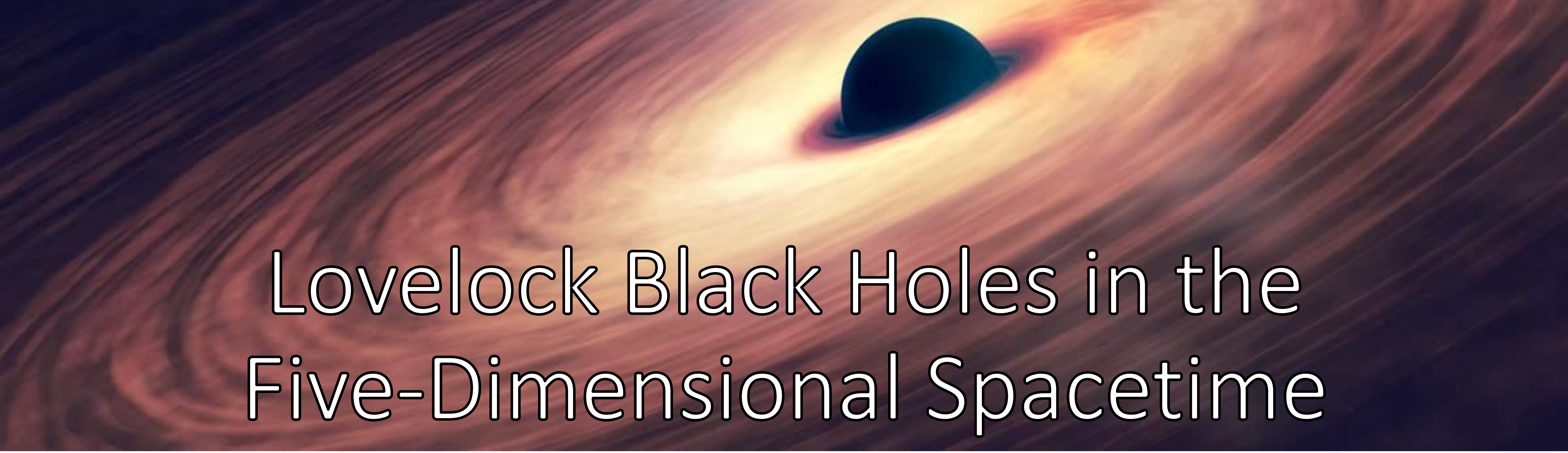
Lovelock Black Holes in the Five-Dimensional Spacetime



Source: Indiatimes

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CASST 2022

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15 August 2022, 11:24 AM (ET)

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Arthur B. McDonald
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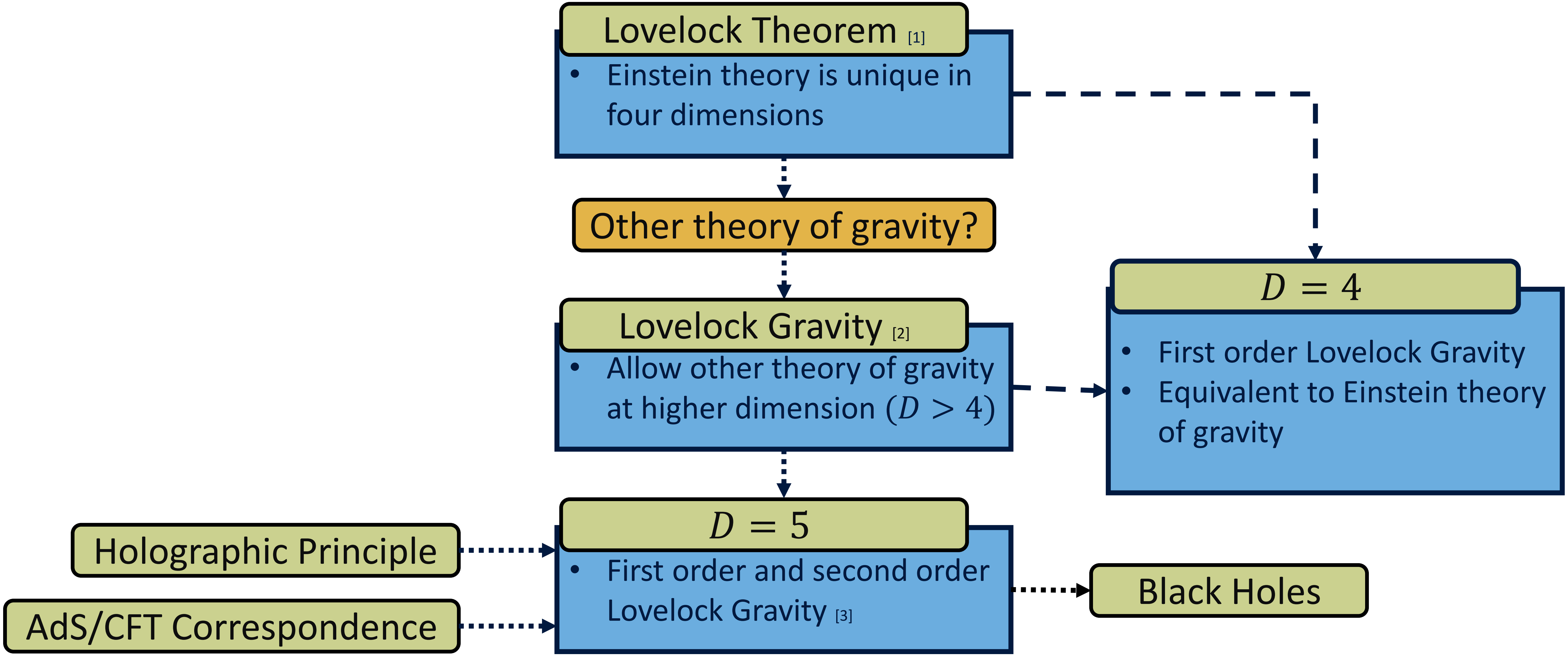
Outline

