Quantum Science with Ultracold Molecules

EPSRC
Engineering and Physical Sciences Research Council

Department of Physics
Clarendon Laboratory

UNIVERSITY OF OXFORD

Thermalisation in few-body systems: Revealing missing charges with quantum fluctuation relations

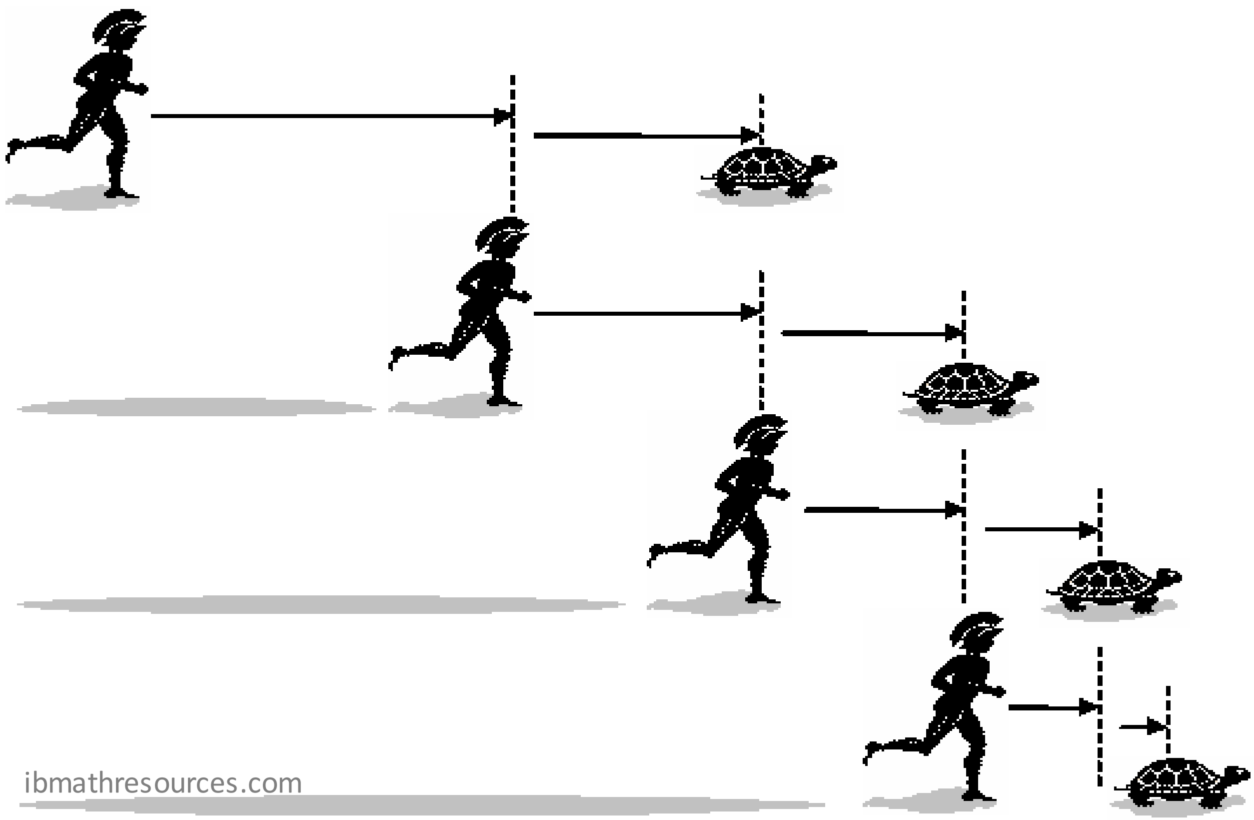
Jordi Mur-Petit

J. Mur-Petit, A. Relaño, R. A. Molina & D. Jaksch, Nature Commun. **9**, 2006 (2018)

Dynamics?

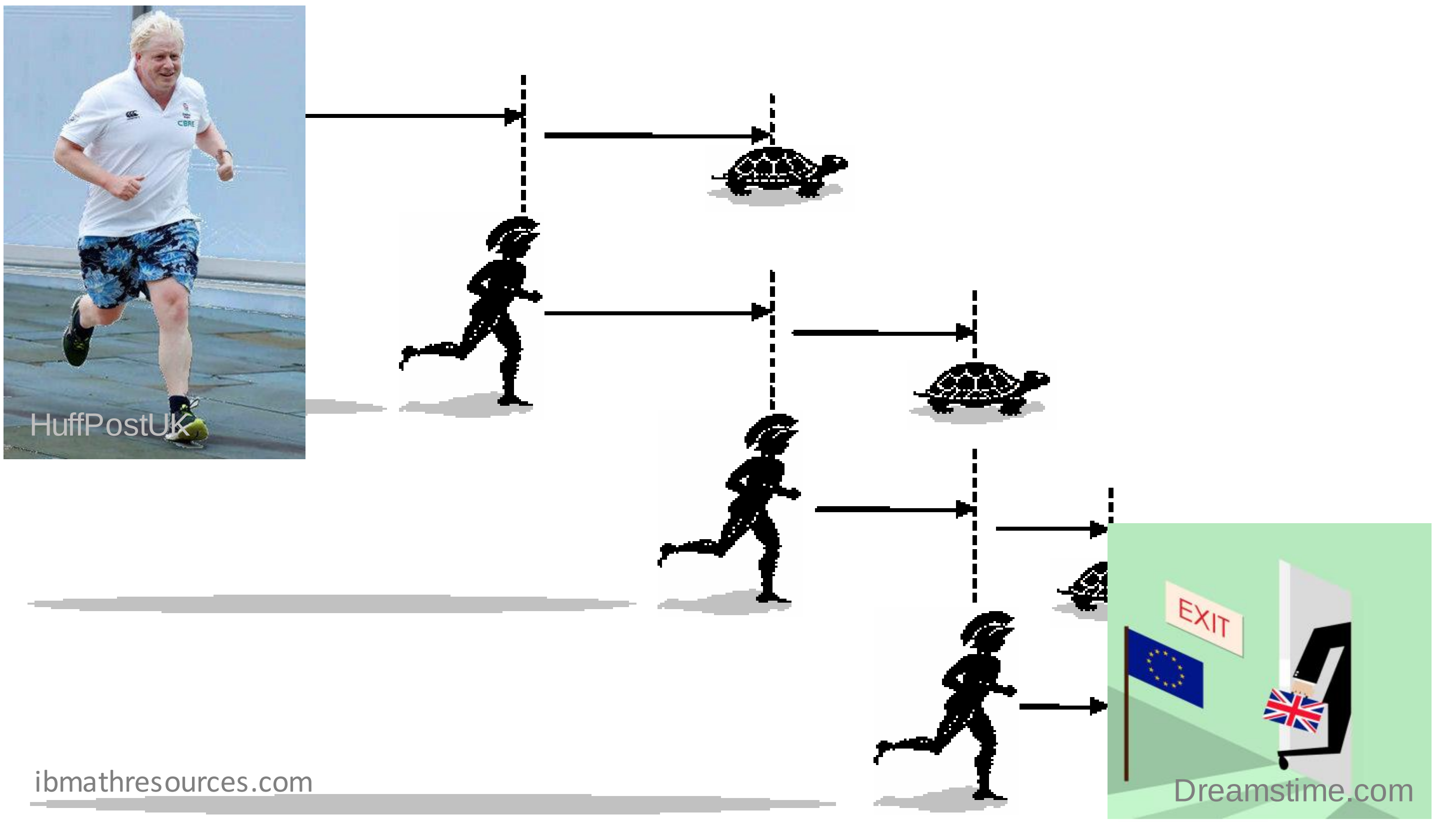
Achilles vs. tortoise

Zeno (5th Cent. BCE)



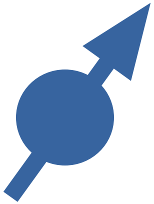
Dynamics?

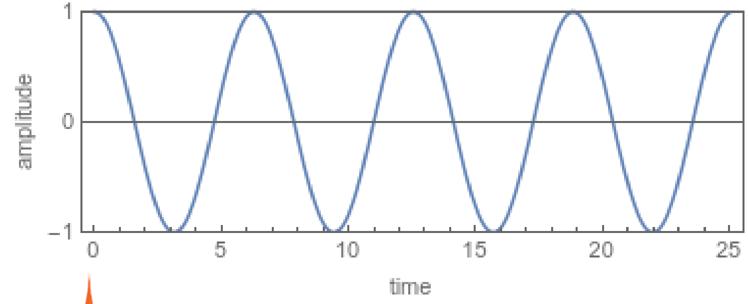
~~Achilles vs. tortoise~~ Boris vs. Parliament UK (2019)



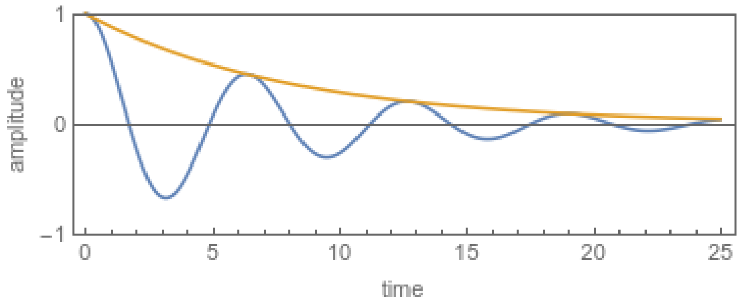
Quantum dynamics vs. Relaxation

Single isolated spin:
Bloch oscillations

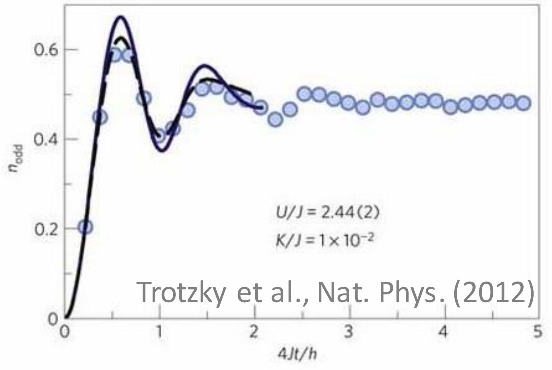




Single spin coupled to **large** bath: Dephasing



Many-body (closed):
Eigenstate Thermalisation Hypothesis (ETH)



