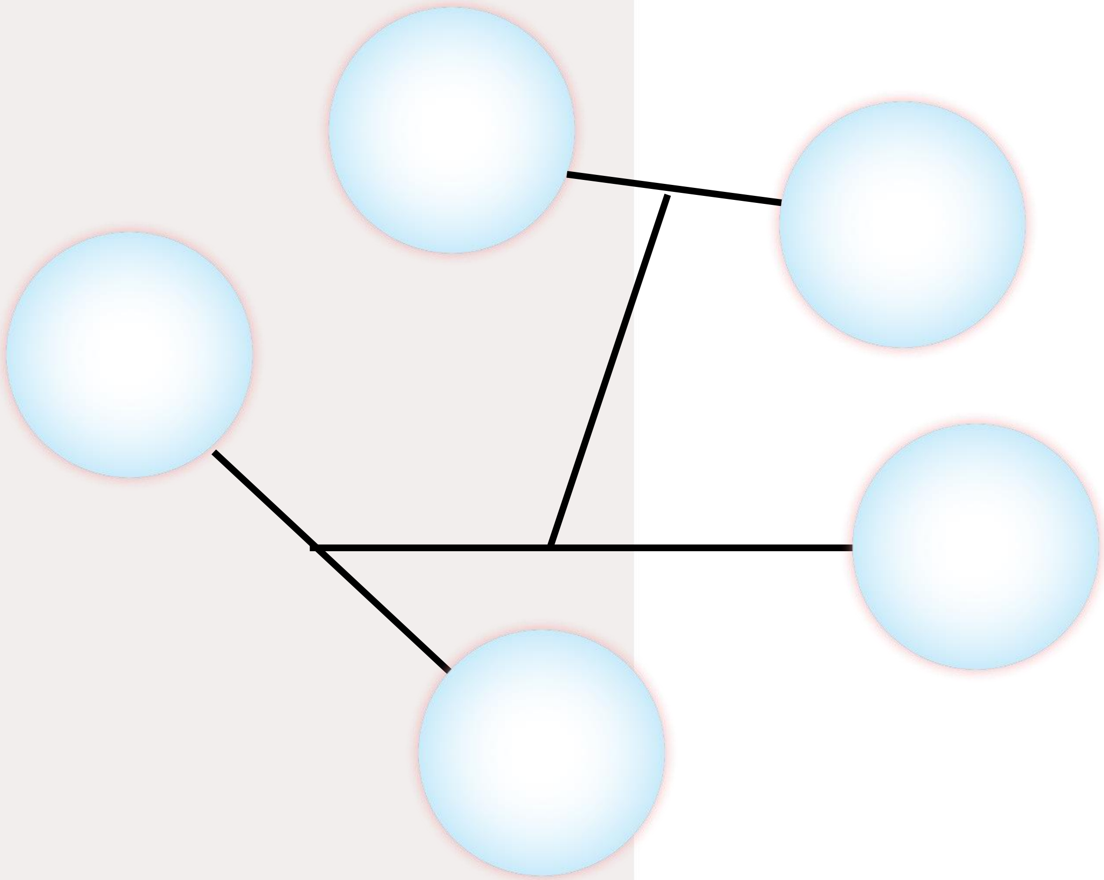
Solutions of the Fadddeev-Yakubovsky equations for five-nucleon systems



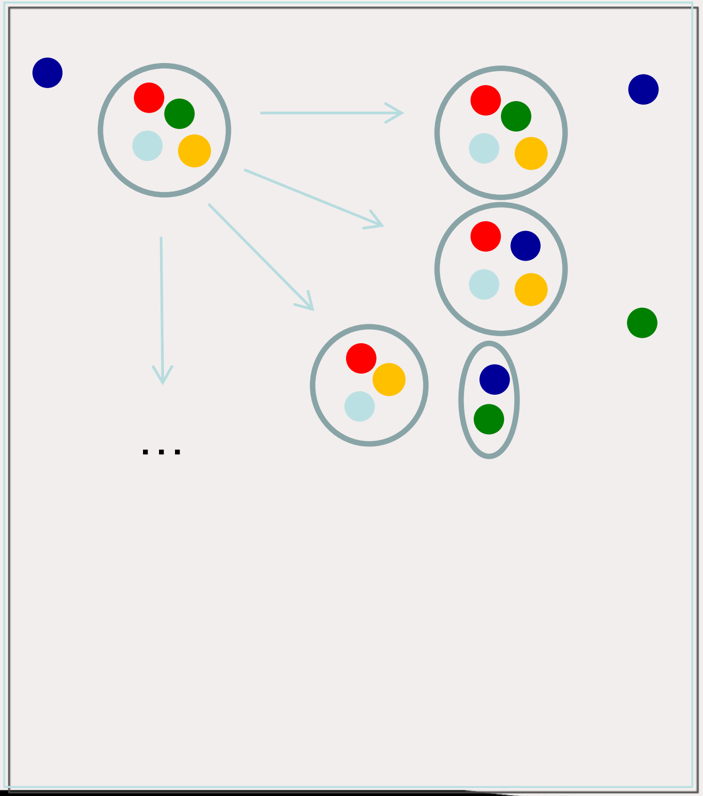
R. Lazauskas

Contents

- 5-body Faddeev-Yakubovsky equations
- Some applications
  - N-^{4}He elastic scattering
  - pv in low-energy n-^{4}He scattering
  - Resonances in ^{5}H

Non-relativistic Collisions

- In configuration space wave functions extend to infinity!
- Increasingly complex asymptotic behaviour for A>2 systems!!



How to take care of the boundary condition?

✓ Conceptual difficulties to uncouple different particle channel, to constrain assymptotes of the solutions in all directions and thus get unique (physical) solution to the Schrödinger eq.

- It is ok, as long as there is single particle channel (elastic plus target/projectile excitations)
- Mathematically ill-conditioned problem when several particle channels are open

✓ Faddeev-Yakubovsky equations efficiently separates asymptotes of the binary channels

L. D. Faddeev, Zh. Eksp. Teor. Fiz. 39, 1459 (1960). [Sov. Phys. JETP 12, 1014(1961)].
O. A. Yakubovsky, Sov. J. Nucl. Phys. 5, 937 (1967).

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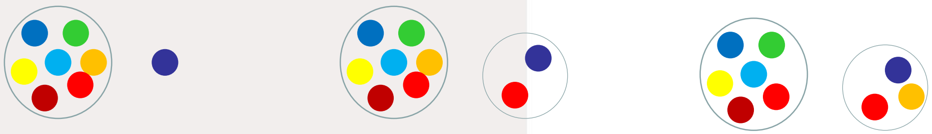
Properties of the rigorous scattering eq.

- Should separate all possible scattering channels to incorporate proper asymptotes! Number of binary channels increases $\sim 2^N$

$$\Psi_N = \sum_{perm} \Psi_{(N-1)(1)} + \sum_{perm} \Psi_{(N-2)(2)} + \sum_{perm} \Psi_{(N-3)(3)} + \cdots$$



- Should be systematically reducible to smaller subsystems, in order to built proper asymptotic solutions and to be consistent to its subsystems: chain of partitions (tree-like structures to break system in clusters & subclusters)

$$\Psi_{(N-i)(i)} = \left(\Psi_{N-i} \bigcup \Psi_i \right)$$

- FY equations are derived following this pattern, reconnecting different partition chains

Very fast growth of components with N!!