Motivation



[1] D. Lovelock, “The Four-Dimensionality of Space and the Einstein Tensor,” *J. Math. Phys.*, vol. 13, no. 6, pp. 874–876, Jun. 1972, doi: 10.1063/1.1666069.
[2] D. J. Lovelock, “The Einstein Tensor and Its Generalizations,” *J Math Phys*, vol. 12, pp. 498–501, 1971, doi: 10.1063/1.1665613.
[3] T. Takahashi, “Instability of charged Lovelock black holes: Vector perturbations and scalar perturbations,” *Prog. Theor. Exp. Phys.*, vol. 2013, no. 1, Jan. 2013, doi: 10.1093/ptep/pts049.

What we're solving

Spacetime [3]

- We assume → **Asymptotically Flat**
  - Which means:
    - $\lim_{r \to \infty} f(r) = 1$
    - $\Lambda = 0$

Black Hole [4]

- Described by:
  - Angular Momentum (J)
  - Charge (Q)
  - Mass (M)

Parameter	Static	Rotating
Neutral	$M \neq 0, Q = 0, J = 0$	$M \neq 0, Q = 0, J \neq 0$
Charged	$M \neq 0, Q \neq 0, J = 0$	$M \neq 0, Q \neq 0, J \neq 0$

We solve for neutral and charged static black hole

Goals

- **Reformulate metric solutions of static Lovelock black holes in the five-dimensional asymptotically flat spacetime and identifying (some of) its properties.**

Five-Dimensional Spacetime

Because we're dealing with static black holes

Spherically Symmetric Metric [4]

General D-Dimensional Metric [5]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_n$$

Five-Dimensional Metric

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\ d\theta^2 + r^2 \sin^2\theta\ d\phi^2 + r^2 \sin^2\theta \sin^2\phi\ d\lambda^2$$

$f(r)$

$$f(r) = 1 - r^2\psi(r)$$

Ansatz

[4] C. Bambi, Introduction to General Relativity: A Course for Undergraduate Students of Physics. Singapore: Springer Singapore, 2018. doi: 10.1007/978-981-13-1090-4.
[5] R.-G. Cai, "A note on thermodynamics of black holes in Lovelock gravity," Phys. Lett. B, vol. 582, no. 3–4, pp. 237–242, 2004.

Lovelock Theorem

Lovelock Theorem: *in* ***four spacetime dimensions****, the* ***unique*** *symmetric rank-2, divergence-free tensors* ***depending*** *only on the metric and its first and second derivatives are the* ***Einstein tensor*** *and* ***the metric tensor*** *itself.* [6]

$$E^{\mu\nu} = aG^{\mu\nu} + bg^{\mu\nu}$$

$a = 1$ $b = \Lambda$

$$E^{\mu\nu} \equiv G^{\mu\nu} + \Lambda g^{\mu\nu}$$

Unique in four dimension

[6] G. Papallo, "Causality and the initial value problem in Modified Gravitys," PhD Thesis, Cambridge University, Cambridge, UK, 2020. doi: 10.17863/CAM.24726.

Lovelock Theory of Gravity

Lovelock Theory of Gravity: Generalization of Lovelock Theorem in D-dimension [6]

Lagrangian of Lovelock Theory [7]

$$\hat{\mathcal{L}}_G = -2\Lambda + \sum_{m=1}^{\bar{m}} \frac{1}{2^m}\frac{\alpha_m}{m}\delta^{\mu_1 v_1 \mu_2 v_2 \ldots \mu_m v_m}_{\lambda_1 \sigma_1 \lambda_2 \sigma_2 \ldots \lambda_m \sigma_m} \times R_{\mu_1 \nu_1}{}^{\lambda_1 \sigma_1} R_{\mu_2 \nu_2}{}^{\lambda_2 \sigma_2} \ldots R_{\mu_m \nu_m}{}^{\lambda_m \sigma_m}$$

- $\bar{m}$ represent order of curvature correction

First Order Lagrangian

$$\hat{\mathcal{L}}_G = -2\Lambda + \alpha_1 R$$

→ Einstein-Hilbert Lagrangian!