Trotzky et al., Nat. Phys. (2012)

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Quantum dynamics vs. Relaxation

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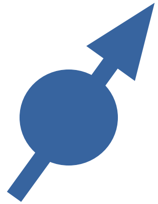

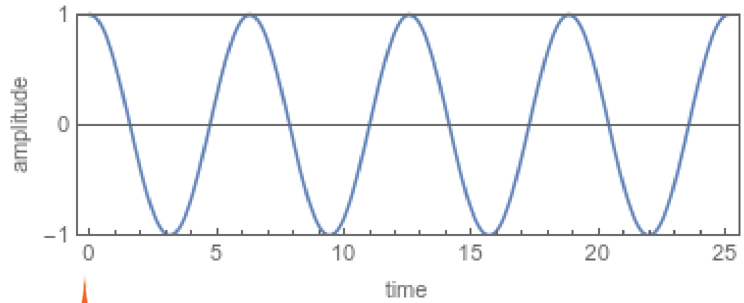

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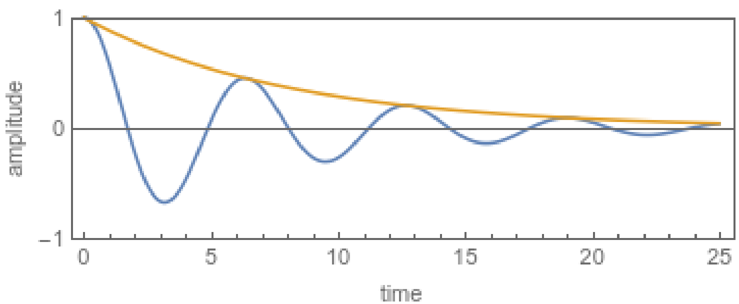

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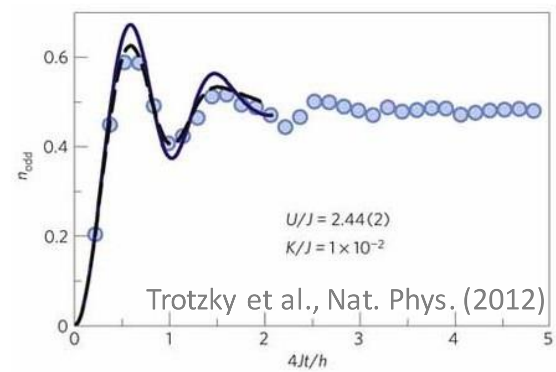
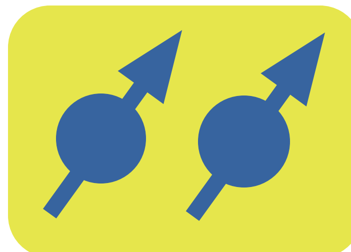


Trotzky et al., Nat. Phys. (2012)

How does relaxation emerge as number of particles increases?

Known exceptions to relaxation

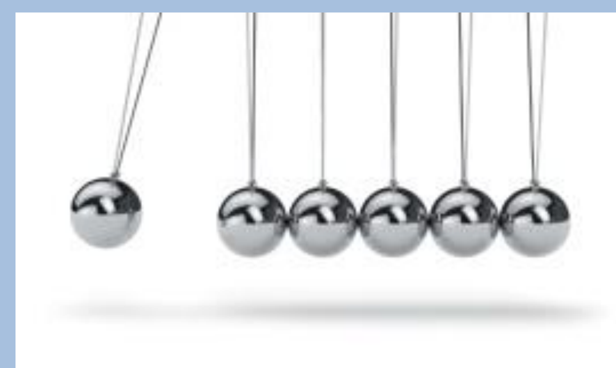
- Decoherence free subspaces



$$|0\rangle|0\rangle \longrightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \longrightarrow e^{i\phi}|0\rangle|1\rangle$$
$$|1\rangle|0\rangle \longrightarrow e^{i\phi}|1\rangle|0\rangle$$
$$|1\rangle|1\rangle \longrightarrow e^{2i\phi}|1\rangle|1\rangle$$

- Reservoir engineering

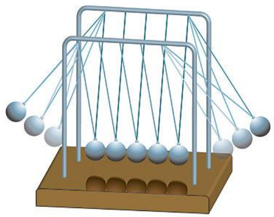
- Integrable systems

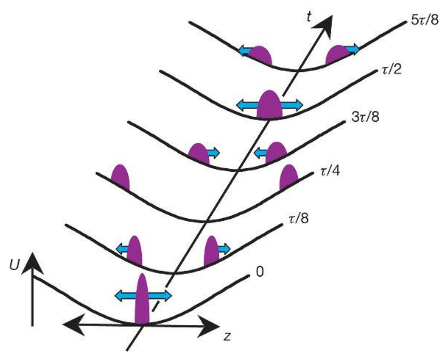


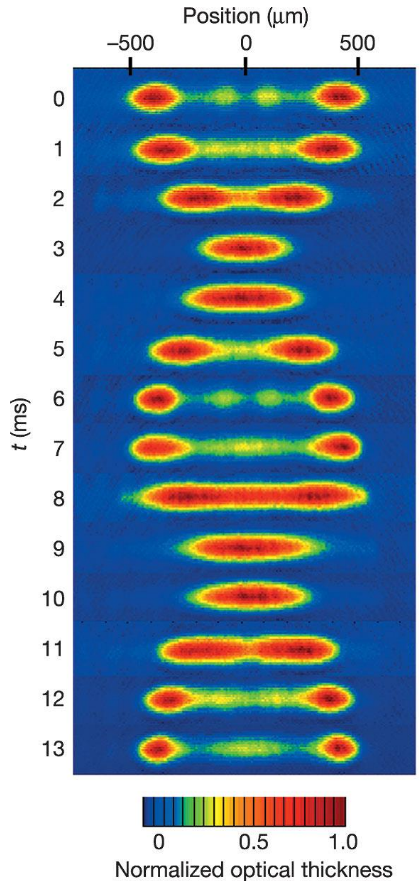
Integrable systems

Strongly interacting bosons in 1D

N~10^6







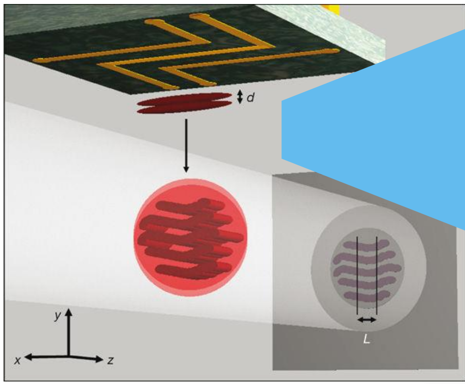
Kinoshita et al., Nature (2006)

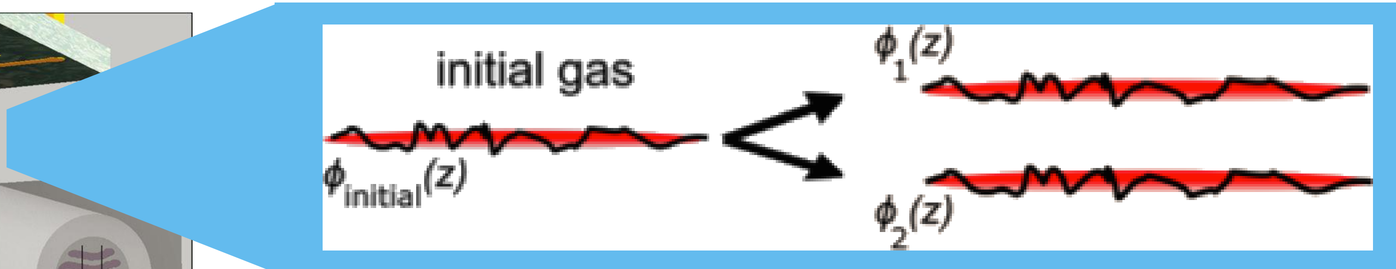
Energy & momentum conservation in each collision preclude relaxation

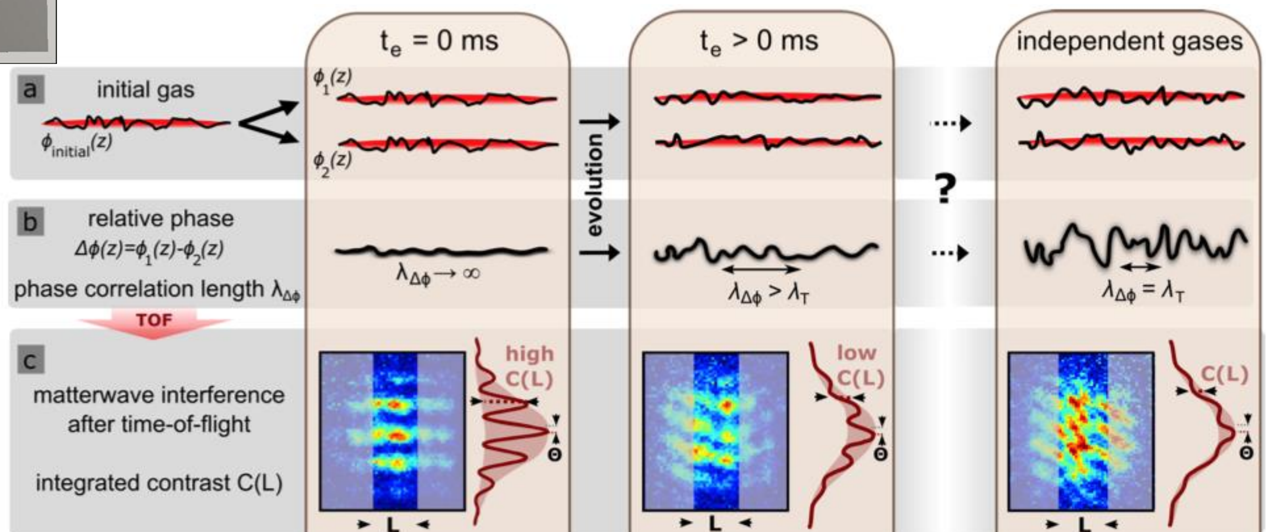
Relaxation in an integrable system

Strongly interacting bosons in 1D x 2

N~10^3







Hofferberth et al., Nature (2007); Gring et al., Science (2012); Langen et al., Science (2015) *(Schmiedmayer lab)*

