PENNSTATE

Energy dependence of 3-body recombination in 1D

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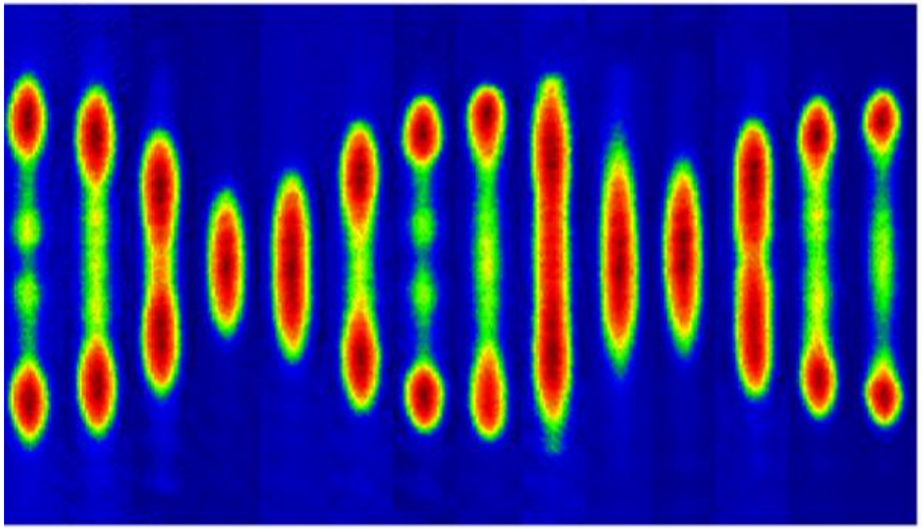
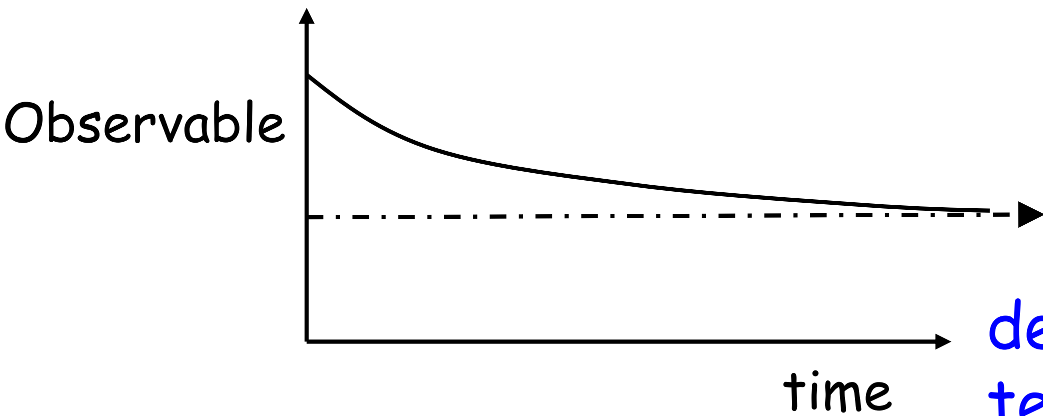


Supported by the ARO and NSF

Our motivation

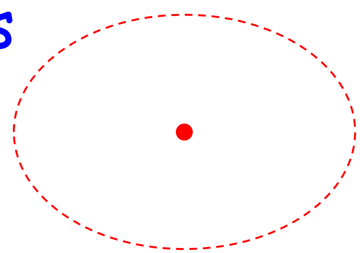
Generically, many-body systems thermalize: they approach a most probable (highest entropy) state



the stat mech answer

depends only on bath temperature (canonical) or total energy (microcanonical).

integrable systems are an exception: there are extra conserved quantities



Are they mathematical singular points, or are they the center of a region where stat mech does not apply?

When does the premise of stat mech hold?

Outline: 1D 3-body recombination

A. Experiments with ground state 1D Bose gases: the Lieb-Liniger model and integrable many-body systems

B. Out-of-equilibrium 1D gases: Quantum Newton's Cradles (QNCs)

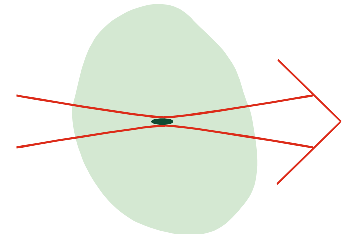
C. Measurement of 3-body recombination in 1D

D. Modeling 3-body loss to determine $K_3(E_{cm})$.

Keep track of all 3-body recombination collisions.

Compare the experiment to theory.

Optical Lattices

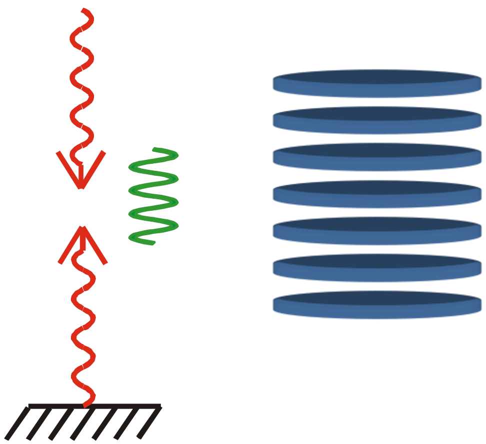


Calculable, versatile atom traps

Far from resonance,
no light scattering

$U_{AC} \propto$ Intensity

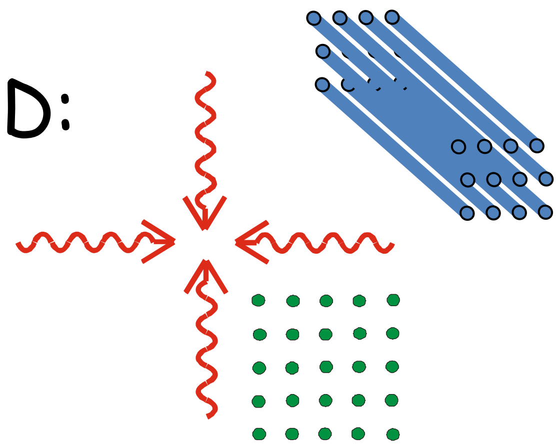
1D:



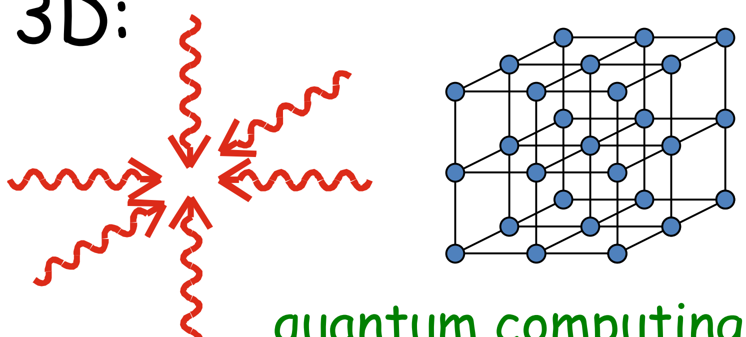
electron
electric dipole
moment search

1D Bose gases

2D:



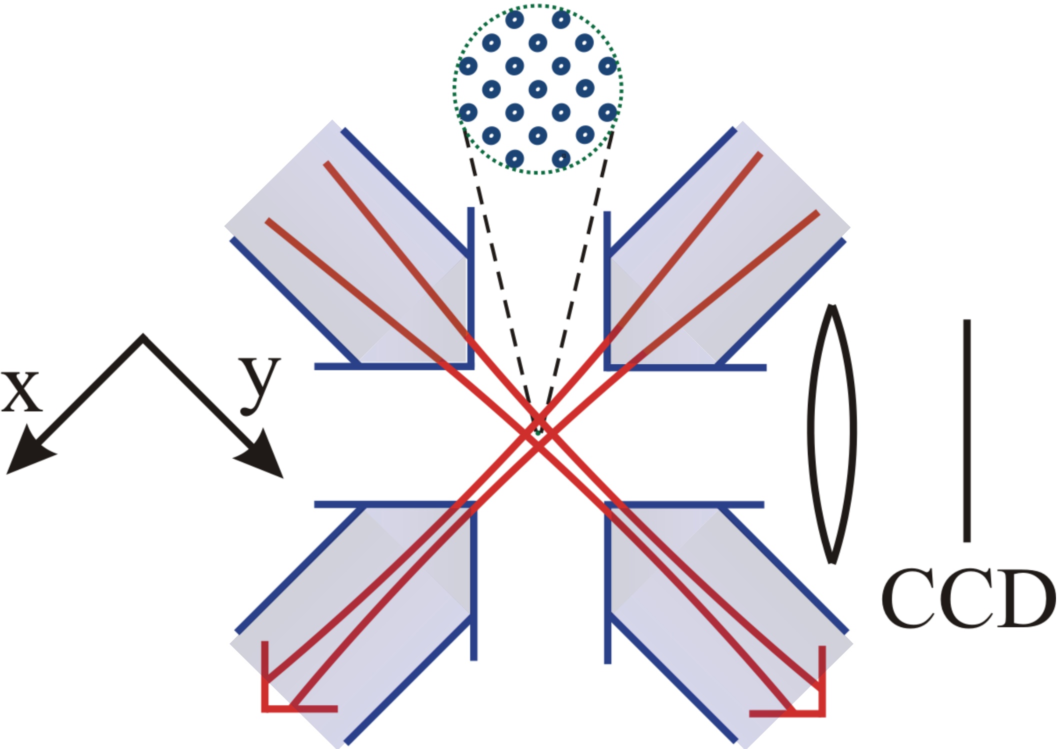
3D:

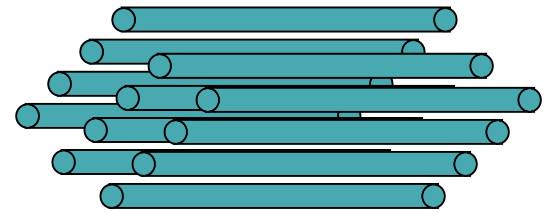


quantum computing

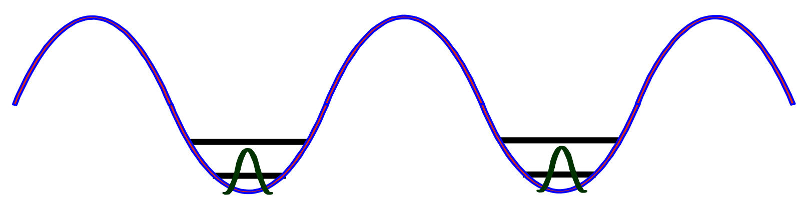
(Maxwell demon
for >50 atoms)

Experimental 1D gases





0-400 atoms/tube
~1,000-8000 tubes



For 1D:all energies < $\hbar\omega_\perp$;
negligible tunneling

1D Bose gases with variable point-like interactions

Elliot Lieb and Werner Liniger, 1963: Exact solutions for 1D Bose gases with arbitrary $\delta(z)$ interactions

A Bethe ansatz approach yields solutions parameterized by

$$\gamma = \frac{m}{\hbar^2}\frac{g_{1D}}{n_{1D}}$$

$$H_{1D} = \sum_{j=1}^{N} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z_j^2} + \sum_{i<j} g_{1D}\,\delta(z_i - z_j)$$

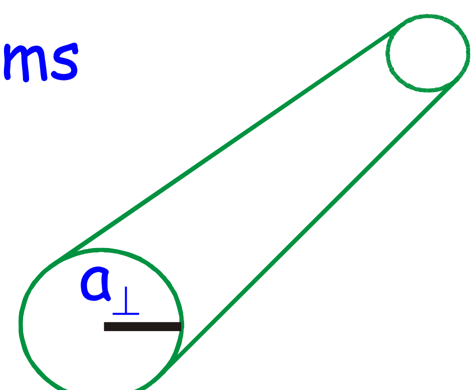
Lieb & Liniger, Phys Rev **130** 1605 (1963)

L-L is integrable $\Rightarrow$ many extra constants of motion.
Wavefunctions and all local properties are exactly calculable.

Maxim Olshanii, 1998: Adaptation to real atoms

$$\gamma \approx \frac{4a_{3D}}{a_\perp^2 n_{1D}}$$

a_{3D} = 3D scattering length
$a_\perp$ = transverse oscillator length



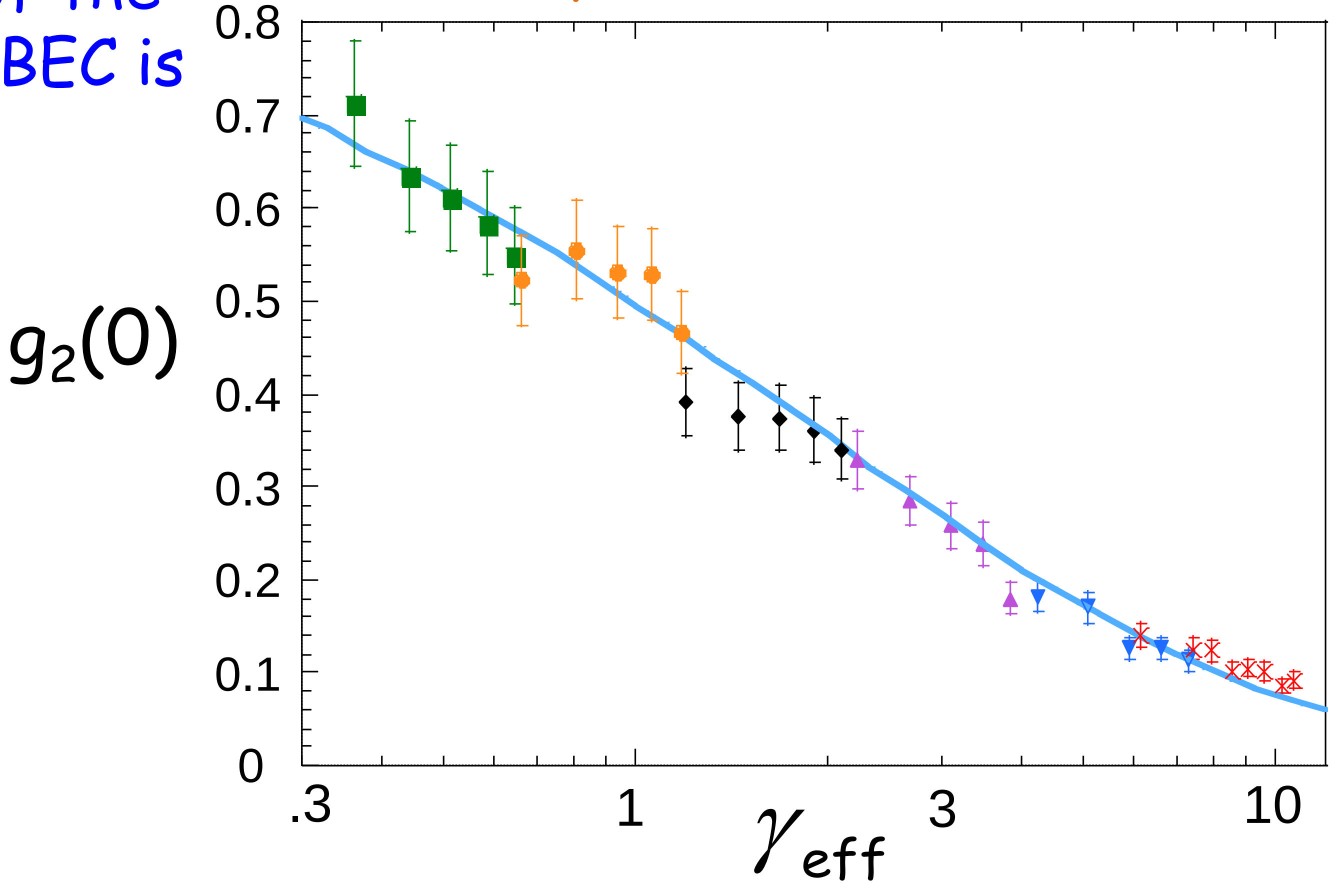
Eg., Normalized Local Pair Correlations

By photo-association

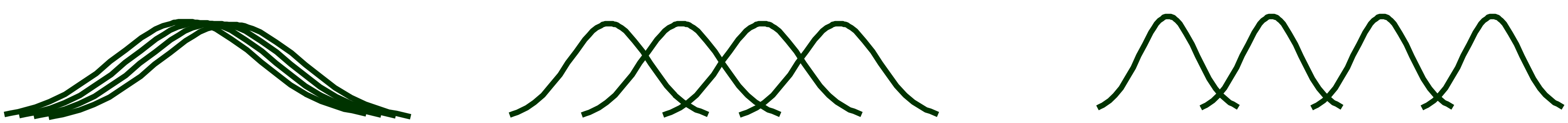
Theory: Gangardt & Shlyapnikov, PRL **90** 010401 (2003)