Second Order Lagrangian

$$\hat{\mathcal{L}}_G = -2\Lambda + \alpha_1 R + \frac{\alpha_2}{2}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}\right)$$

- $-2\Lambda + \alpha_1 R$: Einstein term
- $\frac{\alpha_2}{2}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}\right)$: Gauss-Bonnet term → Non-0 at $D \geq 5$
- Einstein term + Gauss-Bonnet term: Einstein-Gauss-Bonnet Lagrangian

[7] R. A. Konoplya and A. Zhidenko, "Massive particles in the Einstein-Lovelock-anti-de Sitter black hole spacetime," *Class. Quantum Gravity*, vol. 38, no. 4, p. 045015, Feb. 2021, doi: 10.1088/1361-6382/abd302.

Lovelock Tensor ($E_{\mu\nu}$)

We obtain the definition of Lovelock tensor by applying least action principle to the action formed with the first and second order Lagrangian

First Order Lovelock Tensor

$$E_{\mu\nu} \equiv \alpha_1 \left[R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right] + \Lambda g_{\mu\nu}$$

Second Order Lovelock Tensor

$$E_{\mu\nu} \equiv \Lambda g_{\mu\nu} - \alpha_1 \left[\frac{1}{2} g_{\mu\nu} R - R_{\mu\nu} \right] + \frac{\alpha_2}{2} \left[\left(2RR_{\mu\nu} + 2R_\mu^{\rho\sigma\lambda} R_{\nu\rho\sigma\lambda} - 4R_{\mu\lambda} R_\nu^\lambda - 4R^{\rho\sigma} R_{\mu\rho\nu\sigma} \right) - \frac{g_{\mu\nu}}{2} \left(R^2 - 4R_{\alpha\beta} R^{\alpha\beta} + R_{\alpha\beta\rho\sigma} R^{\alpha\beta\rho\sigma} \right) \right]$$

$$E_{\mu\nu} = 0 \quad \rightarrow \quad \text{vacuum (neutral)}$$

$$E_{\mu\nu} = T_{\mu\nu} \quad \rightarrow \quad \text{non-vacuum (charged)}$$

Mixed Form → $E_\mu^\nu = 0$, $E_\mu^\nu = T_\mu^\nu$

Stress-Energy Tensor

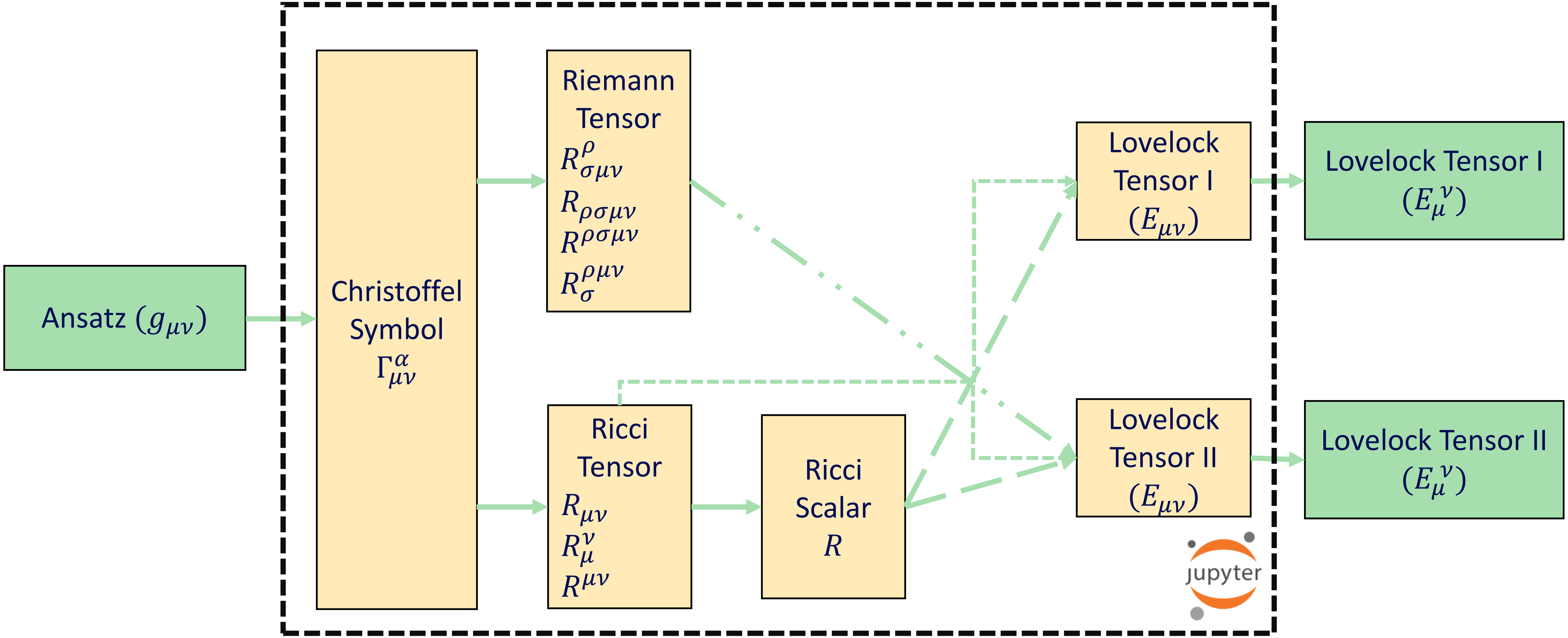
$$T_{\mu\nu} = -F_\mu^\alpha F_{\alpha\nu} + \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

