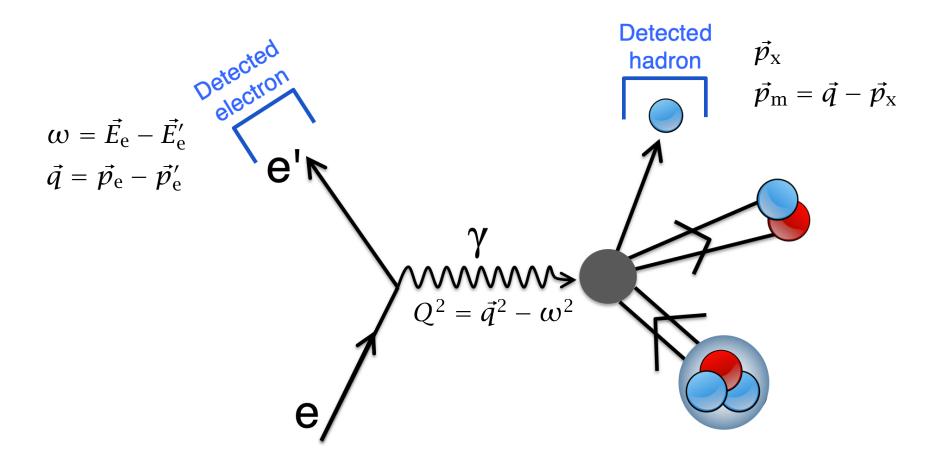
Study of light nuclei by polarization observables in electron scattering

S. Širca, U of Ljubljana, Slovenia

Guildford, 6 Sep 2019

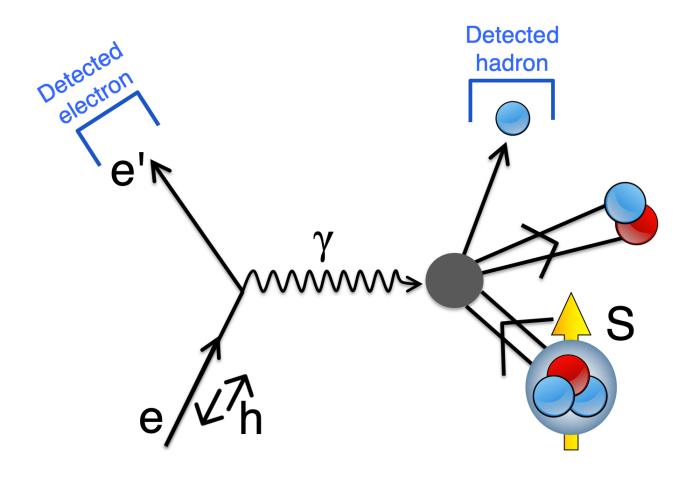


Unpolarized electron scattering



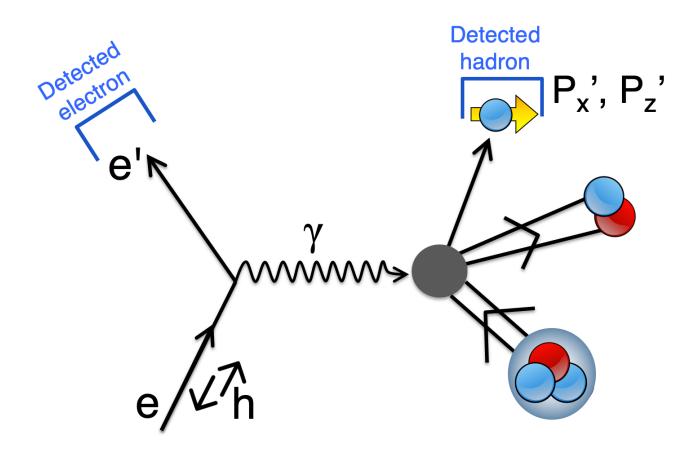
$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_{\mathrm{e'}}\mathrm{d}\Omega_{\mathrm{e'}}\mathrm{d}E_{\mathrm{x}}\mathrm{d}\Omega_{\mathrm{x}}} = \sigma_0 \left[v_L R_L + v_T R_T + v_{LT} R_{LT} + v_{TT} R_{TT} \right]$$

Electron scattering with beam and target polarization



$$A = \frac{\sigma(h_+, \vec{S}) - \sigma(h_-, \vec{S})}{\sigma(h_+, \vec{S}) + \sigma(h_-, \vec{S})} \propto v_{T'} R_{T'} + v_{LT'} R_{LT'}$$

Electron scattering with beam and recoil polarization



$$P'_{z} = P'_{\ell} \propto v_{LT'} R_{LT'}^{\ell} + v_{TT'} R_{TT'}^{\ell}$$

$$P_{n} \propto v_{L} R_{L}^{n} + v_{T} R_{T}^{n} + v_{LT} R_{LT}^{n} + v_{TT} R_{TT}^{n}$$

$$P'_{x} = P'_{t} \propto v_{LT'} R_{LT'}^{t} + v_{TT'} R_{TT'}^{t}$$

Experiments covered in this talk

³He

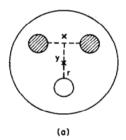
- JLab E05–102 Double-spin asymmetries in quasi-elastic ${}^{3}\vec{\mathrm{He}}(\vec{\mathrm{e}}, \mathbf{e}'\mathbf{d})p$ ${}^{3}\vec{\mathrm{He}}(\vec{\mathrm{e}}, \mathbf{e}'p)d$ ${}^{3}\vec{\mathrm{He}}(\vec{\mathrm{e}}, \mathbf{e}'p)pn$
- JLab E05–015 Target single-spin asymmetry in quasi-elastic ³He[†](e, e')
- JLab E08–005 Target single-spin asymmetry in quasi-elastic ³He(e, e'n) Double-spin asymmetries in quasi-elastic ³He(e, e'n)
- MAMI (Mainz) Project 'N' *Triple*-polarized ³He(e, e'p)

²H and ¹²C

- MAMI (Mainz) Single-spin asymmetries in ¹²C(e[†], e')
- MAMI (Mainz + TAU) joint recoil-polarimetry effort Double-spin asymmetries in ${}^2\vec{H}(\vec{e}, e'\vec{p})$ and ${}^{12}\vec{C}(\vec{e}, e'\vec{p})$

Physics motivation for studying processes on ³He

• Knowledge of ground-state structure of ³He needed to **extract information on the neutron** from ³He(e, e'X) or ³He(e, e'). Examples: G_E^n , G_M^n , A_1^n , g_1^n , g_2^n , GDH.



- Complications: protons in 3 He partly polarized due to presence of S'- and D-state components.
- Addressing differences in $\sqrt{\langle r^2 \rangle}$ (³H, ³He).
- Understanding (iso)spin dependence of reaction mechanisms (MEC, IC).

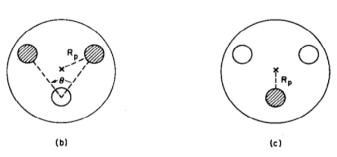
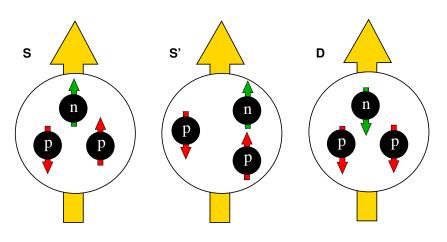


Fig. 1. Schematic picture of trinucleon when all forces are identical is shown in (a). The effect on 3 He and 3 H when the pp or nn force is weaker than the np force is illustrated in (b) and (c). R_P is the "charge radius". Shading indicates a proton.

- Understanding role of *D* and *S'* states is one of key issues in "Standard Model" of few-body theory.
- Persistent discrepancies among theories regarding double-polarization observables most sensitive to ³He ground-state structure.

Polarized ³He: it is easy to draw the cartoon ...