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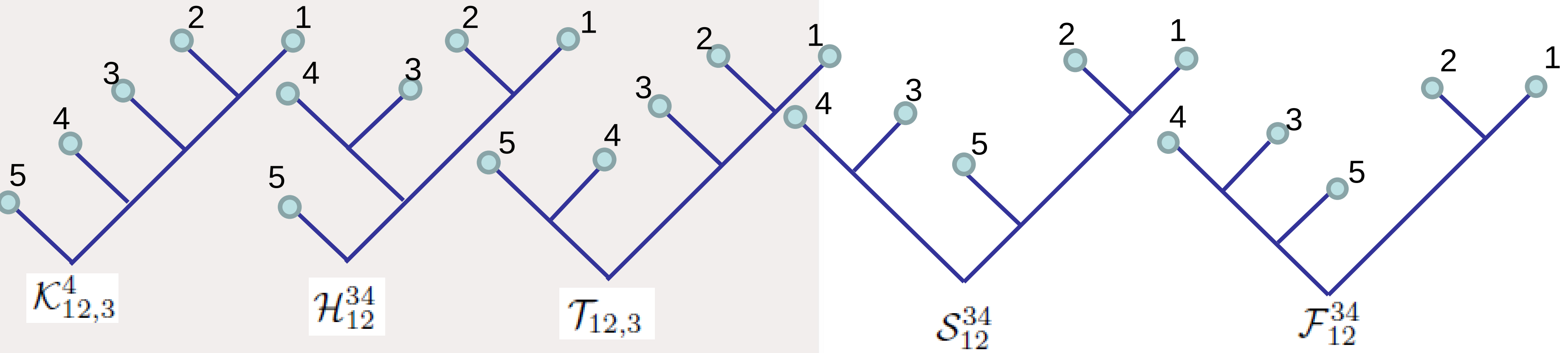
5-body Faddeev-Yakubovski eq



$$
\begin{aligned}
(E-H_0-V_{12})\,\mathcal{K}^4_{12,3} &= V_{12}\,(\mathcal{K}^4_{13,2}+\mathcal{K}^4_{23,1}+\mathcal{K}^5_{13,4}+\mathcal{K}^5_{23,4}+\mathcal{K}^2_{13,4}+\mathcal{K}^1_{23,4}\\
&\quad +\mathcal{T}_{13,4}+\mathcal{T}_{23,4}\\
&\quad +\mathcal{H}^{24}_{13}+\mathcal{H}^{14}_{23}+\mathcal{S}^{24}_{13}+\mathcal{S}^{14}_{23}+\mathcal{F}^{24}_{13}+\mathcal{F}^{14}_{23})\\
(E-H_0-V_{12})\,\mathcal{H}^{34}_{12} &= V_{12}\,(\mathcal{H}^{12}_{34}+\mathcal{K}^2_{34,1}+\mathcal{K}^1_{34,2}+\mathcal{K}^5_{34,1}+\mathcal{K}^5_{34,2}\\
&\quad +\mathcal{T}_{34,1}+\mathcal{T}_{34,2})\\
(E-H_0-V_{12})\,\mathcal{T}_{12,3} &= V_{12}\,(\mathcal{T}_{13,2}+\mathcal{T}_{23,1}\\
&\quad +\mathcal{H}^{45}_{13}+\mathcal{H}^{45}_{23}+\mathcal{S}^{45}_{13}+\mathcal{S}^{45}_{23}+\mathcal{F}^{45}_{13}+\mathcal{F}^{45}_{23})\\
(E-H_0-V_{12})\,\mathcal{S}^{34}_{12} &= V_{12}\,(\mathcal{F}^{12}_{34}+\mathcal{S}^{15}_{34}+\mathcal{S}^{25}_{34}\\
&\quad +\mathcal{F}^{15}_{34}+\mathcal{F}^{25}_{34}\\
&\quad +\mathcal{H}^{15}_{34}+\mathcal{H}^{25}_{34})\\
(E-H_0-V_{12})\,\mathcal{F}^{34}_{12} &= V_{12}\,(\mathcal{S}^{12}_{34}+\mathcal{K}^1_{34,5}+\mathcal{K}^2_{34,5}+\mathcal{T}_{34,5})
\end{aligned}
$$

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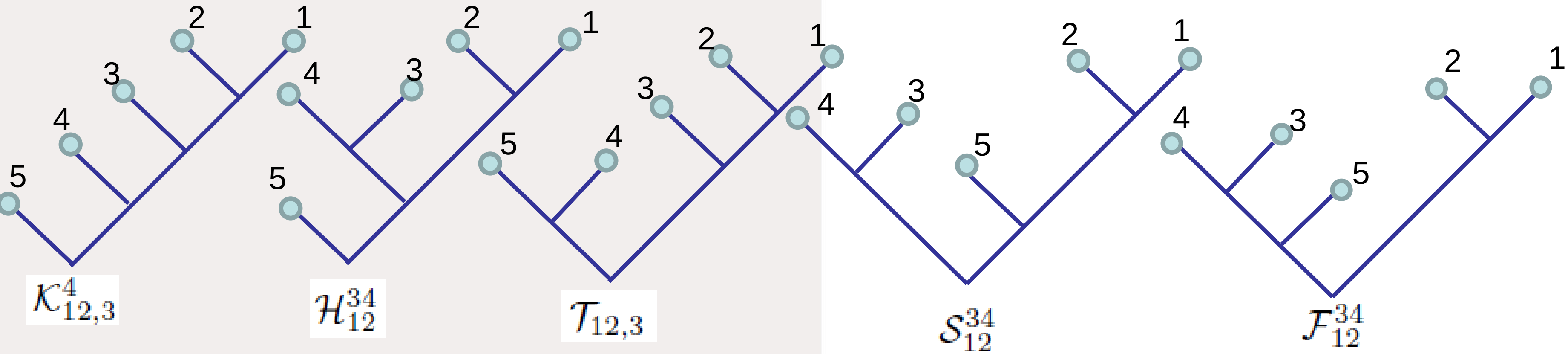


Merits:

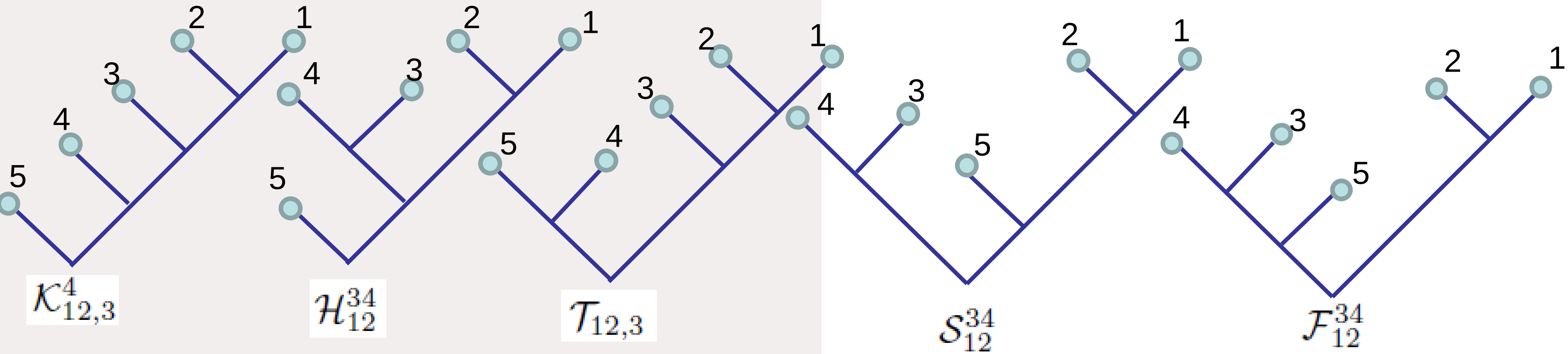
- ✓ Handling of symmetries
- ✓ Boundary conditions for binary channels
- ✓ Easy reduction to subsystems
- ✓ 3BF implemented at reasonable price
- ✓ Built for short-ranged interactions.

