Four-Body Scale in Universal Few-Boson Systems.

בצלאל בזק   *יוהנס קירשר*   סבאסטואן מלך   מנואל פאבון

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[4]Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University

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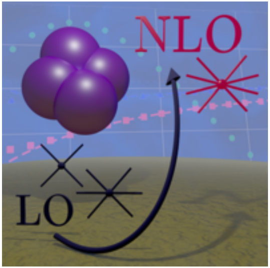
[6]Department of Physics, University of Arizona



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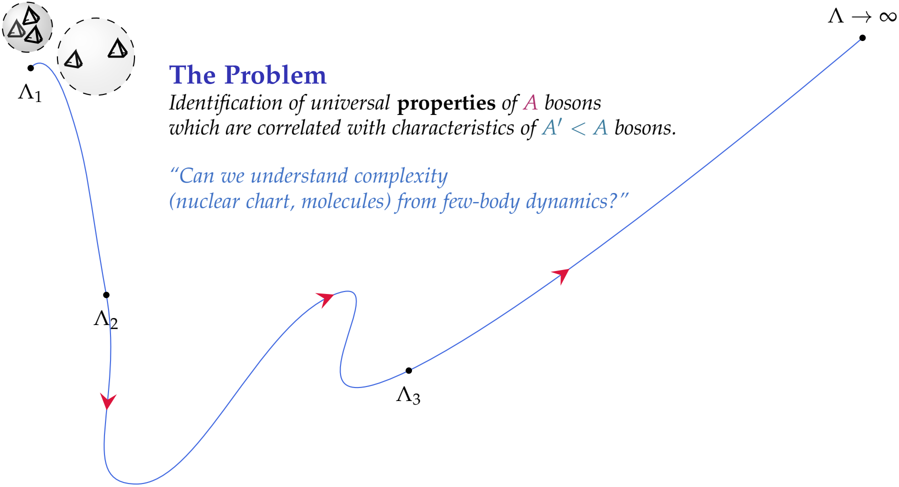
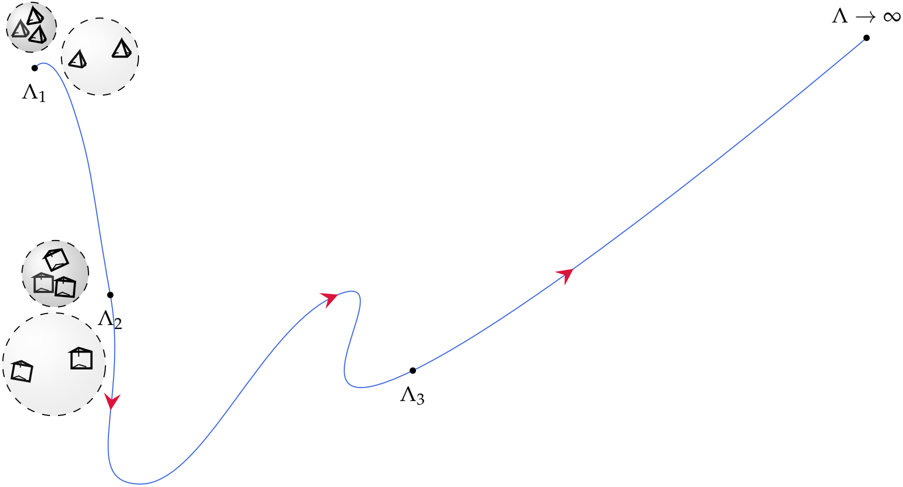
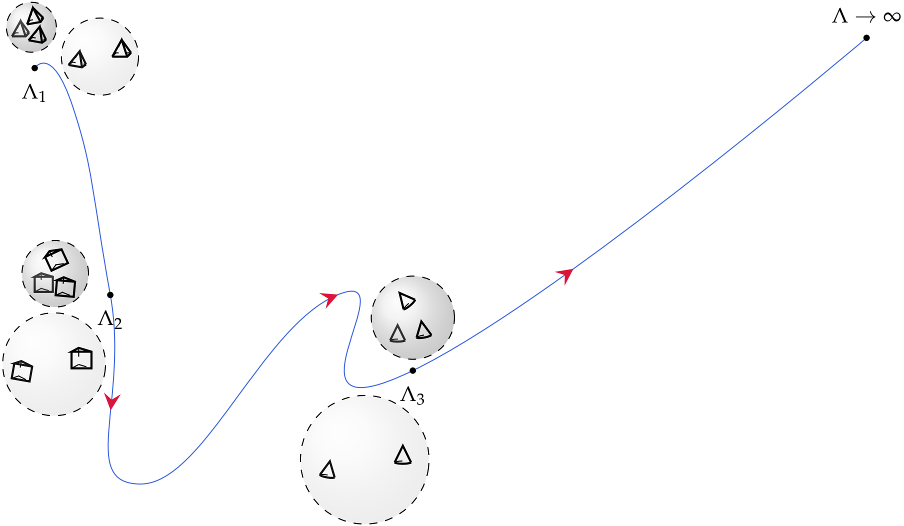
The Problem

