

The charged Z_c and Z_b structures in a constituent quark model approach

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on Few-Body Problems in Physics
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Outline

1. Introduction

2. The model

3. Results

4. Conclusions



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1. Introduction

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The quark model (I)

Volume 8, number 3

PHYSICS LETTERS



André Petermann

Murray Gell-Mann

George Zweig

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN

California Institute of Technology, Pasadena, California

Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dy-

bar $n_L - n_{\bar{L}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^- , s^- , u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{1}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] and the members of the anti-triplet as anti-quarks \bar{q} . Baryons can now be constructed from quarks by using the combinations $(qq\bar{q})$, $(qq\bar{q}\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(q\bar{q}\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration $(qq\bar{q})$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly give just **1** and **8**.

$$3 \times 3 = 9,$$

$$3 \times 3 \times 3 = 27, \dots$$

$$3 \otimes \overline{3} = 8 \oplus 1,$$

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$



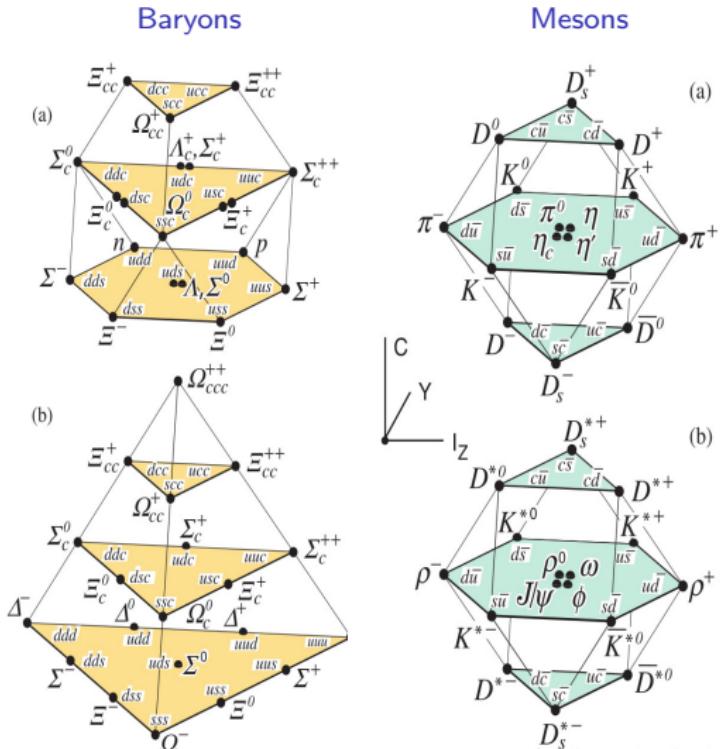
Murray Gell-Mann, 10-year-old, New York, 1939. Now, 25 years later, Caltech





The quark model (II)

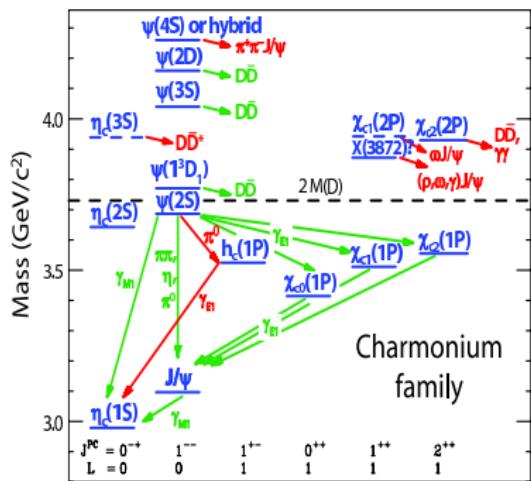
Successful classification scheme organizing the large number of **conventional hadrons**



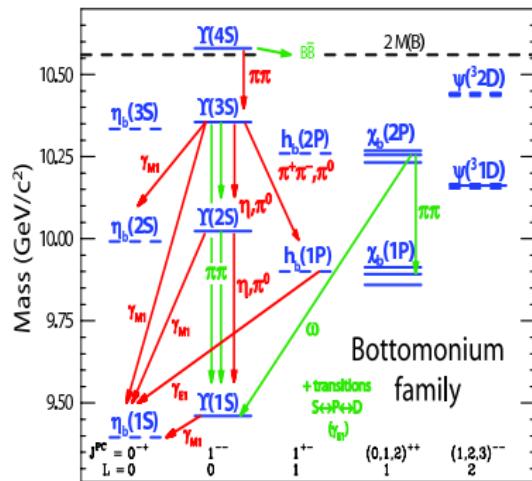


The heavy quarkonia before 2003

Charmonium and bottomonium states were discovered in the 1970s.
Experimentally clear spectrum of narrow states below the open-flavor threshold



Eichten et al., Rev. Mod. Phys. 80, 1161 (2008)

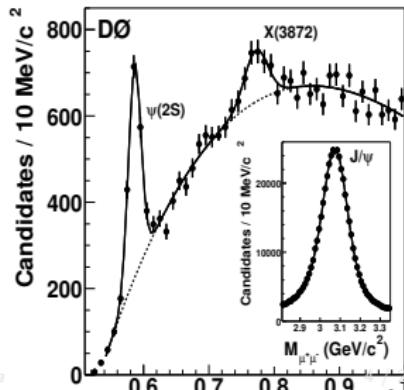
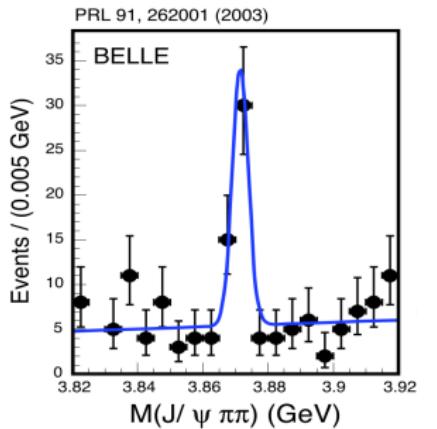
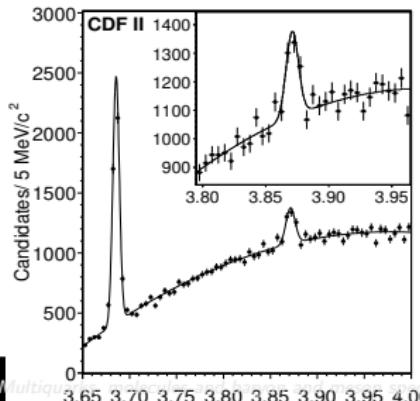
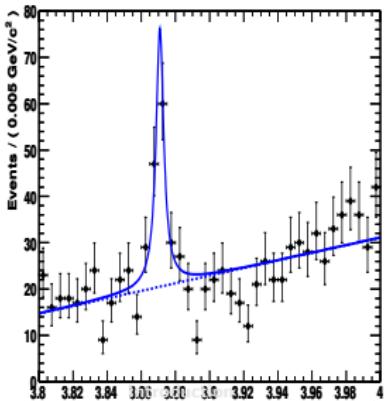


- Heavy quarkonia are bound states made of a heavy quark and its antiquark ($c\bar{c}$ charmonium and $b\bar{b}$ bottomonium).
- They can be classified in terms of the quantum numbers of a nonrelativistic bound state → Reminds positronium [(e^+e^-) -bound state] in QED.



The discovery of the X(3872)

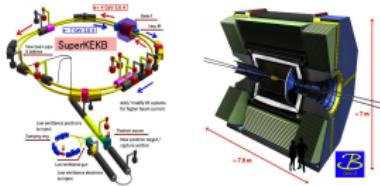
- In 2003, Belle observed an unexpected enhancement in the $\pi^+\pi^-J/\psi$ invariant mass spectrum while studying $B^+ \rightarrow K^+\pi^+\pi^-J/\psi$.
- It was later confirmed by BaBar in B-decays and by both CDF and D0 at Tevatron in prompt production from $p\bar{p}$ collisions.
- Its quantum numbers, mass, and decay patterns make it an unlikely conventional charmonium candidate.



