Generalized Parton Distributions(GPDs) of ρ Meson

Yubing Dong

Institute of High Energy Physics(IHEP)

Chinese Academy of Sciences(CAS), CHINA

Collaborator: (Baodong Sun)

PRD 96 (2017), 036019
Chin.Phys. C 42 (2018), 063104
PRD 99 (2019), 016023

Outline

1, Introduction

2, Spin 1 particle and its basic properties

3, Light-front constituent quark model;
Results (unpolarized and Polarized)
and other observables

4, Summary

1, Introduction

GPDs (generalized parton distributions)

GPDs $H_q(x,\xi,Q^2)$ naturally embody the information of both PDFs and FFs, and, therefore, display the unique properties to present a "3-D" description for a system.

GPDs allow for a unified description of a number of hadronic properties; for example:

(1) In the forward limit they reduce to conventional PDFs

Parton distributions (PDFs)

$$H_q(x,0,0) = q(x)\,,$$

$$\tilde{H}_q(x,0,0) = \Delta q(x)\,.$$

(2) When one integrates GPDs over x they reduce to the usual form factors, e.g. the Dirac form factors[a]

Form Factors(FFs)

$$\sum_q e_q \int dx\, H_q(x,\xi,t) = F_1(t)\,,$$

$$\sum_q e_q \int dx\, E_q(x,\xi,t) = F_2(t)\,.$$

GPDs (generalized parton distributions)

GPDs for pion (S=0),

Broniowski, PLB 574, PRD78; Choi et al., PRD64; Fanelli, EPJC76; ……

for nucleon ***(s=1/2, proton and neutron)***

Diehl et al., EPJC 73; Kroll, EPJA53; Pire et al., PRD79; Selyugin, PRD91;……

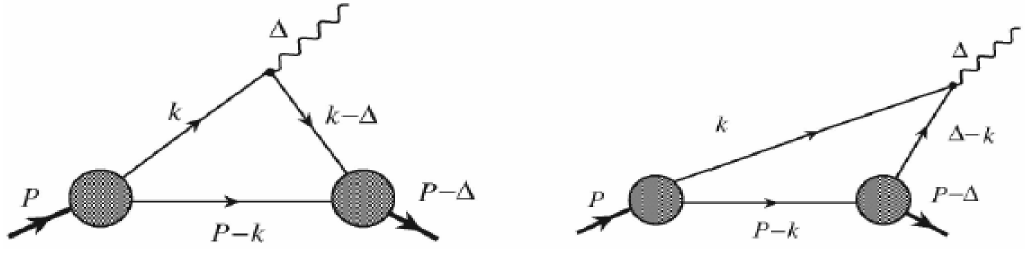
Light Nuclei: He-3

Rinaldi et al., PRC87……

Deuteron (S=1)

Cano et al., PRL87, YBD et al., JPG19,……

Generalized Parton distributions for pion, e. g.



Covariant amplitude with a reduced photon vertex for pion GPD (left diagram) and its nonvalence $x < \zeta$ part (right diagram).

Broniowski, PLB574, In the limit of ξ=0

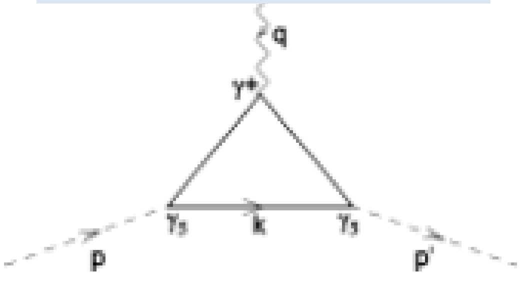
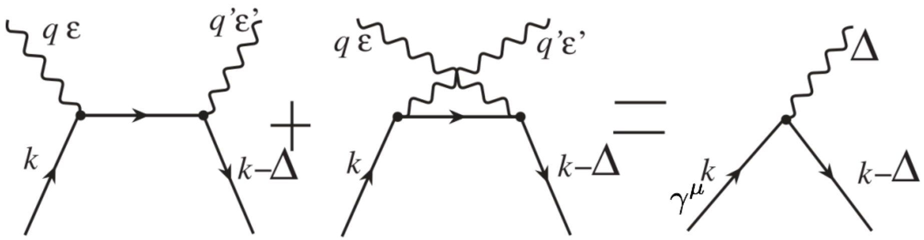


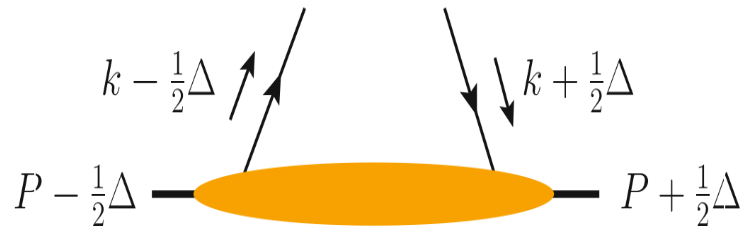
Fig. 1. The diagram for the evaluation of the generalized parton distribution of the pion in chiral quark models.

GPDs (generalized parton distributions)

Deep virtual Compton Scattering

[Chueng-Ryong Ji ’06, Diehl ’16]





A GPD factorization formula:

$$\mathcal{A}(\xi,\Delta^2,Q^2) = \sum_i \int_{-1}^{1} \mathrm{d}x\, C_i\big(x,\xi;\log(Q/\mu)\big) H_i(x,\xi,\Delta^2;\mu)$$

DVCS, TCS, meson productions

Parton correlation function:

$$H(k,P,\Delta) = (2\pi)^{-4}\int d^4z\, e^{izk} \times \langle p(P+\tfrac{1}{2}\Delta)|\bar{q}(-\tfrac{1}{2}z)\Gamma q(\tfrac{1}{2}z)|p(P-\tfrac{1}{2}\Delta)\rangle$$

flavor by flavor

Gauge $A^+=0$

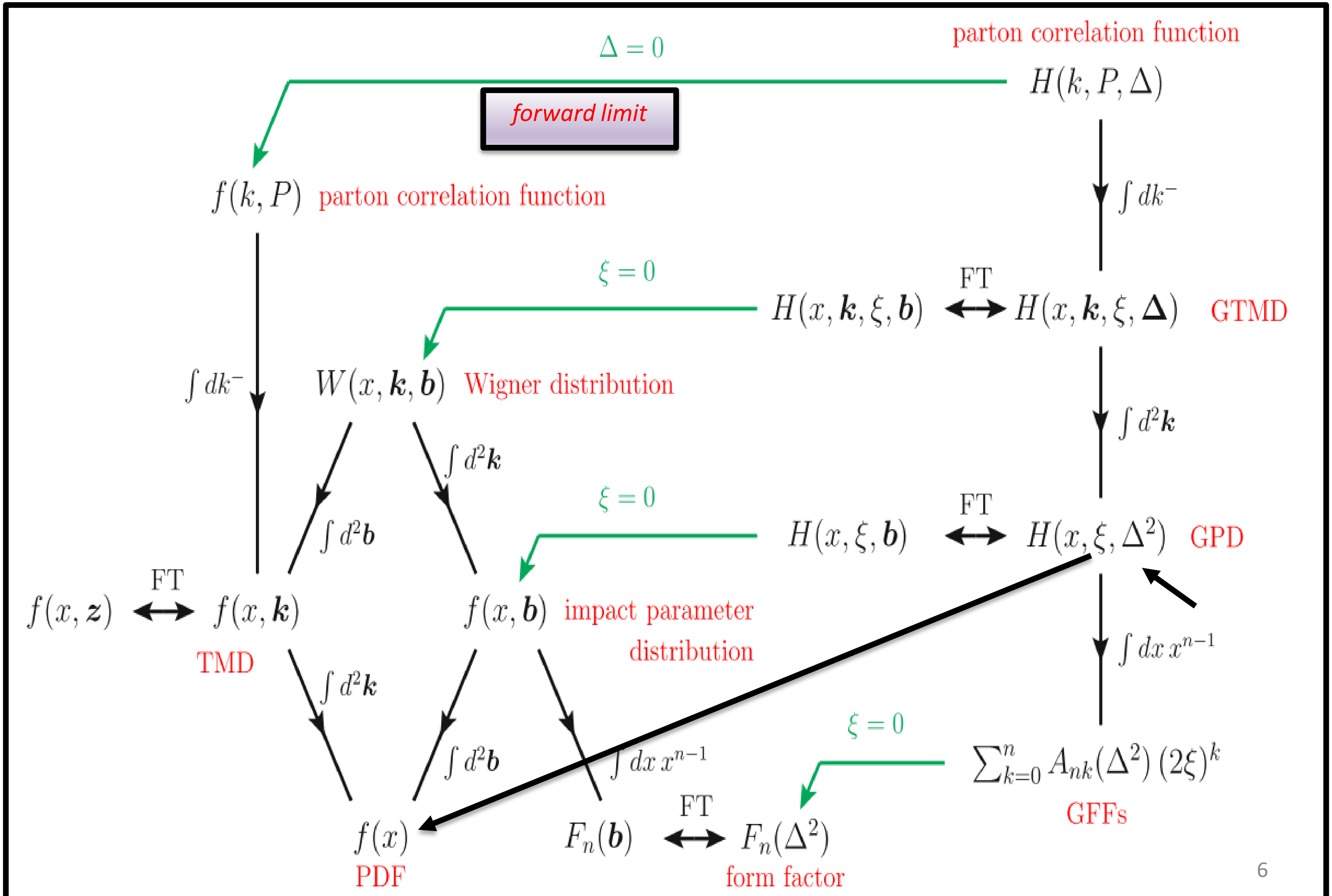
It may be measured by
Deeply virtual Compton scattering
OR
Deeply virtual meson electro-productions

The Dirac matrix Γ selects the twist and the parton spin degrees of freedom.

$$\Gamma^\mu \to \gamma^\mu$$

Schemes

[Diehl '16]



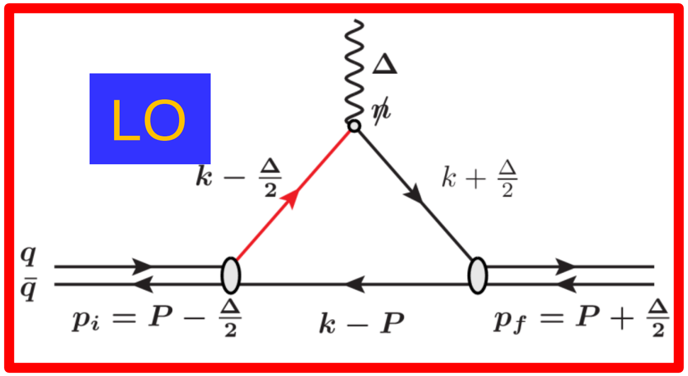
2, Spin-1 particle and basic properties

Definitions of GPDs (spin -1)