Treatment of Coulomb – true adventure, still reasonable for repulsive case.

Price

- ✓ Overcomplexity with N

Problem	Number eq. (identical particles)	Number eq. (different particles)
A=2	1	1
A=3	1	3
A=4	2	18
A=5	5	180
A=6	15	2700
A=N	$\mathrm{nint}(\frac{2(N-1)!}{(\pi/2)^N})$	$\frac{N!\,(N-1)!}{2^{N-1}}$



$$\mathcal{K}^{4}_{12,3}\left(\vec{x},\vec{y},\vec{z},\vec{w},S,L,T\right)=\sum_{\alpha_K=(l_{..},s_{..},t_{..})}\frac{f_{\alpha_K}(x,y,z,w)}{xyzw}\left[\left\{(l_x l_y)_{l_{xy}}\,(l_z l_w)_{l_{zw}}\right\}_L\{...\}_S\right]_{JM}\{...\}_T$$

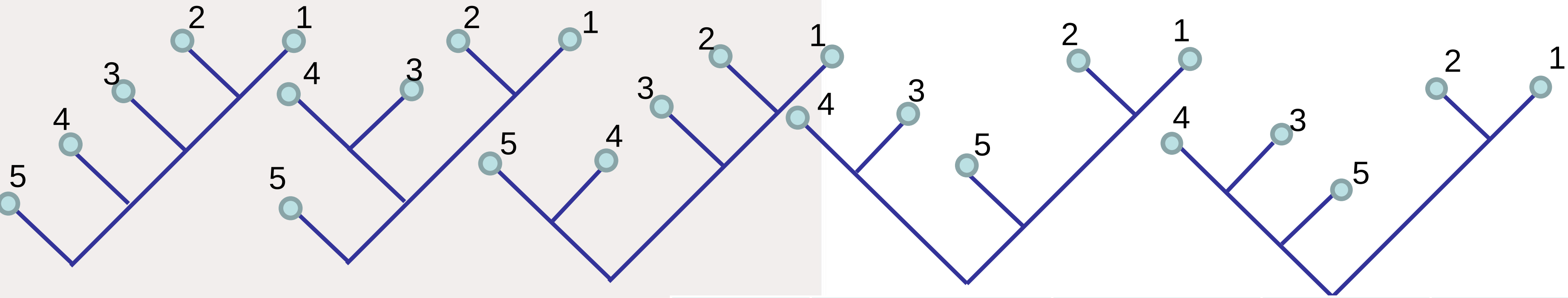
NUMERICAL SOLUTION

*R.L., PhD Thesis, Université Joseph Fourier, Grenoble (2003).

- PW decomposition of the components K,H,T,S,F
- Radial parts expanded using Lagrange-mesh method

D. Baye, Physics Reports 565 (2015) 1

- Resulting linear algebra problem solved using iterative methods
- Observables extracted using integral relations



Problem	Number eq. (ident particles)	Number eq. (diff. particles)	PW basis.	Radial disc.
2N	1	1	2	~N
3N	1	3	~100	$\sim N^2$
4N	2	18	$\sim 10^4$	$\sim N^3$
5N	5	180	$\sim 10^6$	$\sim N^4$

