ASYMPTOTIC SCATTERING THEORY

Few-Body Problems, Surrey, 2019

Per Osland

University of Bergen

Roy Glauber's

ASYMPTOTIC SCATTERING THEORY

Scattering theory provides a framework for understanding the scattering of waves and particles. This book presents a simple physical picture of diffractive nuclear scattering in terms of semi-classical trajectories, illustrated throughout with examples and case studies. Trajectories in a complex impact parameter plane are discussed, and it stresses the importance of the analytical properties of the phase shift function in this complex impact plane in the asymptotic limit. Several new rainbow phenomena are also discussed and illustrated. Written by Nobel Prize winner Roy Glauber, and Per Osland an expert in the field of particle physics, it illustrates the transition from quantum to classical scattering, and provides a valuable resource for researchers using scattering theory in nuclear, particle, atomic and molecular physics.

Roy J. Glauber was Mallinckrodt Professor of Physics (Emeritus) at Harvard University and Adjunct Professor of Optical Sciences at the University of Arizona. In 2005 he received half the Nobel Prize in Physics for his contribution to "the quantum theory of optical coherence". He received many awards, including the Albert A. Michelson Medal from the Franklin Institute (1985), the Max Born Award from the Optical Society of America, (1985) and the Dannie Heinemann Prize for Mathematical Physics from the American Physical Society (1966). He was elected a Foreign Member of the Royal Society (1997), an Honorary Member of the Optical Society of America (2008) and served on the Advisory Board of the Max Plank Institute of Light.

Per Osland is Professor Emeritus at the Department of Physics and Technology, University of Bergen, Norway. He is a member of the Royal Norwegian Society of Sciences and Letters and a Fellow of the Swedish Collegium for Advanced study. He has held research positions at CERN, Harvard, NORDITA and DESY, and served as chair of the High Energy and Particle Physics Division Board of the European Physical Society.

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Glauber and Osland

Asymptotic Diffraction Theory and Nuclear Scattering

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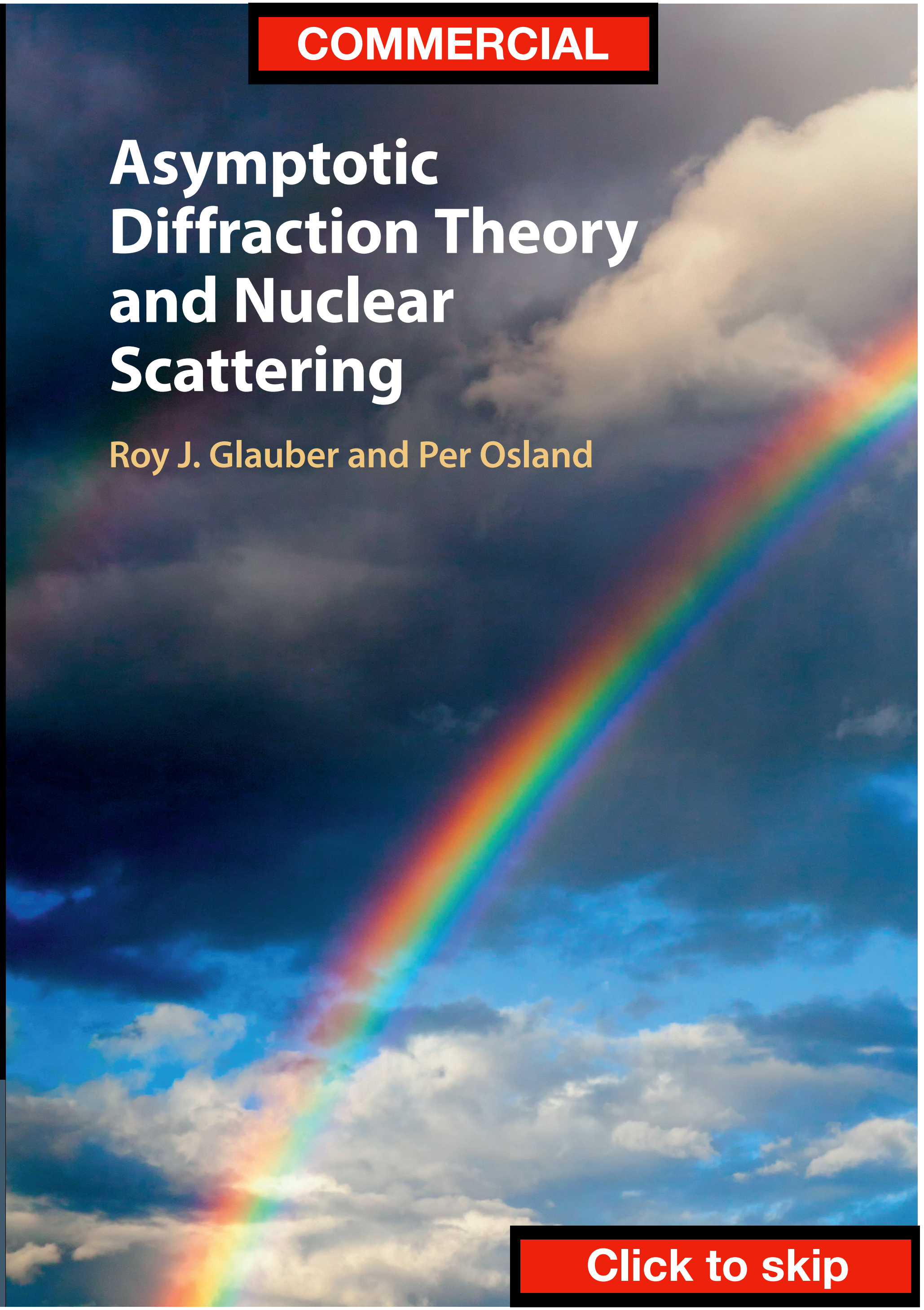

Asymptotic Diffraction Theory and Nuclear Scattering

Roy J. Glauber and Per Osland

Outline

- Brief review of Roy Glauber’s vita
- Asymptotic Scattering Theory (commercial)
- Personal reminiscences

Roy Glauber vita

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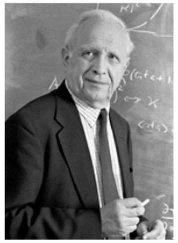

Roy Jay Glauber, Ph.D.

Affiliations: 1952- Physics Harvard University, Cambridge, MA, United States
Area: quantum optics
Website: http://www.nobelprize.org/nobel_prizes/physics/laureates/2005/glauber-bio.html
Google: "Roy Glauber"
Bio:
http://www.nasonline.org/member-directory/members/8938.html
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http://library.nd.edu/chemistry/resources/genealogy/physics/documents/GlauberRJ.pdf
https://inspirehep.net/record/1008226?ln=en
http://www.genealogy.math.ndsu.nodak.edu/id.php?id=15199
http://adsabs.harvard.edu/abs/1949PhDT.........7G
The Nobel Prize in Physics 2005 was divided, one half awarded to Roy J. Glauber "for his contribution to the quantum theory of optical coherence", the other half jointly to John L. Hall and Theodor W. Hänsch "for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique".
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precision spectroscopy, including the optical frequency comb technique".
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Daniel J. Kleitman	grad student	1958	Harvard (MathTree)
Leo Phillip Kadanoff	grad student	1960	Harvard
Daniel Frank Walls	grad student	1969	Harvard
Sudhakar Prasad	grad student	1983	Harvard (Astronomy Tree)
Wenda Shen	research scientist	1993	Harvard

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Leuchs G, Glauber RJ, Schleich WP. (**2015**) Intensity-intensity correlations determined by dimension of quantum state in phase space: P-distribution *Physica Scripta*. 90

Braungardt S, Rodríguez M, Glauber RJ, et al. (**2012**) Particle-counting statistics of time- and space-dependent fields *Physical Review a - Atomic, Molecular, and Optical Physics*. 85

Prasad S, Glauber RJ. (**2011**) Coherent scattering by a spherical medium of resonant atoms *Physical Review a - Atomic, Molecular and Optical Physics*. 83

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Daniel Frank Walls	grad student	1969	Harvard
Sudhakar Prasad	grad student	1983	Harvard (Astronomy Tree)
Wenda Shen	research scientist	1993	Harvard

+ Victor Franco

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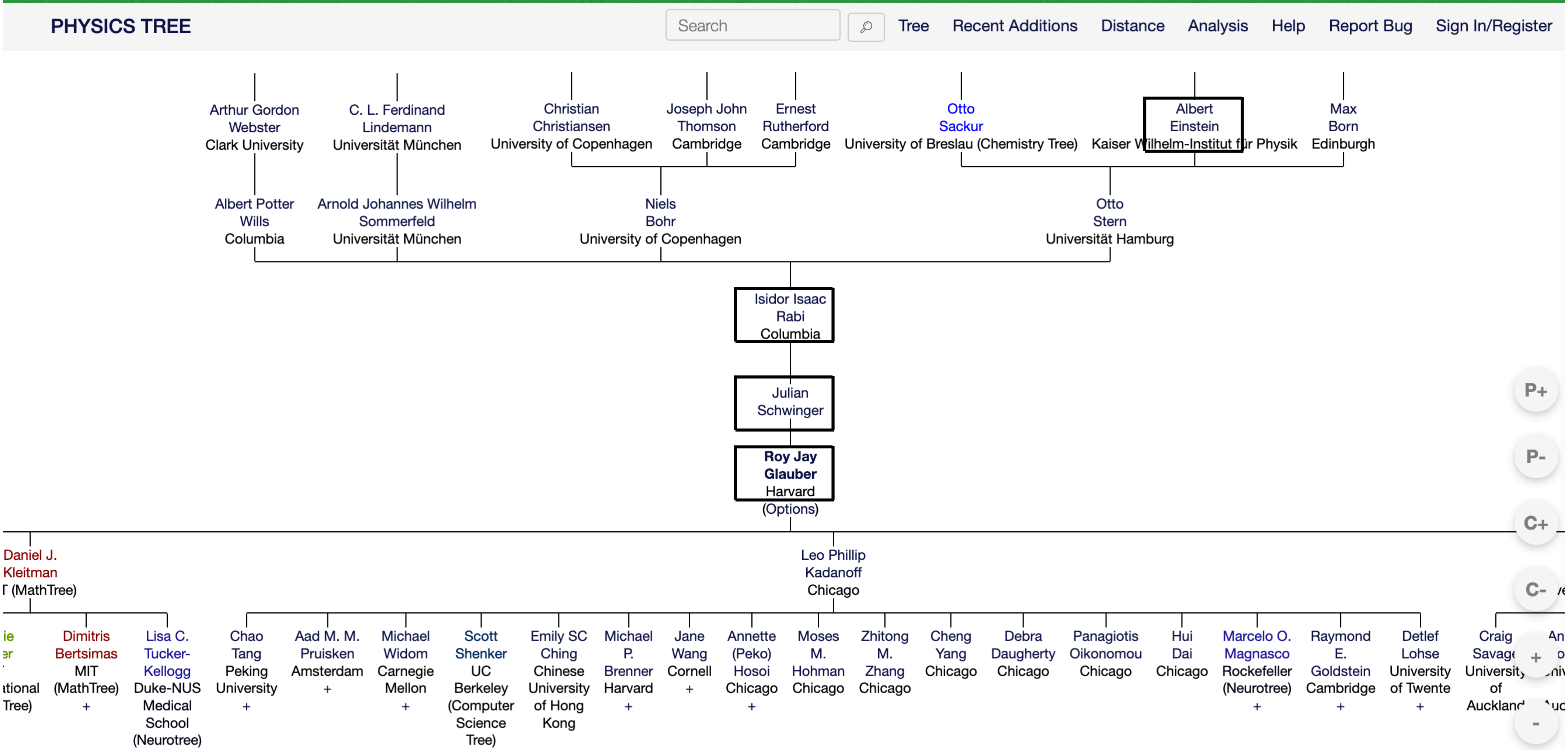

