Effects of an induced three-body force in the incident channel of (d,p) reactions
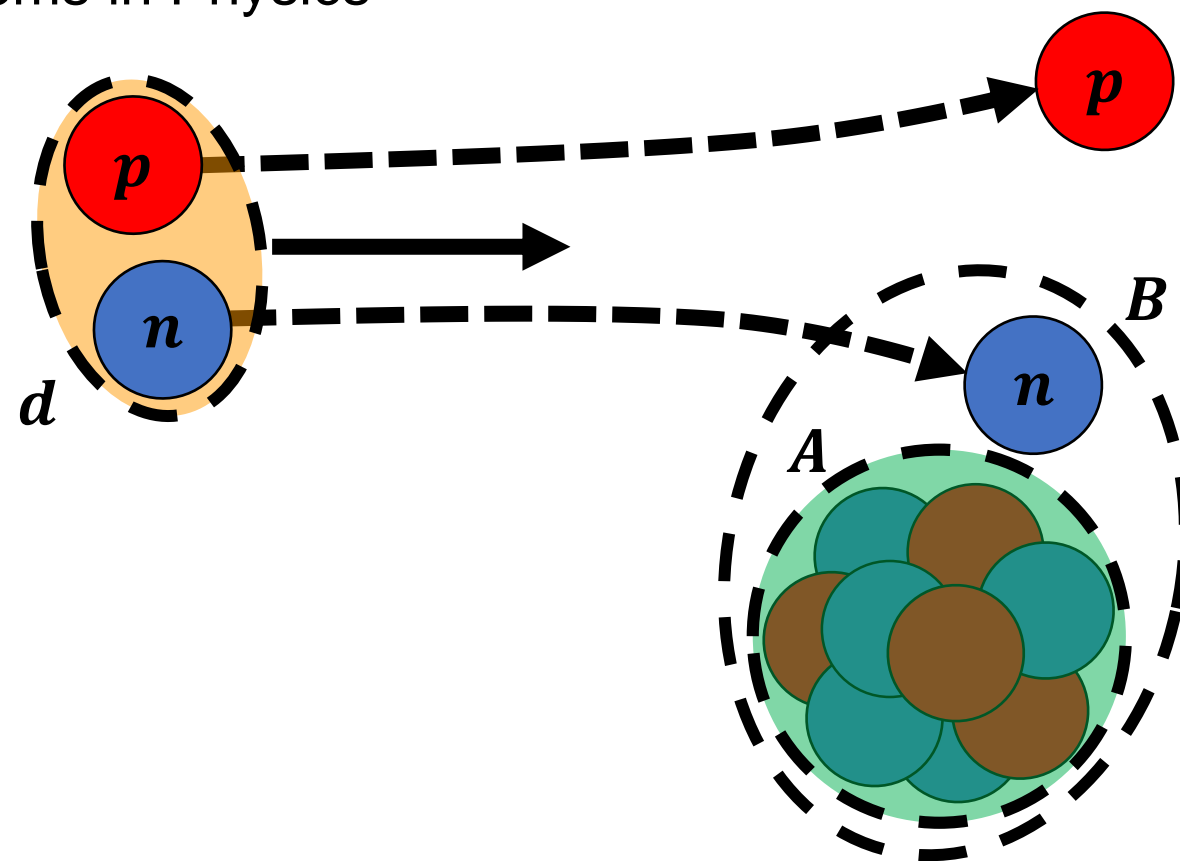
24th European Conference on Few-Body Problems in Physics

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Projecting many-body wave functions onto few-body wave functions

$A(d,p)B$ transfer reactions are inherently many-body problems, with each target nucleon interacting with the incoming neutron and proton separately. This is described by a many-body (MB) Schrödinger equation

$$H_{MB}\Psi_{MB} = E\Psi_{MB}$$

We project this many-body wave function onto a three-body (3B) wave function using Feshbach projection operator technique:

$$H_{\text{eff}}\Psi_{3B} = E\Psi_{3B}$$
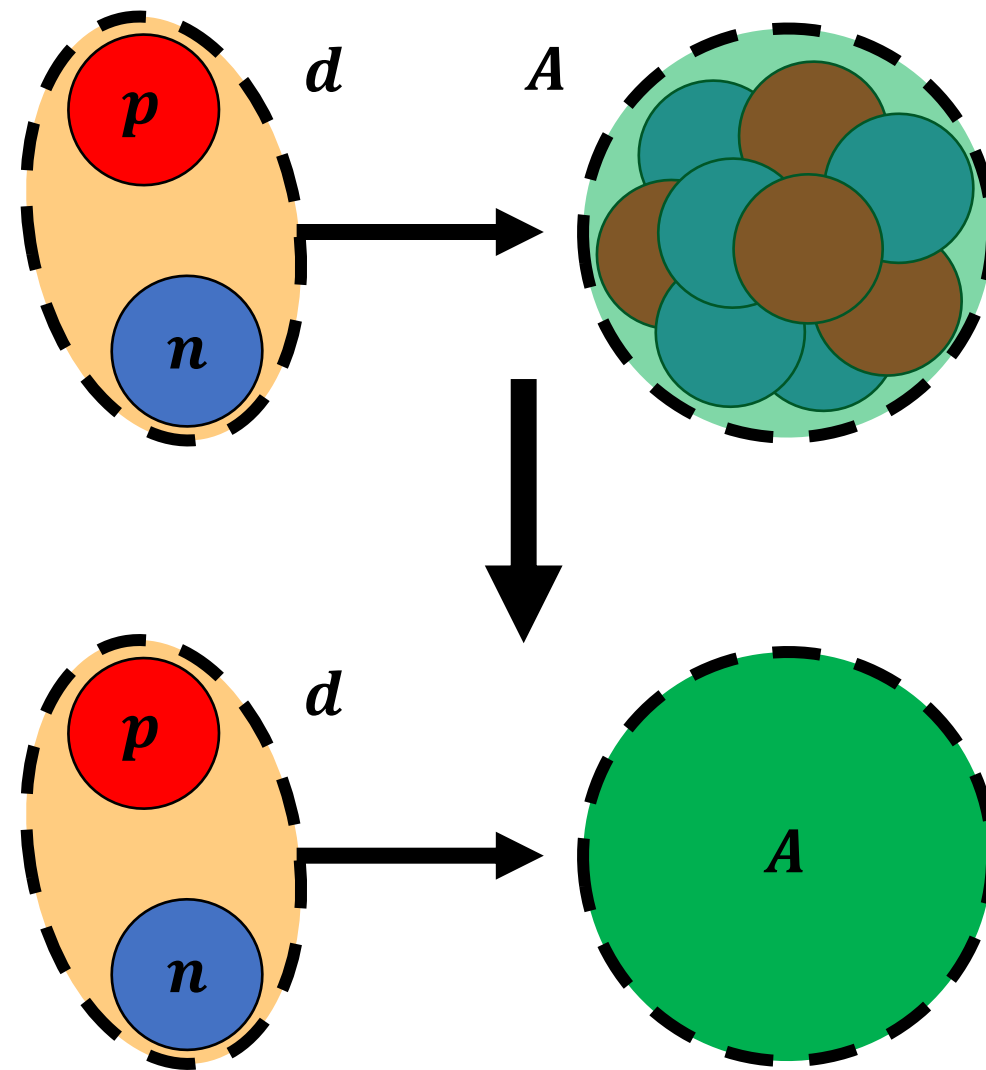
Feshbach projection operator formalism

For a two-body scattering case, the total many-body wave scattering function is split into two parts by operator which project onto the ground and excited states,