Non-0 Component of $F_{\mu\nu}$ [8]

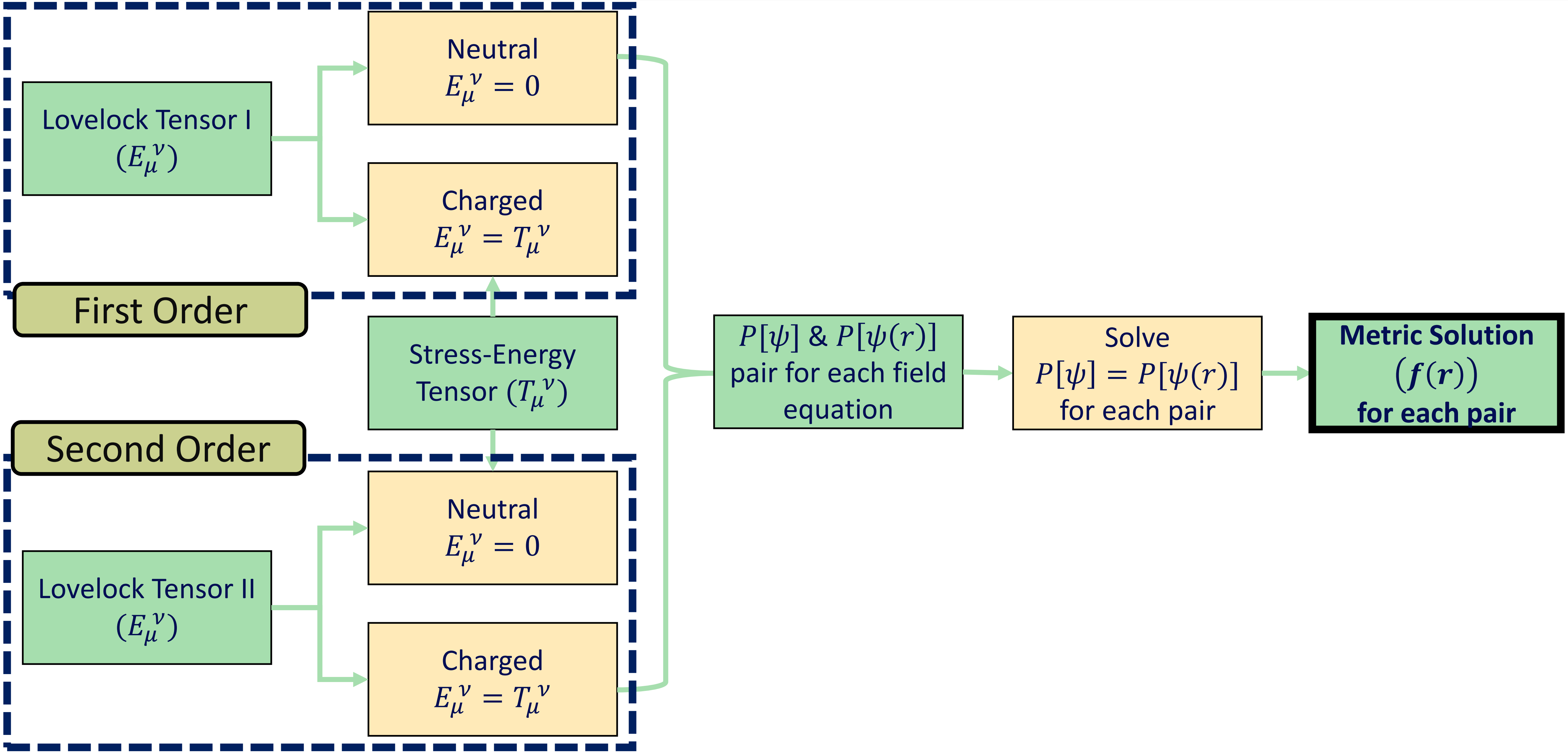
$$F_{tr} = -F_{rt} = -\sqrt{3} \frac{Q}{r^3}$$

[8] R. A. Konoplya and A. Zhidenko, "Black holes in the four-dimensional Einstein-Lovelock gravity," *Phys. Rev. D*, vol. 101, no. 8, p. 084038, Apr. 2020, doi: 10.1103/PhysRevD.101.084038.