Identification of universal **properties** *of A bosons which are correlated with characteristics of $A' < A$ bosons.*

"Can we understand complexity (nuclear chart, molecules) from few-body dynamics?"




$\Lambda \to \infty$
Λ_1
Λ_2
Λ_3


Λ_1
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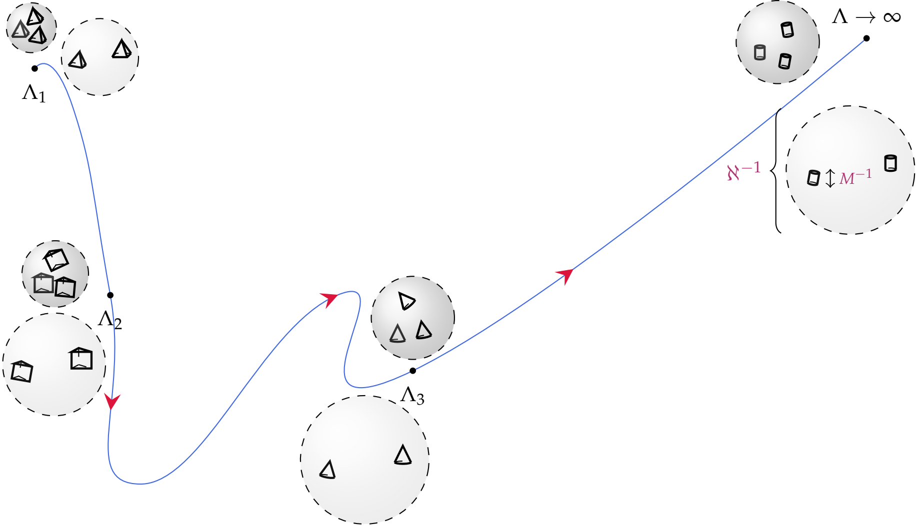
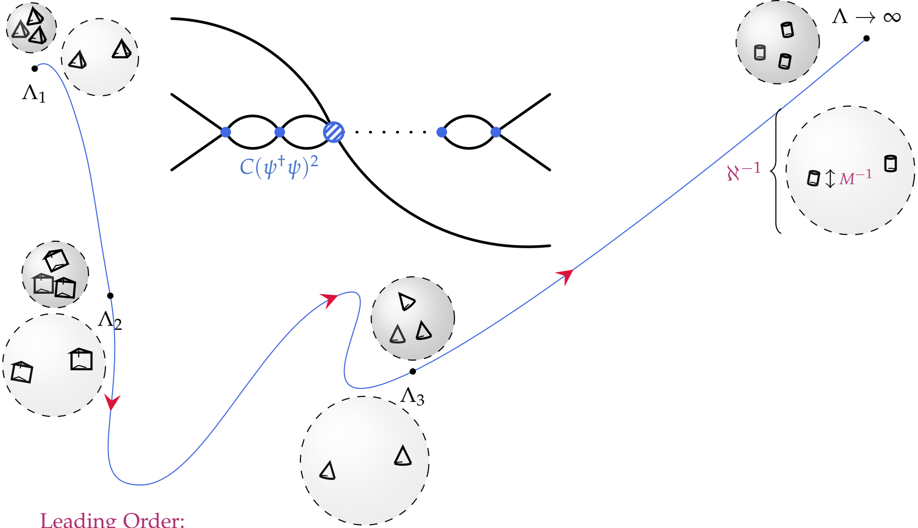