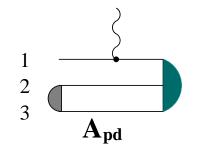
- S: spatially symmetric
 ≈ 90% of spin-averaged WF;
 "polarized neutron"
- *D*: generated by tensor part of NN force, $\approx 8.5\%$.
- S': mixed symmetry component; (spin-isospin)-space correlations, $\approx 1.5\%$. $P'_S \approx E_{\rm h}^{-2.1}$.
- $P_{
 m n}^{
 m eff} pprox +$ **0.86**, $P_{
 m p}^{
 m eff} pprox -$ **0.03**

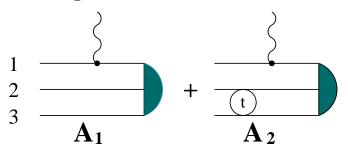
Hamiltonian	S	S'	P	D
AV18	90.10	1.33	0.066	8.51
AV18/TM	89.96	1.09	0.155	8.80
AV18/UIX	89.51	1.05	0.130	9.31
CD-Bonn	91.62	1.34	0.046	6.99
CD-Bonn/TM	91.74	1.21	0.102	6.95
Nijm I	90.29	1.27	0.066	8.37
Nijm I/TM	90.25	1.08	0.148	8.53
Nijm II	90.31	1.27	0.065	8.35
Nijm II/TM	90.22	1.07	0.161	8.54
Reid93	90.21	1.28	0.067	8.44
Reid93/TM	90.09	1.07	0.162	8.68

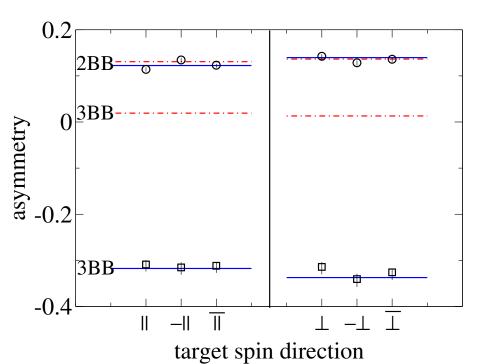
Schiavilla++ PRC 58 (1998) 1263 TM = Tucson-Melbourne π - π exchange 3NF UIX = Urbana 3NF

... supported e. g. by data on $^3\vec{He}(\vec{e}, e'p)d/pn$...

- quasi-elastic ($Q^2 = 0.31$, $\omega = 135$, q = 570)
- 3NF, MEC negligible, FSI small in 2bbu, large in 3bbu







⊳ 2bbu

 $A_{\text{PWIA}} \approx A_{\text{PWIA+FSI}}$ || kinematics + small p_{d} \Rightarrow polarized p target, $P_{\text{p}} \approx -\frac{1}{3}P_{\text{He}}$

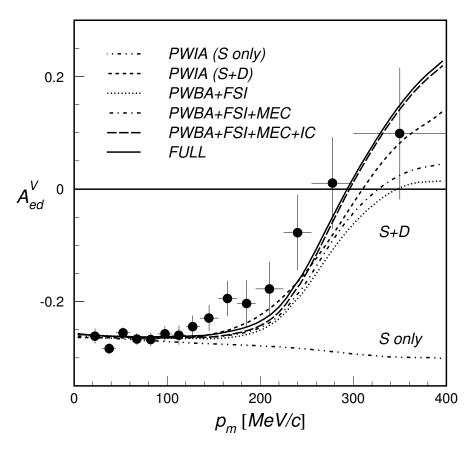
⊳ 3bbu

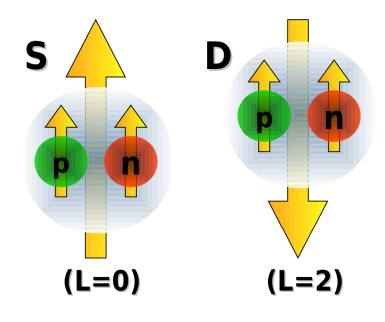
 $A_{\text{PWIA}} \approx 0 \; (\text{p} \uparrow \text{p} \downarrow)$ $A_{\text{PWIA+FSI}} \; \text{large \& negative}$ not a polarized p target

PRC **72** (2005) 054005, EPJA **25** (2005) 177

... and which has a nice analogue in the deuteron ...

$$\vec{d}(\vec{e}, e'p)$$





$$\sigma = \sigma_0 \left(1 + h P_1^{\mathsf{d}} A_{\mathsf{ed}}^{\mathsf{V}} \right)$$

$$P_z^{\mathrm{p}} = \sqrt{\frac{2}{3}} \left(P_S - \frac{1}{2} P_D \right) P_1^{\mathrm{d}}$$

Passchier++ PRL **82** (1999) 4988

Passchier++ PRL **88** (2002) 102302

... but the true ground state of ³He is like lace

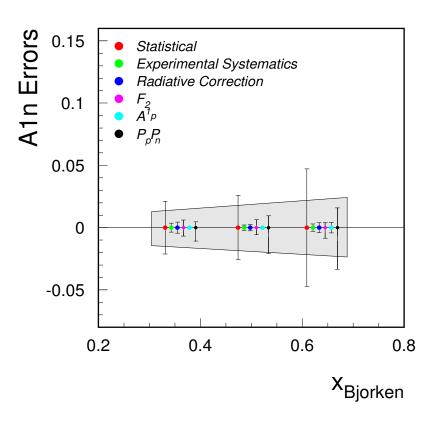
Channel			_				Probability
number	L	S	l_{α}	L_{α}	P	K	(%)
1	0	0.5	0	0	A	1	87.44
2	0	0.5	0	0	M	2	0.74
3	0	0.5	1	1	$oldsymbol{M}$	1	0.74
4	0	0.5	2	2	\boldsymbol{A}	1	1.20
5	0	0.5	2	2	M	2	0.06
6	1	0.5	1	1	M	1	0.01
7	1	0.5	2	2	\boldsymbol{A}	1	0.01
8	1	0.5	2	2	M	2	0.01
9	1	1.5	1	1	M	1	0.01
10	1	1.5	2	2	M	2	0.01
11	2	1.5	0	2	M	2	1.08
12	2	1.5	1	1	M	1	2.63
13	2	1.5	1	3	M	1	1.05
14	2	1.5	2	0	M	2	3.06
15	2	1.5	2	2	M	2	0.18
16	2	1.5	3	1	M	1	0.37

Blankleider, Woloshyn PRC **29** (1984) 538

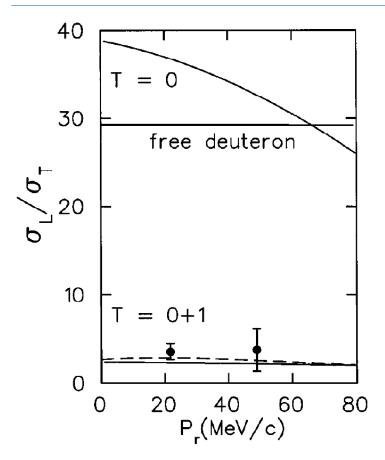
The E05-102 and E08-005 experiments at JLab

- Benchmark measurement of A'_{χ} and A'_{z} asymmetries in ${}^{3}\vec{\text{He}}(\vec{\text{e}},\text{e'd})$, ${}^{3}\vec{\text{He}}(\vec{\text{e}},\text{e'p})$, and ${}^{3}\vec{\text{He}}(\vec{\text{e}},\text{e'n})$.
- Better understanding of ground-state spin structure of polarized ³He —

 S, S', D wave-function components.
 Improve knowledge of ³He rather than using it as an effective neutron target.
 Direct consequences for all polarized ³He experiments.
- Distinct manifestations of S, D, S' with changing p_{miss} in $(e, e'\{p/d/n\})$.
- Data at (almost) identical Q^2 for $(\vec{e}, e'd)$, $(\vec{e}, e'p)$, and $(\vec{e}, e'n)$ simultaneously over a broad range of p_{miss} poses **strong constraints on state-of-the-art calculations**.