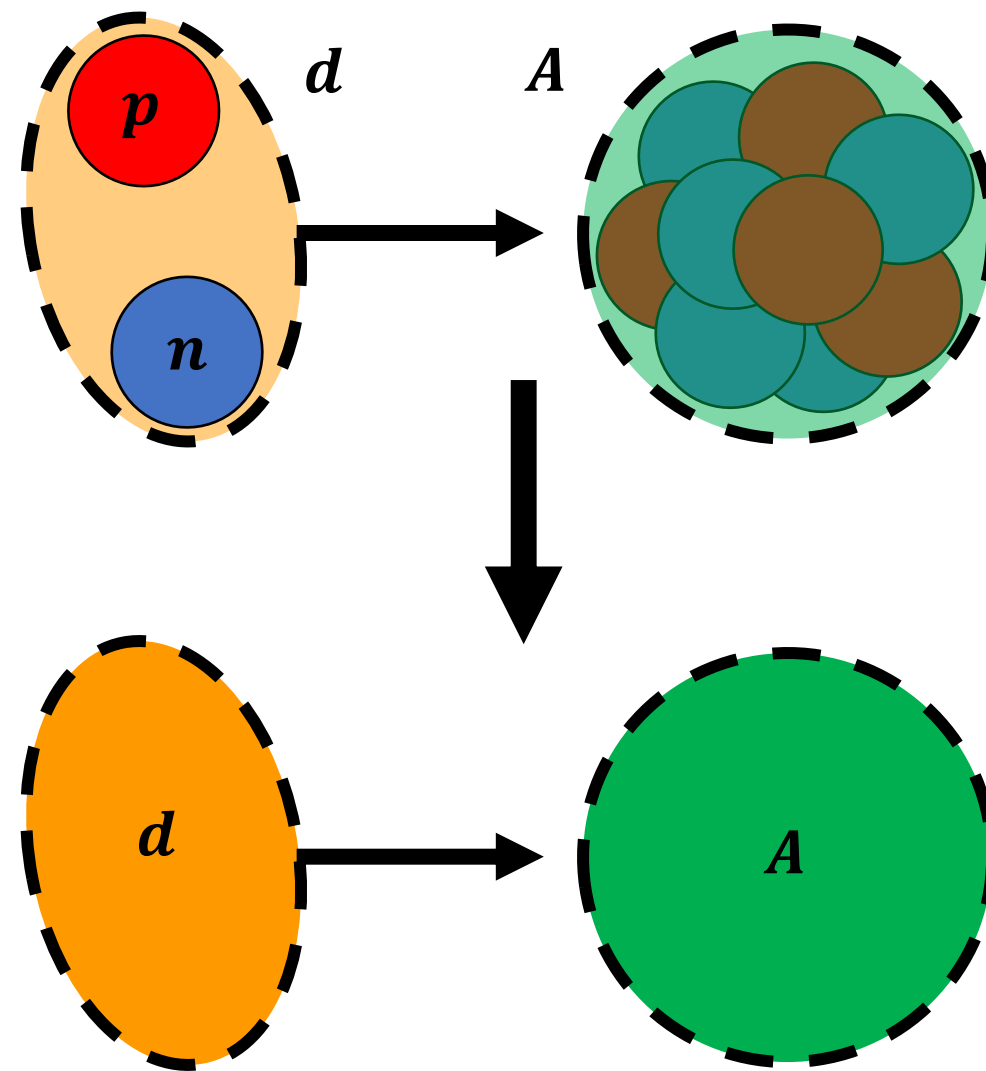$$\Psi_P = P_A |\Psi_N^{(+)}\rangle, \quad \Psi_Q = Q_A |\Psi_N^{(+)}\rangle$$

which appear in a modified Schrödinger equation governed by an effective Hamiltonian

$$(E - H_{PP})\Psi_P = 0, \quad H_{PP} = T_{NA} + V_{NA}^{opt}(E_N).$$

The optical potential V_{NA}^{opt} is related to the optical model operator U_{NA}^{opt} via

$$V_{NA}^{opt}(E_N) = \langle \phi_A | U_{NA}^{opt} | \phi_A \rangle,$$

$$U_{NA}^{opt} = v_{NA} + v_{NA}Q_A \frac{1}{e_N - Q_A v_{NA} Q_A} Q_A v_{NA}.$$

Three-body extension

In previous work (Phys. Rev. C **89,** 024605) this idea was extended for a three-body case. The three-body wavefunction can be considered as the projection Ψ_P of the full many-body wavefunction.

This projection is governed by a different effective Hamiltonian,

$$H_{\text{eff}} = T_3 + V_{np} + \boxed{\langle \phi_A | U | \phi_A \rangle}$$

where the many body operator U cannot be explicitly written solely in terms of pairwise $n\text{-}p$, $A\text{-}n$ and $A\text{-}p$ interactions.

$$U = U_{pA} + U_{nA} + U_{nA} \frac{Q_A}{e} U_{pA} + U_{pA} \frac{Q_A}{e} U_{nA} + \dots$$

$$\varepsilon_d = -2.2245 \text{MeV}$$
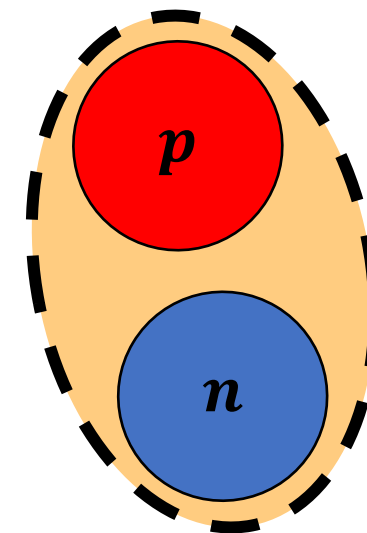
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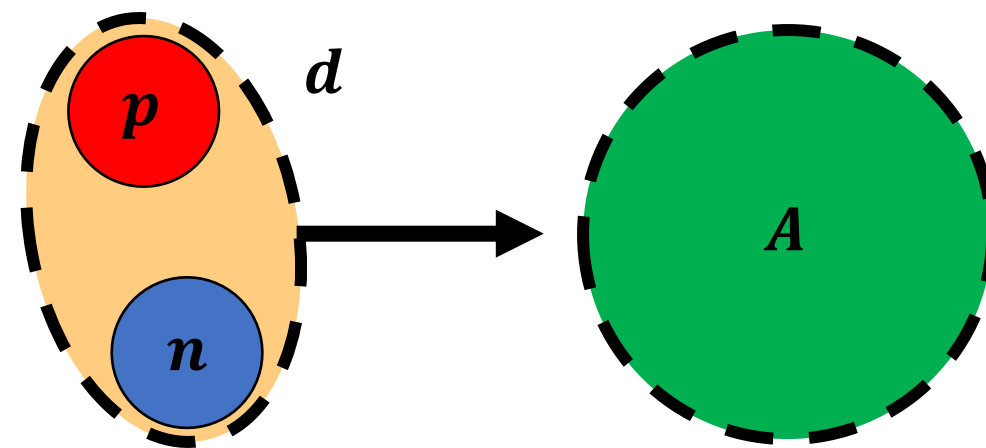
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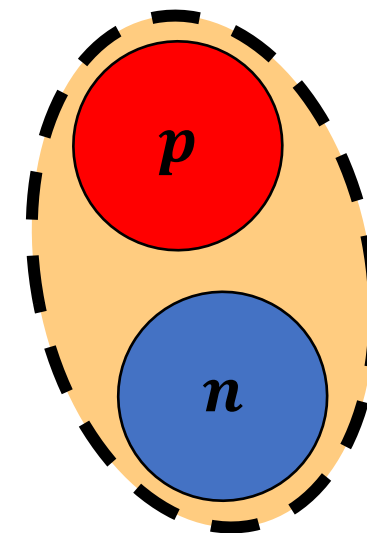
$$\varepsilon_d = -2.2245\text{MeV}$$