Relaxation in an integrable system

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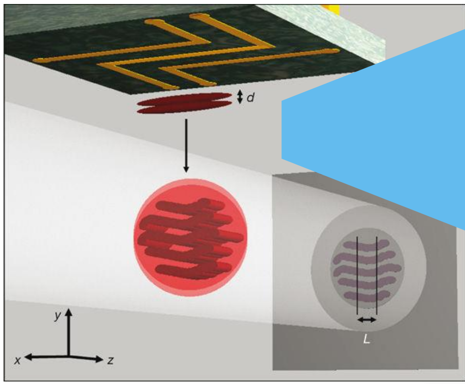
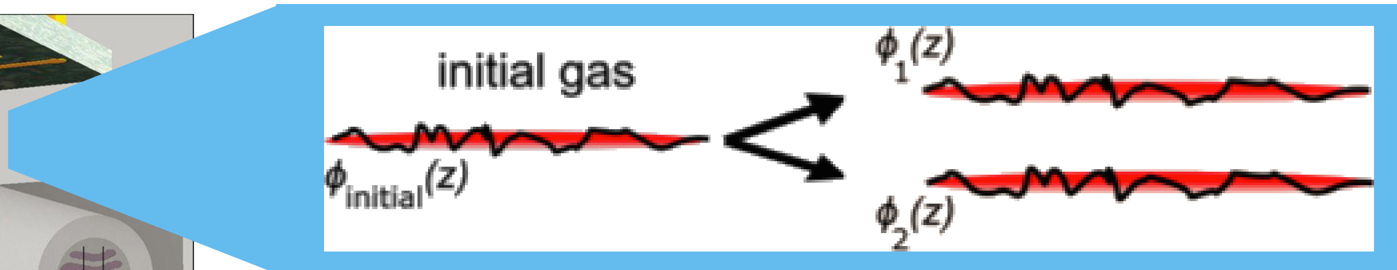
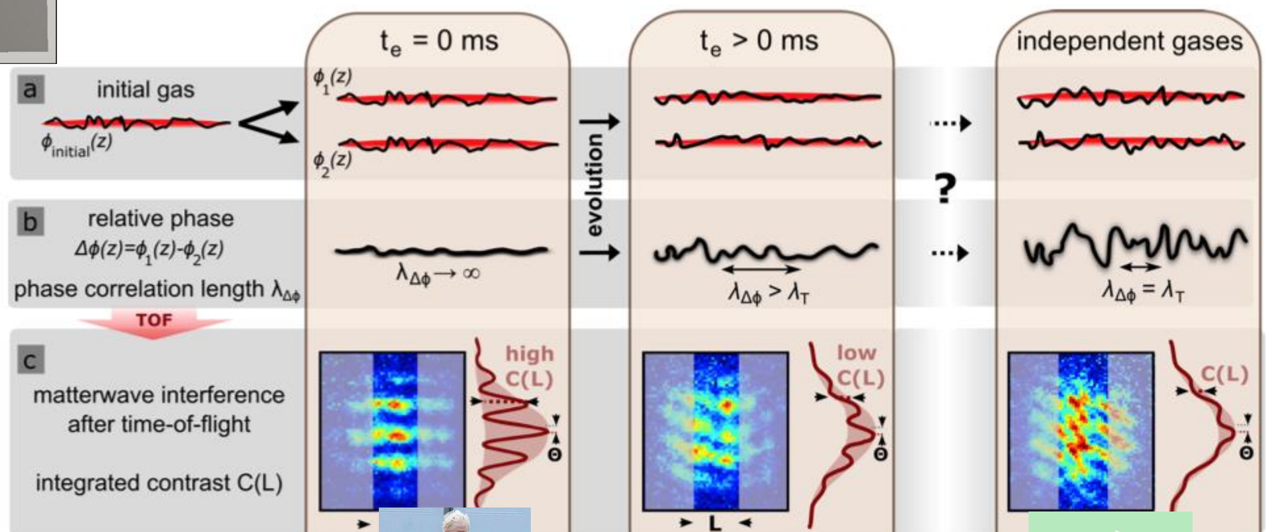


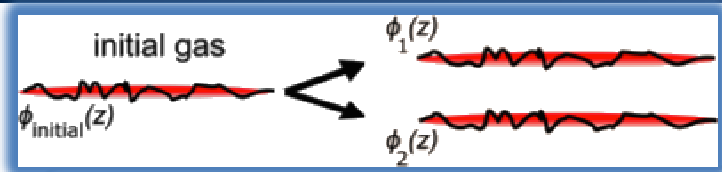




(?)

Hofferberth et al., Nature (2007); Gring et al., Science (… ngen et al., Science (2015) *(Schmiedmayer lab)*

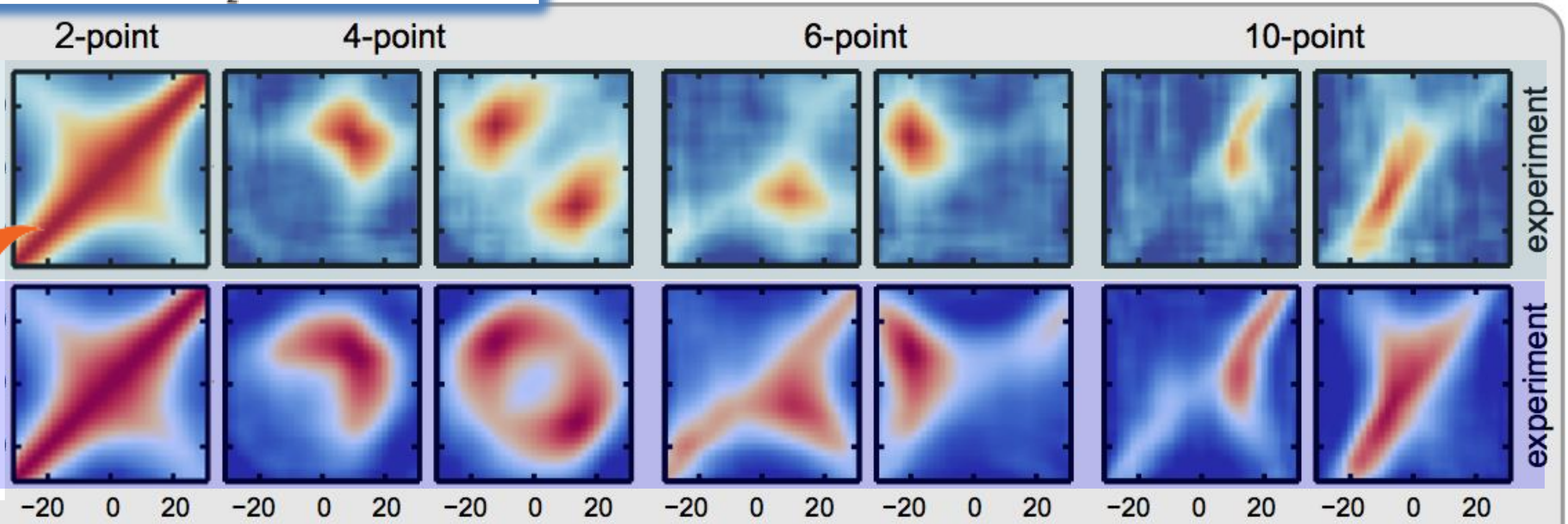
Relaxation in an integrable system



$$C(z_1, \ldots, z_N) = \langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\ldots\Psi_1^\dagger(z_N)\Psi_2(z_N)\rangle$$

‘slow split’

‘fas sp t’