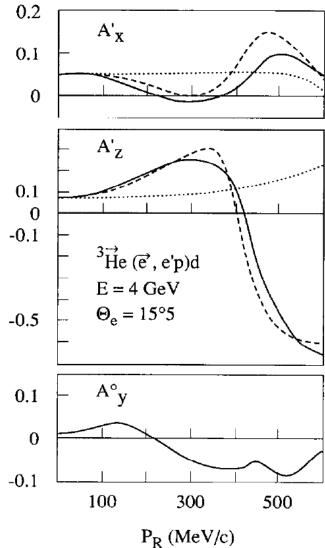


What is so special about ${}^{3}\text{He}(e, e'd)$ and ${}^{3}\vec{\text{He}}(\vec{e}, e'd)$?



unique isoscalar-isovector interference in (e, e'd)

Tripp++ PRL **76** (1996) 885



in (e, e'p) the D/S' effects seen only at high p_{miss}

Laget PLB **276** (1992) 398

Exploiting state-of-the-art calculations

Bochum/Krakow (full Faddeev)

- AV18 NN-potential (+ Urbana IX 3NF, work in progress)
- Complete treatment of FSI, MEC

Hannover/Lisbon (full Faddeev)

• CC extension and refit of CD-Bonn NN-potential

- Includes FSI, MEC
- Δ as active degree-of-freedom providing effective 3NF and 2-body currents
- Coulomb interaction for outgoing charged baryons

PRC **72** (2005) 014001

- AV18 + Urbana IX (or IL7)
- Inclusion of FSI by means of the variational PHH expansion and MEC
- Not Faddeev, but accuracy completely equivalent to it

Trento

Coming up

Basic machinery: Faddeev calculations

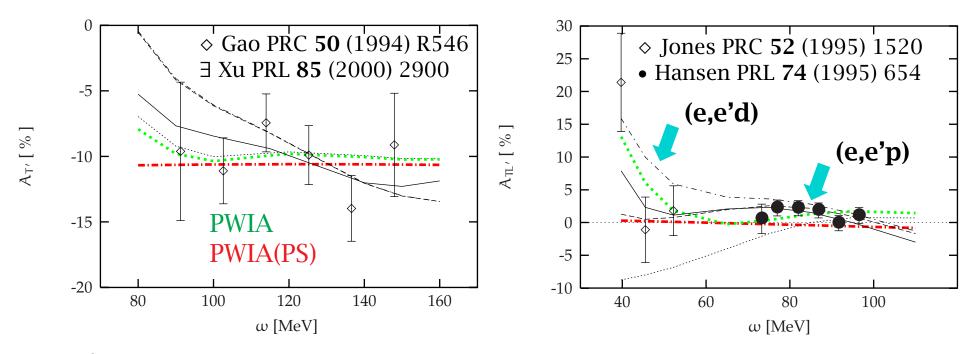
Nuclear transition current for breakup of ${}^{3}\text{He}$: $J^{\mu} = \left\langle \Psi_{\mathrm{f}} \, | \, \hat{\mathcal{O}}^{\mu} \, | \, \Psi_{^{3}\text{He}}(\theta^{*}, \phi^{*}) \right\rangle$ Photon absorption operator: $\hat{\mathcal{O}}^{\mu} = \sum_{i=1}^{3} \left[\hat{J}_{\mathrm{SN}}(i) + \hat{J}_{\mathrm{MEC}}(i) \right]$ Final-state interactions (auxiliary states): $\left\langle \Psi_{\mathrm{f}} \, | \, \hat{\mathcal{O}}^{\mu} \, | \, \Psi_{^{3}\text{He}}(\theta^{*}, \phi^{*}) \right\rangle \longrightarrow \left\langle \Psi_{\mathrm{f}} \, | \, U_{\mathrm{f}}^{\mu} \right\rangle$

$$+ \frac{3}{3} + \frac{$$

Golak++ Phys Rep **415** (2005) 89

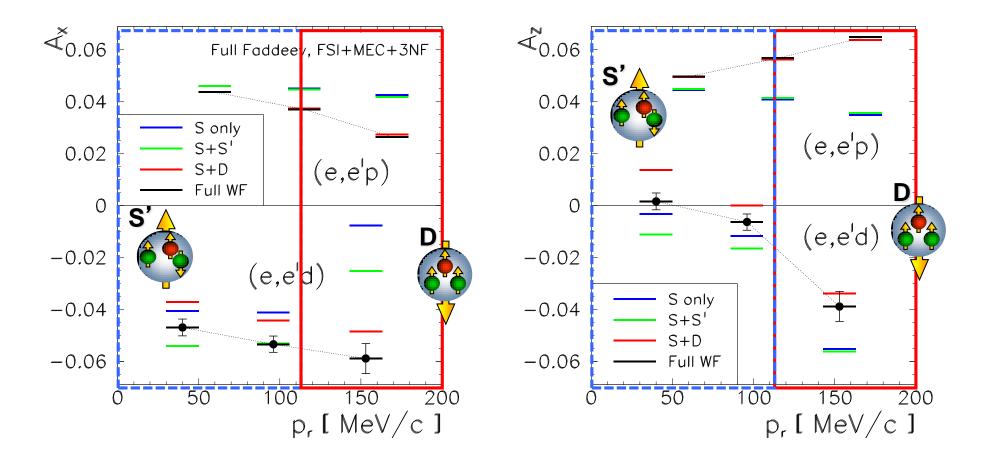
Indication of *D* and *S'* components in ${}^{3}\vec{He}(\vec{e}, e')$

Inclusive $A'_{\rm T}$ (= A_z) and $A'_{\rm LT}$ (= A_x)



- A'_{LT} receives contributions from ingredients which go beyond most simplistic picture [$F_1^{(n)} = 0$]
- sensitive to replacement PWIA(PS) \rightarrow PWIA.
- S'- and D-state pieces contribute very strongly to A'_{LT}

Ishikawa++ PRC **57** (1998) 39



- S' state relevant at small p_r (= p_{miss})
- D state governs variation of A_z at large p_r

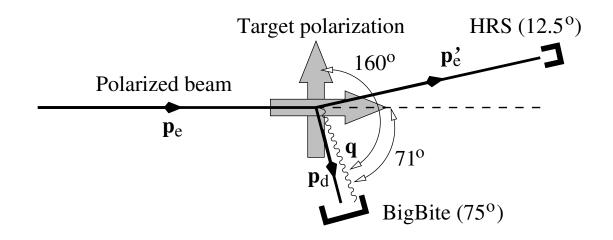
Beam-target asymmetry in QE p/d knockout from ³He

• Cannot disentangle effects of WF components (S, D, S') by measurement of cross-sections alone: *need polarization observables*

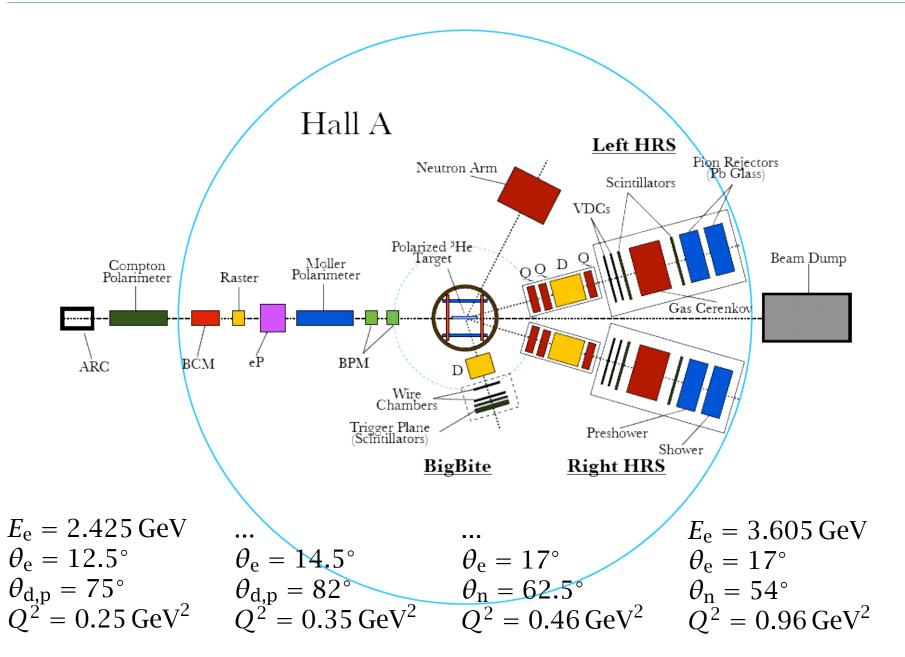
$$\frac{\mathrm{d}\sigma(h,\vec{S})}{\mathrm{d}\Omega_{\mathrm{e}}\,\mathrm{d}E_{\mathrm{e}}\,\mathrm{d}\Omega_{\mathrm{d}}\,\mathrm{d}p_{\mathrm{d}}} = \frac{\mathrm{d}\sigma_{0}}{\cdots} \left[1 + \vec{S} \cdot \vec{A}^{0} + h(A_{\mathrm{e}} + \vec{S} \cdot \vec{A}) \right]$$

$$A(\theta^*, \phi^*) = \vec{S}(\theta^*, \phi^*) \cdot \vec{A} = \frac{[d\sigma_{++} + d\sigma_{--}] - [d\sigma_{+-} + d\sigma_{-+}]}{[d\sigma_{++} + d\sigma_{--}] + [d\sigma_{+-} + d\sigma_{-+}]}$$