Expt: Kinoshita, Wenger, DSW, PRL **95** 190406 (2005)

g_2 of the 3D BEC is 1.



Pauli exclusion for bosons

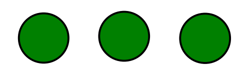


Collisions in 1D

For identical particles, reflection looks just like transmission !

➔ Two-body collisions between distinct bosons cannot change their momentum distribution.

1D Bose gases with δ-fn interactions are integrable systems.



$p_a, p_b, p_c \longrightarrow p_a, p_b, p_c$

(no "diffractive" collisions)

➔ they do not: ergodically sample phase space ≈ become chaotic ≈ thermalize

Imperfect δ-fn interactions lift integrability. Do 1D gases then eventually thermalize?

Approach to studying thermalization

Take the system out of equilibrium and follow time evolution of the momentum distribution.

Processes that drive momentum evolution in 1D

- Density-independent heating: mostly spontaneous emission from lattice light

J.F Riou, A. Reinhard, L. Zundel, D.S. Weiss, *PRA* **86**, 033412 (2012)

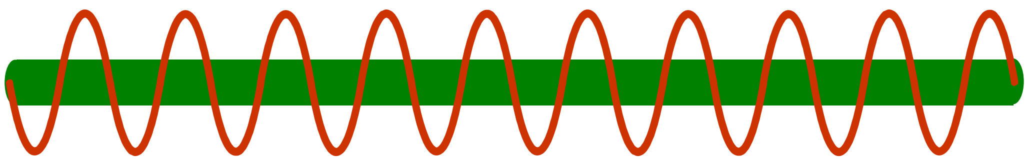
- Heating from 3-body loss

L.A. Zundel, J.M. Wilson, N. Malvania, L. Xia, J.F Riou, D.S. Weiss, *PRL* 122, 013402 (2019)

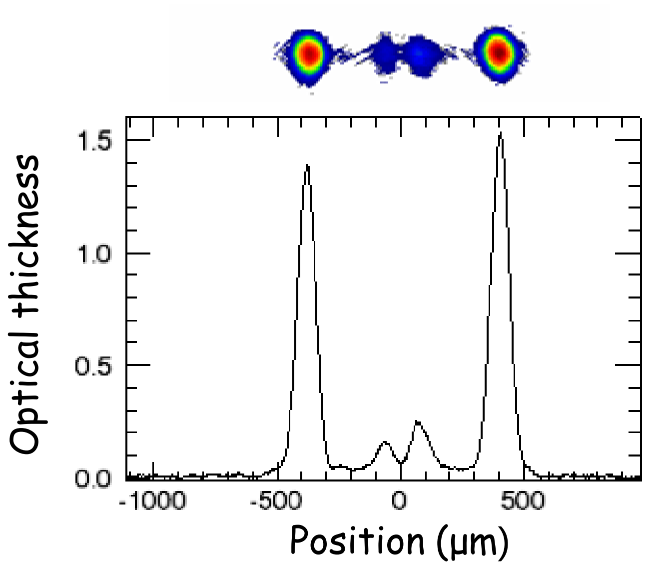
- Diffractive 3-body collisions: evaporative cooling

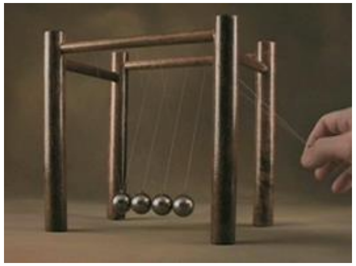
← isolated effect of 3-body elastic collisions

Creating Non-Equlibrium Distributions



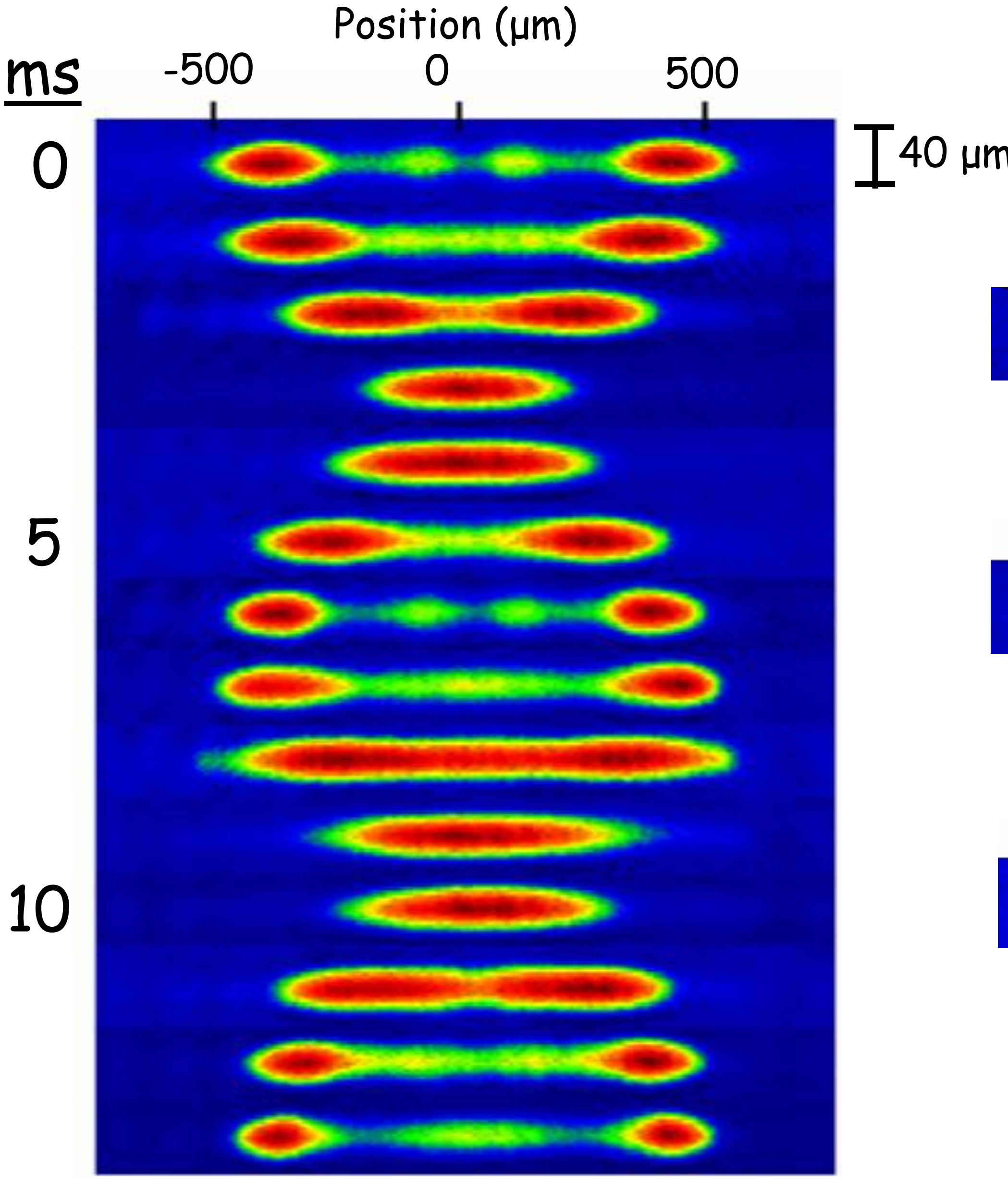
2 standing wave pulses



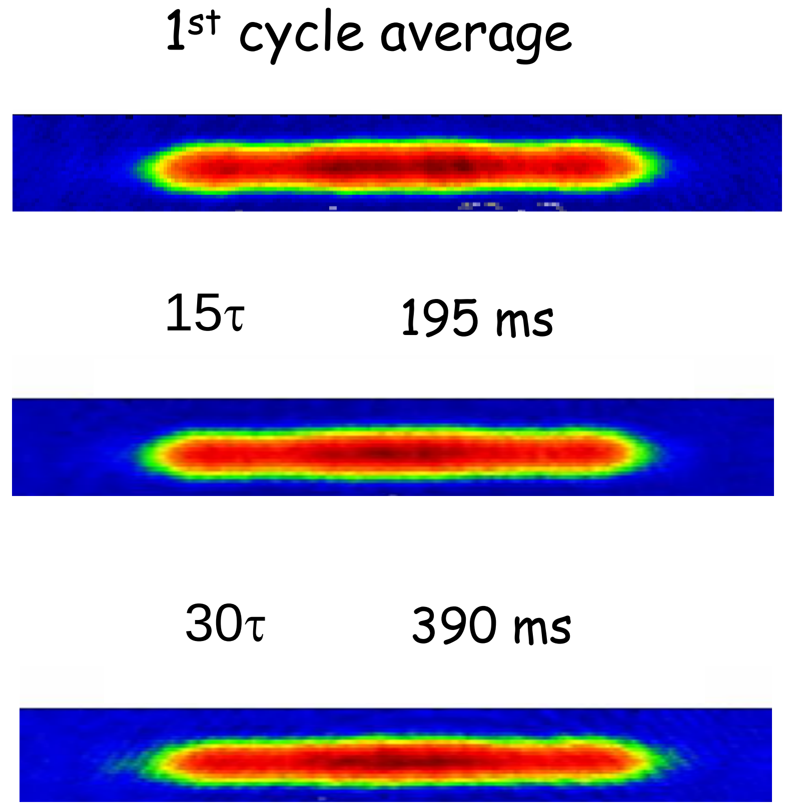


Wang, et al., PRL **94**, 090405 (2005)

Quantum Newton's Cradles



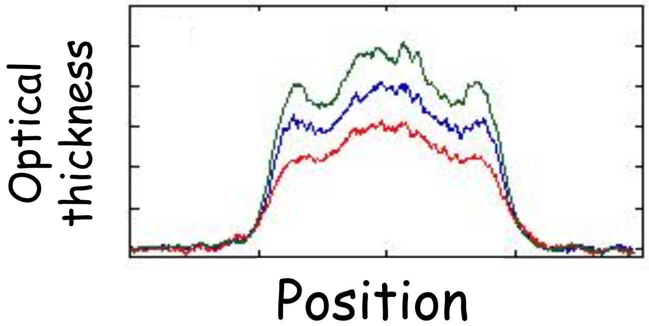
Kinoshita, Wenger, DSW, *Nature* **440**, 900 (2006)



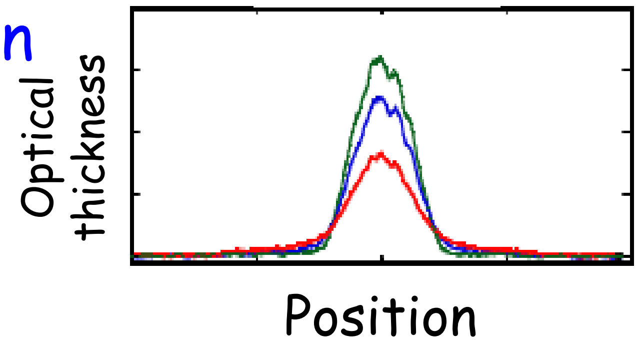
1st cycle average
15 τ distribution
40 τ distribution

