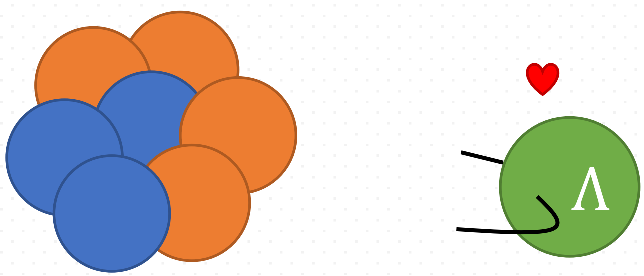
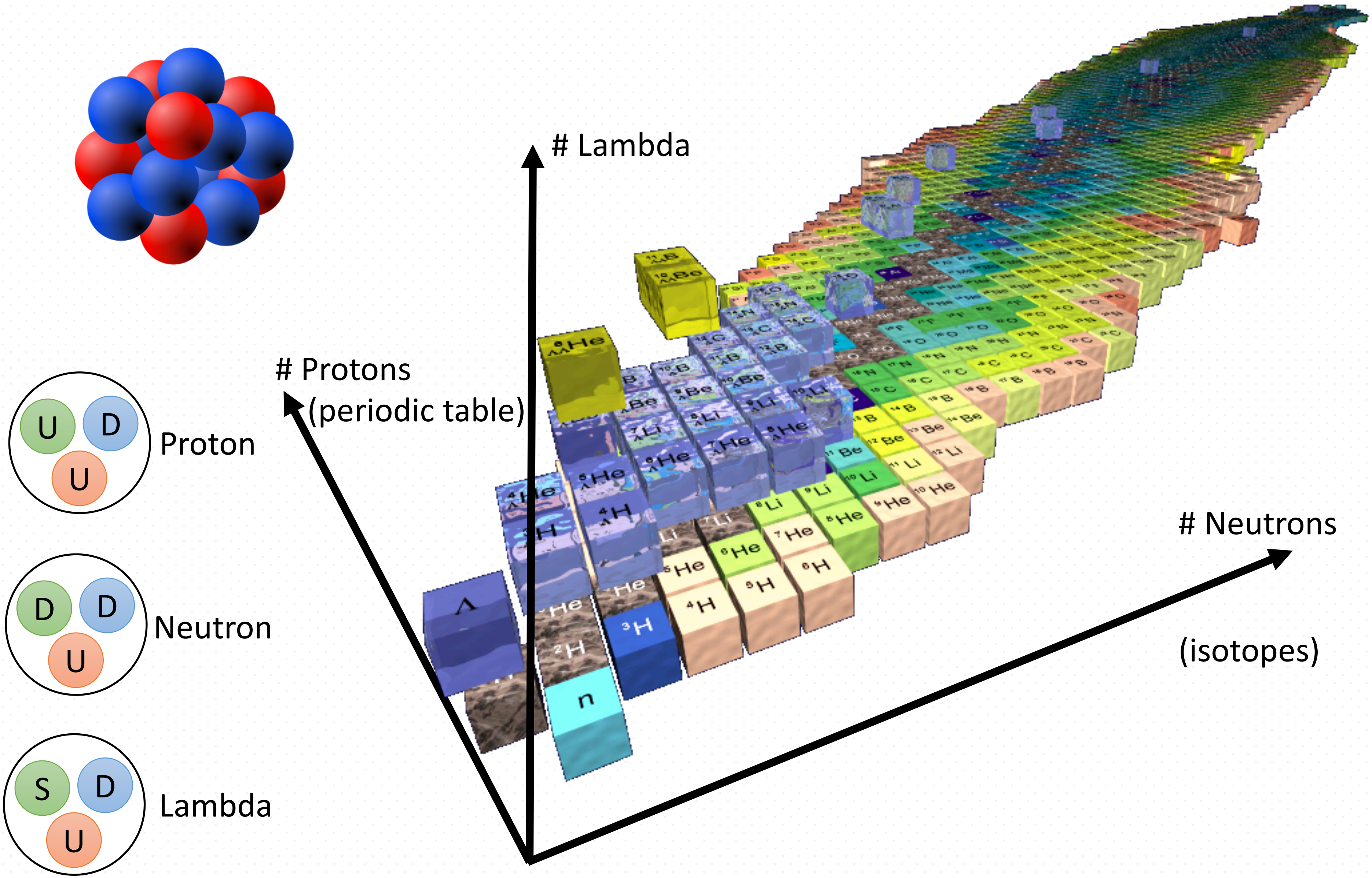
Single- and double-Λ hypernuclei in EFT($\not{\pi}$)



Lorenzo Contessi
Martin Shäfer
Nir Barnea
Avraham Gal
Jiří Mareš

THE HEBREW
UNIVERSITY
OF JERUSALEM



Lambda
Protons
(periodic table)
Neutrons
(isotopes)
U
D
U
Proton
D
D
U
Neutron
S
D
U
Lambda

Focus and goal:

Phaseshift data shortage of

$N-\Lambda$ and $\Lambda-\Lambda$

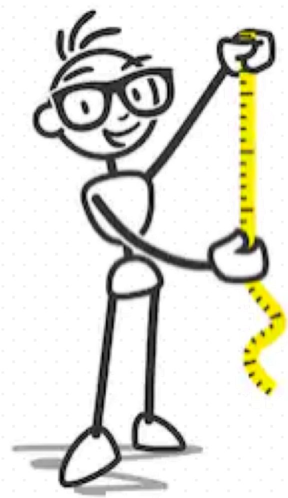
Theoretically difficult to describing all S-wave systems together

Known few-body

$^{3}_{\Lambda}H$ $^{5}_{\Lambda}\mathrm{He}$ ← Overbinding

$^{4}_{\Lambda}H\,(S{=}0)$ $^{4}_{\Lambda}H\,(S{=}1)$

Experimentally **known**
Theoretically not described



Unknown few-body

$nn\Lambda$ $n\Lambda\Lambda$

$nn\Lambda\Lambda$ $np\Lambda\Lambda$

$^{5}_{\Lambda\Lambda}\mathrm{He}$ → J-PARC **P75** proposal

Experimentally **not known**
Theoretically **debated**

Very few Double-Λ data

$^{6}_{\Lambda\Lambda}He$ $^{10}_{\Lambda\Lambda}\mathrm{Be}$

$^{11}_{\Lambda\Lambda}\mathrm{Be}$

Focus and goal:

Phaseshift data shortage of

$N-\Lambda$ and

$\Lambda-\Lambda$

Theoretically difficult to describing all S-wave systems together

Known few-body

Overbinding

${}^{3}_{\Lambda}H$ ${}^{5}_{\Lambda}\mathrm{He}$

${}^{4}_{\Lambda}H\,(S=0)$ ${}^{4}_{\Lambda}H\,(S=1)$

Experimentally **known**

Theoretically not described

Many system that should be studied

--- ---

Few input data to tune theories

↓

Minimal interaction

Unknown few-body

$nn\Lambda$ $n\Lambda\Lambda$

$nn\Lambda\Lambda$ $np\Lambda\Lambda$

${}^{5}_{\Lambda\Lambda}\mathrm{He}$

Experimentally **not**

Theoretically **debated**

Very few Double-Λ data

${}^{6}_{\Lambda\Lambda}He$ ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$

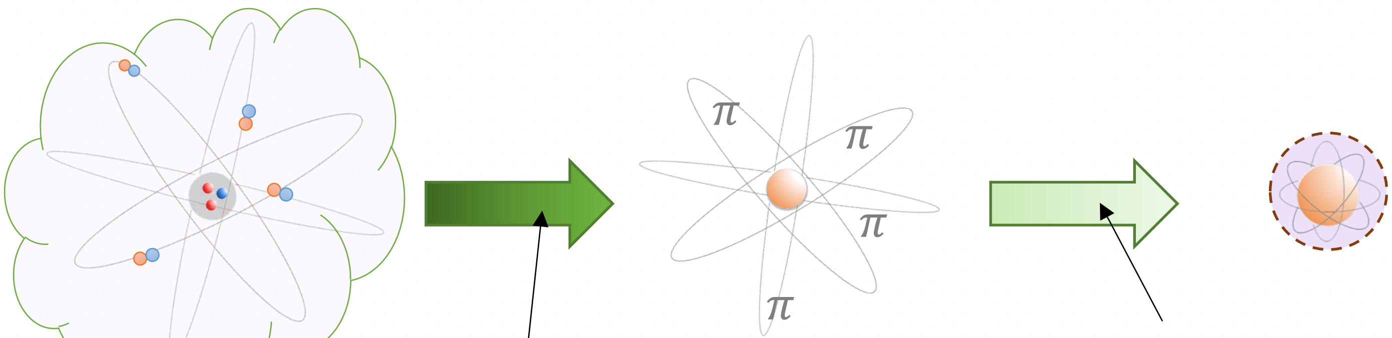
${}^{11}_{\Lambda\Lambda}\mathrm{Be}$

Separation of scales

Quarks and **gluons**
QCD

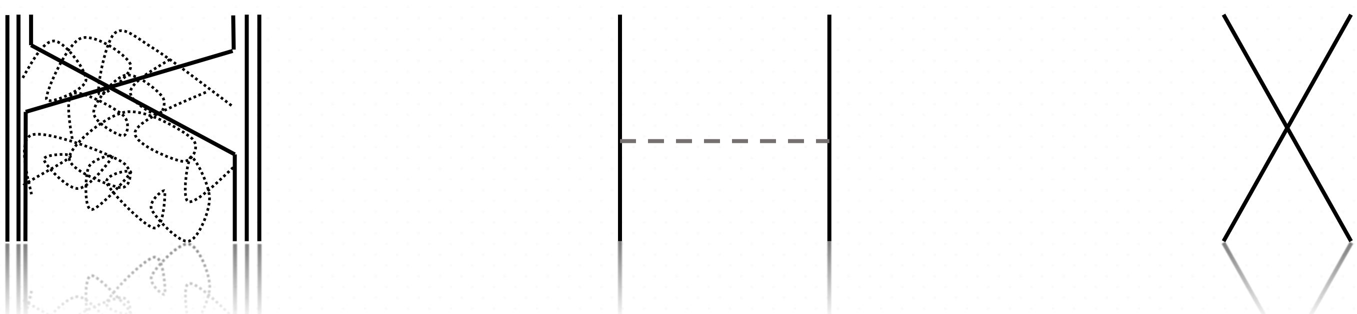
Nucleons and **pions**
Meson exchanges

Nucleons
Contact interactions



- Color confinement

- Low exchanged momentum
- Large pion mass

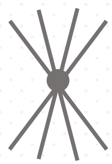


QCD χEFT Contact EFT

expansion δ and derivative

Three body force is necessary to avoid Thomas collapse

LO  Momentumless contacts

NLO  $O\left(\frac{Q}{m_\pi}\right)$

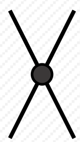
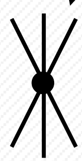
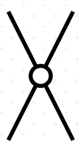
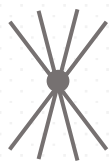
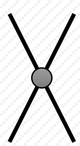
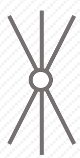
N^2LO  $O\left(\left(\frac{Q}{m_\pi}\right)^2\right)$