Adiabatic Distorted-Waves Approximation (ADWA)

$$T_{(d,p)} = \langle \chi_{pB}\phi_B\phi_p | V_{np} | \Psi_{3B}\phi_A \rangle$$

The Adiabatic Distorted-Waves Approximation (ADWA) accounts for break-up effects with a three-body description of the deuteron channel, Ψ_{3B}, in the (d,p) transition amplitude

It is typical to treat deuteron breakup in $A(d,p)B$ reactions using pairwise interactions.
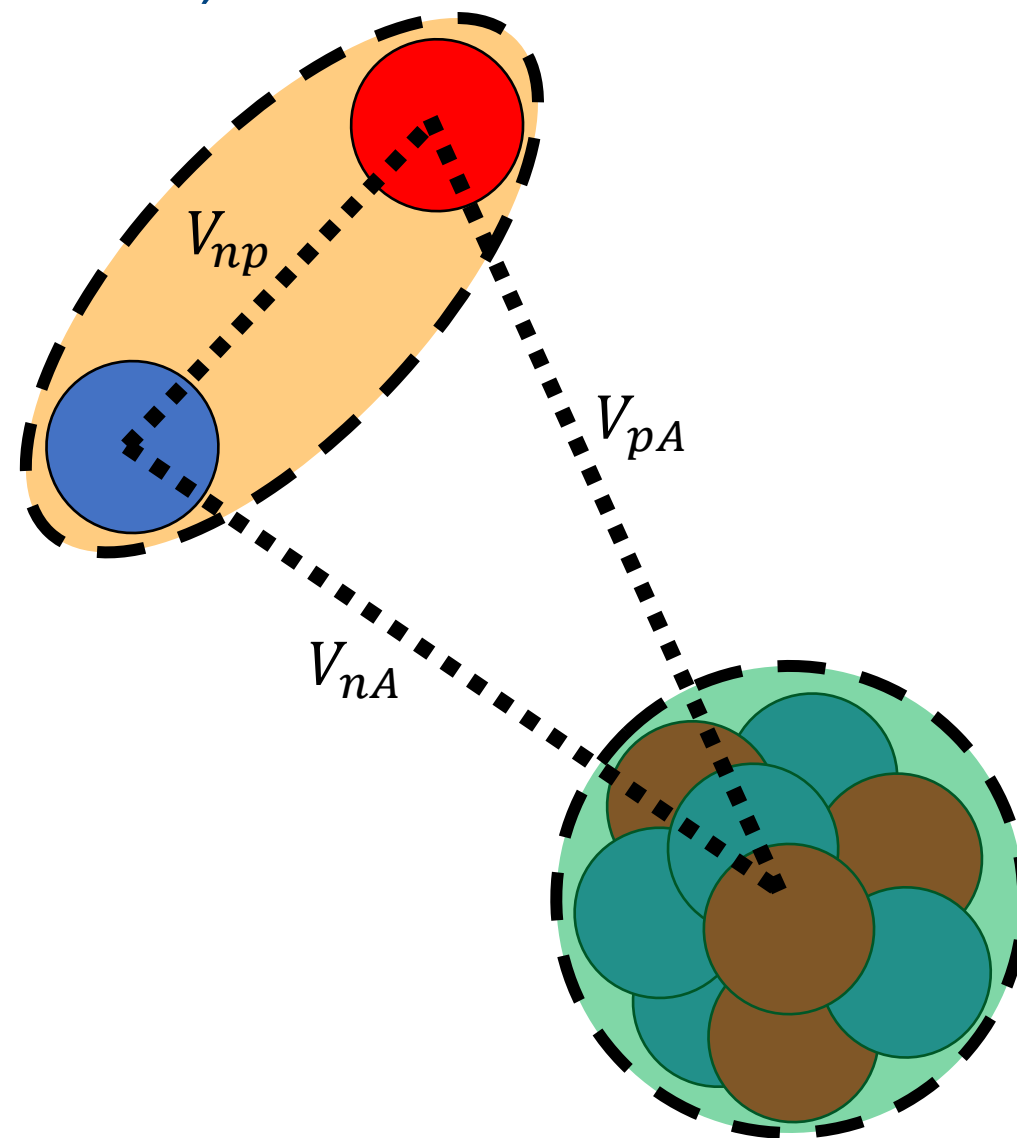
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It is typical to treat deuteron breakup in $A(d,p)B$ reactions using pairwise interactions.

From the next to leading order terms in the operator U terms corresponding to the excitation of the target by the separate nucleons in the deuteron appear.

$$U = U_{pA} + U_{nA} + \boxed{U_{nA} \frac{Q_A}{e} U_{pA} + U_{pA} \frac{Q_A}{e} U_{nA}} + \ldots$$
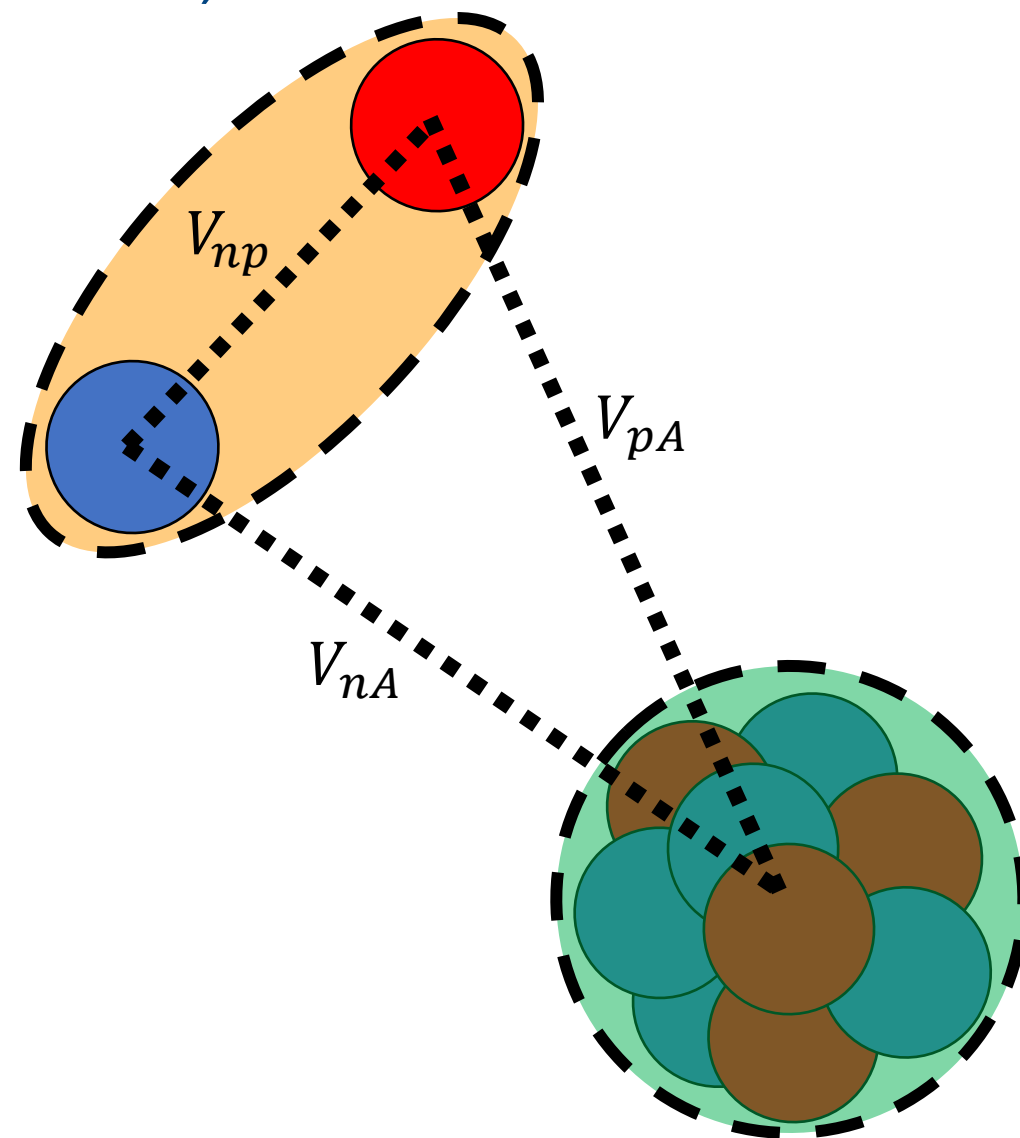
Adiabatic Distorted-Waves Approximation (ADWA)