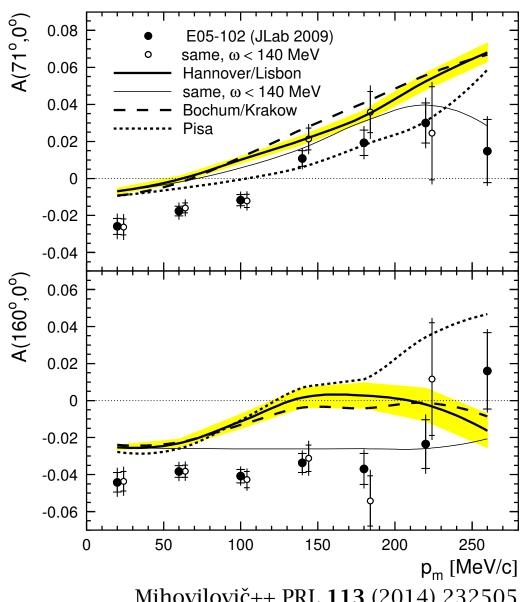
- Access to [effects of] small WF components (D, S')
- E05–102: simultaneous measurement of all break-up channels: 3 He(\vec{e} , e'd)p, 3 He(\vec{e} , e'p)d, 3 He(\vec{e} , e'p)pn ... and also 3 He(\vec{e} , e'n)pp



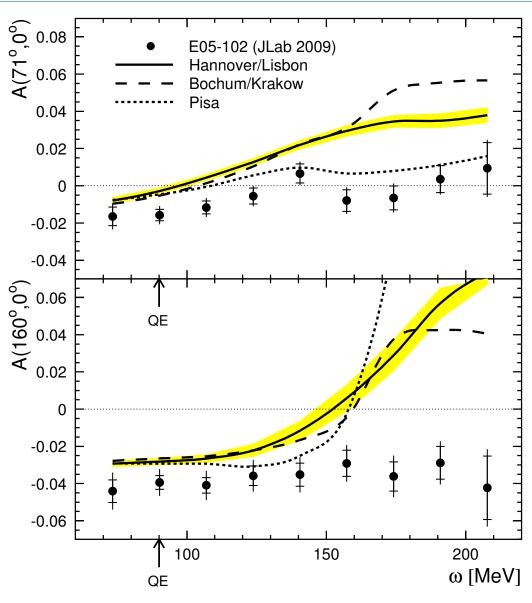
Experimental Setup



- Asymmetries are small (typically a few %), thus hard to reproduce theoretically (cancellations)
- Good agreement on the transverse asymmetry (71°)
- Worse for the longitudinal asymmetry (160°) ... but it improves when ω is restricted to QE peak
- Discrepancy due to
 - incomplete treatment of FSI (?)
 - unaccounted for 3NF (?)
 - underestimated S'component of g.s. WF (?)



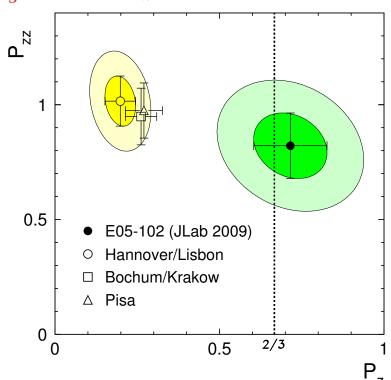
Mihovilovič++ PRL 113 (2014) 232505

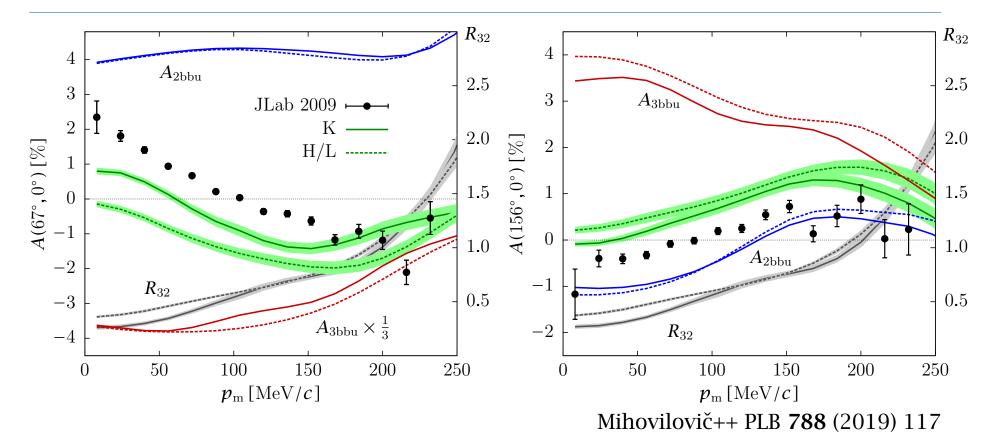


Mihovilovič++ PRL **113** (2014) 232505

Attempt to evaluate P_z and P_{zz}

- Assume 3 He(e, e'd)p at low p_{miss} is like elastic scattering off polarized d
- Use $A_x^{(^3\text{He})}$, $A_z^{(^3\text{He})}$ as if they were $A_x^{(\text{ed})}$, $A_z^{(\text{ed})}$ with appropriate deuteron FFs, and extract P_z and P_{zz}
- Toy model $|^{3}\text{He}\rangle = |\text{d}\rangle + |\text{p}\rangle$
- Spin decomposition $|{}^{3}\text{He}\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}|1,1\rangle \left|\frac{1}{2},-\frac{1}{2}\right\rangle \sqrt{\frac{1}{3}}|1,0\rangle \left|\frac{1}{2},\frac{1}{2}\right\rangle$ gives $P_z = \langle I_z\rangle_{^{3}\text{He}} = \frac{2}{3}$, $P_{zz} = \langle 3I_z^2 2\rangle_{^{3}\text{He}} = 0$

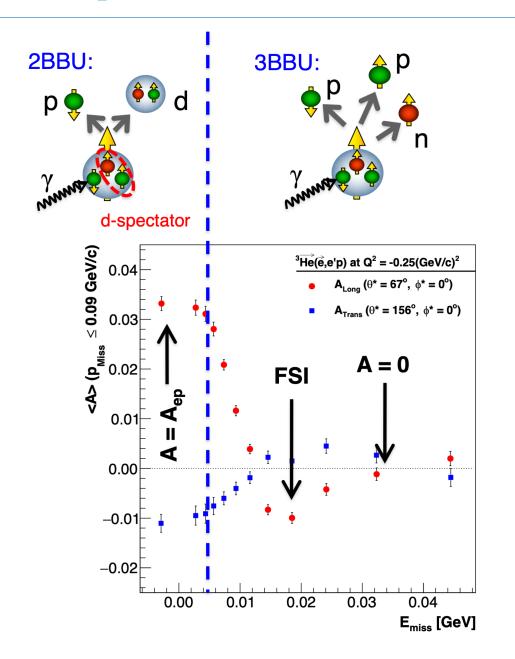




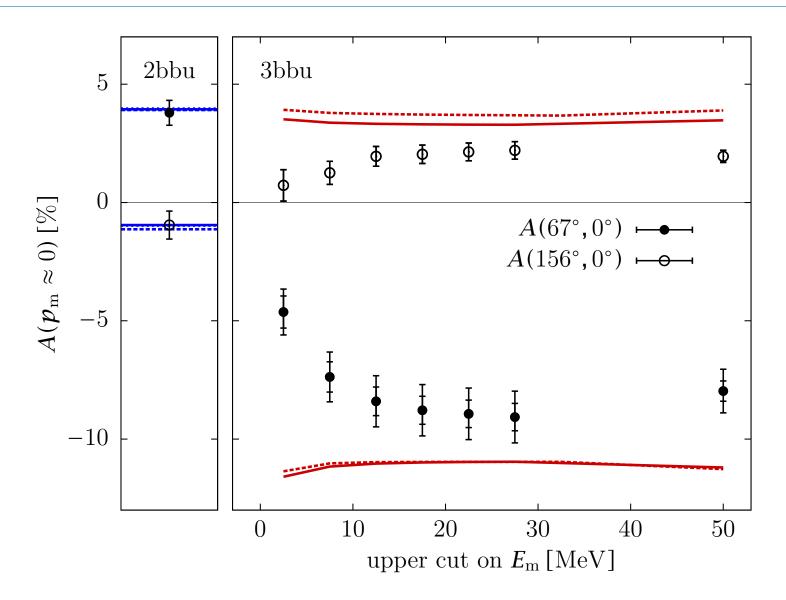
- No 2bbu/3bbu separation possible; rely on MC to disentangle A_{2bbu}/A_{3bbu} \triangleright Unpolarized 2bbu and 3bbu XS as well as A_{2bbu} well established
- Only qualitative agreement of data with theory. Issues:
 - ⊳ Cancellation of 2bbu and 3bbu contributions
 - ⇒ 3bbu asymmetry dominant possibly too much so
 - ▷ Pertinent ingredients: Coulomb, RC, FSI, 3NF (?)