$N^{>2}LO$ … $O\left(\left(\frac{Q}{m_\pi}\right)^{\geq 3}\right)$

B. Bazak, Four-Body Scale in Universal Few-Boson Systems, PRL 122.143001 (2019)
G.P. Lepage, How to renormalize the Schrodinger equation (1997)
van Kolck, U. Nucl.Phys. A645 (1999) 273-302
Chen, Jiunn-Wei et al. Nucl.Phys. A653 (1999)
S. K˙onig, H. W. Grießhammer, H. W. Hammer, and U. van Kolck J. Phys. G43, 055106 (2016)

expansion δ and derivative

Three body force is necessary to avoid Thomas collapse

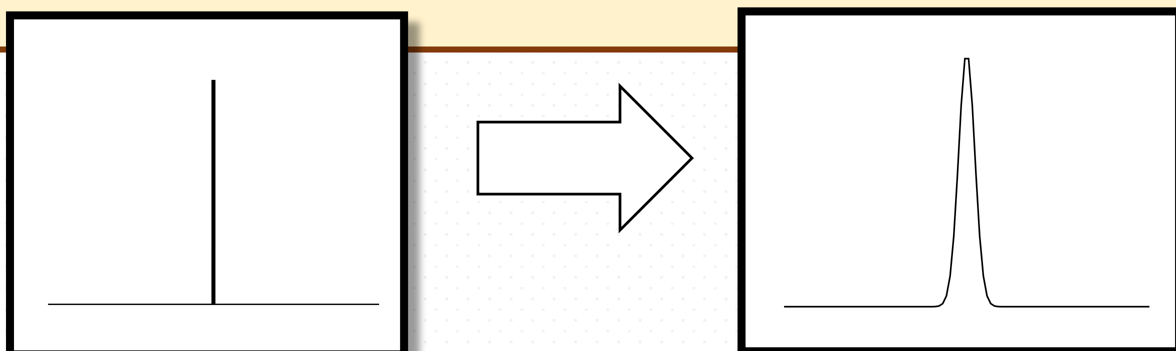
LO			Momentumless contacts
NLO			$O\left(\frac{Q}{m_\pi}\right)$
N^2LO			$O\left(\left(\frac{Q}{m_\pi}\right)^2\right)$
$N^{>2}LO$	…		$O\left(\left(\frac{Q}{m_\pi}\right)^{\geq 3}\right)$

B. Bazak, Four-Body Scale in Universal Few-Boson Systems, PRL 122.143001 (2019)
G.P. Lepage, How to renormalize the Schrodinger equation (1997)
van Kolck, U. Nucl.Phys. A645 (1999) 273-302
Chen, Jiunn-Wei et al. Nucl.Phys. A653 (1999)
S. K˙onig, H. W. Grießhammer, H. W. Hammer, and U. van Kolck J. Phys. G43, 055106 (2016)

Pionless at Leading Order

❖ **Regularization/Renormalization** is required:

$$C\,\delta(\vec{r_i}-\vec{r_j}) \;\rightarrow\; C(\lambda)\left[\frac{\lambda^3}{8\pi^{3/2}}\right]\; e^{\frac{-\lambda^2 r_{ij}^2}{4}}$$



$\lambda \rightarrow \infty$ *:

- Regularization/model independent
- Observables are λ dependent

*Observables are **cut-off dependent:***

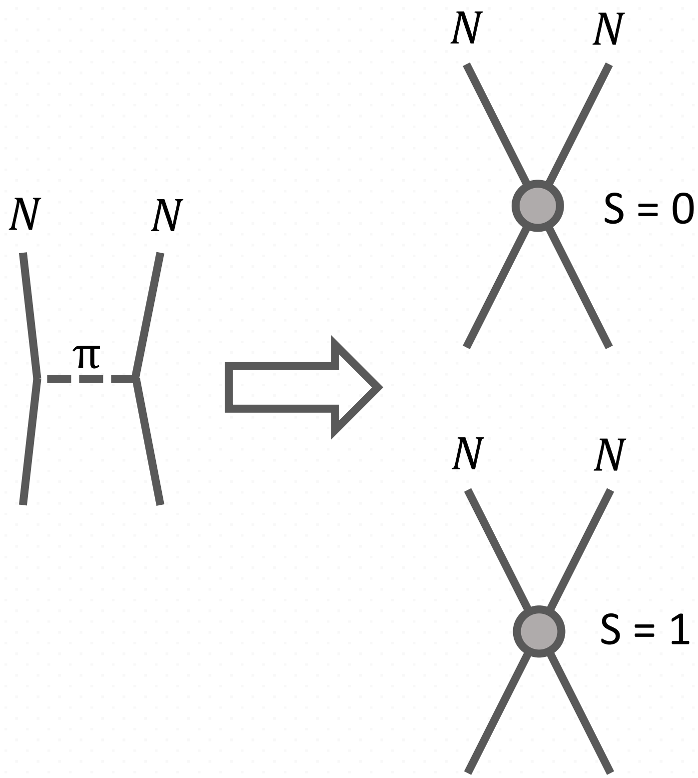
$$O_\lambda = O_\infty + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^3} + \cdots$$

* $\lambda \gg M$

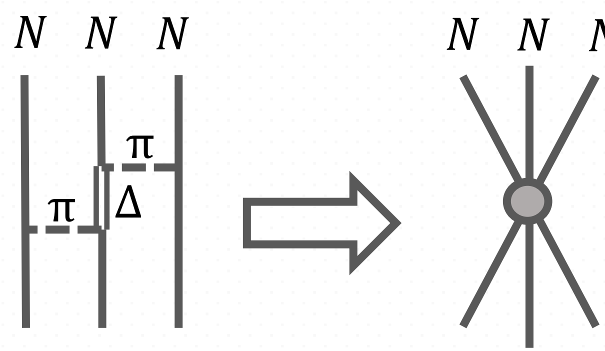
π-EFT (N)

M = Theory break-scale
Q = Typical exchanged momentum
B = Typical binding per particle

2-Body



3-Body



$\mathbf{B \sim 7\ MeV}$

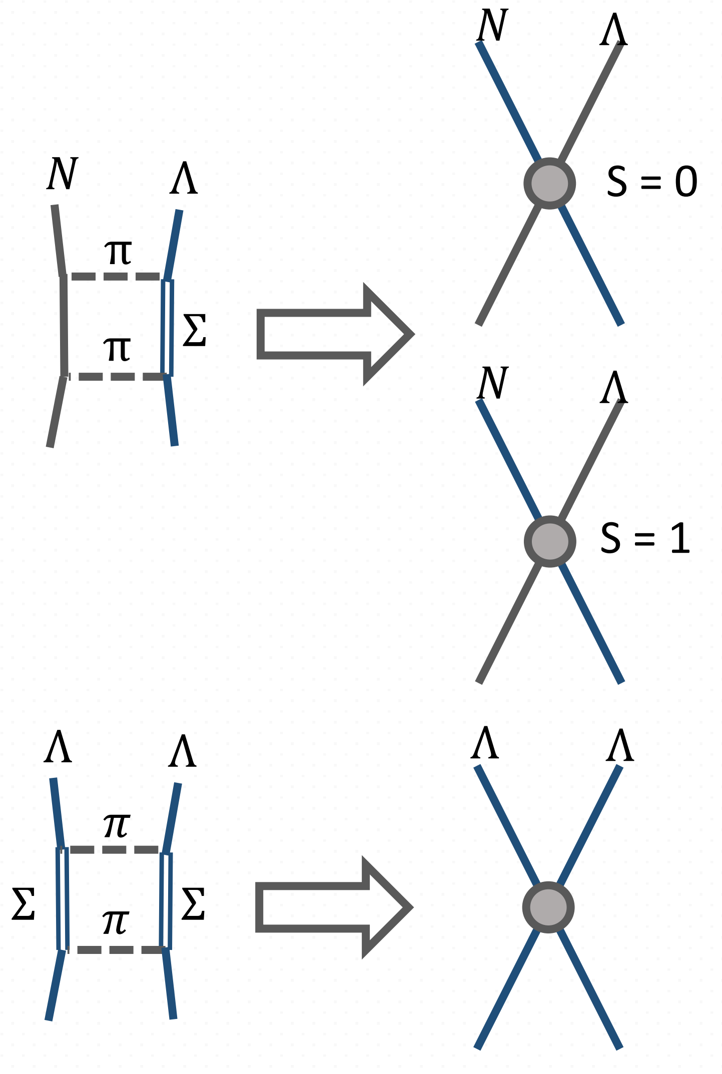
$\boldsymbol{M \sim m_\pi}$

$$\delta LO = \left(\frac{Q}{M}\right) \sim 50\%$$

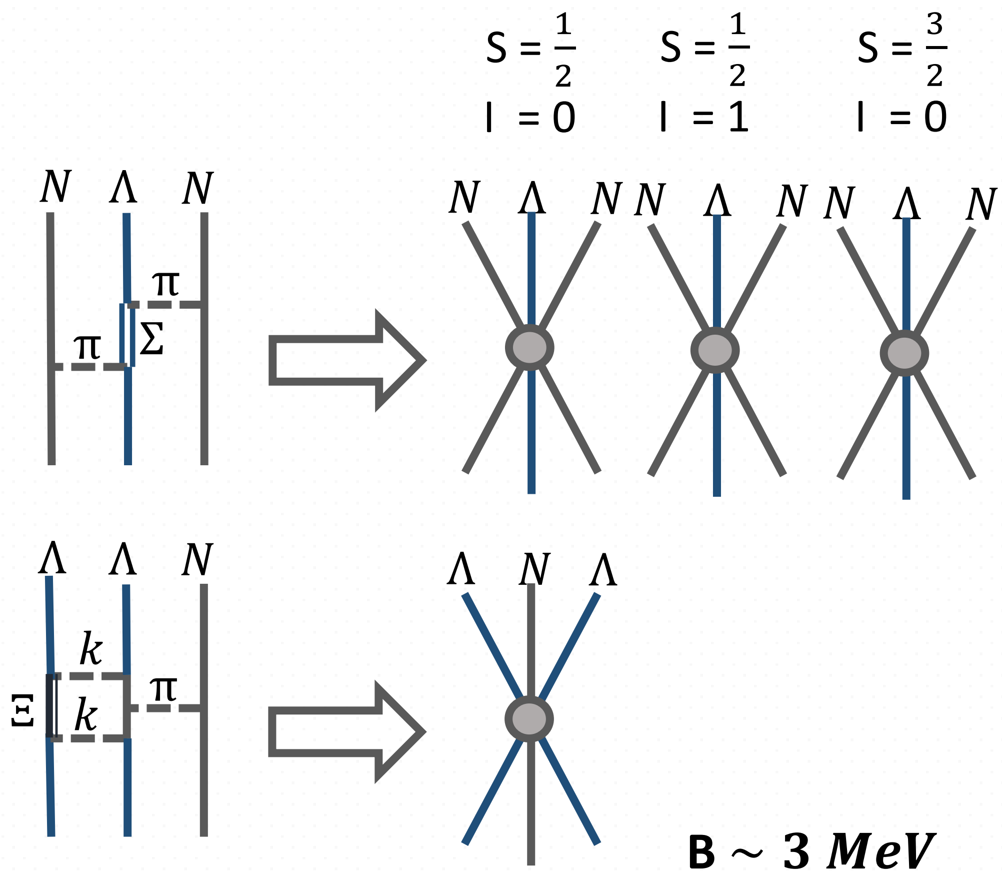
$\not{\pi}$-EFT (Λ)

M = Theory break-scale
Q = Typical exchanged momentum
B = Typical binding per particle

2-Body



3-Body

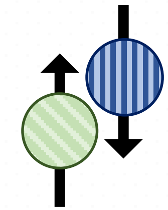


$\mathbf{B \sim 3\ MeV}$
$\boldsymbol{M \sim 2\ m_\pi}$

$$\delta LO = \left(\frac{Q}{M}\right)^2 \sim 9\%$$

Fitting input

Two body

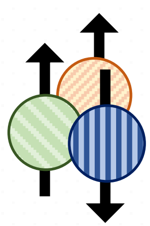


$2\times\ NN$ $\boldsymbol{a_{NN}}$ (Spin singlet and triplet)

$2\times\ N\Lambda$ $\boldsymbol{a_{N\Lambda}}$ (Spin singlet and triplet)