Calculation Scheme



Calculation Scheme



$\boldsymbol{P[\psi]}$ & $\boldsymbol{P[\psi(r)]}$ Pair

First order Black Hole

Neutral

$$P[\psi] = \psi, \;\; P[\psi(r)] = \frac{2\mathcal{M}}{r^4}$$

Charged

$$P[\psi] = \psi, \;\; P[\psi(r)] = \frac{2\mathcal{M}}{r^4} - \frac{\mathcal{Q}}{r^6}$$

Solve for $\psi(r)$ by
$P[\psi] = P[\psi(r)]$

Second Order Black Hole

Neutral

$$P[\psi] = \psi + \tilde{\alpha}_2\psi^2, \;\; P[\psi(r)] = \frac{2\mathcal{M}}{r^4}$$

Charged

$$P[\psi] = \psi + \tilde{\alpha}_2\psi^2, \;\; P[\psi(r)] = \frac{2\mathcal{M}}{r^4} - \frac{\mathcal{Q}}{r^6}$$

Metric Solution (Neutral First Order Black Hole)[9]

Metric Function

$$f(r) = 1 - \left(\frac{2\mathcal{M}}{r^2}\right)$$

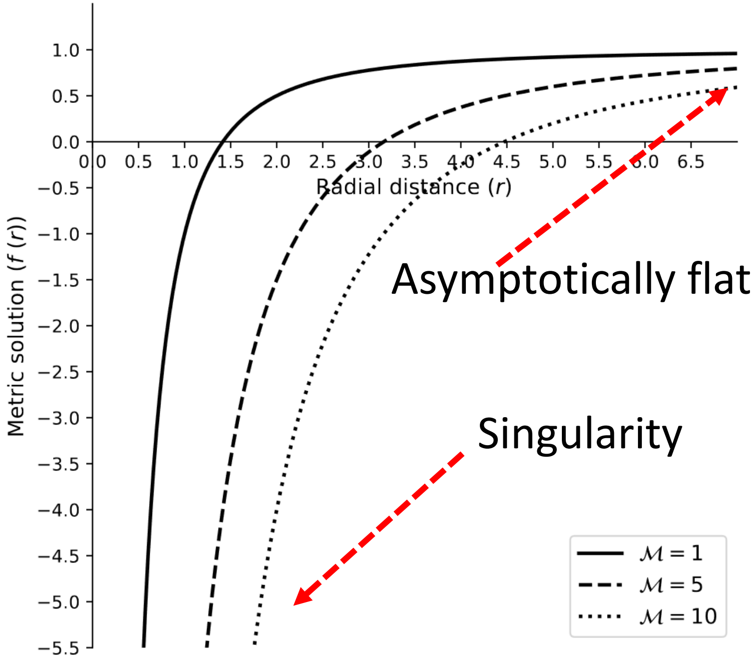
→ **Horizon Radius (r_h)** → $f(r_h) = 0$ →

Horizon Radius

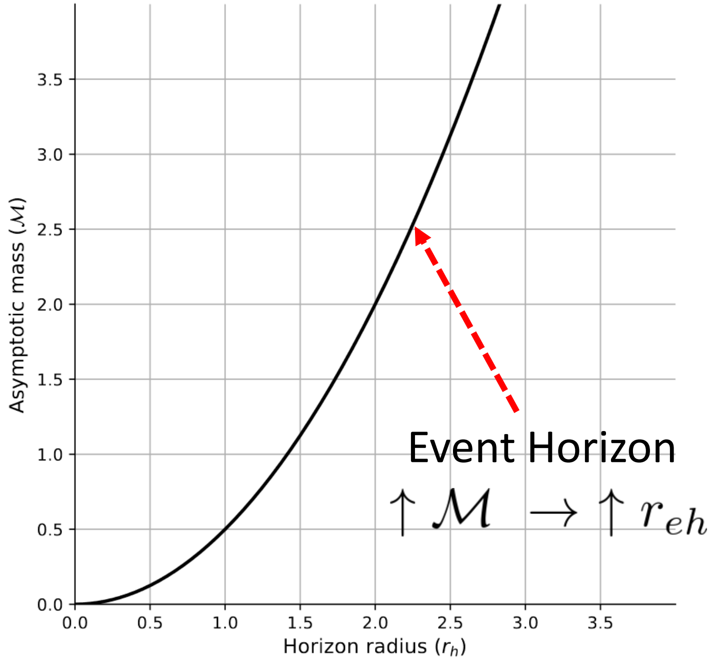
$$r_{eh} = \sqrt{2\mathcal{M}}$$

$\mathcal{M}(r_h)$

$$\mathcal{M} = \frac{r_h^2}{2}$$



Mass Variation



[9] F. R. Tangherlini, "Schwarzschild field in n dimensions and the dimensionality of space problem," *Nuovo Cim*, vol. 27, p. 636, 1963, doi: 10.1007/BF02784569.

Metric Solution (Charged First Order Black Hole)[9]

