AB INITIO CALCULATIONS OF LIGHT HYPERNUCLEI

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OUTLINE

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  - Ab initio no-core shell model
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  - Charge symmetry breaking puzzle in light mirror hypernuclei
  - Nuclear structure uncertainties
  - Hypertriton decay
  - Few-body resonances
- **SUMMARY**

MOTIVATION

Strangeness nuclear physics

MOTIVATION

Strangeness nuclear physics

Interdisciplinary subject connecting particle physics, nuclear physics and astrophysics.

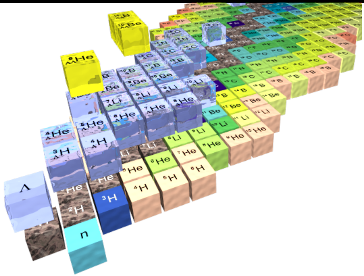
Related topical questions include:

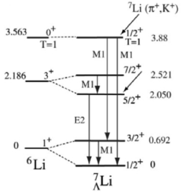
- interaction of (anti)kaons with the nuclear medium
  - possible existence of deeply-bound K^-–nuclear states?
  - antikaons in dense matter?
- interaction of hyperons with the nuclear medium
  - S=-1 Λ hypernuclei, Σ-hypernuclei?
  - S=-2 ΛΛ-hypernuclei, Ξ hypernuclei
  - hyperons in dense nuclear matter and neutron stars?

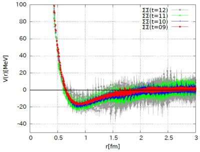
STRANGENESS NUCLEAR PHYSICS

Study of hypernuclei

- Improve understanding of NY interaction
  - strict constraints on NY interaction
  - precise experimental data on hypernuclear spectroscopy
  - supplement (very sparse) hyperon–nucleon scattering data base
- New precision experiments at J-PARC, J-Lab, FAIR, …
- New constraints from heavy ion collisions: production of light hypernuclei, baryon–baryon interactions (femtoscopy)
- Lattice QCD can be a game changer for strangeness nuclear physics
- Modern developments of NY interactions based on SU(3) chiral EFT / $\not{\pi}$EFT
- **Advanced many-body computational methods are required**







FIRST-PRINCIPLES MODELING OF HYPERNUCLEI:

Ab initio no-core shell model

AB INITIO NO-CORE SHELL MODEL

Given a Hamiltonian operator solve the A-body eigenvalue problem:

$$\left[\sum_{i\leq A}\frac{\hat{\mathbf{p}}_i^2}{2m_i} + \sum_{i<j\leq A-1}\hat{V}_{NN;ij} + \sum_{i<j<k\leq A-1}\hat{V}_{NNN;ijk} + \sum_{i<j=A}\hat{V}_{NY;ij}\right]\Psi = E\Psi$$

Ab initio

- all particles are active (no rigid core)
- exact Pauli principle
- realistic internucleon interactions
- controllable approximations

- Hamiltonian is diagonalized in a finite A-particle harmonic oscillator basis

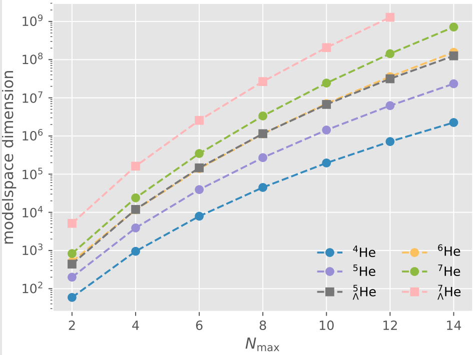
$$\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_A) = \sum_{n\leq N_{max}} \Phi_n^{HO}(\mathbf{r}_1,\ldots,\mathbf{r}_A)$$

(matrix dimensions up to $\sim 10^{10}$ with $\sim 10^{14}$ nonzero elements)