$\Lambda\Lambda$ $\boldsymbol{a_{\Lambda\Lambda}}$

Three body



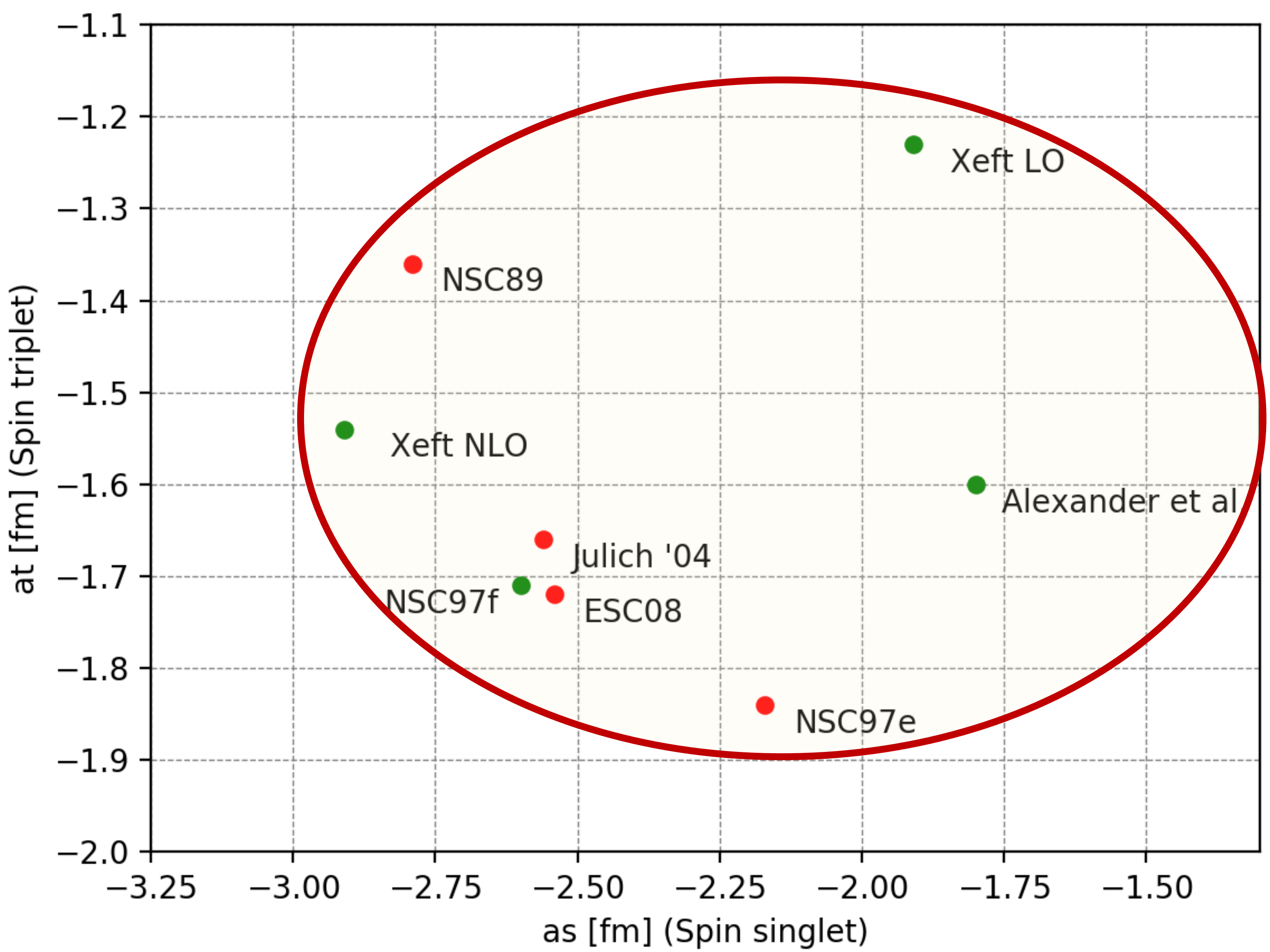
NNN $\mathbf{{}^{3}H}$

$3\times NN\Lambda$:
- $\mathbf{{}^{3}_{\Lambda}H}$
- $\mathbf{{}^{4}_{\Lambda}H(S=0, I=1/2)}$
- $\mathbf{{}^{4}_{\Lambda}H(S=1, I=1/2)}$

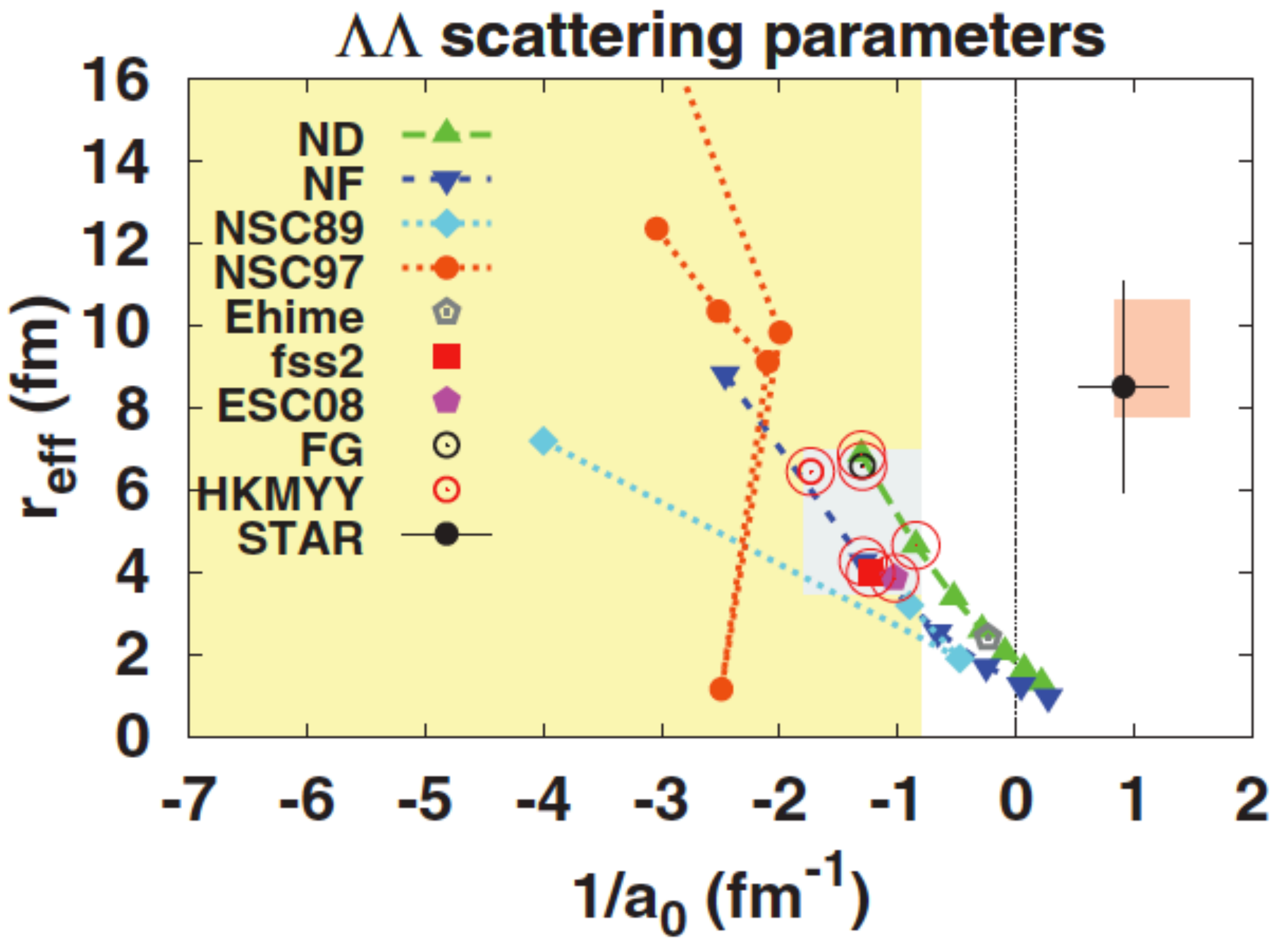
$N\Lambda\Lambda$ $\mathbf{{}^{6}_{\Lambda\Lambda}He}$

N-Λ scattering lengh

A. Gal et al. - Strangeness in nuclear physics - Rev.Mod.Phys. 88 (2016) no.3, 035004



ΛΛ Scattering data



PHYSICAL REVIEW C 91, 024916 (2015)

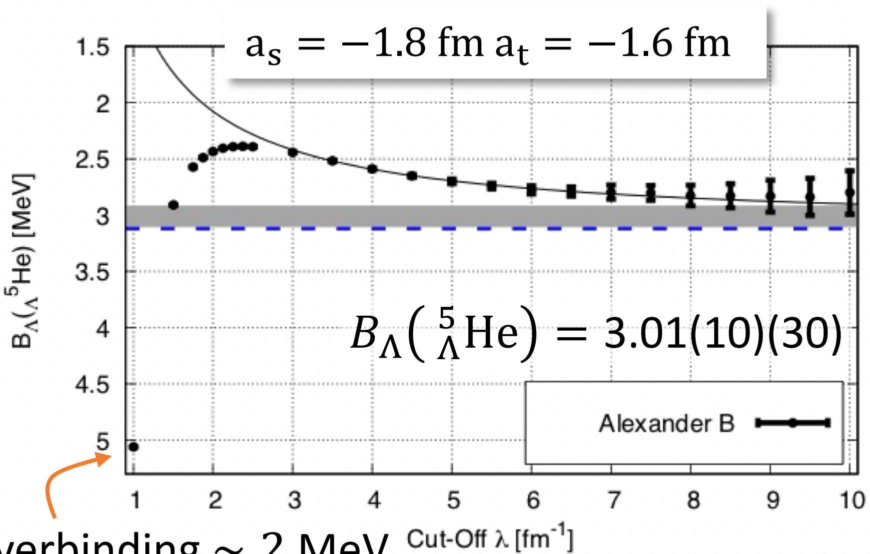
$$a_{\Lambda\Lambda} \sim \{-0.5 \,..\, -1.9\}\ \text{fm}$$

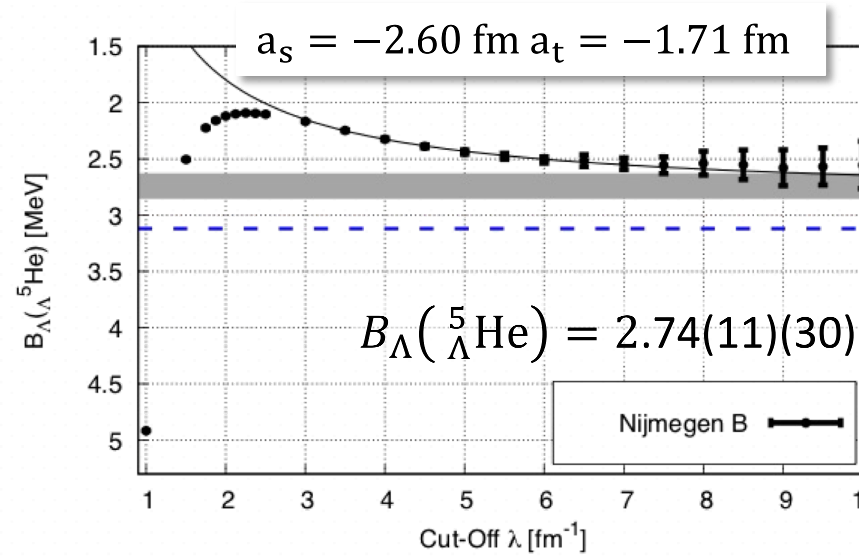
Results

- ${}^{5}_{\Lambda}\mathbf{He}$
- ${}^{5}_{\Lambda\Lambda}\mathbf{H}$
- $\boldsymbol{np\Lambda\Lambda}$

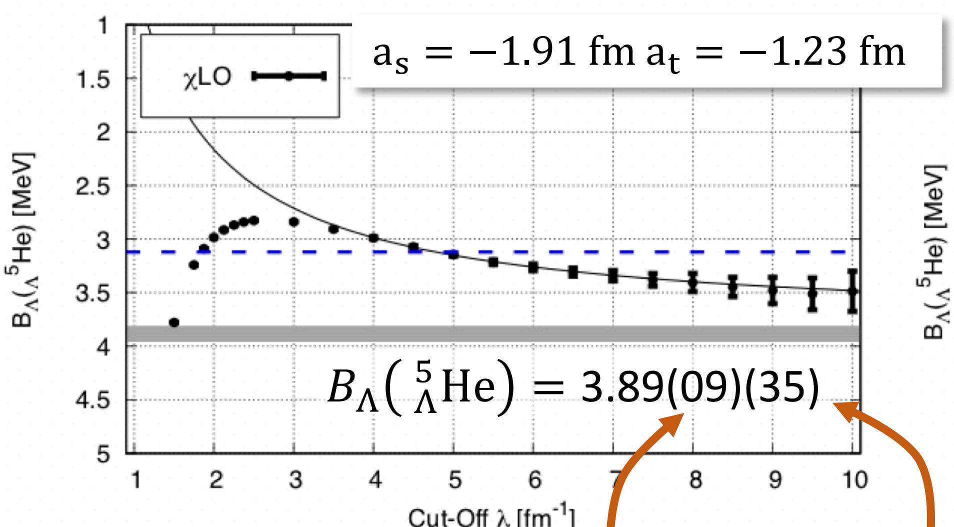
${}^{5}_{\Lambda}\mathrm{He}$: Λ separation energy