- ***Unpolarized***

[PRL: Berger '01 , for the deuteron]

$$V_{\lambda'\lambda} = \frac{1}{2}\int \frac{d\omega}{2\pi}\, e^{ix(Pz)} \langle p',\lambda'|\, \bar{q}(-\tfrac{1}{2}z)\, \not{n}\, q(\tfrac{1}{2}z)\ |p,\lambda\rangle\Big|_{z=\omega n}$$
$$= \sum_i \epsilon'^{*\nu} V^{(i)}_{\nu\mu} \epsilon^{\mu} H^q_i(x,\xi,t)$$



- ***Polarized***

$$A_{\lambda'\lambda} = \frac{1}{2}\int \frac{d\omega}{2\pi}\, e^{ix(Pz)} \langle p',\lambda'|\, \bar{q}(-\tfrac{1}{2}z)\, \not{n}\gamma_5\, q(\tfrac{1}{2}z)\ |p,\lambda\rangle\Big|_{z=\omega n}$$
$$= \sum_i \epsilon'^{*\nu} A^{(i)}_{\nu\mu} \epsilon^{\mu} \tilde{H}^q_i(x,\xi,t)$$

$V_{\mu\nu}$:

$\{g_{\mu\nu}, P_\mu n_\nu, P_\nu n_\mu, P_\mu P_\nu, n_\mu n_\nu\}$

$$P = \frac{p'+p}{2}\ , \quad t = \Delta^2 = (p'-p)^2\ ,$$
$$n^2 = 0\ , \text{(lightlike four-vector)}$$
$$\xi = (n\cdot\Delta)/(n\cdot P)\ , \quad \text{skewness parameter}\ ,$$
$$\epsilon = \epsilon(p,\lambda)\ , \epsilon' = \epsilon'(p',\lambda')\ , \text{polarizations}\ ,$$

Symmetry properties:

T

$$H_i(x,\xi,t) = H_i(x,-\xi,t) \quad (I=1,2,3,5)$$
$$H_4(x,\xi,t) = -H_4(x,-\xi,t)$$
$$\tilde{H}_i(x,\xi,t) = \tilde{H}_i(x,-\xi,t) \quad (I=1,2,4)$$
$$\tilde{H}_3(x,\xi,t) = -\tilde{H}_3(x,-\xi,t)$$

G

$$H^d_{\rho^+}(x,\xi,t) = -H^u_{\rho^+}(x,-\xi,t)$$

- ***Form factor decomposition of Local current***

[Hoodbhoy '89, Frederico '97, Berger '01, Broniowski '08 Cosyn'17]

$$I^{\mu}_{\lambda'\lambda} = \langle p', \lambda' |\, \bar{q}(0)\, \gamma^{\mu}\, q(0)\, | p, \lambda \rangle$$
$$= \epsilon'^{*\beta}\epsilon^{\alpha}\Big[-\Big(G_1^q(t) g_{\beta\alpha} + G_3^q(t)\frac{P_\beta P_\alpha}{2M^2}\Big)P^{\mu} + G_2^q(t)\,\Big(g^{\mu}_{\alpha}P_\beta + g^{\mu}_{\beta}P_\alpha\Big)\Big]$$

FFs in flavor

- ***Sum rules of GPDs***

$$\int_{-1}^{1} dx H_i^q(x,\xi,t) = G_i^q(t) \quad (i=1,2,3)\ ,$$
$$\int_{-1}^{1} dx H_i^q(x,\xi,t) = 0 \quad (i=4,5)\ .$$

- ***Conventional Form factors***

$$G_C(t) = G_1(t) + \frac{2}{3}\eta G_Q(t)\ ,$$
$$G_M(t) = G_2(t)\ ,$$
$$G_Q(t) = G_1(t) - G_2(t) + (1+\eta)G_3(t)\ ,$$

Forward limit

- **GPDs in forward limit**

$$H_1(x,0,0) = \frac{q^1(x) + q^{-1}(x) + q^0(x)}{3}\ ,$$
$$H_5(x,0,0) = q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2}\ ,$$

for x > 0. $\tilde{H}_1(x,0,0) = q_\uparrow^1(x) - q_\uparrow^{-1}(x)$

- DIS structure functions

$$F_1(x) = \frac{1}{2}\sum_q e_q^2 \frac{q^1(x) + q^{-1}(x) + q^0(x)}{3} + \{q \to \bar{q}\},$$
$$b_1(x) = \frac{1}{2}\sum_q e_q^2\Big[q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2}\Big] + \{q \to \bar{q}\}$$
$$g_1(x) = \frac{1}{2}\sum_q e_q^2[q_\uparrow^1(x) - q_\uparrow^{-1}(x)] + \{q \to \bar{q}\}.$$

- Single-flavor $F_1^{q\uparrow(\downarrow)}$, $b_1^{q\uparrow(\downarrow)}$

Quark densities:

$$q^{\lambda}(x) = q_\uparrow^{\lambda}(x) + q_\downarrow^{\lambda}(x)$$
$$q_\uparrow^{\lambda} = q_\downarrow^{-\lambda}$$

-H1 and -H5 for x < 0, antiquark

$$F_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2}H_1^u(x,0,0)$$
$$b_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2}H_5^u(x,0,0)$$

• EMT (Energy--Momentum tensor): Gravitational Form Factors

Polyakov & Schweitzer, 2018

J=0: $\langle p'|\hat{T}^a_{\mu\nu}(x)|p\rangle = \Big[2P_\mu P_\nu\, A^a(t) + \frac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)\, D^a(t) + 2\, m^2\, \bar{c}^a(t)\, g_{\mu\nu}\Big]\, e^{i(p'-p)x}\,,$

J=1/2:

$$\langle p',s'|\hat{T}^a_{\mu\nu}(x)|p,s\rangle = \bar{u}'\Big[A^a(t)\,\frac{P_\mu P_\nu}{m} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t) g_{\mu\nu}\Big] u\, e^{i(p'-p)x}.$$

J=1:

$$\begin{aligned}\langle p',\sigma'|\hat{T}^a_{\mu\nu}(x)|p,\sigma\rangle = \Big[&2P_\mu P_\nu\Big(-\epsilon'^*\cdot\epsilon\, A^a_0(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}\,A^a_1(t)\Big)\\ &+\frac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)\Big(\epsilon'^*\cdot\epsilon\, D^a_0(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}\,D^a_1(t)\Big)\\ &+2\,\big[P_\mu(\epsilon'^*_\nu\,\epsilon\cdot P + \epsilon_\nu\,\epsilon'^*\cdot P) + P_\nu(\epsilon'^*_\mu\,\epsilon\cdot P + \epsilon_\mu\,\epsilon'^*\cdot P)\big]\; J^a(t)\\ &+\Big[\frac{1}{2}(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu)\Delta^2 - (\epsilon'^*_\mu\Delta_\nu + \epsilon'^*_\nu\Delta_\mu)\,\epsilon\cdot P\\ &+(\epsilon_\mu\Delta_\nu + \epsilon_\nu\Delta_\mu)\,\epsilon'^*\cdot P - 4g_{\mu\nu}\,\epsilon'^*\cdot P\,\epsilon\cdot P\Big]\, E^a(t)\\ &+\Big((\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu - \frac{\epsilon'^*\cdot\epsilon}{2}\, g_{\mu\nu}\Big)\, m^2\,\bar{f}^a(t)\\ &+g_{\mu\nu}\Big(\epsilon'^*\cdot\epsilon\, m^2\,\bar{c}^a_0(t) + \epsilon'^*\cdot P\,\epsilon\cdot P\,\bar{c}^a_1(t)\Big)\Big]\, e^{i(p'-p)x}\,,\end{aligned}$$

Cosyn et al., EPJC19
Polyakov, Sun, PRD19

Sum rules of EMT and GPDs:

$$\int_{-1}^{1} x dx H_1^q(x,\xi,t) = A_0^q(t) - \xi^2 D_0^q(t) + \frac{t}{6m^2} E^q(t) + \frac{1}{3}\bar{f}^q(t)$$

$$\int_{-1}^{1} x dx H_2^q(x,\xi,t) = 2J^q(t) \, ,$$

$$\int_{-1}^{1} x dx H_3^q(x,\xi,t) = -\frac{1}{2}\left[A_1^q(t) + \xi^2 D_1^q(t)\right] \, ,$$

$$\int_{-1}^{1} x dx H_4^q(x,\xi,t) = -2\xi E^q(t) \, ,$$

$$\int_{-1}^{1} x dx H_5^q(x,\xi,t) = \frac{t}{2m^2} E^q(t) + \bar{f}^q(t) \, .$$

Mass distribution:

$$m = \int d^3\mathbf{r}\, T^{00}(\vec{r}) = m A_0(0)$$

$$A_0(0) = \sum_a A_0^a(t) = 1$$

Energy density:

$$T^{00}(\vec{r},\sigma',\sigma) = \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} < p',\sigma' \left| \hat{T}^{00}(0) \right| p,\sigma >$$

$$= \varepsilon_0(r)\delta_{\sigma'\sigma} + \varepsilon_2(r) Y_2^{ij} \hat{Q}^{ij}$$

$$Y_2^{ij} = \frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}$$

Gravitational radius

$$\langle r^2 \rangle_{\rm grav} = \frac{\int d^3r\, r^2\, T^{00}(\vec{r})}{\int d^3r\, T^{00}(\vec{r})} = \frac{1}{m}\int d^3\mathbf{r}\; r^2\; T^{00}(\vec{r}) = -6 \frac{dA_0(t)}{dt}\bigg|_{t\to 0}$$

Share--force, pressure

$$T^{ij}(\vec{r},\sigma',\sigma) = \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} < p',\sigma' \left| \hat{T}^{ij}(0) \right| p,\sigma >$$

$$= p_0(r)\delta^{ij}\delta_{\sigma'\sigma} + s_0(r) Y_2^{ij}\delta_{\sigma'\sigma}$$

$$+ p_2(r)\hat{Q}^{ij} + 2 s_2(r)[\ldots \hat{Q}^{ip} \ldots] + \ldots$$

D-term

$$D \equiv -\frac{2}{5} m \int d^3 r (r^2 Y_2^{ij}) \sum_{\sigma'\sigma} \frac{\delta_{\sigma'\sigma}}{3} T^{ij}(\vec{r},\sigma',\sigma >$$

$$= -\frac{4}{15} m \int d^3 r r^2 s_0(r)$$

3, Light-front constituent quark model

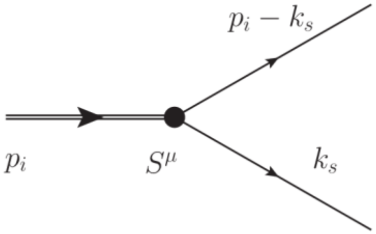
Isospin combinations *[Berger '01, Frederico '09 , Bronioski'03]*