Lattice depth: 63 E_r

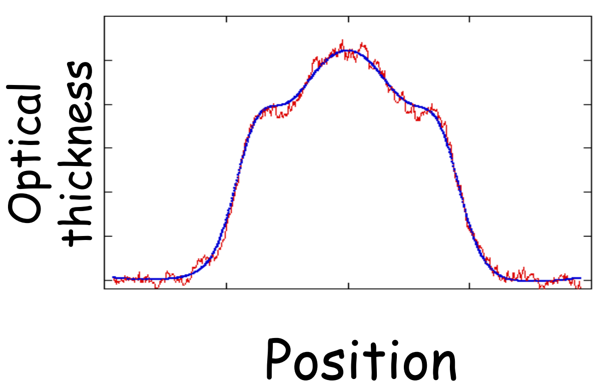
Steady-state Momentum Distributions



Evolution without grating pulses



Project the evolution



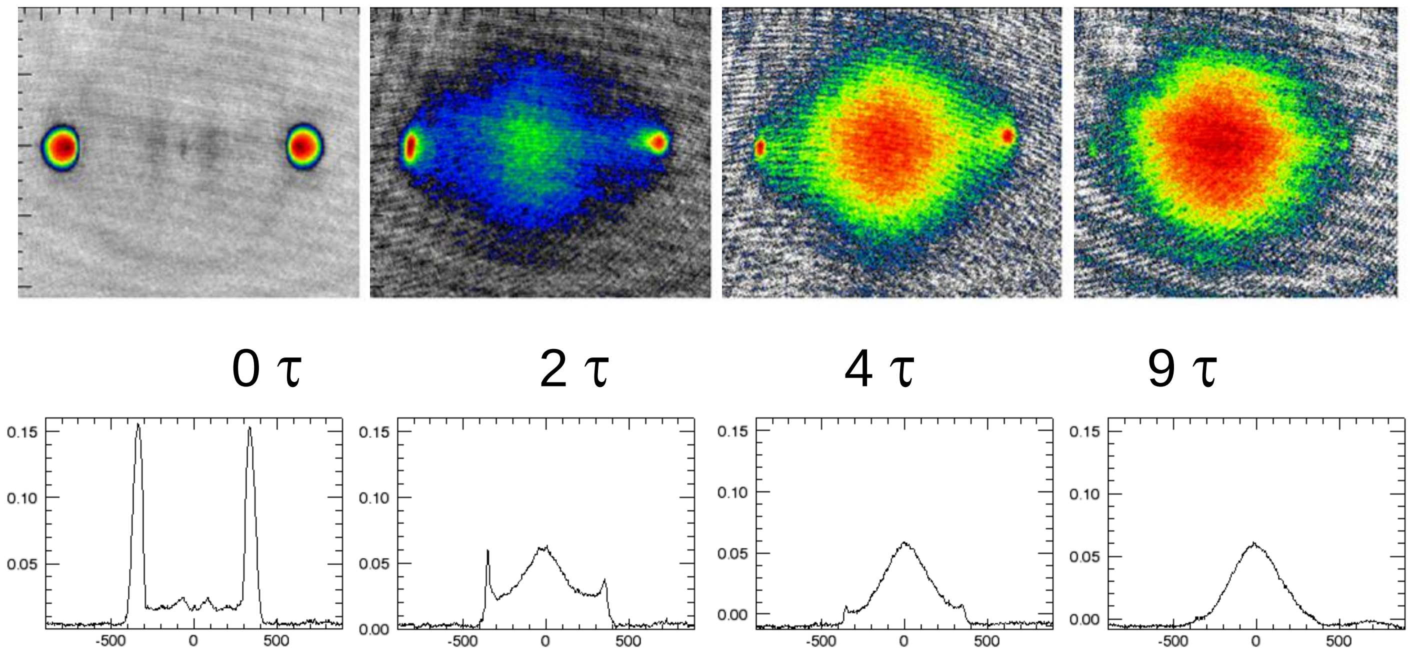
After dephasing (**prethermalization**), the 1D gases reach a steady state that is not thermal equilibrium

Each atom continues to oscillate with the amplitude it has after dephasing

Lower limit: thousands of **2-body** collisions without thermalization

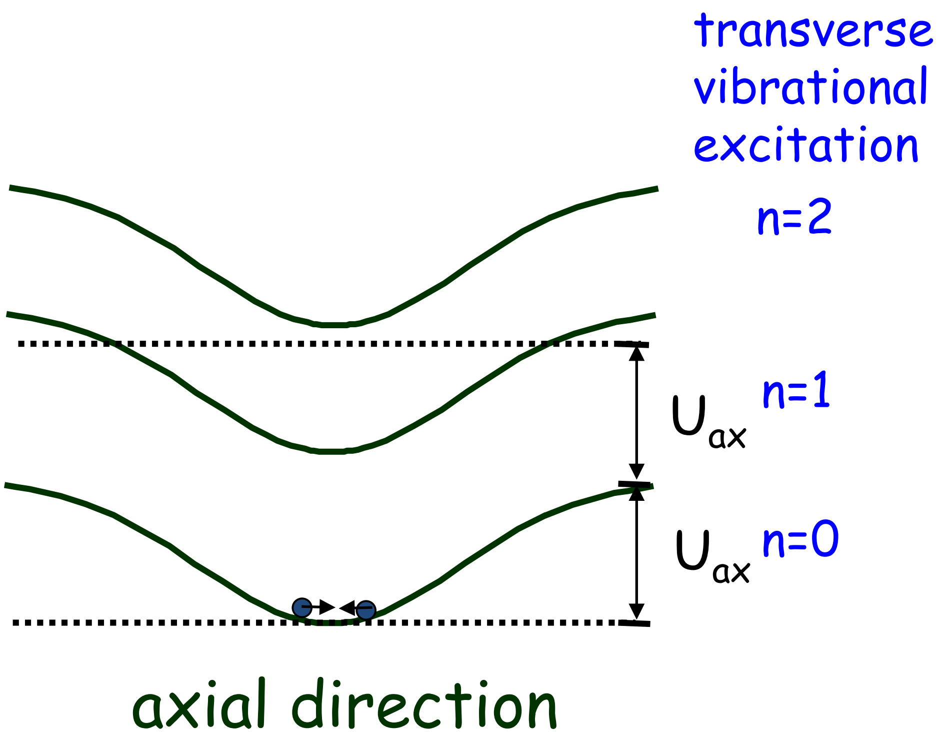
What happens to the QNC in 3D?

Thermalization is known to occur in ~3 collisions.



Enforced 1D dynamics

The blue-detuning of the 2D lattice is critical.



Ground band atoms never have enough energy to pairwise collisionally excite to higher bands.

Excitation to higher bands leads to loss and/or "heating".

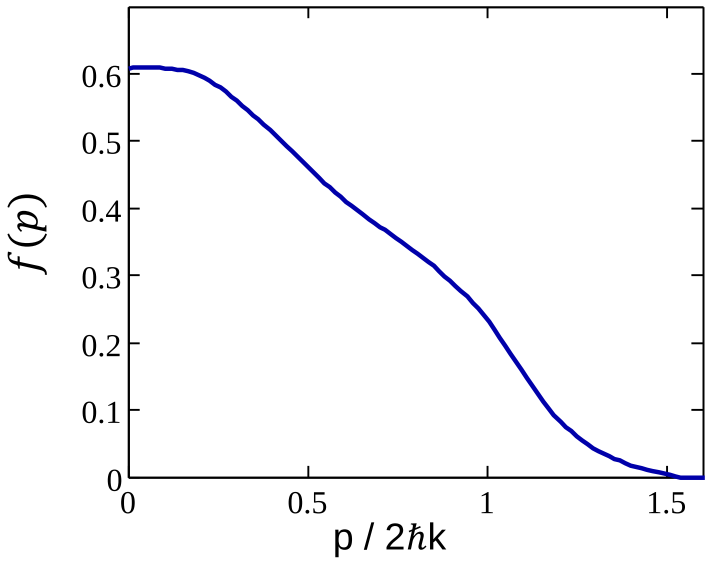
We have a higher density, reduced noise, improved momentum measurements compared to our original QNC.

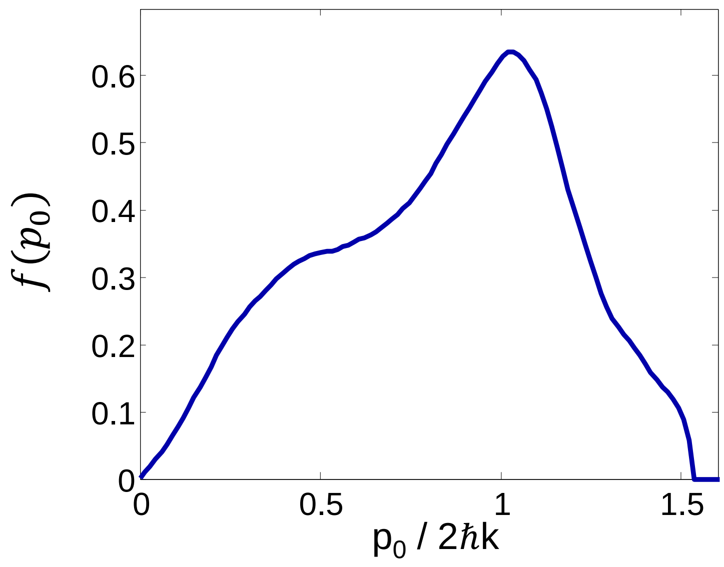
Distribution of peak momenta

Atom energy: $\frac{p_0^2}{2m} = \frac{p^2}{2m} + U(z)$

The QNC has enough energy that the axial motion is ~semi-classical

(half) the momentum distribution, measured by TOF

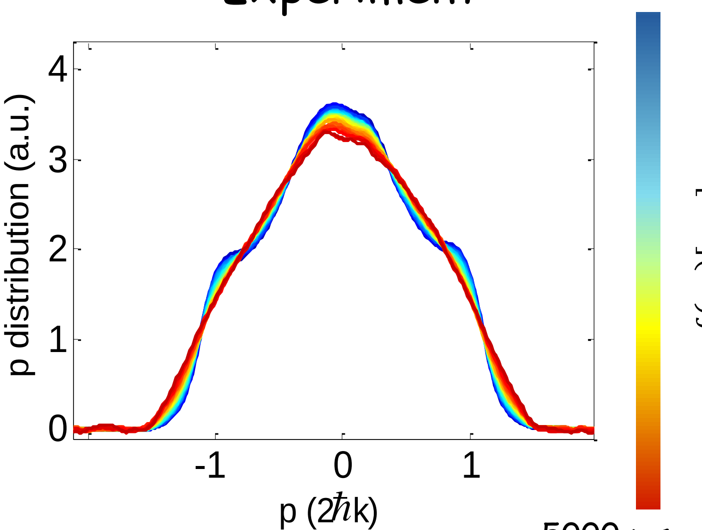




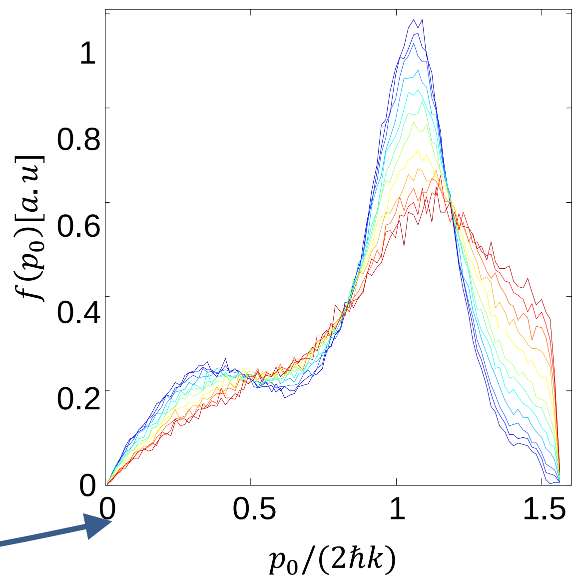
The momentum amplitude, p_o, is the conserved quantity.

