The eikonal model of reactions involving exotic nuclei; Roy Glauber's legacy in today's nuclear physics

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THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

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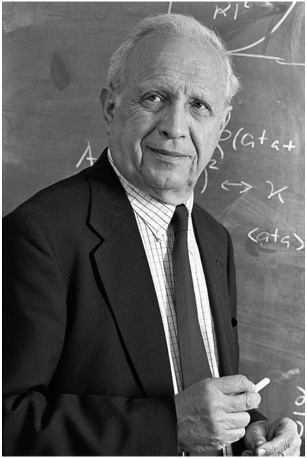
Roy J. Glauber

Roy J. Glauber 1925–2018

2005 Nobel Laureate for “his contribution to the quantum theory of optical coherence”

Development of the eikonal approximation in quantum collision theory

Sweeper of the broom at the Ig Nobel ceremony



Halo nuclei

Exotic nuclear structures are found far from stability

In particular halo nuclei with peculiar quantal structure :

- Light, n-rich nuclei
- Low $S_{\rm n}$ or $S_{\rm 2n}$

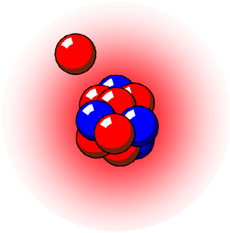


Exhibit large matter radius due to clear few-body structure : neutrons tunnel far from the core and form a halo

One-neutron halo

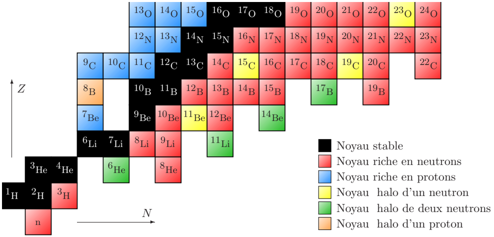
^{11}Be ≡ ^{10}Be + n

^{15}C ≡ ^{14}C + n

Two-neutron halo

^{6}He ≡ ^{4}He + n + n

^{11}Li ≡ ^{9}Li + n + n



Proton halœs are also possible, but less probable

Reactions with exotic nuclei

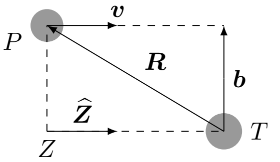
Exotic nuclei, like halo nuclei, are studied through reactions

Elastic scattering
Breakup ≡ dissociation of halo from core
by interaction with target
Knockout ≡ one (or two) nucleon removals
by interaction with (light) target

Need an good understanding of the reaction mechanism
i.e. an accurate theoretical description of reaction
coupled to a realistic model of projectile

The eikonal approximation first developed by Glauber
[Glauber, *Lecture in Theoretical Physics* Vol. 1, p. 315 (1959)]
simple and efficient model of reactions for halo nuclei at high energy

Eikonal approximation in a nuttshell



Collision of a projectile (P) on a target (T)
with interaction simulated by potential V_{PT}
⇒ need to solve the Schrödinger equation

$$[T_R + V_{PT}(R)]\,\Psi(\boldsymbol{R}) = E_T\Psi(\boldsymbol{R})$$

with the initial condition $\Psi(\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}$

Eikonal approximation : factorise $\Psi(\boldsymbol{R}) = e^{iKZ}\,\widehat{\Psi}(\boldsymbol{R})$

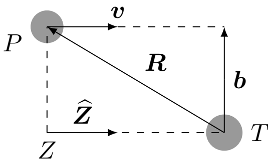
$$T_R\Psi = e^{iKZ}[T_R + vP_Z + \frac{\mu_{PT}}{2}v^2]\widehat{\Psi}$$

Neglecting T_R vs vP_Z and using $E_T = \frac{1}{2}\mu_{PT}v^2$

$$\begin{aligned} i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{b},Z) &= V_{PT}(R)\,\widehat{\Psi}(\boldsymbol{b},Z) \\ \Rightarrow \widehat{\Psi}(\boldsymbol{b},Z) &= \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{Z} V_{PT}(b,Z')\,dZ'\right] \end{aligned}$$

Eikonal approximation in a nuttshell

We thus have



$$\Psi(\boldsymbol{b}, Z) \underset{Z\to+\infty}{\longrightarrow} e^{iKZ}\, e^{i\chi(b)}$$

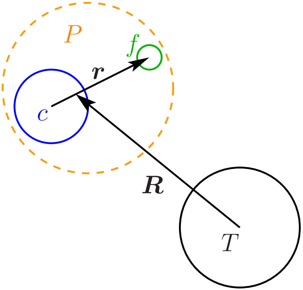
with the eikonal phase $\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} V_{PT}(b, Z)\, dZ$

- easy to compute
- simple semiclassical interpretation :
  P follows a straight-line trajectory
  along which it accumulates a phase due to V_{PT}
- can be extended to few-body projectiles. . .