Effective Chiral Lagrangian:

$$\mathcal{L}_{\rho\to q\bar{q}} = -\imath(M/f_\rho)\bar{q}S^\mu\tau q\cdot\rho_\mu = -\imath(M/f_\rho)\left[\bar{u}S^\mu u\rho^0_\mu + \sqrt{2}\bar{u}S^\mu d\rho^+_\mu + \sqrt{2}\bar{d}S^\mu u\rho^-_\mu + \bar{d}S^\mu d\rho^0_\mu\right]$$

Quark field doublets:

$$q(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}, \quad \tau_3 q(x) = \begin{pmatrix} u(x) \\ -d(x) \end{pmatrix}$$



- 5 <u>un-polarized</u> GPDs: **Isospin combinations**

$$\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix(Pz)}\langle\rho^b(p',\lambda')|\,\bar{q}(-\tfrac{1}{2}z)\,\not{n}\,\tau_3 q(\tfrac{1}{2}z)\,|\rho^a(p,\lambda)\rangle\Big|_{z=\lambda n} = \imath\epsilon_{3ab}\Big\{-(\epsilon'^*\cdot\epsilon)\,H^{I=1}_{1,\rho^b}$$

$$+\frac{(\epsilon\cdot n)(\epsilon'^*\cdot P)+(\epsilon'^*\cdot n)(\epsilon\cdot P)}{P\cdot n}H^{I=1}_{2,\rho^b} - \frac{2(\epsilon\cdot P)(\epsilon'^*\cdot P)}{m^2}H^{I=1}_{3,\rho^b}$$

$$+\frac{(\epsilon\cdot n)(\epsilon'^*\cdot P)-(\epsilon'^*\cdot n)(\epsilon\cdot P)}{P\cdot n}H^{I=1}_{4,\rho^b} + \left[m^2\frac{(\epsilon\cdot n)(\epsilon'^*\cdot n)}{(P\cdot n)^2}+\frac{1}{3}(\epsilon'^*\cdot\epsilon)\right]H^{I=1}_{5,\rho^b}\Big\}$$

$H_1 \to F_1$
$H_5 \to b_1$
$\widetilde{H}_1 \to g_1$

Isospin combinations:

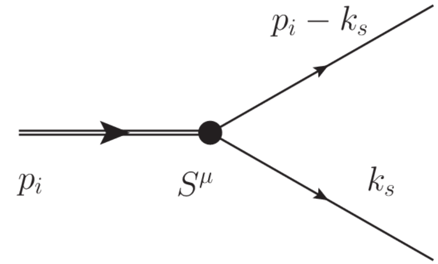
$$H^{I=1}_{i,\rho^\pm}(x,\xi,t) = \frac{1}{2}[H^u_{i,\rho^\pm}(x,\xi,t) - H^d_{i,\rho^\pm}(x,\xi,t)]$$

G parity:

$$H^d_{\rho^+}(x,\xi,t) = -H^u_{\rho^+}(x,-\xi,t)$$

Phenomenological vertex ρ meson

[Choi '04, Frederico '09]



Phenomenal vertex: $S^\mu = \Gamma^\mu \Lambda(k_s, p)$

Bethe-Salpeter amplitude(BSA):

$$\Lambda(k_s,p) = \frac{c}{[k_s^2 - m_R^2 + \imath\epsilon][(p-k_s)^2 - m_R^2 + \imath\epsilon]}$$

S-wave **Meson vertex:**

$$\Gamma^\mu = \gamma^\mu - \frac{(k_q + k_{\bar{q}})^\mu}{M_0 + 2m}$$

Dispersion relation

$$x' = \frac{-k_s^+}{p_i^+}$$

$$\kappa_\perp = k_{s\perp} - \frac{k_s^+}{p_i^+} p_{i\perp}$$

Kinematic invariant mass:

$$M_0^2 = \frac{\kappa_\perp^2 + m^2}{1-x'} + \frac{\kappa_\perp^2 + m^2}{x'}$$

[Choi '04, Frederico '09, Miller'09]

Residuals considerations

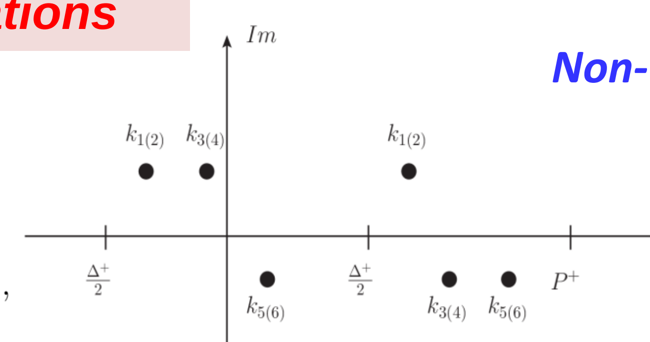
[Prescription： Chueng-Ryong Ji . et al,'06]

Poles (Valence)

$$k^-_{1(2)} = P^- + (k-P)^-_{on(R)} - i\frac{\epsilon}{k^+ - P^+},$$

$$k^-_{3(4)} = \frac{\Delta^-}{2} + (k - \frac{\Delta}{2})^-_{on(R)} - i\frac{\epsilon}{k^+ - \frac{\Delta^+}{2}},$$

$$k^-_{5(6)} = -\frac{\Delta^-}{2} + (k + \frac{\Delta}{2})^-_{on(R)} - i\frac{\epsilon}{k^+ + \frac{\Delta^+}{2}}.$$



Nonvalance
(|x|<|ξ|,ERBL)

Valance
(|x|>|ξ|,DGLAP)

Non-valance kinematic invariant mass

$$M_{0i(v)}^2 = \frac{\kappa_\perp^2 + m^2}{1-x'} + \frac{\kappa_\perp^2 + m^2}{x'}$$

$$\rightarrow \frac{\kappa_\perp^2 + m^2}{x'-1} + \frac{\kappa_\perp^2 + m^2}{x'} = M_{0i(nv)}^2$$

$$x = \frac{n \cdot k}{n \cdot P} = \frac{k^+}{P^+}; \qquad x' = \frac{1-x}{1-|\xi|}$$

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP)
Efremov-Radyushkin-Brodsky-Lepage (ERBL)

Results: Form factors $G_{C,M,Q}$

$$G_C(t) = G_1(t) + \frac{2}{3}\eta G_Q(t) \ ,$$

$$G_M(t) = G_2(t) \ ,$$

$$G_Q(t) = G_1(t) - G_2(t) + (1+\eta) G_3(t) \ ,$$

- *low-energy observables*

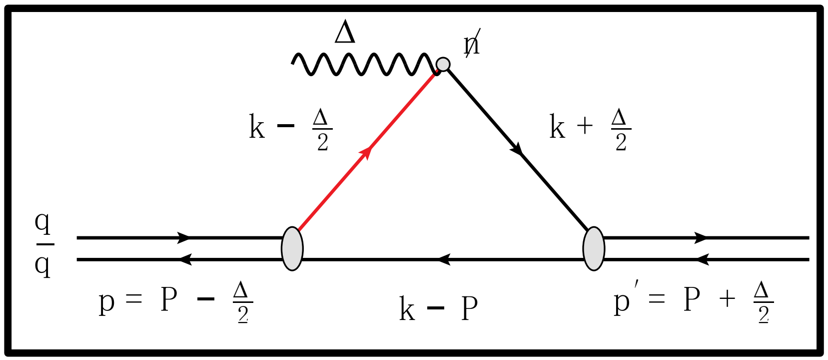
$$G_C(0) = 1 \ ,$$

$$G_M(0) = 2M\mu \ ,$$

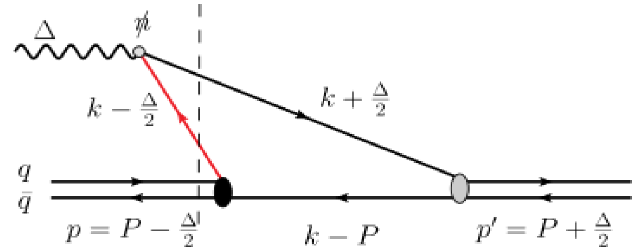
$$G_Q(0) = M^2 Q_\rho \ ,$$

$$< r^2 > = \lim_{t\to 0} \frac{6\,[G_C(t) - 1]}{t} \ .$$

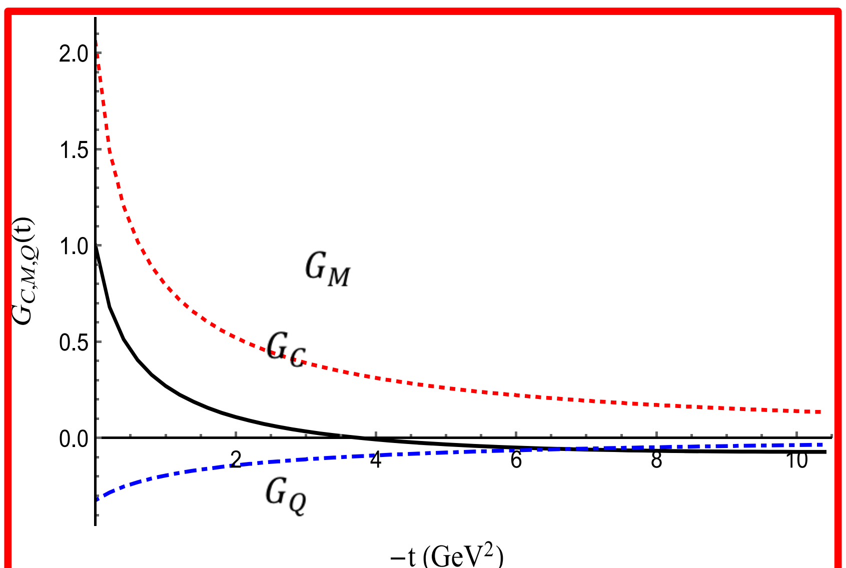
Valance



***Non-valance* pair production**



The struck u quark in the nonvalence regime, yielded by the off-diagonal terms in the Fock space. The black blob represents the non-wave-function vertex. The red line has the negative sign in this regime.