- Systematically improvable: converges to exact results for $N_{max} \to \infty$

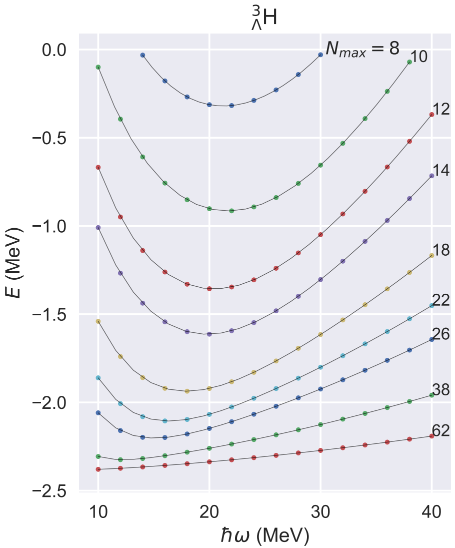
The curse of dimensionality

- Basis dimensions for s- and p-shell (hyper)nuclei:



- Strategies:
  - effective interactions: Lee–Suzuki, similarity RG
  - basis reduction: importance truncation (limit to relevant states), Symmetry-Adapted NCSM (exploit dynamical symmetries)
  - robust extrapolation technique for “$N_{max} \to \infty$”

AB INITIO NO-CORE SHELL MODEL



- Bare interactions used ($NNLO_{sim}$)
- Model space parameters: N_{max}, $\hbar\omega$

Convergence in finite HO spaces

- What is the equivalent of Lüscher formula?
- (N_{max}, $\hbar\omega$) imposes cutoffs in momentum space (UV) and in position space (IR)
- In a regime with negligible UV corrections, IR corrections are universal for short-range interactions

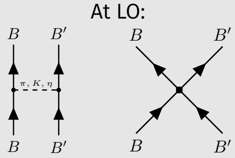
$$E(L_{eff}) = E_\infty + e^{-k_\infty L_{eff}} + \dots$$

- L_{eff} identified as the size of the hyperspherical cavity associated with (N_{max}, $\hbar\omega$) [Wendt et al., PRC 91, 061391 (2015)]

INPUT V_{NN}, V_{NNN} and V_{NY} POTENTIALS

Potentials derived from chiral EFT

- long-range part (π, K, η-exchange) predicted by χPT
- short-range part parametrized by contact interactions, LECs fitted to experimental data



NN+NNN interaction

- chiral N^3LO NN potential [Entem, Machleidt, PRC 68, 041001 (2003)]
  chiral N^2LO NNN potential [Navrátil, FBS 41, 14 (2007)]
- $NNLO_{sim}$ NN + NNN potential family [Carlsson et al., PRX 6, 011019 (2016)]

NY interaction

- chiral LO potential [Polinder et al., NPA 779, 244 (2006)], NLO developed
- $\Lambda N - \Sigma N$ mixing explicitly taken into account:

$$V_{NY} = \begin{pmatrix} V_{\Lambda N-\Lambda N} & V_{\Lambda N-\Sigma N} \\ V_{\Sigma N-\Lambda N} & V_{\Sigma N-\Sigma N} \end{pmatrix} + \Delta m$$

Coupled-channel Λ-hypernucleus – Σ-hypernucleus problem!

APPLICATION:

Structure of s- and p-shell hypernuclei

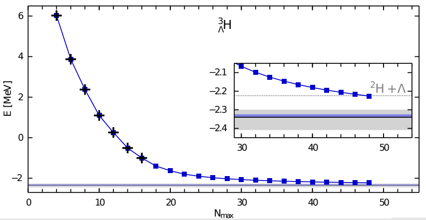
[Gazda, Mareš, Navrátil, Roth, Wirth, FBS 55, 857 (2014)]
[Wirth, Gazda, Navrátil, Calci, Langhammer, Roth, PRL 113, 192502 (2014)]
[Wirth, Gazda, Navrátil, Roth, PRC 97, 064315 (2018)]

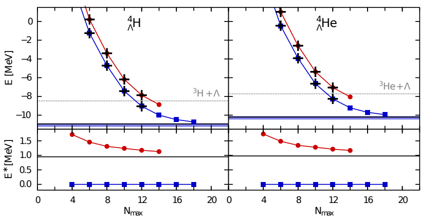
STRUCTURE OF s- AND p-SHELL HYPERNUCLEI

Aims

- Develop an ab initio computational technique for A≥5 hypernuclei
- Test the performance of existing NY interaction models