Λ_1
Λ_2
Λ_3
$\Lambda \to \infty$
$\aleph^{-1}$
M^{-1}

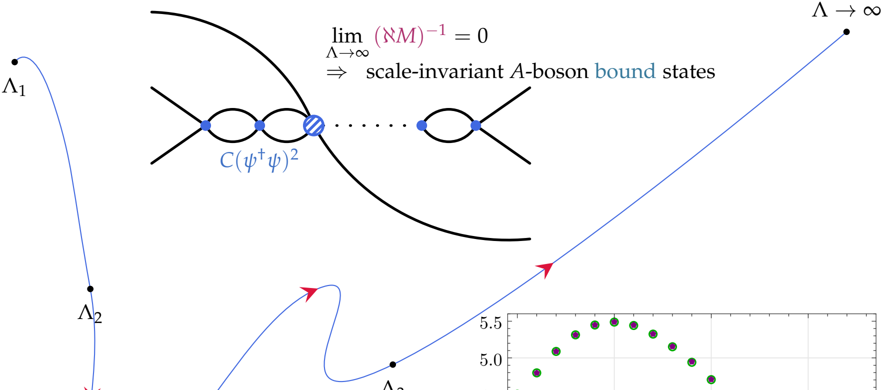


Leading Order:

$$\mathcal{P}\left(A + n \begin{array}{c}\text{2-/3-body}\\ \text{collisions}\end{array}\right) = \mathcal{P}\left(A \begin{array}{c}\text{2-/3-body}\\ \text{collisions}\end{array}\right) \quad \forall |n| \leq A$$

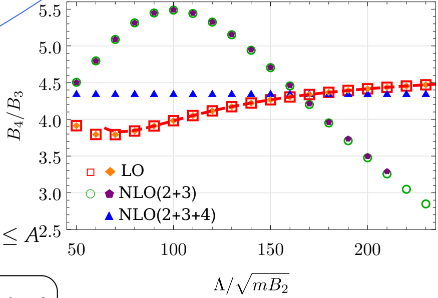
$$\boxed{\mathcal{L} = \psi^\dagger \left(\mathrm{i}\partial_0 + \frac{\nabla^2}{2m}\right)\psi - C(\psi^\dagger\psi)^2 - D(\psi^\dagger\psi)^3}$$

P. F. Bedaque, H.-W. Hammer, and U. van Kolck, NPA ***646****, 444 (1999).*



$$\lim_{\Lambda\to\infty} (\aleph M)^{-1} = 0$$

$\Rightarrow$ scale-invariant A-boson bound states



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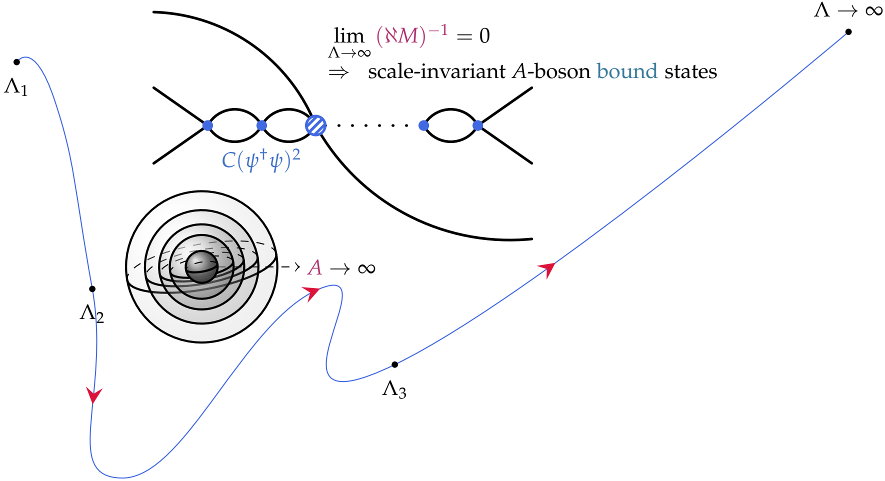
$\boldsymbol{\Psi} = 12\cdot\mathbf{Y} + 6\cdot\mathbf{H} \overset{\text{id.}}{\to} \mathbf{Y}+\mathbf{H}$ (S. König, accurate calibration); $\boldsymbol{\Psi} = \sum_n c_n \cdot e^{-\boldsymbol{\eta}^\dagger \mathbf{A}_n \boldsymbol{\eta}}$ (B. Bazak, $A > 4$).