[Melo '97, Gudino '14]

	This work	Melo1 1997	Exp. [Gudino2014]
$\langle r^2 \rangle$(fm^2)	0.52	0.37	--
μ	2.06	2.19	2.1(5)
Q_2(fm^2)	0.021	0.050	--

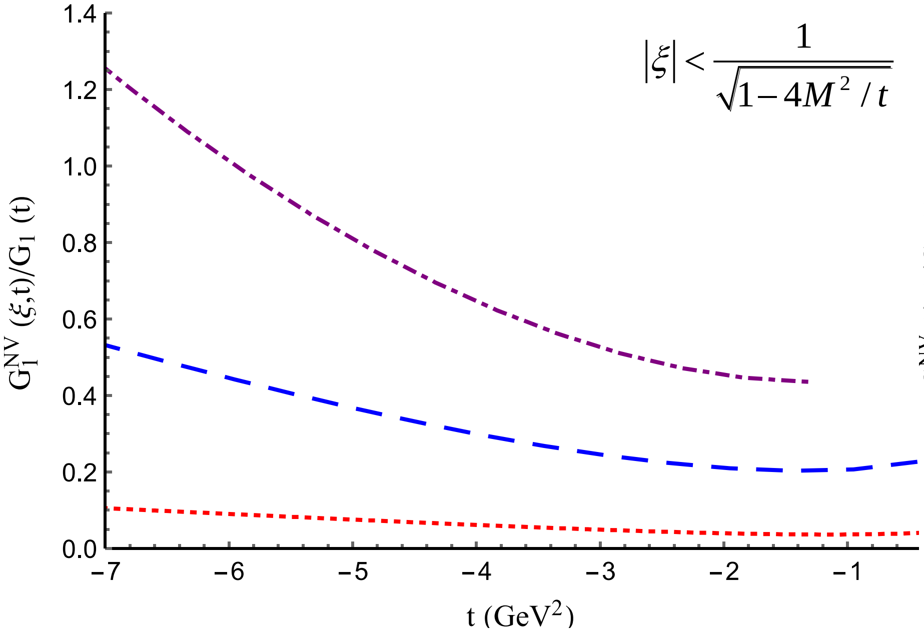
m (constituent mass)	mR (regulator mass)
0.403GeV	1.61GeV

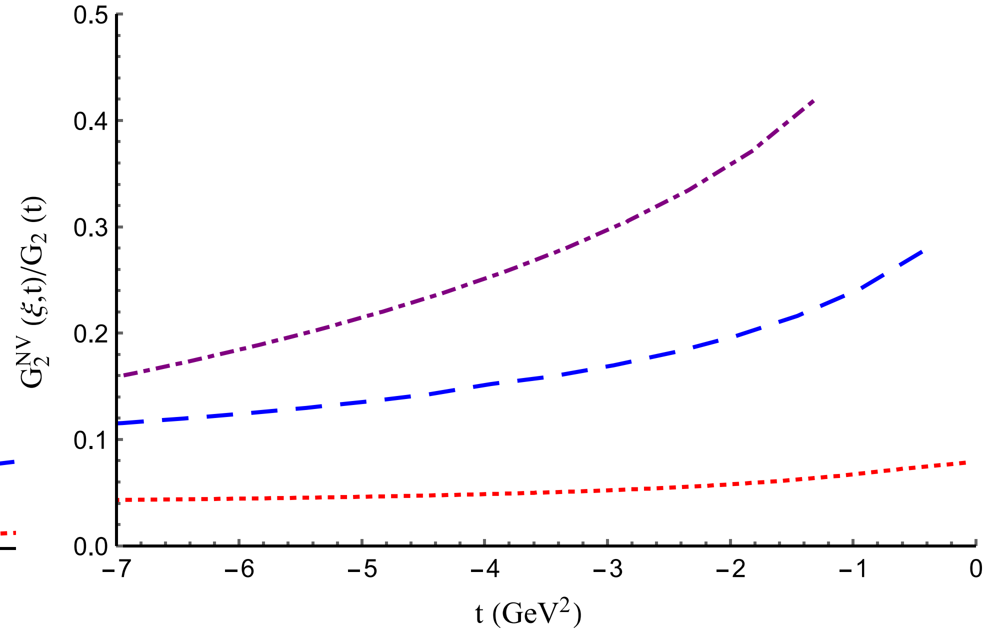
TABLE I. The comparison of the results for the magnetic moment μ_ρ (in natural magnetons $e/2M_\rho$) in different approaches.

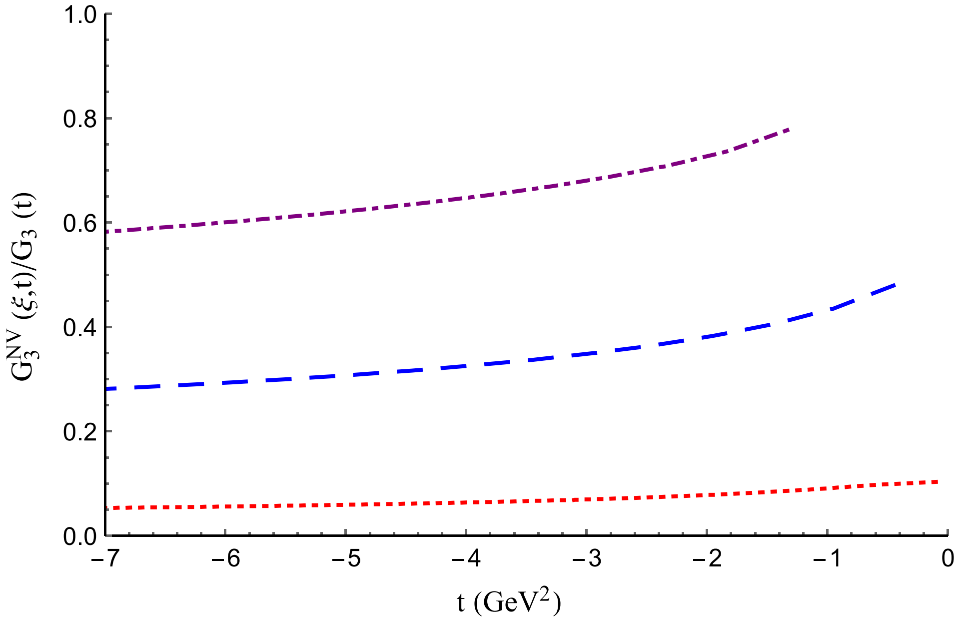
Model	μ_ρ
This work, mIF RHD	2.16 ± 0.03
Cardarelly, LF RHD [1]	2.26
Melo, LF RHD [2]	2.14
Bakker, LF RHD [3]	2.1
Jaus, LF RHD [4]	1.83
Choi, LF RHD [5]	1.92
He, LF, IF RHD [6]	1.5
He, PF RHD [6]	0.9
Biernat, PF RHD [7]	2.20
Sun, LF CQM [8]	2.06
Hawes, Dyson-Schwinger equation (DSE) [9]	2.69
Ivanov, DSE [10]	2.44
Bhagwat, DSE [11]	2.01
Roberts, DSE [12]	2.11
Pitschmann, DSE [13]	2.11
Carrillo-Serrano, Nambu–Jona-Lasinio model (NJL) [14]	2.59
Luan, NJL [15]	2.1
Samsonov, QCD sum rules [16]	2.0 ± 0.3
Aliev, QCD sum rules [17]	2.4 ± 0.4
Melikhov, LF triangle [18]	2.35
Šimonis, bag model [19]	2.06
Bagdasaryan, relativistic CQM [20]	2.3
Badalian, relativistic Hamiltonian [21]	1.96
Djukanovic, effective field theory [22]	2.24
Andersen, lattice [23]	2.25 ± 0.34
Hedditch, lattice [24]	2.02
Lee, lattice [25]	2.39 ± 0.01
Owen, lattice [26]	2.21 ± 0.08
Lushevskaya, lattice [27]	2.11 ± 0.10
Gudinõ, experiment [28]	2.1 ± 0.5

[Krutov, Polezhaev, and Troitsky, PRD97, 033007]

Form Factors $G_{1,2,3}$, Non-valence contributions







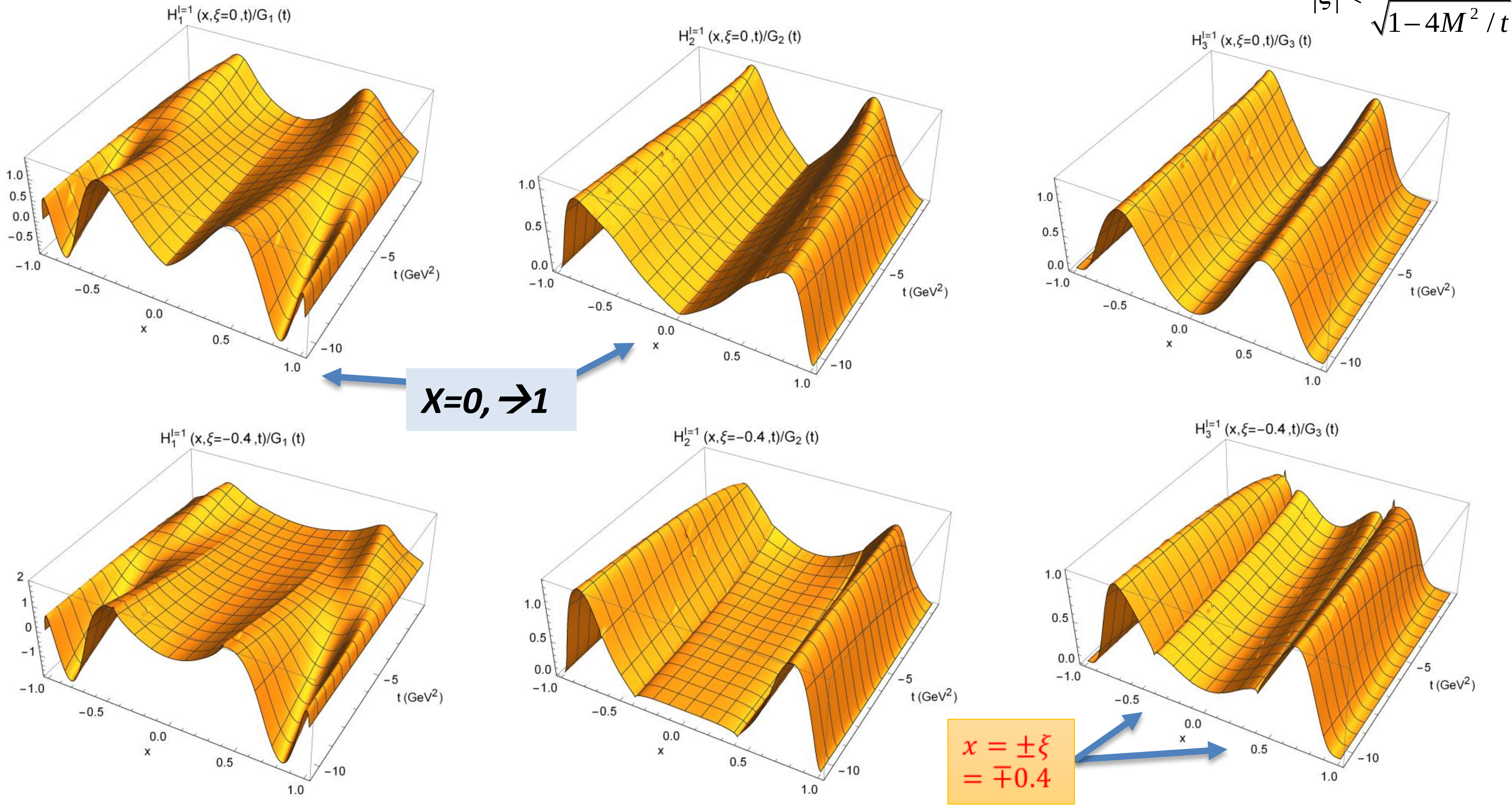
The non-valence contributions to FFs $G_{1,2,3}$ at ξ = -0.2 (dotted), -0.4 (dashed), and -0.6 (dotted-dashed).

