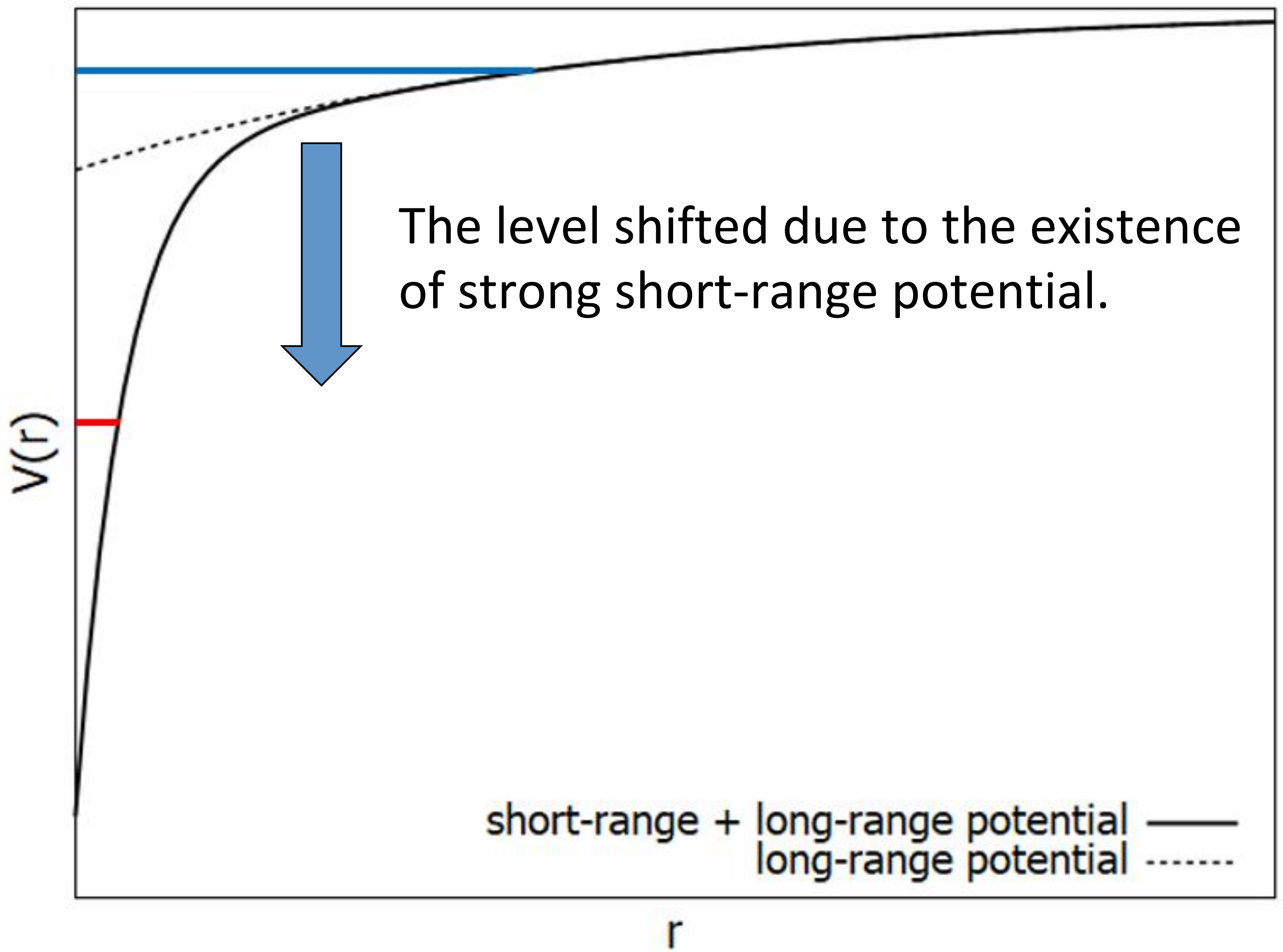
Introduction

- Particles are interacting with long- and short-range potential in mesonic atom systems.
  - K^-p ,K^-pp, K^-pn
- Due to the existence of the short-range potential, the levels are shifted from these with only long-range potential.
  - $\bar{K}N$ interaction is strong short-range attraction[1].