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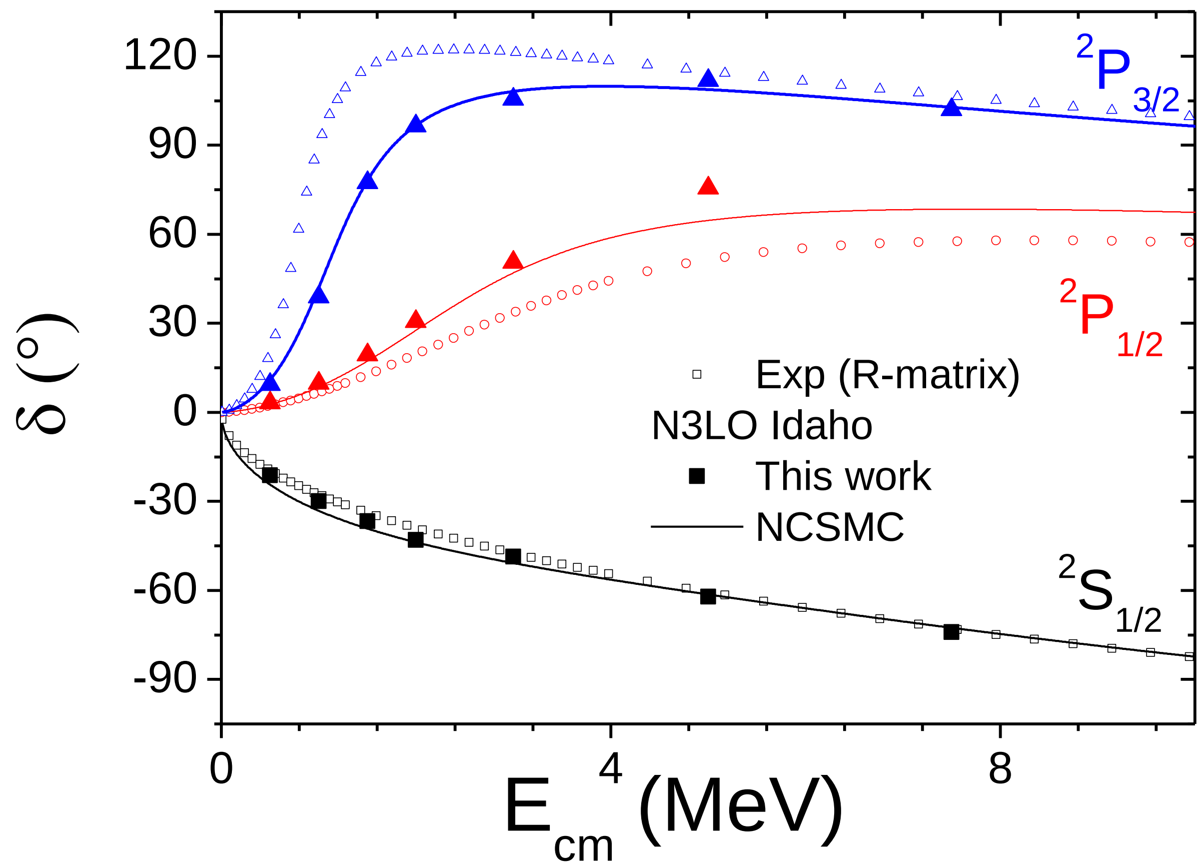
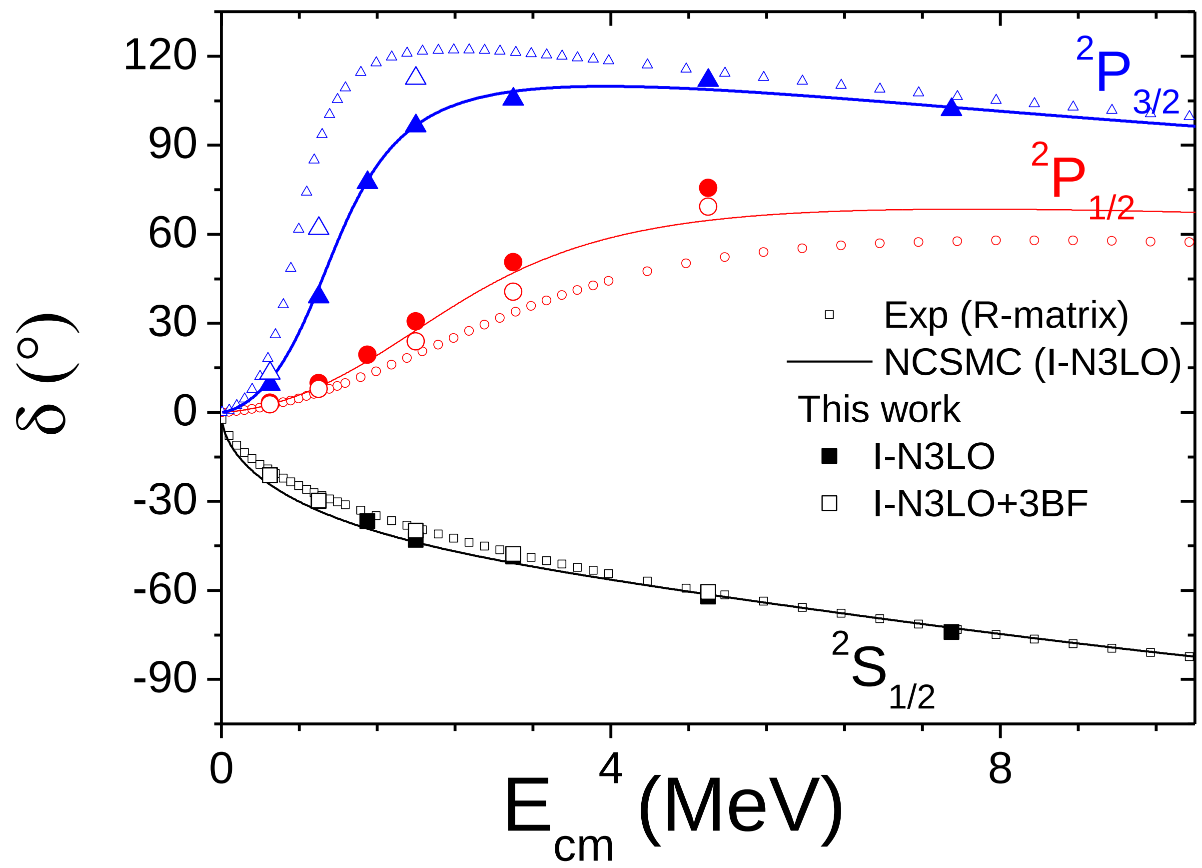


NCSMC: P. Navratil et al., Physica Scripta **91** (2016) 053002

n-^{4}He scattering



NCSMC: P. Navratil et al., Physica Scripta **91** (2016) 053002

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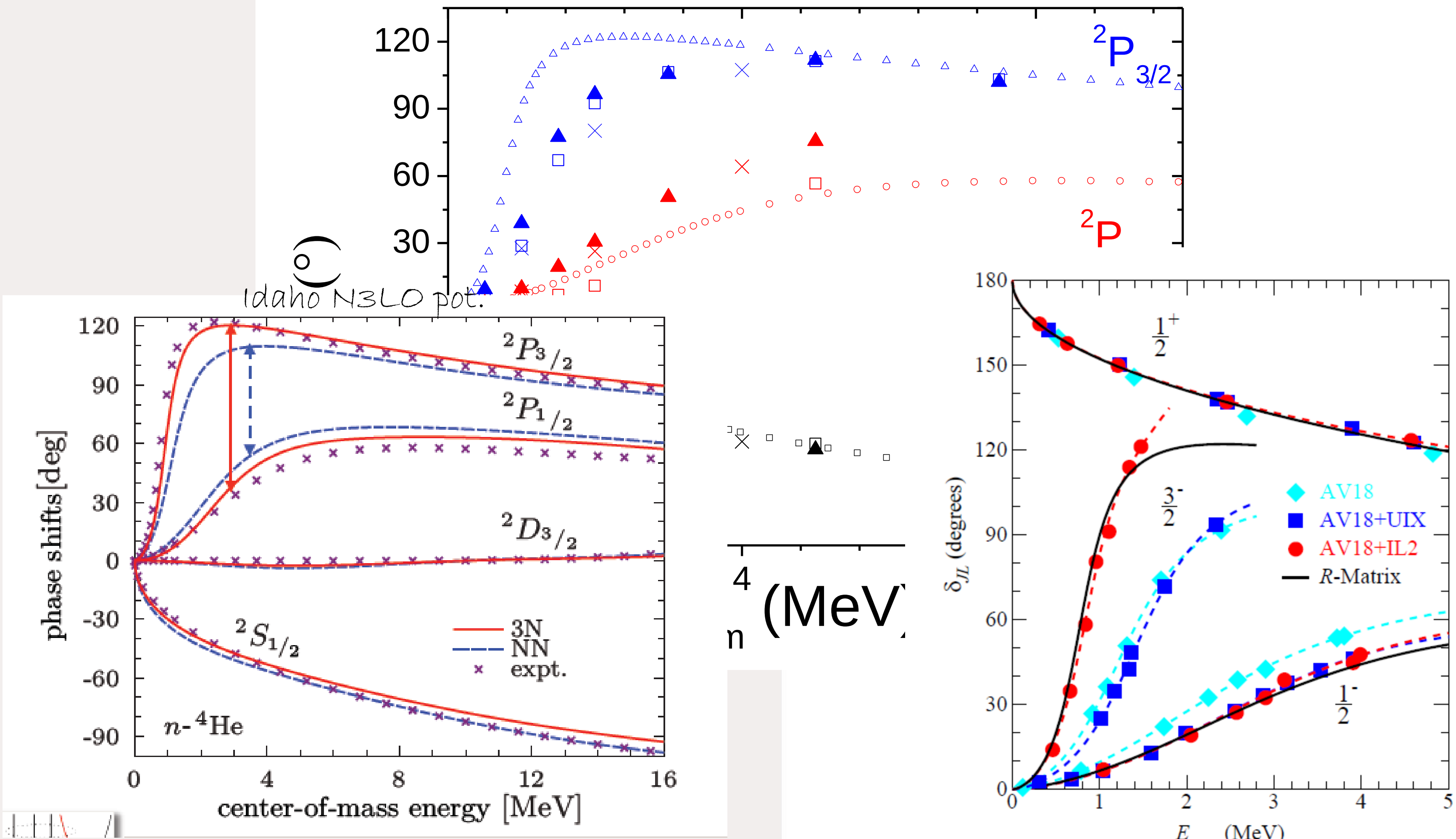
n-^{4}He scattering