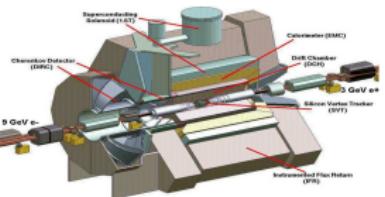


Discoveries at *B*-factories

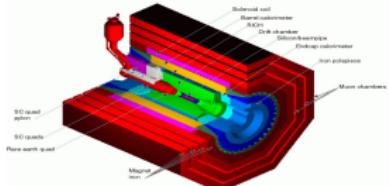
BELLE@KEK (Japan)



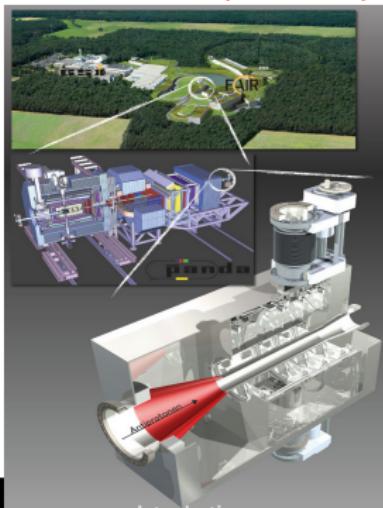
BABAR@SLAC (USA)



CLEO@CORNELL (USA)

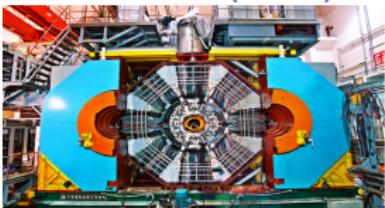


PANDA@GSI (Germany)

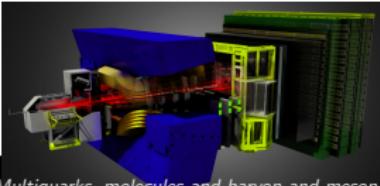


Explosion of related experimental activity:
Signals of exotic structures?
Standard $q\bar{q}$ or qqq ?
Threshold cusps?

BES@IHEP (China)



LHCb@CERN
(Switzerland)

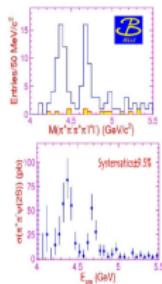
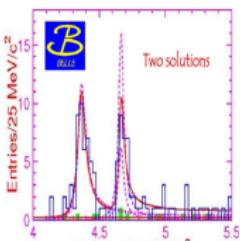
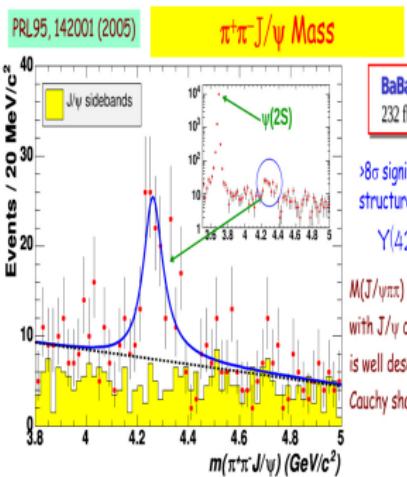


GLUEx@JLAB (USA)

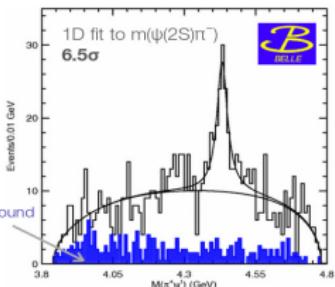




The XYZ particles – A new particle zoo?

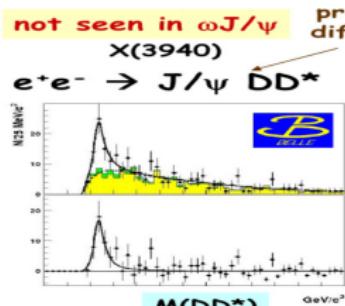


- Belle observed $Z(4430)^-$ from sample of $\sim 2k$ $B^0 \rightarrow \psi(2S)K^{+0}$
- Charged state \Rightarrow minimal quark content of $cc\bar{u}\bar{d}$



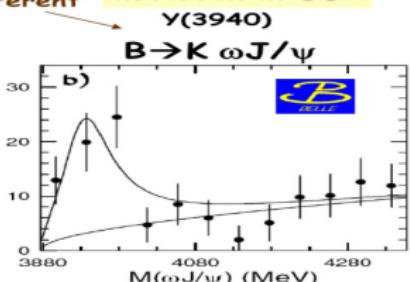
$$M = 4433 \pm 4 \pm 2 \text{ MeV}/c^2$$

$$\Gamma = 45^{+18+30}_{-13-13} \text{ MeV}/c^2$$



probably different

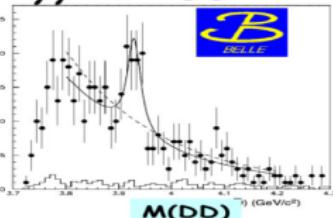
not seen in DD^*



Probably the χ_{c2}'

Z(3930)

$\gamma\gamma \rightarrow DD$



$$M = 3942 \pm 7 \pm 6 \text{ MeV}$$

$$M = 3942 \pm 11 \pm 13 \text{ MeV}$$

$$M = 3929 \pm 5 \pm 2 \text{ MeV}$$



In this work...



- We evaluate the molecular nature of some XYZ states using a constituent quark model

① $X(3872)$ state:

- Quantum numbers: $J^{PC} = 1^{++}$
- Mass slightly below $D^0 \bar{D}^{*0}$ threshold.
- Large isospin breaking.

② $Z_c(3900)^\pm$ and $Z_c(4020)^\pm$ states:

- $J^{PC} = 1^{+-}$ charged states.
- Close to $D\bar{D}^*$ and $D^*\bar{D}^*$ thresholds.
- Absence of $D\bar{D}$ peaks → Evidence in favor of a role for pion exchange in forming molecules of open-flavor pairs.

③ $Z_b(10610)^\pm$ and $Z_b(10650)^\pm$ states:

- Bottom partners of $Z_c(3900)^\pm$ and $Z_c(4020)^\pm$.
- $J^{PC} = 1^{+-}$ charged states.
- Close to $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds.
- Absence of $B\bar{B}$ peaks.



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2. The model

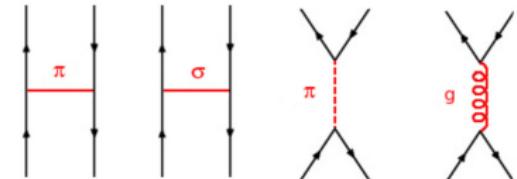
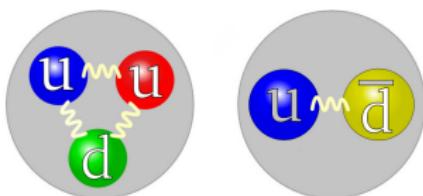
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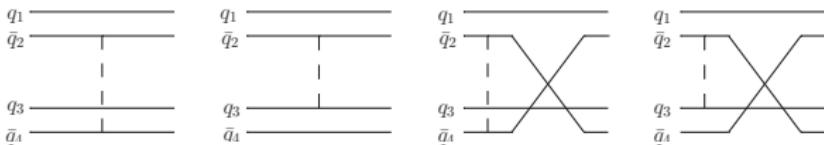


Roadmap

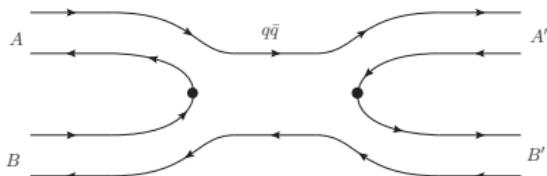
- Meson and Baryon spectra from constituent quark models.