[1] M. Iwasaki et al. Phys. Rev. Lett. 78, 3067 (1997)

Level rearrangement



Deser-Teueman formula

- The shift of the spectrum of the long-range potential is expressed by Deser-Trueman formula (DT) in the case of the two-body systems. [2,3]

$$\delta E^{(2)} = \frac{2\pi}{\mu} |\Psi_{LR}^{(2)}(0)| a_{SR}$$

- DT is the product of the contribution of long-range potential and short-range potential.
  - $\Psi_{LR}^{(2)}(0)$: wave function of the long-range potential
  - a_{SR} : scattering length by the short-range potential

[2] S. Deser et al., Phys. Rev. 96, 774-776 (1954)
[3] T. Truman, Nucl. Phys. A 26, 57-67 (1961)

Deser-Trueman formula

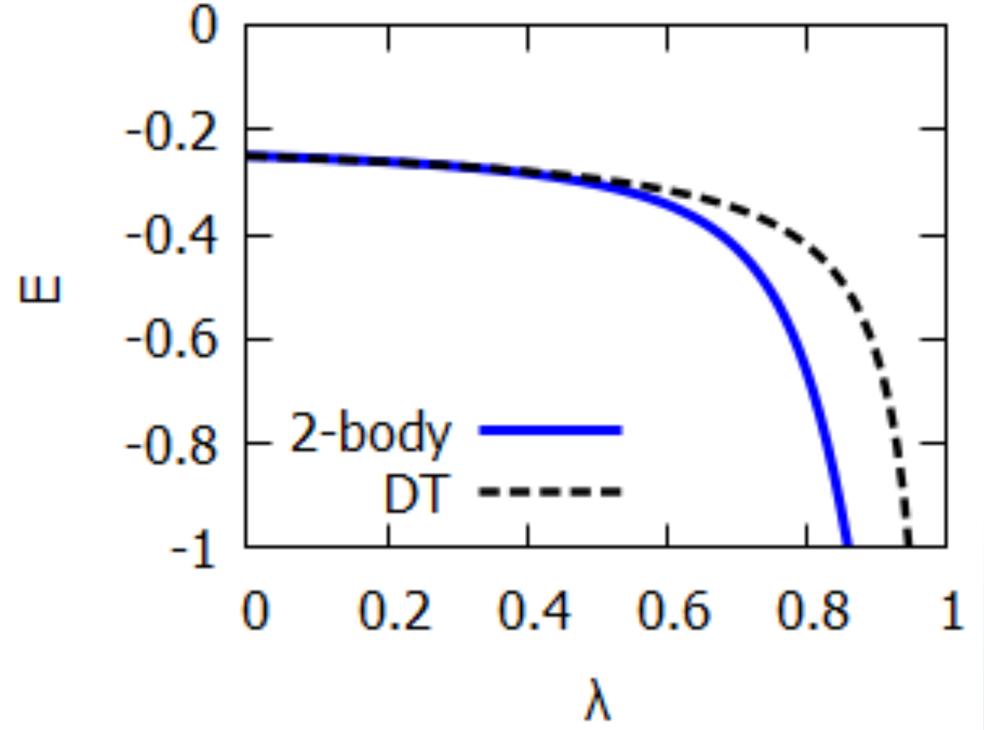
- When the short-range potential is strong enough, the DT does not reproduce the level shift.

$$H = \sum_{i}^{2} T_i - T_{cm} + V_{12}^{LR} + \lambda V_{12}^{SR}$$

$$V_{ij}^{SR} = -C_{SR}\mu_{SR}^3 \exp\left(-\mu_{SR}^2 r_{ij}^2\right)$$

$$V_{ij}^{LR} = -\frac{\mathrm{erf}(\mu_{LR} r_{ij})}{r_{ij}}$$

$$\mu_{SR} = \mu_{LR} = 30$$



Determinant method

- The determinant method [4] is adopted to evaluate whether the DT works well quantitatively.