Low density evolution

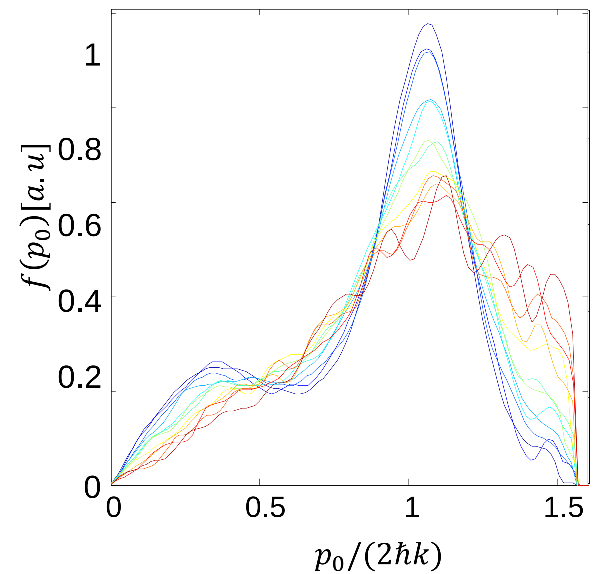
Experiment



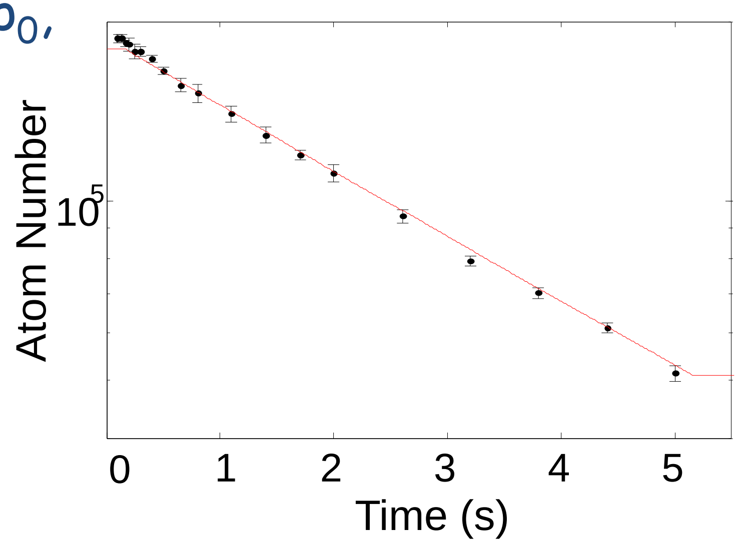
Experiment

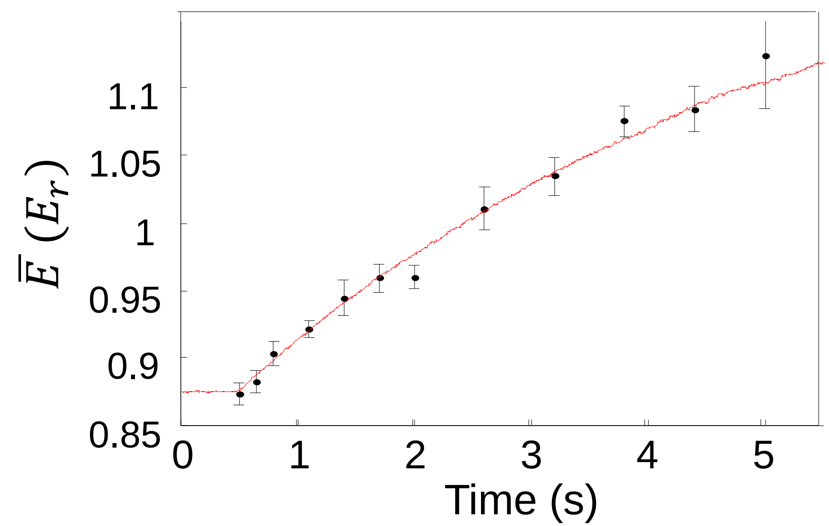


Monte Carlo

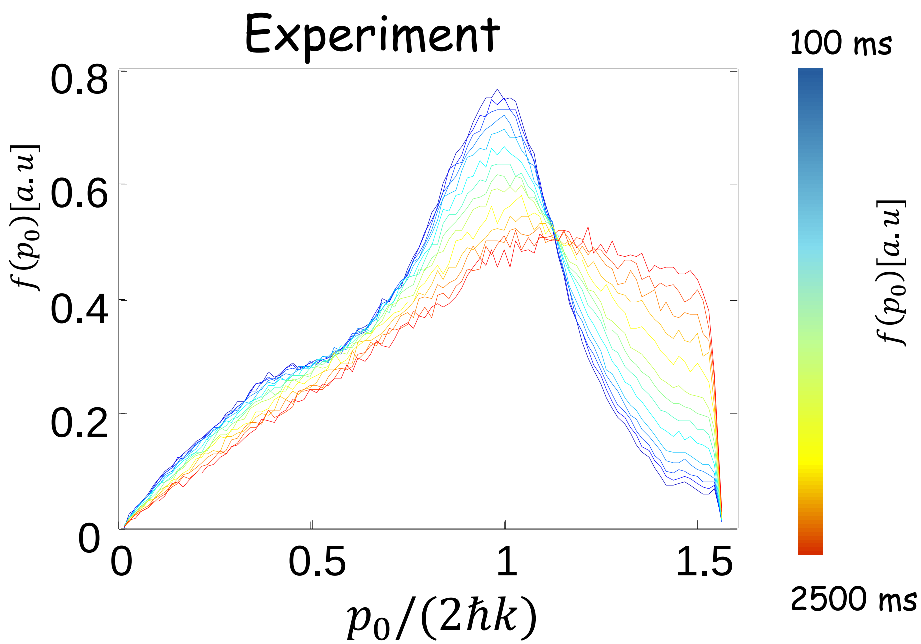


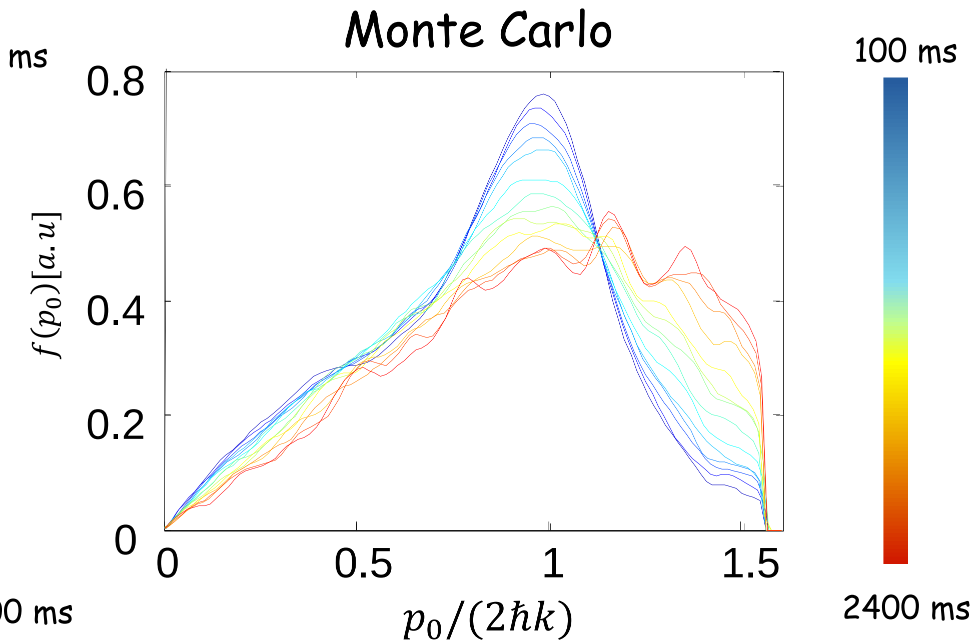
Deconvolve from p to p_0, the peak momentum



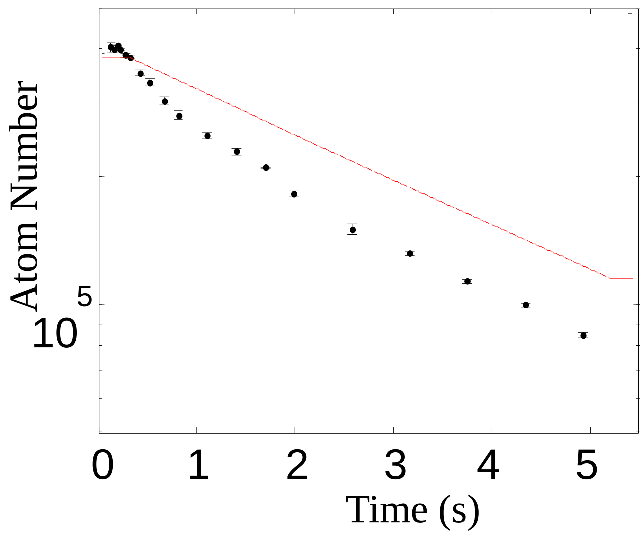


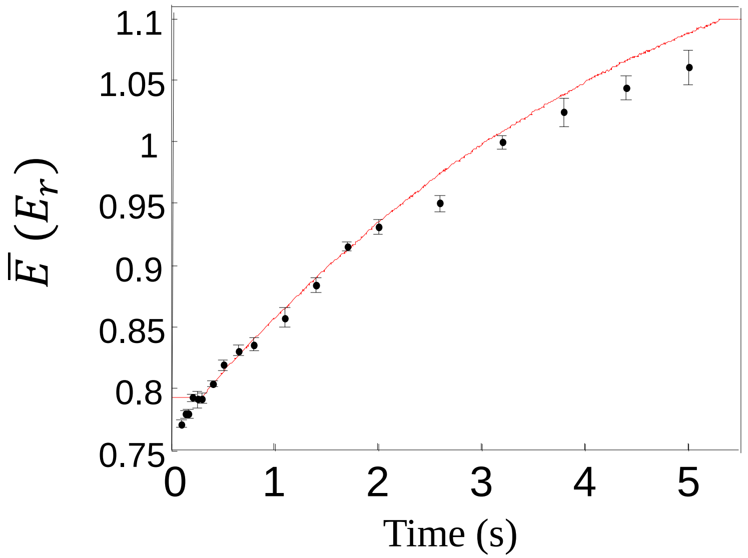
High density evolution





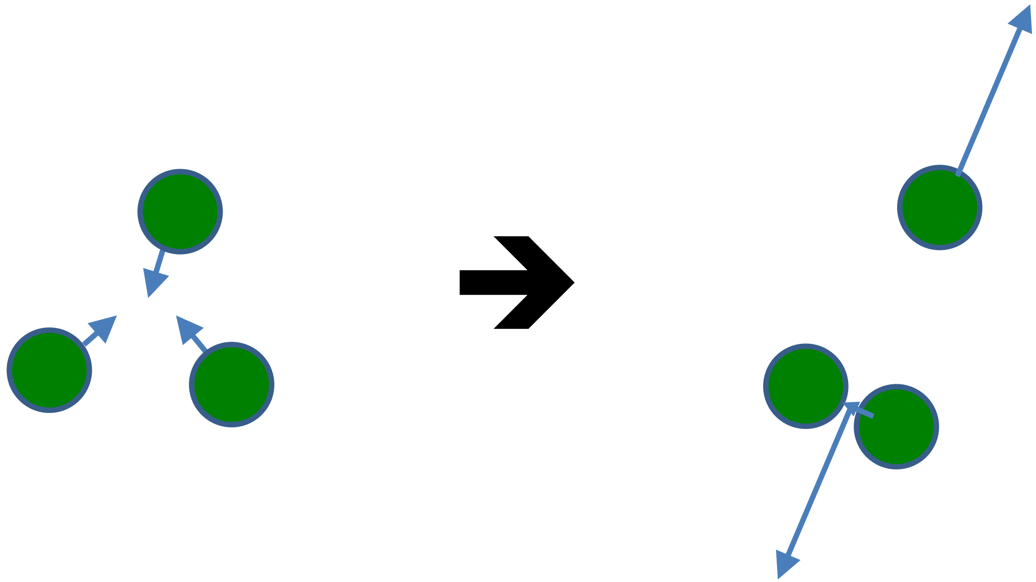
There is extra 3-body loss





3-body recombination

Esry, Greene, Burke,
PRL **83**, 1751 (1999)



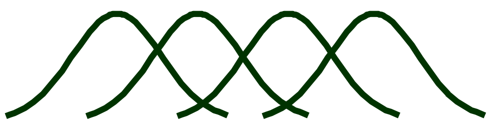
In 3D at low energy, the loss coefficient, K_3, is energy independent.

Suppression of 3-body recombination in 1D

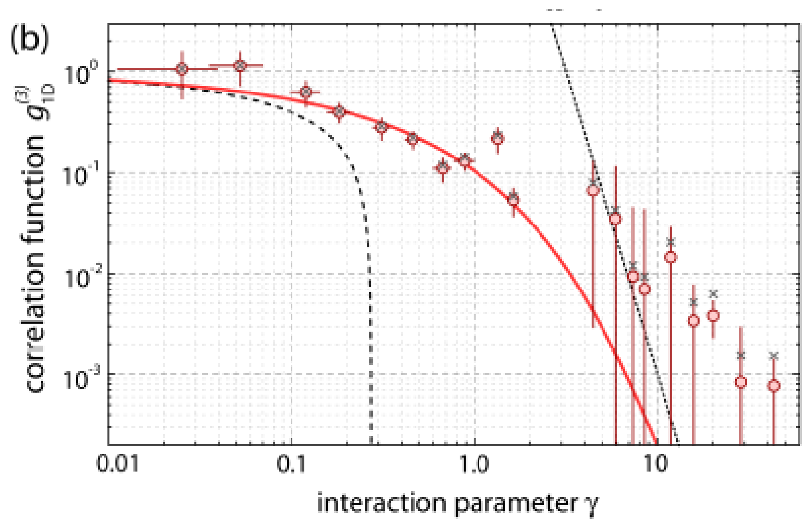
Two reasons for suppression

Reduced $g_3(0)$ due to increased correlations

Laburthe Tolra et al. PRL **92**, 190401 (2004)



Applies to a gas in thermal equilibriuim



Haller et al. PRL **107**, 230404 (2011)

Threshold scattering for isolated 3-body collisions

$$K_3 \propto \frac{\hbar}{\mu}(ka)^6$$

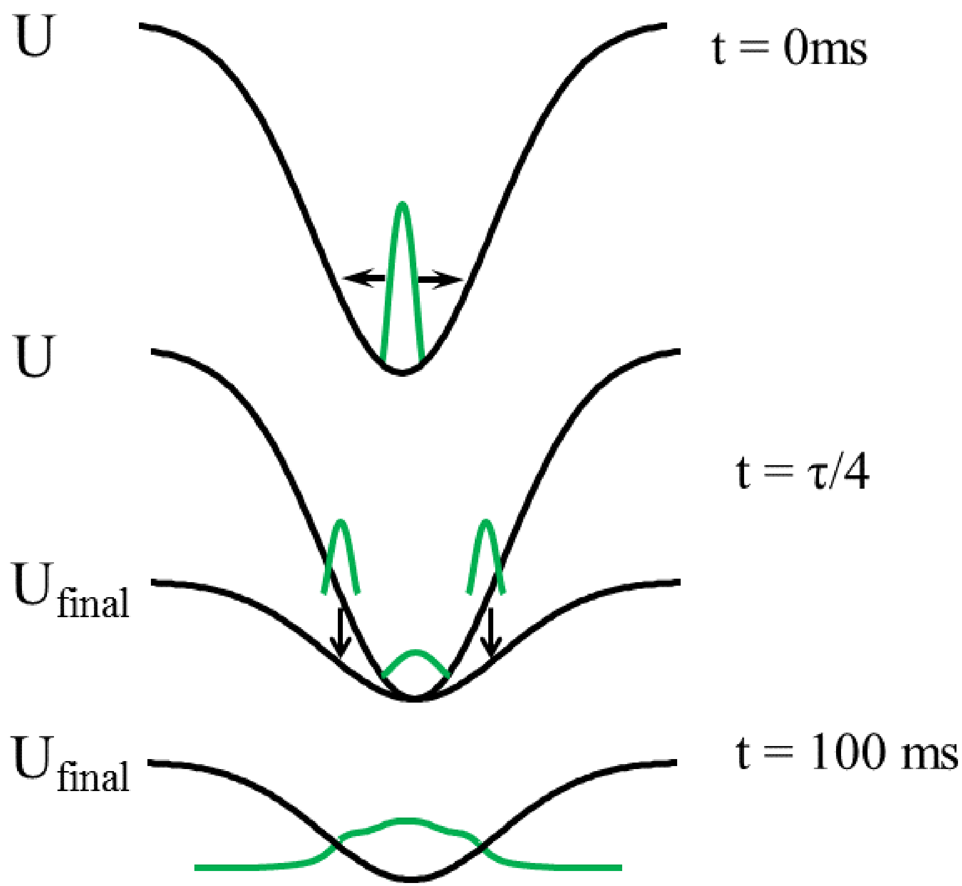
Mehta, Esry, Greene PRA **76**, 022711 (2007)