Reaction model for two-body projectiles (1-nucleon halo)

Projectile (P) modelled as a two-body system :
core (c)+loosely bound nucleon (f) described by

$H_0 = T_r + V_{cf}(\boldsymbol{r})$



V_{cf} adjusted to reproduce bound state Φ_0 and resonances

Target T seen as structureless particle

P-T interaction simulated by optical potentials

$\Rightarrow$ breakup reduces to three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with initial condition $\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}\Phi_0(\boldsymbol{r})$

Eikonal approximation for two-body projectiles

Three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with condition $\Psi \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}\Phi_0$

Eikonal approximation : factorise $\Psi = e^{iKZ}\,\widehat{\Psi}$

$$\Rightarrow i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - \epsilon_0 + V_{cT} + V_{fT}]\,\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

solved for each $\boldsymbol{b}$ with condition $\widehat{\Psi} \underset{Z\to-\infty}{\longrightarrow} \Phi_0(\boldsymbol{r})$

(usual) eikonal includes the adiabatic approximation : $(H_0 - \epsilon_0) \approx 0$

$$\Rightarrow \Psi(\boldsymbol{r},\boldsymbol{b},Z) \underset{Z\to+\infty}{\longrightarrow} e^{iKZ}\ \exp\left[i\left(\chi_{cT}(\boldsymbol{r},\boldsymbol{b}) + \chi_{fT}(\boldsymbol{r},\boldsymbol{b})\right)\right]\Phi_0(\boldsymbol{r})$$

Used to study spectroscopy of halo nuclei through reactions

[Hansen & Tostevin, Ann. Rev. Nucl. Part. Sc. 53, 219 (2003)]

One-neutron knockout

One neutron removed in high-energy collision on light target

neutron either breaks up from the core or is absorbed by the target

Only the core is measured : $^{11}\mathrm{Be} + \mathrm{Be} \rightarrow\ ^{10}\mathrm{Be} + X$

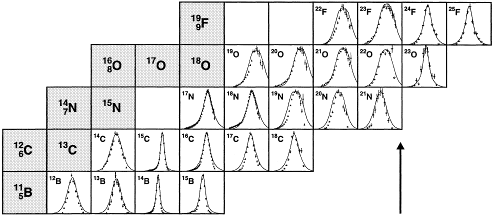
Eikonal approximation well suited to analyse KO data

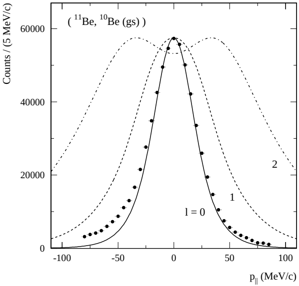
[Hansen & Tostevin, Ann. Rev. Nucl. Part. Sc. 53, 219 (2003)]

Halo nuclei have large spacial expansion

$\Rightarrow$ narrow momentum distribution

$d\sigma_{\mathrm{KO}}/dp_{c\|}$ helps identify halos and gives spectroscopic information





[Sauvan *et al.* PLB 491, 1 (2000)]

[Aumann *et al.* PRL 84, 35 (2000)]

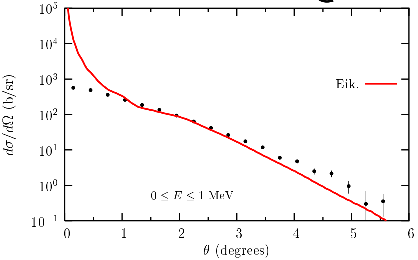
Breakup

Halo neutron(s) dissociate from core in collision with target

Both core and neutron are detected : $^{11}\mathrm{Be} + \mathrm{Pb} \rightarrow\ ^{10}\mathrm{Be} + \mathrm{n} + \mathrm{Pb}$

Eikonal approximation does not properly treat the Coulomb breakup

$$^{11}\mathrm{Be} + \mathrm{Pb} \rightarrow\ ^{10}\mathrm{Be} + \mathrm{n} + \mathrm{Pb} \ @\ 69A \text{ MeV}$$



Exp. : [Fukuda *et al.* PRC 70, 054606 (2004)]

Th. : [Goldstein, Baye, P.C. PRC 73, 024602 (2006)]

Adiabatic approximation not valid for infinitely ranged Coulomb force

Mathematically : $\chi^C(b) \underset{b\rightarrow\infty}{\longrightarrow} \frac{1}{b}$ ⇒ diverges at large b ⇔ forward angle

Idea : avoid adiabatic approximation. . .