	λ_1	λ_2	λ_3	$\cdots$
LR_{I}	$\delta E(LR_{\mathrm{I}}, \lambda_1)$	$\delta E(LR_{\mathrm{I}}, \lambda_2)$	$\delta E(LR_{\mathrm{I}}, \lambda_3)$	$\cdots$
LR_{II}	$\delta E(LR_{\mathrm{II}}, \lambda_1)$	$\delta E(LR_{\mathrm{II}}, \lambda_2)$	$\delta E(LR_{\mathrm{II}}, \lambda_3)$	$\cdots$
LR_{III}	$\delta E(LR_{\mathrm{III}}, \lambda_1)$	$\delta E(LR_{\mathrm{III}}, \lambda_2)$	$\delta E(LR_{\mathrm{III}}, \lambda_3)$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	

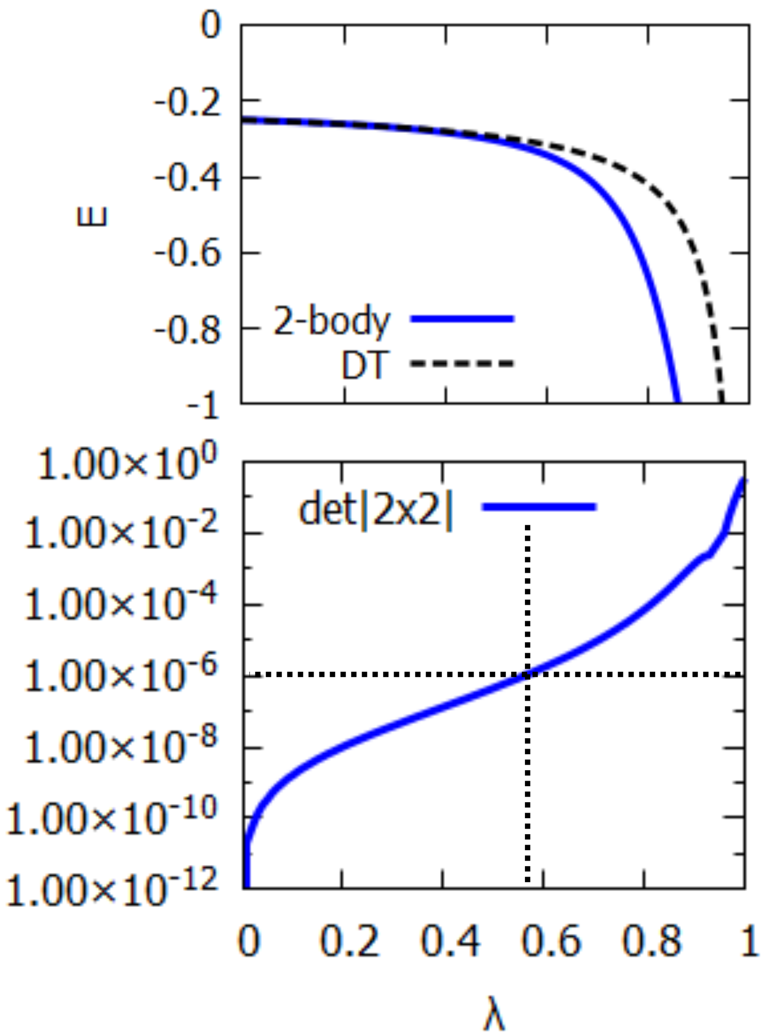
- If the energy shift is expressed by the DT, the determinant of the blued sub matrix is always 0

$$\mathrm{S}_2 = \begin{vmatrix} \delta E(LR_{\mathrm{I}}, \lambda_1) & \delta E(LR_{\mathrm{I}}, \lambda_2) \\ \delta E(LR_{\mathrm{II}}, \lambda_1) & \delta E(LR_{\mathrm{II}}, \lambda_2) \end{vmatrix} = \begin{vmatrix} A_{LR_{\mathrm{I}}} B_{SR}(\lambda_1) & A_{LR_{\mathrm{I}}} B_{SR}(\lambda_2) \\ A_{LR_{\mathrm{II}}} B_{SR}(\lambda_1) & A_{LR_{\mathrm{II}}} B_{SR}(\lambda_2) \end{vmatrix} = 0$$

[4] J.-M. Richard, C. Fayard, Phys. Lett. A 381, 3217-3221 (2017)

Determinant method

- Determinant method analysis on the two-body system.