$$T_{(d,p)} = \langle \chi_{pB} \phi_B \phi_p | V_{np} | \Psi_{3B} \phi_A \rangle$$

To calculate $V_{np}|\Psi_{3B}\rangle$ the ADWA expands the three-body wavefunction in a discrete set of states and Ψ_{3B} is expanded as

$$\Psi_{3B}(\mathbf{R}, \mathbf{r}) \approx \phi_d(\mathbf{r})\chi(\mathbf{R})$$

where $\chi(\mathbf{R})$ is found as the solution to the two-body Schrödinger equation

$$(E_d - T_R - \langle \phi_1 \phi_A | U | \phi_0 \phi_A \rangle)\chi(\mathbf{R}) = 0$$
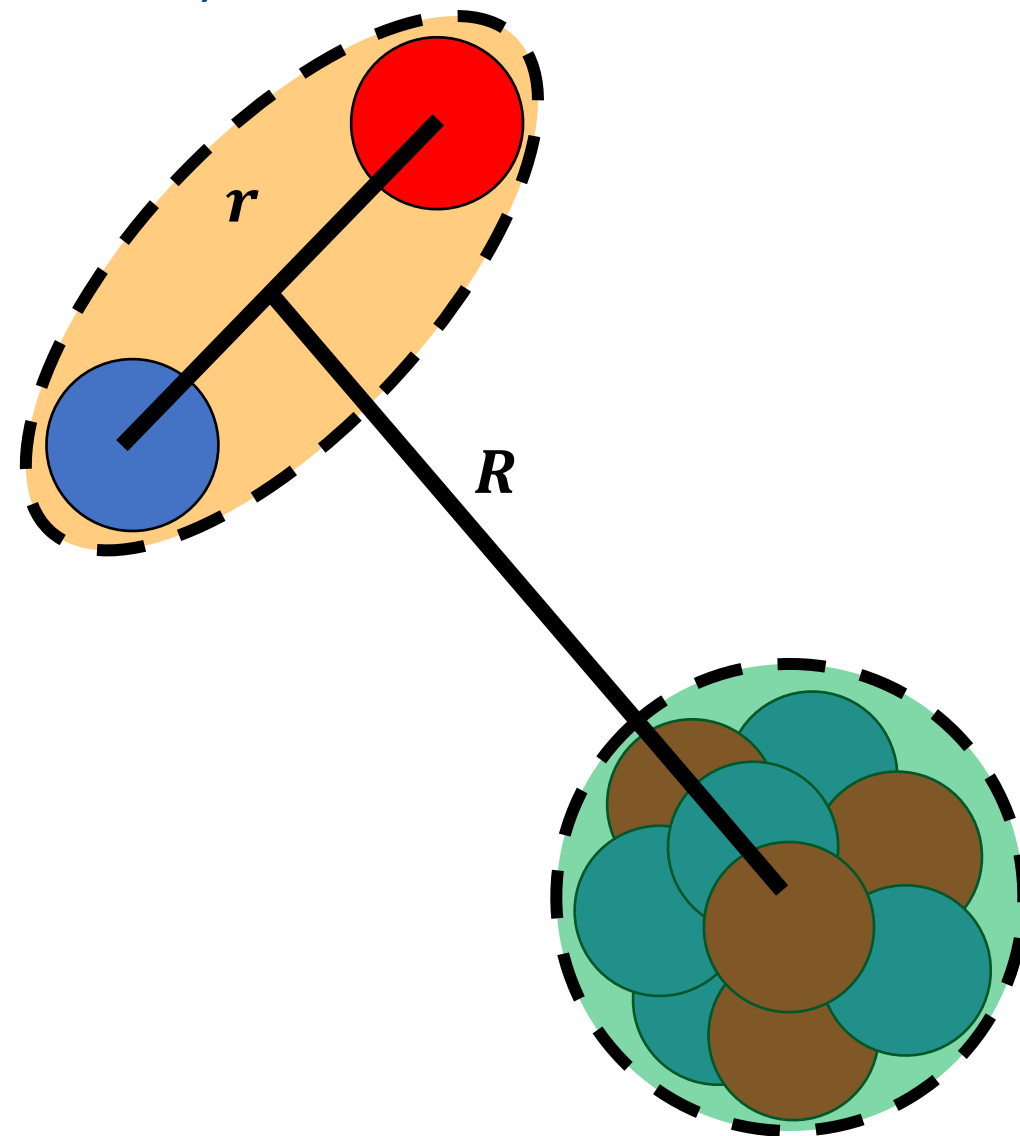
The many-body operator U

$$U = U_{pA} + U_{nA} + U_{nA}\frac{Q_A}{e}U_{pA} + U_{pA}\frac{Q_A}{e}U_{nA} + \dots$$

The operator U can be expressed in terms of operators U_{pA} and U_{nA}, which define excitations of A by p and n separately. Up to second order these terms can be written

$$U^{(0)} = U_{pA} + U_{nA},$$

$$U^{(1)} = U_{nA}\frac{Q_A}{e}U_{pA} + U_{pA}\frac{Q_A}{e}U_{nA},$$

where

$$U_{NA} = v_{NA} + v_{NA}\frac{Q_A}{e - Q_A v_{NA} Q_A}v_{NA}.$$

The many-body operator U: LO terms

$$U = U_{pA} + U_{nA} + U_{nA}\frac{Q_A}{e}U_{pA} + U_{pA}\frac{Q_A}{e}U_{nA} + \dots$$

Assuming that $U \approx U^{(0)}$, it was shown previously (Phys. Rev. C **89,** 024605) that

$$\langle\phi_1\phi_A|U^{(0)}|\phi_A\phi_0\rangle \approx \sum_{N=n,p} \langle\phi_1\phi_A|v_{NA}|\phi_0\phi_A\rangle$$
$$+ \langle\phi_1\phi_A|v_{NA}\frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A}v_{NA}|\phi_0\phi_A\rangle$$

and is taken at a shifted energy

$$E_{\text{eff}} = \frac{1}{2}E_d + \frac{1}{2}\langle T_r\rangle \quad \textcolor{green}{\textit{np}\text{ kinetic energy in range of } V_{np}}$$

The many-body operator U: NLO terms

$$U = U_{pA} + U_{nA} + U_{nA}\frac{Q_A}{e}U_{pA} + U_{pA}\frac{Q_A}{e}U_{nA} + \dots$$