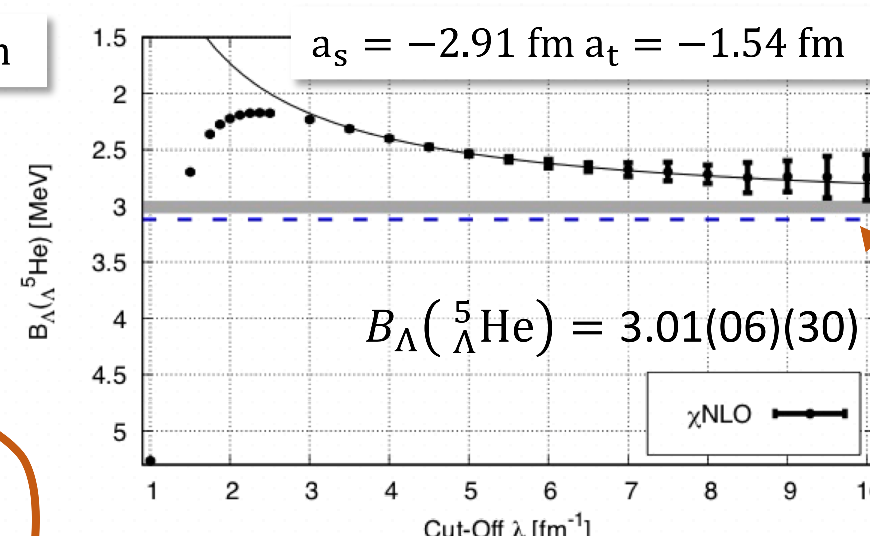
No sign of bound **NNΛ**, other than ${}^{3}_{\Lambda}\mathrm{H}\left(S=\frac{1}{2},\ I=0\right)$;