$$\int_{-1}^{1} dx H_i^q(x,\xi,t) = G_i^q(t) \quad (i=1,2,3) \ ,$$

$$\int_{-1}^{1} dx H_i^q(x,\xi,t) = 0 \quad (i=4,5) \ .$$

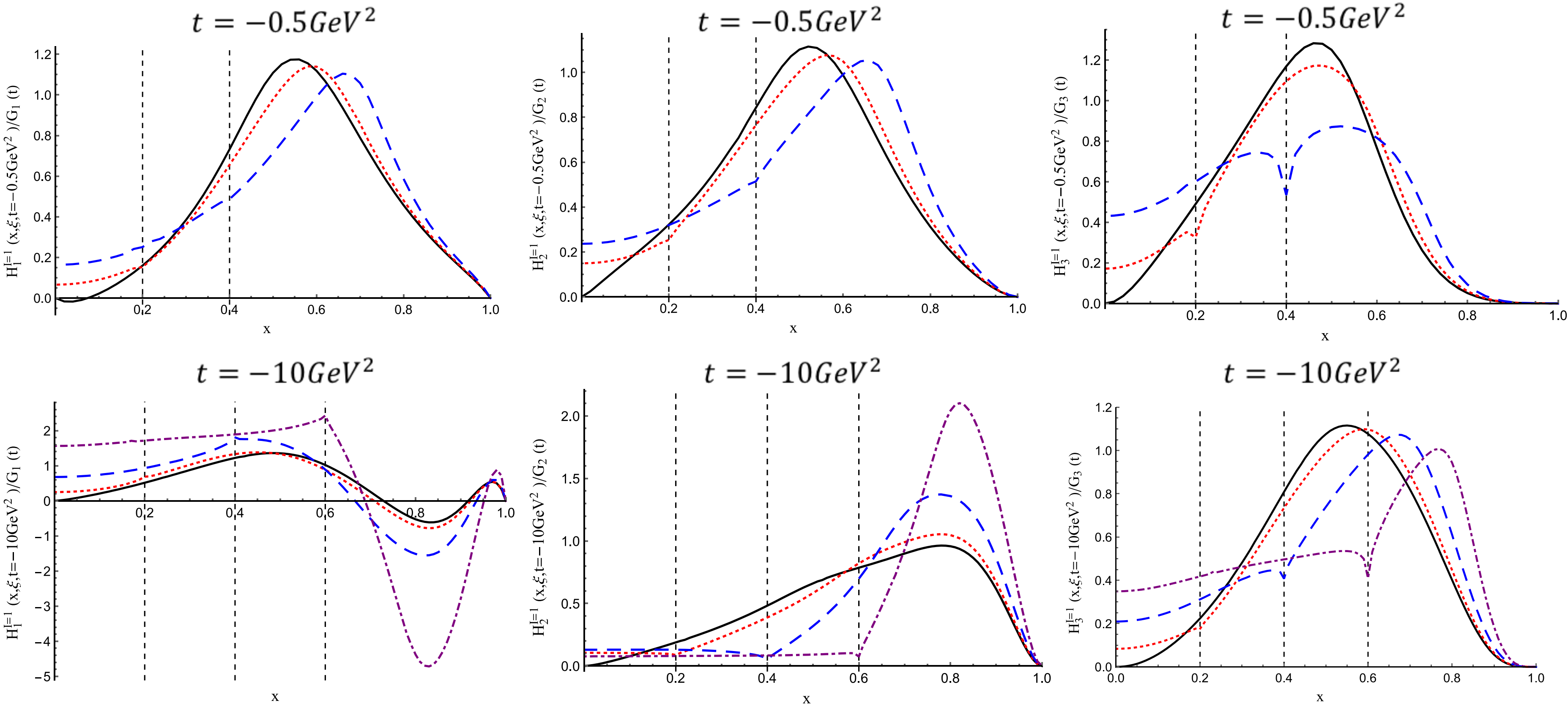
Results: Unpolarized GPDs $H_{1,2,3}$ (x,ξ_0,t)

$$|\xi| < \frac{1}{\sqrt{1-4M^2/t}}$$



Results: Un-polarized GPDs $H_{1,2,3}(x,\xi_0,t)$

$$|\xi| < \frac{1}{\sqrt{1-4M^2/t}}$$



$\xi = 0$ (solid black line), -0.2 (dotted red line), -0.4 (dashed blue line), -0.6 (dot-dashed purple line)

Polarized GPDs of ρ-meson

$$A_{\lambda'\lambda} = \frac{1}{2}\int \frac{d\omega}{2\pi}\, e^{ix(Pz)} \langle p',\lambda'|\,\bar{q}(-\tfrac{1}{2}z)\,\not{n}\gamma_5\, q(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z=\omega n}$$
$$= \sum_i \epsilon'^{*\nu} A^{(i)}_{\nu\mu} \epsilon^{\mu} \tilde{H}^q_i(x,\xi,t)$$

[PRL: Berger '01 , for the deuteron]

$$\int_{-1}^{1} dx\, \tilde{H}^q_i(x,\xi,t) = \tilde{G}^q_i(t) \qquad (i=1,2),$$

with matrix elements of

$$\langle p'|\,\bar{q}(0)\,\gamma^\mu\gamma_5\, q(0)\,|p\rangle = -2i\,\epsilon^{\mu}{}_{\alpha\beta\gamma}\,\epsilon'^{*\alpha}\epsilon^{\beta}P^{\gamma}\,\tilde{G}^q_1(t)$$
$$+\,4i\,\epsilon^{\mu}{}_{\alpha\beta\gamma}\,\Delta^{\alpha}P^{\beta}\,\frac{\epsilon^{\gamma}(\epsilon'^{*}P)+\epsilon'^{*\gamma}(\epsilon P)}{M^2}\,\tilde{G}^q_2(t).$$

For other two GPDs, time reversal invariance gives

$$\int_{-1}^{1} dx\, \tilde{H}^q_3(x,\xi,t) = 0\ ,$$

and the Lorenz invariance constraints

$$\int_{-1}^{1} dx\, \tilde{H}^q_4(x,\xi,t) = 0\ .$$

$$\Delta q \equiv \int_0^1 \left[g_1^u(x) + g_1^d(x)\right] dx = \int_0^1 \Delta u(x)\, dx$$

$$\Delta q = 0.86$$

Wandzura-Wilcze relation

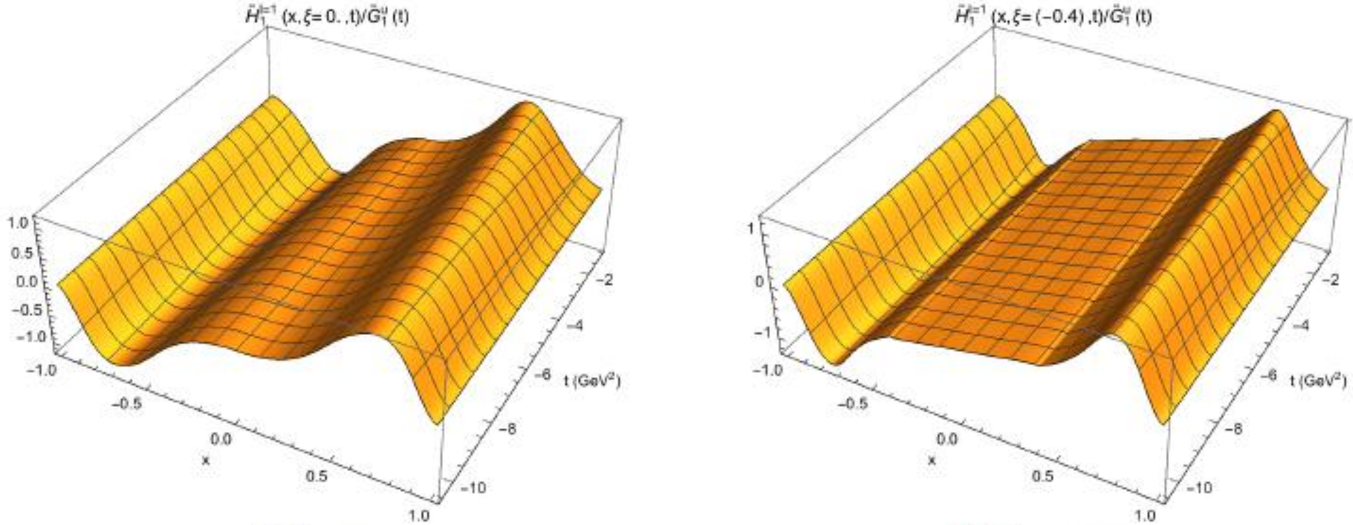
$$g_2^{WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y).$$

$$\int_0^1 g_2(x)\, dx = 0\ .$$

Transverse spin density

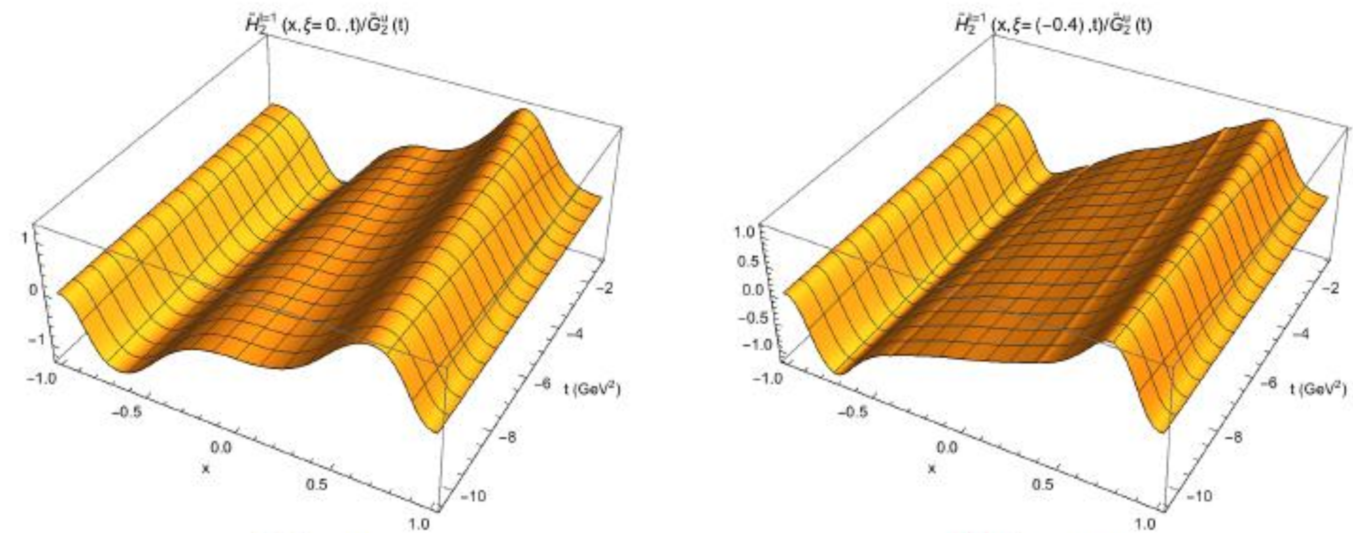
$$g_T(x) = g_1(x) + g_2(x) \sim \int_x^1 \frac{dy}{y} g_1(y).$$