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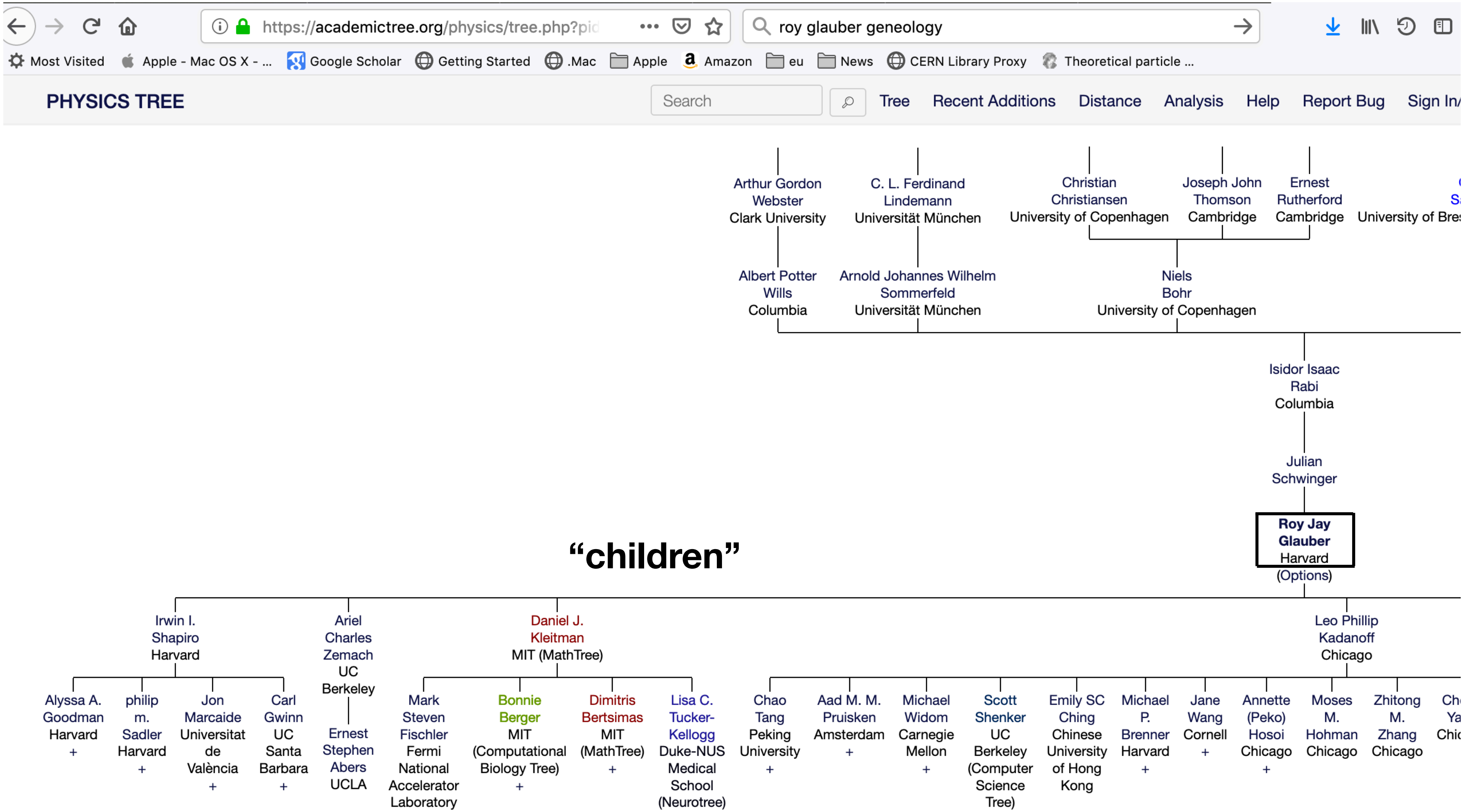

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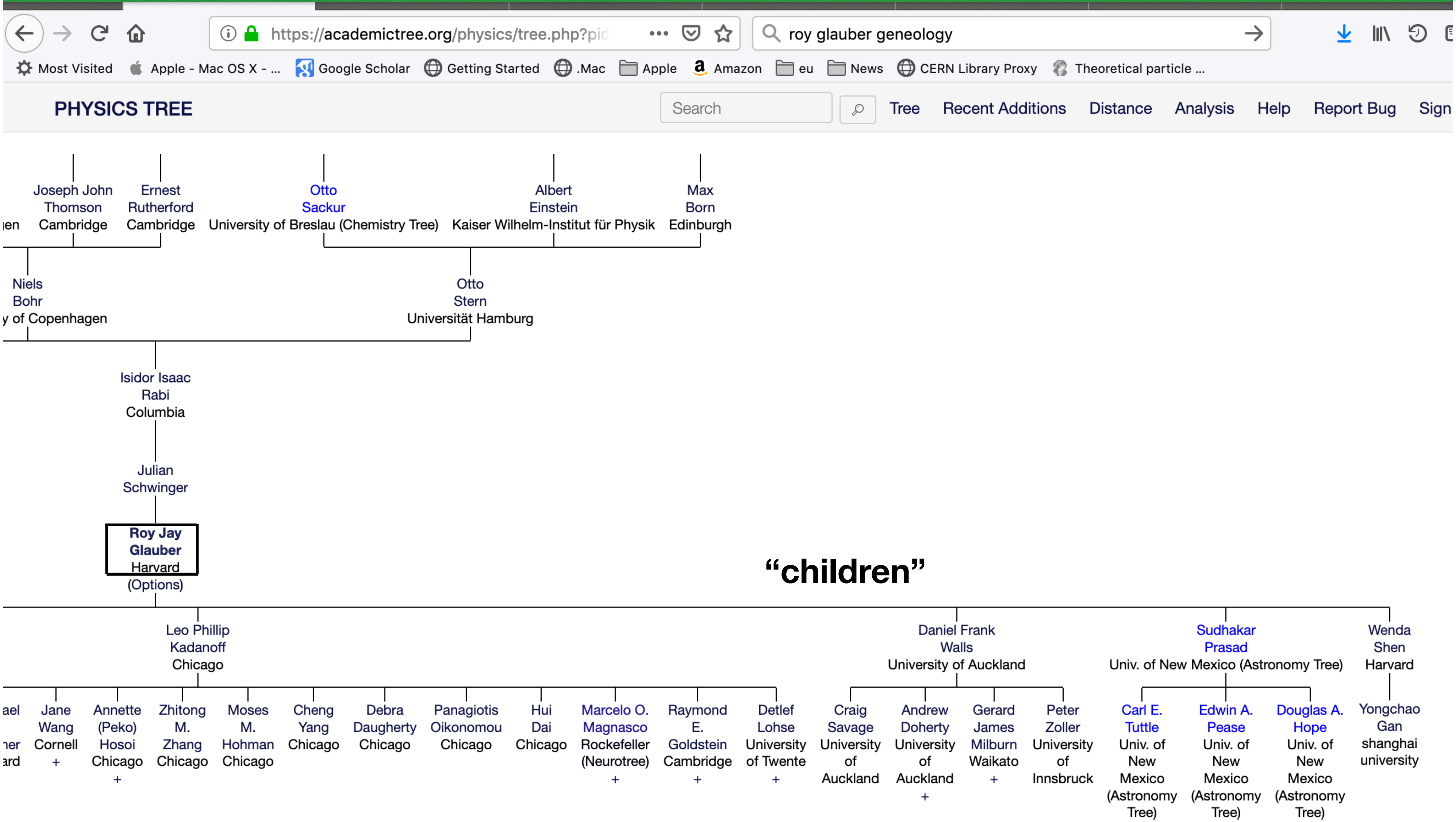

Roy Glauber vita

Curriculum Vitae

Roy J. Glauber

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Education:

- S.B., *summa cum laude*, Harvard University, 1946
- A.M., Harvard University, 1947
- Ph.D., Harvard University, 1949

Positions:

- 1944–46 Staff member, theory division, Manhattan Project, later Los Alamos Scientific Laboratory, New Mexico
- 1949–50 Member, Institute for Advanced Study, Princeton, on Atomic Energy Commission Postdoctoral Fellowship
- 1950–51 Member, Institute for Advanced Study, on Jewett Fellowship
- 1951–52 Lecturer, California Institute of Technology, Pasadena
- 1952–54 Lecturer on Physics and Bayard Cutting Fellow for Research in Physics, Harvard
- 1954–56 Assistant Professor of Physics, Harvard
- 1956–61 Associate Professor Physics, Harvard
- 1962–76 Professor of Physics, Harvard
- 1976– Mallinckrodt Professor of Physics, Harvard

CV date: 1997

Roy Glauber vita

Selected Publications

[1] The Critical Masses of Tamped Spheres, (Work done in 1944.) Introduction of the use of the spherical harmonic method for these calculations.) Los Alamos Report: LA–174 (1944-45).

[2] The Stopping of Multiplication in Expanding, Tamped Sheres (Possibly still classified.) Los Alamos Report: LA–333 (1944–45).

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[17] Nucleon Scattering at High Energies (with I. I. Shapiro) (Abstract), *Phys. Rev.* **99**, 629(1955).

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[22] Dynamics of Neutron Scattering by Molecules (with A. C. Zemach), *Phys. Rev.* **101**, 118(1956).

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[24] Relativistic and Screening Effects in Radiative Electron Capture (with P. C. Martin, T. Linqvist, and C. S. Wu), *Phys. Rev.* **101**, 905(1956).

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[26] The Influence of the Exclusion Principle on the Refractive Index of the Nucleus at High Energies, (Abstract) *Physica* **22**, 1185 (1956).

[27] Relativistic Theory of Radiative Orbital Electron Capture (with P. C. Martin), *Phys. Rev.* **109**, 1307(1958).

★ [28] High Energy Collision Theory Lectures in *Theoretical Physics*, (New York: Interscience Publishers, Inc., 1959), Vol. I, p. 315.