Related to Efimov physics

Varying the Axial Energy

Vary U over a factor of 5

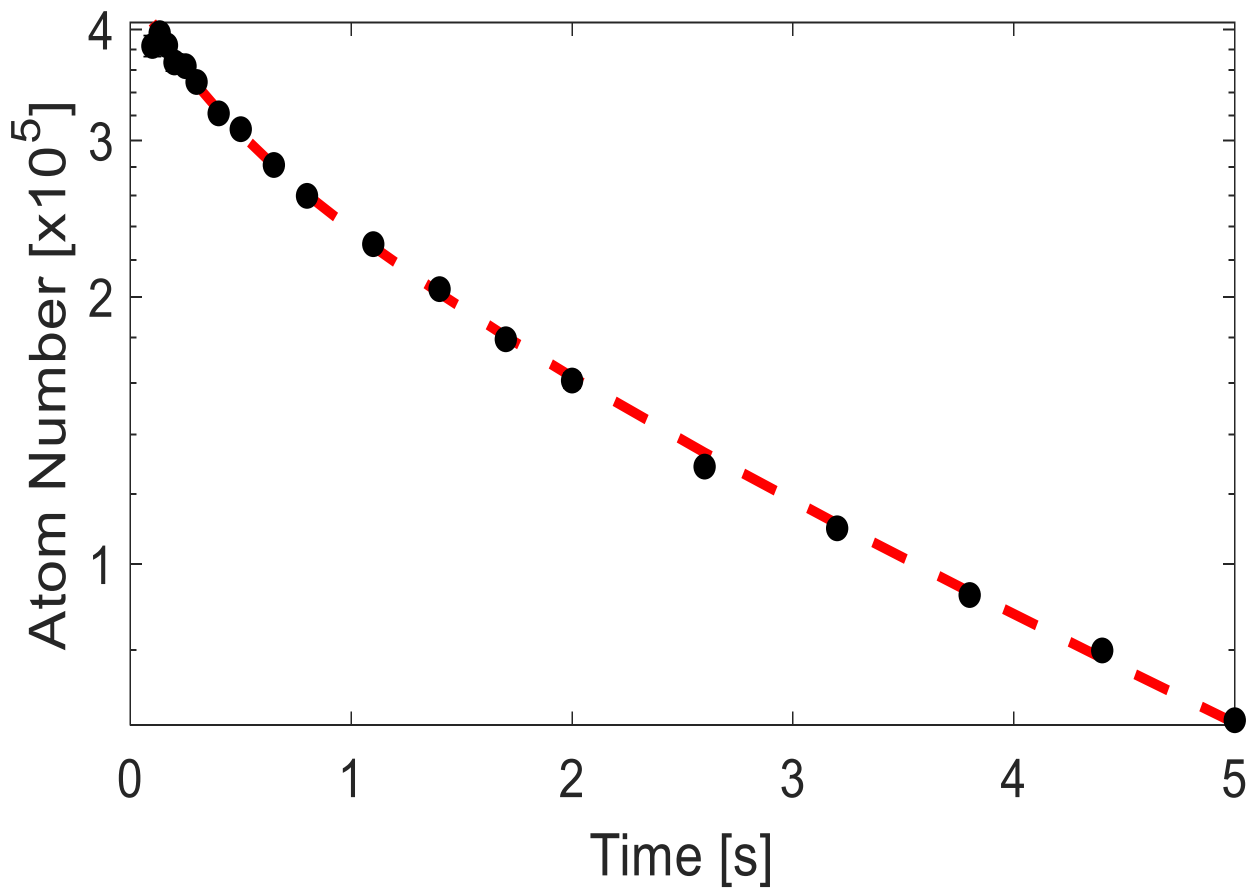
Choose U_{final} to ensure 1D for each of 4 different lattice depths



Simple fit of 3-body loss

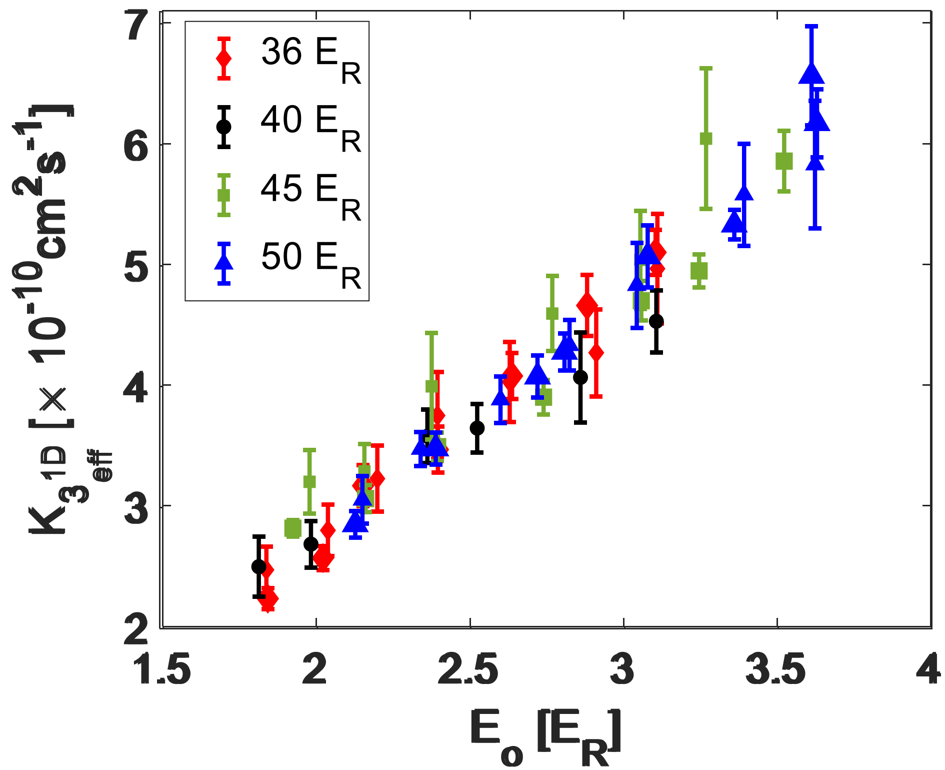
To compare the 3-body loss for all data sets in one figure, we fit loss data to:

$$\frac{dN}{dt} = -K_1 N - K_{3eff}^{1D} \int n_{1D}^3 dz$$



The loss depends on the average energy

L.A. Zundel, J.M. Wilson, N. Malvania, L. Xia, J.F Riou, D.S. Weiss, *PRL* 122, 013402 (2019)



Two different initial densities, 4 different lattice depths, ~7 different initial average energies

More energetic ensembles of atoms have a higher ${K_{3eff}}^{1D}$

In 3D, this curve would be a horizontal line.

Adapting a strictly 1D theory

Mehta, Esry, Greene PRA **76**, 022711 (2007)

Olshanii, PRL **81**, 938 (1998)

Strictly 1D theory

$$K_3^{1D} \propto (ka_{1D})^6$$

Quasi-1D theory

$$a_{1D} = -\frac{2a_\perp^2}{a_{3D}}$$

$$K_3^{1D} \propto E_{cm}^3 a_\perp^{12}$$

Bound K_3^{1D} by the thermal value of K_3^{3D}:

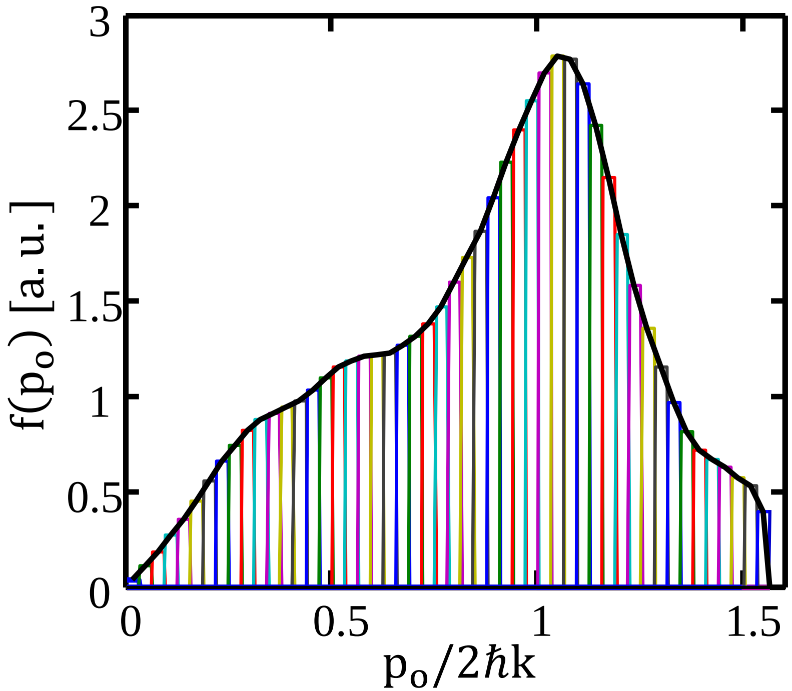
$$K_{3_{max}}^{1D} = \frac{6K_3^{3D}}{3\pi^2 a_\perp^4}$$

Introduce E_l to characterize crossover between 1D and 3D.

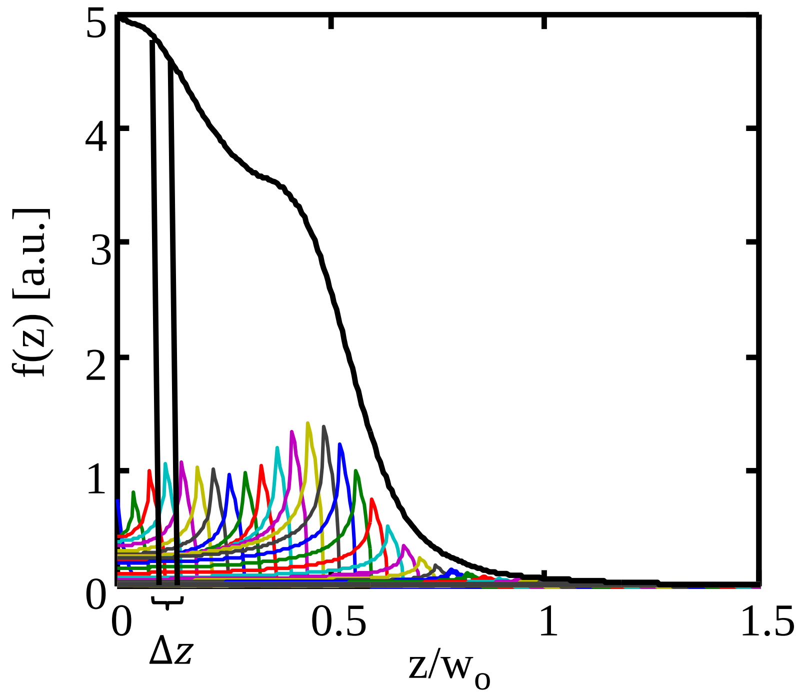
$$K_3^{1D}(E_{cm}) = C' a_\perp^{12} E_{cm}^3 \left(\frac{C' a_\perp^{12} E_{cm}^3}{K_{3_{max}}^{1D} \left(\frac{E_{cm}}{E_{cm} + E_l} \right)} + 1 \right)$$

Calculating the Loss

Keep track of all 3-body collisions $E_{cm}^3 = (p_{1com}^2 + p_{2com}^2 + p_{3com}^2)^3$



⇨



Loss within each Δz is given by:

$$\frac{dN}{dt} = -K_1 N - N^3 \sum_{p_{o_i}=p_{o_1}}^{p_{o_f}} \sum_{p_{o_j}=p_{o_i}}^{p_{o_f}} \sum_{p_{o_k}=p_{o_j}}^{p_{o_f}} K_3^{1D}(E_{cm}) \int_z^{z+\Delta z} f(z', p_{o_i}, t) f\left(z', p_{o_j}, t\right) f(z', p_{o_k}, t) dz'$$

Fitting the Loss

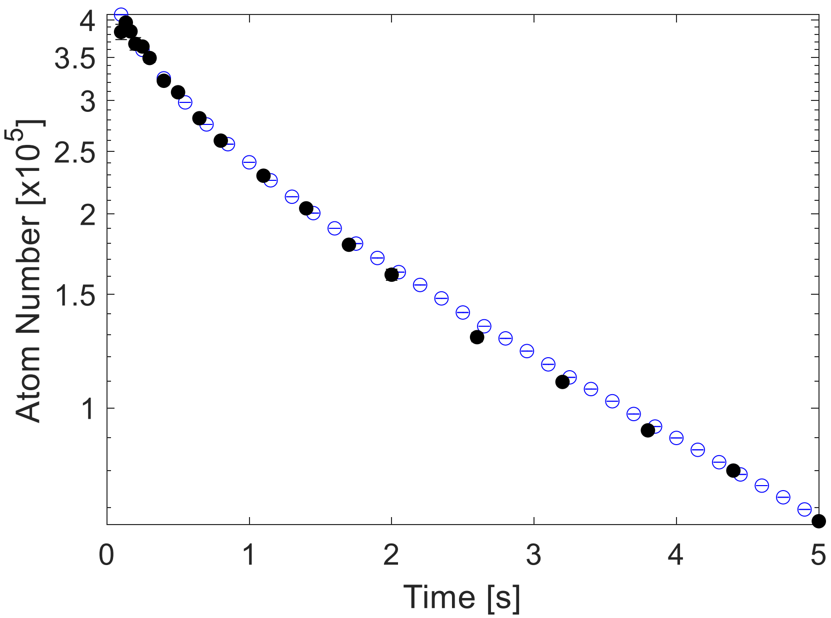
48 decay curves

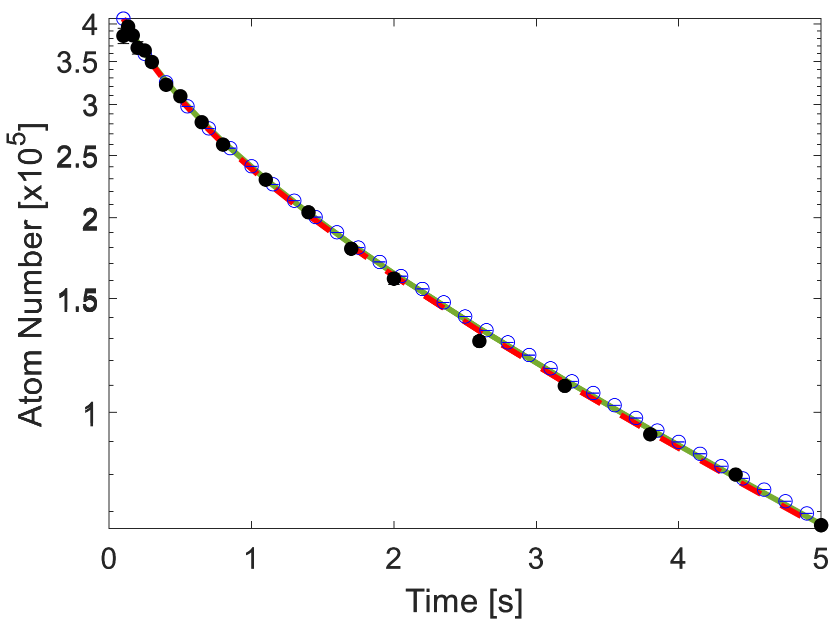
Free parameters:

- C'
- E_l

Do a global fit of all the data.

Eg.