- Residual meson-meson interaction.



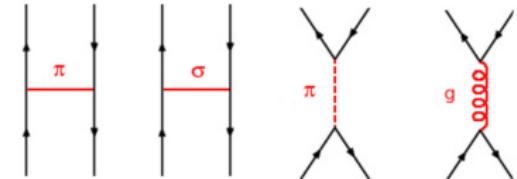
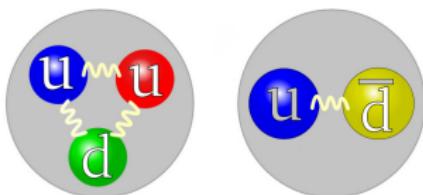
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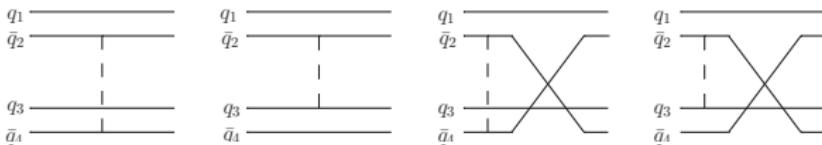


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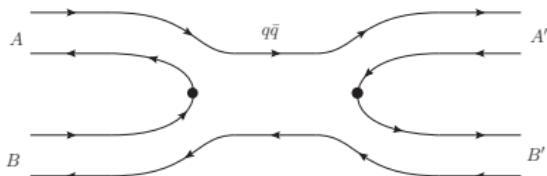
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Constituent quark model

The model includes:

- Spontaneous breaking of chiral symmetry \rightarrow Constituent mass and Pseudo-Goldstone bosons.
- QCD perturbative effects \rightarrow Gluon exchange.
- Confinement \rightarrow Linear screened potential.
- All parameters constrained from low-lying meson and baryon spectra.

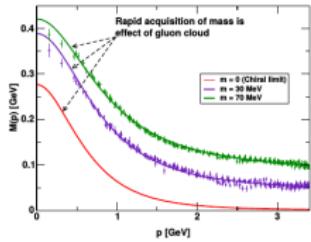




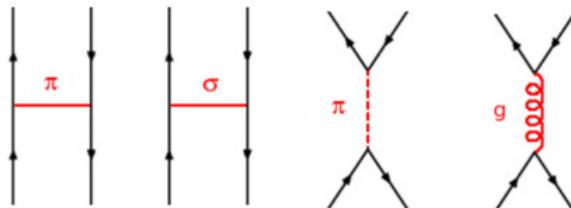
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C. D. Roberts, arxiv:1109.6325v1 [nucl-th]

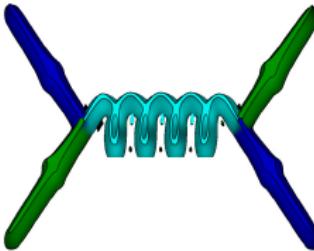




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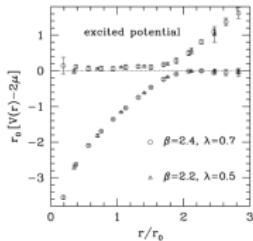
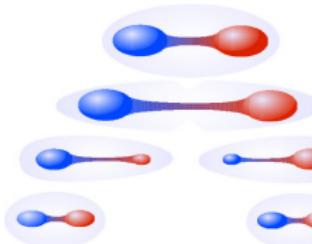




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Constituent quark model

- Nucleon-Nucleon interaction:

- D. R. Entem, F. Fernández, A. Valcarce, **PRC62**, 034002 (2000).
- A. Valcarce, A. Faessler, F. Fernández, **PLB345**, 367 (1995).
- F. Fernández, A. Valcarce, U. Straub, A. Faessler, **JPG19**, 2013 (1993).

- Baryon spectrum:

- A. Valcarce, H. Garcilazo, and J. Vijande, **PRC72**, 025206 (2005).
- H. Garcilazo, A. Valcarce, F. Fernández, **PRC64**, 058201 (2001).

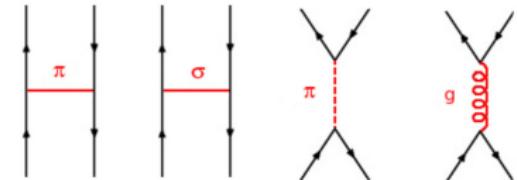
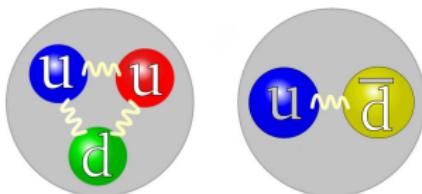
- Meson spectrum:

- J. Vijande, F. Fernández y A. Valcarce, **JPG31**, 481 (2005).
- J. Segovia, A. M. Yasser, D. R. Entem, F. Fernández, **PRD78**, 114033 (2008).
- J. Segovia, P. G. Ortega, D. R. Entem, F. Fernández, **PRD90**, 074027 (2016).

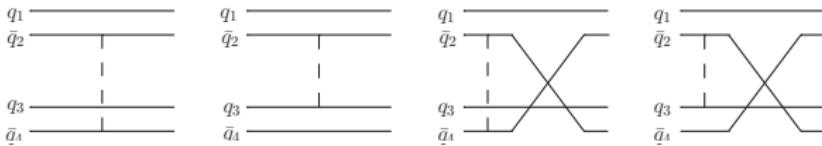


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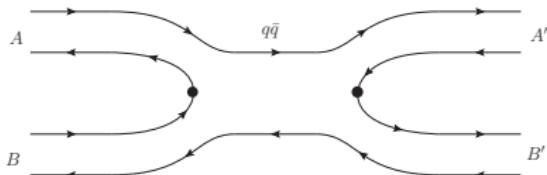
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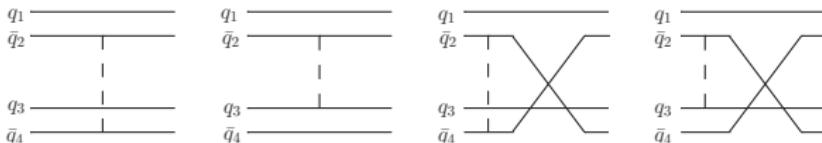


Solving the two body problem

- We want to explore meson-meson interactions.
- Meson wave function \rightarrow Gaussian Expansion Method:

- $\psi_{lm}(\vec{p}) = \sum_{n=1}^{n_{max}} C_{nl} Y_{lm}(\hat{p}) \phi_{nl}(p)$, with $\phi_{nl}(p) = (-i)^l \frac{N_{nl}}{(2\eta_n)^{l+3/2}} p^l e^{-\frac{p^2}{4\eta_n}}$
- GEM free parameters: $\{n_{max}, r_1, a\}$
- Rayleigh-Ritz variational principle:
$$\sum_{n'=1}^{n_{max}} \left[(T_{nn'}^\alpha - EN_{nn'}^\alpha) c_{n'l}^\alpha + \sum_{\alpha'}^{n^{ch nl}} V_{nn'}^{\alpha\alpha'} c_{n'l}^{\alpha'} \right] = 0$$

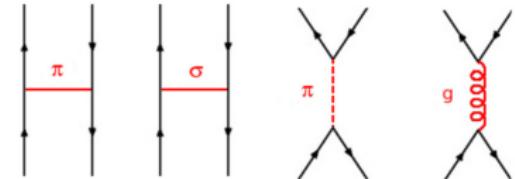
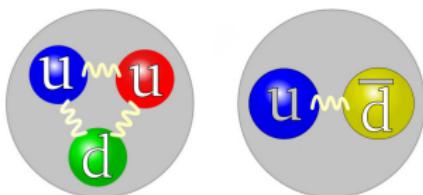
- Resonating Group Method:
 - Interaction at quark level \rightarrow Interaction between clusters
 - Direct and exchange potentials:



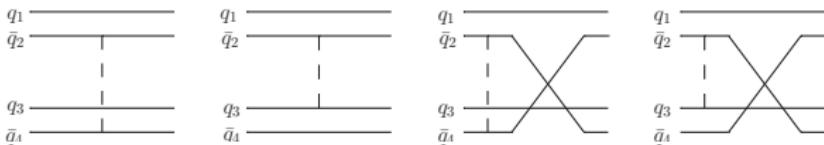


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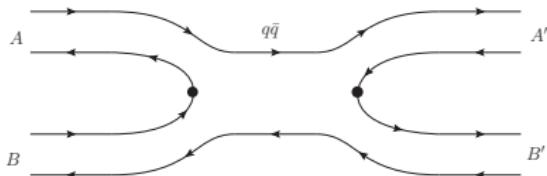
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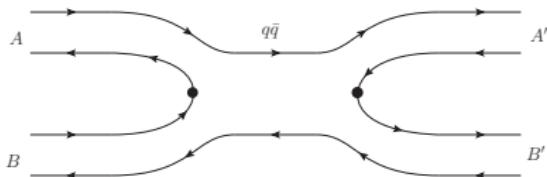


Coupling between $q\bar{q}$ and $q\bar{q} - q\bar{q}$ sectors

- Study of $\mathcal{H} = \mathcal{H}_{Q\bar{Q}} \oplus \mathcal{H}_{Q\bar{Q}q\bar{q}}$
- Effect of $q\bar{q}$ on meson-meson states \rightarrow Molecular states, mass-shifts, threshold cusps, ...
- Mixed states: $|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\psi\rangle + \sum_{\beta} \chi_{\beta}(P) |\phi_{M1}\phi_{M2}\beta\rangle$
- Solving the coupling with the $q\bar{q}$ meson spectrum \rightarrow Schrödinger-type equation:

$$\sum_{\beta} \int \left(H_{\beta'\beta}^{M_1 M_2}(P', P) + V_{\beta'\beta}^{\text{eff}}(E; P', P) \right) \chi_{\beta}(P) P^2 dP = E \chi_{\beta'}(P')$$

with $V_{\beta'\beta}^{\text{eff}}(E; P', P)$:

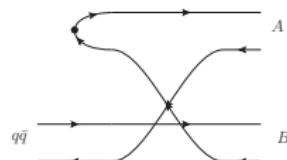




3P_0 interaction

- 3P_0 model used as coupling mechanism.
- Pair creation hamiltonian:

$$\mathcal{H} = g \int d^3x \bar{\psi}(x) \psi(x)$$

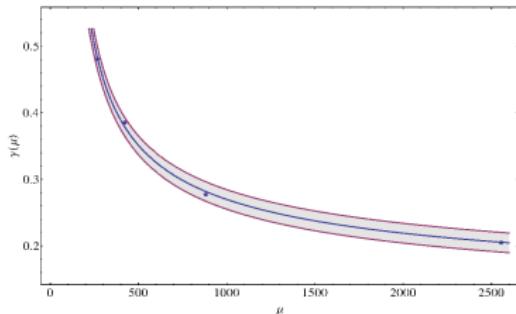


- Non relativistic reduction:

$$T = -3\sqrt{2}\gamma' \sum_{\mu} \int d^3p d^3p' \delta^{(3)}(p + p') \left[\mathcal{Y}_1 \left(\frac{p-p'}{2} \right) b_{\mu}^{\dagger}(p) d_{\nu}^{\dagger}(p') \right] C=1, I=0, S=1, J=0$$

with $\gamma' = 2^{5/2}\pi^{1/2}\gamma$ and $\gamma = \frac{g}{2m}$

- Running of the 3P_0 strength $\gamma \rightarrow$ [J. Segovia et al., arXiv:1205.2215](#)



$$\gamma(\mu) = \frac{\gamma_0}{\log(\mu/\mu_\gamma)}$$

- $\gamma_0 = 0.81 \pm 0.02$
- $\mu_\gamma = 49.84 \pm 2.58$ MeV
- Solid line is the fit
- Shaded area \rightarrow 90% C.L.



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1. Introduction

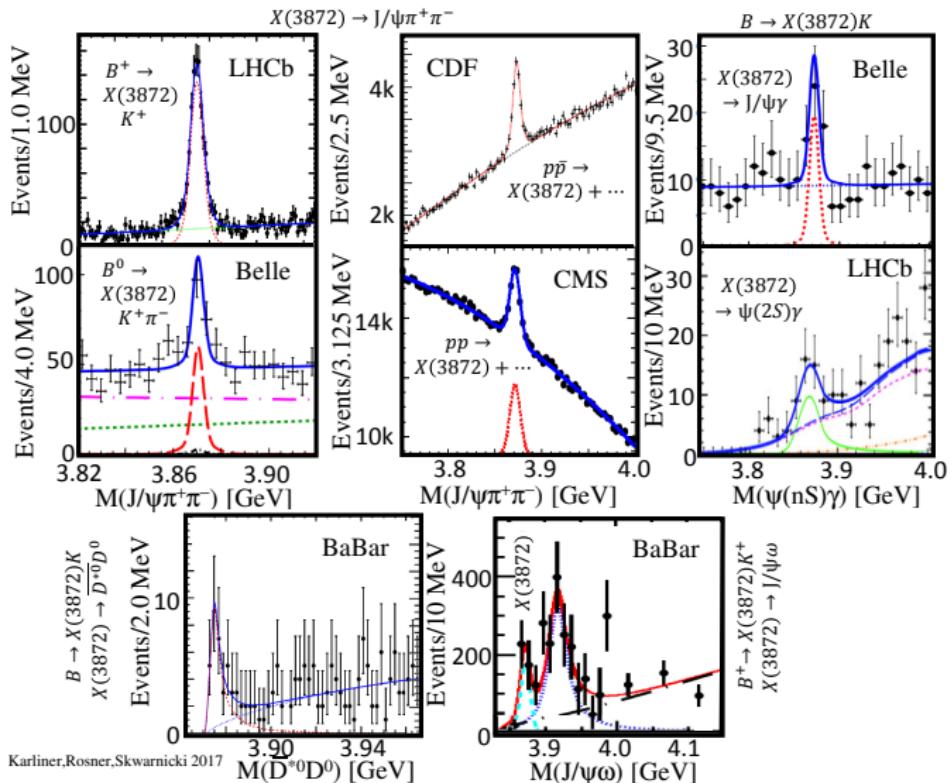
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$X(3872)$ state





$X(3872)$ state

- Quantum numbers: $J^{PC} = 1^{++}$
- Width : $\Gamma < 1.2$ (90% C.L.)
- Mass : $M_X = 3871.69 \pm 0.17 \text{ MeV}/c^2 \rightarrow$
Close & slightly below $D^0\bar{D}^{*0}$ threshold.
- $R_1 = \frac{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 1.0 \pm 0.4 \pm 0.3 & (\text{Belle}) \\ 0.8 \pm 0.3 & (\text{BaBar}) \end{cases},$
- $R_2 = \frac{\mathcal{B}(X \rightarrow J/\psi \gamma)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 0.33 \pm 0.12 & (\text{BaBar}) \\ 0.14 \pm 0.05 & (\text{Belle}) \end{cases},$
- $R_3 = \frac{\mathcal{B}(X \rightarrow \psi(2S) \gamma)}{\mathcal{B}(X \rightarrow J/\psi \gamma)} = 2.6 \pm 0.6$ (LHCb).