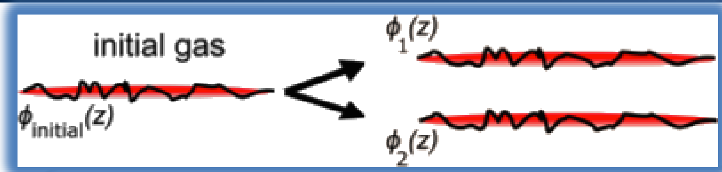
Slow split: Correlations match **Gibbs ensemble** with effective T=1/β

$$\hat{\rho}(t \to \infty) \to \hat{\rho}_\beta = \exp(-\beta H)/Z$$

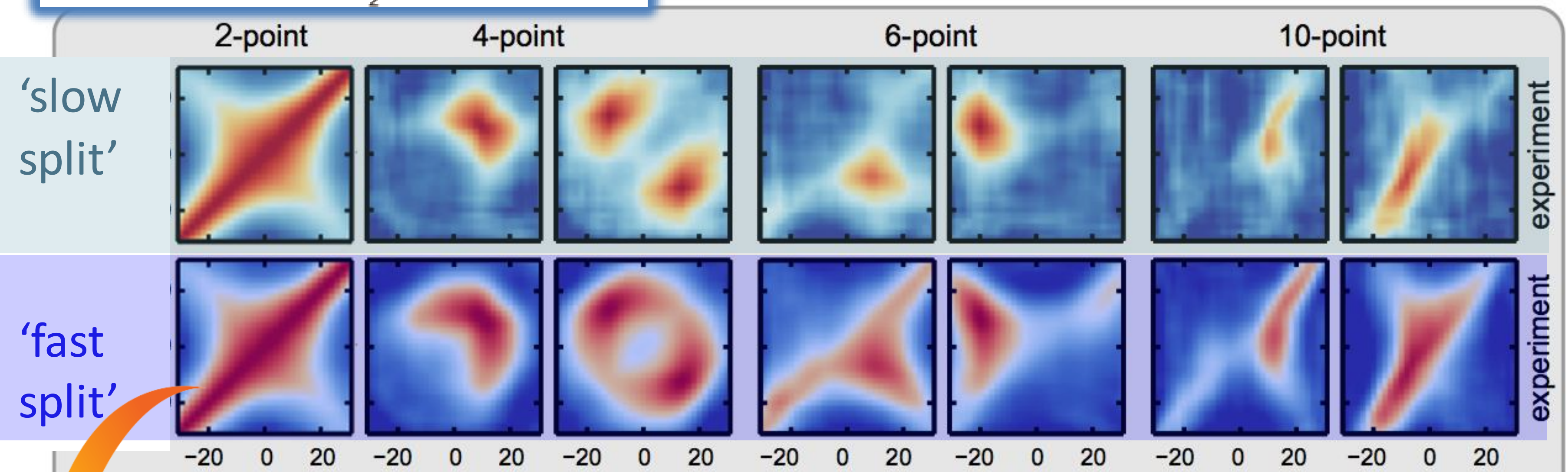
(BRINO)

Langen et al., Science 348, 207 (2015)

Relaxation in an integrable system



$$C(z_1, \dots, z_N) = \langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\dots\Psi_1^\dagger(z_N)\Psi_2(z_N)\rangle$$



Fast split: Up to 10 different 'temperatures' to match!

$$\hat{\rho}(t \to \infty) \to \hat{\rho}_{GGE} = \frac{1}{Z}\exp(-\beta \hat{H} - \sum_k \beta_k \hat{M}_k) \;,\; [\hat{M}_k, \hat{H}] = 0$$

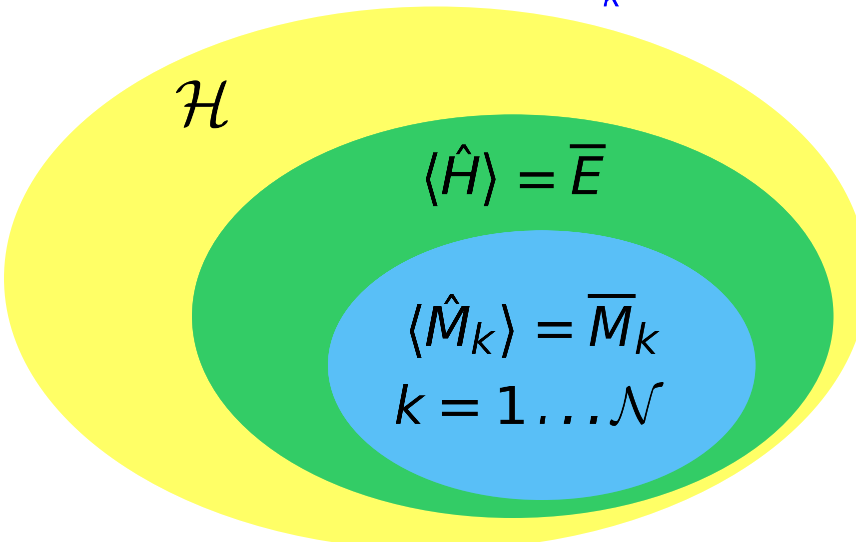
$\beta \hat{H}$ → energy conservation

$\beta_k \hat{M}_k$ → other conserved charges

Langen et al., Science 348, 207 (2015)

Pre-thermalised states

$$\hat{\rho}(t \to \infty) \to \hat{\rho}_{\mathrm{GGE}} = \frac{1}{Z}\exp\left(-\beta \hat{H} - \sum_k \beta_k \hat{M}_k\right) \;\; , \;\; [\hat{M}_k, \hat{H}] = 0$$



- Generalized Gibbs ensemble **(GGE)**
- Conserved charges prevent relaxation to 'true' thermal equilibrium: **→ Pre-thermalised state**

How do we harness this for small-*N* systems?

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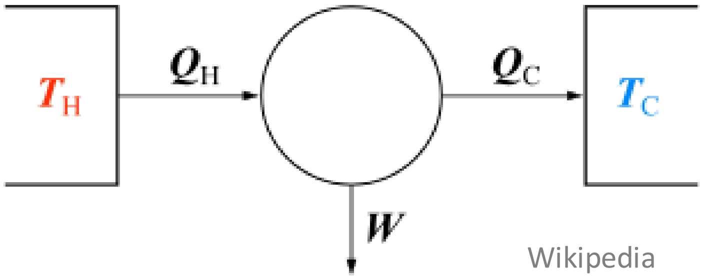
Thermodynamic Laws

Macroscopic systems: N≥10^{24}

$$w \leq \Delta F \,, \quad \Delta S \geq 0$$



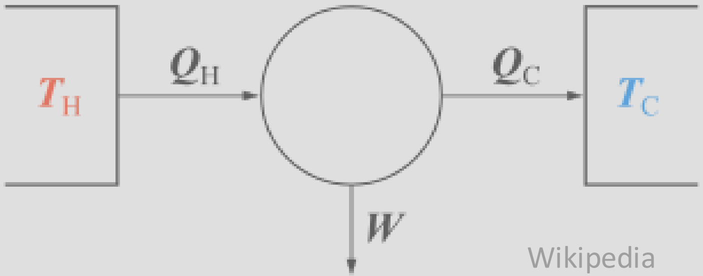
Wikipedia

'Puffing Billy' (1813), Pockerley Waggonway, Beamish museum

Fluctuation relations: classical

Macroscopic systems: N≥10²⁴

$$w \leq \Delta F\,, \quad \Delta S \geq 0$$



Wikipedia

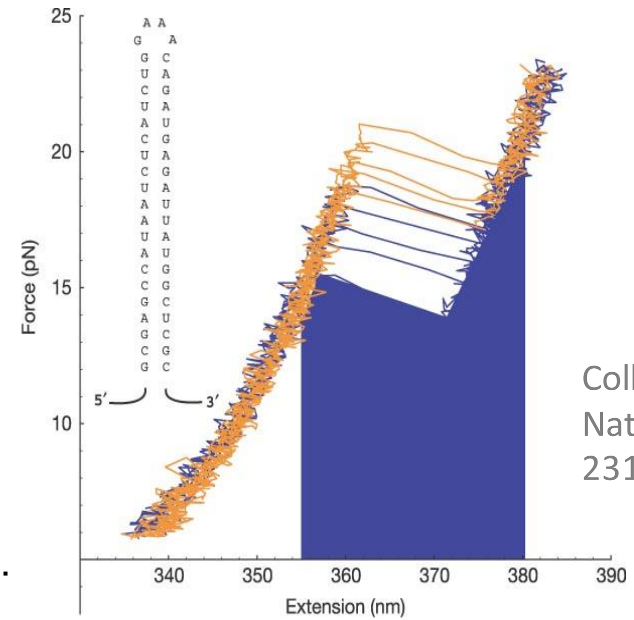
Classical systems

Jarzynski equality

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

Crooks fluctuation relation

$$P_f(w) = e^{\beta(w-\Delta F)} P_b(-w)$$



Collin et al., Nature 437, 231 (2005)

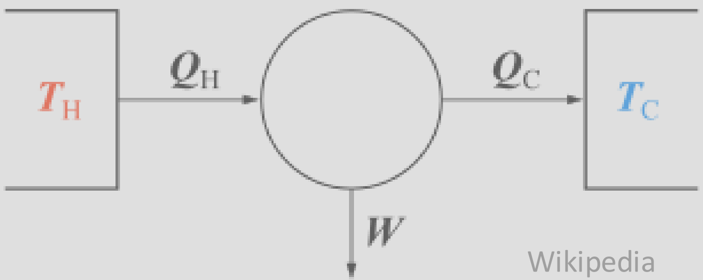
- Constrain PDF: $P(w) \mapsto \langle w \rangle\,, \langle w^2 \rangle\,, \langle w^3 \rangle\,, \ldots$
- Equilibrium properties from non-equil. measurements

Jarzynski (1997); Crooks (1999)

Fluctuation relations: quantum

Macroscopic systems: N≥10^24

$$w \leq \Delta F\,, \quad \Delta S \geq 0$$



Classical systems

Jarzynski equality

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Crooks fluctuation relation

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Quantum systems

Quantum Jarzynski equality (QJE)

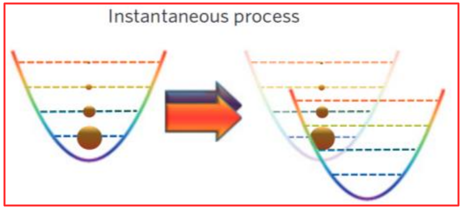
$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