*The DT represents the two-body correlation

*The criteria whether the determinant is zero or not depends on the calculation accuracy

$$\lambda_c \sim 0.6$$

The DT is applicable to three-body systems?

- Extended the DT for the three-body systems[3].

$$\sum_{i>j=1}^{3} \frac{2\pi}{\mu} |\Psi^{(3)}_{LR,ij}(0)|^2 a_{SR,ij}$$

- Summing up the DT for all pair in a three-body system.
  - $\Psi^{(3)}_{LR,ij}(0)$: three-body wave function of long-range potential when the two particles labeled *i* and *j* contact.
  - $a_{SR,ij}$: scattering length of *i* and *j* particles by short-range potential.
  - Including only two-body correlations
- The determinant method is still useful under the extention.

[3] J.-M. Richard, C. Fayard, Phys. Lett. A 381, 3217-3221 (2017)

Models

- To overview the behavior we firstly tackle to these two simple models consisting of bosons.

Model1

$$H_{\mathrm{I}} = \sum_{i}^{3} T_i - T_{cm} + \sum_{i>j=1}^{3} V_{ij}^{LR} + \lambda \sum_{i>j=1}^{3} V_{ij}^{SR}$$

$$V_{ij}^{LR} = -\frac{\mathrm{erf}(\mu_{LR} r_{ij})}{r_{ij}} \qquad V_{ij}^{SR} = -C_{SR}\mu_{SR}^3 \exp\left(-\mu_{SR}^2 r_{ij}^2\right)$$

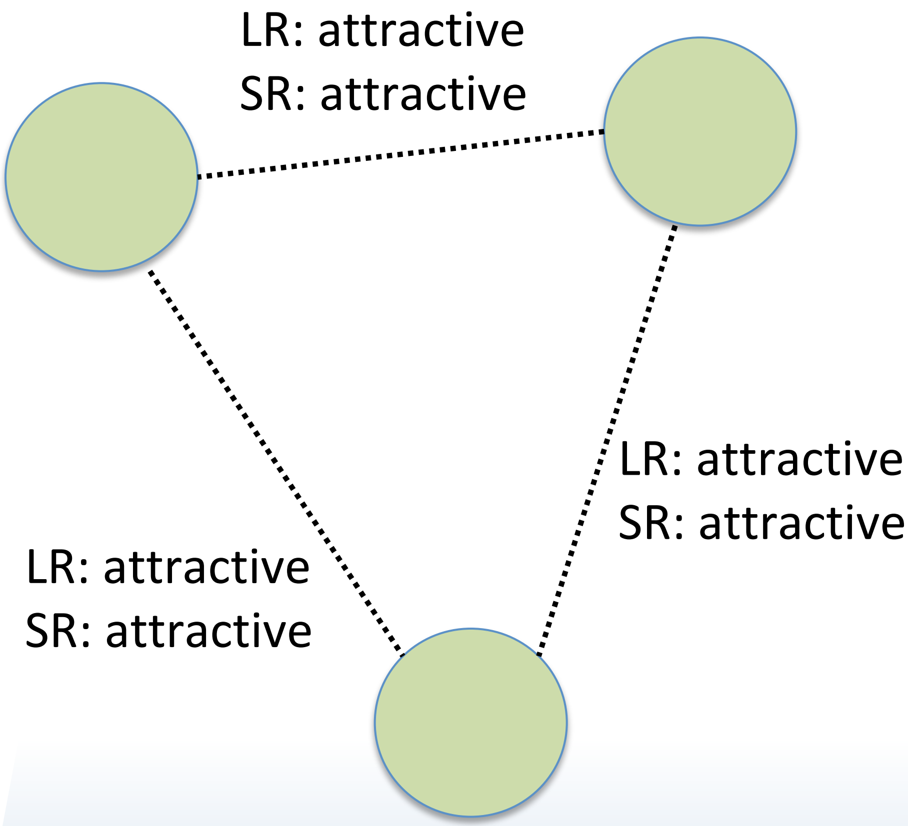
Model2

$$H_{\mathrm{II}} = \sum_{i}^{3} T_i - T_{cm} + \sum_{i>j=1}^{3} V_{ij}^{LR} + \lambda \sum_{i=1}^{2} V_{i3}^{SR}$$