Experimental data suggest a loosely-bound $D^0\bar{D}^{*0}$ molecule coupled to $2P$ $c\bar{c}$ states.

P. G. Ortega *et al.*, PRD 81, 054023 (2010).



Results for the $X(3872)$ coupled calculation

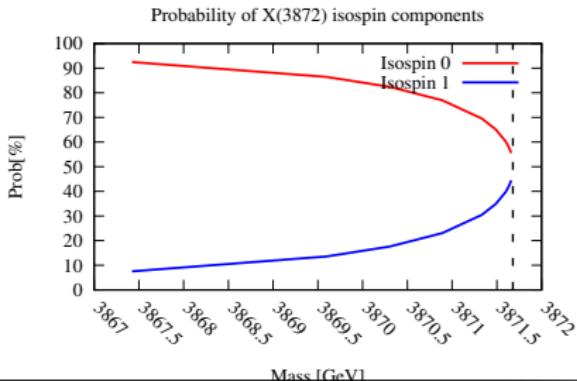
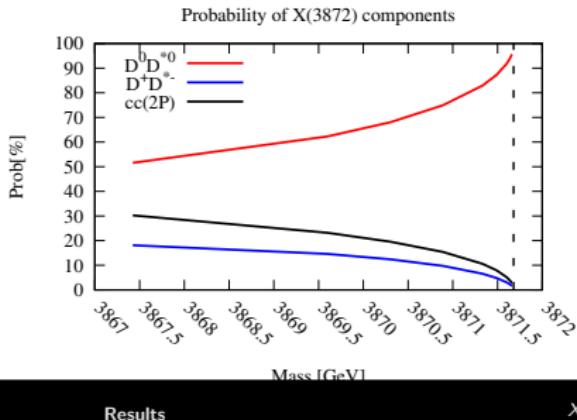
- $D^0 D^{*0}$ and $D^\pm D^{*\mp}$ meson-meson channels are included
- Coupled to $c\bar{c}(2^3P_1)$ meson state \rightarrow Theoretical bare mass = 3947.4 MeV
- Inclusion of $J/\psi\rho$ y $J/\psi\omega$ channels, needed for describing the strong decays \rightarrow Rearrangement diagrams
Small contribution to the mass
- Experimental mass of the $X(3872)$ obtained for the 3P_0 strength parameter γ , constrained from $Q\bar{Q}$ strong decay studies.

γ	E_{bind}	$c\bar{c}(2^3P_1)$	$D^0 D^{*0}$	$D^\pm D^{*\mp}$	$J/\psi\rho$	$J/\psi\omega$
0.231	-0.60	12.40	79.24	7.46	0.49	0.40
0.226	-0.25	8.00	86.61	4.58	0.53	0.29



Results for the $X(3872)$ coupled calculation

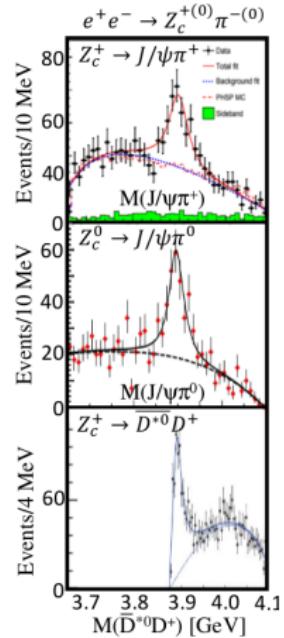
- $D^0 D^{*0}$ and $D^\pm D^{*\mp}$ meson-meson channels are included
- Coupled to $c\bar{c}(2^3P_1)$ meson state \rightarrow Theoretical bare mass = 3947.4 MeV
- Inclusion of $J/\psi\rho$ y $J/\psi\omega$ channels, needed for describing the strong decays \rightarrow Rearrangement diagrams
Small contribution to the mass
- Experimental mass of the $X(3872)$ obtained for the 3P_0 strength parameter γ , constrained from $Q\bar{Q}$ strong decay studies.





$Z_c(3900)^\pm$ and $Z_c(4020)^\pm$

- $J^{PC} = 1^{+-}$ charged states.
- Close to $D\bar{D}^*$ and $D^*\bar{D}^*$ thresholds.
- $Z_c(3900)^\pm$, with ave. mass (3891.2 ± 3.3) MeV, seen in:
 - $e^+e^- \rightarrow \pi\pi J/\psi$ as a peak in $M(\pi J/\psi)$
 - $e^+e^- \rightarrow \pi D\bar{D}^*$ as a peak in $M(D\bar{D}^*)$.
 - $e^+e^- \rightarrow \pi\pi\psi(3868)$ as a peak in $M(\pi\psi(3868))$.
- $Z_c(4020)^\pm$, with ave. mass (4022.9 ± 2.8) MeV, seen in:
 - $e^+e^- \rightarrow \pi\pi h_c$ as a peak in $M(\pi h_c)$
 - $e^+e^- \rightarrow \pi D^*\bar{D}^*$ as a peak in $M(D^*\bar{D}^*)$.
 - $e^+e^- \rightarrow \pi\pi\psi(3868)$ as a peak in $M(\pi\psi(3868))$.
- Absence of $D\bar{D}$ peaks → Evidence in favor of a role for pion exchange in forming molecules of open-flavor pairs.



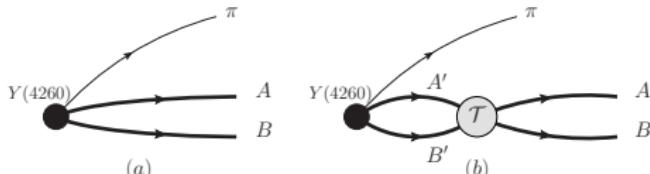
Karliner,Rosner,Skwarnicki 2017



$Z_c(3900)^\pm$ and $Z_c(4020)^\pm$

- Coupled-channels calculation of $J^{PC} = 1^{+-}$ sector with $I = 1$.
- No coupling with meson spectrum. Only direct+exchange interaction from CQM.
- Including the following thresholds:
 - $\pi J/\psi$ (3234.19 MeV)
 - $\rho\eta_c$ (3755.79 MeV)
 - $D\bar{D}^*$ (3875.85 MeV)
 - $D^*\bar{D}^*$ (4017.24 MeV)
- Poles of the S-matrix and production lineshapes.

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{k_{AB} k_{\pi Z_c}}{4M_Y^2} |\overline{\mathcal{M}^\beta(m_{AB})}|^2 dm_{AB}$$

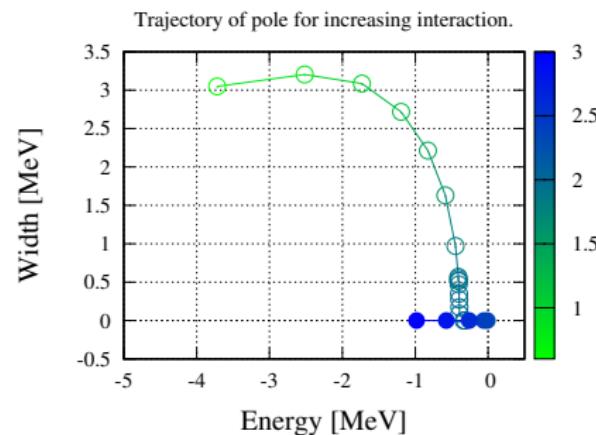
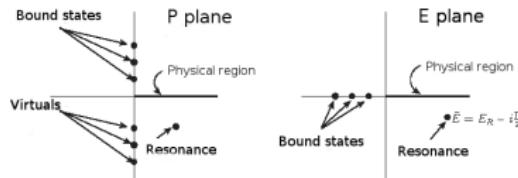




$Z_c(3900)^\pm$ and $Z_c(4020)^\pm$ poles

- $D^{(*)}\bar{D}^*$ attractive, but not strong enough to bind the meson pairs.
- States found as **virtual poles** in S-matrix.
- Poles below the $D^{(*)}\bar{D}^*$ threshold in 2nd Riemann sheet → Enhancement in production lineshapes.