Overbinding ~ 2 MeV



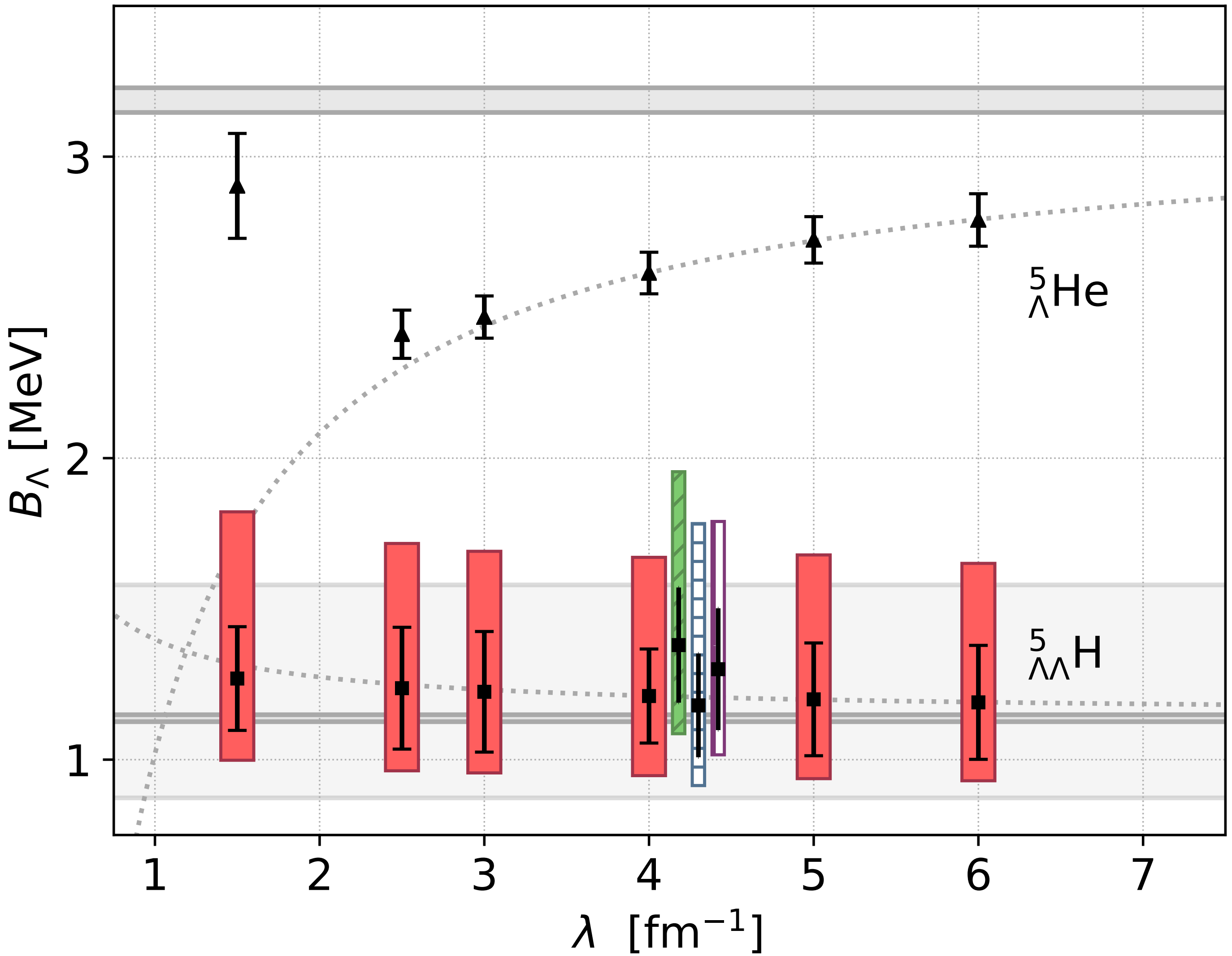


Numerical uncertainty

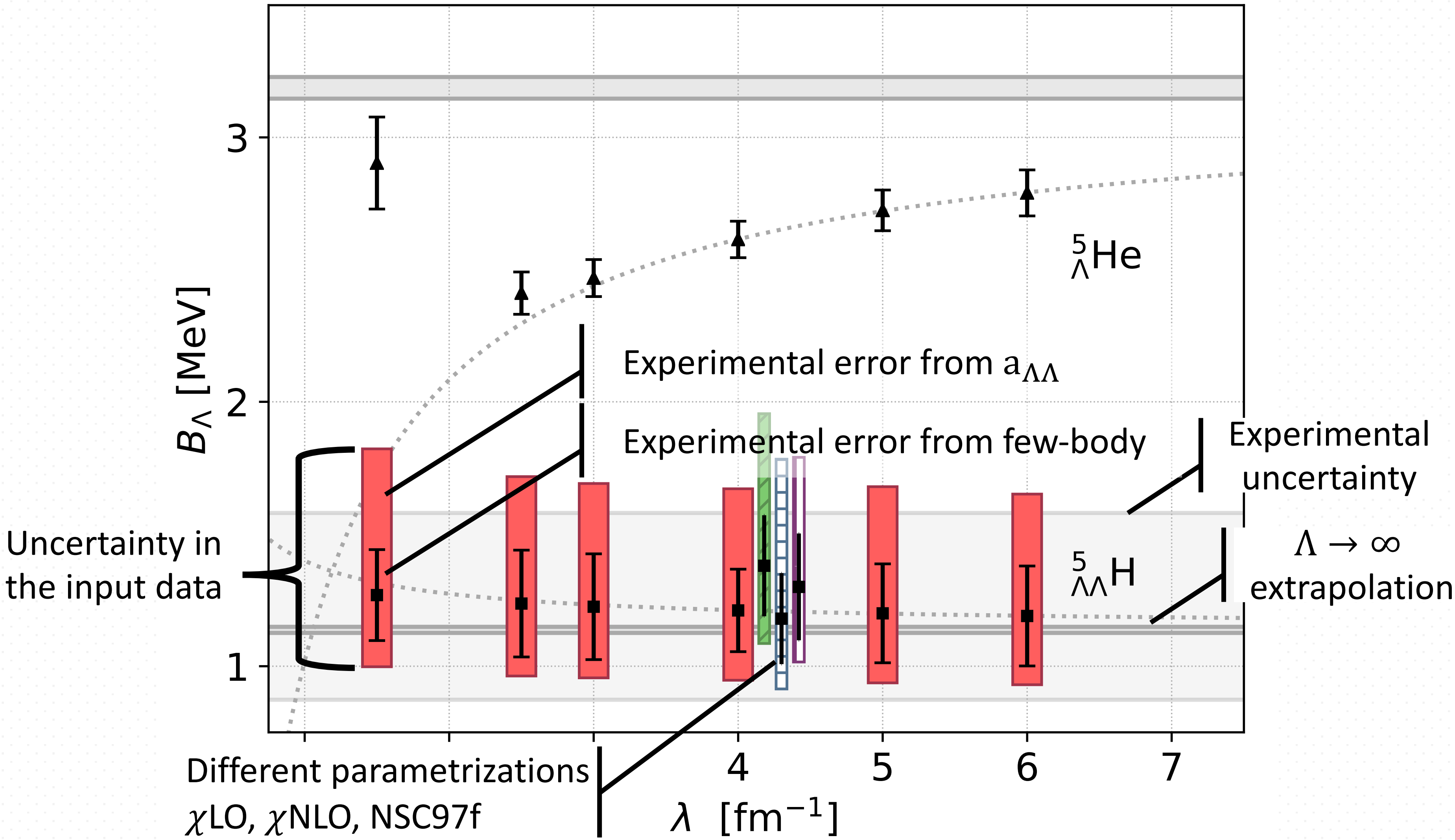
Theory uncertainty

Experimental: 3.12(2) MeV

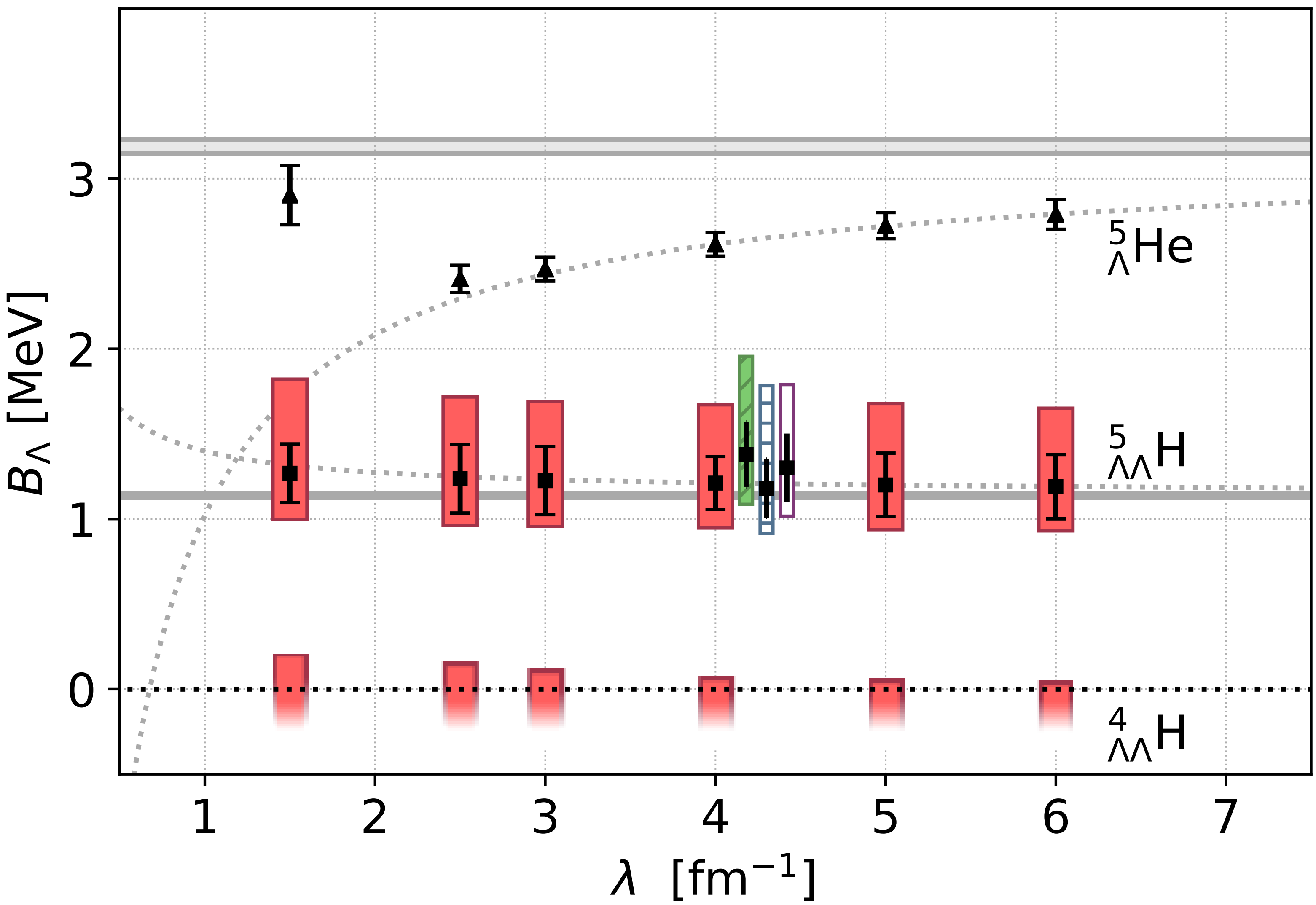
$^{5}_{\Lambda\Lambda}H$



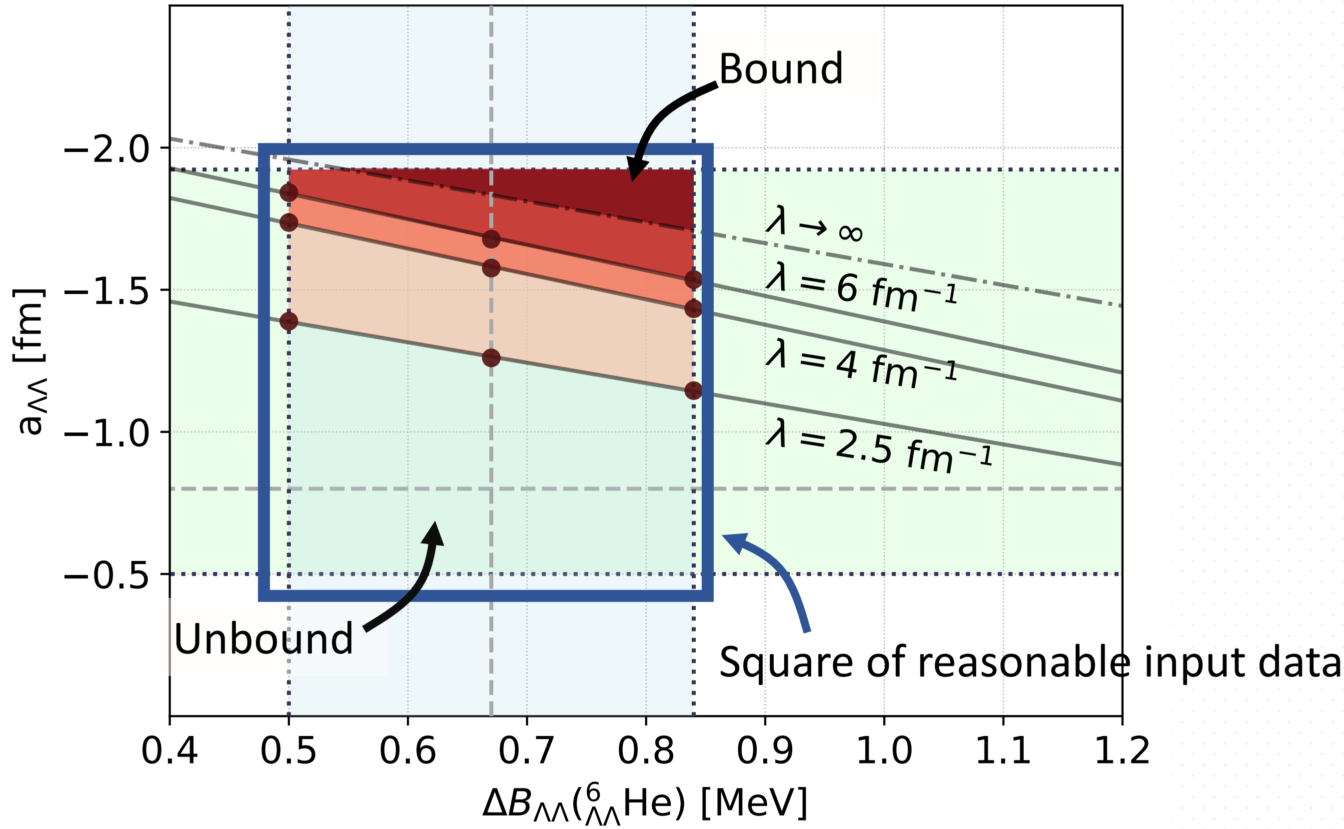
$B_\Lambda({}^{5}_{\Lambda\Lambda}\mathrm{H}) = 1.14(1)^{+(44)}_{-(26)}$ MeV



$^{4}_{\Lambda\Lambda}\mathrm{H}$



$^{4}_{\Lambda\Lambda}\mathrm{H}$



$$\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He}) = B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He}) - 2\, B_{\Lambda}(^{5}_{\Lambda}\mathrm{He})$$

$^{4}_{\Lambda\Lambda}\mathrm{H}$ is bound/unbound depending to the theory input

In a nutshell

$^{3}_{\Lambda}\mathrm{H}$

$^{4}_{\Lambda}\mathrm{H}$ $^{4}_{\Lambda}\mathrm{H}^{*}$

$^{5}_{\Lambda}\mathrm{He}$

It is possible to describe them all together.
(No overbinding problem!)

$^{5}_{\Lambda\Lambda}\mathrm{He}$

Solidly bound!

$$B_{\Lambda}\left({}^{5}_{\Lambda\Lambda}\mathrm{He}\right) = 1.14(1)^{+(44)}_{-(26)}$$

$np\Lambda\Lambda$

Bound?
Need more precise
$\Lambda\Lambda$ scattering data!

$nn\Lambda$

$nn\Lambda\Lambda$

$n\Lambda\Lambda$

Unbound

Conclusions

General

- $\pi\!\!\!/$-EFT can be applied **successfully to Λ hypernuclei**:
  (no catastrophic failure, truncation error of $\sim 10\%$ at LO).
- **7** new input data that **can** be fix on **experimental** data!
- **Overcomes overbinding** problem (comprehensive description of $A \leq 5$ Λ-hyperons)

Predictions

- **No** boundstate in $\boldsymbol{nn\Lambda}, \boldsymbol{np\Lambda}\left(S = \frac{3}{2}\right), \boldsymbol{n\Lambda\Lambda}$ or $\boldsymbol{nn\Lambda\Lambda}$
- $\boldsymbol{np\Lambda\Lambda}$ might be bound for large $a_{\Lambda\Lambda} < -1.5$ fm
- $\boldsymbol{{}^{5}_{\Lambda\Lambda}\mathrm{He}}$ **bound** ($\boldsymbol{B\left({}^{5}_{\Lambda\Lambda}\mathrm{He}\right) = 1.14(1)^{+(44)}_{-(26)}}$ MeV)

Prospective

- **Extend** this approach to **A > 6** systems.
- Include **subleading contributions** (explicit Ξ mixing, effective range, ..).

${}^{5}_{\Lambda}$He: Overbinding problem

	$B_\Lambda({}^{3}_{\Lambda}H)$	$B_\Lambda({}^{4}_{\Lambda}H_{g.s.})$	$B_\Lambda({}^{4}_{\Lambda}H_{exc.})$	$B_\Lambda({}^{5}_{\Lambda}He)$
Exp.	0.13(5) [4]	2.16(8) [5]	1.09(2) [6]	3.12(2) [4]
DHT [7]	0.10	2.24	0.36	$\geq$ 5.16
AFDMCa	-	1.97(11) [8]	-	5.1(1) [9]
AFDMCb'	0.23(9) [13]	1.95(9) [13]	-	2.60(6) [13]
χEFTa	0.11 [10]	2.31 (3) [11]	0.95(15) [11]	5.82(2) [12]
χEFTb	-	2.13 (3) [11]	1.39(15) [11]	4.43(2) [12]

All the energies are in MeV.

[7] R.H. Dalitz, R.C. Herndon, and Y.C. Tang, Nucl. Phys. B 47, 109 (1972).
[8] D. Lonardoni, F. Pederiva, and S. Gandolfi, Phys. Rev. C 89, 014314 (2014).
[9] D. Lonardoni, S. Gandolfi, and F. Pederiva, Phys. Rev. C 87, 041303(R) (2013).
[10] R. Wirth et al., Phys. Rev. Lett. 113, 192502 (2014).
[11] D. Gazda and A. Gal, Phys. Rev. Lett. 116, 122501 (2016); D. Gazda and A. Gal, Nucl. Phys. A 954, 161 (2016).
[12] R. Wirth and R. Roth, Phys. Lett. B 779, 336 (2018). We thank Roland Wirth for providing us with these values.
[13] D. Lonardoni arXiv:1711.07521v2 & Private comunication.
[15] H. Nemura, Y. Akaishi, and Y. Suzuki, Phys. Rev. Lett. 89, 142504 (2002); see also Y. Akaishi, T. Harada.

N-Λ scattering data

Alexander et al. : $a_s = -1.8$ fm
$a_t = -1.6$ fm

G. Alexander, U. Karshon, A. Shapira, et al. Phys. Rev. 173, 1452 (1968)

Sechi-Zorn et al. : $0 > a_s > -9$ fm
$-0.8 > a_t > -3.2$ fm

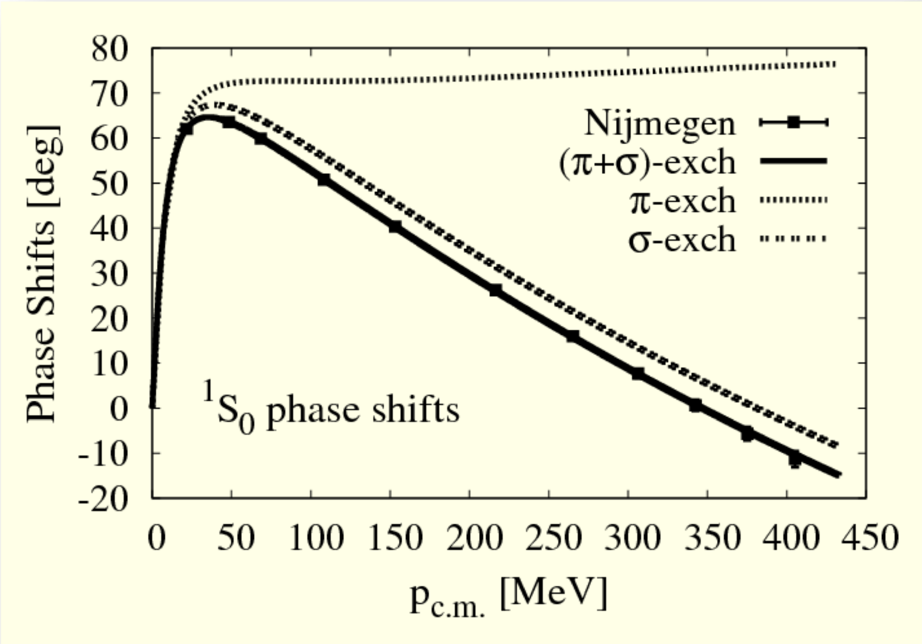
Sechi-Zorn, B., B. Kehoe, J. Twitty, and R. A. Burnstein, 1968, Phys. Rev. 175, 1735.

