Guildford, September 2-6, 2019

24th European Conference on Few-Body Problems in Physics (EFB24)

Description of continuum structures in a discrete basis: Three-body resonances and two-nucleon decays

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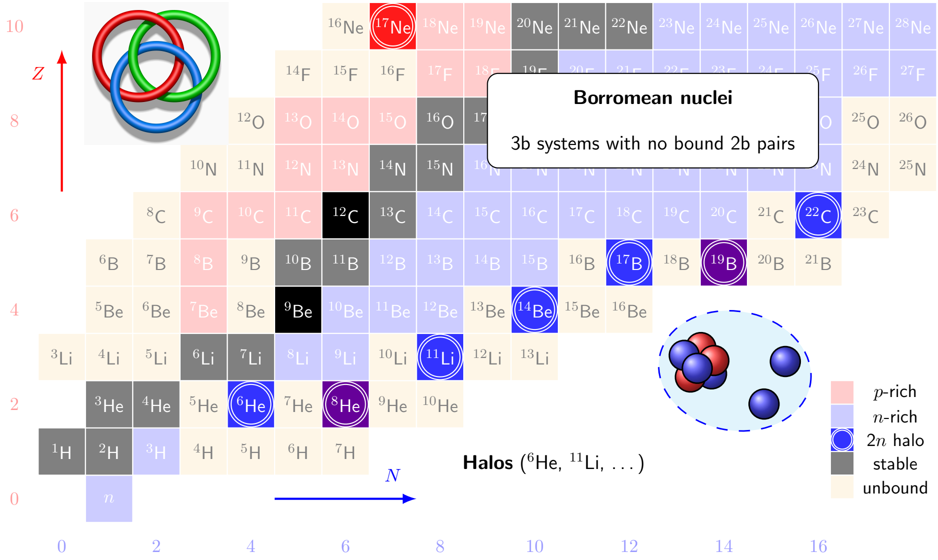
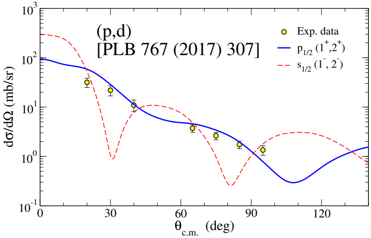
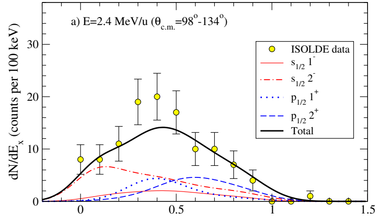
INFN

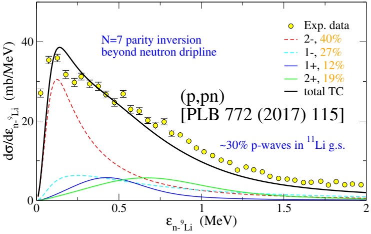

Borromean nuclei
3b systems with no bound 2b pairs
Halos (^{6}He, ^{11}Li, . . .)
Z
N
^{16}Ne ^{17}Ne ^{18}Ne ^{19}Ne ^{20}Ne ^{21}Ne ^{22}Ne ^{23}Ne ^{24}Ne ^{25}Ne ^{26}Ne ^{27}Ne ^{28}Ne
^{14}F ^{15}F ^{16}F ^{17}F ^{25}F ^{26}F ^{27}F
^{12}O ^{13}O ^{14}O ^{15}O ^{16}O ^{25}O ^{26}O
^{10}N ^{11}N ^{12}N ^{13}N ^{14}N ^{15}N ^{24}N ^{25}N
^{8}C ^{9}C ^{10}C ^{11}C ^{12}C ^{13}C ^{14}C ^{15}C ^{16}C ^{17}C ^{18}C ^{19}C ^{20}C ^{21}C ^{22}C ^{23}C
^{6}B ^{7}B ^{8}B ^{9}B ^{10}B ^{11}B ^{12}B ^{13}B ^{14}B ^{15}B ^{16}B ^{17}B ^{18}B ^{19}B ^{20}B ^{21}B
^{5}Be ^{6}Be ^{7}Be ^{8}Be ^{9}Be ^{10}Be ^{11}Be ^{12}Be ^{13}Be ^{14}Be ^{15}Be ^{16}Be
^{3}Li ^{4}Li ^{5}Li ^{6}Li ^{7}Li ^{8}Li ^{9}Li ^{10}Li ^{11}Li ^{12}Li ^{13}Li
^{3}He ^{4}He ^{5}He ^{6}He ^{7}He ^{8}He ^{9}He ^{10}He
^{1}H ^{2}H ^{3}H ^{4}H ^{5}H ^{6}H ^{7}H
n
10 8 6 4 2 0
0 2 4 6 8 10 12 14 16
p-rich
n-rich
$2n$ halo
stable
unbound