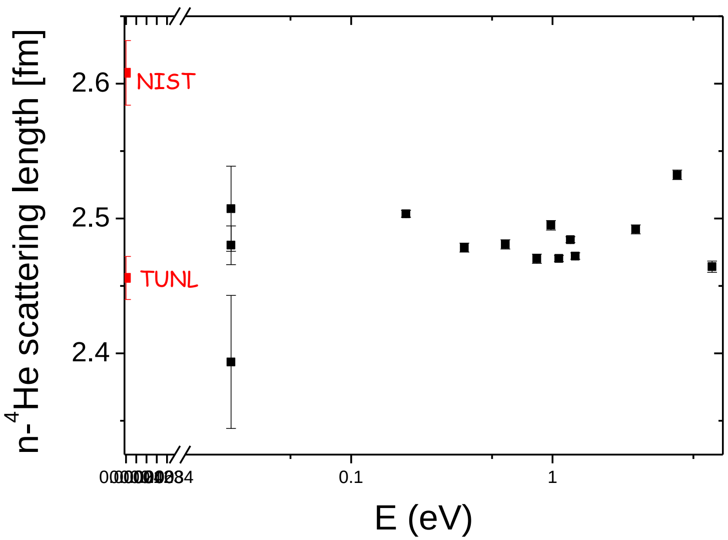
P. Navratil et al., Physica Scripta 91 (2016) 053002

K.M. Nollett et. al., Phys.Rev.Lett.99:022502,2007

Case of little interest: S-wave

Experimental n-^{4}He scattering length …

nothing should be as easy to measure…



NIST (Neutron News 3, 1992)

	Coh a (fm)	Inc b (fm)
^{1}H	-3.7406(11) -3.79406(11)	25.274(9)
^{2}H	6.671(4)	4.04(3)
^{3}H	4.792(27)	-1.04(17)
^{3}He	5.74(7)-1.483(2)*i*	-2.5(6)+2.568(3)*i*
^{4}He	3.26(3)	

TUNL: D.R. Tilley et al., Nucl. Phys. **A708** (2002) 3

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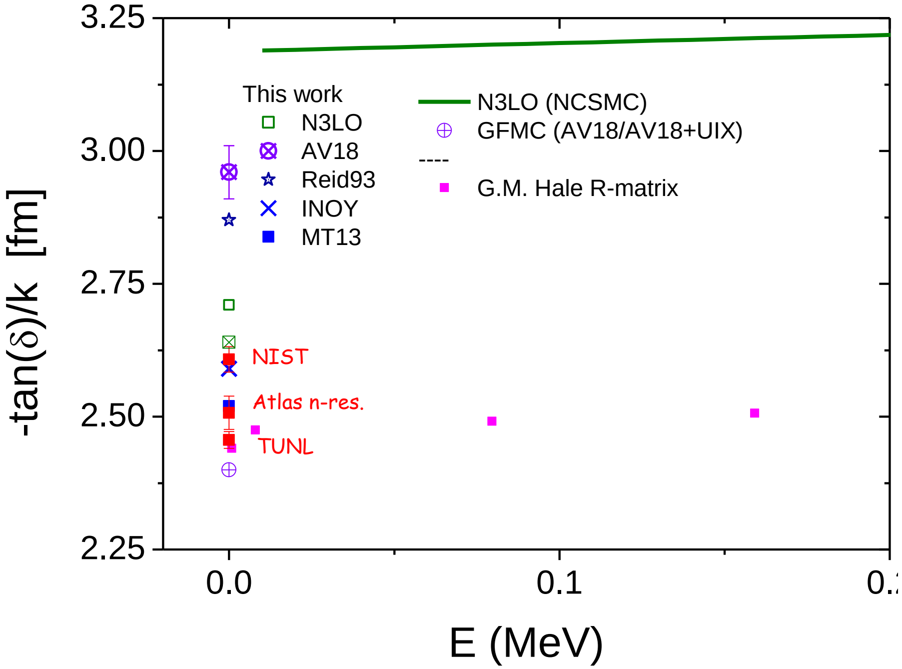
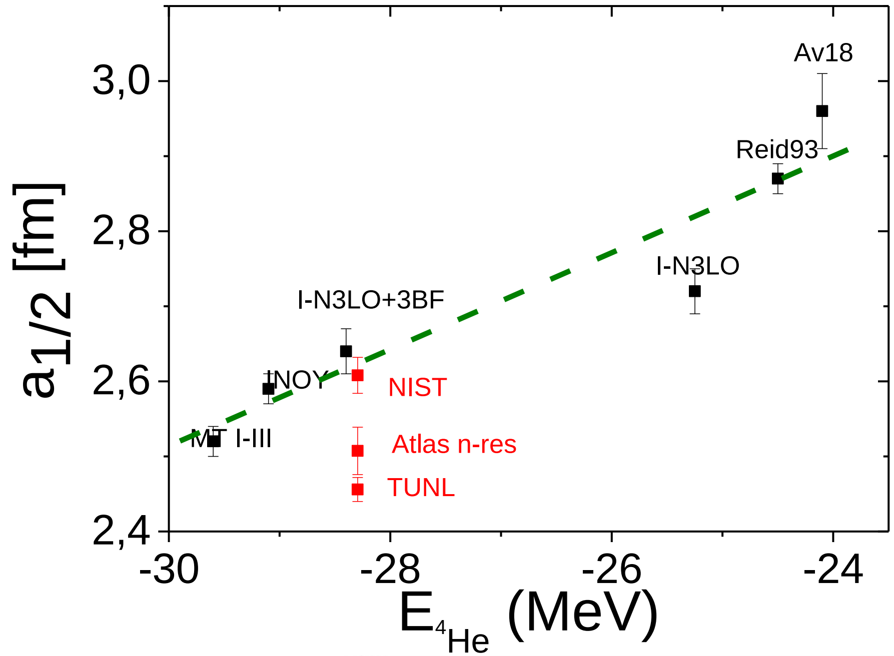
Case of little interest: S-wave





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S. Ali PSA: S. Ali et al., Rev. Mod. Phys. 57 (1985) 923
Bang-Gignoux pot: J. Bang, C. Gignoux, Nucl. Phys. A 313 (1979)
NCSMC: P. Navratil et al., Physica Scripta 91 (2016) 053002
GFMC: K.M. Nollett, PRL99, 022502 (2007)

R. Lazauskas

PV violation for $\vec{n}$-^{4}He

Slow $\vec{n}$ spin rotation studyt at NIST $\frac{d\phi}{dz} = 2.1 \pm 8.3(stat.)^{+2.9}_{-0.2}(sys) \times 10^{-7}$ rad/m

E. Swanson et al. PRC **100** (2019) 015204

- ✓ Weak process $V^{weak} \ll V^{strong}$

1st order perturbation:

$$R^{weak}_{f\leftarrow i} \propto \left\langle \Psi^{strong}_f \middle| V^{weak} \middle| \Psi^{strong}_i \right\rangle$$

- ✓ The last expression one may calculate within FY framework, without passing directly to total system's wave function

  R. Lazauskas, Y.H. Song, PRC **99** (2019) 054002

Input: V^{strong} I−N3LO+3BF

V^{weak} DDH meson exchange pot. $(\pi, \rho, \omega, \rho')$. B. Desplanques et al, Ann.Phys. **124** (1980) 449 .