Results: Polarized GPDs $\tilde{H}_{1,2}(x,\xi,t)$



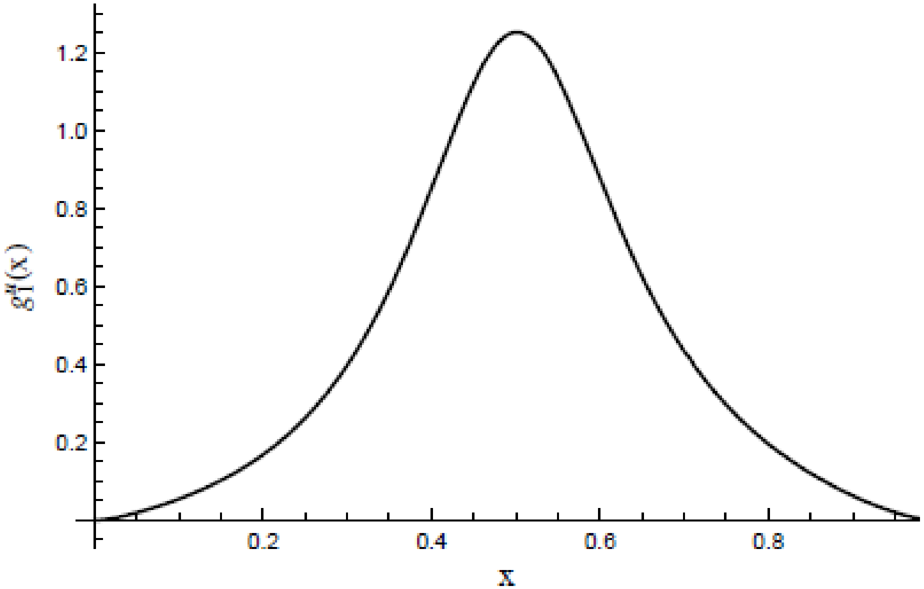
(a) $\xi = 0$ (b) $\xi = -0.4$

Fig. 3. ρ^+ GPD $\tilde{H}_1$ with $\xi = 0$ and -0.4.

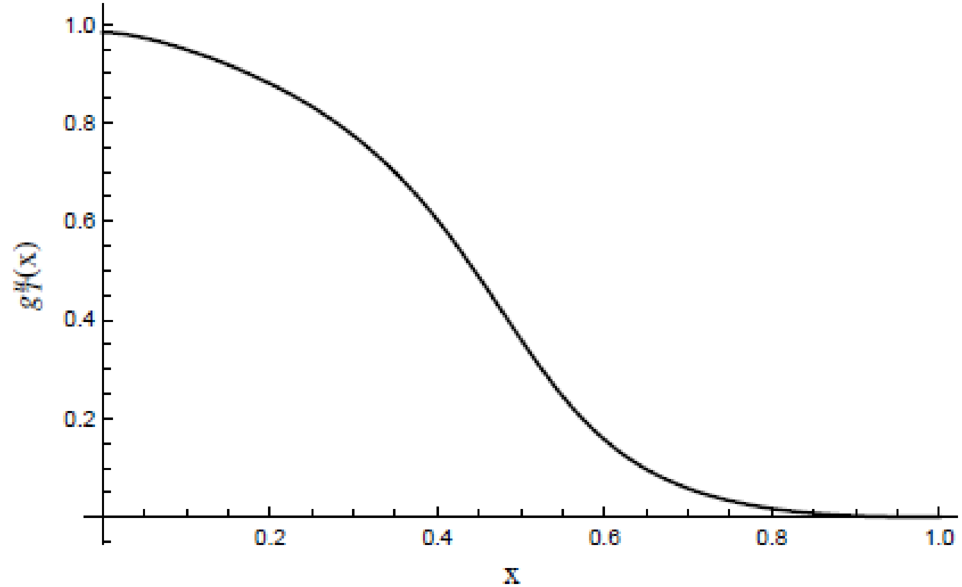


(a) $\xi = 0$ (b) $\xi = -0.4$

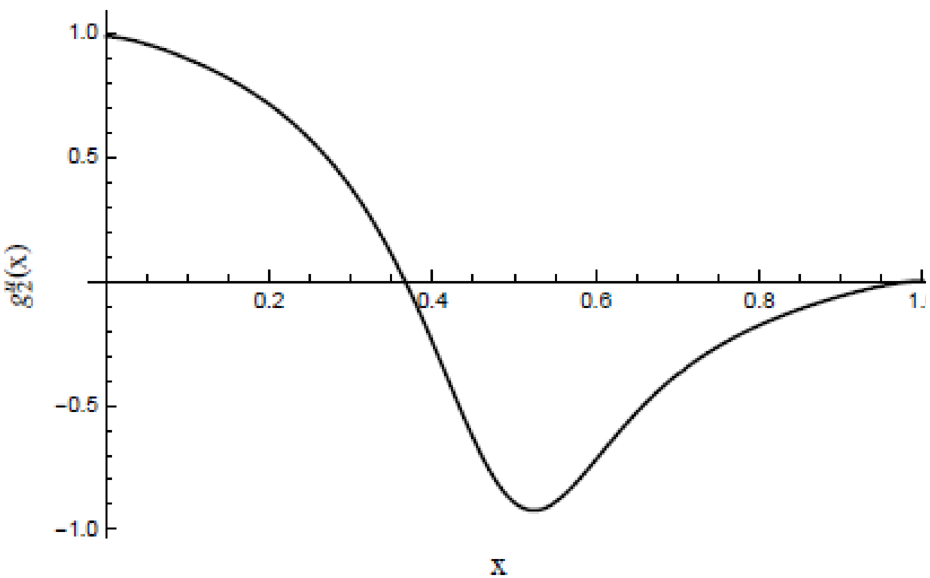
ρ^+ GPD $\tilde{H}_2$ with $\xi = 0$ and -0.4.



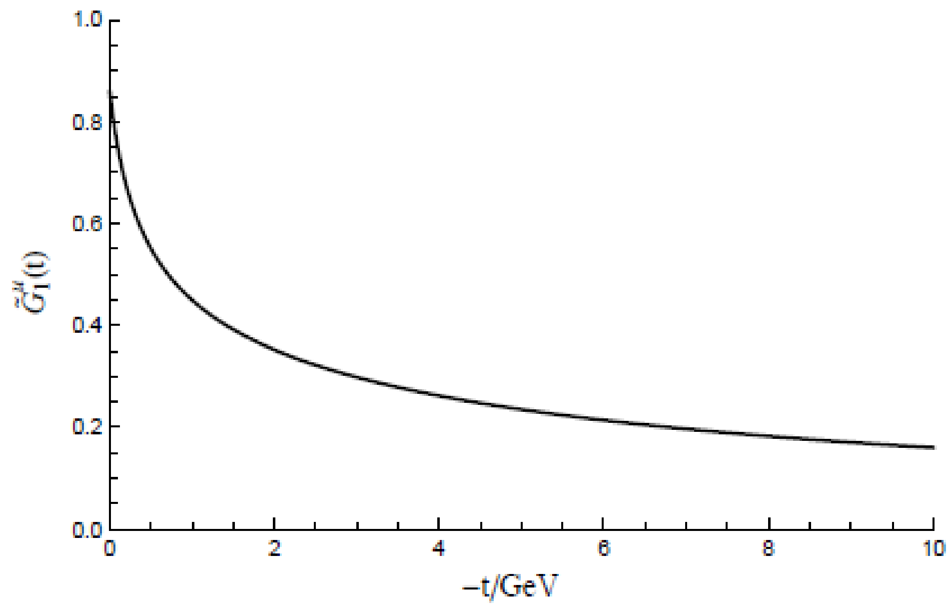
The u quark structure function $g_1^u(x)$



$g_T^u(x)$



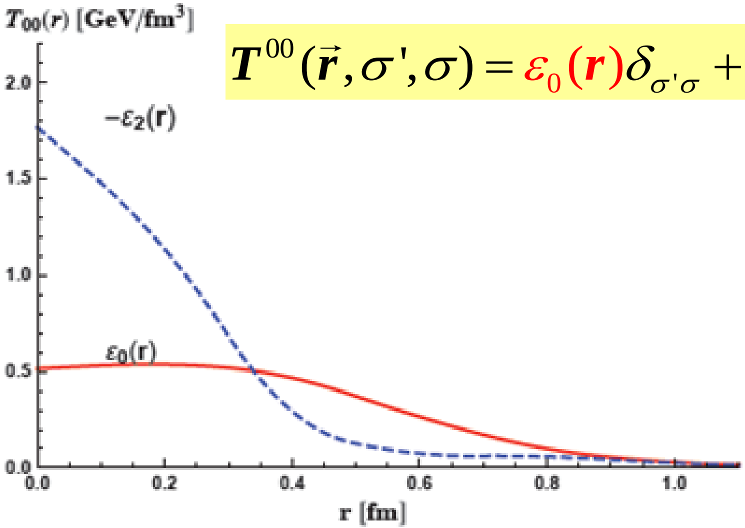
The u quark structure function $g_2^u(x)$