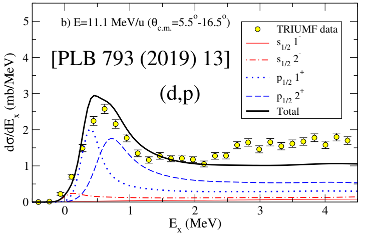
Some publicity: **Continuum spectrum of ^{10}Li** (our lithium "trilogy")

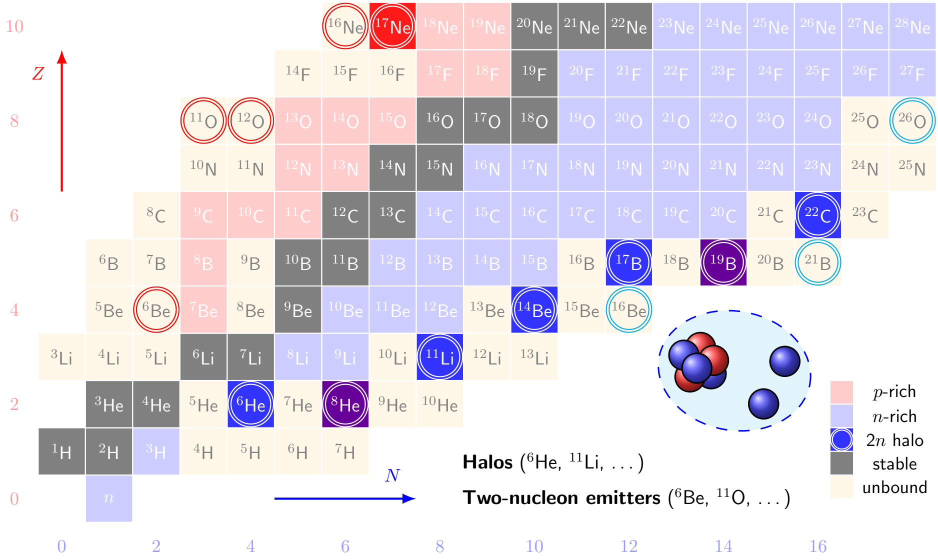
- reaction dynamics: ^{11}Li$(p,d)^{10}$Li; ^{11}Li$(p,pn)^{10}$Li; ^{9}Li$(d,p)^{10}$Li
- study g.s. properties of two-neutron halo ^{11}Li
- explore unbound ^{10}Li (^{9}Li $+ n$) states





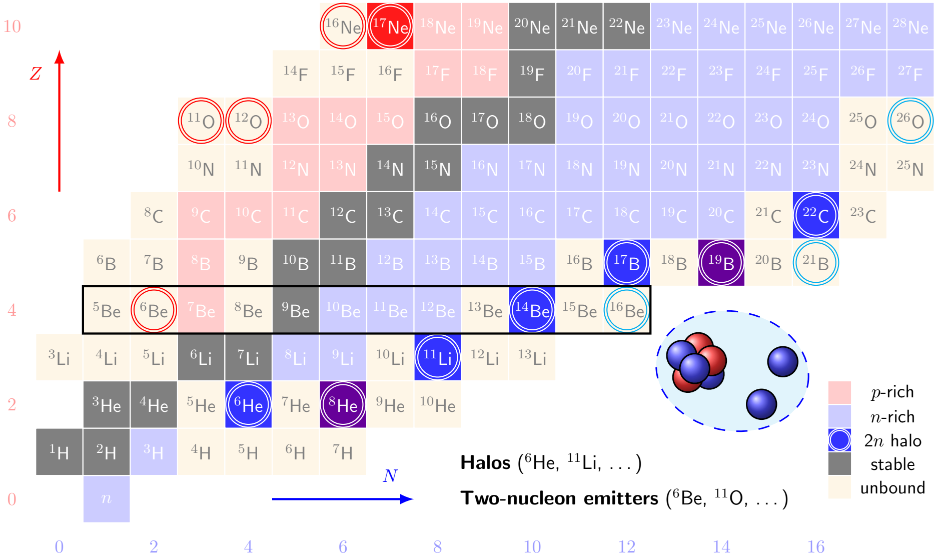






Halos (^{6}He, ^{11}Li, ...)

Two-nucleon emitters (^{6}Be, ^{11}O, ...)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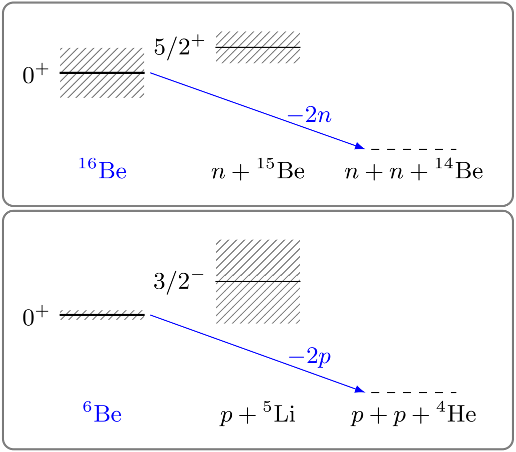

Possible $2N$ decay paths:

- Sequential
- Simultaneous ("Dinucleon"-like)
- "Democratic"

^{6}Be, ^{16}Be

$1N$ bound
$2N$ unbound

two-nucleon correlations in the ground state?