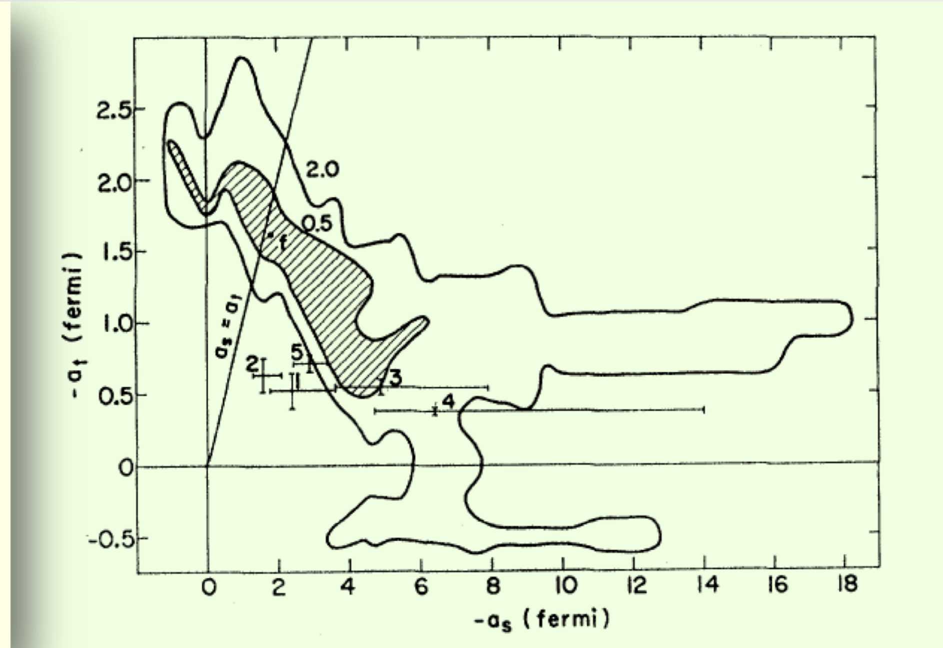
TABLE VII ΛN scattering lengths and effective ranges (in fm) for several YN interaction models. For the EFT models, these refer to Λp and to cutoff parameter of 600 MeV.

Model	Reference	a^s	r_0^s	a^t	r_0^t
NSC89	Maessen, Rijken, and de Swart (1989)	-2.79	2.89	-1.36	3.18
NSC97e	Rijken, Stoks, and Yamamoto (1999)	-2.17	3.22	-1.84	3.17
NSC97f	Rijken, Stoks, and Yamamoto (1999)	-2.60	3.05	-1.71	3.33
ESC08c	Nagels, Rijken, and Yamamoto (2015b)	-2.54	3.15	-1.72	3.52
Jülich '04	Haidenbauer and Meißner (2005)	-2.56	2.75	-1.66	2.93
EFT (LO)	Polinder, Haidenbauer, and Meißner (2006)	-1.91	1.40	-1.23	2.20
EFT (NLO)	Haidenbauer *et al.* (2013)	-2.91	2.78	-1.54	2.72

A. Gal et al. - Strangeness in nuclear physics - Rev.Mod.Phys. 88 (2016) no.3, 035004

Regular Strange





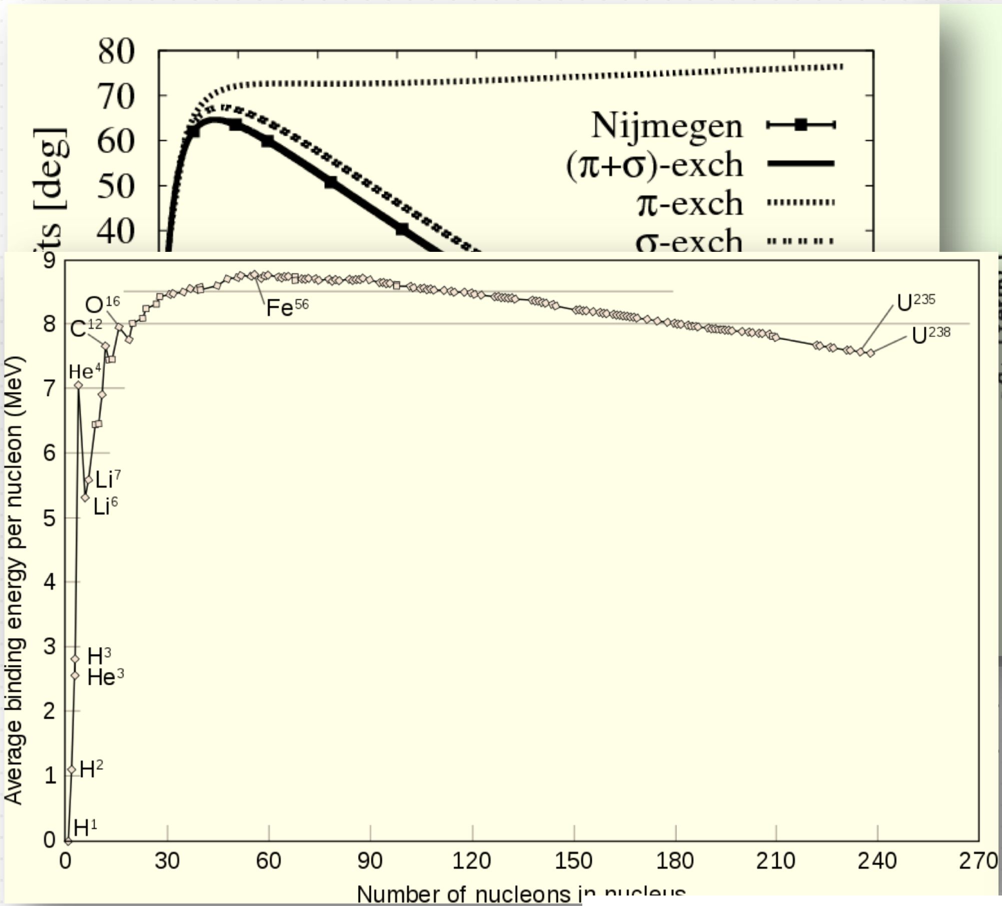
Phys. Rev. 175, 1735. (1968)
Phys. Rev. 173, 1452 (1968)

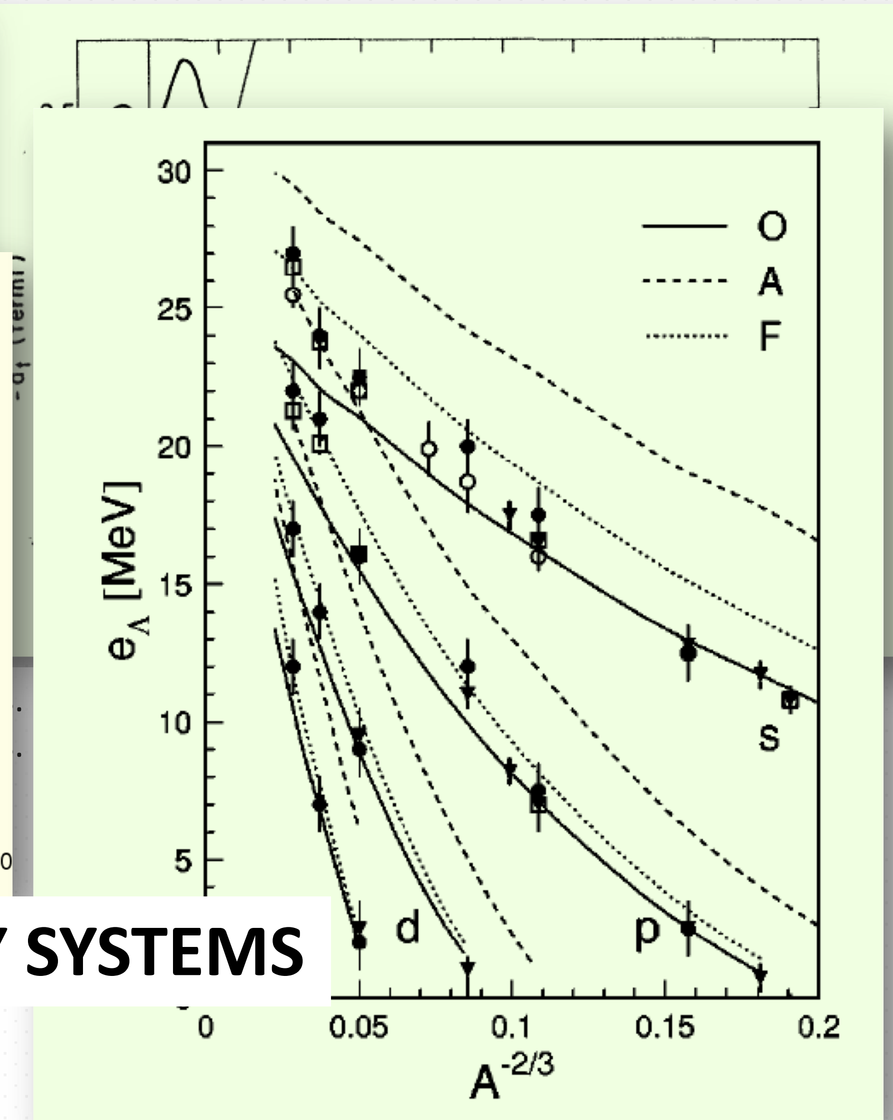
TWO-BODY SCATTERING

ALL PICTURES SHOWN ARE FOR ILLUSTRATION PURPOSE ONLY.