Simple interpretation of 3 He $(\vec{e}, e'p)$

- Valid for $p_{\rm m} \approx 0$
- Assume PWIA
- *S*-state dominates
- Missing energy: $E_{\rm m} = \omega T_{\rm p} T_{\rm d}$
- Low- $E_{\rm m}$ region dominated by 2bbu: $A \approx A(\vec{e} - \vec{p} \text{ elastic})$
- High- $E_{\rm m}$ region dominated by 3bbu: $A \approx 0$
- Non-zero asymmetry in 3bbu probably caused by FSI

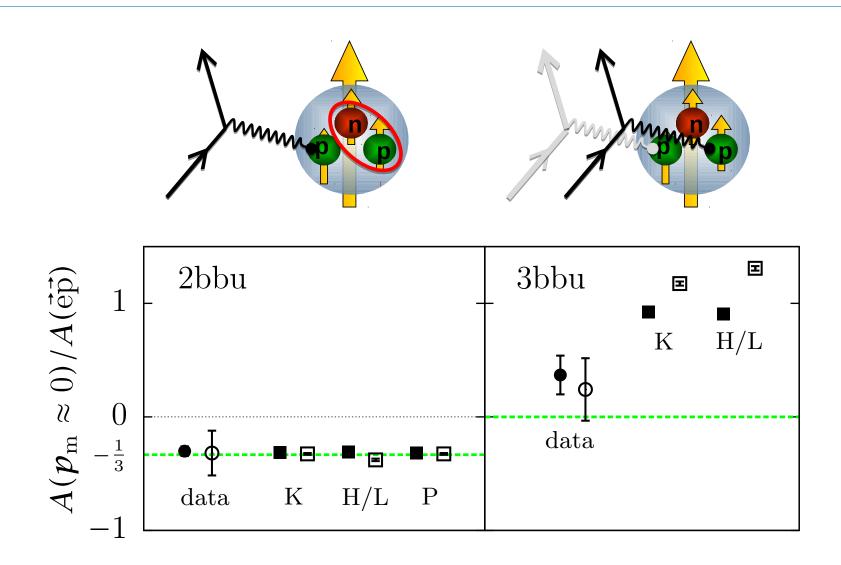


Extraction of 2bbu and 3bbu asymmetries in 3 He $(\vec{e}, e'p)$



Mihovilovič++ PLB **788** (2019) 117

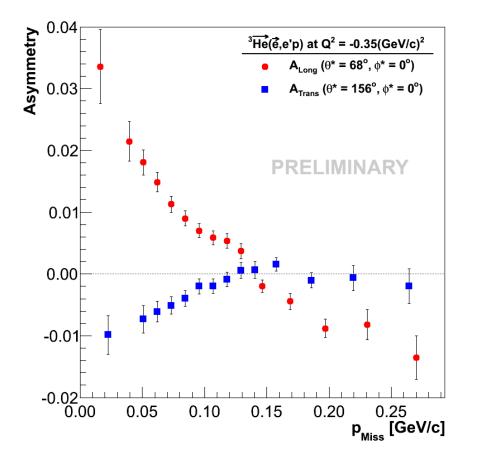
Message on 2bbu and 3bbu asymmetries in 3 He $(\vec{e}, e'p)$

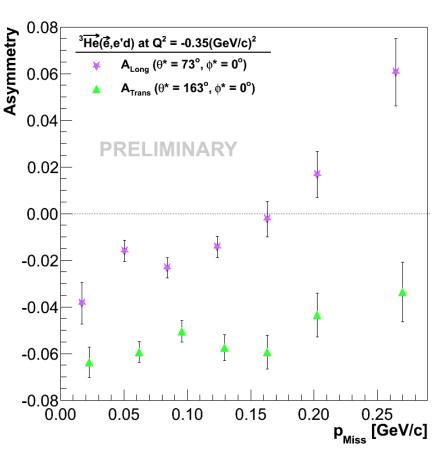


Mihovilovič++ PLB **788** (2019) 117

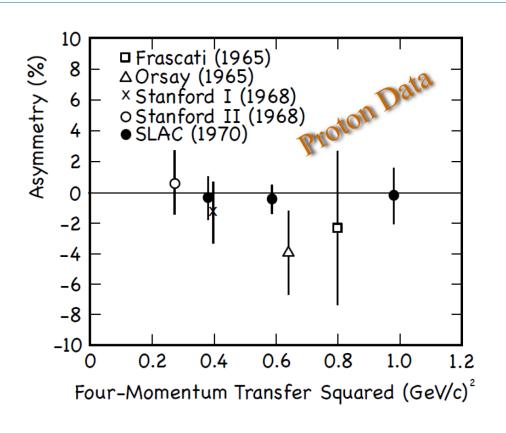
More $^{3}\vec{He}(\vec{e}, e'd)$ and $^{3}\vec{He}(\vec{e}, e'p)$...

• High-statistics data also available at $Q^2 \approx 0.35 \, \text{GeV}^2$ in all channels



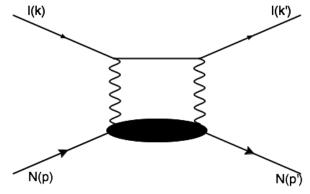


- Opportunity to study Q^2 -dependence of asymmetries
- Theoretical calculations pending

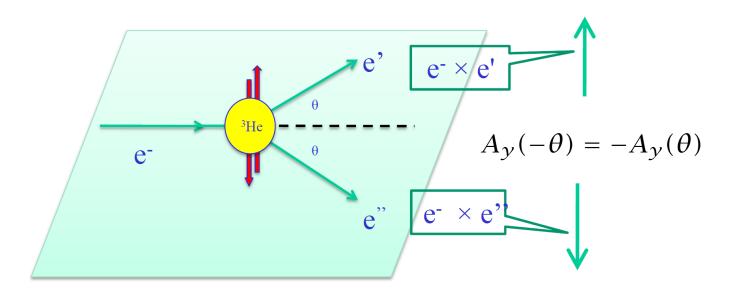


$$A_{\mathcal{Y}} = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

$$\propto \vec{s} \cdot (\vec{k} \times \vec{k}')$$

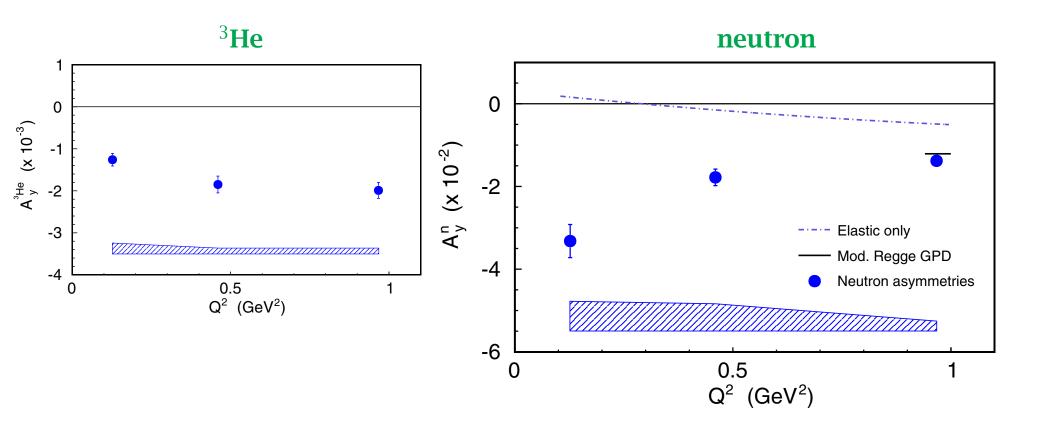


- $A_y = 0$ in Born approximation (*T*-invariance)
- $A_{\mathcal{Y}} \neq 0$ indicative of $2\mathcal{Y}$ effects, $\propto \text{Im}\{T_{1\mathcal{Y}}T_{2\mathcal{Y}}^*\}$ interference; relevant for G_E^p/G_M^p , GPDs
- no measurement of comparable precision on **neutron**