J.C. [PRC 97 (2018) 034613] arXiv:1801.01280

Description of unbound few-body systems

Solution of an A-body scattering problem with proper asymptotics

Alternative: **Pseudostate (PS) method**

Diagonalize $\mathcal{H}$ in a given basis of $\mathcal{L}^2$ functions. Positive-energy eigenstates provide a discrete representation of the actual continuum

Description of unbound few-body systems

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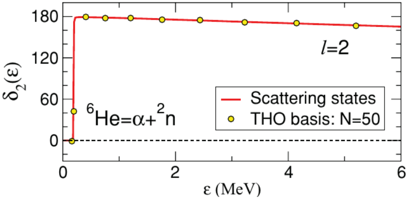
Stabilization approach

Hazi & Taylor [PRA 1 (1970) 1109]

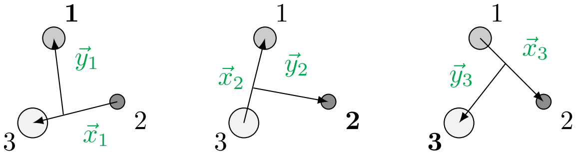
stable eigenstates close to resonance energies provide a good approximation of the inner part of the exact scattering wave function*

single-channel two-body case



*: we need stabilization parameter

J. A. Lay [PRC 82 (2010) 024605]



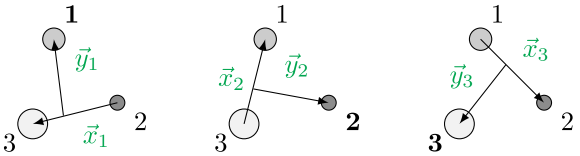
Jacobi coordinates

$\{x, y, \widehat{x}, \widehat{y}\}$

Hyperspherical coordinates

$$\{\rho, \alpha, \widehat{x}, \widehat{y}\}$$

$$\rho = \sqrt{x^2 + y^2} \qquad \alpha = \arctan \frac{x}{y}$$



Jacobi coordinates $\{x, y, \widehat{x}, \widehat{y}\}$

Hyperspherical coordinates $\{\rho, \alpha, \widehat{x}, \widehat{y}\}$ $\quad \rho = \sqrt{x^2+y^2} \quad \alpha = \arctan\frac{x}{y}$

$$\Psi^{j\mu}(\rho,\Omega) = \rho^{-5/2}\sum_\beta \chi_\beta^j(\rho)\mathcal{Y}_\beta^{j\mu}(\Omega) \qquad \beta \equiv \{K, l_x, l_y, l, S_x, J; I\}$$

Hyperspherical Harmonics (HH) expansion $\qquad$ hypermomentum $\widehat{K}$

$$\mathcal{Y}_\beta^{j\mu}(\Omega) = \left[\left(\Upsilon_{Klm_l}^{l_x l_y}(\Omega)\otimes\kappa_{S_x}\right)_J \otimes \phi_I\right]_{j\mu}$$

$$\Upsilon_{Klm_l}^{l_x l_y}(\Omega) = \varphi_K^{l_x l_y}(\alpha)\left[Y_{l_x}(\widehat{x})\otimes Y_{l_y}(\widehat{y})\right]_{lm_l}$$

$$\varphi_K^{l_x l_y}(\alpha) = N_K^{l_x l_y}(\sin\alpha)^{l_x}(\cos\alpha)^{l_y} P_n^{l_x+\frac{1}{2}, l_y+\frac{1}{2}}(\cos 2\alpha)$$

Pseudo-State (PS) method

$$\chi_{\beta}^{j}(\rho) = \sum_{i=0}^{N} C_{i\beta}^{j} U_{i\beta}(\rho)$$

expanded in $\mathcal{L}^2$ basis

N: number of hyperradial excitations included

$$\mathcal{H}\Psi_n^{j\mu} = \varepsilon_n \Psi_n^{j\mu}$$

- $\varepsilon_n < 0$ **bound states**
- $\varepsilon_n > 0$ **discretized continuum**

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$$V_{\beta'\beta}^{j\mu}(\rho) = \left\langle \mathcal{Y}_\beta^{j\mu}(\Omega) \middle| V_{12} + V_{13} + V_{23} \middle| \mathcal{Y}_{\beta'}^{j\mu}(\Omega) \right\rangle$$