Validation for A = 3, 4 hypernuclei





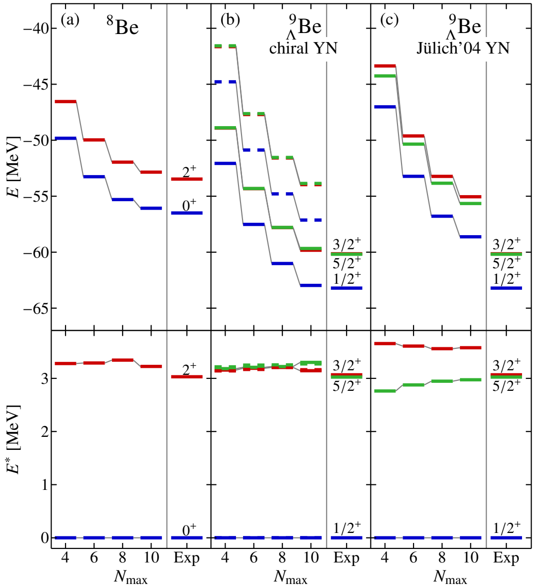
- two formulations of NCSM developed: in relative Jacobi-coordinate HO basis (squares) and Slater-determinant s.p. HO basis (crosses)
- calculations agree with exact Faddeev results [Nogga et al., NPA 914, 140 (2013)]

First applications

- systematic study from A = 3 $^3_\Lambda$H to A = 13 $^{13}_\Lambda$C

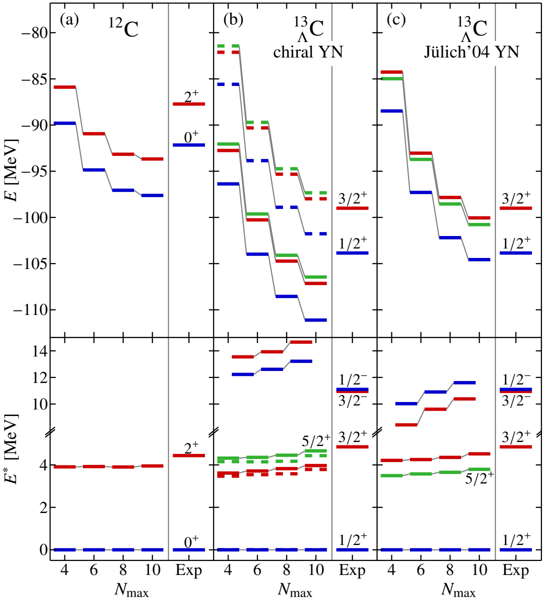
  [Gazda et al., FBS 55, 857 (2014); Wirth, Gazda et al., PRL 113, 192502 (2014); Wirth, Gazda, et al., PRC 97, 064315 (2018)]

STRUCTURE OF s- AND p-SHELL HYPERNUCLEI: $^{9}_{\Lambda}$Be



- calculations with SRG-evolved NN+NNN and bare NY potentials
- surprisingly good performance of chiral LO NYpotentials for low-lying states
- reveals deficiencies of the phenomenological potential

STRUCTURE OF s- AND p-SHELL HYPERNUCLEI: $^{13}_{\Lambda}C$



- calculations with SRG-evolved NN+NNN and bare NY potentials
- surprisingly good performance of chiral LO NYpotentials for low-lying states
- reveals deficiencies of the phenomenological potential, as well as deficiencies of the chiral LO potential at higher partial waves

APPLICATION:

Charge symmetry breaking puzzle in light mirror hypernuclei

[Gazda, Gal, PRL 116, 122501 (2016)]

[Gazda, Gal, NPA 954, 161 (2016)]

CHARGE SYMMETRY BREAKING PUZZLE IN LIGHT MIRROR HYPERNUCLEI

Charge symmetry in hadron physics

- invariance of the strong interaction under the interchange of up and down quarks (protons and neutrons)
- broken in QCD by the **up** and **down** light quark mass differences and their QED interactions, expected to break down at $(m_u - m_d)/M \sim 10^{-3}$