Regular

Strange



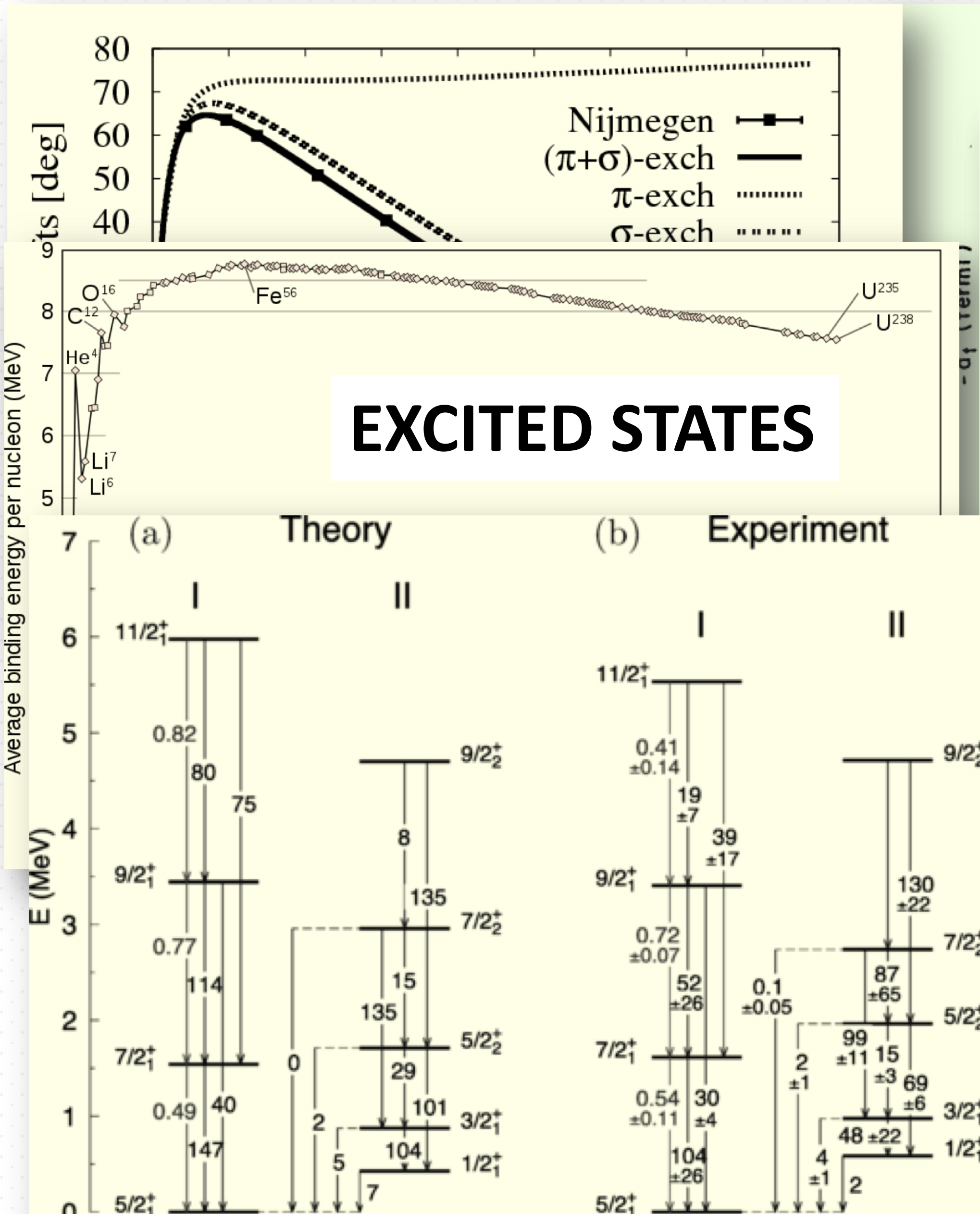


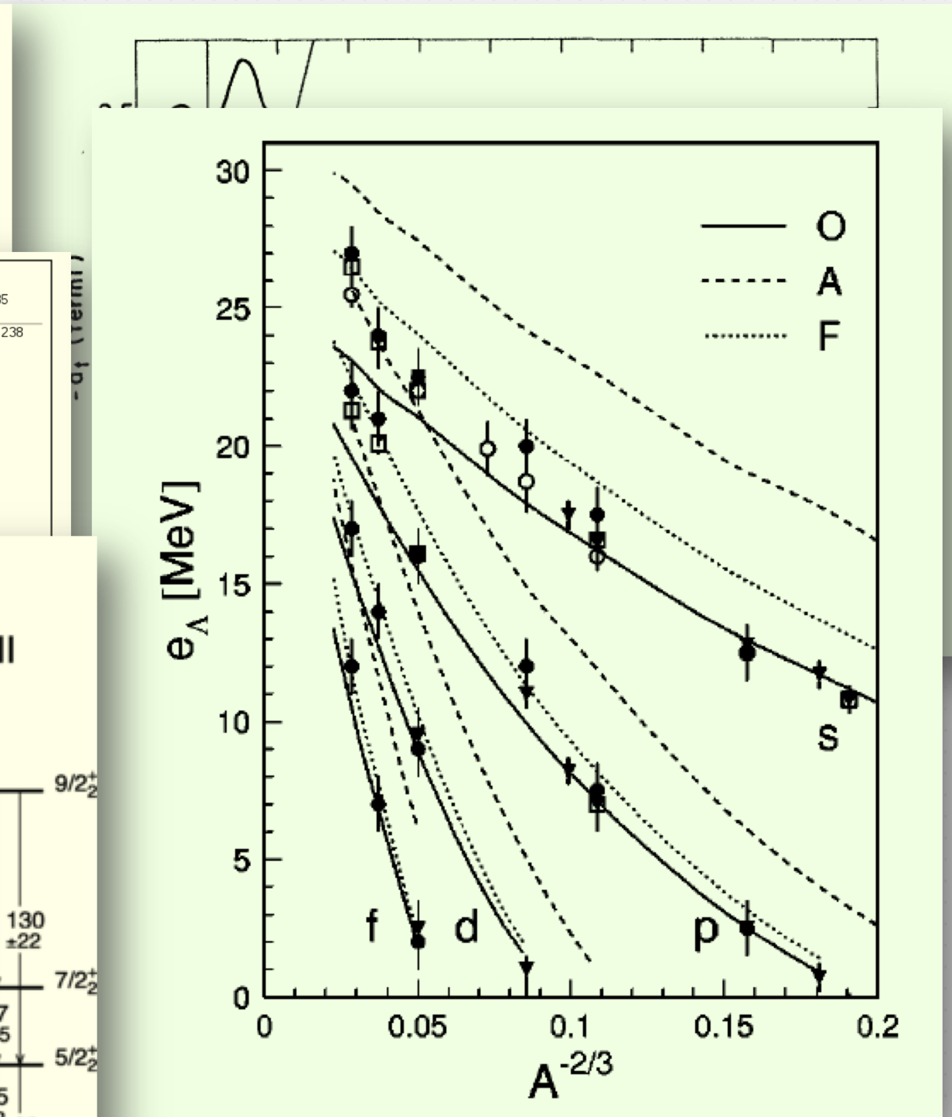
MANY BODY SYSTEMS

Physical Review C. 64. 10.1103/PhysRevC.64.044301.

Regular

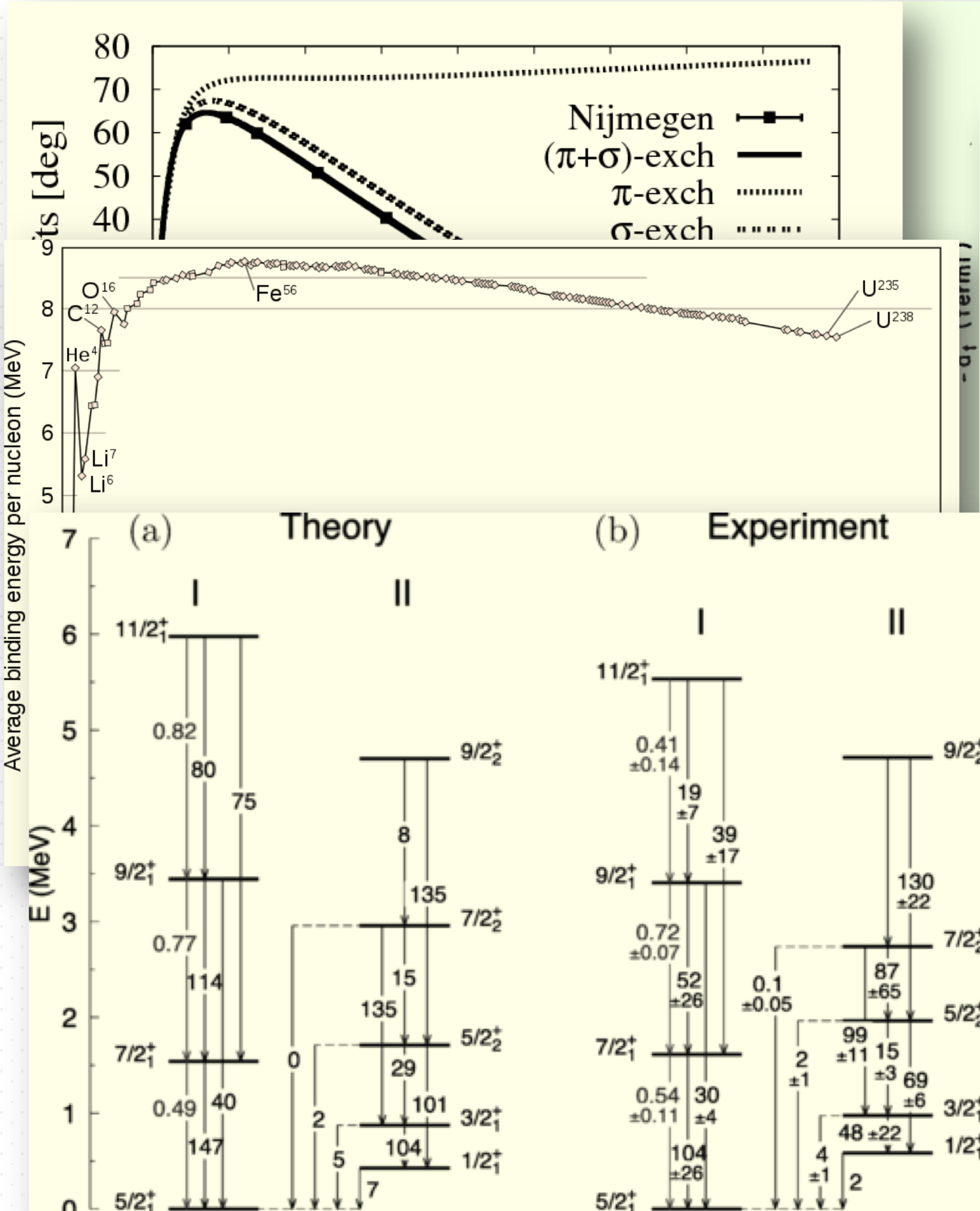
Strange



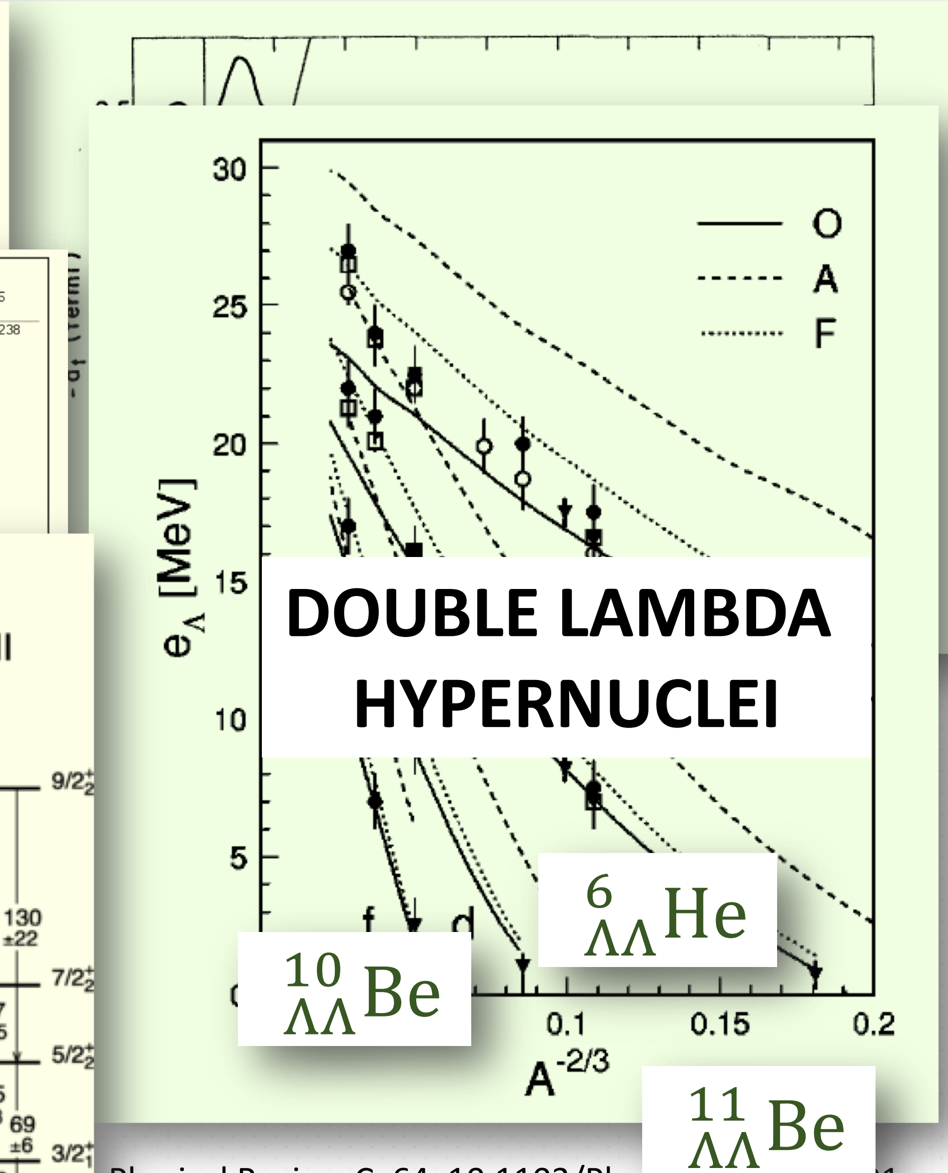


Physical Review C. 64. 10.1103/PhysRevC.64.044301.

Regular



Strange



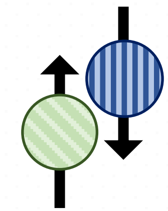
Physical Review C. 64. 10.1103/PhysRevC.64.044301.