➢ V_{ij} interaction between pairs

central, spin-orbit, spin-spin, tensor. To reproduce binary subsystem

Pseudo-State (PS) method

$$\chi^j_\beta(\rho) = \sum_{i=0}^{N} C^j_{i\beta} U_{i\beta}(\rho)$$

expanded in $\mathcal{L}^2$ basis

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$$\mathcal{H}\Psi^{j\mu}_n = \varepsilon_n \Psi^{j\mu}_n$$

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- $\varepsilon_n > 0$ **discretized continuum**

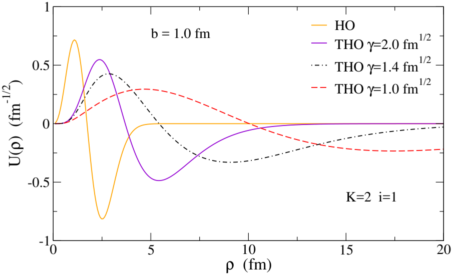
$$V^{j\mu}_{\beta'\beta}(\rho) = \left\langle \mathcal{Y}^{j\mu}_\beta(\Omega) \middle| V_{12} + V_{13} + V_{23} \middle| \mathcal{Y}^{j\mu}_{\beta'}(\Omega) \right\rangle + \delta_{\beta\beta'} V_{\mathrm{3b}}(\rho)$$

- V_{ij} interaction between pairs
  central, spin-orbit, spin-spin, tensor. To reproduce binary subsystem
- V_{3b} phenomenological three-body force
  diagonal term. Fixed to fine-tune the three-body energies

Analytical **T**ransformed **H**armonic **O**scillator **(THO)** basis

$$U_{i\beta}^{\mathrm{THO}}(\rho) = \sqrt{\frac{ds}{d\rho}} U_{iK}^{\mathrm{HO}}[s(\rho)]$$

$$s(\rho) = \frac{1}{\sqrt{2}\boldsymbol{b}} \left[\frac{1}{\left(\frac{1}{\rho}\right)^4 + \left(\frac{1}{\boldsymbol{\gamma}\sqrt{\rho}}\right)^4} \right]^{\frac{1}{4}}$$



[PRC**88**(2013)014327]

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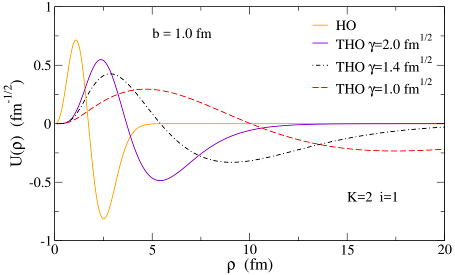


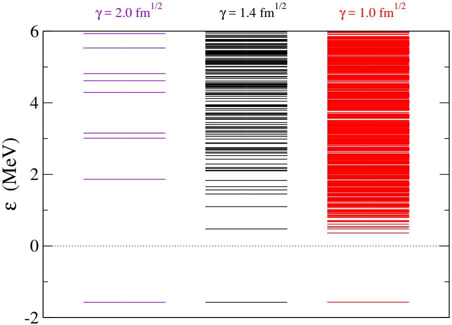
[PRC**88**(2013)014327]

Example:

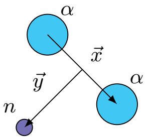
$\Psi_n^{j\mu}(\rho,\Omega)$ PS spectra, ε_n

$b = 0.7$ fm

The ratio γ/b controls the density of PS as a function of the energy.

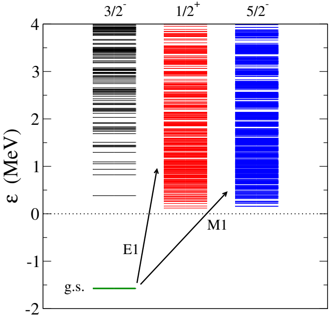


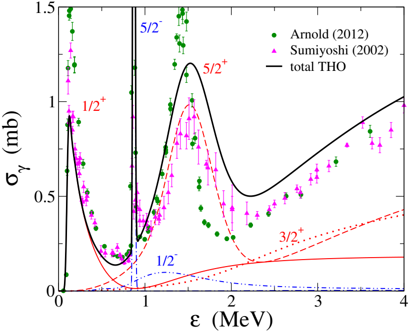
Example: ^{9}Be photodissociation



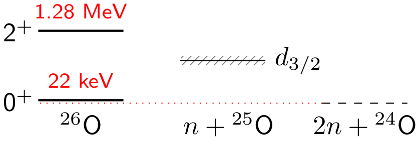
(α-α, α-n potentials fixed to ^{8}Be, ^{5}He)

$$\sigma_\gamma^{(\mathcal{O}\lambda)}(\varepsilon_\gamma) = \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\left(\frac{\varepsilon_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(\mathcal{O}\lambda)}{d\varepsilon}$$





