

# Conservation, Non-conservation Laws and Gravitational Energy momentum in Curved Spacetime

*Saturday, June 17, 2023 2:00 PM (30 minutes)*

The spacetime  $M$  in general relativity (GR) is curved. Parallel transportations of vectors in  $M$  depend on path. This leads to the fact that vectors distributed on different points in  $M$  can not be added up to get a sum vector unambiguously. Geometry does not allow talking about matter energy momentum distributed on (passing through) a finite or infinite spacelike (timelike) hypersurface, does not allow talking about the net increase of matter energy momentum in a finite or infinite 4-dimensional spacetime region. Even for a simple physical system consisting of only two uncharged mass points, geometry does not allow talking about its energy momentum. Therefore, it is pressing to explore the meaning of conservation of matter energy momentum in GR from a geometric perspective.

We show, in a curved spacetime  $M$ , when limited to an infinitesimal spacetime region, vectors distributed at different points can still be added up to get a sum vector unambiguously, if neglecting higher order infinitesimals. For an  $(r,s)$ -tensor  $Q$ , denoting by  $J$  its flux density  $(r+1,s)$ -tensor field, the conservation law of  $Q$  in curved spacetime  $M$  is “the covariant divergence of  $J$  vanishes everywhere”. It reads, “the net increase of tensor  $Q$  in any infinitesimal 4-dimensional neighborhood is zero”.

In particular, “the covariant divergence of  $T$  (flux density of matter energy momentum) vanishes everywhere” is the conservation law of matter energy momentum in GR.

Noether’s theorem for GR is re-visited. It is shown, all the Noether’s conserved currents corresponding to generating smooth vector fields on  $M$  are vector fields on  $M$ , hence they are conservation laws of scalars. It is also shown, all these Noether’s conservation laws together are equivalent to the equations of motion of GR.

**Author:** Prof. WU, Zhao Yan (Centre for Theoretical Physics, Jilin University, China)

**Presenter:** Prof. WU, Zhao Yan (Centre for Theoretical Physics, Jilin University, China)

**Session Classification:** Relativity, Gravitation and Cosmology

**Track Classification:** Relativity, Gravitation and Cosmology