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Boulder Lectures

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Roy Glauber vita

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Conjecture:
nucleon scattering work grew out of Los Alamos experience

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optics

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CV date: 1997

THE QUANTUM MECHANICAL THEORY OF COLLISIONS

Roy GLAUBER

Department of Physics

Harvard University

Quantum Mechanical Theory of Collisions (Les Houches, 1954)

Université de Grenoble

Cours professé à

l'Ecole d'Eté de Physique Théorique

Les Houches (Haute-Savoie), France

Eté 1954

The Quantum Mechanical Theory of Collissions. 1954

I.- In the present course we shall consider the quantum mechanical treatment of problems of particle interactions in which at least one of the particles is unbound and therefore able to travel large distances freely. In effect such particles are able to communicate the results of their interaction directly to macroscopic detection apparatus. The interpretation of the resulting data is in general the most direct way of forming conclusions about the interactions.

A considerable variety of physical phenomena involves unbound particles. Among the simplest are scattering processes ; in particular the deflection of particles from a collinated beam. More general collision processes which induce reactions of various sorts also lie within the class we shall examine. Our chief concern will be with the means of solving problems in which well-defined models have been postulated to describe the basic interactions. We shall restrict ourselves, in fact, to the treatment of fairly simple models, and

LECTURES IN THEORETICAL PHYSICS

1. Boulder, 1958.

VOLUME I LECTURES DELIVERED AT THE SUMMER INSTITUTE FOR THEORETICAL PHYSICS, UNIVERSITY OF COLORADO, BOULDER, 1958

1958

Edited by
Professor WESLEY E. BRITTIN
and LITA G. DUNHAM
Department of Physics, University of Colorado

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HIGH-ENERGY COLLISION THEORY

R. J. Glauber
Department of Physics
Harvard University

Few trends are more striking nowadays than the increase of attention being devoted to the collisions of particles accelerated to high energies. The reasons for such studies lie basically in the information they furnish about the interactions of the colliding particles, and about the reaction products they generate. Both types of information may usually be obtained more readily at high than at low energies. The study of low-energy collisions ordinarily tells us only a certain measure of the strength of an interaction. At high energies, on the other hand, the shorter wave-length of the incident particles makes them sensitive probes of the region of interaction. When the wavelengths are sufficiently short, for example, the angular distribution of elastically scattered particles becomes, in a sense, a detailed map of the region of interaction. Inelastic collisions are capable of furnishing much the same information too, and as particle energies rise, the importance of such collisions grows and their variety multiplies.

The type of problem we should like to treat, say the collision of an incident particle with a nucleus consisting of many particles, is not an easy one to formulate at any energy. But at high energies the complication of the problem as evidenced, for example, by the huge number of final states available to the system, makes the prospect of reaching exact solutions quite dim indeed. Fortunately, however, the physical conditions which hold at high energies are in a number of ways well suited to the introduction of approximation methods. The major part of these lectures will be devoted to the development of such techniques.

The approximation methods we shall describe are quite elementary in structure. They all bear a certain family resemblance to the approximations used in the diffraction theory of physical optics. That is not to say that they are too familiar, however, since the situations encountered in collision theory are usually quite different from those of optics. For example, the target particles in a nucleus are free to move about in a bound state while the obstacles of diffraction theory are always fixed. It will be necessary, therefore, to develop mathematical methods for treating the quantum mechanical problem which are of much greater generality than those of physical optics. But the mathematics required is very sim-

... collision of an incident particle with a nucleus...
... at high energies...

... approximation methods ...

plane wave

scattering

From the integral equation

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} - \frac{2m}{4\pi\hbar^2}\int \frac{e^{ik|r-r'|}}{|\vec{r}-\vec{r}'|} V(\vec{r}')\psi(\vec{r}')\,d\vec{r}'$$

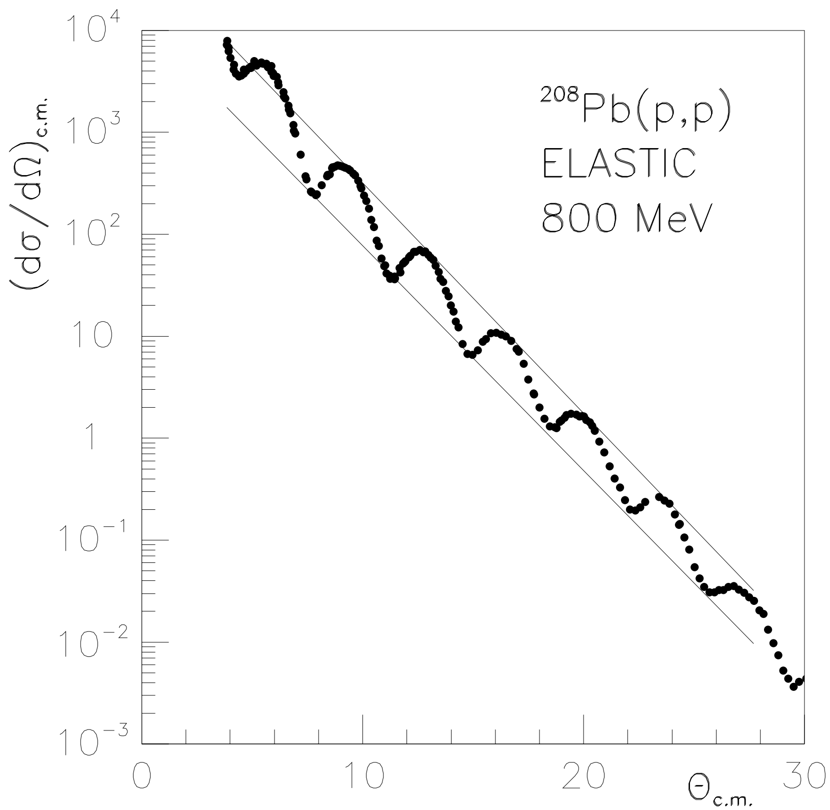
we obtain as $r \to \infty$

$$\psi(\vec{r}) \rightarrow e^{i\vec{k}\cdot\vec{r}} - \frac{2m}{4\pi\hbar^2}\frac{e^{ikr}}{r}\int e^{-i\vec{k}_r\cdot\vec{r}'} V(\vec{r}')\psi(\vec{r}')\,d\vec{r}'.$$

Asymptotic Diffraction Theory

1980

Experiments at
Los Alamos



Description:

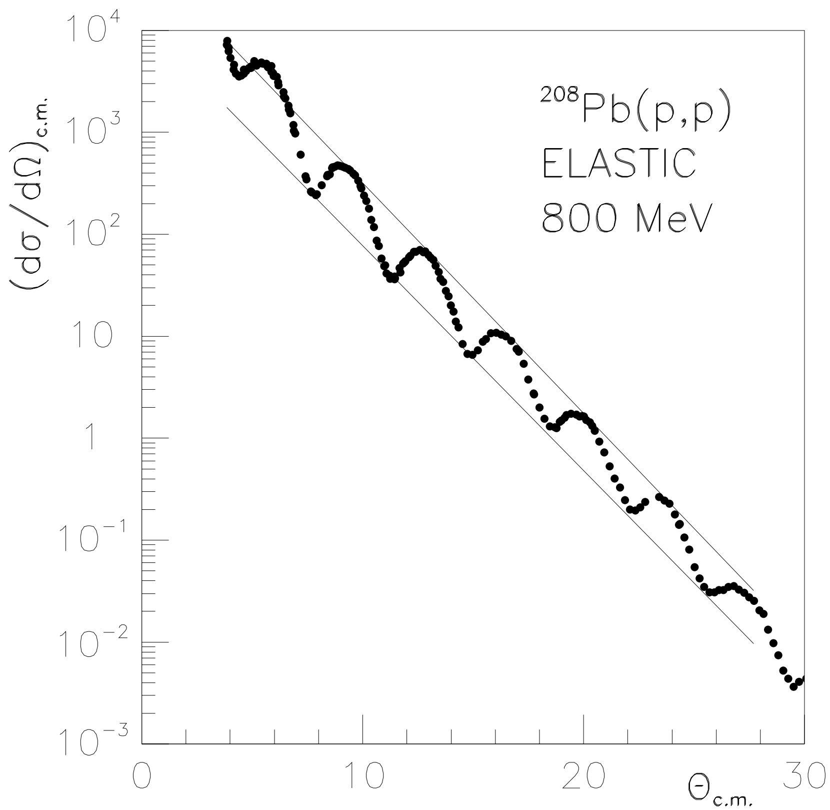
$$f(\boldsymbol{k}', \boldsymbol{k}) = \frac{ik}{2\pi} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \{1 - e^{i\chi(\boldsymbol{b})}\} \, \mathrm{d}^2 b$$

Gross Features:

1. exponentially falling envelope
2. periodic oscillations

Asymptotic Diffraction Theory

1980



seven orders of magnitude

Description:

$$f(\boldsymbol{k}', \boldsymbol{k}) = \frac{ik}{2\pi} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \{1 - e^{i\chi(\boldsymbol{b})}\} \, \mathrm{d}^2 b$$

Roy: “But what does it mean?”

Asymptotic Diffraction Theory

Conventional two-slit diffraction: $$I \propto \left(\frac{\sin\theta}{\theta}\right)^2$$

Oscillations with a power-law fall-off

slit has sharp edges!

Roy: consider a slit with diffuse edges:

General diffraction amplitude: $$f(\boldsymbol{k}', \boldsymbol{k}) \sim \int e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{b}} A(\boldsymbol{b})\, \mathrm{d}^2 b$$

A straight slit with a diffuse edge:

$$A(x) = \frac{\beta}{\pi}\frac{1}{x^2+\beta^2}$$

x in scattering plane

$$f(\boldsymbol{k}', \boldsymbol{k}) \sim \frac{\beta}{\pi}\int_{-\infty}^{\infty} \frac{e^{-iqx}}{x^2+\beta^2}\,\mathrm{d}x$$

poles at $x = \pm i\beta$

$$\sim \frac{1}{2\pi i}\int_{-\infty}^{\infty} e^{-iqx}\left\{\frac{1}{x-i\beta} - \frac{1}{x+i\beta}\right\}\mathrm{d}x$$

Asymptotic Diffraction Theory

Let q be positive. Close contour in lower half-plane.

$$f(\boldsymbol{k}', \boldsymbol{k}) \sim e^{-\beta|\boldsymbol{q}|}$$

Reminiscent of envelope of experimental data plot shown.