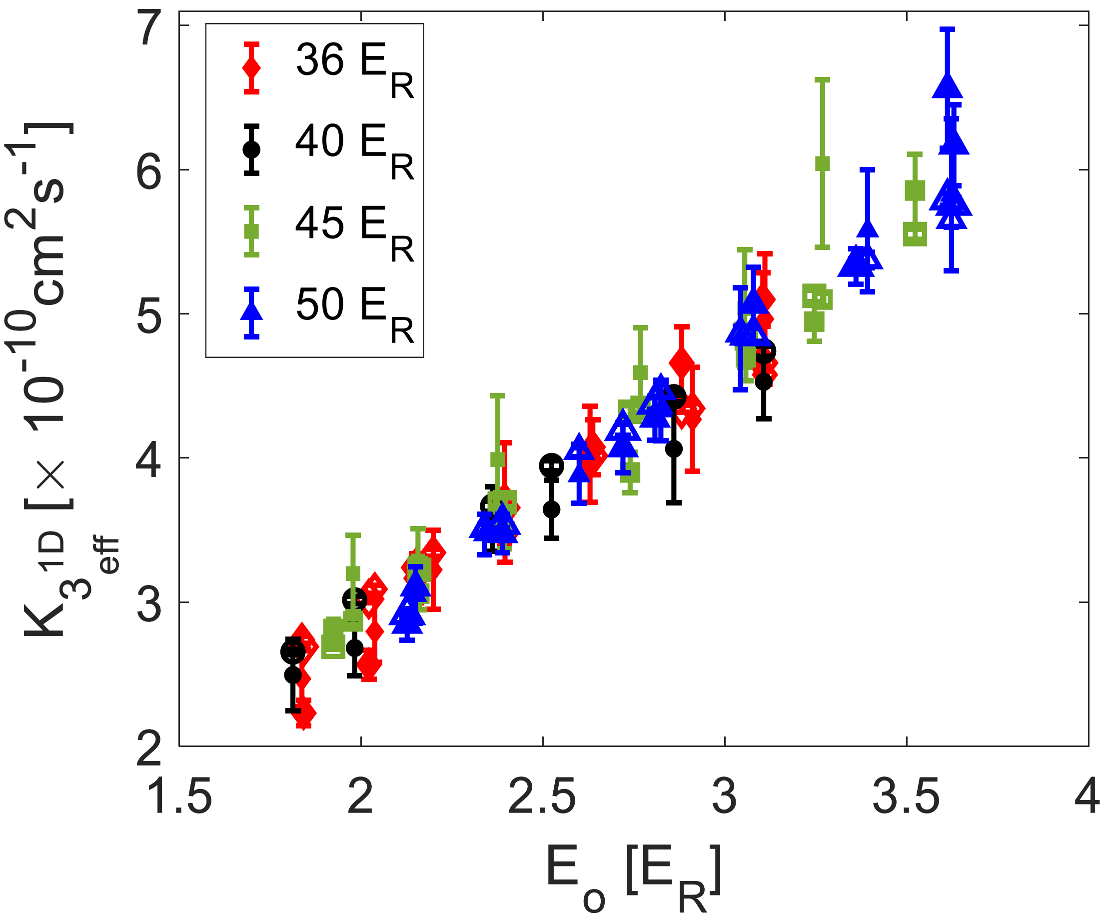




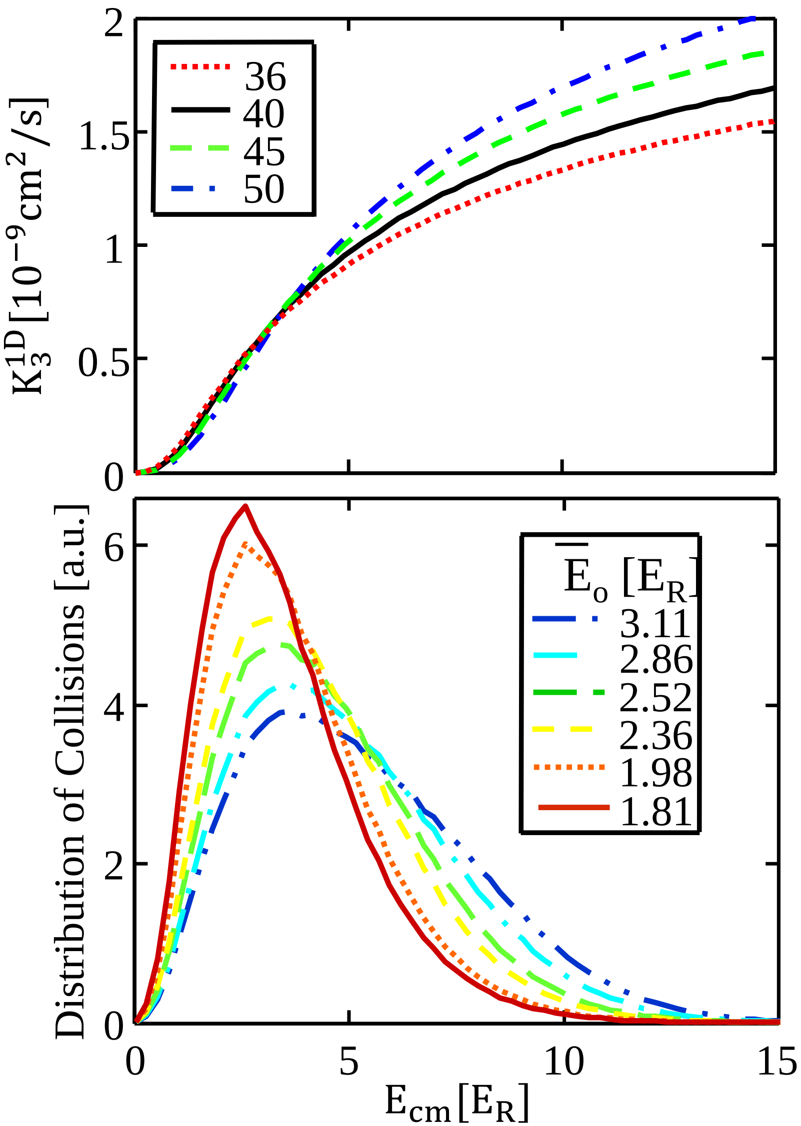
We the take each of the 48 global fit decay curves and do a least squares fit to find K_{3eff} for that curve (as we did for the experimental decay curves).

$$\frac{dN}{dt} = -K_1 N - K_{3eff}^{1D} \int n_{1D}^3 dz$$

All the data with all the theory



The results of the fit



K_3^{1D} rolls over to K_3^{3D} , which dramatically reduces the dependence of loss on lattice depth.

Much of the loss occurs in this rollover region.

$E_{cm} > 20E_R$ is needed for any transverse excitation.

The system is 1D until the inelastic process occurs.

$K_3^{1D}(E_{cm})$ or $g_3(0)$?

Isolated 3-body collisions

$K_3^{1D} \propto E_{cm}^3$

Correlated gas

$K_3^{1D} \propto g_3(0)\, K_3^{3D}$

They can't simultaneously apply.

Two views of the same underlying physics?

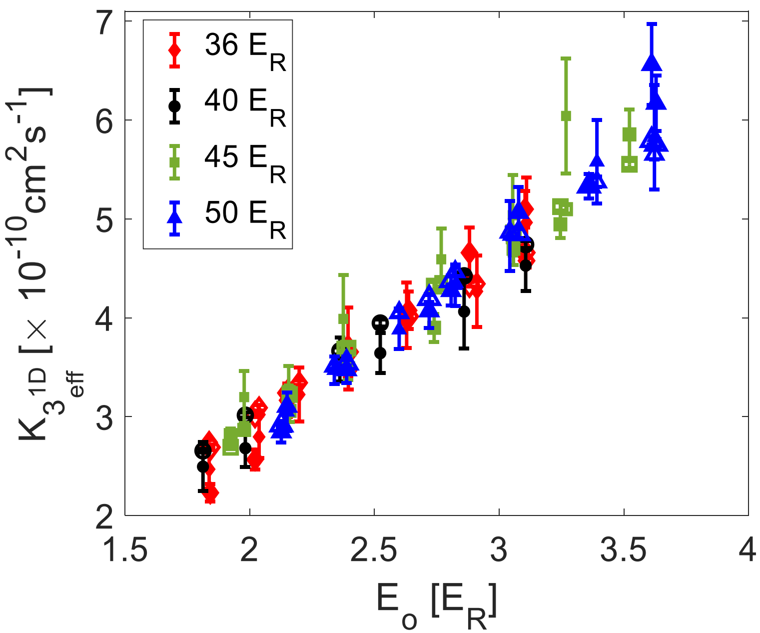
Summary

- 1D Bose gas experiments are close enough to integrable systems that it is possible to keep track of all three body collisions.

- We have seen the predicted strong dependence of 3-body recombination on collision energy.

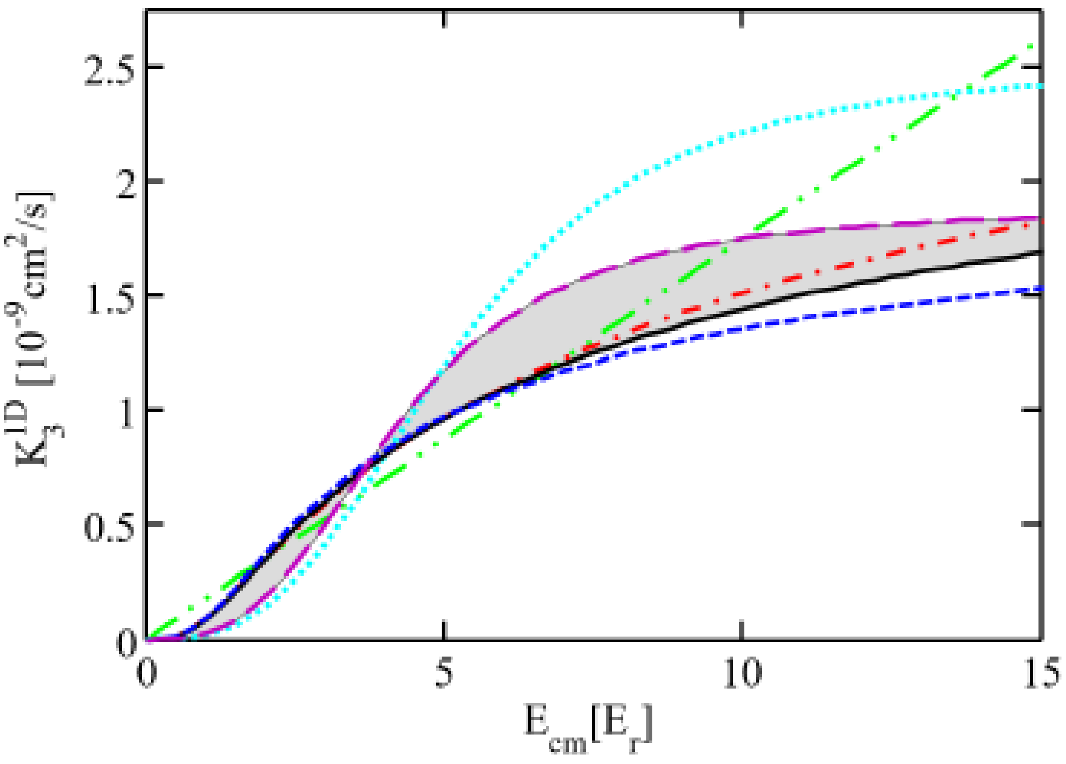
- The results fit a model with two free parameters.

- More theory is needed.



With regard to thermalization near integrability, we see the onset at a rate that suggests stat mech holds.

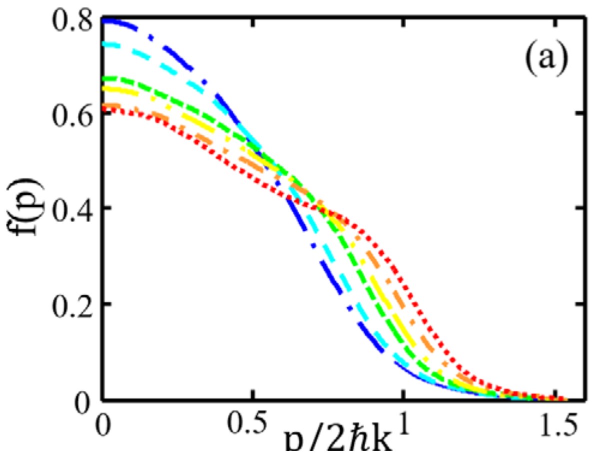
Robustness of the model



Alternative roll-off shapes are possible, but the fits are significantly worse when the best fit is ~outside of the shaded area. E_{cm}^3 is best, but the experiment is most sensitive to the roll-off region,

Complete space and momentum distributions

What we measure (momentum distributions)

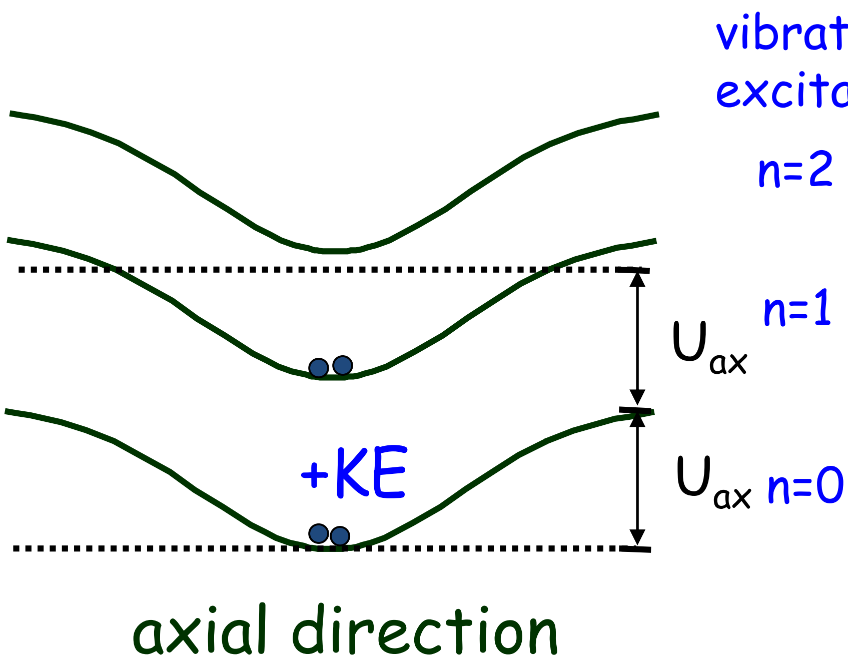


nferred
spatial
distributions

inferred peak momentum distributions

inferred peak amplitude distributions

Heating in a 2D optical lattice



Transverse excitations due to dipole fluctuations and spontaneous emission can be calculated a priori.

We treat the axial motion semi-classically, and Monte-Carlo simulate how all heating processes ultimately deposit their energy.

Benchmark the heating at low density.
Any additional evolution at high density is due to density-dependent processes.

C.

The Classical KAM theorem

Kolomogorov, Arnold and Moser (1954-1963)

If a non-integrable classical system is sufficiently close to integrable, it will not ergodically sample phase space.