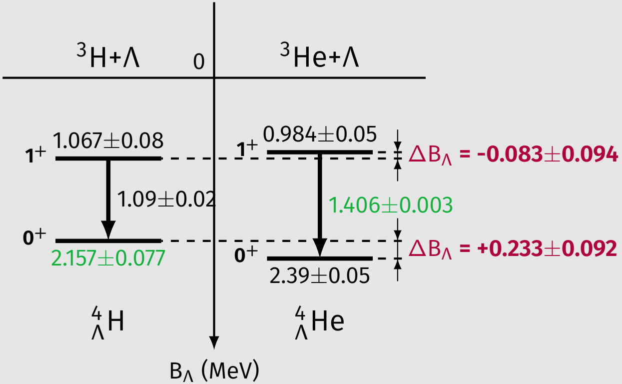
Charge symmetry breaking in nuclear physics

- manifest in pp and nn scattering lengths, well understood
- $^3\mathrm{He} - {}^3\mathrm{H}$: $\Delta E_{SI}^{CSB} \approx 70$ keV

Charge symmetry breaking in hypernuclear physics

- poor pΛ and no nΛ scattering data
- highly suppressed in ${}^3_\Lambda\mathrm{H}$
- ${}^4_\Lambda\mathrm{He} - {}^4_\Lambda\mathrm{H}$ energy level splittings, $\Delta B_\Lambda^{CSB} \approx 200$ keV

CSB in A = 4 hypernuclei



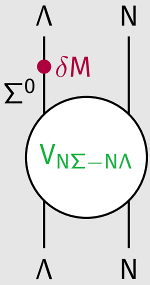
- Recently reaffirmed by J-PARC E13 observation of $^4_\Lambda$He$(1^+_{\mathrm{exc.}} \rightarrow 0^+_{\mathrm{g.s.}})$ γ-ray transition [Yamamoto et al., PRL 115, 222501 (2015)] and MAMI-A1 determination of $B_\Lambda(^4_\Lambda\mathrm{H})$ [Esser et al., PRL 114, 232501 (2015)]
- Until recently, no calculation was able to reproduce large ΔB_Λ
- CSB due to $\Lambda - \Sigma^0$ mixing and related to $\Lambda N - \Sigma N$ coupling

  [Gazda, Gal, PRL 116, 122501 (2016); Gazda, Gal, NPA 954, 161 (2016)]

Electromagnetic $\Lambda-\Sigma^0$ mixing

- Physical Λ and Σ^0 hyperons have mixed isospin composition in terms of the SU(3) pure-isospin Λ (I=0) and Σ (I=1) hyperons
- mixing angle proportional to EM mass matrix element $\langle\Sigma^0|\delta m|\Lambda\rangle$

Relating $\Lambda-\Sigma^0$ CSB mixing to $\Lambda N-\Sigma^0 N$ coupling



- For NY interaction models with explicit $N\Sigma - N\Lambda$ coupling, the electromagnetic $\Lambda-\Sigma^0$ mixing relates matrix elements of $V^{CSB}_{N\Lambda}$ with $V_{N\Sigma-N\Lambda}$:

$$\langle N\Lambda|V^{CSB}_{N\Lambda}|N\Lambda\rangle = -2\frac{\langle\Sigma^0|\delta M|\Lambda\rangle}{M_{\Sigma^0}-M_\Lambda}\tau_{N3}\frac{1}{\sqrt{3}}\langle N\Sigma|V_{N\Sigma-N\Lambda}|N\Lambda\rangle$$

[Gal, PLB 744, 352 (2015)]

- The first microscopic model which generates large $\Delta B_\Lambda(0^+_{g.s.}) \approx 200$ keV in A = 4 hypernuclei!

[Gazda, Gal, PRL 116, 122501 (2016)]