Dynamical eikonal approximation (DEA)

Three-body scattering problem with condition $\Psi \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\Phi_0$

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

Eikonal factorisation $\Psi = e^{iKZ}\widehat{\Psi}$

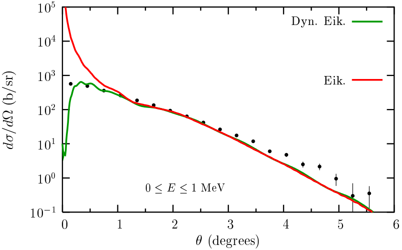
$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - \epsilon_0 + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

solved for each $\boldsymbol{b}$ with condition $\widehat{\Psi} \underset{Z\to-\infty}{\longrightarrow} \Phi_0(\boldsymbol{r})$

Dynamical Eikonal Approx. [Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

Same in coupled-channel approach [Ogata *et al.* PRC 68, 064609 (2003)]

^{11}Be + Pb → ^{10}Be + n + Pb 69A MeV

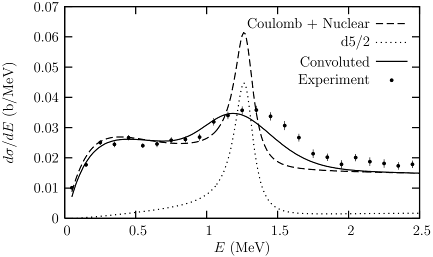


Dynamical calculation provides correct treatment of Coulomb (excellent agreement with data)

⇒ tool to study breakup on both light and heavy targets

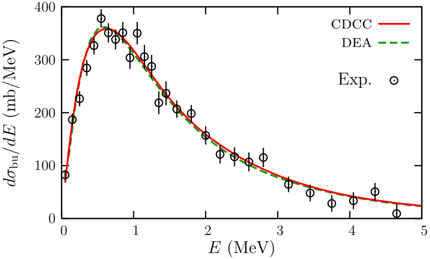
^{11}Be+C @ 67AMeV & ^{15}C+Pb @ 68AMeV

^{11}Be + C → ^{10}Be + n + C @ 67A MeV



Exp. : [Fukuda *et al.* PRC 70, 054606 (2004)]
Th. : [P.C., Goldstein, Baye, PRC 70, 064605 (2004)]

^{15}C + Pb → ^{14}C + n + Pb @ 68A MeV



Exp. : [Nakamura *et al.* PRC 79, 035805 (2009)]
Th. : [P.C., Esbensen, Nunes, PRC 85, 044604 (2012)]

- Excellent agreement with data on both light and heavy targets
  ⇒ confirms the halo structure of ^{11}Be and ^{15}C
- Nuclear breakup provides information on core-n resonances
- Same result as purely quantal CDCC
  ⇒ validates the eikonal approximation at intermediate energy
- But : DEA more computationally expensive than usual eikonal

Coulomb-Corrected Eikonal

The eikonal Coulomb phase reads

$$\chi_C(\boldsymbol{r}, b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} \frac{Z_c Z_T e^2}{R_{cT}} dZ \propto \frac{1}{b}$$

$\Rightarrow e^{i\chi_C} = 1 + i\chi_C - \frac{1}{2}\chi_C^2 + \ldots$ diverges when $\int db$

Idea : replace χ_C by χ_{FO} from perturbation theory

[Margueron, Bonaccorso, Brink, NPA 720, 337 (2003)]

$$\chi_{FO}(\boldsymbol{r}, b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} e^{i\omega Z}\frac{Z_c Z_T e^2}{R_{cT}} dZ \propto \frac{e^{-\omega b}}{b}$$

i.e. correct asymptotics

The Coulomb-corrected eikonal (CCE) then reads

$$e^{i\chi} = e^{i\chi_N}\left(e^{i\chi_C} - i\chi_C + i\chi_{FO}\right)$$

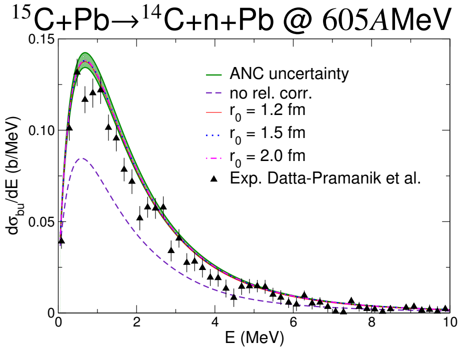
- Simple correction, computationally cheap
- Includes higher-order effects, interference with nuclear potential
- Good agreement with DEA calculations at $70A$MeV

[P.C., Baye, Suzuki, PRC 78, 054602 (2008)]

Relativistic energies

CCE well suited to include relativistic corrections

[Moschini, P.C. PLB 790, 367 (2019)]



Exp : [Datta Pramanik *et al.* PLB 551, 63 (2003)]

Th : [Moschini, Yang, P.C. arXiv :1907.11753 (2019)]