Consider two such (diffuse) slits:

$$A(x) = \frac{\beta}{2\pi}\left\{\frac{1}{(x-c)^2+\beta^2} + \frac{1}{(x+c)^2+\beta^2}\right\}$$

Resulting Fraunhofer diffraction amplitude:

$$f(\boldsymbol{k}', \boldsymbol{k}) \sim e^{-\beta|\boldsymbol{q}|}\cos(cq)$$

periodic oscillations, exponential damping

Scattering from the whole impact plane is summarized by the contributions from two pole singularities located in the complex plane:

$$x = \pm c - i\beta$$

Asymptotic Diffraction Theory

“Kirchhoff integral” for “Fraunhofer diffraction” (Roy’s terminology):

$$f(\boldsymbol{k}', \boldsymbol{k}) = \frac{ik}{2\pi} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \left\{ 1 - e^{i\chi(\boldsymbol{b})} \right\} \mathrm{d}^2 b$$

The “1” only contributes a delta-function in the forward direction

The rest can be evaluated approximately,
stationary phase at “large” values of q

$$\nabla_b \{ -\boldsymbol{q}\cdot\boldsymbol{b} + \chi(\boldsymbol{b}) \} = 0$$

$$\boldsymbol{q} = \nabla_b \chi(\boldsymbol{b})$$

Asymptotic Diffraction Theory

Classical considerations

Let particle at the position $\boldsymbol{r} = \boldsymbol{b} + \hat{\boldsymbol{\kappa}} z$ be subject to a potential $V(\boldsymbol{b} + \hat{\boldsymbol{\kappa}} z)$.
It experiences a transverse force $-\nabla_b V(\boldsymbol{b} + \hat{\boldsymbol{\kappa}} z)$.

The integral of this force over time is given by $\hbar \nabla_b \chi(\boldsymbol{b})$ since

$$\chi(\boldsymbol{b}) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} V(\boldsymbol{b} + \hat{\boldsymbol{\kappa}} z)\, \mathrm{d}z$$

That transverse impulse then must represent the transfer of momentum to the scattered particle,

$$\hbar(\boldsymbol{k}' - \boldsymbol{k}) = \hbar \boldsymbol{q} = \hbar \nabla_b \chi(\boldsymbol{b}).$$

which is precisely the condition of stationary phase!

Asymptotic Diffraction Theory

Stationary phase approximation

Transverse coordinates:

$$\boldsymbol{b} - \boldsymbol{b}_0 = \hat{\boldsymbol{q}} x + \hat{\boldsymbol{n}} y$$

Stationary phase conditions:

$$\frac{\partial}{\partial b_x} \chi(b_x, b_y) = q$$

$$\frac{\partial}{\partial b_y} \chi(b_x, b_y) = 0$$

Rotational invariance: $\chi(\boldsymbol{b}) = \chi(b) = \chi\left(\sqrt{b_x^2 + b_y^2}\right)$

Stationary phase conditions then become:

$$\frac{b_x}{b} \chi'(b) = q$$

$$\frac{b_y}{b} \chi'(b) = 0 \quad \longrightarrow \quad b_y = 0.$$

Asymptotic Diffraction Theory

Stationary phase approximation

Phase of integrand:

$$\chi(\boldsymbol{b}) - \boldsymbol{q}\cdot\boldsymbol{b} = \chi(\boldsymbol{b}_0) - \boldsymbol{q}\cdot\boldsymbol{b}_0 + \alpha_x x^2 + \alpha_y y^2$$

with

$$\alpha_x \equiv \tfrac{1}{2}\chi''(|b_{0\,x}|)$$
$$\alpha_y = \chi'(|b_{0\,x}|)/2|b_{0\,x}| = q/2b_{0\,x}$$

Amplitude:

$$f_0(\boldsymbol{k}',\boldsymbol{k}) = \frac{k}{2\pi i}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}_0 + i\chi(b_0)}\, \mathcal{I}_0(\boldsymbol{q})$$

$$\mathcal{I}_0(\boldsymbol{q}) = \frac{\pi}{(-\alpha_x\alpha_y)^{\frac{1}{2}}}$$

may introduce a convergence factor

with

$$= 2\pi\left(\frac{-b_{0\,x}}{\chi'(b_{0\,x})\chi''(b_{0\,x})}\right)^{\frac{1}{2}}$$

$$f_0(\boldsymbol{k}',\boldsymbol{k}) = \frac{k}{i}\left(\frac{-b_{0\,x}}{q\chi''(b_{0\,x})}\right)^{\frac{1}{2}} e^{-iqb_{0\,x} + i\chi(b_{0\,x})}$$

Asymptotic Diffraction Theory

Stationary phase approximation

Phase of integrand:

$$\chi(\boldsymbol{b}) - \boldsymbol{q}\cdot\boldsymbol{b} = \chi(\boldsymbol{b}_0) - \boldsymbol{q}\cdot\boldsymbol{b}_0 + \alpha_x x^2 + \alpha_y y^2$$

Azimuthal symmetry:

$$f_0(\boldsymbol{k}', \boldsymbol{k}) = \frac{k}{i}\left(\frac{-b_{0\,x}}{q\chi''(b_{0\,x})}\right)^{\frac{1}{2}} e^{-iqb_{0\,x} + i\chi(b_{0\,x})}$$

General result (no symmetry):

$$f_0(\boldsymbol{k}', \boldsymbol{k}) = \frac{k}{\left\{\det \nabla_b \nabla_b \chi(\boldsymbol{b})\big|_{\boldsymbol{b}_0}\right\}^{\frac{1}{2}}} \exp\{i[-\boldsymbol{q}\cdot\boldsymbol{b}_0 + \chi(\boldsymbol{b}_0)]\}$$

Asymptotic Diffraction Theory

Classical cross section

Consider particles impinging on the area

$$\mathrm{d}\sigma = \mathrm{d}^2\boldsymbol{b}$$

Scattered into a cone given by transverse momenta $\mathrm{d}^2\boldsymbol{q}$

$$\mathrm{d}^2\boldsymbol{b} = \frac{\partial(\boldsymbol{b})}{\partial(\boldsymbol{q})}\,\mathrm{d}^2\boldsymbol{q}$$

↖ 2×2 Jacobian determinant

$$\mathrm{d}^2\boldsymbol{q} = k^2\,\mathrm{d}\Omega.$$

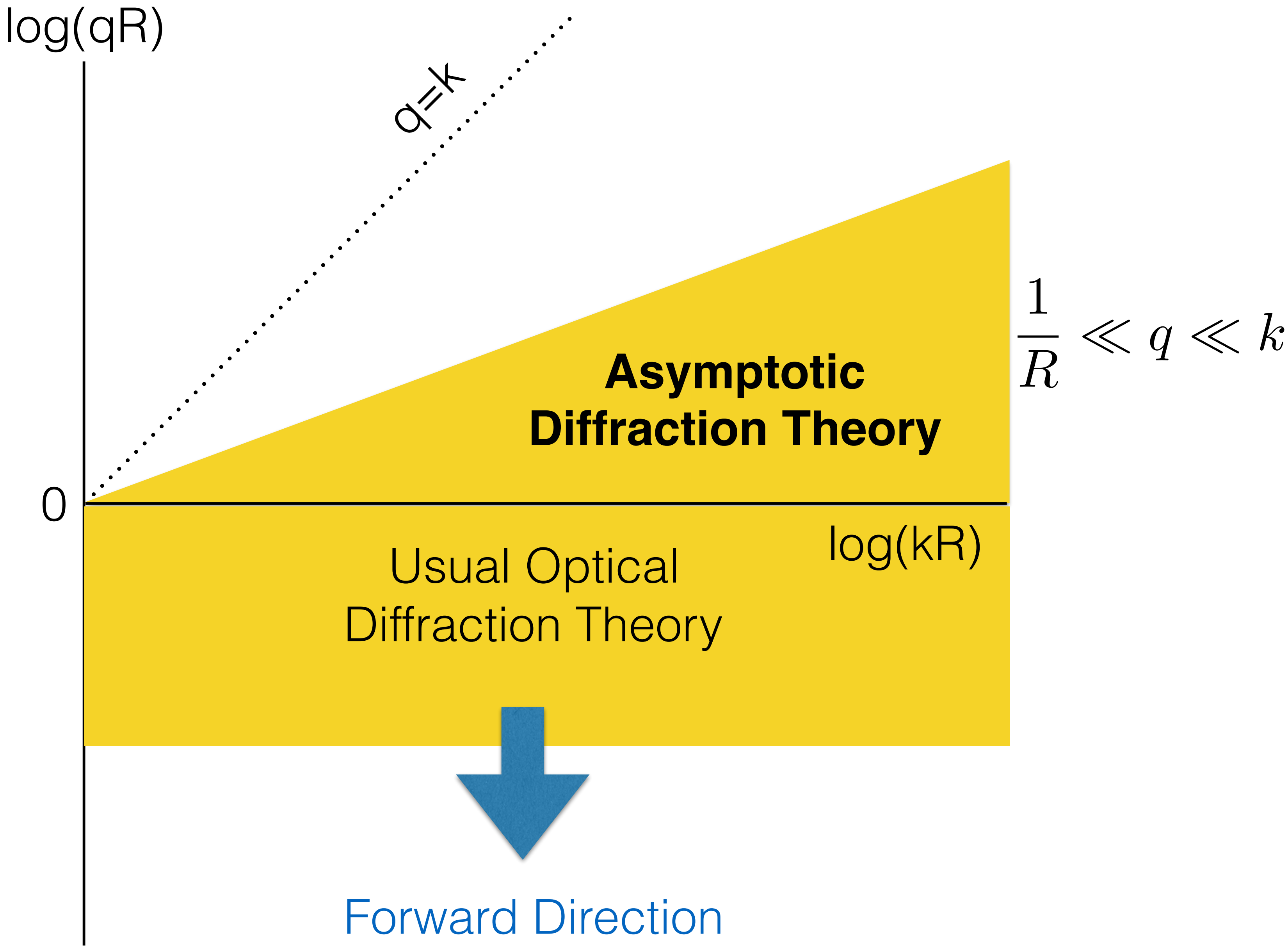
Classical cross section

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega} = k^2 \frac{\partial(\boldsymbol{b})}{\partial(\boldsymbol{q})}\bigg|_{\boldsymbol{b}_0} = \frac{k^2}{|\det \nabla_b \nabla_b \chi(\boldsymbol{b})|_{\boldsymbol{b}_0}}$$

|Stationary-phase amplitude|^2

Asymptotic Diffraction Theory

Region of validity



Asymptotic Diffraction Theory

Simple example #1: Coulomb scattering

Point-charge potential:

$$V(\boldsymbol{r}) = \frac{Ze^2}{4\pi r}$$

Phase-shift function:

$$\chi(b) = 2\eta \log\left(\frac{b}{2R}\right) + \mathcal{O}\left(\frac{b^2}{R^2}\right) \qquad \eta = \frac{Ze^2}{4\pi\hbar v}$$

screening radius

Stationary-phase point:

$$\frac{2\eta}{b_x} = q$$

Amplitude:

$$f(\boldsymbol{k}', \boldsymbol{k}) = \frac{2\eta k}{q^2} \exp\left\{-2i\eta \log\frac{qR}{\eta} - 2i\eta + \frac{i\pi}{2}\right\}$$

identical up to phase

Rutherford cross section:

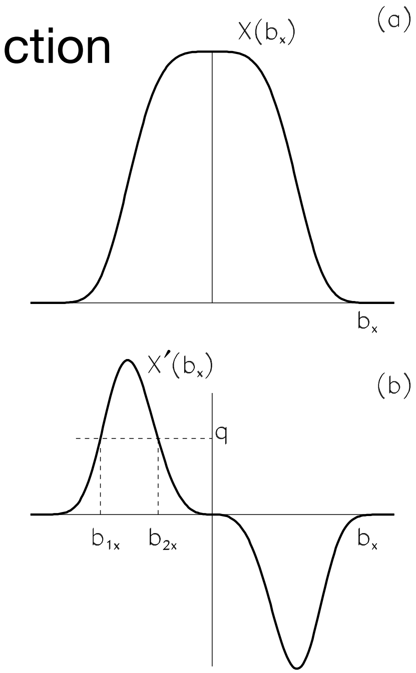
$$|f(\boldsymbol{k}', \boldsymbol{k})|^2 = \left(\frac{2\eta k}{q^2}\right)^2 = \frac{(2\eta k)^2}{|\boldsymbol{k} - \boldsymbol{k}'|^4}$$

Asymptotic Diffraction Theory

Simple example #2: Paired trajectories

Real, symmetric phase-shift function

Two solutions to stationary-phase condition:



$$F(b_{ix}) = \frac{k}{i}\sqrt{\frac{-b_{ix}}{qX''(b_{ix})}}\, e^{-iqb_{ix}+iX(b_{ix})}$$

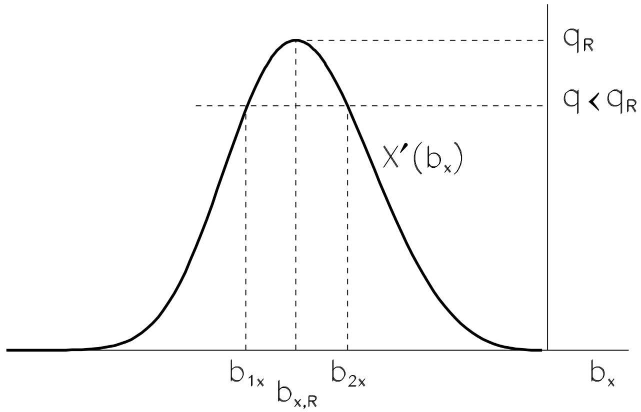
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(\boldsymbol{k}', \boldsymbol{k})|^2 = |F(b_{1\,x}) + F(b_{2\,x})|^2$$

Compare:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{class.}} = |F(b_{1\,x})|^2 + |F(b_{2\,x})|^2$$

Asymptotic Diffraction Theory

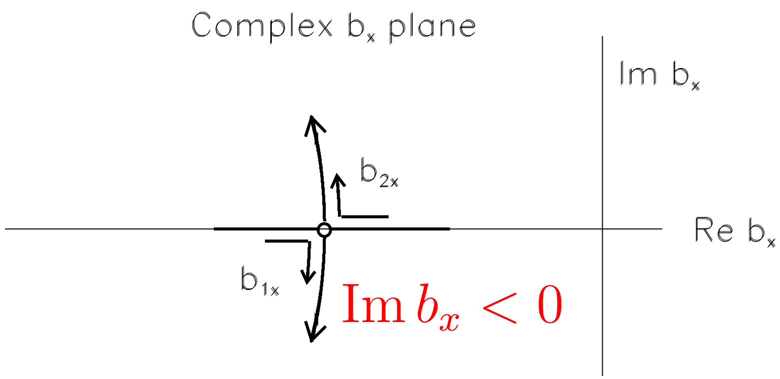
Rainbow:



At rainbow point, two real solutions (real phase shift function) run together

Cross section diverges as $(q_{\rm R} - q)^{-\frac{1}{2}}$

Beyond rainbow point, two complex solutions,
only one of which is encountered along path of integration.



(sign convention)

Cross section falls off exponentially! Classically forbidden region.

Asymptotic Diffraction Theory

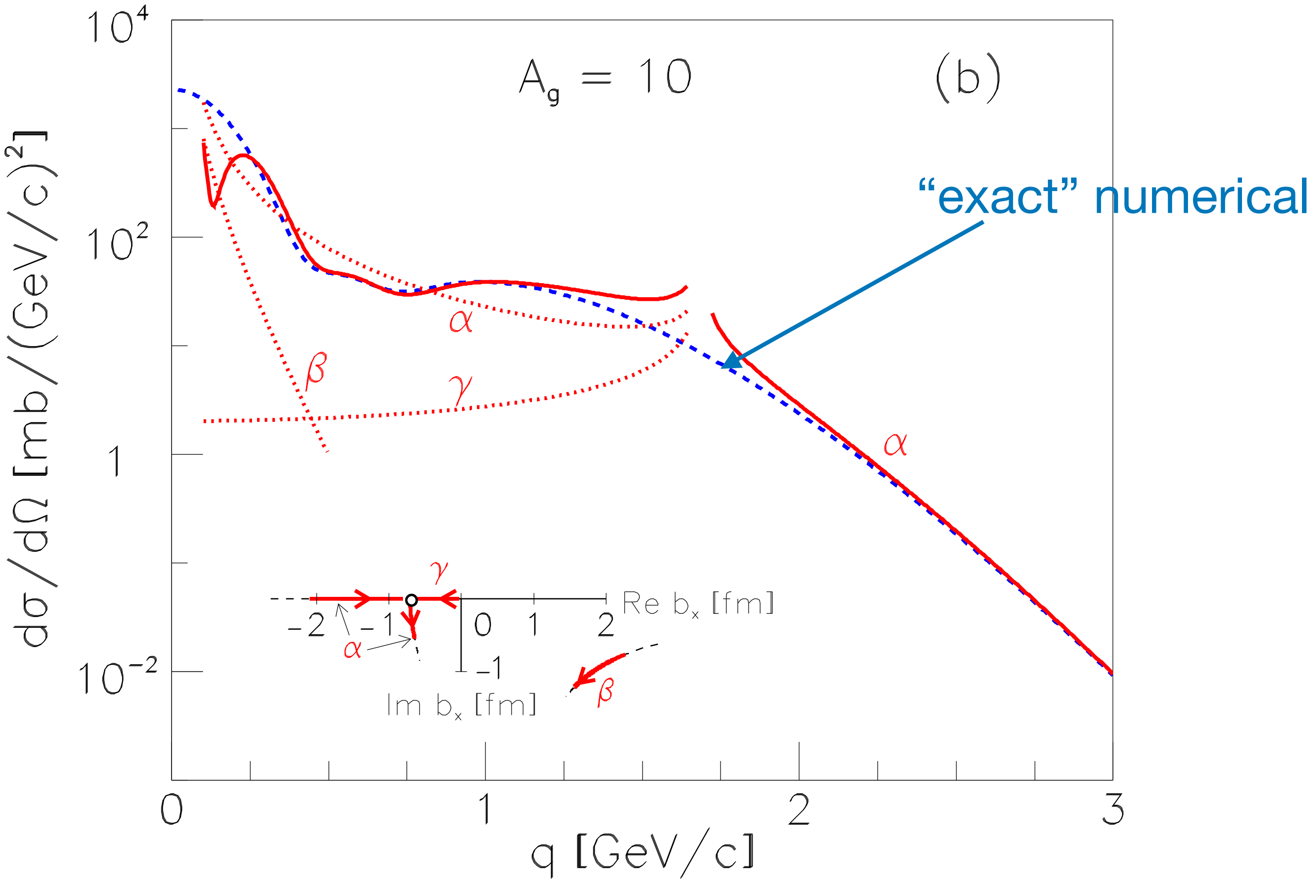
A less trivial example

Real gaussian

$$X(b_x) = A_{\rm g} \exp(-b_x^2/\beta^2)$$

inside rainbow: 3 stat. points

outside rainbow: 2 stat. points

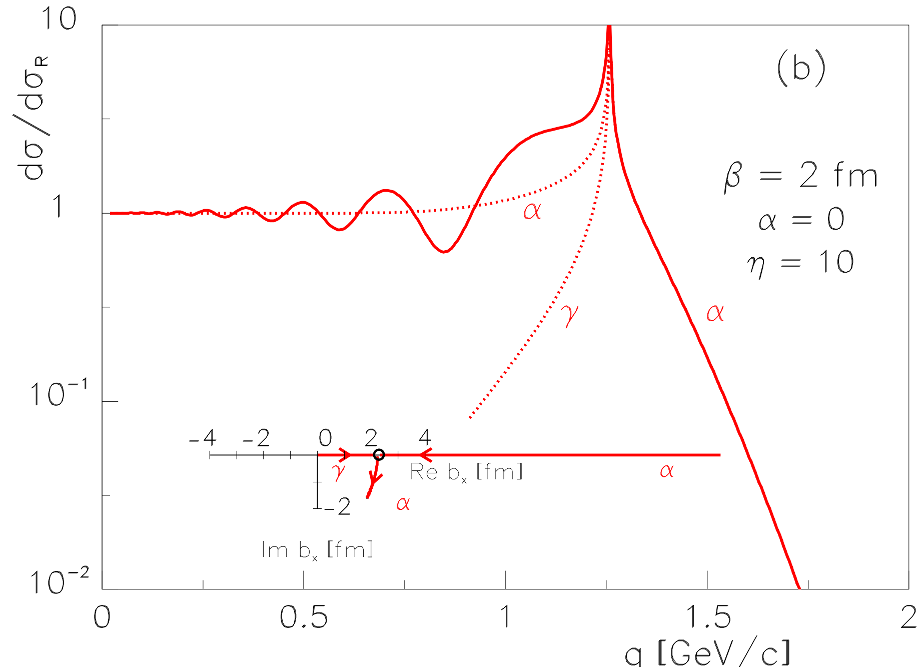


Asymptotic Diffraction Theory

Coulomb scattering, extended charge distribution

$$\rho_2(r) = \frac{1}{\pi^{3/2}\beta^3}\frac{1}{1+\frac{3}{2}\alpha}\left(1+\alpha\frac{r^2}{\beta^2}\right)e^{-r^2/\beta^2}$$

Display ratio to Rutherford cross section:

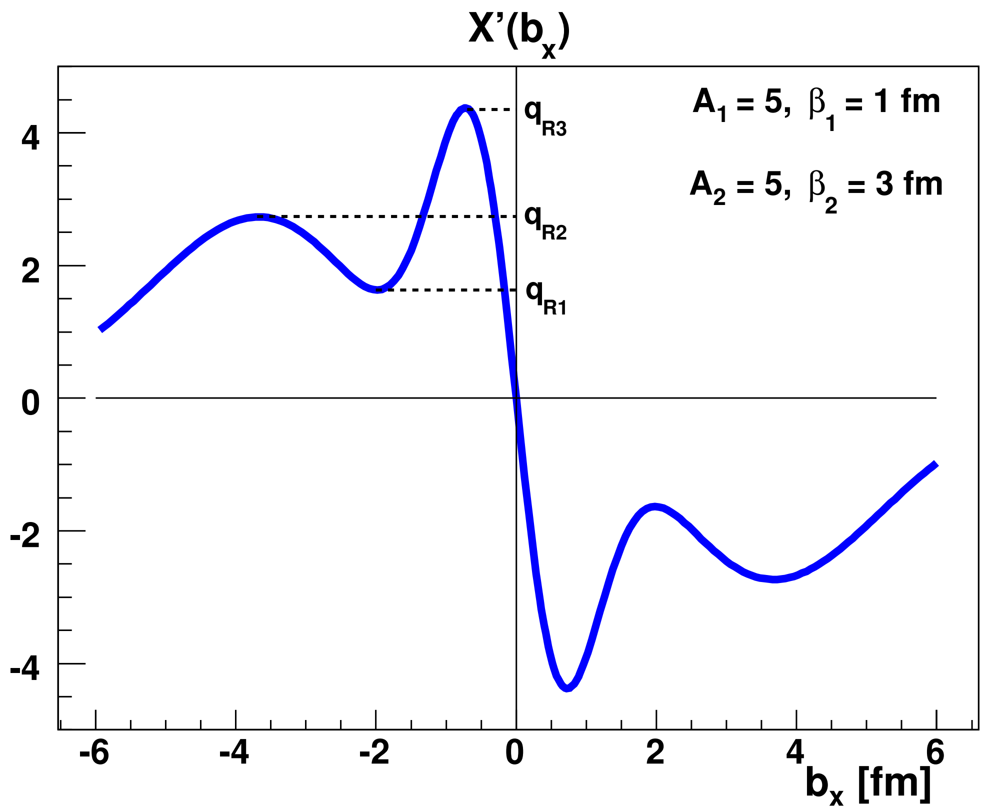


Interference of two amplitudes: oscillations

Asymptotic Diffraction Theory

A case with three rainbows:

$$X'(b_x) = -(b_x/\beta_1^2)A_1 \exp(-b_x^2/\beta_1^2) - (b_x^3/\beta_2^4)A_2 \exp(-b_x^2/\beta_2^2)$$

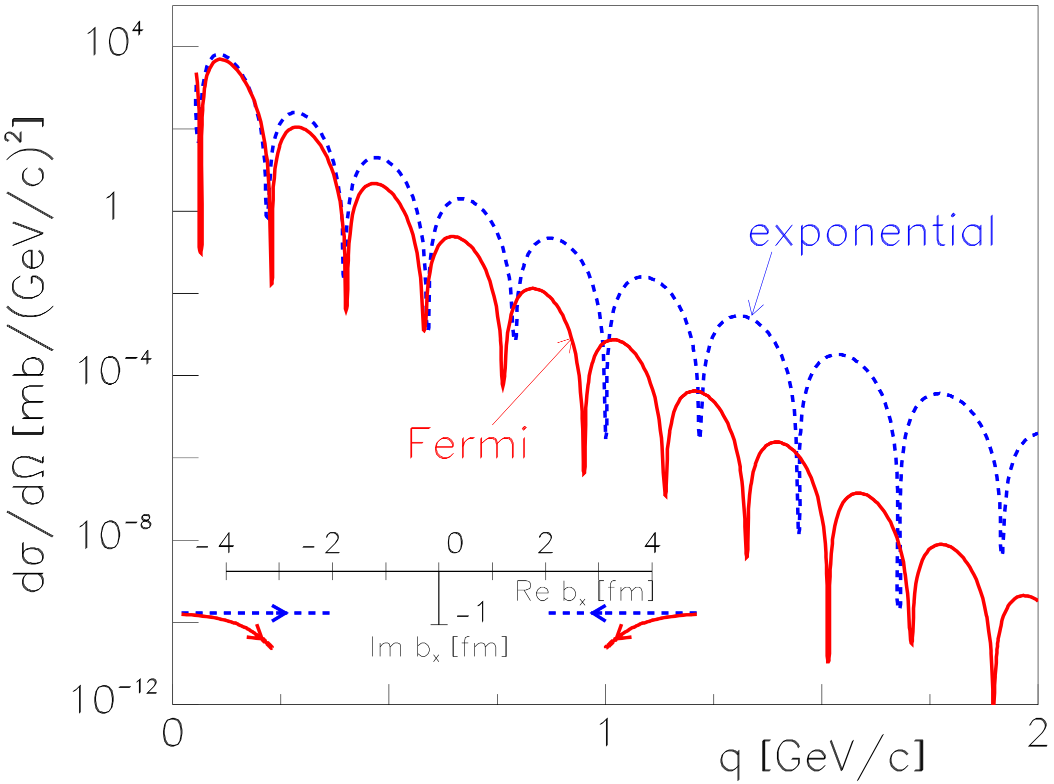


3 extrema > 0

Asymptotic Diffraction Theory

Absorption (simple case: pure imaginary)

$$X_{\rm exp}(b_x) = A_{\rm exp}\, e^{-b_x/\beta}, \qquad \mathrm{Re}\, b_x > 0,$$
$$X_{\rm exp}(-b_x) = X_{\rm exp}(b_x),$$
$$X_{\rm F}(b_x) = \frac{A_{\rm F}}{1+\exp[(b_x - c)/\beta]}, \qquad \mathrm{Re}\, b_x > 0$$
$$X_{\rm F}(-b_x) = X_{\rm F}(b_x).$$

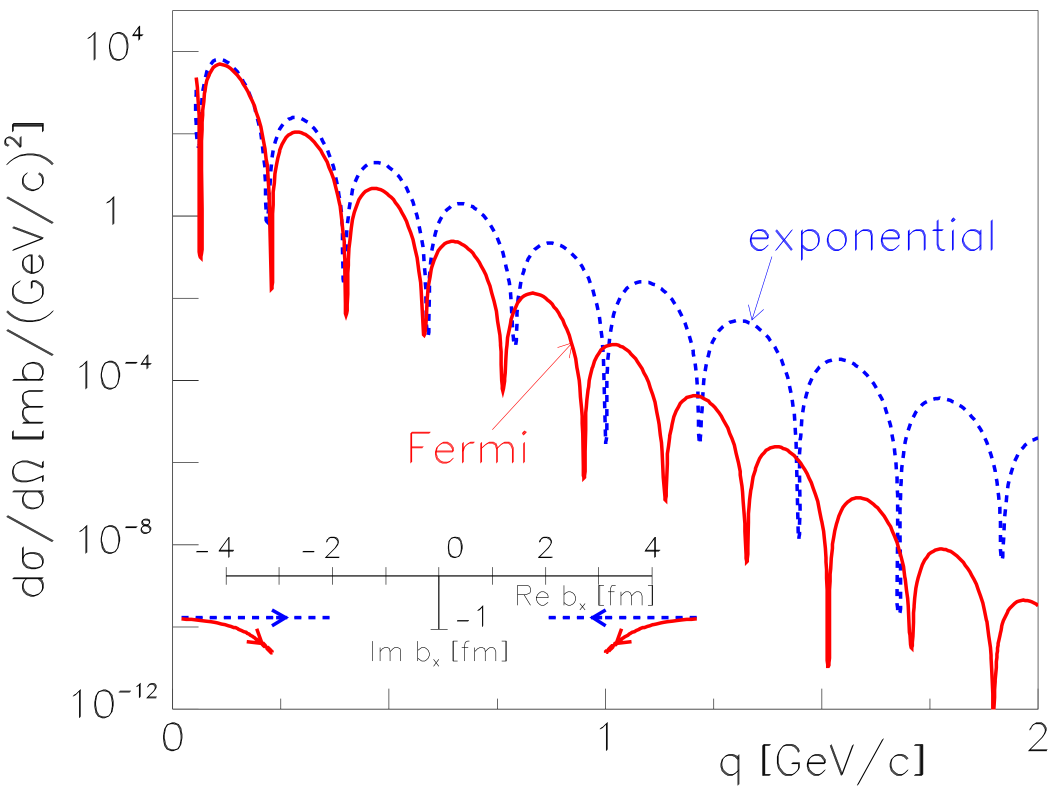


Asymptotic Diffraction Theory

Absorption (simple case: X pure imaginary)

Typically two points of stationary phase

Symmetrically located with respect to imaginary axis



Asymptotic Diffraction Theory

Technical note:

For simple functions $X'(b_x)$, may find explicit solutions for stationary points.

For “complicated” functions $X'(b_x)$, may draw a map in complex b_x plane:

1. Distinguish regions where Re $X'(b_x)$ is positive vs negative

Possible solutions where Re $X'(b_x) > 0$

2. Draw contours where Im $X'(b_x) = 0$

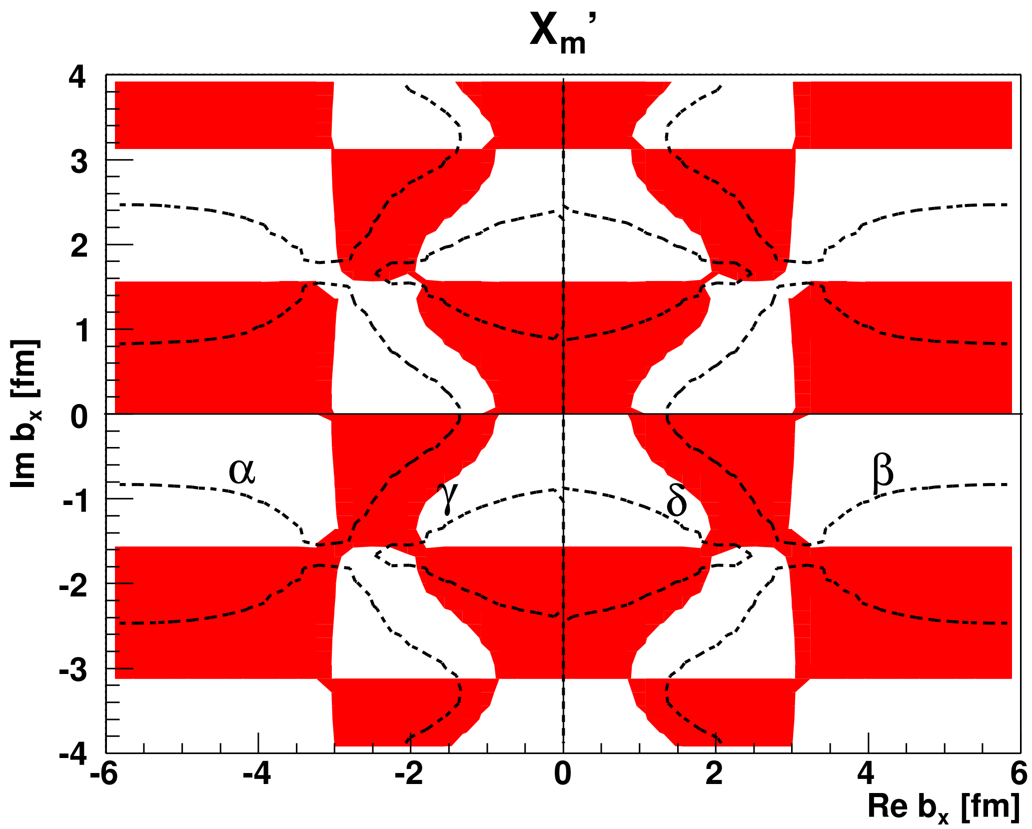
Stationary points have to lie where Re $X'(b_x) > 0$, Im $X'(b_x) = 0$ and Im b_x non-positive