Taking $U \approx U^{(0)} + U^{(1)}$, and approximating U_{NA} by its leading value v_{NA} we get,

$$\langle\phi_1\phi_A, \mathbf{R}|U^{(1)}|\phi_A\phi_0, \mathbf{R}'\rangle \approx \langle\phi_1\phi_A, \mathbf{R}|v_{nA}\frac{Q_A}{e}v_{pA}|\phi_A\phi_0, \mathbf{R}'\rangle$$

$$+ \langle\phi_1\phi_A, \mathbf{R}|v_{pA}\frac{Q_A}{e}v_{nA}|\phi_A\phi_0, \mathbf{R}'\rangle$$

which is non-local in space $\mathbf{R}$. Replacing the energy denominator in the same way

$$\langle\phi_1\phi_A, \mathbf{R}|U^{(1)}|\phi_A\phi_0, \mathbf{R}'\rangle = \langle\phi_1\phi_A, \mathbf{R}|v_{NA}\frac{Q_A}{e}v_{NA}|\phi_A\phi_0, \mathbf{R}'\rangle$$

$$= \langle\phi_1\phi_A|v_{NA}\frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA}Q_A}v_{NA}|\phi_0\phi_A\rangle.$$

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- Assume *p-A* and *n-A* interactions are equal
- Spin independent interactions
- Ignore Coulomb contributions

$$= \langle\phi_1\phi_A|v_{NA}\frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A}v_{NA}|\phi_0\phi_A\rangle.$$

The many-body operator U: NLO terms

$$U = U_{pA} + U_{nA} + U_{nA}\frac{Q_A}{e}U_{pA} + U_{pA}\frac{Q_A}{e}U_{nA} + \ldots$$

Comparing this to the LO term, there is an obvious similarity

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The many-body operator U: Induced three-body (I3B) forces

So we obtain our final form for the optical potential,

$$\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle \approx \langle \phi_1 \phi_A | v_{NA} + 2 v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA} Q_A | \phi_0 \phi_A \rangle,$$

or

$$\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle \approx 2 \langle \phi_1 \phi_A | U^{(0)} | \phi_0 \phi_A \rangle - \langle \phi_1 \phi_A | v_{NA} | \phi_0 \phi_A \rangle.$$

This accounts for effects such as *n-A* and *p-A* excitations that cannot be expressed as a sum of **separate** *n* and *p* components (such as the target being excited by the *n* and de-excited by the *p).*

These additional terms create an effective three-body interaction that cannot be treated with *n-A, p-A* and *n-p* pairwise interactions.

Connection to Dispersive Optical Model (DOM) potentials

It has now been shown that the induced three-body forces that arise due from $U^{(1)}$ can be accounted for in the ADWA by doubling part of the adiabatic deuteron optical potential

$$\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle \approx \langle \phi_1 \phi_A | v_{NA} + 2 v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA} Q_A | \phi_0 \phi_A \rangle,$$

The optical potential used to construct adiabatic distorting potentials should be nonlocal, energy dependent and it has been shown to fulfill a dispersion relation (Nucl. Phys. A **484**, 205)

$$V_{NA}^{opt}(E) = V_{NA}^{HF} + \Delta V_{NA}^{dyn}(E),$$

where

$$\Delta V_{NA}^{dyn}(E) = i W_{NA}(E) + \frac{\mathcal{P}}{\pi} \int_{-\infty}^{\infty} dE' \frac{W_{NA}(E')}{E - E'}.$$

Connection to Dispersive Optical Model (DOM) potentials

We can relate our optical potential to the constituent parts of the DOM potential,

$$\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle \approx \langle \phi_1 \phi_A | v_{NA} + 2 v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA} Q_A | \phi_0 \phi_A \rangle,$$

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$$V_{NA}^{opt}(E) = \qquad V_{NA}^{HF} \qquad + \qquad \Delta V_{NA}^{dyn}(E)$$

so we can say

$$\langle \phi_A | U^{(0)} + U^{(1)} | \phi_A \rangle = V_{nA}^{HF} + 2\Delta V_{nA}^{dyn}(E),$$
$$+ V_{pA}^{HF} + 2\Delta V_{pA}^{dyn}(E).$$

This provides an approximate practical approach to estimating the effect of I3B forces if V_{NA}^{HF} and $\Delta V_{NA}^{dyn}(E)$ are known.

Non-local Dispersive Optical Model (NLDOM) potential

Parameterized for ^{40}Ca using elastic scattering data (Phys. Rev. Lett. **112**, 162503), and adjusted for (d,p) reactions (Phys. Rev. C **94**, 034609), we can use the phenomenological energy dependent NLDOM potential,

$$\mathcal{U}^{\mathrm{NLDOM}}(\boldsymbol{R}, \boldsymbol{R}') = \sum_{i=1}^{7} U_i \left(\frac{|\boldsymbol{R} + \boldsymbol{R}'|}{2} \right) H_i(s), \qquad H_i(s) = \frac{\exp(-s^2/\beta_i^2)}{\pi^{\frac{3}{2}} \beta_i^3}.$$

All seven terms have different non-locality ranges, and each contributes to one of the two parts of the DOM optical potential

Hartree-Fock (V_{NA}^{HF})	Dynamic ($\Delta V_{NA}^{dyn}(E)$)
Vol1	Sur+
Vol2	Sur-
Wine	Vol+
	Vol-

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	Hartree-Fock (V_{NA}^{HF})	Dynamic ($\Delta V_{NA}^{dyn}(E)$)
Real	Vol1	Sur+
	Vol2	Sur-
	Wine	Vol+
		Vol-

Complex due to dispersion relation

$$\Delta V_{NA}^{dyn}(E) = iW_{NA}(E) + \frac{\mathcal{P}}{\pi} \int_{-\infty}^{\infty} dE' \frac{W_{NA}(E')}{E - E'}$$

Non-local Dispersive Optical Model (NLDOM) potential

The net effect of our additional I3B forces is to double the contribution from the dynamic terms in the potential

$$\langle\phi_A|U^{(0)} + U^{(1)}|\phi_A\rangle = \boxed{V_{nA}^{HF}} + \boxed{2\Delta V_{nA}^{dyn}(E)}$$
$$+ \boxed{V_{pA}^{HF}} + \boxed{2\Delta V_{pA}^{dyn}(E)}$$

Hartree-Fock (V_{NA}^{HF})	Dynamic ($\Delta V_{NA}^{dyn}(E)$)
Vol1	Sur+
Vol2	Sur-
Wine	Vol+
	Vol-

Real

Complex due to dispersion relation

$$\Delta V_{NA}^{dyn}(E) = iW_{NA}(E) + \frac{\mathcal{P}}{\pi}\int_{-\infty}^{\infty} dE' \frac{W_{NA}(E')}{E - E'}$$