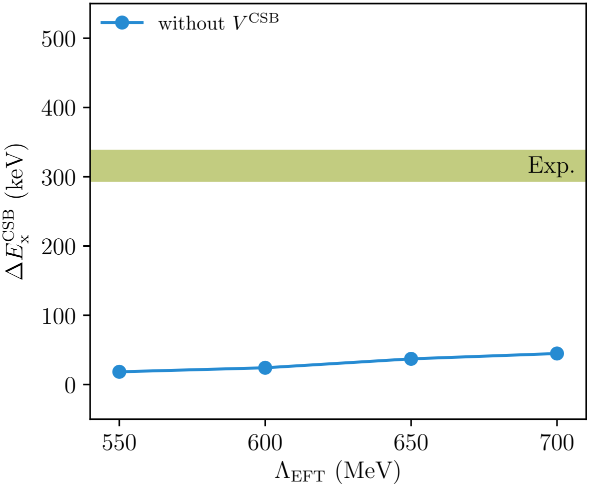


Figure 1: Cutoff momentum (Λ_{EFT}) dependence of the difference $\Delta E_{x}^{\mathrm{CSB}}$ of the excitation energies $E_x(0^+_{\mathrm{g.s.}} \rightarrow 1^+)$ in $^{4}_{\Lambda}$He and $^{4}_{\Lambda}$H in ab initio NCSM calculations **without** $V_{\Lambda N}^{\mathrm{CSB}}$ generated by $\Lambda N - \Sigma N$ conversion from LO chiral NY interactions.

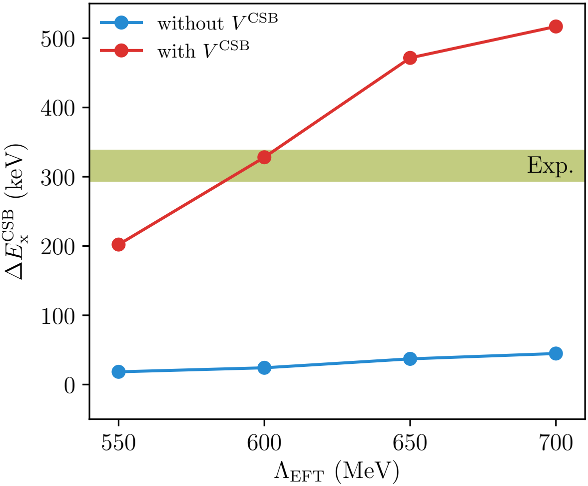


Figure 1: Cutoff momentum (Λ_{EFT}) dependence of the difference $\Delta E_x^{\mathrm{CSB}}$ of the excitation energies $E_x(0^+_{\mathrm{g.s.}} \to 1^+)$ in $^4_\Lambda$He and $^4_\Lambda$H in ab initio NCSM calculations **without** and **with** $V^{\mathrm{CSB}}_{\Lambda N}$ generated by $\Lambda N - \Sigma N$ conversion from LO chiral NY interactions.

APPLICATION:

Nuclear Structure uncertainties

[unpublished]

NUCLEAR STRUCTURE UNCERTAINTIES IN HYPERNUCLEI

Aim

What are the theoretical uncertainties of hypernuclear properties resulting from the remaining freedom in the constructions of nuclear NN+NNN interactions?

The NNLO_{sim} family of NN+NNN potentials

- Parameters fitted to reproduce simultaneously πN, NN, and NNN low-energy observables
- family of 42 Hamiltonians where the experimental uncertainties propagate into LECs

$$\left.\begin{array}{ll} T_{NN}^{\text{lab,max}} & \leq 125, \ldots, 290 \text{ MeV} \\ \Lambda_{\text{EFT}} & \leq 450, \ldots, 600 \text{ MeV} \end{array}\right\} 42\ V_{NN} + V_{NNN} \text{ potentials}$$

- All Hamiltonians give equally good description of the fit data
- Note that $\Delta E^{(^3\text{He}/^3\text{H})} \approx 0$ (fitted) while $\Delta E_{\text{g.s.}}^{(^4\text{He})} \approx 1.5$ MeV

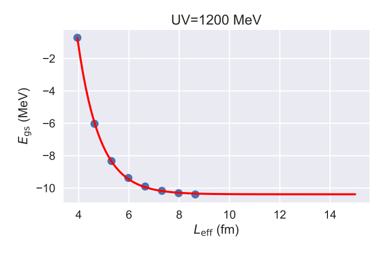
[Carlsson et al., PRX 6, 011019 (2016)]

${}^{3}_{\Lambda}$H $J^{\pi} = 1/2^{+}$

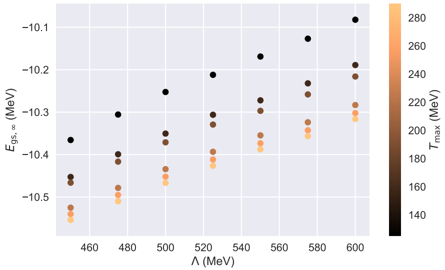
- converged NCSM calculations (N_{max}=70)
- $\Delta E_{g.s} \approx 0.1$ MeV $\approx \Delta B_{\Lambda}$ ($B_{\Lambda} \approx 0.13$ MeV)