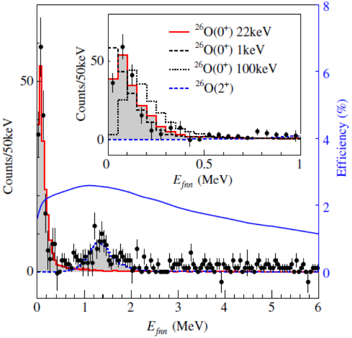
good description of resonant spectra [PRC**90**(2014)044304]

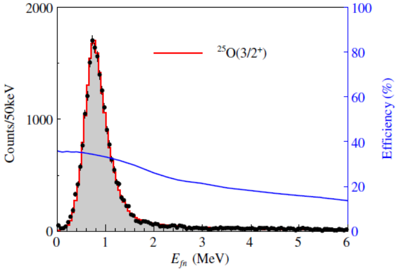


^{26}O (^{24}O $+ n + n$)

barely $2n$ unbound
Kondo [PRL 116 (2016) 102503]

Proton removal from ^{27}F on C target @ 201 MeV/u (RIKEN-RIBF)

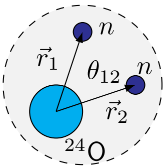




Sequential decay forbidden for the 0^+

Very narrow resonance! Ideal for pseudostates and stabilization approach

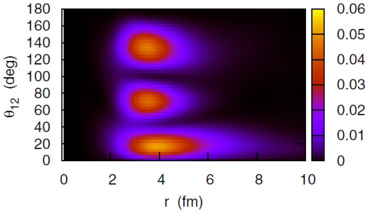
➢ Three-body model by Hagino and Sagawa [PRC 93 (2016) 034330]



V_{cn}: central + spin-orbit term
adjust ^{25}O $d_{3/2}$ resonance

V_{nn}: density-dependent contact interaction
fix ^{26}O g.s. energy

Dominant dineutron component



Radius suggests halo structure

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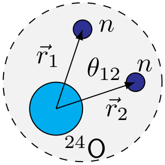

V_{cn}: central + spin-orbit term
adjust ^{25}O $d_{3/2}$ resonance

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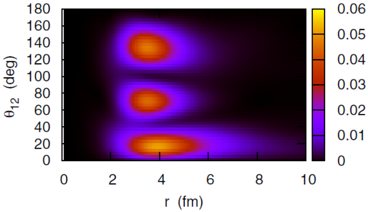
^{27}F wave function in a ^{25}F $+ n + n$ model
same core $+ n$ potential except for a smaller V_{LS}

Dominant dineutron component



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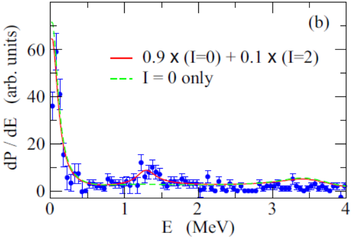
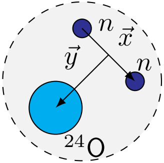


Decay-energy spectrum using
Green's function method

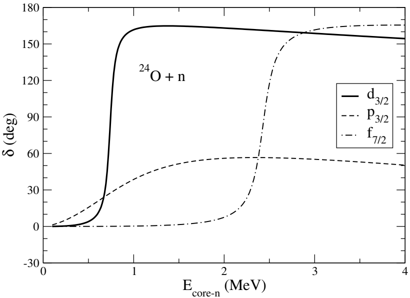
➢ **This work: pseudostates within the hyperspherical formalism**



V_{cn}: same potential as Hagino and Sagawa [PRC 93 (2016) 034330]

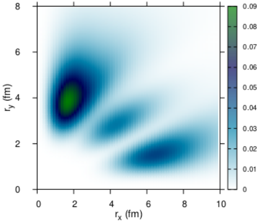
V_{nn}: GPT nucleon-nucleon interaction (fixed!)

V_{3b}: small correction to fix ^{26}O energy



core $+ n$: $d_{3/2}$, also $p_{3/2}$, $f_{7/2}$ res.

0^+ g.s. in ^{26}O as single PS close to the ^{24}O $+ n + n$ threshold



dominant dineutron; $\sim 60\%$ $d_{3/2}$ components

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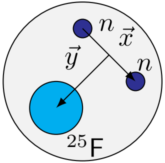

V_{cn}: same potential as Hagino and Sagawa [PRC 93 (2016) 034330] smaller V_{LS} for ^{27}F

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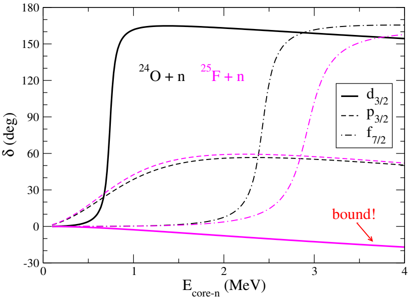
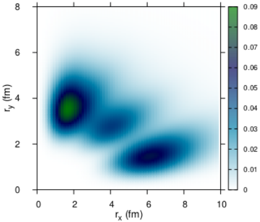


core $+ n$: $d_{3/2}$, also $p_{3/2}$, $f_{7/2}$ res.