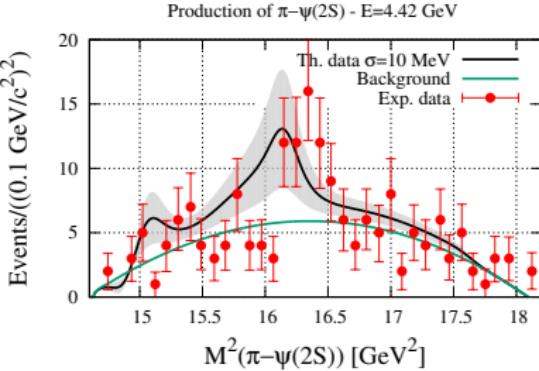
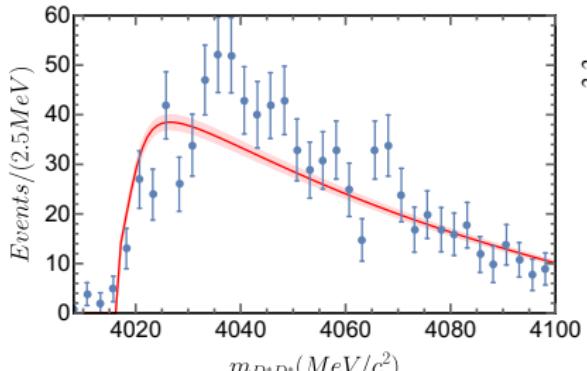
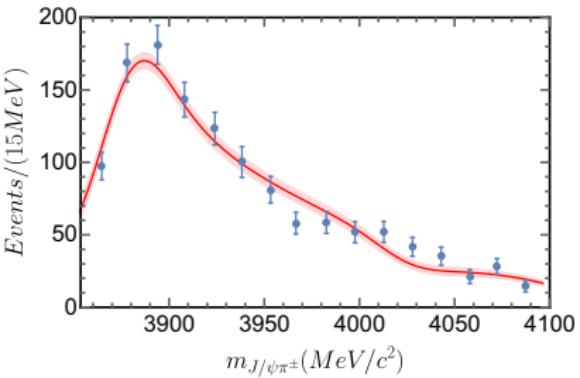
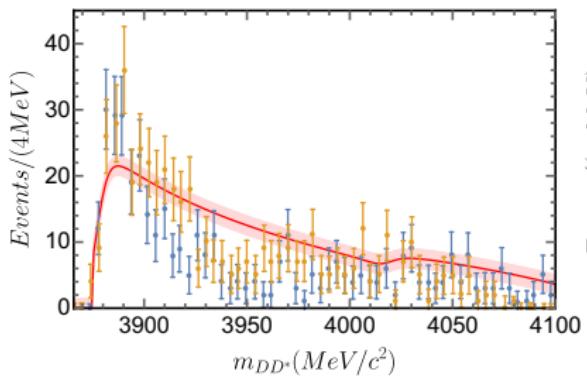
	$Z_c(3900)$	$Z_c(4020)$
Pole position	3871.74	4013.21



- Good description of production lineshapes.



$Z_c(3900)^\pm$ and $Z_c(4020)^\pm$ production



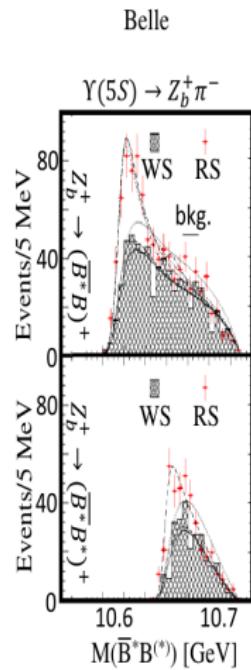
Results

Charged states: $Z_c(3900)^\pm$ and $Z_c(4020)^\pm$



$Z_b(10610)^\pm$ and $Z_b(10650)^\pm$

- Bottom partners of $Z_c(3900)^\pm$ and $Z_c(4020)^\pm$.
- $J^{PC} = 1^{+-}$ charged states.
- Close to $B\bar{B}^*$ (10604 MeV) and $B^*\bar{B}^*$ (10649 MeV) thresholds.
- $Z_b(10610)^\pm$, with ave. mass (10607.2 ± 2.0) MeV, seen in:
 - $e^+e^- \rightarrow \pi\pi\Upsilon(nS)$ ($n = 1, 2, 3$).
 - $e^+e^- \rightarrow \pi B\bar{B}^*$.
 - $e^+e^- \rightarrow \pi\pi h_b(nP)$ ($n = 1, 2$).
- $Z_b(10650)^\pm$, with ave. mass (10652.2 ± 1.5) MeV, seen in:
 - $e^+e^- \rightarrow \pi\pi\Upsilon(nS)$ ($n = 1, 2, 3$).
 - $e^+e^- \rightarrow \pi B^*\bar{B}^*$.
 - $e^+e^- \rightarrow \pi\pi h_b(nP)$ ($n = 1, 2$).
- Absence of $B\bar{B}$ peaks → Evidence in favor of a role for pion exchange in forming molecules of open-flavor pairs.





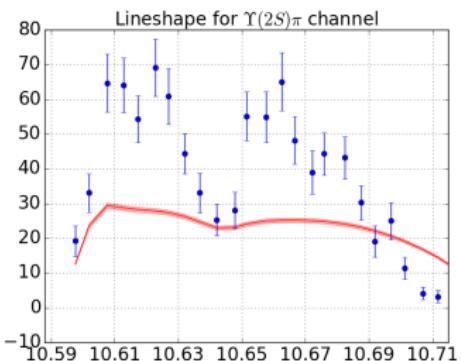
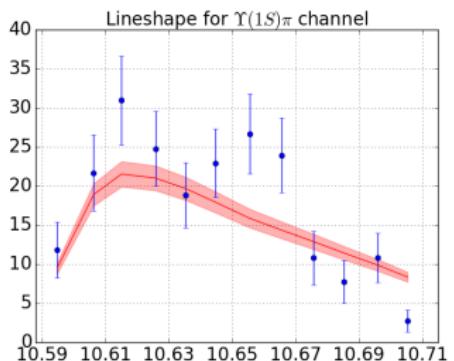
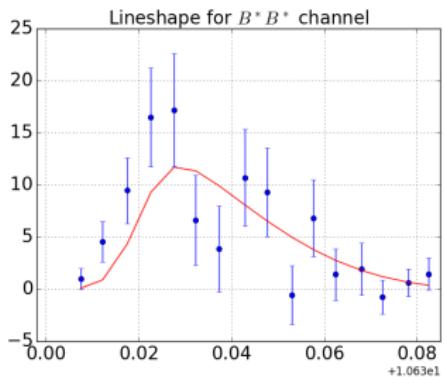
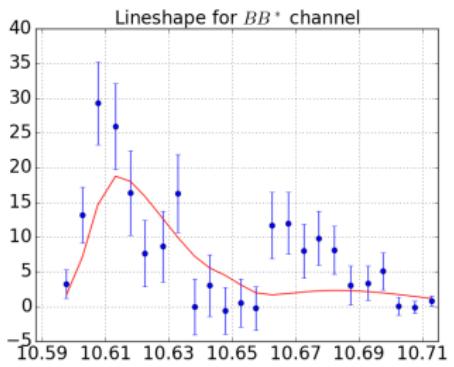
$Z_b(10610)^\pm$ and $Z_b(10650)^\pm$