${}^{4}_{\Lambda}$He(H) $J^{\pi} = 0^{+}, 1^{+}$

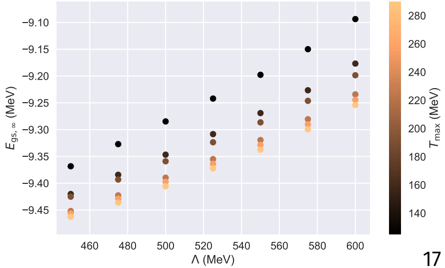
- $N_{max} = 20(16)$ for $J^{\pi} = 0^{+}(1^{+})$
- e.g. for $\Lambda = 500$ MeV, $T_{max} = 290$ MeV:



${}^{4}_{\Lambda}$He $J^{\pi} = 0^{+}$ g.s.: $\Delta E_{g.s} \approx 0.5$ MeV

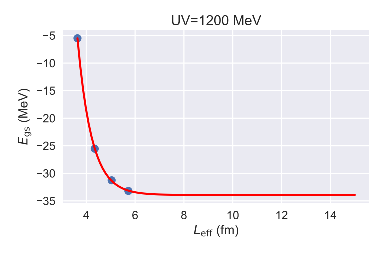


${}^{4}_{\Lambda}$He $J^{\pi} = 1^{+}$ exc.: $\Delta E_{exc.} \approx 0.6$ MeV



${}^{5}_{\Lambda}$He $J^\pi = 1/2^+$

- $N_{max} = 10$ for $J^\pi = 1/2^+$
- e.g. for $\Lambda = 500$ MeV, $T_{max} = 290$ MeV:

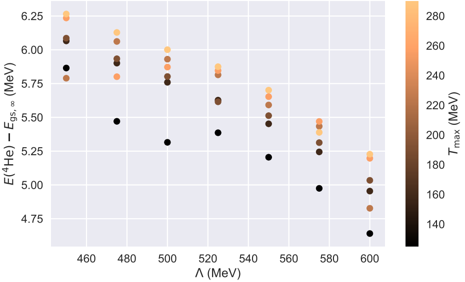


${}^{5}_{\Lambda}$He overbinding problem

- exp. $B_\Lambda({}^{5}_{\Lambda}\text{He}) = 3.12(2)$ MeV, $B_\Lambda = E({}^4\text{He}) - E({}^{5}_{\Lambda}\text{He})$
- Hard to reproduce by any NY interaction model
- Evidence of missing ΛNN forces?

${}^{5}_{\Lambda}$He $J^\pi = 1/2^+$ g.s.:

$\Delta B_\Lambda({}^{5}_{\Lambda}\text{He}) \approx 1.75$ MeV



LO NY χEFT cutoff dependence

For $\Lambda_{NY} = 550, \ldots, 700$ MeV:

$\Delta B_\Lambda({}^{5}_{\Lambda}\text{He}) \approx 3.7$ MeV

Altogether

$B_\Lambda \approx 2.27 - 7.62$ MeV!

APPLICATION:

Mesonic decay of the hypertriton

[unpublished] (preliminary)

MESONIC DECAY OF THE HYPERTRITON

${}^{3}_{\Lambda}$H lifetime puzzle

- The weakly-bound ${}^{3}_{\Lambda}$H ($B_\Lambda \approx 0.13$ MeV) is expected to have lifetime within few % of the free Λ hyperon lifetime
- Faddeev calculation: $\tau = 0.94\tau_\Lambda$ [Kamada et al., PRC 57, 1595 (1998)]
- Recent heavy-ion ${}^{3}_{\Lambda}$H production experiments yield lifetimes shorter by $\gtrsim 30\%$ (wo. avg.): $\tau = 142^{+24}_{-21} \pm 29$ ps ($0.54^{+0.09}_{-0.08}\tau_\Lambda$)
  [ALICE collab., PLB 754, 360 (2016); STAR collab. PRC 97, 054909 (2018)]