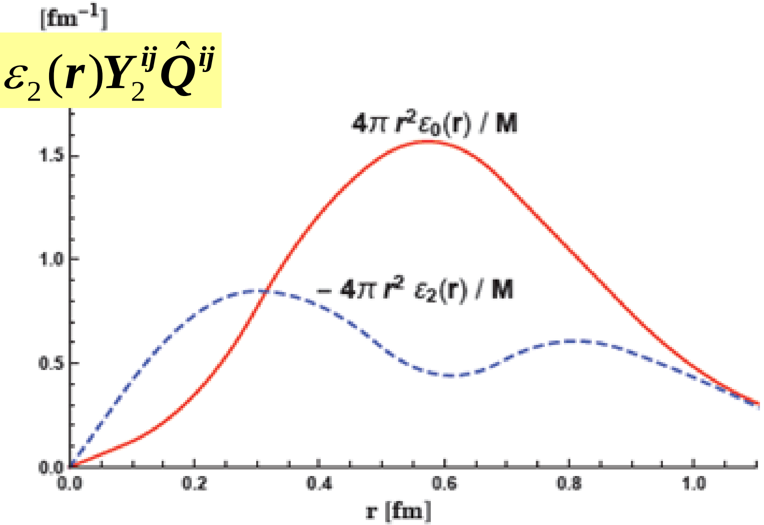


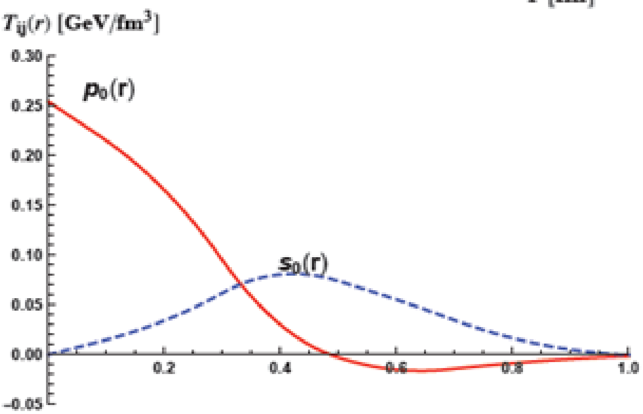
The u quark axial form factor $\tilde{G}_1^u(t)$.

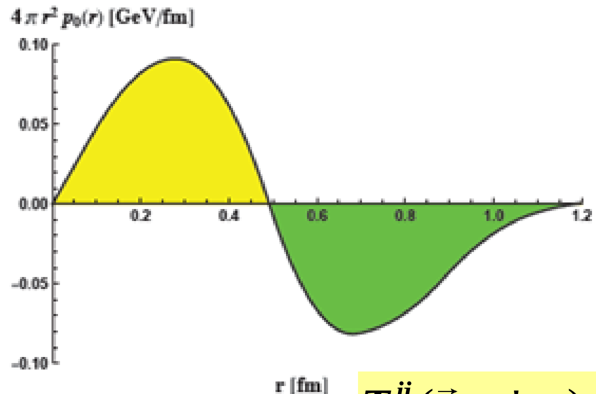
Energy density

$$T^{00}(\vec{r},\sigma',\sigma) = \varepsilon_0(r)\delta_{\sigma'\sigma} + \varepsilon_2(r)Y_2^{ij}\hat{Q}^{ij}$$



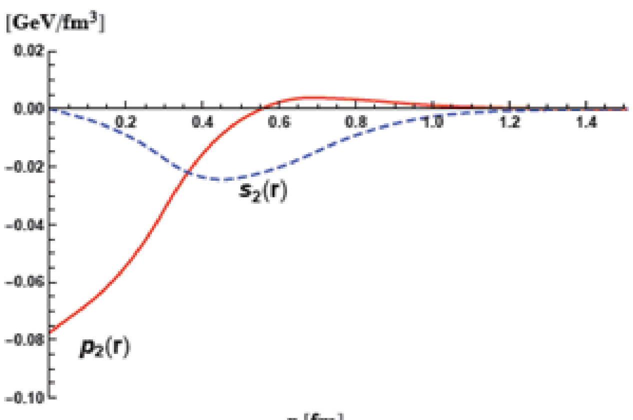


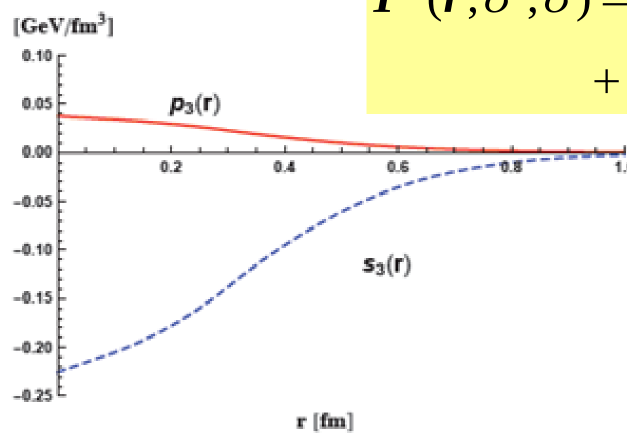




$$D = -0.21 < 0$$

$$T^{ij}(\vec{r},\sigma',\sigma) = p_0(r)\delta^{ij}\delta_{\sigma'\sigma} + s_0(r)Y_2^{ij}\delta_{\sigma'\sigma}$$
$$+ p_2(r)\hat{Q}^{ij} + 2s_2(r)[\ldots\hat{Q}^{ip}\ldots] + \ldots$$



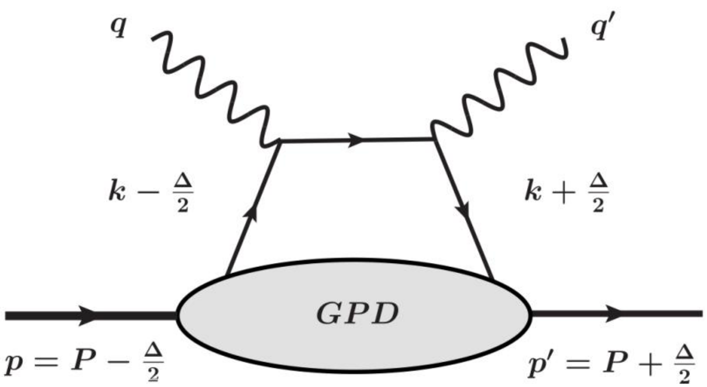


4, Summary

- GPDs for ρ meson (spin-1)
- Phenomenological approach for ρ meson
- ρ meson FFs / GPDs and others
- Results are reasonable comparing to other calculation (models, Lattice)

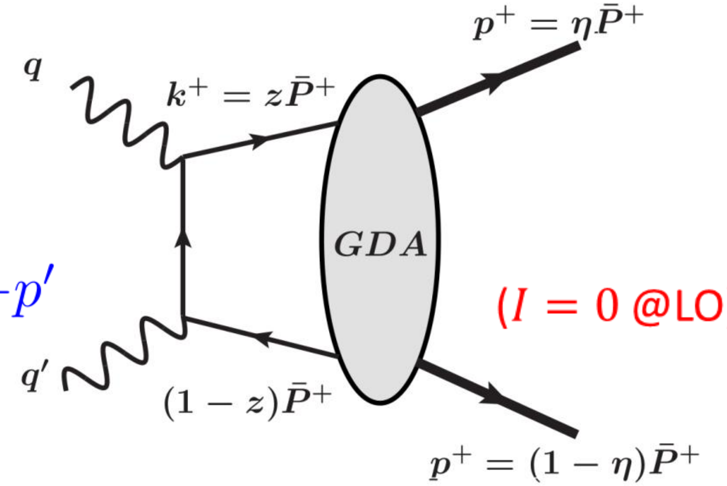
GDA (Generalized Distribution Amplitude)

[PRL: Diehl '98, '03, Kumano '17]



$t \leftrightarrow s$

$p', p \leftrightarrow p, -p'$



($I = 0$ @LO)

[Kawamura '13, Kumano '17, '18 for pion]

Outlook

- GDAs
- Double parton distributions (DPDs)
- In particular for Deuteron

Thanks!

BACKUP

QCD evolution of the structure functions

The moments of the structure functions at different scale

$$\frac{\tilde{V}_n^u(\mu)}{\tilde{V}_n^u(\mu_0)} = \left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\gamma_n^{(0)}/(2\beta_0)},$$

where the single quark spin fractions

$$\tilde{V}_n^u = 2M_{n+1}\left[g_1^u(x)\right] \sim r_{n+1}$$

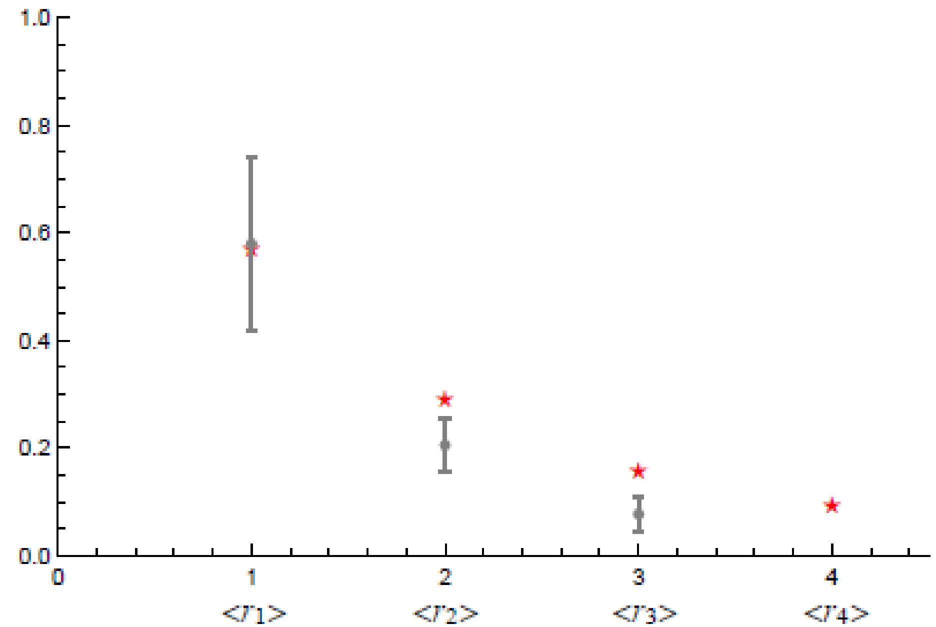
and the running coupling constant is

$$\alpha(\mu) = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda_{QCD}^2)},$$

where $\beta_0 = 11N_c/3 - 2N_f/3$ with $N_c = N_f = 3$ and

$$\Lambda_{QCD} = 0.226\ GeV$$

For the polarized structure function



r_n for u quark. The red stars are our results and the gray ones with errors are the Lattice QCD results [14].