0^+ g.s. in ^{27}F becomes bound

$\varepsilon_{2n} \simeq 3$ MeV



more compact than ^{26}O;

$\sim 90\%$ $d_{3/2}$ components

Decay-energy spectrum of ^{26}O after one proton removal from ^{27}F

- Assume sudden reaction (high energies!)
- Cross section proportional to $\langle ^{26}\mathrm{O} | ^{27}\mathrm{F} \rangle$ overlaps

  ^{26}O: set of $(^{24}\mathrm{O} + n + n)$ 0^+ PS at discrete energies E_n
  ^{27}F: 0^+ $(^{25}\mathrm{F} + n + n)$ ground state at ~ -3 MeV

- Comparison with data after convolution with exp. resolution

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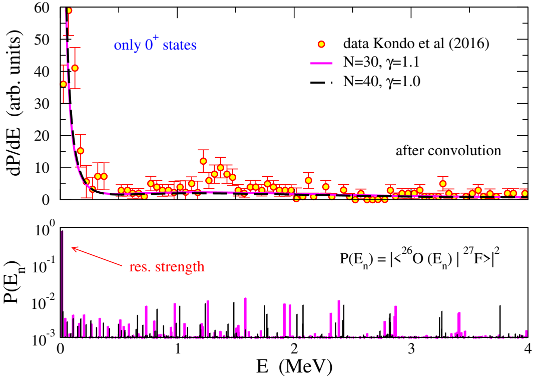
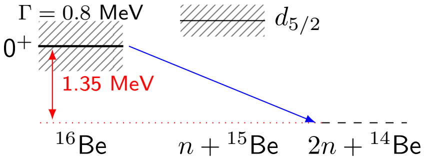


Good agreement using a discrete basis (simple approach)

This is preliminar!

Ongoing:

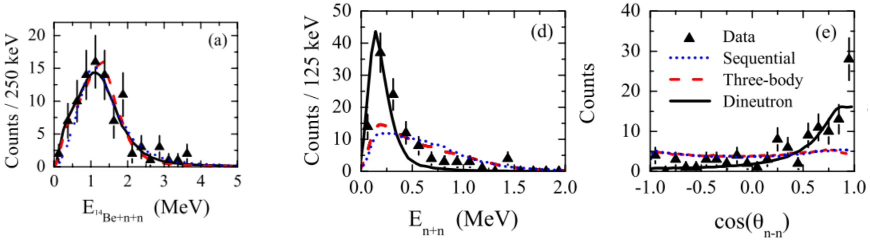
- role of nn corr.
- 2^+ contribution
- transfer on ^{24}O



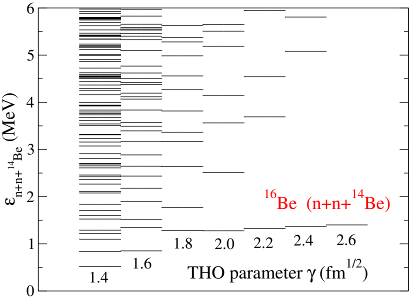
^{16}Be (^{14}Be $+ n + n$)

"Known" $2n$ emitter
Spyrou [PRL 108 (2012) 102501]

Proton removal from ^{17}B on Be target @ 53 MeV/u (MSU)



new RIKEN data - Miguel Marqués and Belén Monteagudo (LPC Caen)

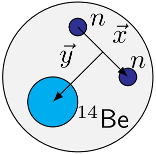


Stabilization in a discrete basis:

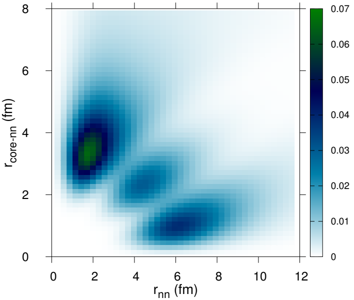
Look for stable pseudostates (PS) under changes in the basis parameters

⇒ PS around 1.3 MeV captures resonant behavior

J.C. [PRC **97** (2018) 034613]



Dineutron dominates

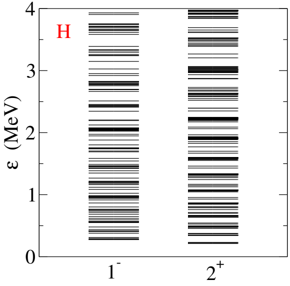


⇒ It favors correlated emission

Can we describe the decay?

(width, nn rel. energy, ...)

Identifying and characterizing few-body resonances: a novel approach

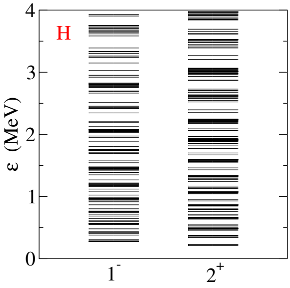


Ex: ^{6}He $(\alpha + n + n)$
non-res. 1^-
2^+ resonance

$\widehat{H}|n\rangle = \varepsilon_n|n\rangle$
mix res. and non-res.