<i>E</i> ₀ [GeV]	E' [GeV]	θ _{lab} [Deg]	Q ² [GeV] ²	<i>q</i> [GeV]	$ heta_q$ [Deg]
1.25	1.22	17	0.13	0.359	71
2.43	2.18	17	0.46	0.681	62
3.61	3.09	17	0.98	0.988	54

Figure & table courtesy of Yawei Zhang, Rutgers

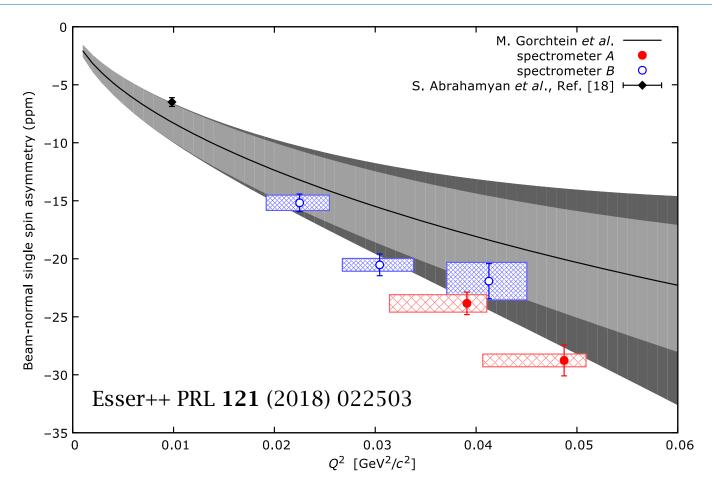


- ullet First measurement of $A^{
 m n}_{\mathcal{Y}}$ (extracted from transversely polarized $A^{
 m 3He}_{\mathcal{Y}}$)
- Uncertainty several times better than previous proton data
- Asymmetry clearly non-zero and negative

Zhang++ PRL **115** (2015) 172502

Single-spin asymmetries in $^{12}C(e^{\uparrow},e')$

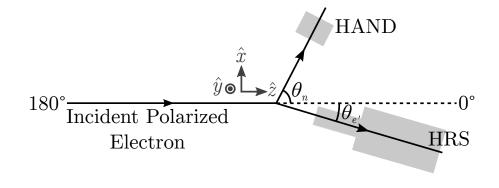
MAMI/A1

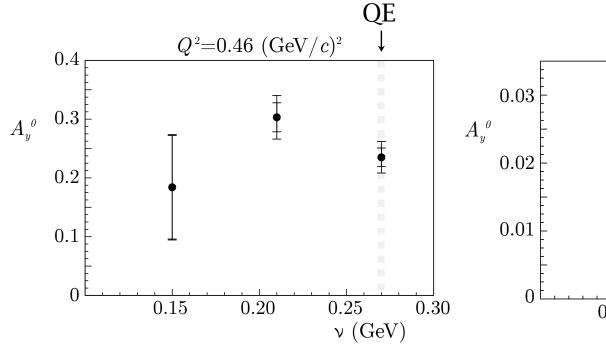


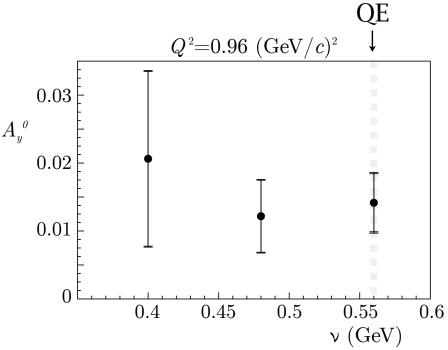
- Several calculations for A_{ν} in $p(e^{\uparrow}, e)$, very few on nuclei
- Generalization of forward inclusive model to nuclear targets:

$$A_{\mathcal{Y}} \sim C_0 \log \left(\frac{Q^2}{m_{\rm e}^2 c^2}\right) \frac{F_{\rm Compton}(Q^2)}{F_{\rm charge}(Q^2)}$$

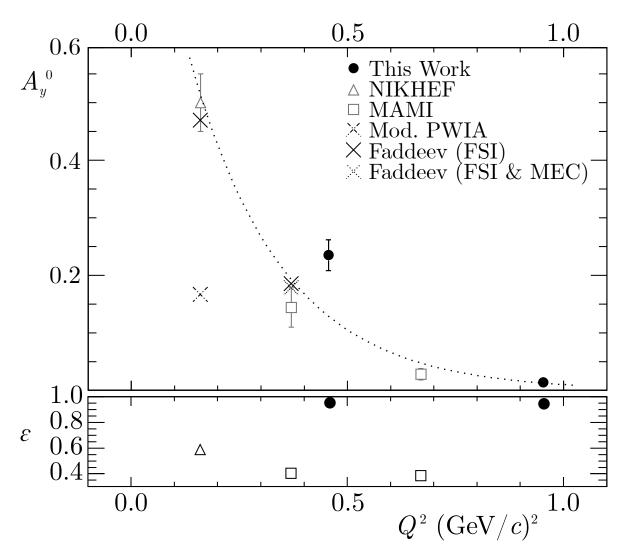
- Ideal probe of **FSI** and **MEC**
- Should be zero in PWIA and should die out at high Q^2
- Difficult calculations at high Q^2





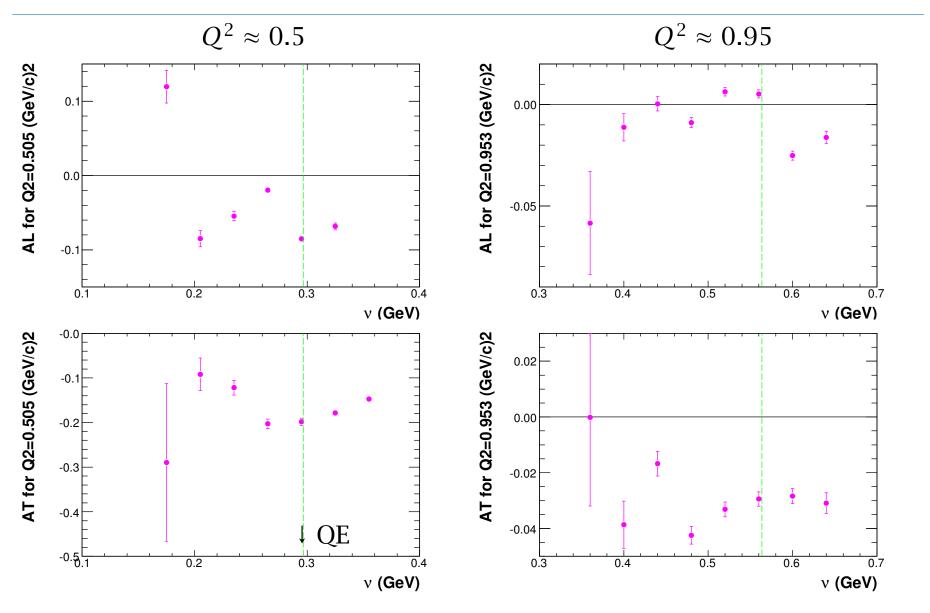


Long++ PLB (in press, 2019)



Long++ PLB (in press, 2019)

 \Rightarrow PWIA good enough for high- Q^2 experiments at JLab 12 GeV!



• Calculations?