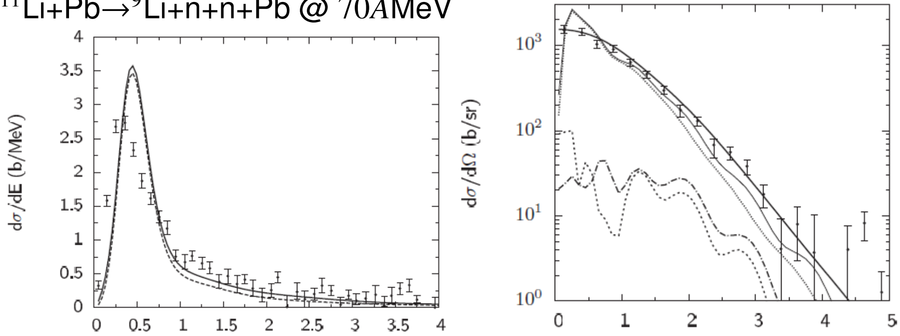
- Excellent agreement with experiment
- Relativistic corrections are needed

Extension to 3-b projectiles

CCE can also be extended to 3-body projectiles (2-n halœs)

[Baye *et al.* PRC 79, 024607 (2009)]

^{11}Li+Pb→^{9}Li+n+n+Pb @ $70A$MeV



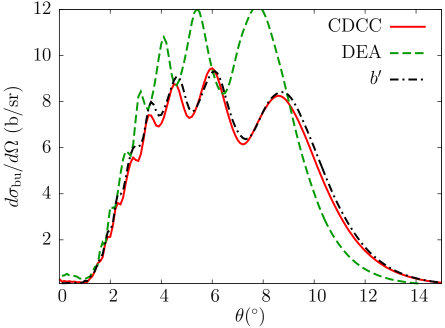
Exp : [Nakamura *et al.* PRL 96, 252502 (2006)]

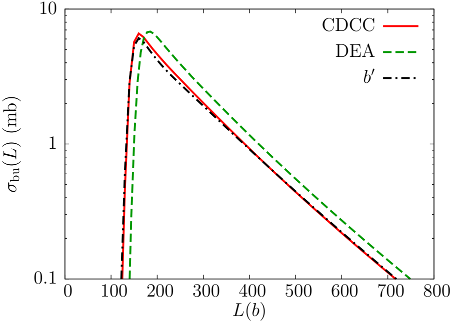
Th : [Pinilla *et al.* PRC 85, 054610 (2012)]

- Good agreement with data
  ⇒ enables study of Borromean nuclei

Extension to low energies

^{15}C+Pb→ ^{14}C+n+Pb @ $20A$MeV





Shift in θ translates into a shift in $L \leftrightarrow b$

⇒ semi-classical correction $b \to b'$ (classical closest approach)

$b \to b'$ corrects $\sigma_{\rm bu}(L)$ and hence $d\sigma_{\rm bu}/d\Omega$

[Fukui, Ogata, P.C. PRC 90, 034617 (2014)]

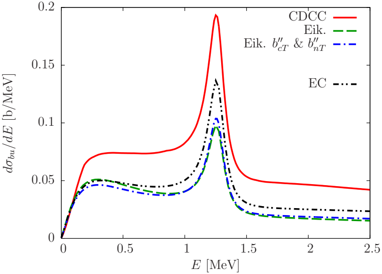
⇒ Improves eikonal model to the level of CDCC…

at least for Coulomb-dominated reactions

Low energy on light target

Can such semiclassical correction be extended to nuclear breakup ?

^{11}Be+C→ ^{10}Be+n+C @ $20A$MeV [Hebborn, P.C. PRC 98, 044610 (2018)]



- Eikonal underestimates CDCC (quantal) calculations
- $b \to b'' \in \mathbb{C}$: distance of closest approach of optical potential
  [Aguiar, Zardi, Vitturi, Phys. Rev. C 56, 1511 (1997)]
- Exact Continued S-matrix correction : $e^{i\chi(b)} \to e^{2i\delta_l}$
  [Brooke, Al-Khalili, Tostevin, Phys. Rev. C 59, 1560 (1999)]

⇒ no efficient correction found for nuclear-dominated reactions

Summary

- Roy Glauber has developed the eikonal approximation
  to describe high-energy collisions
  [Glauber, *Lecture in Theoretical Physics* Vol. 1, p. 315 (1959)]

- Eikonal model used to study few-body structure of halo nuclei
  - Knockout reactions
    [Hansen & Tostevin, Ann. Rev. Nucl. Part. Sc. 53, 219 (2003)]
  - Breakup : requires proper treatment of Coulomb
    DEA [Baye, P. C., Goldstein, PRL 95, 082502 (2005)]
    CCE [Margueron, Bonaccorso, Brink, NPA 720, 337 (2003)]

- Simple and elegant model of reaction

- Difficult to extend to low beam energy ($< 50A$MeV)

- Provides valuable information on halo structure
  in high and intermediate energy reactions

Thanks to my collaborators

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