[J.C., J. Gómez-Camacho, PRC **99** (2019) 014604]

Identifying and characterizing few-body resonances: a novel approach



Ex: ^{6}He $(\alpha + n + n)$
non-res. 1^-
2^+ resonance

$\widehat{H}|n\rangle = \varepsilon_n |n\rangle$
mix res. and non-res.

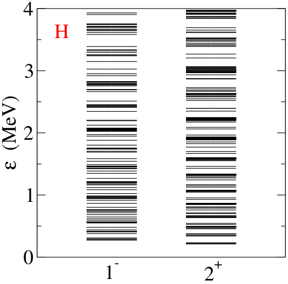
⇒ Diagonalize a **resonance operator** in a PS basis $\{|n\rangle\}$

$$\widehat{M} = \widehat{H}^{-1/2}\widehat{V}\widehat{H}^{-1/2}, \qquad \widehat{M}|\psi\rangle = m|\psi\rangle; \qquad |\psi\rangle = \sum_n \mathcal{C}_n |n\rangle$$

- It separates resonant states, which are strongly localized, from non-resonant continuum states, which are spatially spread.
- The expansion in terms of $|n\rangle$ allows to build energy distributions.

[J.C., J. Gómez-Camacho, PRC **99** (2019) 014604]

Identifying and characterizing few-body resonances: a novel approach



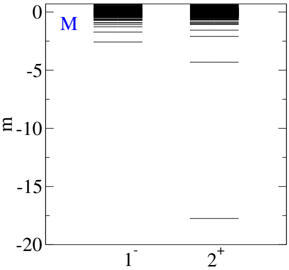
Ex: ^{6}He $(\alpha+n+n)$

non-res. 1^-

2^+ resonance

$\widehat{H}|n\rangle = \varepsilon_n|n\rangle$

mix res. and non-res.



⇒ Diagonalize a **resonance operator** in a PS basis $\{|n\rangle\}$

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Decay ⇒ time evolution:

$$|\psi(t)\rangle = \sum_n e^{-i\varepsilon_n t}|n\rangle$$

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For "ideal" BW:

$$a_r(t) = e^{-i\varepsilon_r t - \frac{\Gamma}{2}t}$$

Resonance quality parameter:

$$\delta^2(\varepsilon_r, \Gamma) = \frac{\int_0^\infty e^{-xt}|a(t) - a_r(t)|^2 dt}{\int_0^\infty e^{-xt}|a(t)|^2 dt}$$

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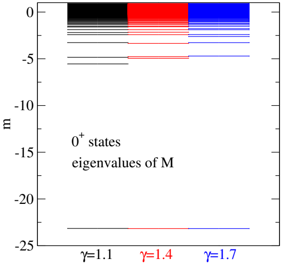
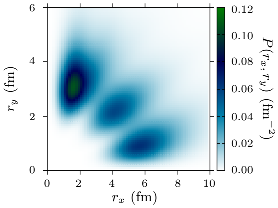
In order to find the resonance parameters ε_r and Γ which best describe the time evolution $a(t)$, we perform a minimization

$$\frac{\partial}{\partial\varepsilon_r}\delta^2\left(\varepsilon_r, \Gamma\right) = 0, \quad \frac{\partial}{\partial\Gamma}\delta^2\left(\varepsilon_r, \Gamma\right) = 0$$

$\Rightarrow$ as a function of x, i.e., $\varepsilon_r(x), \Gamma(x)$ $\qquad$ $x \to 0$ limit means long times

Application to 0^+ ground-state resonance in ^{16}Be





dineutron dominates wf

Lowest eigenstate of $\widehat{M}$:

stable under basis changes

localized state

Resonance parameters:

$\varepsilon_r = 1.34$ MeV
$\Gamma = 0.16$ MeV

width agrees with three-body scattering calc. [PRC 95 (2017) 034605]

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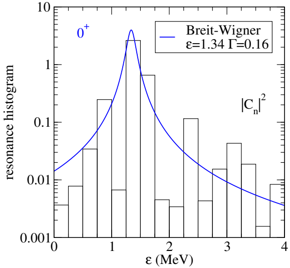

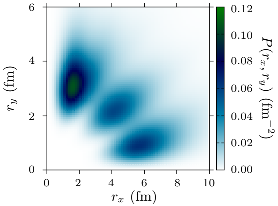

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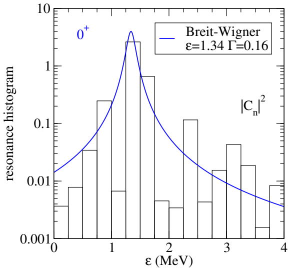

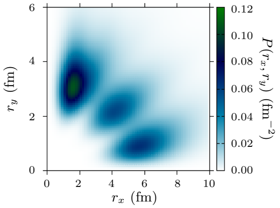

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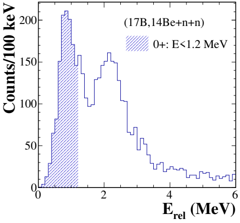
IN PROGRESS:

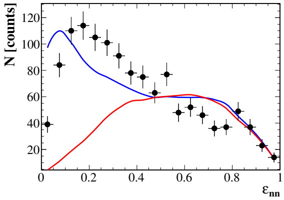
E_{nn} relative energy dist.