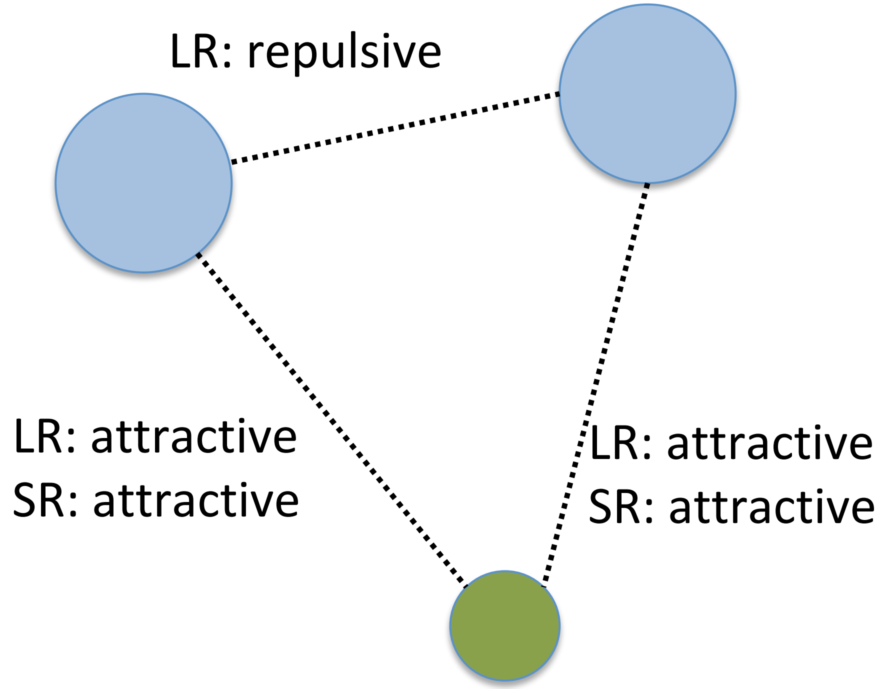
$$V_{ij}^{LR} = -\frac{\mathrm{erf}(\mu_{LR} r_{ij})}{r_{ij}} \qquad V_{ij}^{SR} = -C_{SR}\mu_{SR}^3 \exp\left(-\mu_{SR}^2 r_{ij}^2\right)$$

Model

Model1



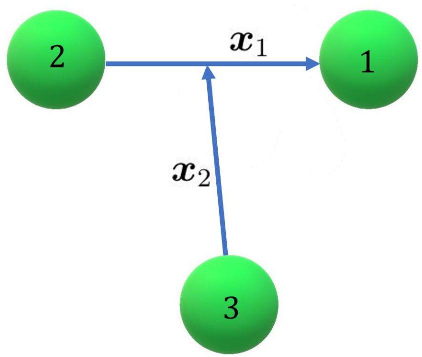
Model2



How three-body calc. is done

- Three-body calculation is done by stochastic variation method with the correlated Gaussian[5].
  - The wave function is expanded by the correlated Gaussian

$$|\Psi^{(3)}\rangle = \sum_k c_k \mathcal{S} \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}} A_k \boldsymbol{x}\right)$$



$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{pmatrix} \qquad \begin{aligned} \boldsymbol{x}_1 &= \boldsymbol{r}_1 - \boldsymbol{r}_2 \\ \boldsymbol{x}_2 &= \frac{m_1 \boldsymbol{r}_1 + m_2 \boldsymbol{r}_2}{m_1 + m_2} - \boldsymbol{r}_3 \end{aligned}$$

  - The variation parameters are determined to minimize the binding energy.