Possible Lattice calculation with quench approximation at μ=2.4 GeV,
Best'97, PRD56, 2743

Lattice QCD Scale Q_0=2.4 GeV

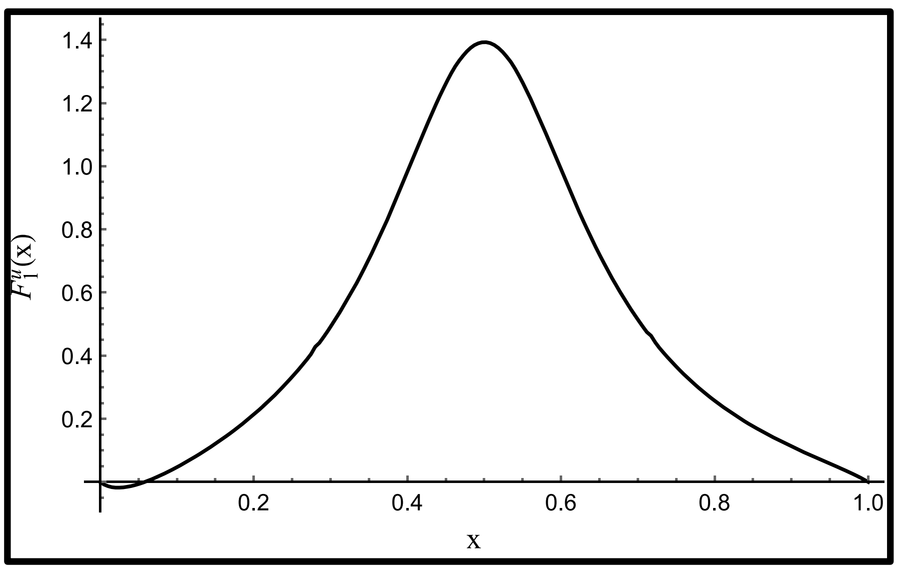
Model Scale $Q_0 = 528^{+77}_{-62}$ MeV

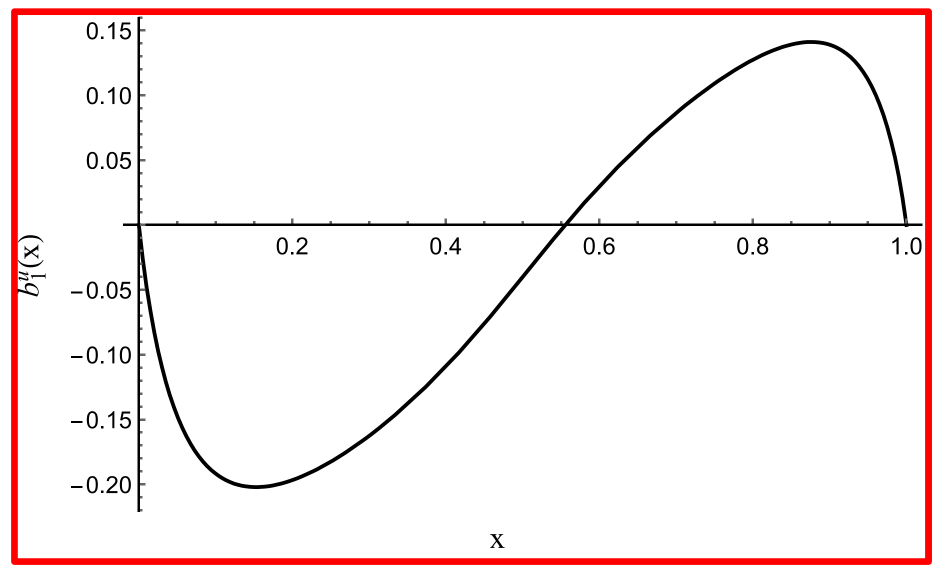
Forward Limit: Single-Flavor F_1^q, b_1^q

$$F_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2} H_1^u(x,0,0)$$

$$b_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2} H_5^u(x,0,0)$$

$$u_{\rho^+}(x) = \bar{d}_{\rho^+}(1-x)$$





Impact Parameter Space

- **Spin ½** *[Burkardt '03, Hoodbhoy '89]*

$$W^{\mu\nu}_{1/2} \sim F_1, F_2, g_1, g_2$$

$$q_N(x,\mathbf{b}) = |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} \times \langle p^+, \mathbf{p}'_\perp, \lambda| \left[\int \frac{dz^-}{4\pi} \bar{q}(-\frac{z^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{z^-}{2}, \mathbf{b}_\perp) e^{-ixp^+z^-} \right] |p^+, \mathbf{p}_\perp, \lambda\rangle$$

$$= |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} H_q(x, \xi = 0, -(\mathbf{p}_\perp - \mathbf{p}'_\perp)^2) e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

$$= \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} H_q(x, 0, -\boldsymbol{\Delta}^2_\perp) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp},$$

Fourier transformation

Density interpretation

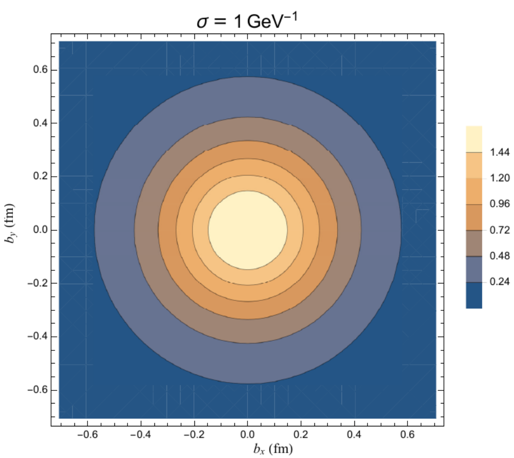
- **Spin 1**

$$q(x,\mathbf{b}) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} H_1(x, 0, -\boldsymbol{\Delta}^2_\perp) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$$

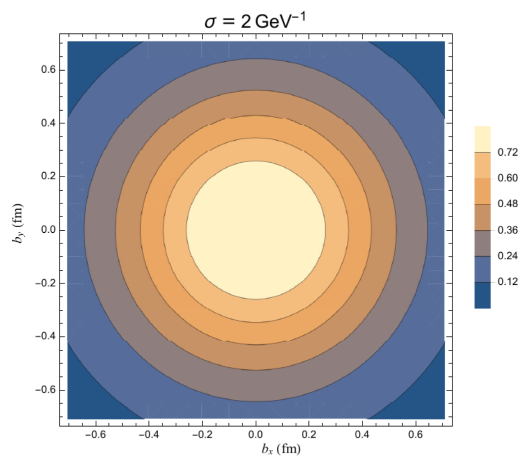
$$= \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H_1(x, 0, -\boldsymbol{\Delta}^2_\perp)$$

$$W^{\mu\nu}_{1/2} \sim F_1, F_2, g_1, g_2$$
$$b_1, b_2, b_3, b_4$$

Gaussian Package $q_\sigma(b)$



(a) $q_\sigma(b)$ (fm^{-2}) with packet width $\sigma = 1$ GeV^{-1}.



(b) $q_\sigma(b)$ (fm^{-2}) with packet width $\sigma = 2$ GeV^{-1}.

1, Introduction

Electromagnetic probes

- Electric and magnetic proton form factors
- Proton and Neutron charge distributions
- Nucleon spin structure
- Nucleon-Delta transition (other resonances)
- Quark-hadron duality in structure functions
- Generalized parton distributions
- Pion and deuteron form factors ……

$\gamma^*\gamma \to \rho\rho$: experiment

Exp:

PLUTO/ TASSO/ CELLO/ ARGUS @ DESY, ’82-’91

L3 @ LEP, ’03-’06

STAR @ RHIC, ’07-’09

Babar @ PEP-II, ‘08

LHCb, ‘12 (TeV, double charm)

L3 Collaboration

Exp.	$e^+e^- \to$	Q^2/GeV^2	$W_{\gamma\gamma^*}/\mathrm{GeV}$
L3(2003)	$\rho^0\rho^0$	$1.2 \sim 30$	$1.1 \sim 3$
L3(2004)	$\rho^0\rho^0$	$0.2 \sim 0.85$	$1.1 \sim 3$
& older exp.	$\rho^0\rho^0$	$0.2 \sim 30$	$1.1 \sim 3$
L3(2004)	$\rho^+\rho^-$	$1.2 \sim 30$	$1.1 \sim 3$
L3(2005)	$\rho^+\rho^-$	$0.2 \sim 0.85$	$1.1 \sim 3$

○ GDAs & $\rho\rho$ production

Exp:

PLUTO/ TASSO/ CELLO/ ARGUS
@ DESY, ’82-’91
L3 @ LEP, ’03-’06
STAR @ RHIC, ’07-’09
Babar @ PEP-II, ‘08
LHCb, ‘12 (TeV, double charm)

L3 Collaboration

ARGUS Collaboration etc.

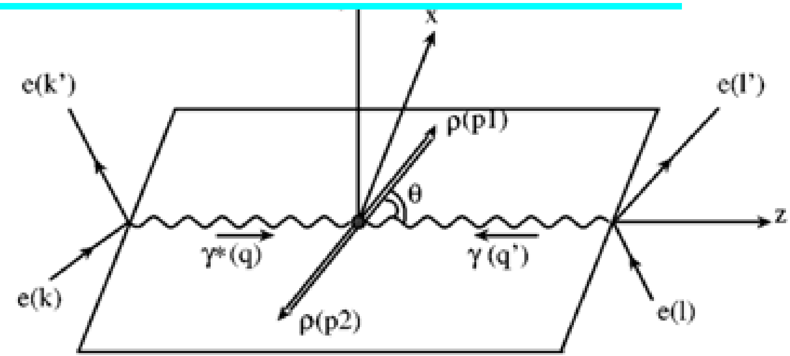
[Albrecht ’90, ’91]

$$\sigma(e^+e^- \to \rho^+\rho^-) = 8.3 \pm 0.7(\mathrm{stat}) \pm 0.8(\mathrm{syst})\ \mathrm{fb}$$

$$\gamma^*\gamma \to \rho\rho$$

- Full reaction: **[Anikin ’04, ‘05]**

$$2e \to 2e + \rho^0\rho^0(\rho^+\rho^-)$$



- @LO (twsit-2), $I = 0$
- charged/neutral cross sec. NOT independent (CG coefs)
- but charged has bremsstrahlung

- Also related to: **[García ’15, Kłusek-Gawenda ’17, Kumano ’17, ‘18]**

$$2e \to 2e + \rho^0 + 2\pi \qquad \to AA + \pi^+\pi^-\pi^+\pi^-$$

$$\to 4\pi \qquad \to AA + \pi^+\pi^- 2\pi^0$$