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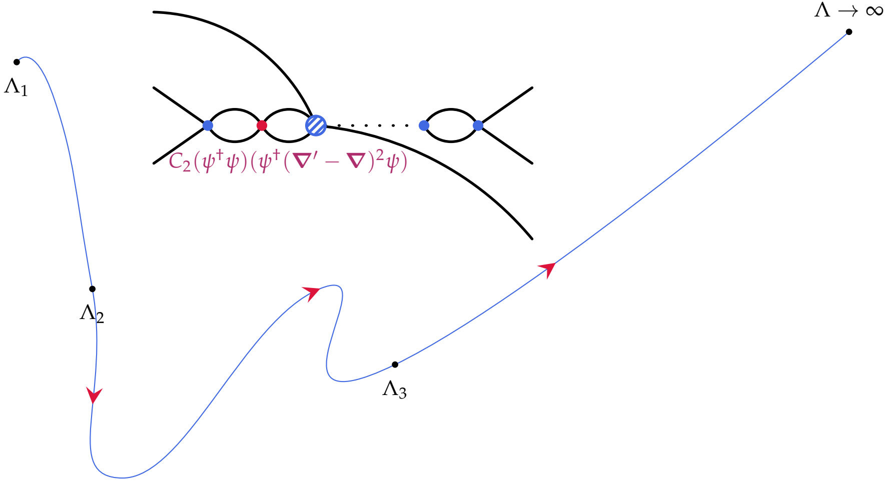
J. von Stecher, PRL ***107****, 200402 (2011); M. Gattobigio, A. Kievsky, and M. Viviani, PRA* ***84****, 052503 (2011);*
B. Bazak, M. Eliyahu, and U. van Kolck, PRA ***94****, 052502 (2016).*



Next-to-leading Order (3 bodies, 3 constraints):

Two long-range constraints, one short-range constraint.

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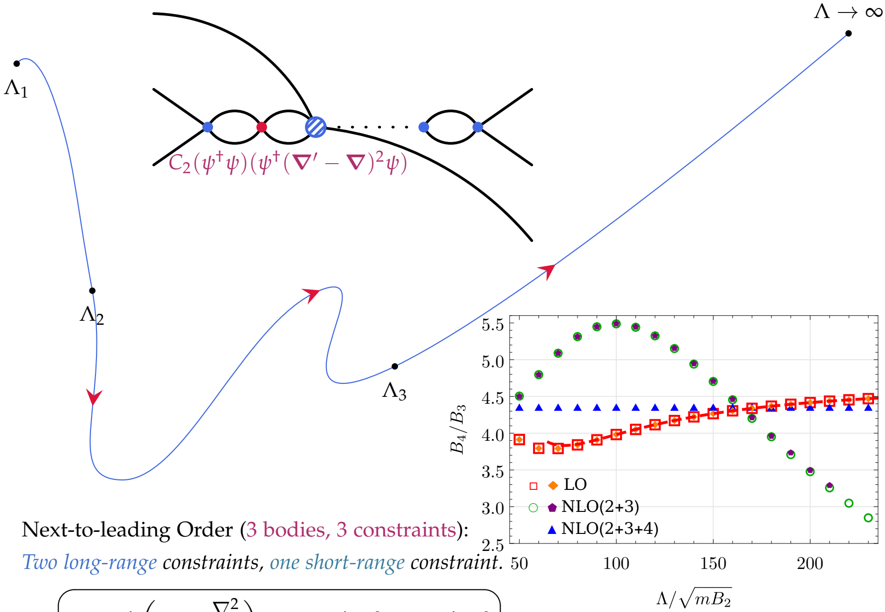