Results: Exact ^{40}Ca(d,p)^{41}Ca cross sections with NLDOM
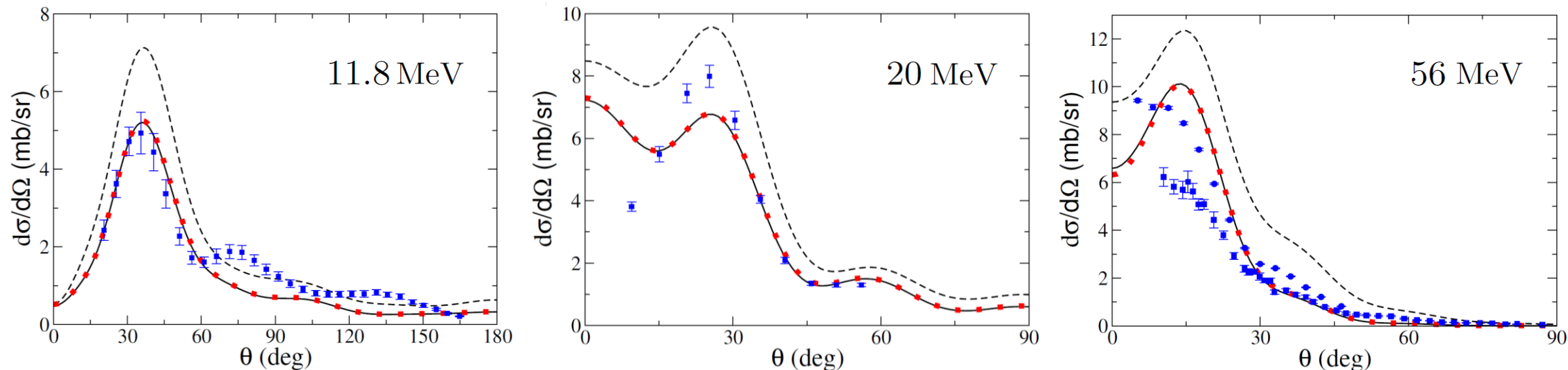
FIG. 1. Comparison of ^{40}Ca$(d,p)^{40}$Ca cross sections for the ground state at $E_d = 11.8$ MeV (top), 20 MeV (middle) and 56 MeV (bottom) using the DOM optical potentials. Cross sections are found with (solid lines) and without (dashed lines) I3B effects. We also present results for when $\mathrm{Re}\left[\Delta V_{NA}^{dyn}\right]$ is left unmodified (dotted lines).

Ratio of $\sigma_{I3B}^{peak}/\sigma^{peak}$

E_d (MeV)	NLDOM
11.8	0.725
20	0.691
56	0.821

11.8 MeV data: Nuc. Phys. **53**, 77
20 MeV data   : Nuc. Phys. A **506**, 159
56 MeV data   : Nuc. Phys. A **419**, 530 & Phys. Rev. C **50**, 263

Results: Importance of the dispersive term
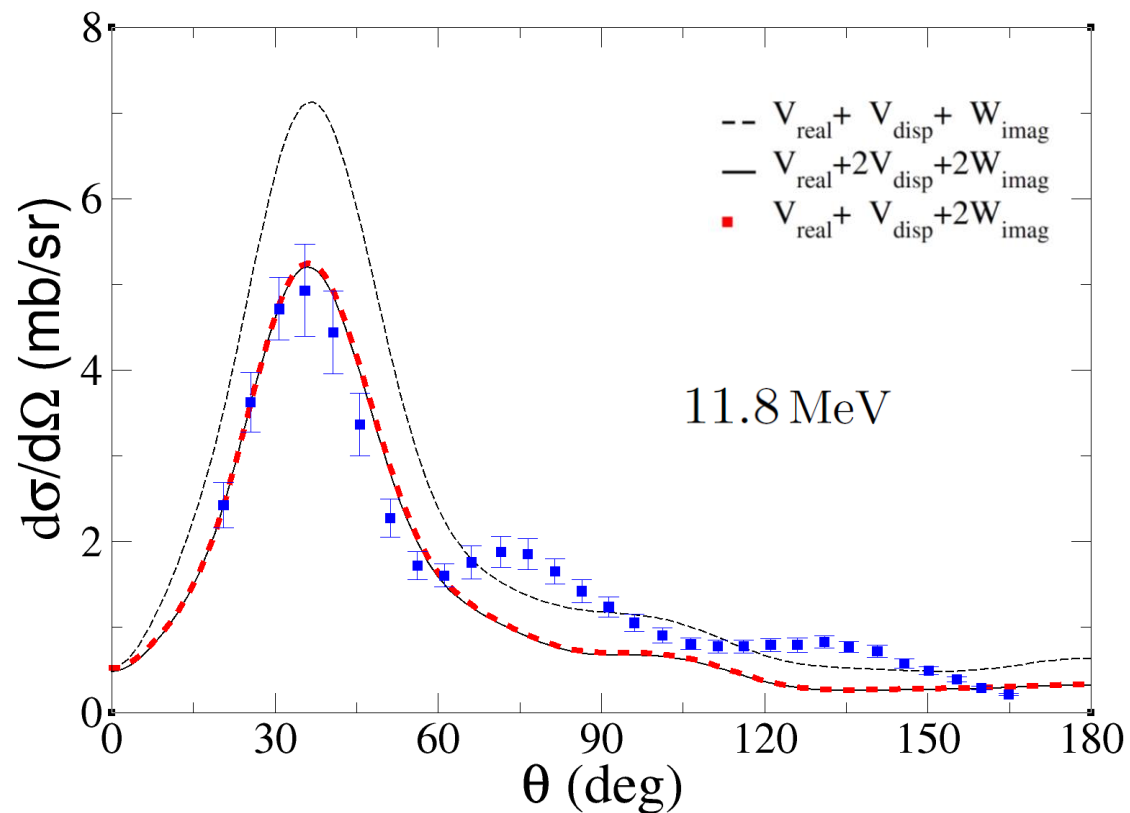
A consequence of including I3B terms by doubling the dynamic part of the potential, $\Delta V_{NA}^{dyn}(E)$, is that the dispersive term within it becomes proportionally less important.

$$\Delta V_{NA}^{dyn}(E) = iW_{NA}(E) + \frac{\mathcal{P}}{\pi}\int_{-\infty}^{\infty} dE' \frac{W_{NA}(E')}{E - E'}.$$

This is due to the dominance of the increased absorption.

NLDOM well depths (MeV) for E_d=11.8 MeV

	Real	Imaginary
Without I3B	-87	-18
With I3B	-95 (~10% increase)	-36 (100% increase)

Results: Importance of the dispersive term

	NLDOM	GR (Ann. Phys. **102**, 458)	GRZ (Ann. Phys. **124**, 208)	MSU (Phys. Rev. C **96**, 051601)
Global?	No, ^{40}Ca and ^{48}Ca only	Yes	Yes	Yes
Energy dependence?	Imaginary volume and surface terms	None	Imaginary surface terms	Imaginary volume and surface terms
Non-local?	Yes	Yes	Yes	Yes
Dispersive terms?	Yes	No	No	No
Is E_{eff} inside the energy range?	Yes	Yes	Yes	Outside of energy range