${}^{3}_{\Lambda}$H decay

- mesonic modes (not Pauli blocked as in heavier hypernuclei):
  ${}^{3}_{\Lambda}\mathrm{H} \to \pi^-(\pi^0) + {}^3\mathrm{He}({}^3\mathrm{H})$ / $\pi^-(\pi^0) + d + p(n)$ / $\pi^-(\pi^0) + p + n + p(n)$
- rare non-mesonic modes: ${}^{3}_{\Lambda}\mathrm{H} \to n + d$ / $n + n + p$

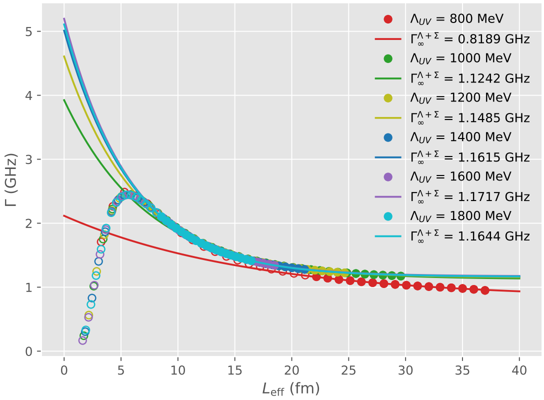
Our aim

- Pionic FSI in ${}^{3}_{\Lambda}\mathrm{H} \to \pi^- + {}^3\mathrm{He}$
  (closure calculation yields $\tau = 1.23\tau_\Lambda$ [Gal, Garcilazo, PLB 791, 48 (2018)])

Decay rate

$$\Gamma({}^3_\Lambda\mathrm{H} \to {}^3\mathrm{He} + \pi^-) \propto \int \langle \Psi_{^3\mathrm{He}} \phi_\pi | \hat{\mathrm{O}} | \Psi_{^3_\Lambda\mathrm{H}} \rangle$$

- ϕ_π plane wave (distorted waves in progress)
- $\Psi_{^3\mathrm{He}}$, $\Psi_{^3_\Lambda\mathrm{H}}$ from NCSM



- Γ sensitive to IR properties of $\Psi_{^3_\Lambda\mathrm{H}}$
- good convergence for $\Lambda_{\mathrm{UV}} > 800$ MeV
- $\Sigma^{0,-}$ hyperons decrease Γ by $\sim 20\%$

APPLICATION:

Few-body resonances

[unpublished] (preliminary)

Few-body resonances

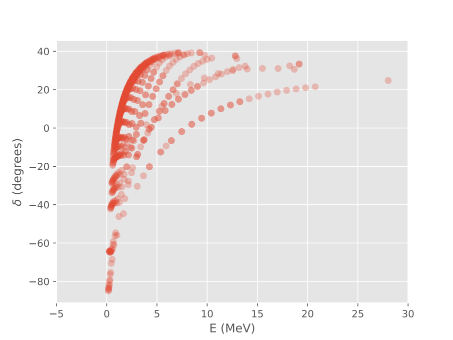
Harmonic-oscillator representation of the scattering equation

- varying N_{max}, $\hbar\omega \to$ NCSM eigenenergies E_i, eigenfunctions Ψ_i
- phase shifts $\delta_{\mathcal{L}}(E_i) = -\arctan S_N^{\mathcal{L}}(E_i)/C_N^{\mathcal{L}}(E_i)$
- Λnn, ${}^{3}_{\Lambda}H$ $J^{\pi} = 3/2^{+}$, ...

[Shirokov et al., PRC 94, 064320 (2016)]

Λnn resonance state

- Signal of particle-stable Λnn [C. Rappold et al. (HypHI collab. PRC 88, 041001 (2013))]
- Three-body $\Lambda + n + n \to \Lambda + n + n$ scattering in hyperspherical HO basis
- No bound subsystems, "democratic" decay
- Preliminary!



SUMMARY

SUMMARY

Ab initio calculations of light hypernuclei

- No-core shell model is a powerful and reliable technique to study s- and p-shell hypernuclei
- Allow to test hyperon–nucleon interaction models
- High precision allows to address important questions of hypernuclear physics, such as:
  - charge-symmetry breaking in mirror hypernuclei
  - quantification of systematic theoretical uncertanties of hypernuclear observables
- New applications:
  - hypernuclear decays
  - exotic few-ody resonances

Thank you!