Metric Function

$$f(r) = 1 - \left(\frac{2\mathcal{M}}{r^2} - \frac{Q^2}{r^4}\right)$$

$\mathcal{M}(r_h)$

$$\mathcal{M} = \frac{1}{2}\left(r_h^2 + \frac{Q^2}{r_h^2}\right)$$

$$\mathcal{M}'(r_{ex}) = 0$$

Horizon Radius

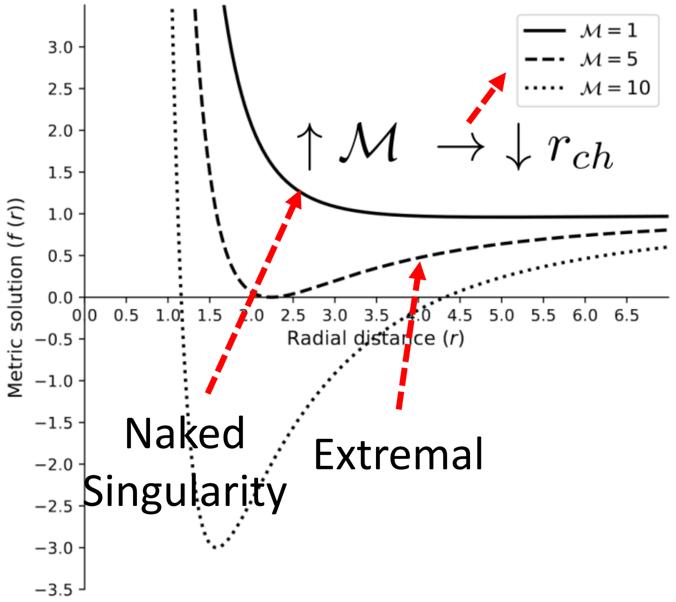
$$r_{eh} = \sqrt{-\mathcal{M} + \sqrt{\mathcal{M}^2 - Q^2}}$$

$$r_{ch} = \sqrt{-\mathcal{M} - \sqrt{\mathcal{M}^2 - Q^2}}$$

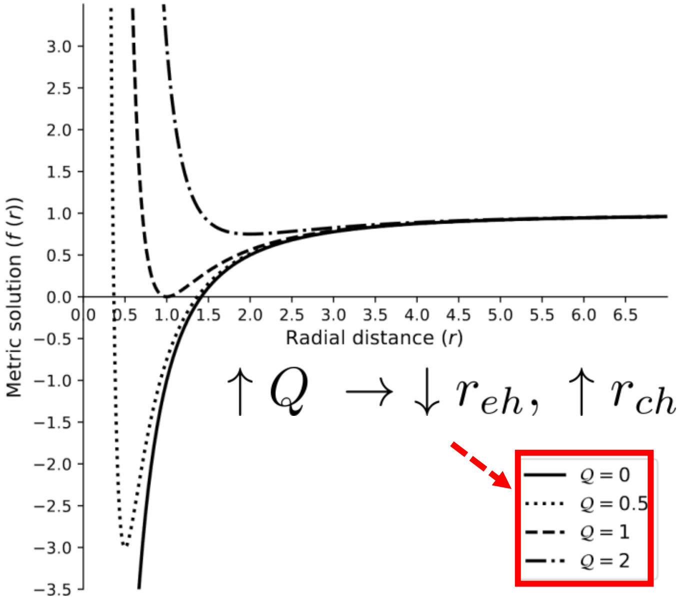
Extremal mass ($\mathcal{M}_{ex}$)

$\mathcal{M}_{ex}$

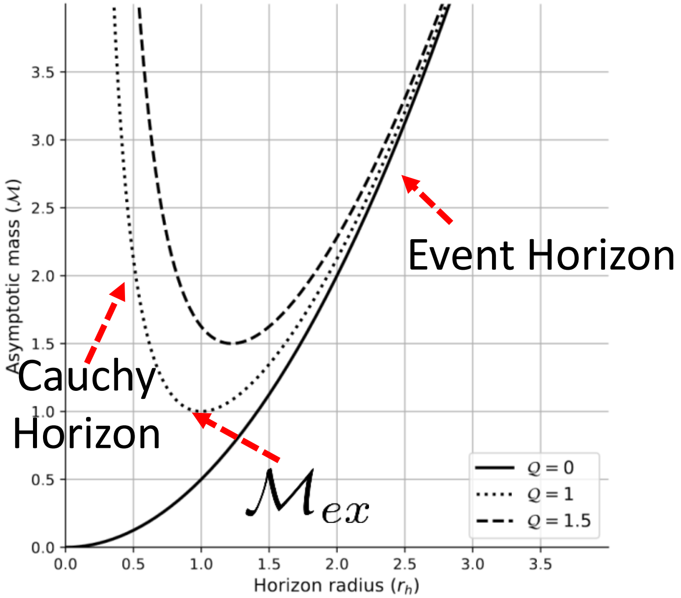
$$\mathcal{M}_{ex} = \frac{1}{2}\left(1 + Q^2\right)$$



Mass Variation



Charge Variation



Charge Variation

Metric Solution (Neutral Second Order Black Hole) [10]

Metric Function

$$f(r) = 1 - r^2\left(\frac{-1\pm\sqrt{1+4\tilde{\alpha}_2\left(\frac{2\mathcal{M}}{r^4}\right)}}{2\tilde{\alpha}_2}\right)$$