Results: Exact ^{40}Ca(d,p)^{41}Ca cross sections with GR and GRZ
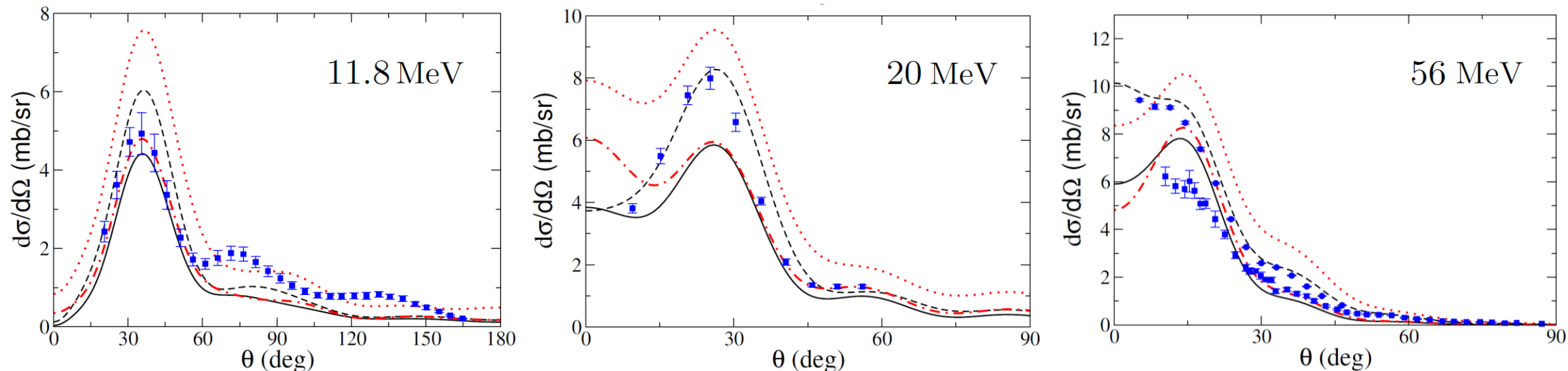
FIG. 2. Comparison of ^{40}Ca$(d, p)^{40}$Ca cross sections for the ground state at $E_d = 11.8$ MeV (top), 20 MeV (middle) and 56 MeV (bottom) using the GR potential with (solid lines) and without (dashed lines) I3B effects, along with cross sections found with the GRZ optical potential with (dot-dashed lines) and without (dotted lines) I3B effects. Experimental data from [23–26].

E_d (MeV)	Ratio of $\sigma_{I3B}^{peak}/\sigma^{peak}$		
	NLDOM	GR	GRZ
11.8	0.725	0.729	0.630
20	0.691	0.706	0.621
56	0.821	0.842	0.786

Conclusions

- We derive a simple modification to existing potentials to include induced three-body forces from $U^{(0)}$ in the ADWA for (d, p) reactions.

- Including induced three-body (I3B) forces reduces cross sections by 20-30% across a range of energies.

- When I3B effects are included the contribution from the dispersive part of the optical potential is minimal.

- Global phenomenological potentials (GR and GRZ) produce similar results to NLDOM when including I3B forces.

Future work

- Extend this work to nuclei of astrophysical significance: ^{26}Al, ^{30}P, ^{34}Cl.

- Inclusion of three nucleon forces.

Thanks to

- Supervisors and co-authors

Natasha Timofeyuk   Jim Al-Khalili   Ron Johnson

- Funding and studentship

Additional slides

Effects of an induced three-body force in the incident channel of (d,p) reactions

The many-body operator U

$$U = U_{pA} + U_{nA} + U_{nA}\frac{Q_A}{e}U_{pA} + U_{pA}\frac{Q_A}{e}U_{nA} + \ldots$$

The operator U can be expressed in terms of operators U_{pA} and U_{nA}, which define excitations of *A* by *p* and *n* separately. Up to second order these terms can be written

$$U^{(0)} = U_{pA} + U_{nA},$$

$$U^{(1)} = U_{nA}\frac{Q_A}{e}U_{pA} + U_{pA}\frac{Q_A}{e}U_{nA},$$

where

$$U_{NA} = v_{NA} + v_{NA}\frac{Q_A}{e - Q_A v_{NA} Q_A}v_{NA}.$$

In previous work NLO terms were neglected, and it was assumed that

$$U \approx U_{pA} + U_{nA}.$$

The many-body operator U: LO terms

Assuming that $\langle \phi_1 | U | \phi_0 \rangle \approx \langle \phi_1 | U^{(0)} | \phi_0 \rangle$, it was shown previously (Phys. Rev. C **89,** 024605) that

$$\langle \phi_1 \phi_A | U^{(0)} | \phi_A \phi_0 \rangle \approx \sum_{N=n,p} \langle \phi_1 \phi_A | U_{NA}^{opt}(E_{\text{eff}}) | \phi_0 \phi_A \rangle,$$

where
$$U_{NA}^{\text{opt}}(E_{\text{eff}}) = v_{NA} + v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA},$$

and is taken at a shifted energy $E_{\text{eff}} = \frac{1}{2} E_d + \frac{1}{2} \langle T_r \rangle.$

$$\langle \phi_1 \phi_A | U^{(0)} | \phi_A \phi_0 \rangle \approx \sum_{N=n,p} \langle \phi_1 \phi_A | v_{NA} | \phi_0 \phi_A \rangle$$
$$+ \langle \phi_1 \phi_A | v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA} | \phi_0 \phi_A \rangle$$

The many-body operator U: NLO terms

Using these same idea we can recover contributions from NLO terms in U,

$$U^{(1)} = U_{nA} \frac{Q_A}{e} U_{pA} + U_{pA} \frac{Q_A}{e} U_{nA}.$$

Approximating $U_{NA}^{\text{opt}}(E_{\text{eff}})$ by its leading value v_{NA} we get,

$$\langle \phi_1 \phi_A, \mathbf{R} | U^{(1)} | \phi_A \phi_0, \mathbf{R}' \rangle \approx \langle \phi_1 \phi_A, \mathbf{R} | v_{nA} \frac{Q_A}{e} v_{pA} | \phi_A \phi_0, \mathbf{R}' \rangle$$

$$+ \langle \phi_1 \phi_A, \mathbf{R} | v_{pA} \frac{Q_A}{e} v_{nA} | \phi_A \phi_0, \mathbf{R}' \rangle$$

$$\equiv U_{np}^{(1)}(\boldsymbol{R}, \boldsymbol{R}') + U_{pn}^{(1)}(\boldsymbol{R}, \boldsymbol{R}').$$

This potential is non-local in space $\mathbf{R}$.

The many-body operator U: NLO terms

To estimate its magnitude we consider the case of local, spin independent interactions v_{NA}, and ignore Coulomb contributions. We can rewrite $U_{np}^{(1)}(\boldsymbol{R}, \boldsymbol{R}')$ as

$$\boxed{\begin{aligned}\boldsymbol{r}_n &= \boldsymbol{R} - \frac{\boldsymbol{r}}{2}\\ \boldsymbol{r}_p &= \boldsymbol{R} + \frac{\boldsymbol{r}}{2}\end{aligned}}$$

$$U_{np}^{(1)}(\boldsymbol{R}, \boldsymbol{R}') = \langle \phi_1 \phi_A, \mathbf{R} | v_{nA} \frac{Q_A}{e} v_{pA} | \phi_A \phi_0, \mathbf{R}' \rangle$$

$$= \int d\xi_A d\boldsymbol{r} \, \phi_1^*(\boldsymbol{r}) \phi_A^*(\xi_A) v_{nA}(\boldsymbol{R} - \frac{\boldsymbol{r}}{2}, \xi_A) \tilde{\Psi}_p(\boldsymbol{r}_n, \boldsymbol{r}_p, \boldsymbol{R}', \xi_A),$$

where $\tilde{\Psi}_p(\boldsymbol{r}_n, \boldsymbol{r}_p, \boldsymbol{R}', \xi_A) = \langle \boldsymbol{r}_n, r_p, \xi_A | \frac{Q_A}{e} v_{pA} | \phi_0 \phi_A, \boldsymbol{R}' \rangle.$

If *p-A* and *n-A* interactions are equal,

$$\langle \phi_1 \phi_A, \mathbf{R} | v_{nA} \frac{Q_A}{e} v_{pA} | \phi_A \phi_0, \mathbf{R}' \rangle = \langle \phi_1 \phi_A, \mathbf{R} | v_{pA} \frac{Q_A}{e} v_{nA} | \phi_0 \phi_A, \mathbf{R}' \rangle.$$

Effects of an induced three-body force in the incident channel of (d,p) reactions