*** PRELIMINARY *** Figures courtesy of Elena Long, UNH

- PWIA: σ_L , σ_T , $\sigma_{T'}$ yield spin-dependent momentum distribution
- FSI, MEC preclude direct access except at $p_d \lesssim 2 \, \mathrm{fm}^{-1}$
- Rich interplay \triangleright **final-state symmetrization**: large effect in C_3
 - \triangleright **FSI**: largest in C_2
 - ightharpoonup MEC: most prominent in C_1

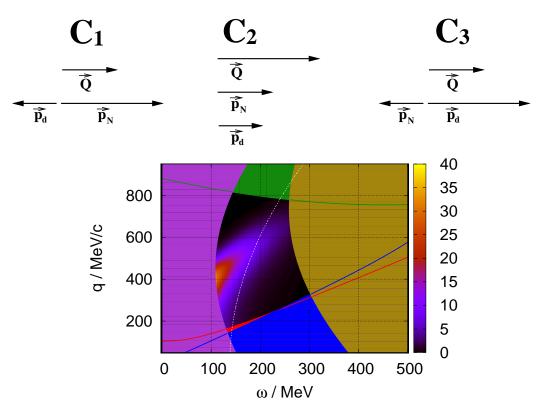
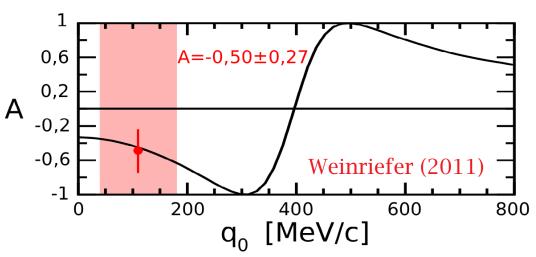


Fig. courtesy of M. Distler, JGU Mainz

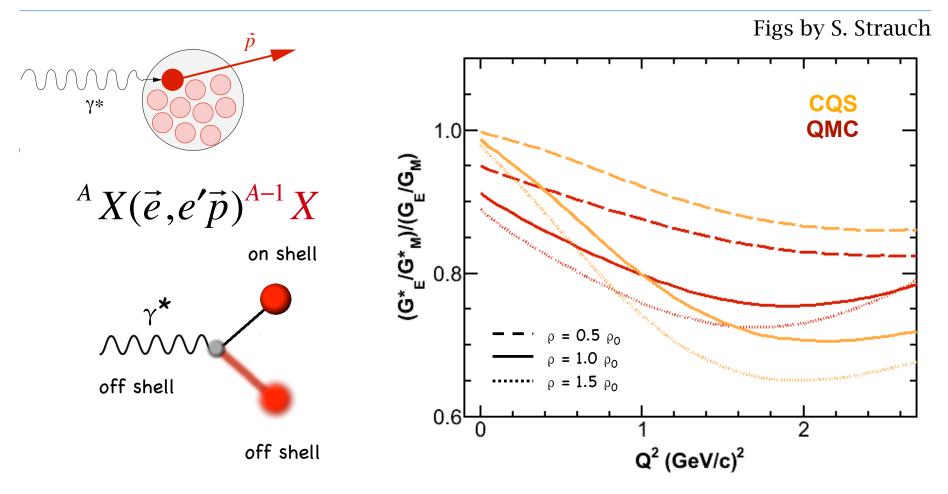
• Spin-dependent momentum distributions of pd clusters in polarized ³He Golak++ PRC **65** (2002) 064004

$$\begin{split} N_{\mu} &= \left\langle \Psi_{\mathrm{pd}}^{(-)} M_{\mathrm{d}} m \, | \, \hat{j}_{\mu}(\vec{q}) \, | \Psi M \right\rangle \\ \mathcal{Y} \left(M = \frac{1}{2}, M_{\mathrm{d}} = 0, m = +\frac{1}{2} \right) \propto \left| N_{-1}^{\mathrm{spin PWIA}} \left(\frac{1}{2}, 0, -\frac{1}{2} \right) \right|^{2} \overset{\mathbb{Z}}{\underset{10^{-5}}{\stackrel{10^{-5}}{\sim}}} \overset{10^{-1}}{\underset{10^{-7}}{\sim}} \\ \mathcal{Y} \left(M = \frac{1}{2}, M_{\mathrm{d}} = 1, m = -\frac{1}{2} \right) \propto \left| N_{+1}^{\mathrm{spin PWIA}} \left(\frac{1}{2}, 1, +\frac{1}{2} \right) \right|^{2} \overset{\mathbb{Z}}{\underset{10^{-7}}{\stackrel{10^{-7}}{\sim}}} \overset{10^{-1}}{\underset{10^{-7}}{\sim}} \overset{10^{-1}}{\underset{10^{-7}}{\sim}} \\ A &= \frac{\mathcal{Y}(1/2, 0, 1/2) - \mathcal{Y}(1/2, 1, -1/2)}{\mathcal{Y}(1/2, 0, 1/2) + \mathcal{Y}(1/2, 1, -1/2)} \end{split}$$

$$\sigma_{\rm L} \propto |N_0|^2$$
 $\sigma_{\rm T} \propto |N_{+1}|^2 + |N_{-1}|^2$
 $\sigma_{\rm T'} \propto |N_{+1}|^2 - |N_{-1}|^2$



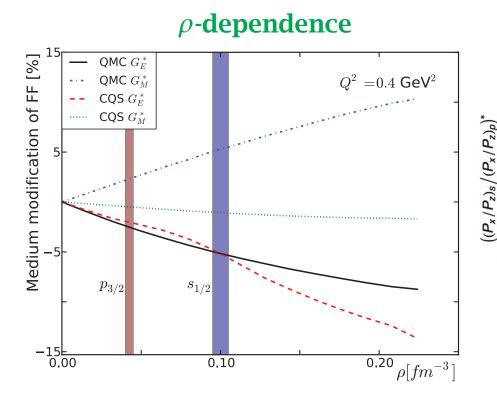
Form-factor modification in medium



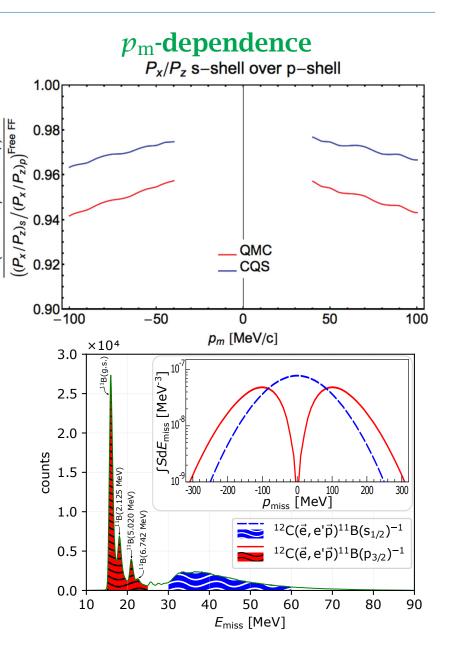
- Observable Q^2 and ρ -dependent effects predicted by various models
- Exploit polarization-transfer technique in \approx QE proton knock-out:

$$\frac{G_E^p}{G_M^p} = -\frac{P_\chi'}{P_z'} \frac{E_e + E_e'}{2M_p} \tan \frac{\theta_e}{2} \qquad \Longrightarrow \qquad \left(\frac{P_\chi'}{P_z'}\right)_A / \left(\frac{P_\chi'}{P_z'}\right)_p$$

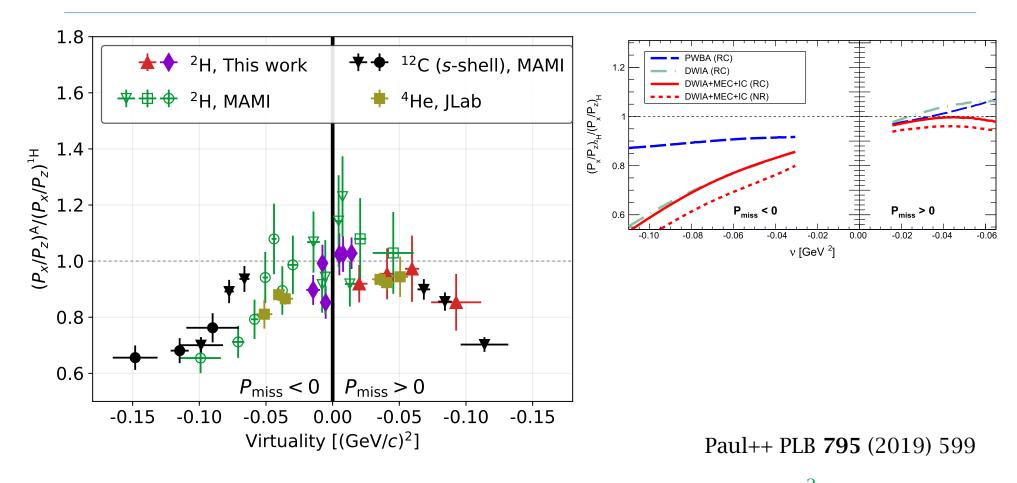
Form-factor modification: calculations for ¹²C



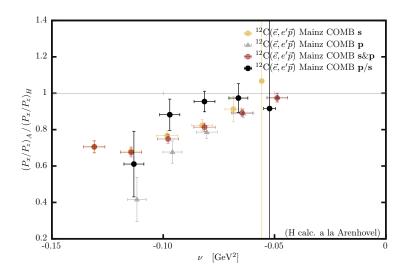
- Different shells ⇔ different local densities // Ron++ PRC **87** (2013) 028202
- Disentangle via $E_{\rm m}$ cuts
- Need to explore $\pm p_{\rm m}$ and $\pm \nu$ regions (no a priori symmetry)

