: This block shows the second system of the piano part. The treble staff has a quarter note D, followed by a quarter note D, and a quarter rest. The bass staff has a quarter note B-flat, followed by a quarter note A, a quarter rest, a quarter note B-flat, an eighth note G, an eighth note A, and a quarter note G. Fingering numbers are provided: 7 for the first D in the treble, 5 for the B-flat, 3 for the A, 2 for the second D, 5 for the B-flat, 3 for the A, 2 for the second B-flat, 3 for the G, 4 for the A, and 3 for the final G.

Pno.

3

1

3

2

3

4

3

1

Detailed description: This block shows the third system of the piano part. The treble staff has a quarter note G, followed by a quarter note D, and a quarter note C. The bass staff has a quarter note A, followed by a quarter note D, a quarter note A, a quarter note B-flat, an eighth note G, an eighth note A, and a quarter note G. Fingering numbers are provided: 3 for the G, 1 for the D, 3 for the A, 2 for the B-flat, 3 for the G, 4 for the A, 3 for the second G, and 1 for the final G.

© SilverTonalities 2015