Ultracold $\vec{n}$-^{4}He spin rotation angle in 10^{-7} rad/m:

$$\frac{d\phi}{dz} = -0.144(1)h^1_\pi + 0.058(8)h^0_\omega - 0.402(1)h^0_\rho + \mathbf{0.0298}h^1_\omega + \mathbf{0.0296}h^1_\rho + \mathbf{0.0061}h^1_{\rho'}$$

$$\frac{d\phi}{dz} = \begin{cases} 3.7 & \text{DDH-best} \\ 3.0 & \text{DZ} \\ 0.8 & \text{FCDH} \\ 12. & \text{large } N_c \end{cases}$$

R. Lazauskas, Y.H. Song, PRC 99 (2019) 054002.

S. Gardner et al., Ann. Rev. Nucl. Part. Sci. 67 (2017) 69

^{5}H resonances: experiment

TABLE I. Summary of experimental results for ^{5}H. Resonance energies are given relative to ^{3}H + 2n.

Reference	Reaction	Detected	E_R (MeV)	Γ (MeV)	E_{beam} (A MeV)
[17]	^{3}H(t,p)^{5}H	p	≈ 1.8	≈ 1.5	7.42
[18]	^{6}He(p,2p)^{5}H	2p	1.7 ± 0.3	1.9 ± 0.4	36
[19]	^{3}H(t,p)^{5}H	t,p,n	1.8 ± 0.1	< 0.5	19.2
[21]	^{3}H(t,p)^{5}H	t,p,n	≈ 2	–	19.2
[22]	^{3}H(t,p)^{5}H	t,p,n	≈ 2	≈ 1.3	19.2
[24]	^{6}He(^{12}C,X + 2n)^{5}H	t,2n	≈ 3	≈ 6	240
[25]	^{6}He(d,^{3}He)^{5}H	^{3}He,t	1.8 ± 0.1	< 0.6	22
[26]	^{6}He(d,^{3}He)^{5}H	^{3}He,t	1.8 ± 0.2	1.3 ± 0.5	22
[27]	^{6}He(d,^{3}He)^{5}H	^{3}He,t	1.7 ± 0.3	≈ 2.5	22
[28]	^{9}Be(π^-,pt)^{5}H	p,t	5.2 ± 0.3	5.5±0.5	E_π < 30 MeV
[28]	^{9}Be(π^-,dd)^{5}H	p,t	6.1 ± 0.4	4.5±1.2	E_π < 30 MeV

[17] P. G. Young, Richard H. Stokes, and Gera
Rev. 173, 949 (1968).
[18] A. A. Korsheninnikov, M. S. Golovkov, Rodin, A. S. Fomichev, S. I. Sidorchuk, S. Chelnokov, V. A. Gorshkov, D. D. Bogdan Ter-Akopian *et al.*, Phys. Rev. Lett. 87, 09
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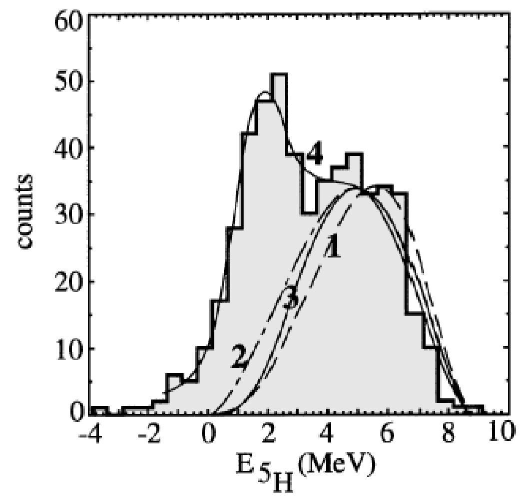

FIG. 3. Spectrum of ^{5}H from the reaction p(^{6}He, ppt). Curves show calculations explained in the text.

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r, D. V. Aleshkin, B. A. Chernyshev,
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sky, and M. V. Telkushev, Eur. Phys.

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^{5}H resonances?

TABLE II. Summary of some theoretical results for ^{5}H. Resonance energies are given relative to ^{3}H + 2n.

Reference	Method	E_R (MeV)	Γ (MeV)
[7]	Cluster, model with source	2–3	4–6
[23]	Three-body cluster	2.5–3	3–4
[31,35]	Cluster, J-matrix, resonating group model	1.39	1.60
[36]	Cluster, complex scaling adiabatic expansion	1.57	1.53
[32]	Cluster, generator coordinate method	≈3	≈1–4
[33]	Cluster, complex scaling	1.59	2.48
[34]	Cluster, analytic coupling in continuum constant	1.9 ± 0.2	0.6 ± 0.2

[7] L. V. Grigorenko, N. K. Timofeyuk, and M. V. Zhukov, Eur. Phys. J. A **19**, 187 (2004).

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[36] R. de Diego, E. Garrido, D. V. Fedorov, and A. S. Jensen, Nucl. Phys. A **786**, 71 (2007).

Predictivity?
^{5}H states does not appear naturally

Cluster models, involvig approximations for 5-body dynamics

- ^{3}H+n+n models: without n-antisymetrization between the core &valence
- ^{3}H+n+n models: including n-antisymmetrization, however by freezing ^{3}H core

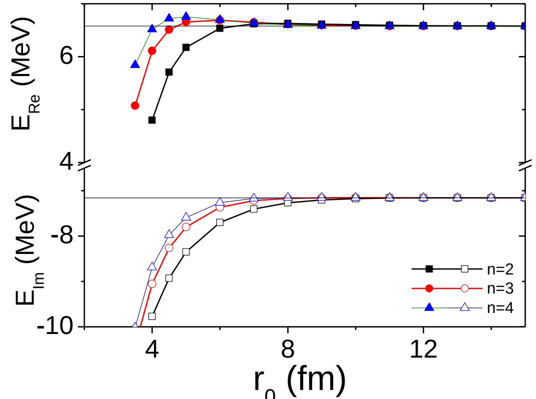
How to handle resonances?