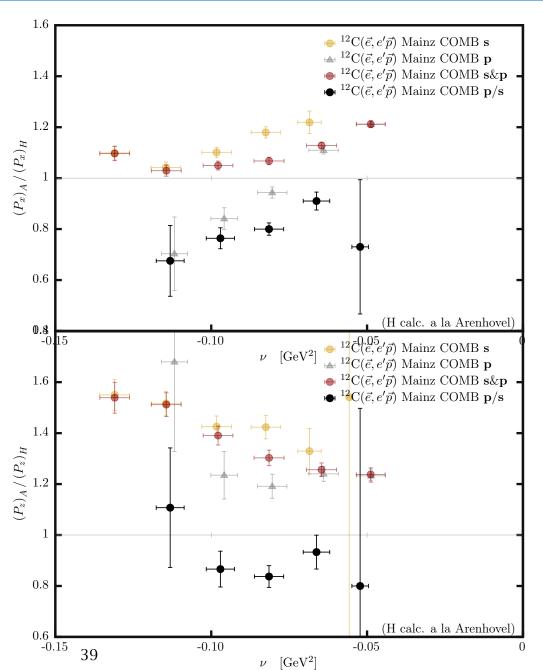


Form-factor modification in medium: "universality"

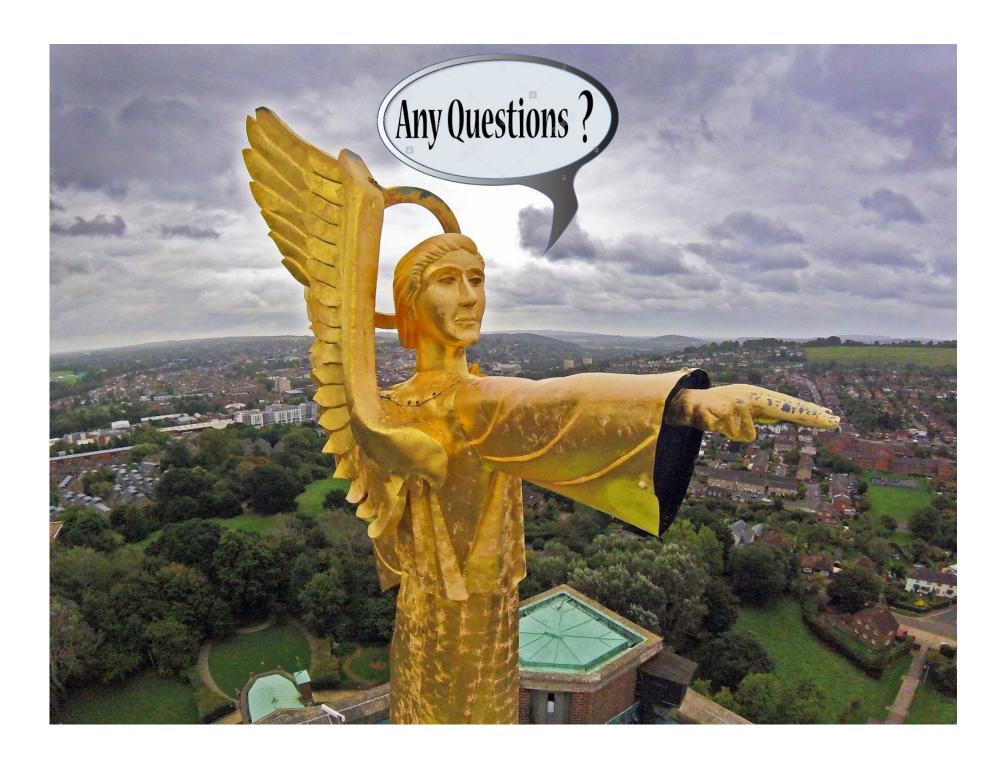


- Virtuality: $v = p^2 m_p^2$ or, better, $v = \left(m_A \sqrt{m_{A-1}^2 + p_{\rm m}^2}\right)^2 p_{\rm m}^2 m_p^2$
- Relevant variable: ν . No A-dependence ("universality")
- Largest effects due to FSI and WF of proton in nucleus, not due to FF modification hard to disentangle \Rightarrow new JLab proposal at higher Q^2



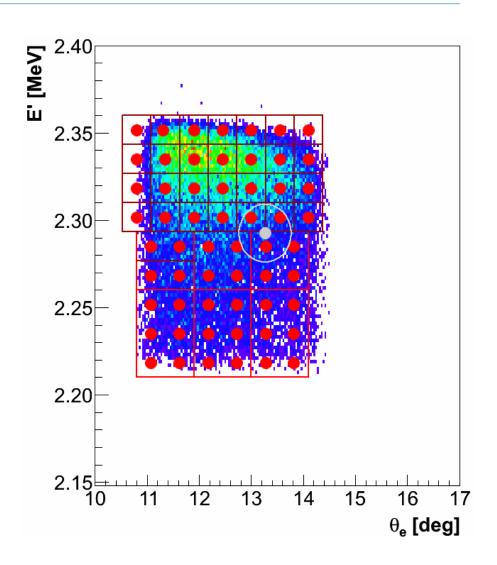


Figs courtesy of T. Kolar



Acceptance-averaging of ${}^{3}\vec{He}(\vec{e}, e'p)$ and ${}^{3}\vec{He}(\vec{e}, e'd)p$

- Calculations available only on a discrete kinematic mesh acceptance averaging needed
- Decision:
 Manipulate calculations not data
- Asymmetry for each (E', θ_e) at each (p_m, θ_{xq}) determined by calculating the weighted mean of the nearest points
- Weak dependence on cell size
- Data at $Q^2 = 0.25$ and $0.35 \,\text{GeV}^2$, only first set published: PRL **113** (2014) 232505

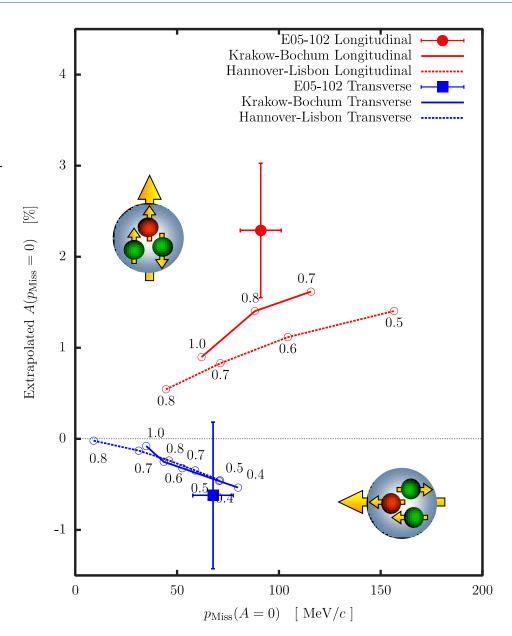


"Fine-tuning" the calculations for $^{3}\vec{He}(\vec{e}, e'p)$

• Rescale 3bbu calculations to roughly match magnitude and zero-crossing of *A*

$$\bullet \ A = \frac{\sigma_2 A_2 + \sigma_3 A_3}{\sigma_2 + \sigma_3} = \frac{A_2 + A_3 R_{32}}{1 + R_{32}}$$

• $\approx 30-40\%$ reduction needed



Extraction of $A_{\gamma}^{\rm n}$ from $A_{\gamma}^{\rm 3He}$ — effective polarization approximation:

$$A_{\mathcal{Y}}^{3\text{He}} = P_{n}f_{n}A_{\mathcal{Y}}^{n} + P_{p}(1 - f_{n})A_{\mathcal{Y}}^{p}$$

$$f_{\rm n} = \frac{\sigma^{\rm n}}{\sigma^{\rm 3He}} = \frac{\sigma^{\rm n}}{2\sigma^{\rm p} + \sigma^{\rm n}}$$

$$P_{\rm n} = 0.86 \pm \cdots$$
 $P_{\rm p} = -0.028 \pm \cdots$

high Q^2 : f_n computed with Kelly's parameterization of nucleon FFs low Q^2 : theoretical estimate (due to FSI): $f_n = 0.042$ (A. Deltuva) $A^p_{\mathcal{Y}}$ computed by Afanasev et al.