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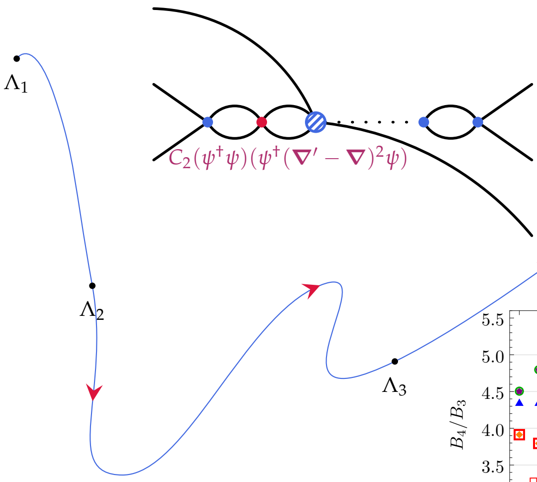
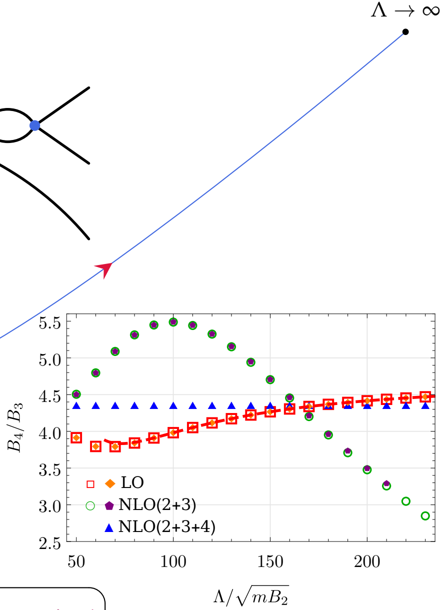
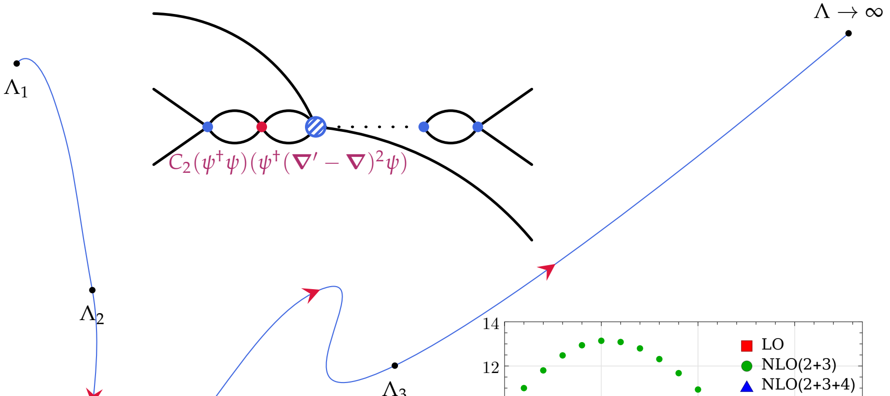


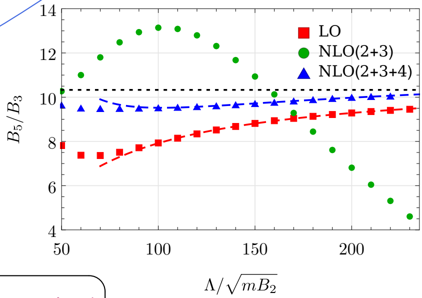


(3 bodies, 3 constraints):

$\Rightarrow$ four-body details are resolved!