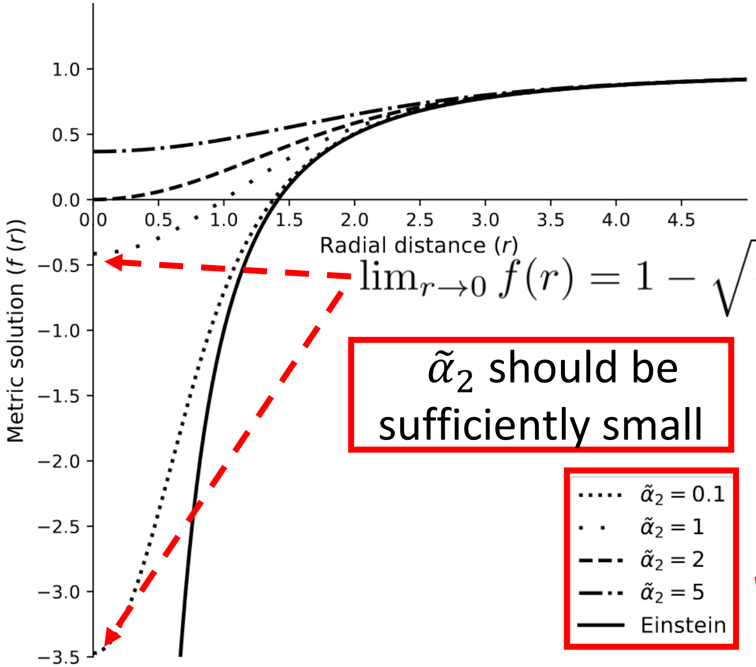
Negative branch is not asymptotically flat

$\mathcal{M}(r_h)$

$$\mathcal{M} = \frac{r_h^2+\tilde{\alpha}_2}{2}$$

Horizon Radius

$$r_{eh} = \sqrt{2\mathcal{M}-\tilde{\alpha}_2}$$

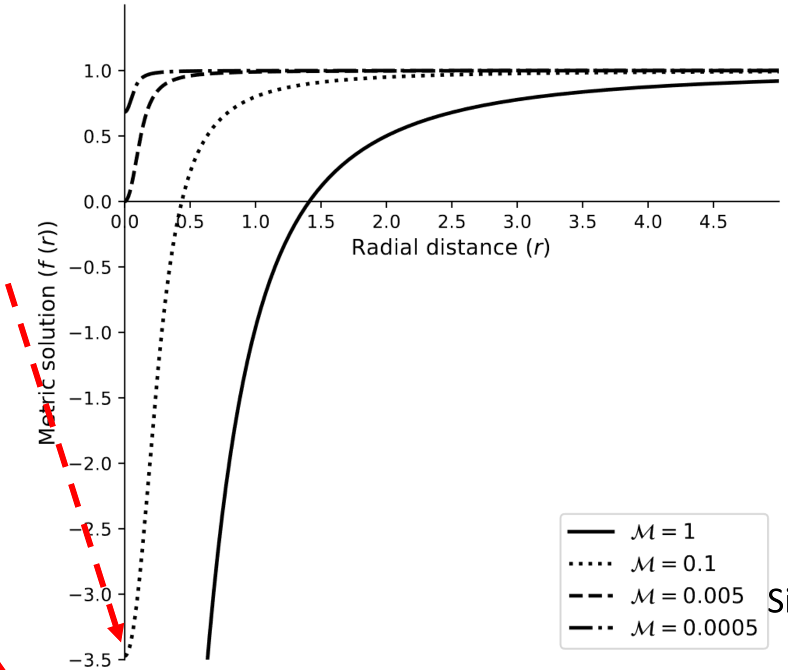


$$\lim_{r\to 0} f(r) = 1 - \sqrt{\frac{2\mathcal{M}}{\tilde{\alpha}_2}}$$

$\tilde{\alpha}_2$ should be sufficiently small

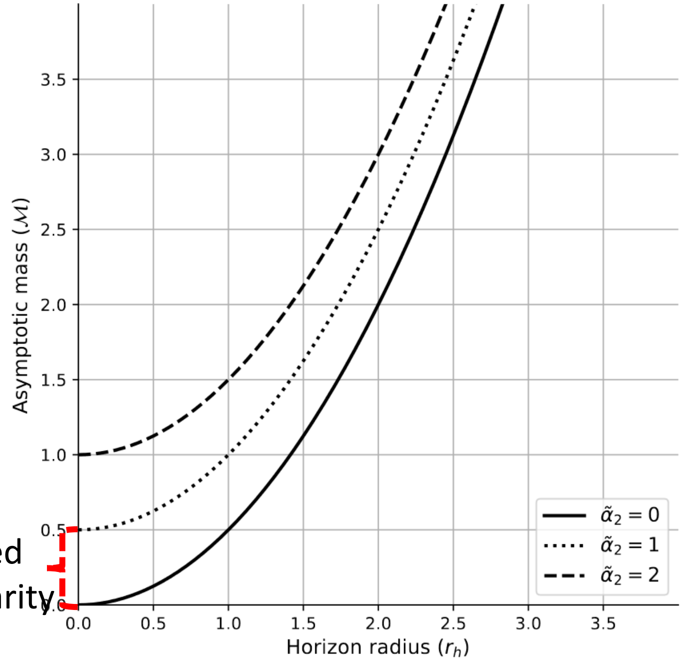
Coupling Constant Variation

$\uparrow \tilde{\alpha}_2 \to \downarrow r_{eh}$



Mass Variation

Naked Singularity



Coupling Constant Variation

[10] D. G. Boulware and S. Deser, "String-Generated Gravity Models," *Phys. Rev. Lett.*, vol. 55, no. 24, pp. 2656–2660, Dec. 1985, doi: 10.1103/PhysRevLett.55.2656.