Asymptotic Diffraction Theory

Technical note:

Difference of two “Fermi” functions:

$$X_m(b_x) = \frac{A_\mathrm{F}}{1+\exp[(b_x-c)/\beta]} - \frac{A'_\mathrm{F}}{1+\exp[(b_x-c')/\beta]}, \qquad \mathrm{Re}\, b_x > 0,$$



Asymptotic Diffraction Theory

The lessons:

Scattering from the whole impact plane represented by contributions from a few stationary points (often just two)

Slope and period will be determined by the location of the stationary points

Asymptotically, the stationary points will move towards singularities

References:

1. "Asymptotic Diffraction Theory", Cambridge University Press

2. Bleszynski, Glauber and Osland, Phys. Lett. B, 1981

Precursor:

Hoffmann, Ray, Barlett et al, Phys. Rev. C, 1980

Personal reminiscences

Hercegnovi 1969

Personal reminiscences

My overlaps with Roy:

1969: Hercegnovi lectures

1972: CERN

1976-1978: postdoc at Harvard

1981,1982: postdoc at Harvard

1991,1993: visits to Bergen

2009, 2010, 2012, 2013, 2014: met at CERN, NORDITA, Bergen, ICTP

2015, 2016: met in Cambridge

Personal reminiscences

Stockholm 2005

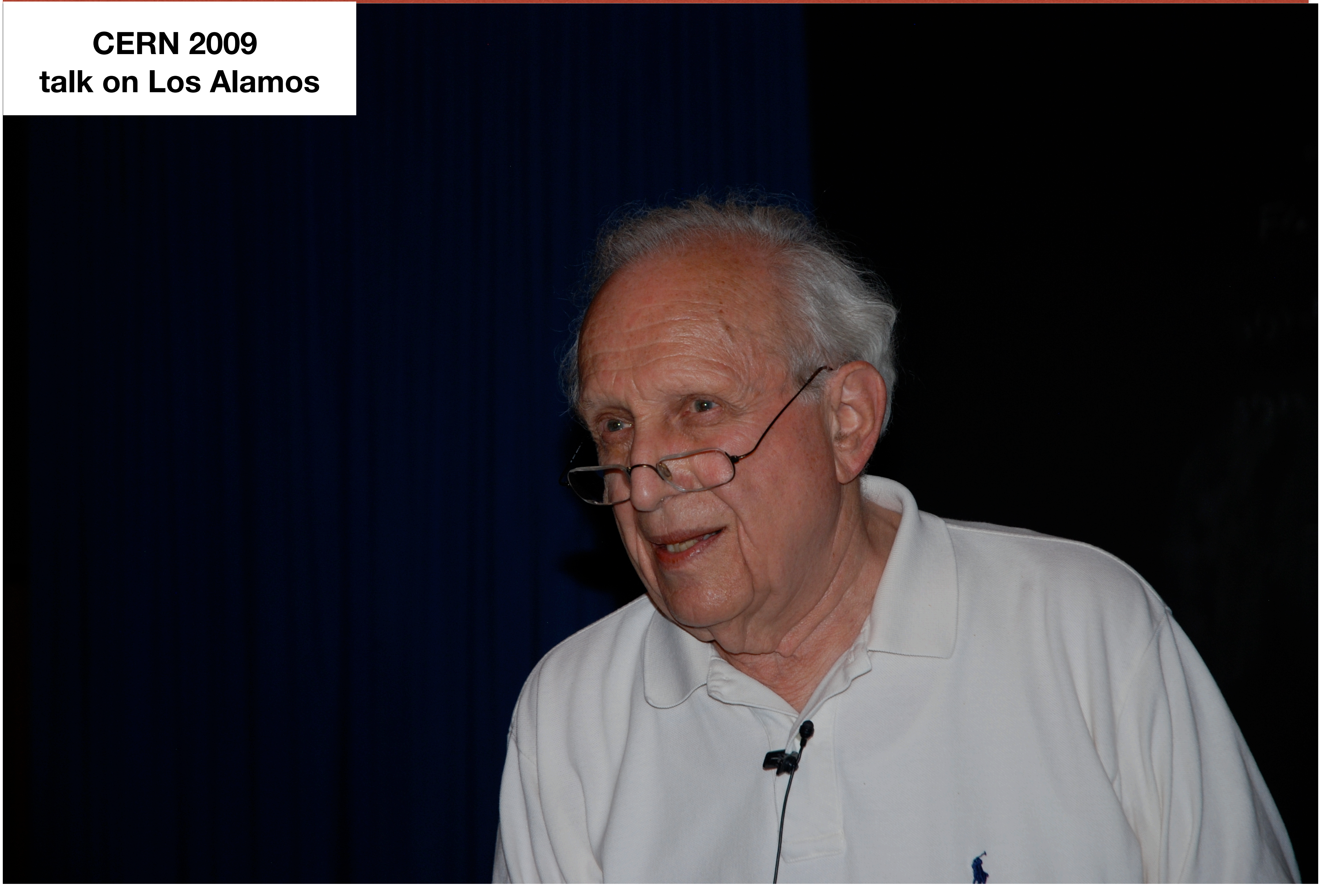
Personal reminiscences

Garching 2009

Personal reminiscences

CERN 2009

Personal reminiscences



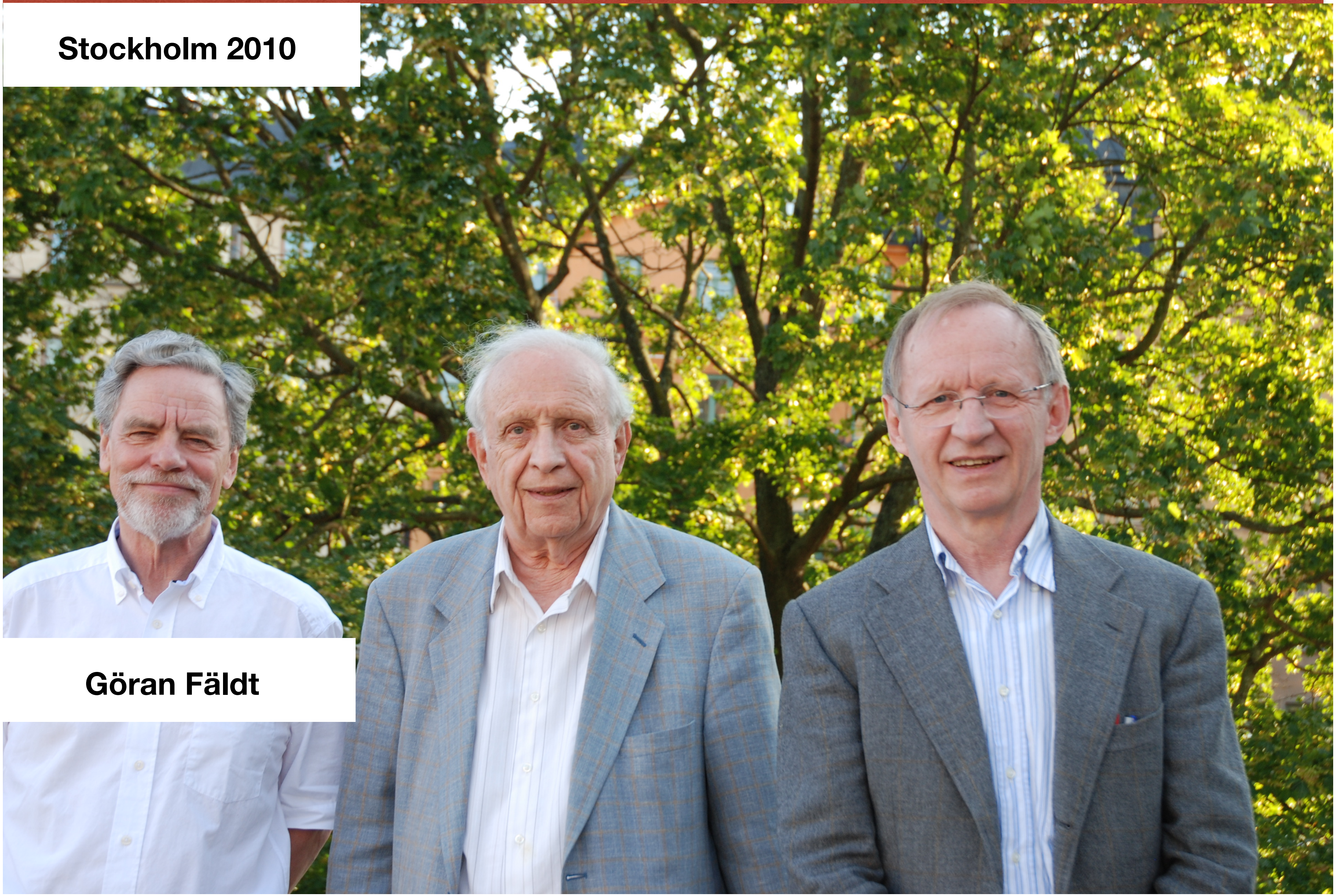
CERN 2009
talk on Los Alamos

Personal reminiscences

Stockholm 2010

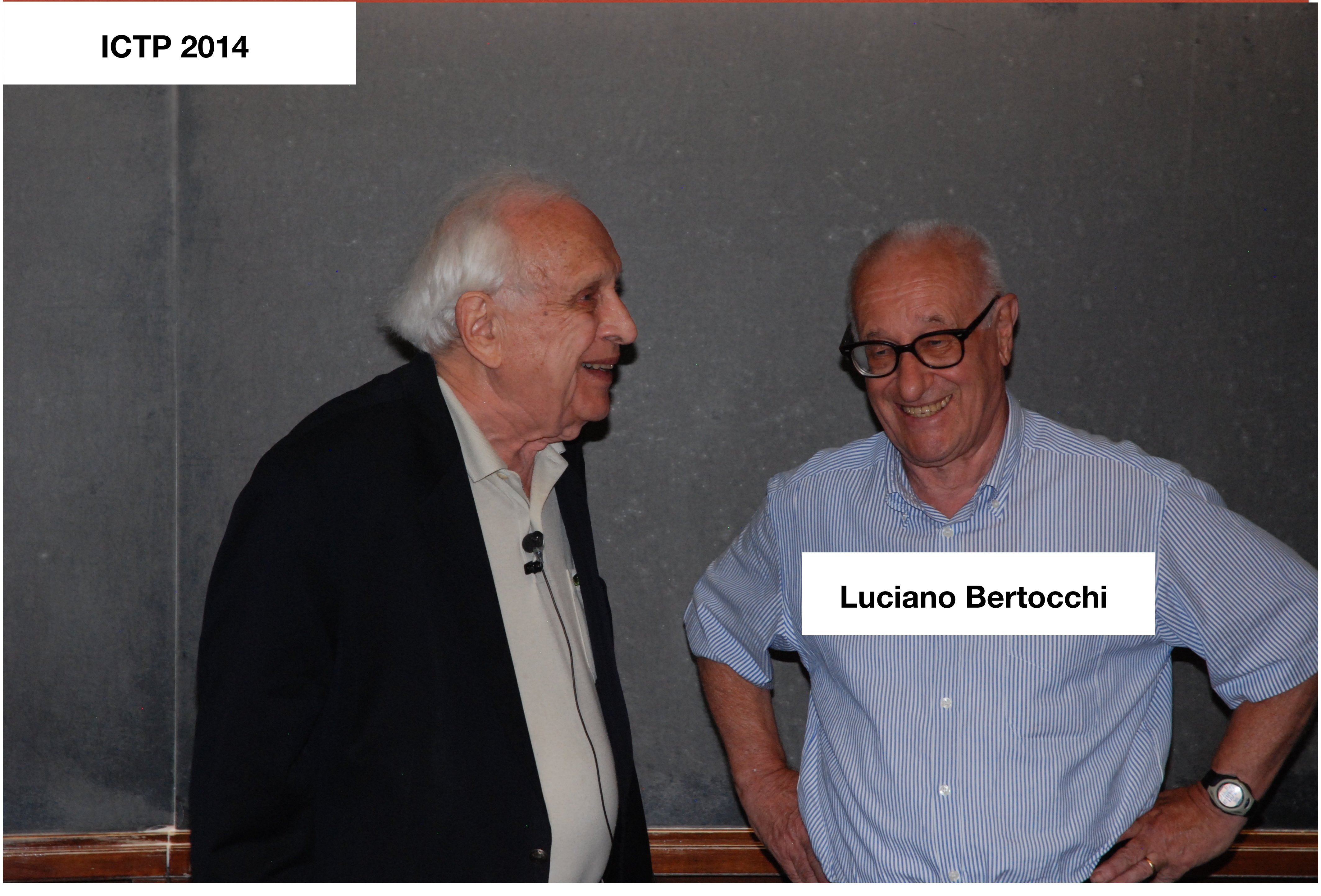
Personal reminiscences

Stockholm 2010



Göran Fäldt

Personal reminiscences



Personal reminiscences

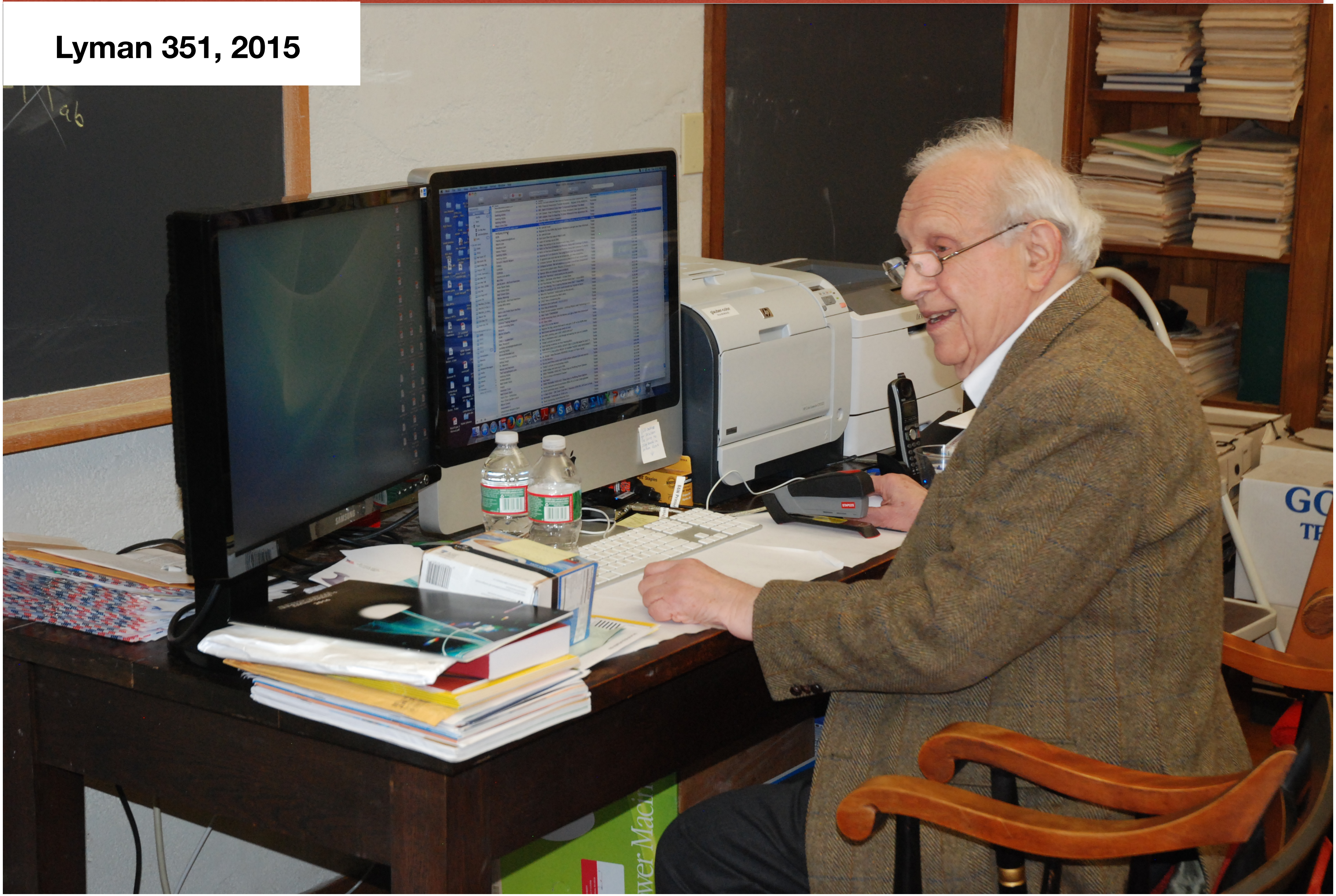
ICTP 2014

Personal reminiscences

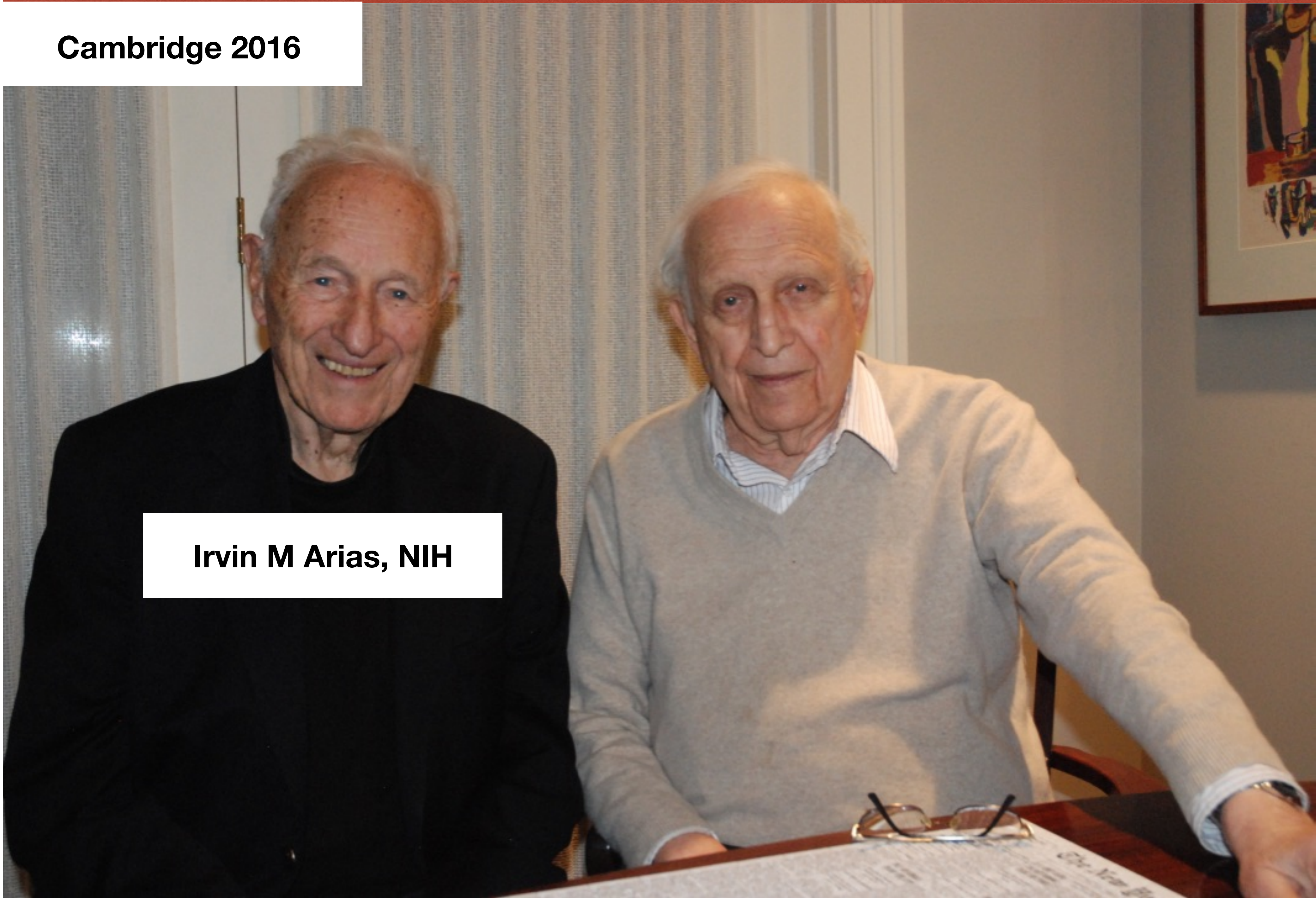
ICTP 2014

Personal reminiscences

Lyman 351, 2015



Personal reminiscences



Personal reminiscences

Final comments

Roy had met all the famous people of his era

He had a fantastic memory!

and could tell detailed stories about most of them

There exist recordings of him telling about the Manhattan Project

Personal reminiscences

Thank you!