$$\boxed{\mathcal{L} = \psi^\dagger\left(\mathrm{i}\partial_0 + \frac{\nabla^2}{2m}\right)\psi - C(\psi^\dagger\psi)^2 - D(\psi^\dagger\psi)^3 - F(\psi^\dagger\psi)^4}$$





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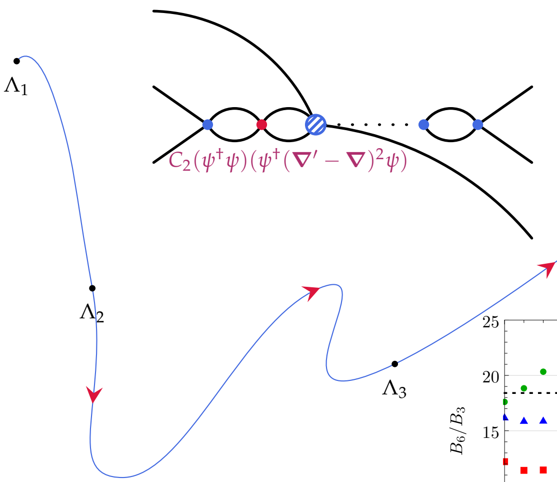

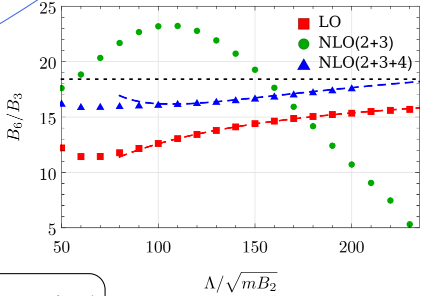

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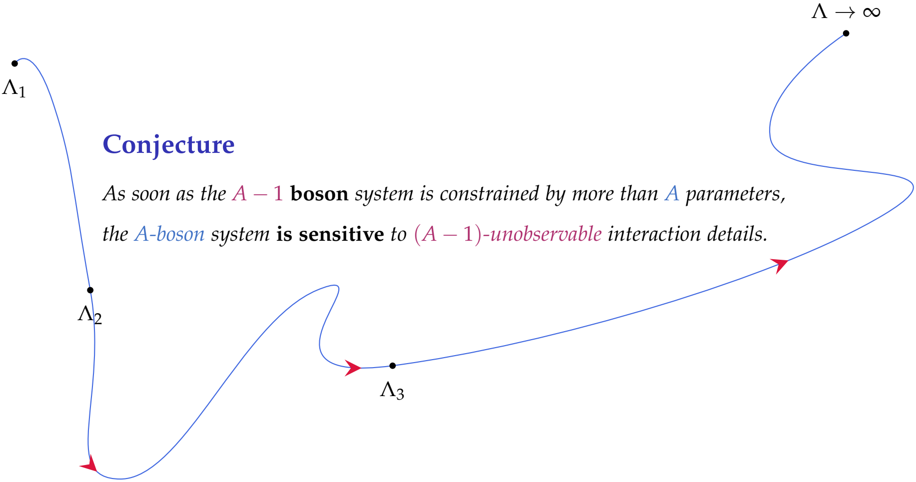
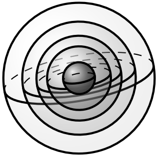


Conjecture

As soon as the $A-1$ **boson** *system is constrained by more than A parameters, the A-boson system* **is sensitive** *to $(A-1)$-unobservable interaction details.*

$$\lim_{|\boldsymbol{r}_{nm}|\to 0}\left\{\vphantom{\prod_{i<j}}\right.$$



$$\left.\vphantom{\prod_{i<j}}\right\} = \frac{u(\boldsymbol{r}_{12}\,,\;\ldots\;,\,\boldsymbol{r}_{A-1,A})}{\prod\limits_{i<j}|\boldsymbol{r}_{ij}|} \quad \forall\; n,\; m$$

and with $0 < |u| < \infty$ for any $|r_{nm}| \to 0$ (no finite polynomial)

$\Lambda \to \infty$

Little Fugue in G Minor

Johann Sebastian Bach 1685 - 1750