- ACCC : **Analytic continuation in the coupling constant** method (V.I. Kukulin et al., « Theory of resonances », Kluwer AP 1989)
  - Artificialy bind ^{5}H with some additional potential $V = \lambda V_0$ (we use 5-body pot not to affect ^{3}H threshold!!)
  - Study $B_{^5H}(\lambda)$ and determine λ_0 such that $B_{^5H}(\lambda_0) = B_{^3H}$
  - Smartly extrapomate $B_{^5H}(\lambda)=f(\lambda-\lambda_0)$ to determine $E_{^5H} = B_{^5H}(0)$

- « Dirty » smooth exterior complex scaling method (DEXCSM)

  B. Simon. Phys. Letters A, 71 (1979) 211

  - Choose sharp transformation function, which almost does not affect r in r<r_o
  - Fix r_o beyond the physical interaction region
  - Ignore inconsisitencies in transformation between different Jacobi bases



2b-example

$$V(l=1) = \frac{1.8}{r}(1438.72e^{-3.11r} - 626.885e^{-1.55r})$$

$$r \to (1-f(r))r + f(r)re^{i\theta} \qquad f(r) = \exp(-\left(\frac{r_0}{r}\right)^n)$$

^{5}H(J=1/2$^+$)

- nn interaction described by the MT I-III potential
- auxilliary potential for ACCC

$$V_{5b}(\rho) = \lambda \rho^p exp(-\rho^2/\rho_0^2).$$

$$\rho^2 = x^2 + y^2 + z^2 + w^2 = 2\sum_{i=1}^{5} r_i^2$$

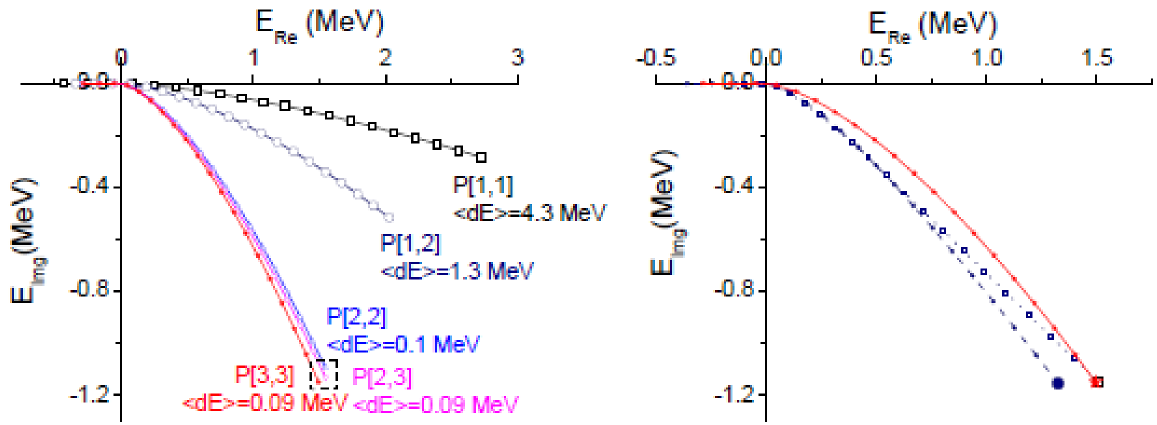


Fig. 3 Resonance trajectories for a $J^\pi = 1/2^+$ state of ^{5}H with respect to ^{3}H threshold. Each trajectory is split by points in 20 intervals of equal step in λ, starting at the position where ^{5}H nucleus is still weakly bound. The endpoint of the trajectory indicates extrapolated value for the bare NN interaction, corresponding $\lambda = 0$ case. In the left panel convergence of the results with respect to order of Padé expansion is presented; calculation is based on auxiliary potential defined in eq. (13) with ρ_0^2=78.4 fm^2 and $p = 0$. In the right panel converged results for three different external potentials are presented.

$E(^5H)-E(^3H)=1.4(1)-i1.2\ (1)$

$E(^5H)-E(^3H)=1.7(2)-i1.2\ (1)$

R. Lazauskas

- nn interaction described by the MT I-III potential

$J=1/2^+$ ($L=0^+$, $S=1/2$)

ACCC:

$E(^5H)-E(^3H)=1.4(1)-i1.2(1)$

DEXCSM:

$E(^5H)-E(^3H)=1.6(2)-i1.2(1)$

$J=5/2^+$ ($L=2^+$, $S=1/2$)

DEXCSM:

$E(^5H)-E(^3H)=2.50(15)-i1.90(15)$

Negative parity states & ones with S=3/2 are much more broader

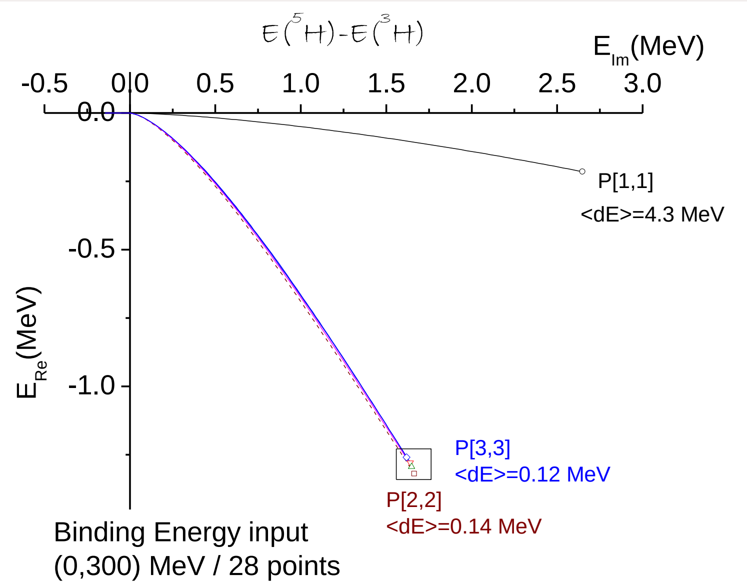
To compare with ^4H resonances:

$E(^4H)-E(^3H)=$ $1.08(1)-i2.04(2)$ $(S=1, L=1^-)$
$0.88(3)-i2.20(4)$ $(S=0, L=1^-)$

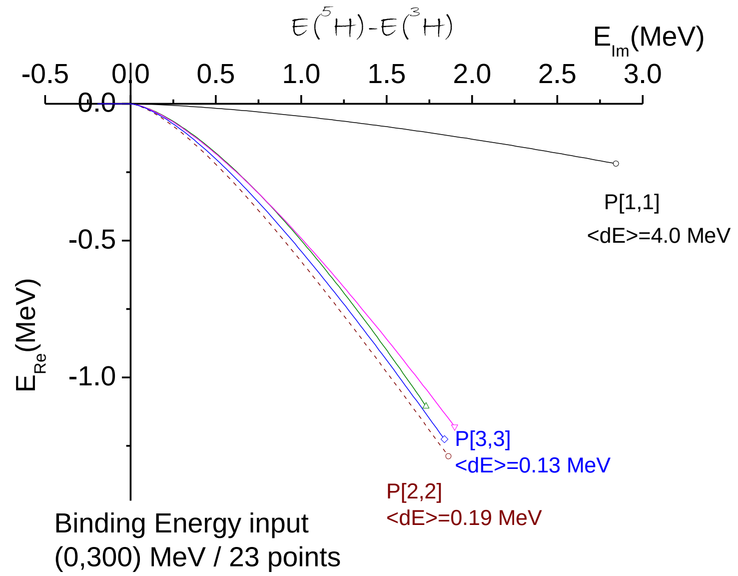
R. Lazauskas, Few-Body Syst. 59 (2018) 13.