- Calculation of $(I)J^{PC} = (1)1^{+-}$ analogous to that of the Z_c states.
- Including $B\bar{B}^*$, $B^*\bar{B}^*$, $\Upsilon(1S)\pi$, $\Upsilon(2S)\pi$ and $\Upsilon(3S)\pi$ channels.
- $B^{(*)}\bar{B}^*$ interaction $\sim D^{(*)}\bar{D}^*$ interaction (HFS).
- Reduction of $B^{(*)}\bar{B}^*$ kinetic energy \rightarrow **Bound state & Resonance**

	$Z_b(10610)^\pm$	$Z_b(10650)^\pm$
Mass	10600.45	$10644.74 - 4.34 i$
Width	2.80	8.88
\mathcal{P}_{\max}	95.76% (BB^*)	64.85% (B^*B^*)
Γ_{BB^*}	—	8.68
$\Gamma_{\Upsilon(1S)\pi}$	0.94	0.08
$\Gamma_{\Upsilon(2S)\pi}$	0.65	0.002
$\Gamma_{\Upsilon(3S)\pi}$	1.21	0.12



$Z_b(10610)^\pm$ and $Z_b(10650)^\pm$ production





Outline

1. Introduction

2. The model

3. Results

4. Conclusions



Conclusions

- Hadron physics in heavy quark sector is indeed a Few Body Problem.
- Heavy quarkonium shows a rich phenomenology, including effects of four-quark structures near thresholds.
- Use of Constituent Quark Model plus a coupled-channels calculation explain the $Z_c(3900)^{\pm}$ and $Z_c(4020)^{\pm}$ peaks as virtual states, and $Z_b(10610)^{\pm}$ and $Z_b(10650)^{\pm}$ ones as a bound state and a resonance.
- The Z_c 's, Z_b 's and $X(3872)$ can be satisfactorily explained within the same model, with no parameter tuning.



Thanks for your attention.

portega@usal.es

Further details at:

- *The Z_c structures in a coupled-channels model* – Eur.Phys.J. C79 (2019) no.1, 78
- *Molecular Structures in Charmonium Spectrum: The XYZ Puzzle* – JPG 40 (2013) 065107.
- *Coupled channel approach to the structure of the $X(3872)$* – PRD 81 (2010) 054023.



Backslides



Coupling formalism with T matrix

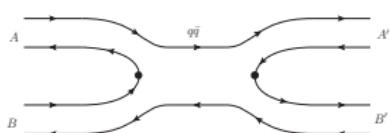
- But... inadequate formalism for states above the threshold
- Resonances, Virtuals, Bound states \rightarrow Poles of the Scattering Matrix: $S_\alpha^{\alpha'} = 1 - 2\pi i \sqrt{\mu_\alpha \mu_{\alpha'} k_\alpha k_{\alpha'}} T_\alpha^{\alpha'}(E + i0; k_{\alpha'}, k_\alpha)$
- T matrix obtained with Lippmann-Schwinger:

$$T^{\beta'\beta}(E; P', P) = V_T^{\beta'\beta}(P', P) + \sum_{\beta''} \int dP'' P''^2 V_T^{\beta'\beta''}(P', P'') \frac{1}{E - E_{\beta''}(P'')} T^{\beta''\beta}(E; P'', P)$$

- With $V_T^{\beta'\beta}(P', P) = V^{\beta'\beta}(P', P) + V_{\text{eff}}^{\beta'\beta}(P', P)$

where

$$V_{\text{eff}}^{\beta'\beta}(P', P) = \sum_{\alpha} \frac{h_{\beta'\alpha}(P') h_{\alpha\beta}(P)}{E - M_{\alpha}}$$



The complete T matrix factorizes like V_T :

$$T^{\beta'\beta}(E; P', P) = T_V^{\beta'\beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta'\alpha'}(E; P') \Delta_{\alpha'\alpha}(E)^{-1} \bar{\phi}^{\alpha\beta}(E; P)$$



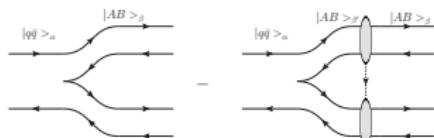
Coupling elements

From $T^{\beta'\beta}(E; P', P) = T_V^{\beta'\beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta' \alpha'}(E; P') \Delta_{\alpha' \alpha}(E)^{-1} \bar{\phi}^{\alpha \beta}(E; P)$:

- Modified vertex:

$$\phi^{\alpha \beta'}(E; P) = h_{\alpha \beta'}(P) - \sum_{\beta} \int \frac{T_V^{\beta' \beta}(E; P, q) h_{\alpha \beta}(q)}{q^2/2\mu - E} q^2 dq,$$

$$\bar{\phi}^{\alpha \beta}(E; P) = h_{\alpha \beta}(P) - \sum_{\beta'} \int \frac{h_{\alpha \beta'}(q) T_V^{\beta' \beta}(q, P, E)}{q^2/2\mu - E} q^2 dq$$

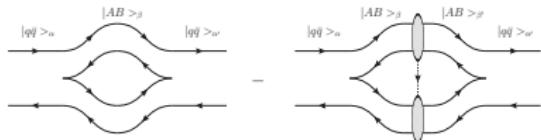


- Complete propagator:

$$\Delta^{\alpha' \alpha}(E) = \left\{ (E - M_{\alpha}) \delta^{\alpha' \alpha} + \mathcal{G}^{\alpha' \alpha}(E) \right\}$$

- Exact mass-shift of the state:

$$\mathcal{G}^{\alpha' \alpha}(E) = \sum_{\beta} \int dq q^2 \frac{\phi^{\alpha \beta}(q, E) h_{\beta \alpha'}(q)}{q^2/2\mu - E}$$





$X(3872)$ strong and radiative decay results

- Experimental results

$$R_1 = \frac{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 1.0 \pm 0.4 \pm 0.3 \\ 0.8 \pm 0.3 \end{cases},$$
$$R_2 = \frac{\mathcal{B}(X \rightarrow J/\psi \gamma)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 0.33 \pm 0.12 \\ 0.14 \pm 0.05 \end{cases},$$
$$R_3 = \frac{\mathcal{B}(X \rightarrow \psi(2S) \gamma)}{\mathcal{B}(X \rightarrow J/\psi \gamma)} = 2.6 \pm 0.6$$

- Theoretical decays [keV]:

E_{bind}	$\Gamma_{\pi^+ \pi^- J/\psi}$	$\Gamma_{\pi^+ \pi^- \pi^0 J/\psi}$	$\Gamma_{J/\psi \gamma}^M$	$\Gamma_{J/\psi \gamma}^{c\bar{c}}$	$\Gamma_{\Psi(2S) \gamma}^{c\bar{c}}$
-0.60	27.61	14.40	0.070	8.15	9.80
-0.25	24.18	10.64	0.056	5.25	6.31

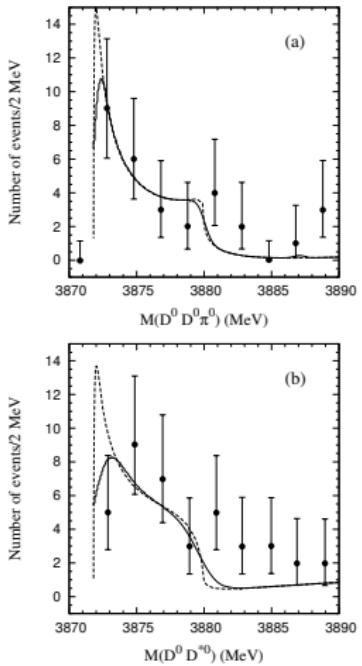
- Theoretical ratios:

E_{bind}	R_1	R_2	R_3
-0.60	0.52	0.30	1.20
-0.25	0.44	0.22	1.20

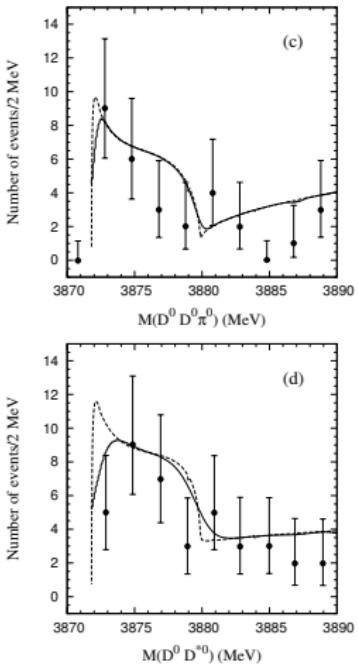


Lineshapes for $X(3872)$

$c\bar{c}$ Production



$D\bar{D}^*$ Production



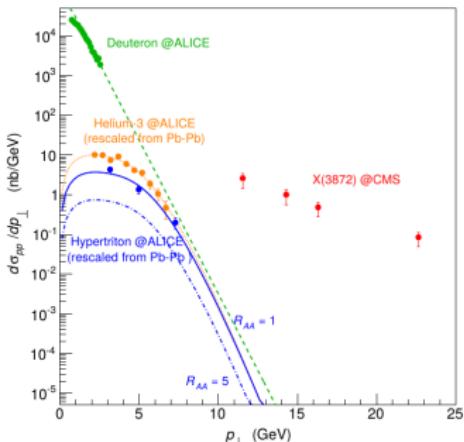
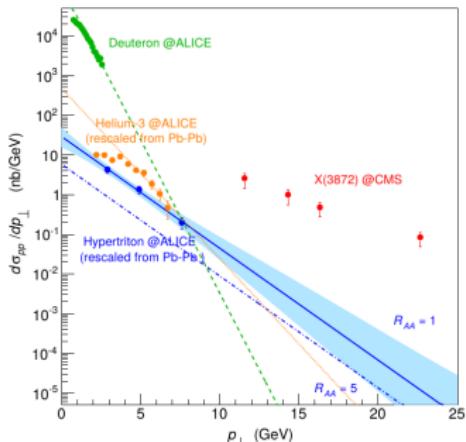
Belle
 $B \rightarrow K D^0 \bar{D}^0 \pi^0$

BaBar
 $B \rightarrow K D^0 \bar{D}^{*0}$



$X(3872)$ pp production [arXiv:1907.01441]

- A recent paper (A. Esposito PRD 92 (2015), 034028) questioned the loosely bound nature of the $X(3872)$ based on its production in high-energy pp collisions.
- They compared with light nuclei production data by ALICE with $p_T \leq 10\text{GeV}$ in Pb-Pb collisions at 2.76TeV , using Boltzmann statistics to conclude no hadronic molecule should be created with such an abundance for large p_T .
- But pp is a non-extensive system, so we cannot use Boltzmann statistics.



$X(3872)$ pp production [arXiv:1907.01441]



- Tsallis distribution works better for pp collisions (ALICE Coll. PRC97 (2018), 024615).
- Tsallis distribution allows to describe the deuteron and $X(3872)$ with the same q and T , and gives info on the production abundance.

$$\frac{d^3 N}{d^3 p} = \frac{gV}{(2\pi)^3} \left(1 + (q-1) \frac{E(p)}{T}\right)^{-\frac{q}{q-1}} \xrightarrow{q \rightarrow 1} \frac{gV}{(2\pi)^3} e^{-\frac{E(p)}{T}}$$

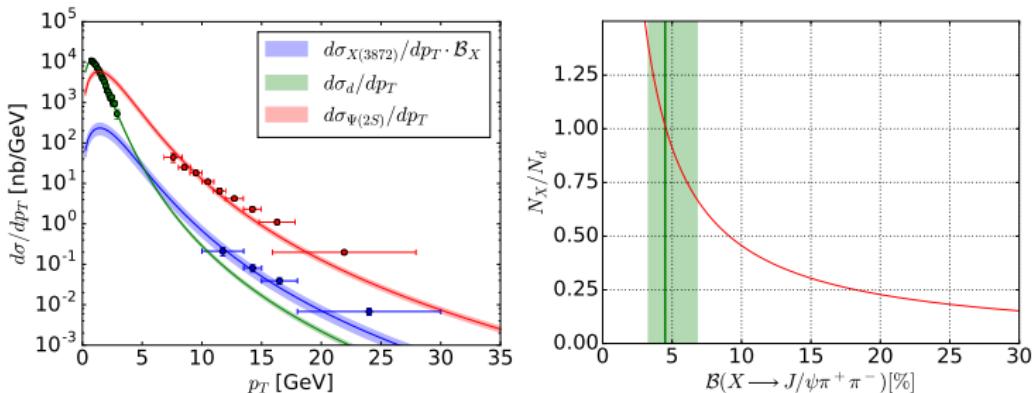
$$\text{with } E(p_T, y) = \sqrt{p_T^2 + m^2} \cosh y$$

	$X(3872) + d$	$X(3872) + \Psi(2S) + d$
$\ln(\mathcal{N}_X \mathcal{B}_X)$	41.4 ± 0.4	41.4 ± 0.4
$\ln(\mathcal{N}_d)$	40.35 ± 0.09	40.35 ± 0.09
$\ln(\mathcal{N}_\Psi)$	-	44.3 ± 0.2
q	1.122 ± 0.001	1.122 ± 0.001
T [MeV]	7.017 ± 0.07	7.018 ± 0.07



$X(3872)$ pp production [arXiv:1907.01441]

- Deuteron/ X production ratio ranging between 0.3 – 1.9 for $\mathcal{B}(X \rightarrow J/\psi\pi^+\pi^-) = 4.5^{+2.3}_{-1.2} \%$ (BESIII constrains).

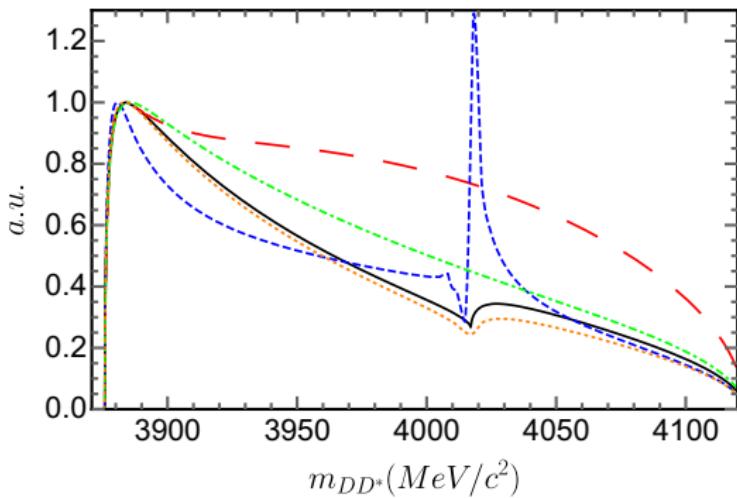


	$X(3872) + d$	$X(3872) + \Psi(2S) + d$
N_d	$(2.02 \pm 0.02) \cdot 10^{-4}$	$(2.01 \pm 0.02) \cdot 10^{-4}$
$N_X \mathcal{B}_X$	$(9 \pm 3) \cdot 10^{-6}$	$(9 \pm 3) \cdot 10^{-6}$
N_Ψ	-	$(2.2 \pm 0.3) \cdot 10^{-4}$



$Z_c(3900)^{\pm}$ and $Z_c(4020)^{\pm}$ production

- $D\bar{D}^*$ line shape for different coupled-channels calculations.



- Red: Only $D\bar{D}^*$.
- Blue: $D\bar{D}^* + D^*\bar{D}^*$.
- Green: $\rho\eta_c + D\bar{D}^*$.
- Orange: $\rho\eta_c + D\bar{D}^* + D^*\bar{D}^*$
- Black: $\pi J/\psi + \rho\eta_c + D\bar{D}^* + D^*\bar{D}^*$