The many-body operator U: NLO terms

We can follow Phys. Rev. C **89** 024605 and replace the energy denominator in the same way,

$$\langle \phi_1 \phi_A, \mathbf{R} | U^{(1)} | \phi_A \phi_0, \mathbf{R}' \rangle = \langle \phi_1 \phi_A, \mathbf{R} | v_{NA} \frac{Q_A}{e} v_{NA} | \phi_A \phi_0, \mathbf{R}' \rangle$$

$$= \langle \phi_1 \phi_A | v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA} | \phi_0 \phi_A \rangle.$$

If we compare this to the LO term there is an obvious similarity

$$\langle \phi_1 \phi_A | U^{(0)} | \phi_A \phi_0 \rangle = \langle \phi_1 \phi_A | v_{NA} | \phi_0 \phi_A \rangle$$

$$+ \langle \phi_1 \phi_A | v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA} | \phi_0 \phi_A \rangle.$$

The many-body operator U: Induced three-body (I3B) forces

So we obtain our final form for the optical potential,

$$\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle \approx \langle \phi_1 \phi_A | v_{NA} + 2 v_{NA} \frac{Q_A}{E_{\text{eff}} - T_N - H_A - Q_A v_{NA} Q_A} v_{NA} Q_A | \phi_0 \phi_A \rangle,$$

or

$$\langle \phi_1 \phi_A | U^{(0)} + U^{(1)} | \phi_A \phi_0 \rangle \approx 2 \langle \phi_1 \phi_A | U^{(0)} | \phi_0 \phi_A \rangle - \langle \phi_1 \phi_A | v_{NA} | \phi_0 \phi_A \rangle.$$

This accounts for effects such as *n-A* and *p-A* excitations that cannot be expressed as a sum of **separate** *n* and *p* components (such as the target being excited by the *n* and de-excited by the *p*).

These additional terms create an effective three-body interaction that cannot be treated with *n-A, p-A* and *n-p* pairwise interactions.

Effects of an induced three-body force in the incident channel of (d,p) reactions

Method differences

$$T_{(d,p)} = \sqrt{S} \langle \chi_p^{(-)} \phi_n | V_{np} | \phi_d \chi_d^{(+)} \rangle$$

Using *TWOFNR* we use two different approaches to calculate the transition amplitude.

Solve χ_d and χ_p exactly	Generate adiabatic distorting potentials
$$(T_d^{(L)} + U_c(R_d) - E_d)\chi_{L'L}^J(R_d) =$$ $$- \sum_{L''} \int_0^\infty dR_d' \, R_d R_d' \, \mathcal{U}_{L'L''}^J(R_d, R_d') \, \chi_{L'}^J(R'),$$ $$(T_p^{(L)} + U_c(R_p) - E_p)\chi_L^J(R_p) =$$ $$- \int_0^\infty dR_p' \, R_p R_p' \, \mathcal{U}_L^J(R_p, R_p') \, \chi_L^J(R_p'),$$	$$U_{loc} = \sum_{i=1}^{7} U_i \exp\left(\frac{\mu_d \beta_i^2}{2\hbar^2} (E_d - U_c - U_{loc}) \right)$$ which *TWOFNR* will use to solve the Schrödinger equation $$(E_d - T_R - \langle \phi_1 \phi_A | U | \phi_0 \phi_A \rangle)\chi_d^{(+)}(\boldsymbol{R}) = 0$$

Method differences - Results

The non-local wavefunctions obtained with the exact method are smaller than those obtained from the local Schrödinger equation by the Perey factor

$$f(r) = \exp\left(\frac{\mu_d \beta^2}{4\hbar^2} U_{loc}\right).$$

With the increased absorption from the I3B terms the contribution from the nuclear interior becomes less important.
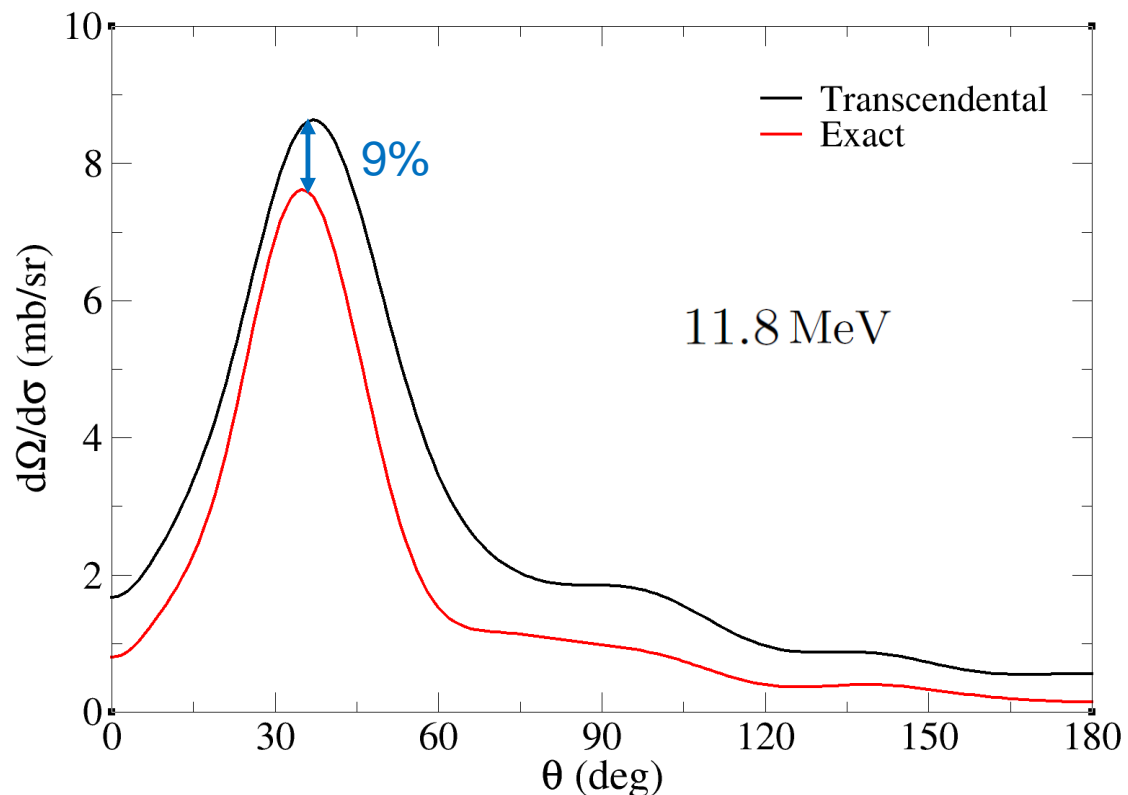
This reduces the difference between methods.

$$\langle \phi_A | U^{(0)} + U^{(1)} | \phi_A \rangle = V_{nA}^{HF} + 2\Delta V_{nA}^{dyn}(E),$$
$$+ V_{pA}^{HF} + 2\Delta V_{pA}^{dyn}(E).$$

Hartree-Fock (V_{NA}^{HF})	Dynamic ($\Delta V_{NA}^{dyn}(E)$)
Vol1	Sur+
Vol2	Sur-
Wine	Vol+
	Vol-

Method differences - Results

Previous work with exact methods (Phys. Rev. C **95**, 024603) showed a reduction of 10% in differential cross sections from the transcendental methods

Percentage difference at main peak

	Without I3B forces	
Energy	d-channel	p-channel
11.8 MeV	9%	10%
20 MeV	8%	7%
56 MeV	8%	7%
	With I3B forces	
Energy	d-channel	p-channel
11.8 MeV	5%	4%
20 MeV	5%	4%
56 MeV	2%	5%