Two body

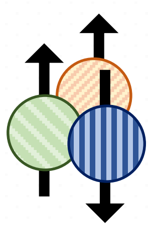
LEC	State	Fitting	
C_{02}	$S=1, I=0$	^{2}H	✓
C_{20}	$S=0, I=1$	$N-N$	✓
C_{01}	$S=1, I=\frac{1}{2}$	$\Lambda-N$	~
C_{21}	$S=0, I=\frac{1}{2}$	$\Lambda-N$	~
C_{00}	$S=0, I=0$	$\Lambda-\Lambda$	~

Boundstate (C_{02})

Scattering (C_{20}, C_{01}, C_{21}, C_{00})



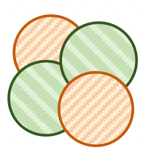
Three body

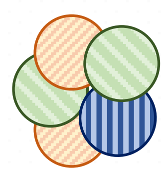


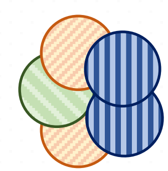
Boundstates

LEC	State		Fitting	
D_{11}	$S=\frac{1}{2}$,	$I=\frac{1}{2}$	^{3}H	✓
D_{01}	$S=\frac{1}{2}$,	$I=0$	$^{3}_{\Lambda}H$	✓
D_{03}	$S=\frac{1}{2}$,	$I=1$	$^{4}_{\Lambda}H_{S=0, I=\frac{1}{2}}$	✓
D_{21}	$S=\frac{3}{2}$,	$I=0$	$^{4}_{\Lambda}H_{S=1, I=\frac{1}{2}}$	✓
$D_{11}^{\Lambda\Lambda N}$	$S=\frac{1}{2}$,	$I=\frac{1}{2}$	$^{6}_{\Lambda\Lambda}He$	✓

Predictions

$B(^{4}He)$ 

$B(^{5}_{\Lambda}He)$ 

$B(^{5}_{\Lambda\Lambda}He)$ 

$$V_{2b}^{\lambda} = \sum_{ij} e^{-\left(\frac{r_{ij}\,\lambda}{2}\right)^2} \left[\, C_{10}^{\lambda}\; P_{[S=1,I=0]}^{NN} + C_{01}^{\lambda}\; P_{[S=0,I=1]}^{NN} \right]$$

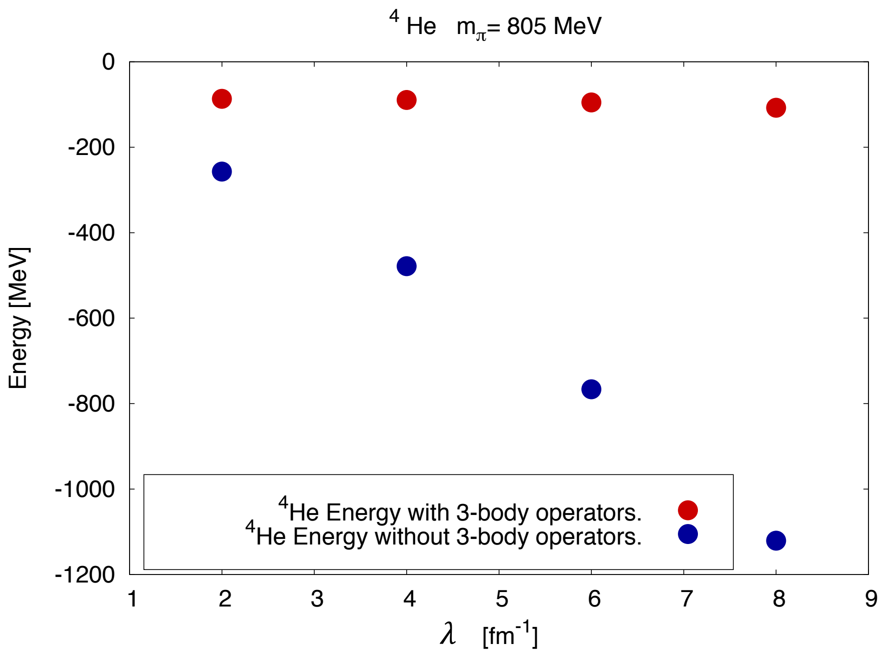
Three body force

The energy of the $A \geq 3$ depends on λ

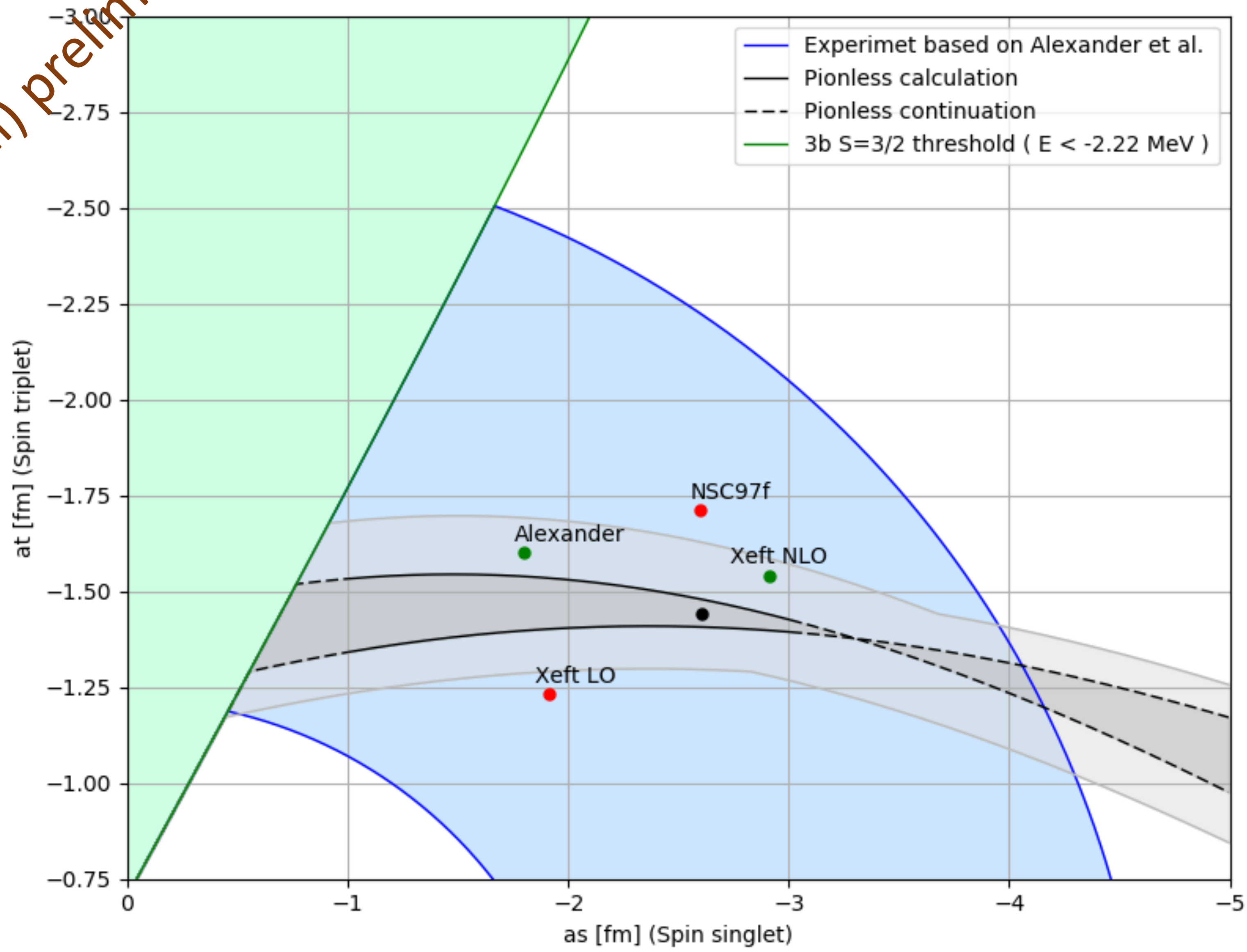
When $\lambda \rightarrow \infty \longrightarrow \mathrm{E}_{3\mathrm{b}} \rightarrow -\infty$
NO PREDICTIVE POWER!

Three-body force stabilize
$A \geq 3$ systems

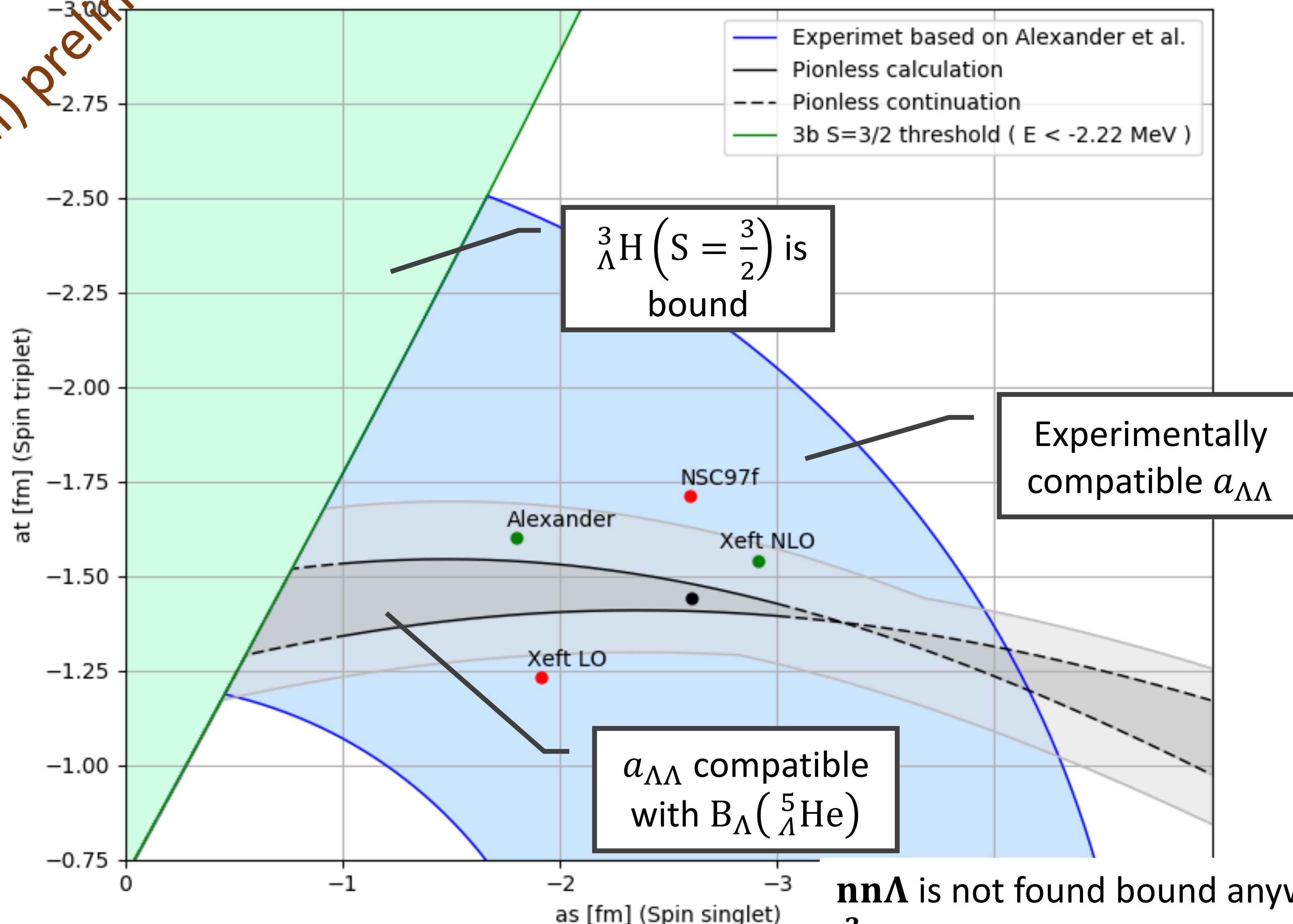
The three body force require
Regularization/Renormalization as well.



(Still) preliminary



(Still) preliminary



nnΛ is not found bound anywhere

$^{3}_{\Lambda}\mathbf{H}$ is bound for “unusual” a_s and a_t