Compute currents
(flux leaving the potential)
Use Jacobi momenta
PRELIMINAR!!

RIKEN data F.M. Marqués

B. Monteagudo's PhD





0^+ with and without nn inter.

Summary and outlook

- Light nuclear systems at the limit of stability exhibit exotic properties and decay modes (e.g., Borromean halos, $2N$-emitters).
- The **continuum of three-body systems**, such as $2n$ emitters, can be described using pseudostates within the hyperspherical framework.

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$\sim 60\%$ $d_{3/2}$ components, mixing with $p_{3/2}$ and $f_{7/2}$.

Spectrum of ^{26}O via proton-removal from ^{27}F estimated using $\langle^{26}\text{O}|^{27}\text{F}\rangle$ overlaps; good agreement with data.

➢ 2^+ states; role of nn correlation; population via transfer

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(2) ^{16}Be (^{14}Be$+n+n$): 0^+ g.s. unbound by ~ 1 MeV

Diagonalization of a resonance operator in a basis of Hamiltonian pseudostates to identify and characterize few-body resonances.

Decay properties from time evolution (resonance amplitudes).

Importance of nn correlations for E_{nn} rel. energy distribution.

➢ 2^+ resonance; $E_{\text{core-}n}$ distribution; refinement of the theory

Collaborators:

J. Gómez-Camacho[1,2], F. M. Marqués[3], B. Monteagudo[3], M. Rodríguez-Gallardo[1], J. M. Arias[1], L. Fortunato[4], A. Vitturi[4]

[1]: Universidad de Sevilla
[2]: Centro Nacional de Aceleradores (CNA)
[3]: LPC Caen
[4]: Università degli Studi di Padova and INFN

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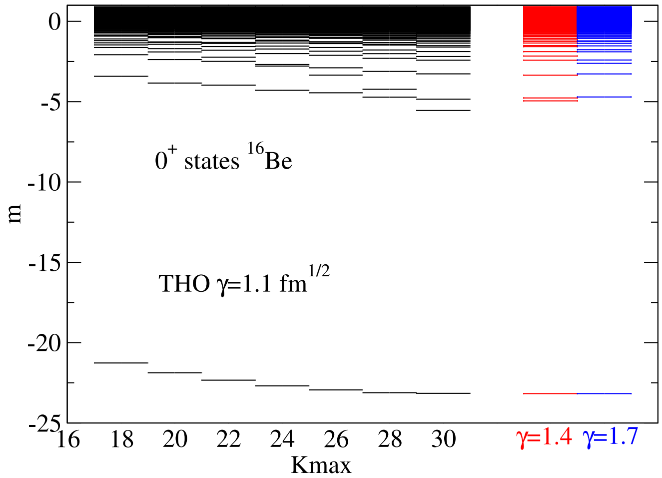
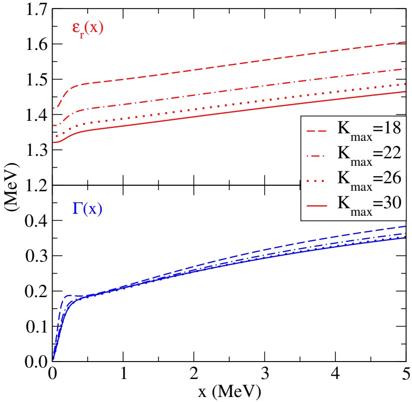
Horizon 2020

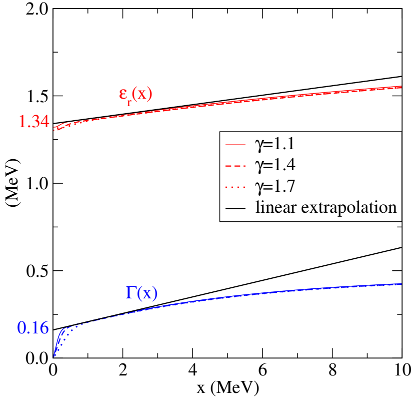
Grant agreement 654002

Resonance operator: ^{16}Be; Convergence of lowest eigenstate



Resonance operator: ^{16}Be; Resonance parameters





Resonance operator: ^{16}Be; 1^-, 2^+ states