Sufficiently large non-integrability → Chaos, ergodicity, microcanonical distributions

How can an isolated quantum system thermalize at all?

$$\Psi = \sum_{\substack{all \\ eigenstates}} a_i \phi_i e^{-i\omega_i t}$$

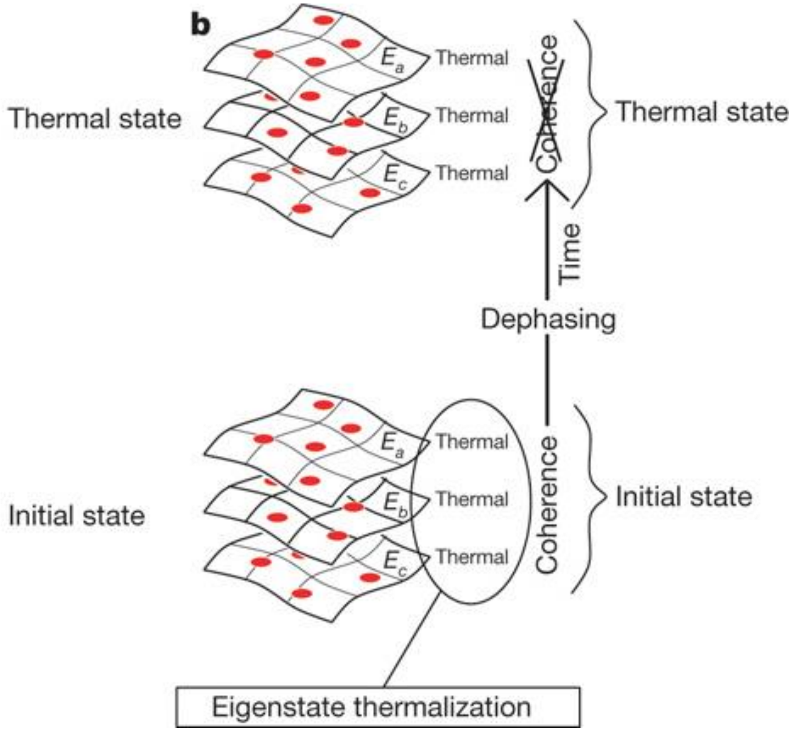
Eigenstate Thermalization Hypothesis

J. M. Deutsch, Phys. Rev. A 43, 2046 (1991);
M. Srednicki, Phys. Rev. E 50, 888 (1994).
M. Rigol, V. Dunjko & M. Olshanii, Nature **452**, 854 (2008).

After a long time, for a simple observable, O:

$$\langle O \rangle = \sum_{\substack{all \\ eigenstates}} a_i^2 \langle \phi_i | O | \phi_i \rangle$$

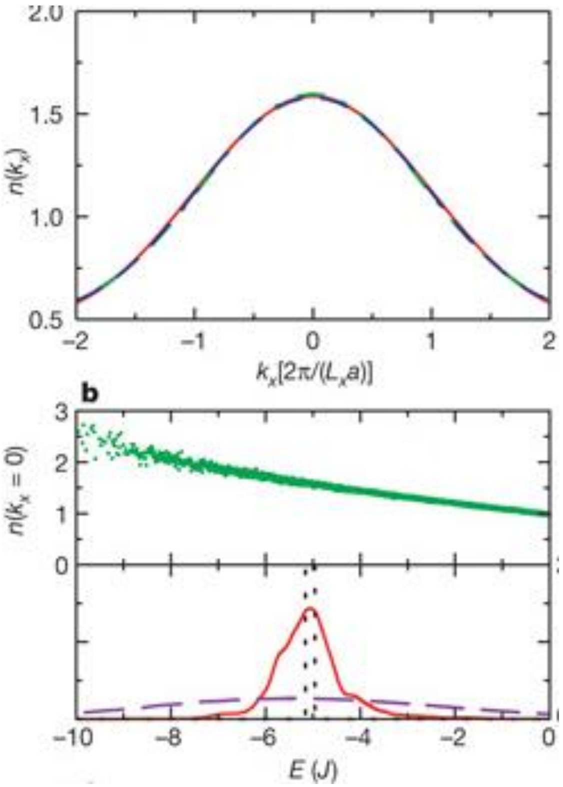
ETH: For states of approximately equal energy, the expectation values of all few body observables are about the same for all eigenstates.



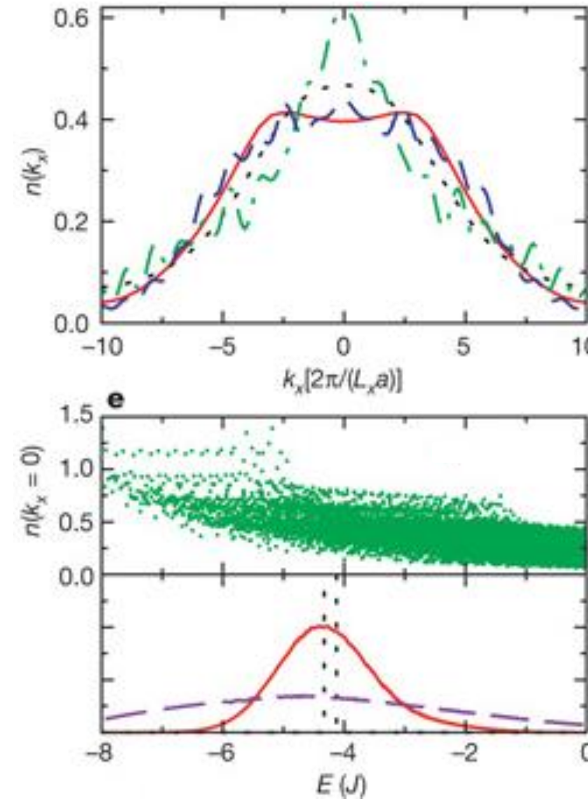
M. Rigol, V. Dunjko & M. Olshanii, Nature **452**, 854 (2008).

Simulations with 5 bosons, nearest neighbor hopping and interactions

(5×5 lattice)



(13 lattice site chain)



?

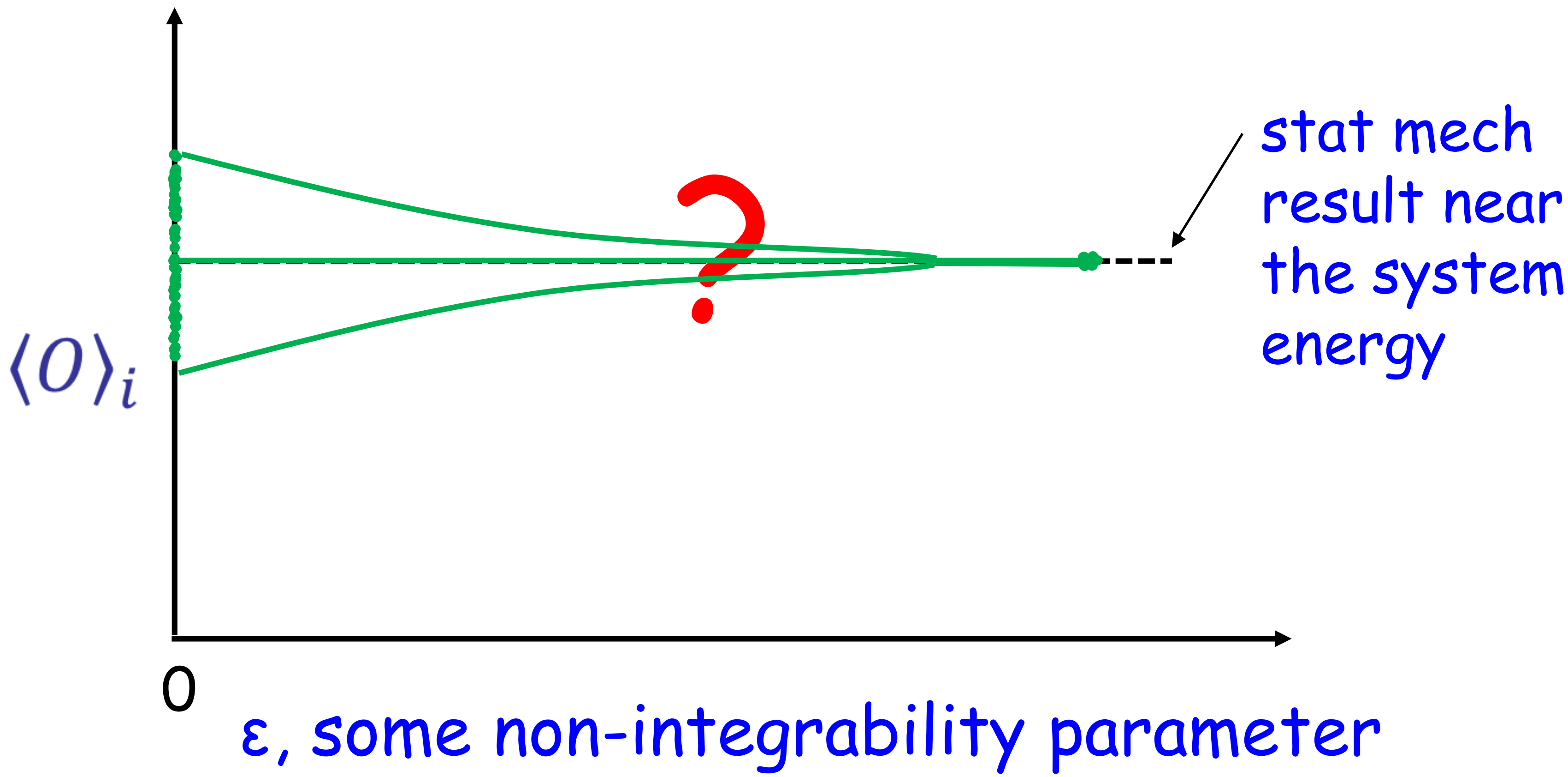
Non-integrable

Slightly non-integrable

Integrable

M. Rigol, PRL **103**, 100403 (2009).

Slightly non-integrable systems



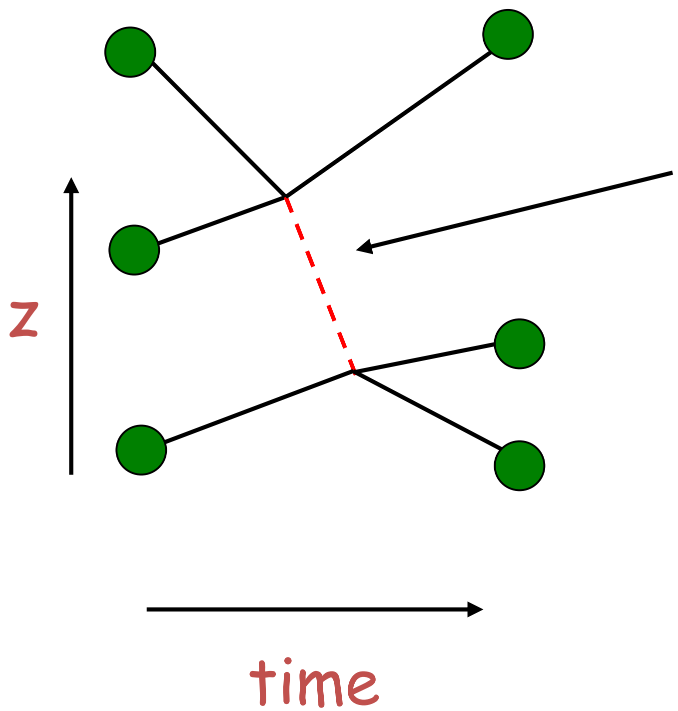
An ETH-based answer gives the answer at infinite time.

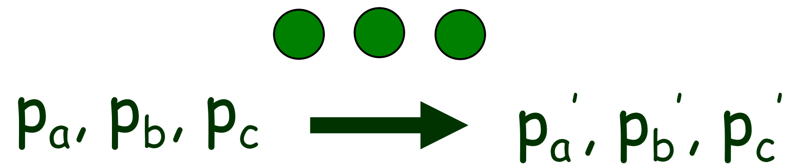
D.

Compromised δ-interactions

Mazets, Schumm, and Schmiedmayer, *PRL* **100**, 210403 (2008)

Tan, Pustilnik, and Glazman, *PRL* **105**, 090404 (2010)





$$\Gamma_{diff} = 6.88 \frac{\hbar}{m} \frac{a_{3D}^4}{a_\perp^2}$$

for a thermal gas

We want to measure the diffraction collision rate and compare it to this theoretical prediction.