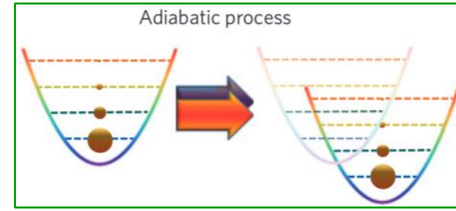
Tasaki-Crooks relation (TCR)

$$P_f(w) = e^{\beta(w-\Delta F)} P_b(-w)$$

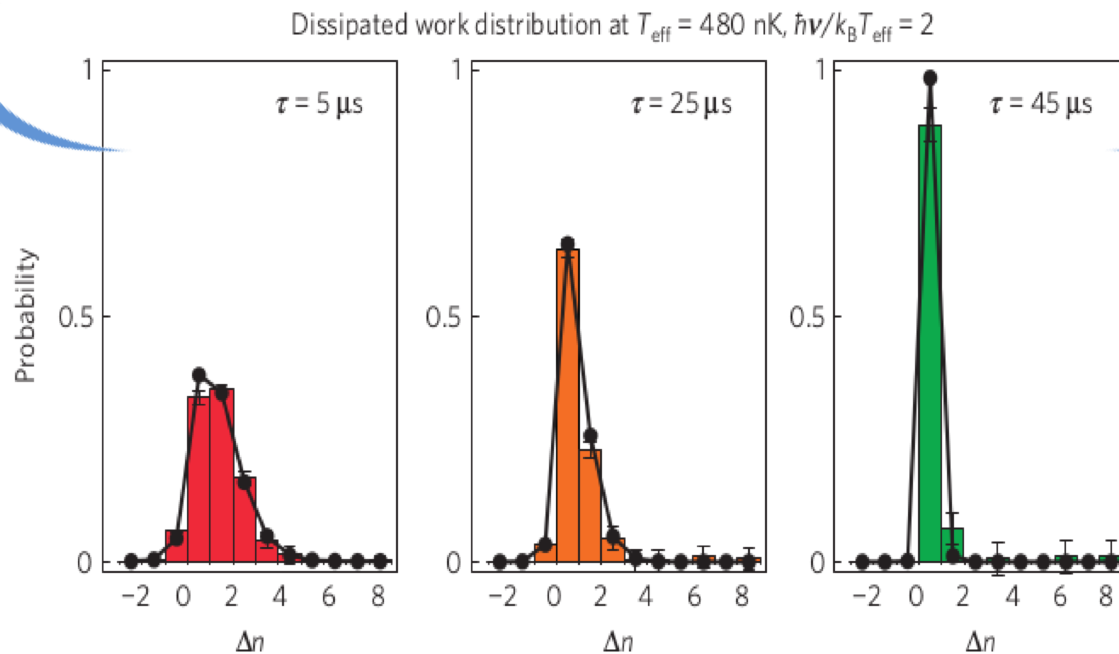
QJE: Tasaki (2000), Kurchan (2000), Yukawa (2000); *TCR:* Tasaki (2000), Monnai (2005)

Testing the QJE with N=1 ion





Fast quench ⟷ Slow quench



$\Delta n \propto w$

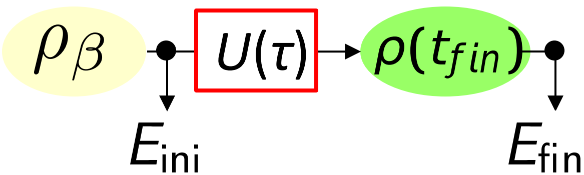
Bars: experiment

Lines: calculation

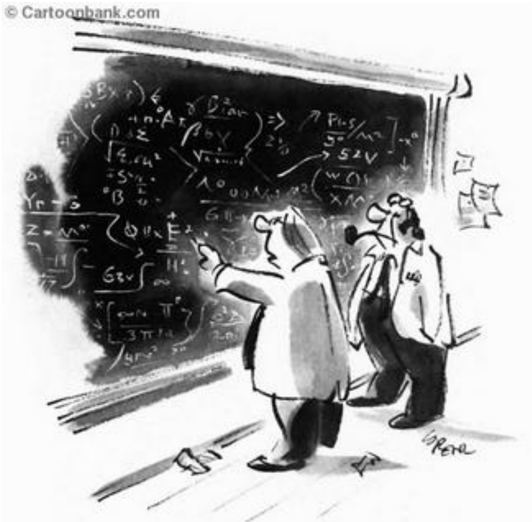
An et al. (Kim lab), Nature Phys. **11**, 193 (2015)

QFRs: The small print

1. Work defined via
two energy-projection measurements



$$w = E_{\text{fin}} - E_{\text{ini}}$$
$$= \text{Tr}[U\rho_\beta U^{-1} H_{\text{fin}}] - \text{Tr}[\rho_\beta H_{\text{ini}}]$$



"Here's your problem—you forgot the sleaze factor."

Talkner, Hänggi et al., J. Phys. A 40, F569 (2007);
Talkner & Hänggi, PRE 93, 022131 (2016)

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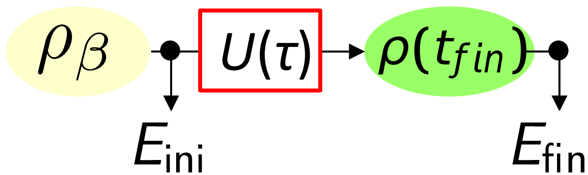

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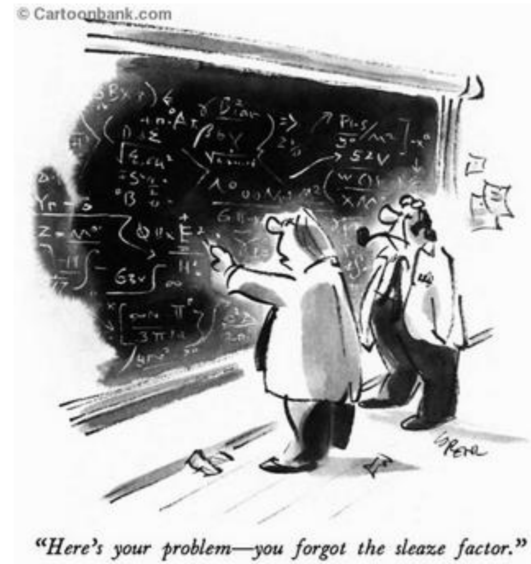
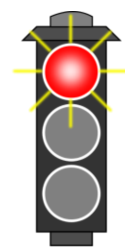


2. Initial state = canonical (Gibbs) equilibrium state

$$\rho(t=0) = \rho_\beta = \frac{1}{Z}\exp[-\beta H_{\text{ini}}]$$



~~GGEs~~

Talkner, Hänggi et al., J. Phys. A 40, F569 (2007);
Talkner & Hänggi, PRE 93, 022131 (2016)

QFRs for GGEs

Quantum systems: Gibbs

Q. Jarzynski equality (QJE)

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

Tasaki-Crooks relation (TCR)

$$P_f(w) = e^{\beta(w-\Delta F)} P_b(-w)$$

Quantum systems: GGE

Generalised QJE

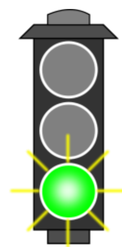
$$\langle e^{-W} \rangle = e^{-\Delta F}$$

Generalised TCR

$$P_{FW}(W) = e^{W-\Delta F} P_{BW}(-W)$$

$$A_{\text{ini}} = \beta E_{\text{ini}} + \sum_k \beta_k M_{k,\text{ini}}, \; A_{\text{fin}} = \beta' E'_{\text{fin}} + \sum_l \beta'_l M'_{l,\text{fin}}$$

$$w = E_{\text{fin}} - E_{\text{ini}} \mapsto W = A_{\text{fin}} - A_{\text{ini}}$$ *Generalised work*



GGEs!

Hickey & Genway, PRE (2014);
Guryanova et al.; Yunger Halpern et al., Nature Commun. (2016)

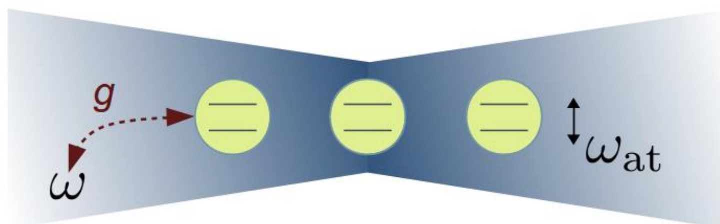
JMP, Relaño, Molina & Jaksch,
Nature Commun. **9**, 2006 (2018)

Testing ground: Dicke model

$$\hat{H} = \hbar\omega_{\text{com}}\hat{a}^\dagger\hat{a} + \hbar\omega_{\text{at}}\hat{J}_z + \hat{H}_{\text{int}}\ , \qquad \hat{J}_{x,y,z} = \sum_{j=1}^{N}\frac{1}{2}\sigma^{(j)}_{x,y,z}$$

***N*=7**

$$\hat{H}_{\text{int}} = \frac{2g}{\sqrt{N}}\left[(1-\alpha)(\hat{J}_+\hat{a} + \hat{J}_-\hat{a}^\dagger) + \alpha(\hat{J}_+\hat{a}^\dagger + \hat{J}_-\hat{a})\right]$$



Two phases $g_{cr} = \sqrt{\omega\omega_{at}}/2$

- g>g → Superradiant
- g<g → Subradiant

Two regimes

- ***α*={0, 1}** → Integrable (TCM)

$$\hat{M} = \hat{J} + \hat{J}_z + \hat{a}^\dagger\hat{a}$$

- Otherwise → Not integrable

Testing ground: Dicke model

$$\hat{H} = \hbar\omega_{\text{com}}\hat{a}^\dagger\hat{a} + \hbar\omega_{\text{at}}\hat{J}_z + \hat{H}_{\text{int}}\,, \qquad \hat{J}_{x,y,z} = \sum_{j=1}^{N}\frac{1}{2}\sigma^{(j)}_{x,y,z}$$