$^5H(J=1/2^+)$

INOY Potential



N_3LO Potential



$E(^5H)-E(^3H)=1.65(5)-i1.26(6)$

$E(^5H)-E(^3H)=1.8(1)-i1.15(15)$

DEXCSM: 1.8(1)-i1.2(1)

DEXCSM: 1.85(10)- i1.20(5)

To compare with ^4H resonances $J=2^-$:

$E(^4H)-E(^3H)=1.31(3)-2.08(2)$

$E(^4H)-E(^3H)=1.17(3)-1.99(3)$

R. Lazauskas, E. Hiyama, J. Carbonell, Phys. Lett. **B 791** (2019) 335

Conclusion

• FY eq. formalism remains reference in few-body scattering calculations. The first solutions of 5-body FY equations are presented.

• Reliable results have been obtained for n-^{4}He scattering at low energies using realistic interactions. Satisfactory description is obtained when using Idaho N3LO NN +N2LO NNN interactions.

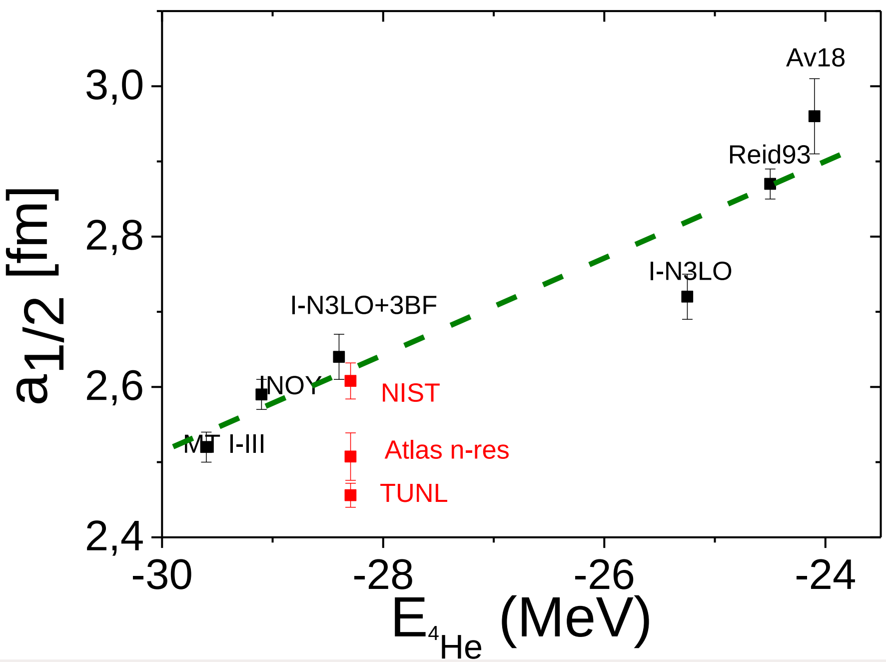
• The first fully realistic calculation of weak process in 5-nucleon sector is performed.

• Description of the ^{5}H resonant states have been performed for the first time using fully realistic description and two different methods to calculate resonance positions. Presence of broad resonant states is confirmed!

Acknowledgements: The numerical calculations have been performed at IDRIS (CNRS, France). We thank the staff members of the IDRIS computer center for their constant help.

Experimental n-^{4}He scattering length …

nothing should be as easy to measure…



TUNL: D.R. Tilley et al., Nucl. Phys. **A708** (2002) 3

NIST: https://www.ncnr.nist.gov

Experimental data:

D.C.Rorer et al., Nucl. Phys. **A 133** (1969) 410

S.F.Mughabghab, Atlas of Neutron Resonances (2006)

R.Genin et al., Journal de Physique **24** (1963) 21

NIST (Neutron News 3, 1992)

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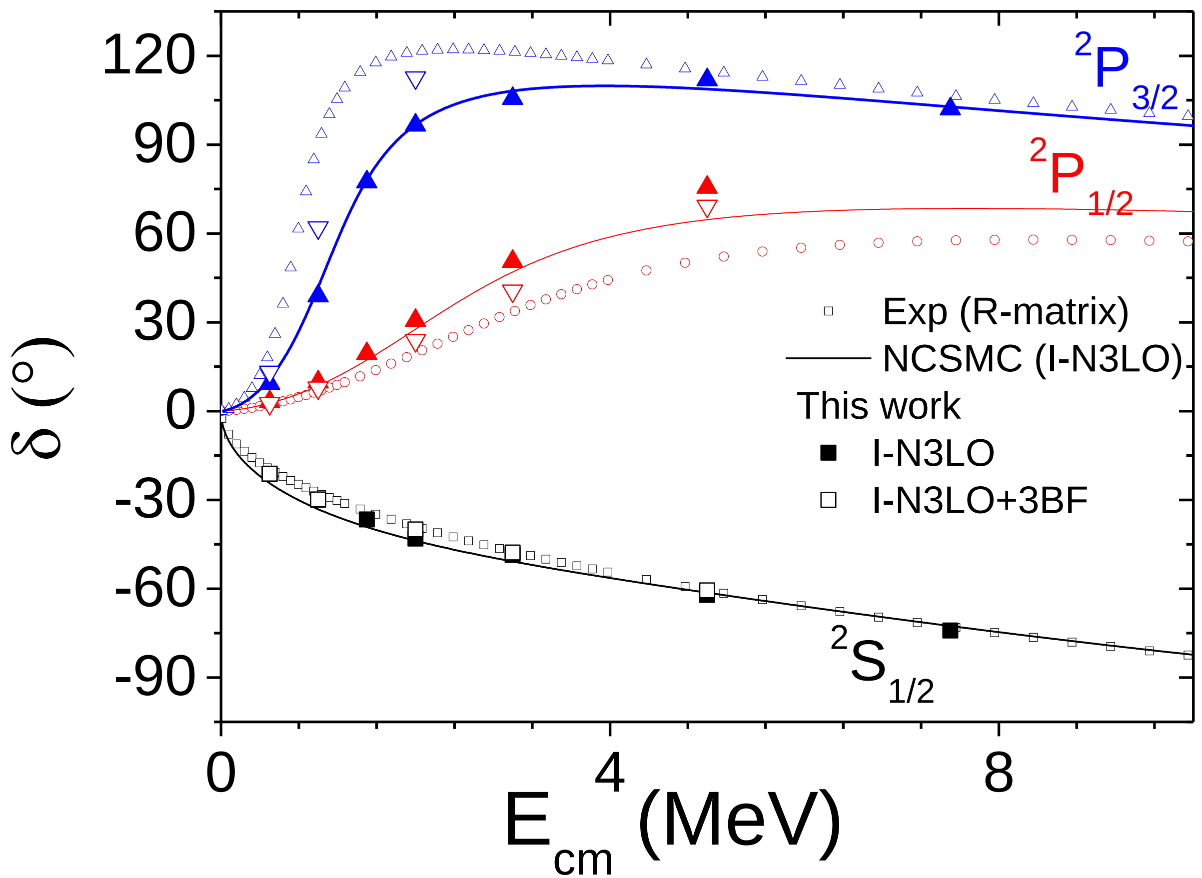
^{3}He	5.74(7)-1.483(2)*i*	-2.5(6)+2.568(3)*i*
^{4}He	3.26(3)	


120
90
60
30
0
-30
-60
-90
δ (°)
0
4
8
E cm (MeV)
2P3/2
2P1/2
2S1/2
Exp (R-matrix)
NCSMC (I-N3LO)
This work
I-N3LO
I-N3LO+3BF

R. Lazauskas