"heating" from all mechanisms, 3-body loss, evaporative cooling

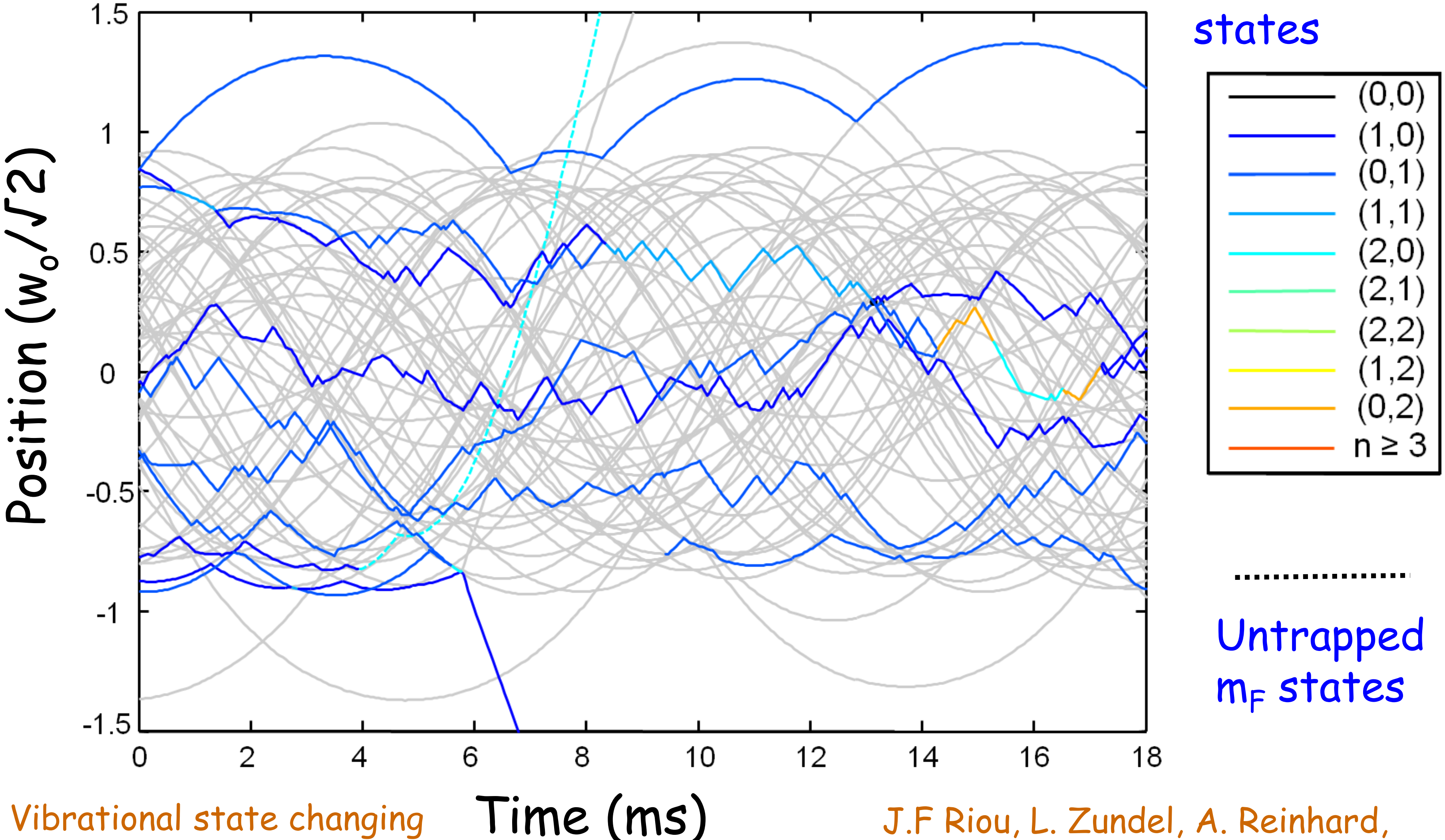
"Heating" Mechanisms

1. Lattice dipole fluctuations
2. Lattice jiggling — T. Savard, K. O'Hara, and J. Thomas, *PRA* **56**, R1095 (1997)
3. Spontaneous emission
   - axial
   - transverse — J.F Riou, A. Reinhard, L. Zundel, DSW, *PRA* **86**, 033412 (2012)
   - m_F-changing
   - spontaneous Raman scattering in optical fibers
4. Lattice intensity noise
5. Atom loss
6. Axial trap intensity noise
7. Axial trap position noise
   - Crossed dipole pointing
   - B-field gradient fluctuations

How much energy is deposited?

How is the energy deposited?

Monte Carlo



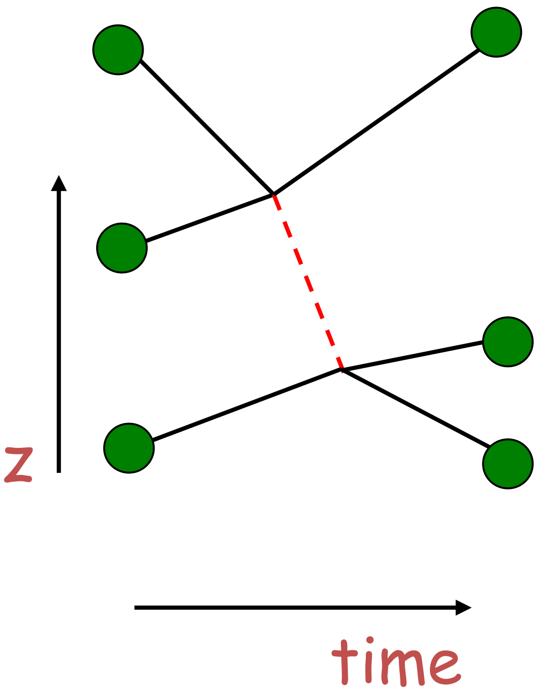
Vibrational states

Untrapped m_F states

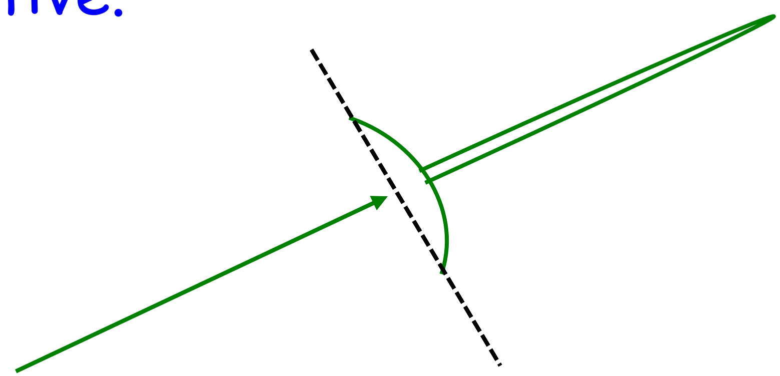
Vibrational state changing rates from Mike Moore

J.F Riou, L. Zundel, A. Reinhard, DSW, *PRA* **90**, 033401 (2014)

What might be wrong with the theory?



The theory ignores the integrable collisions, since they are non-diffractive.

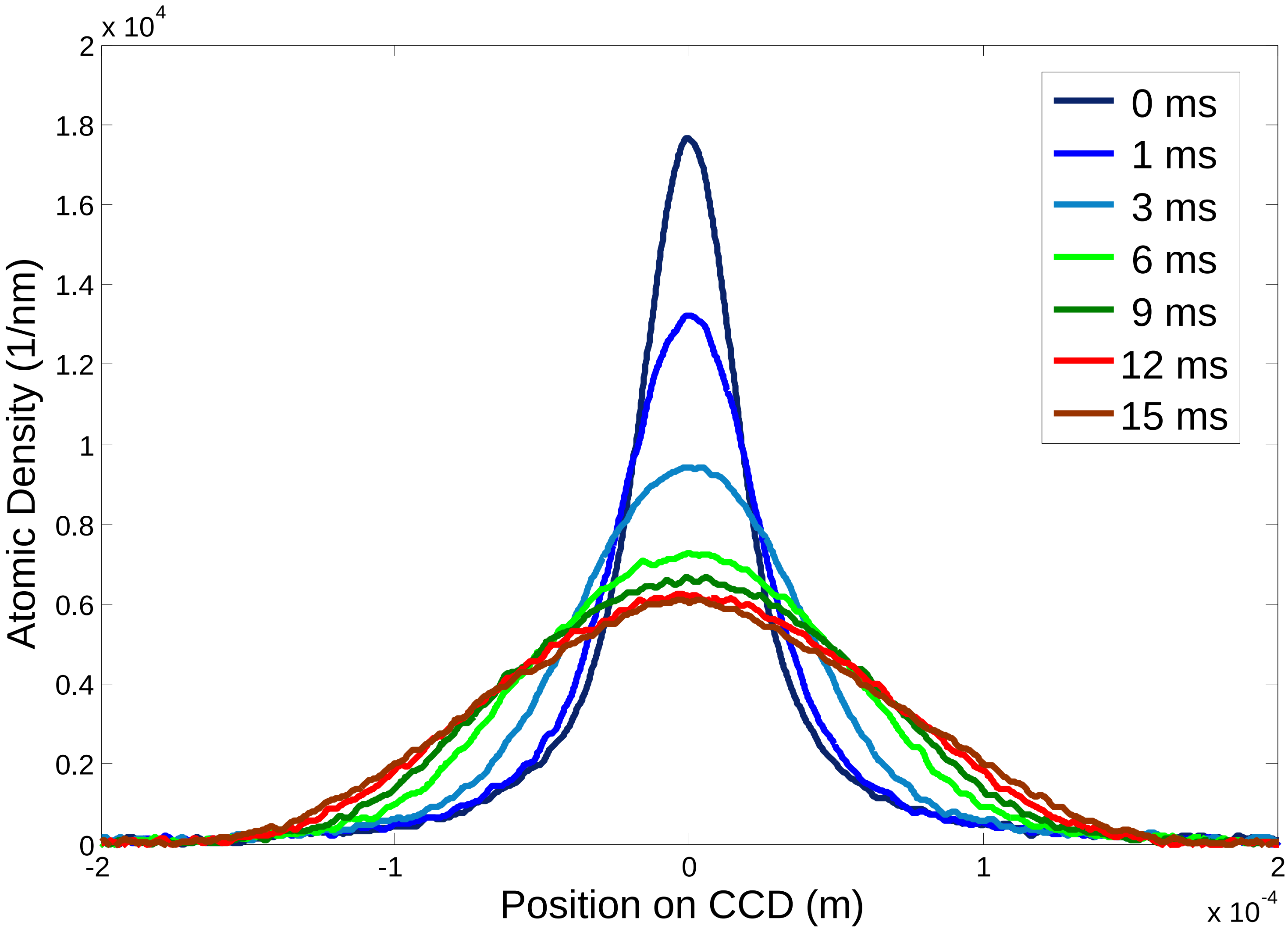


Perhaps they constructively interfere with the non-diffractive part of the diffractive output. By unitarity, the momentum-changing outputs would be suppressed.

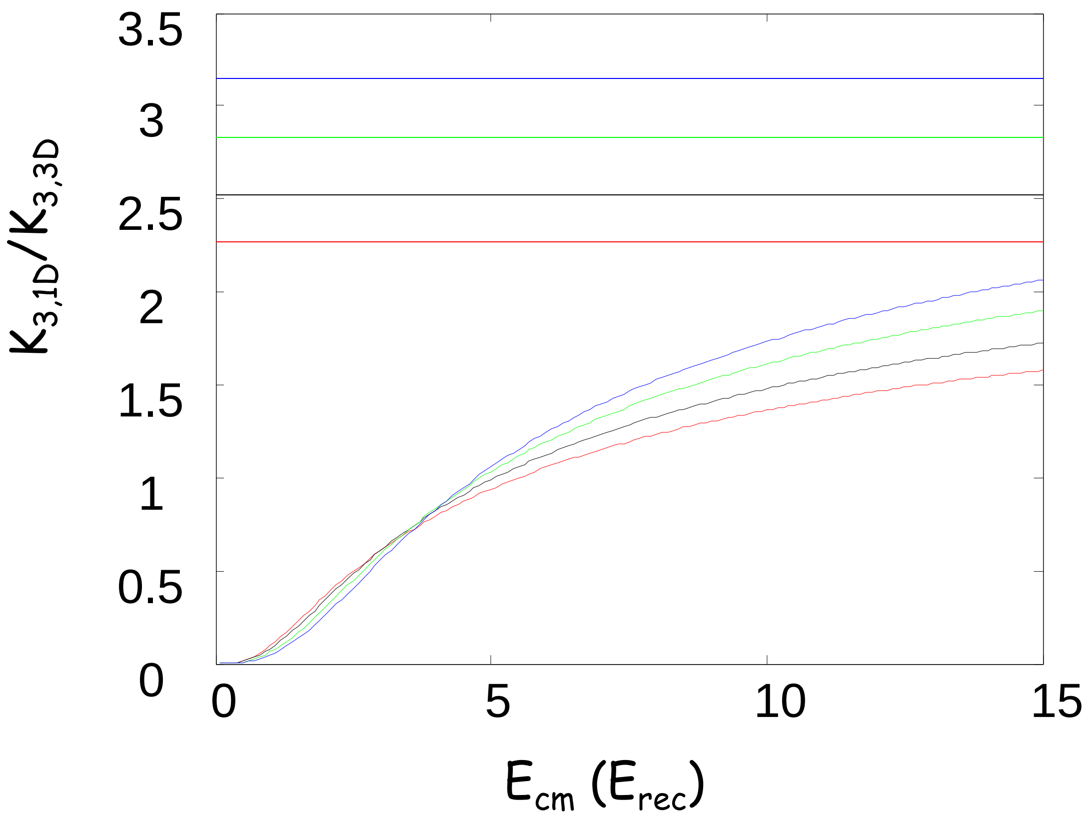
A violation of Fermi's golden rule?

Dynamical Fermionization (new)

with Marcos Rigol

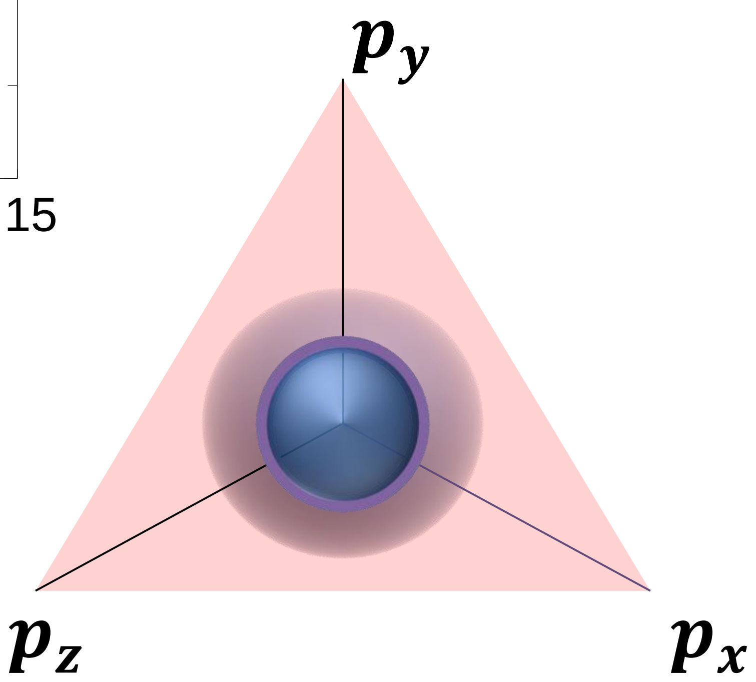


COM Energy-dependent collisions



Loss model: fit loss data with 2 free parameters

Evaporation model: Use the loss model to keep track of all predicted diffractive collisions, and hence evaporative cooling



The Lieb-Liniger limits

$$\gamma = \frac{m}{\hbar^2}\frac{g_{1D}}{n_{1D}}$$

$\gamma \gg 1$
Tonks-Girardeau gas

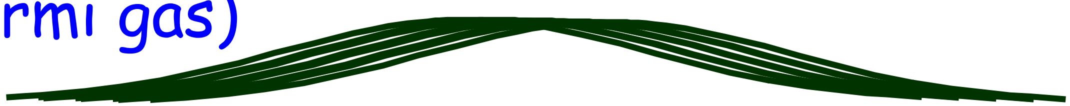
kinetic energy dominates

large g_{1D}
low density

$\gamma \ll 1$
mean field theory
(Thomas-Fermi gas)

mean field energy dominates

small g_{1D}
high density



1D energy parameterized by γ

Exact theory:

Lieb & Liniger, PR **130**, 1605 (1963); Dunjko, Lorent, Olshanii, PRL **86**, 5413 (2001)

Expt: Kinoshita, Wenger, DSW, *Science* **305**, 1125 (2004)

no free parameters