N=7

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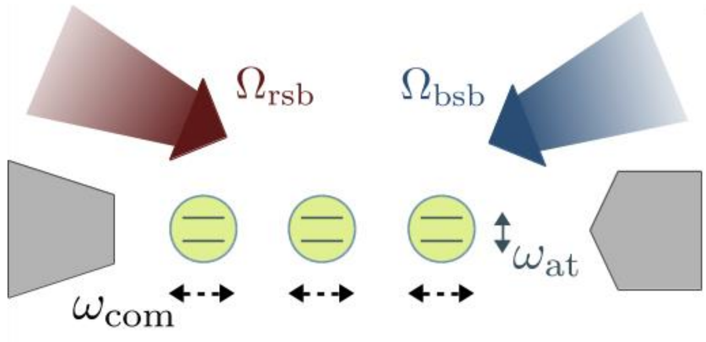

$$g = (\Omega_{\text{rsb}} + \Omega_{\text{bsb}})/2$$
$$\alpha = \Omega_{\text{bsb}}/(\Omega_{\text{rsb}} + \Omega_{\text{bsb}})$$

Two phases $g_{\text{cr}} = \sqrt{\omega\omega_{\text{at}}}/2$

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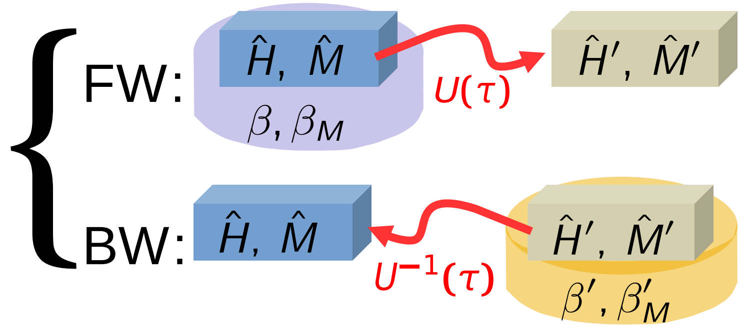
Nature Commun. **9**, 2006 (2018)

Dicke model: Protocol

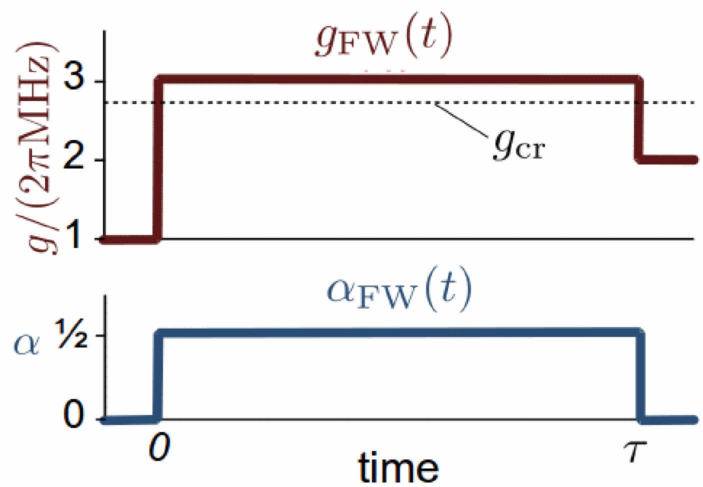
1. Prepare system in GGE state, α=0
2. Non-equilibrium protocol: quench α
3. Repeat many times to collect statistics of work [*]



Repeat with time-reversed protocol (BW)

$$g = (\Omega_{\text{rsb}} + \Omega_{\text{bsb}})/2$$

$$\alpha = \Omega_{\text{bsb}}/(\Omega_{\text{rsb}} + \Omega_{\text{bsb}})$$



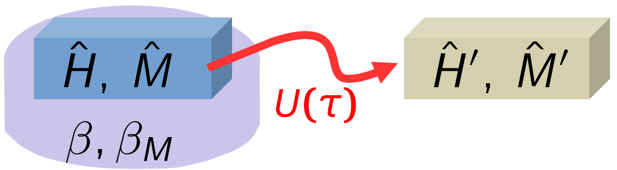
[*] Filtering protocol: Huber et al., PRL 101, 070403 (2008);
An et al. (Kim lab), Nature Phys. 11, 193 (2015)

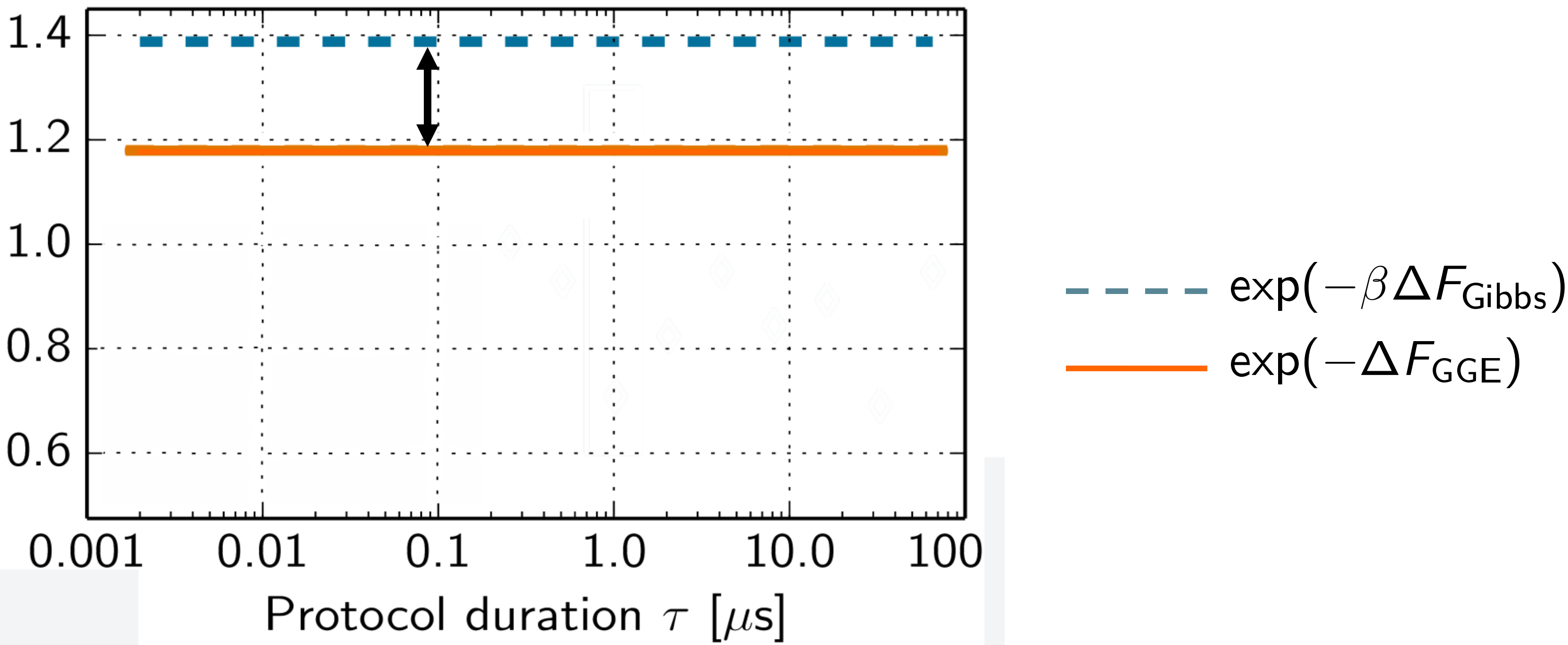
JMP, Relaño, Molina & Jaksch,
Nature Commun. **9**, 2006 (2018)

QJE: Varying protocol duration τ

– std: $\langle \exp(-\beta w) \rangle = \exp(-\beta \Delta F_{\text{Gibbs}})$

– gen: $\langle \exp(-W) \rangle = \exp(-\Delta F_{\text{GGE}})$





$\beta = 0.1, \ \beta_M = 0.3$

JMP, Relaño, Molina & Jaksch,
Nature Commun. **9**, 2006 (2018)

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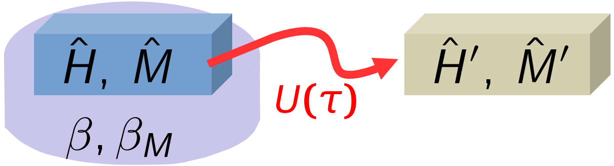
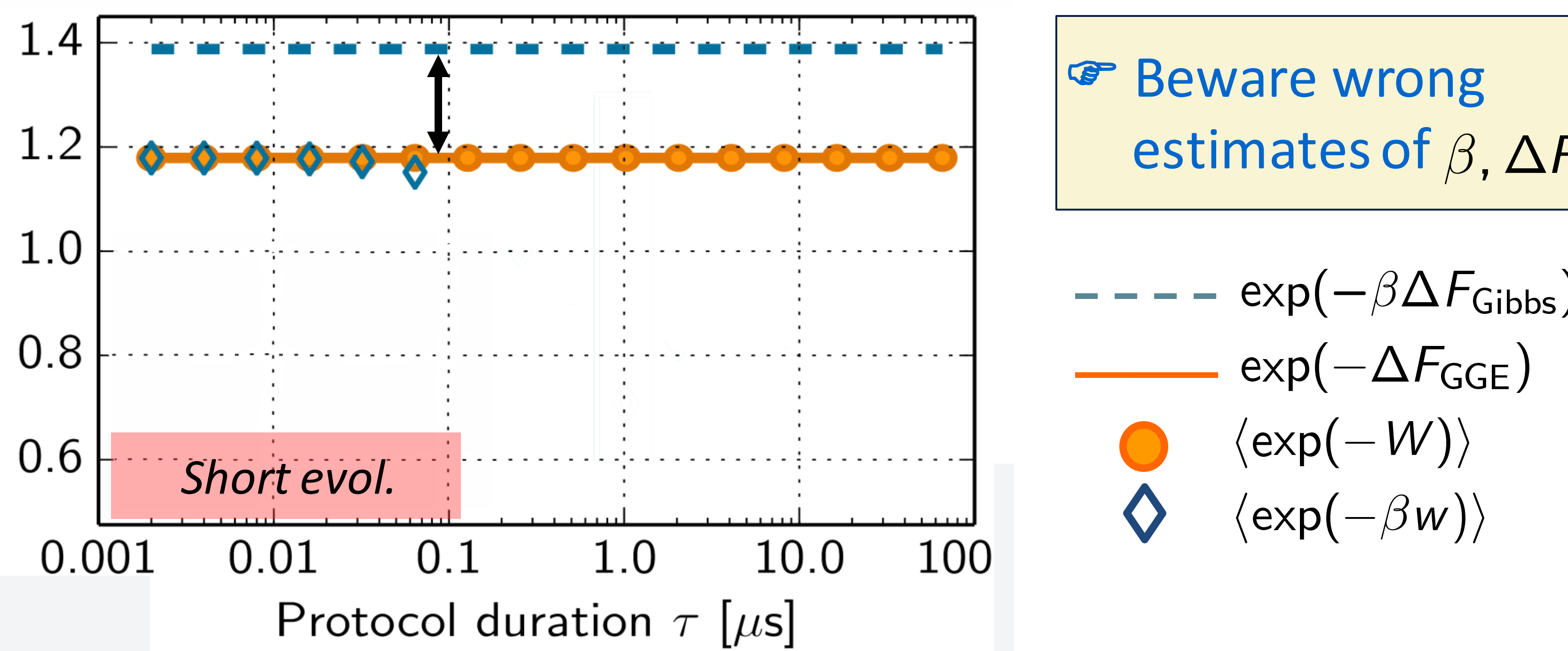