$$\delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0$$

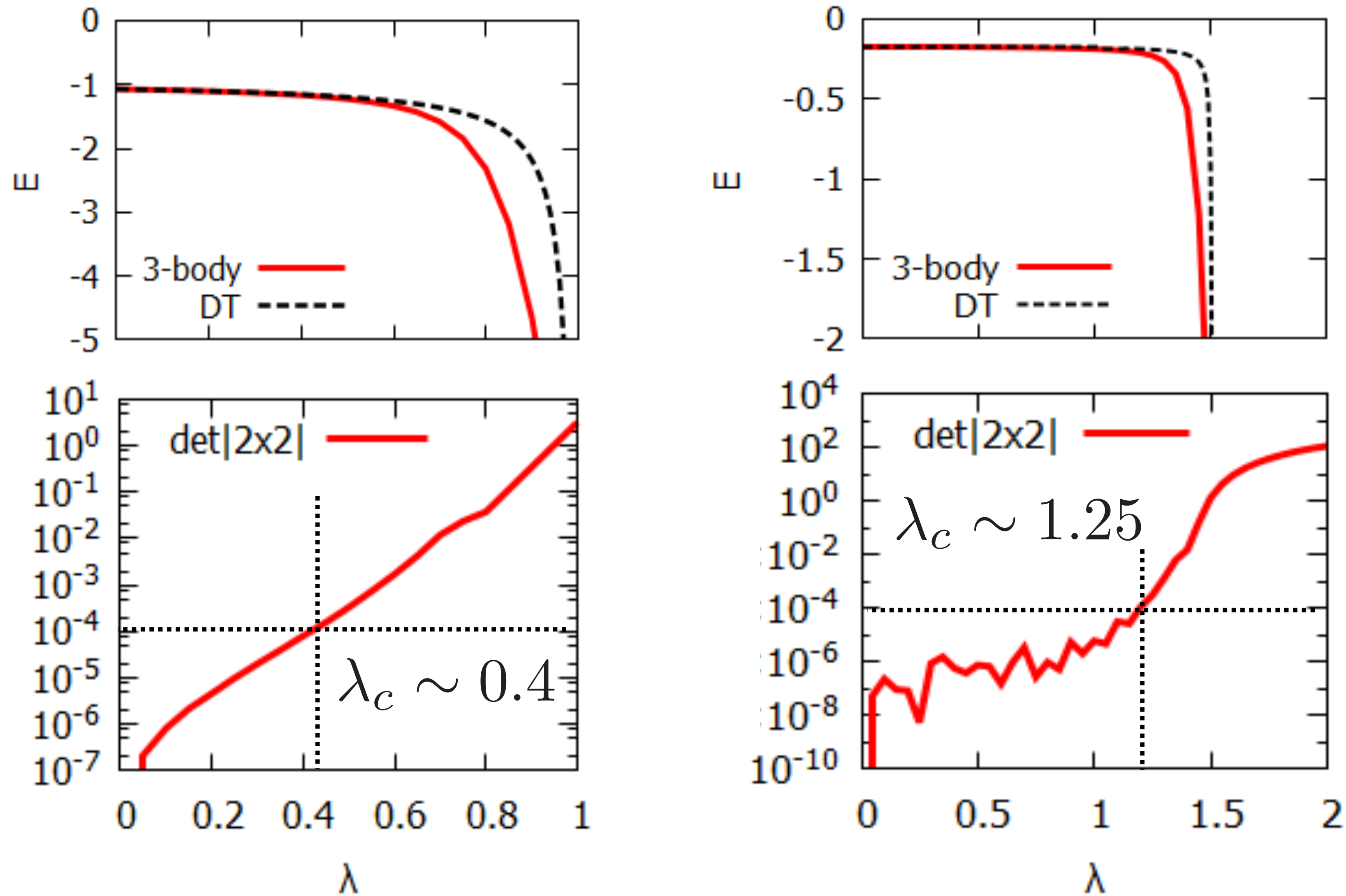
[5] K. Varga and Y. Suzuki, Phys. Rev. C52, 6 (1995)

How three-body calc. is done

- The difficulty of the calculation is that both long-range and short-range model space have to be treated simultaneously.
- The energies are converged until ~10^{-4}.

Results of three-body systems

- Level shift and determinant method of model 1,2



Summary

- The ranges of the strength of the short-range potential where Deser-Trueman formula is available are evaluated by the determinant method.
- 2body: $\lambda_c \sim 0.6$
- 3body
  - Model1 $\lambda_c \sim 0.4$
  - Model2 $\lambda_c \sim 1.25$

- The application to physical system e.g. KNN gives the prediction on to the KN interaction.