☞ Beware wrong estimates of β, ΔF

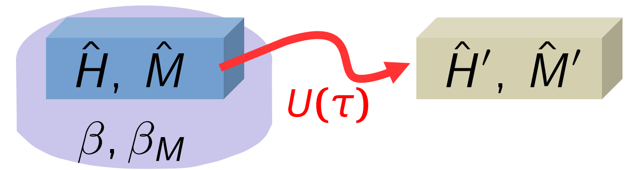
$\beta = 0.1,\ \beta_M = 0.3$

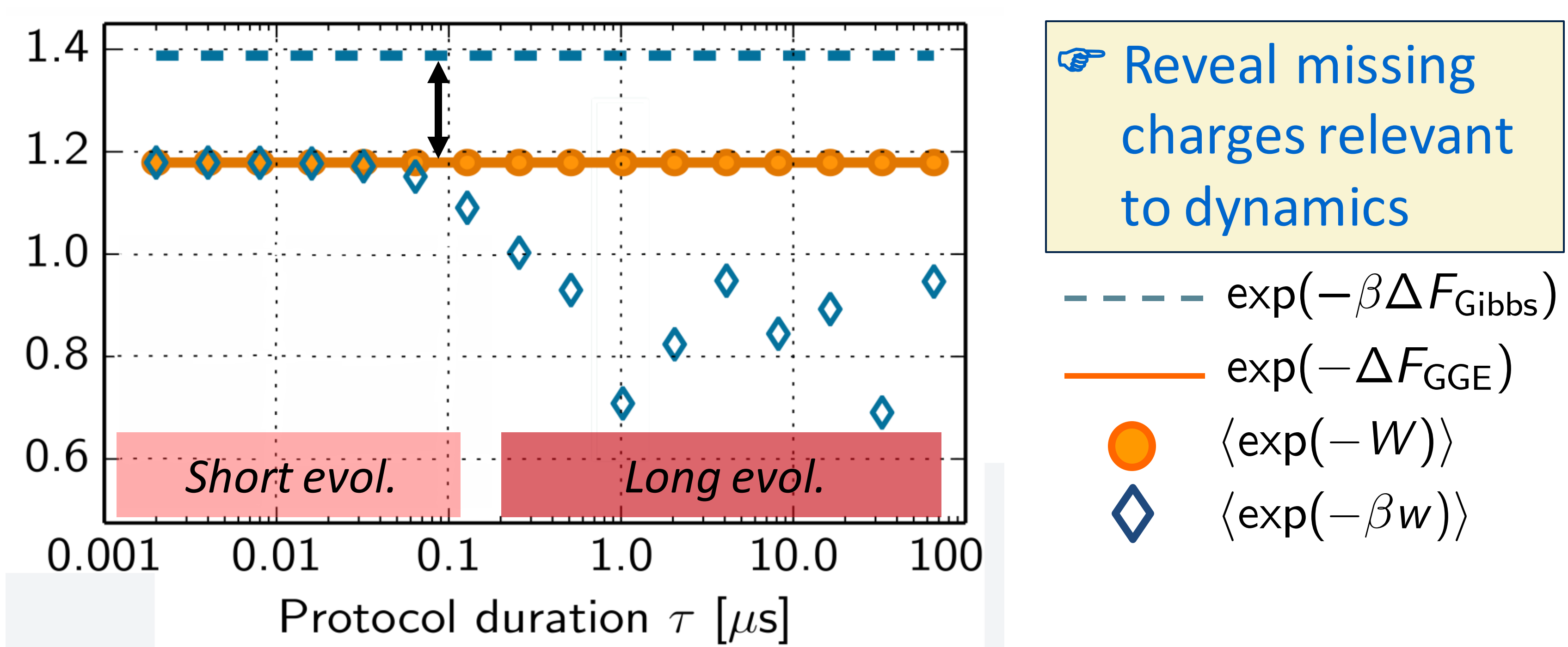
JMP, Relaño, Molina & Jaksch,
Nature Commun. **9**, 2006 (2018)

QJE: Varying protocol duration τ

– std: $\langle \exp(-\beta w) \rangle = \exp(-\beta \Delta F_{\text{Gibbs}})$

– gen: $\langle \exp(-W) \rangle = \exp(-\Delta F_{\text{GGE}})$





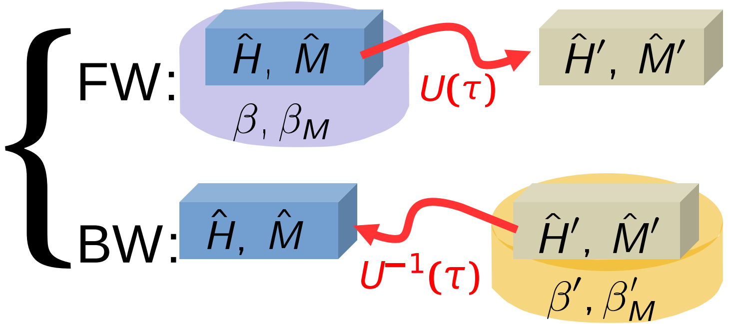
☞ Reveal missing charges relevant to dynamics

$\beta = 0.1,\ \beta_M = 0.3$

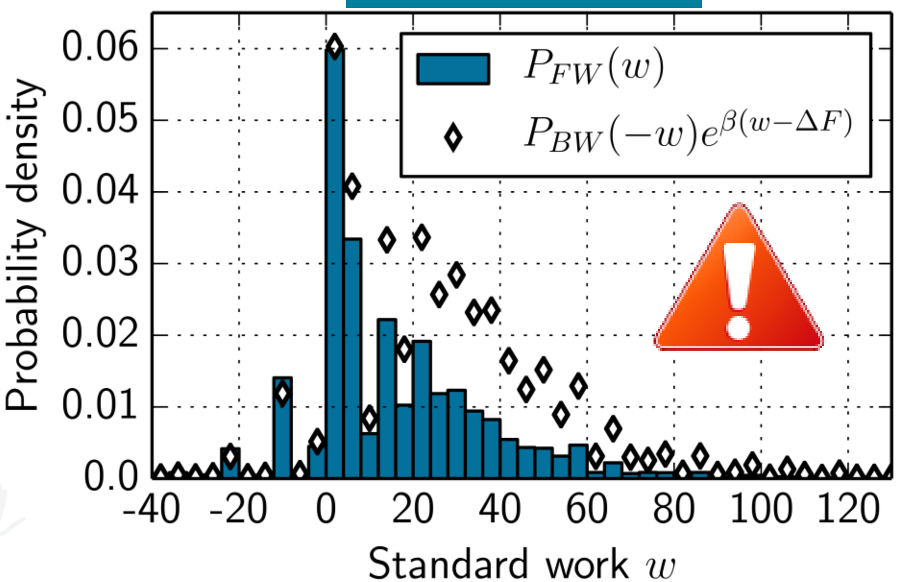
JMP, Relaño, Molina & Jaksch, Nature Commun. **9**, 2006 (2018)

Dicke model: TCR for τ≈1 μs

$$(*)\ P_{FW}(w) = e^{\beta(w-\Delta F_{Gibbs})} P_{BW}(-w)$$



Standard TCR



- Detect if ρ_{ini} is missing charges
- Beware wrong estimates of β, ΔF

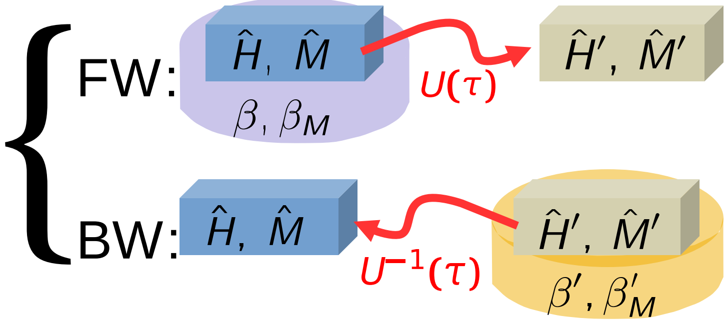
$$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_M = -0.1)$$

Nature Commun. **9**, 2006 (2018)

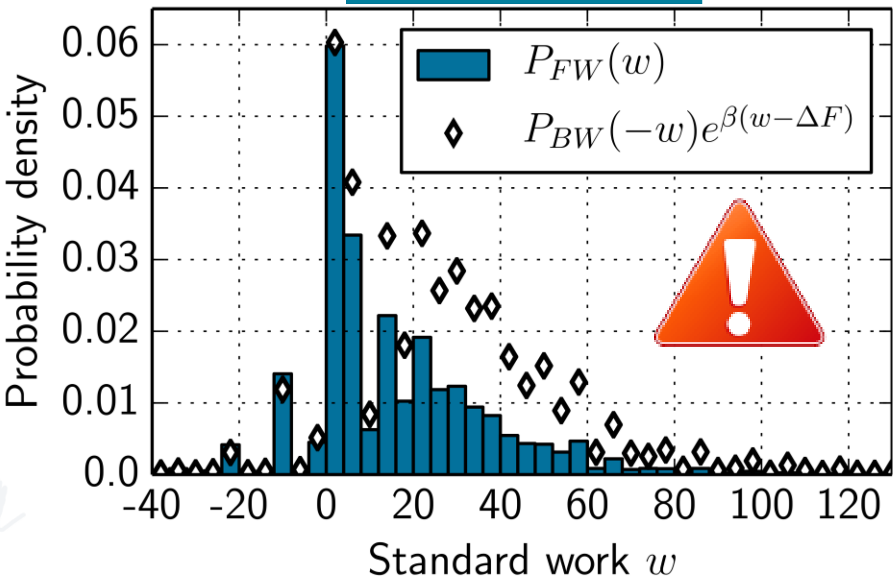
Dicke model: TCR for τ≈1 μs

$$(*)\ P_{FW}(w) = e^{\beta(w-\Delta F_{Gibbs})} P_{BW}(-w)$$

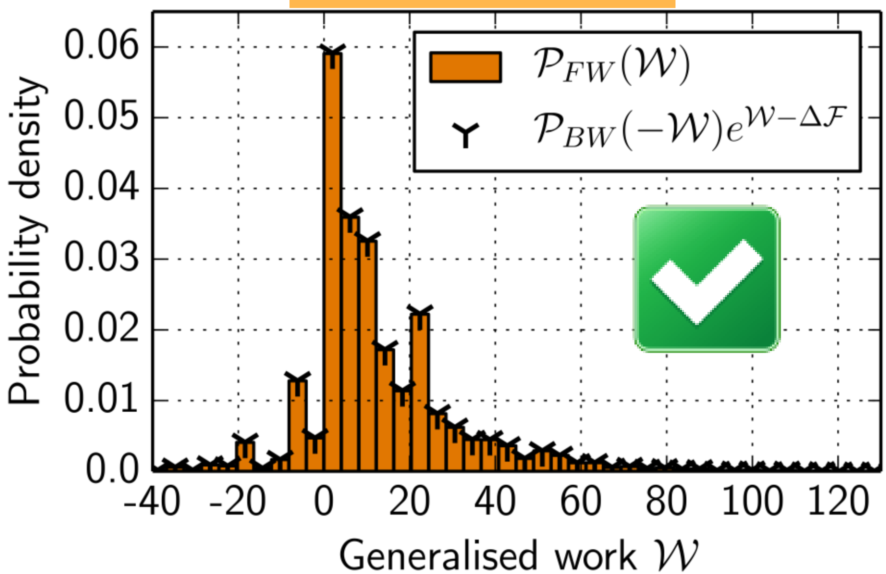
$$P_{FW}(W) = e^{W-\Delta F_{GGE}} P_{BW}(-W)$$



Standard TCR



Generalised TCR



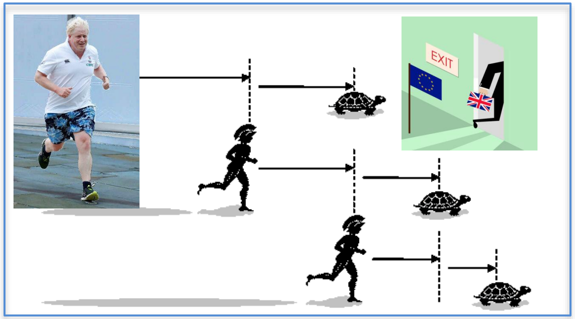
$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_M = -0.1)$

Nature Commun. **9**, 2006 (2018)

Summary & Outlook

Few-body q. systems:

- Ideal testing ground for Q. Thermo.
  - Relaxation vs. conservation laws

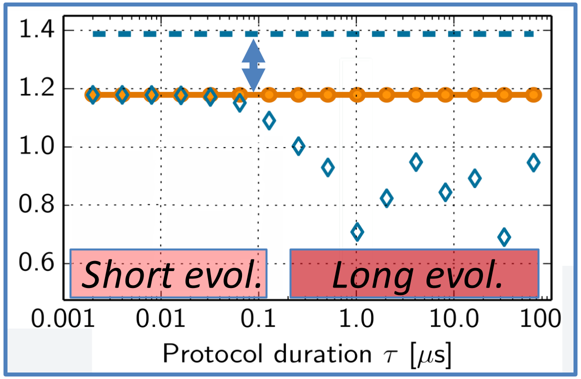


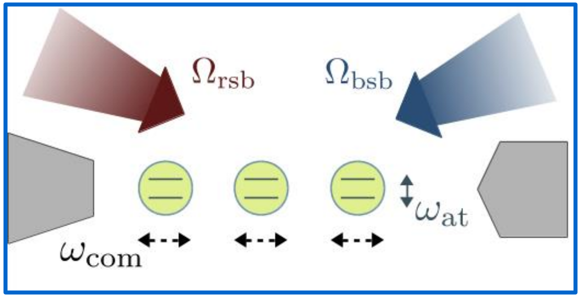
New QFRs for systems with charges:

$$\langle e^{-W} \rangle = e^{-\Delta F}$$

- Reveal hidden charges
- Avoid biased temperature estimates
- Readily testable in experiments

Nature Commun. **9**, 2006 (2018)



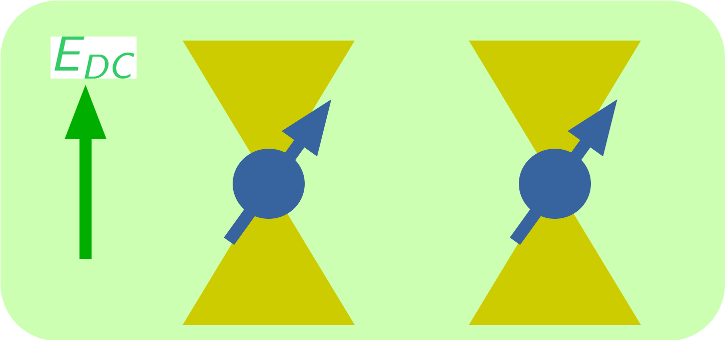


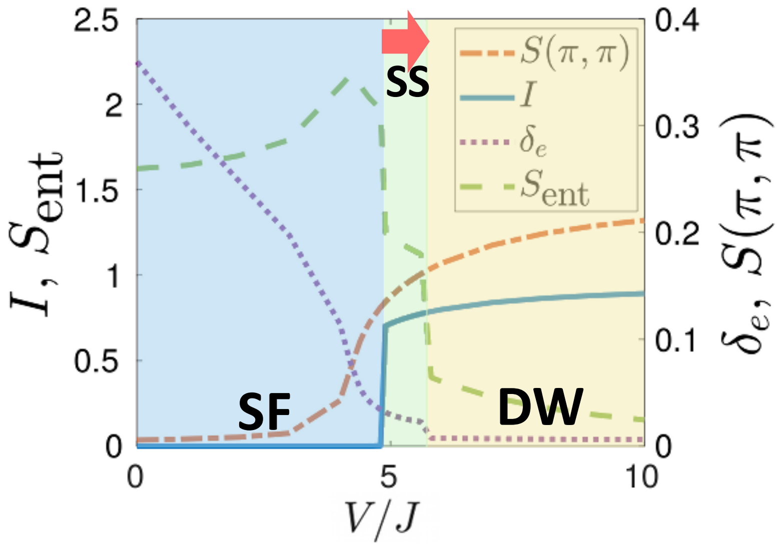
Outlook

- Multiple charges: XXZ…
- Efficiency of quantum nanodevices?

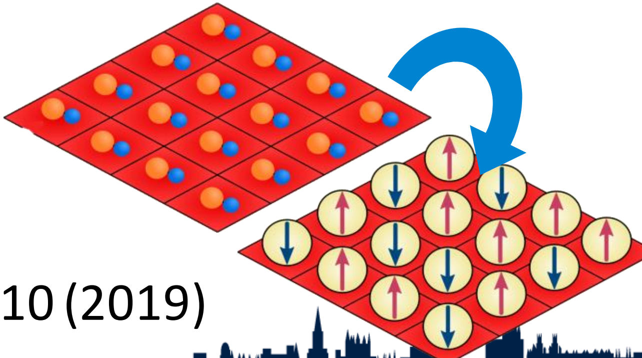
Ongoing research projects

- Controlling dipolar interaction between polar molecules in tweezers
  M. Hughes et al., to be submitted





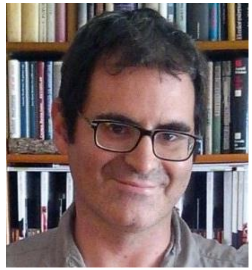
- From few- to many-body: Quantum phases in small cold-matter systems (10-100 part.)
  P. Rosson, M. Kiffner, JMP, D. Jaksch, to be submitted



- Polar molecules for quantum simulation *with Uni. Durham & Imperial College*
  J. Blackmore et al., Quantum Sci Tech **4**, 014010 (2019)

Thank you!

Armando Relaño



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