Metric Solution (Charged Second Order Black Hole)[11]

Negative branch is not asymptotically flat

Metric Function

$$f\left(r\right)=1-r^2\left(\frac{-1\pm\sqrt{1+4\tilde{\alpha}_2\left(\frac{2\mathcal{M}}{r^4}-\frac{Q^2}{r^6}\right)}}{2\tilde{\alpha}_2}\right)$$

$\mathcal{M}(r_h)$

$$\mathcal{M}=\frac{1}{2}\left(r_h^2+\frac{Q^2}{r_h^2}+\tilde{\alpha}_2\right)$$

$\mathcal{M}_{ex}$

$$\mathcal{M}_{ex}=\frac{1}{2}\left(r_h^2+\frac{Q^2}{r_h^2}+\tilde{\alpha}_2\right)$$

Horizon Radius

$$r_{eh}=\sqrt{\frac{(2\mathcal{M}-\tilde{\alpha}_2)+\sqrt{(\tilde{\alpha}_2-2\mathcal{M})^2-4Q^2}}{2}}$$

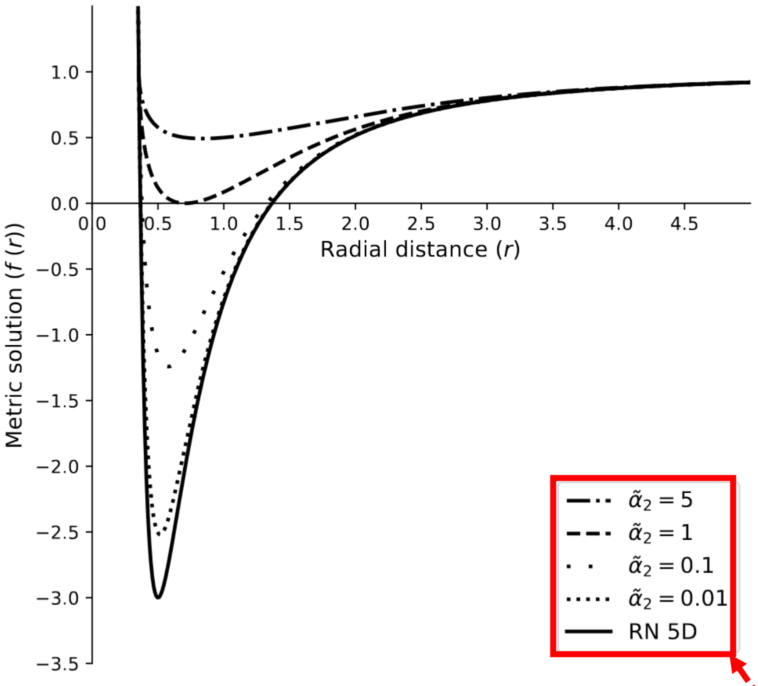
$$r_{ch}=\sqrt{\frac{(2\mathcal{M}-\tilde{\alpha}_2)-\sqrt{(\tilde{\alpha}_2-2\mathcal{M})^2-4Q^2}}{2}}$$

[11] R. Zegers, "Birkhoff's theorem in Lovelock gravity," *J. Math. Phys.*, vol. 46, no. 7, p. 072502, Jul. 2005, doi: 10.1063/1.1960798.

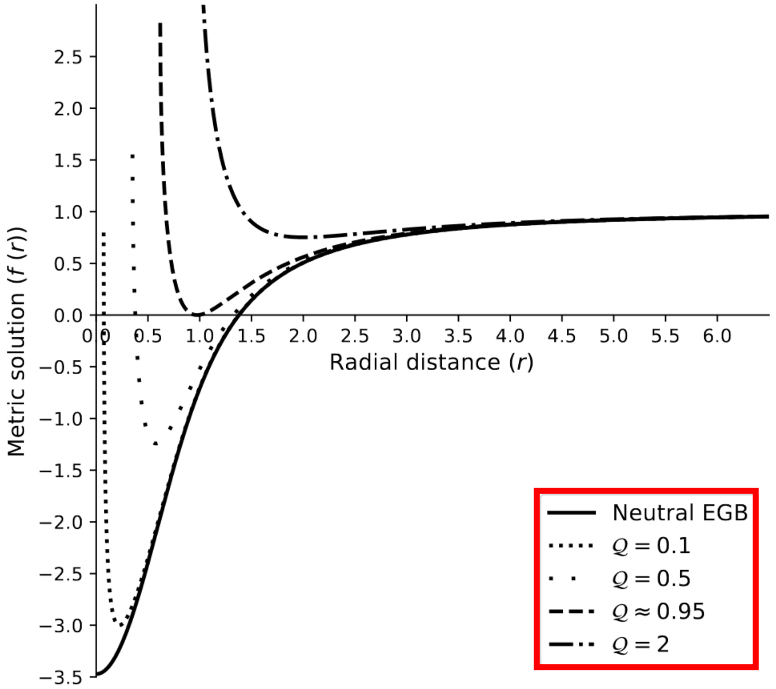
Metric Solution (Charged Second Order Black Hole)[11]

When $\tilde{\alpha}_2 \rightarrow 0$ solution returns to charged first order solution

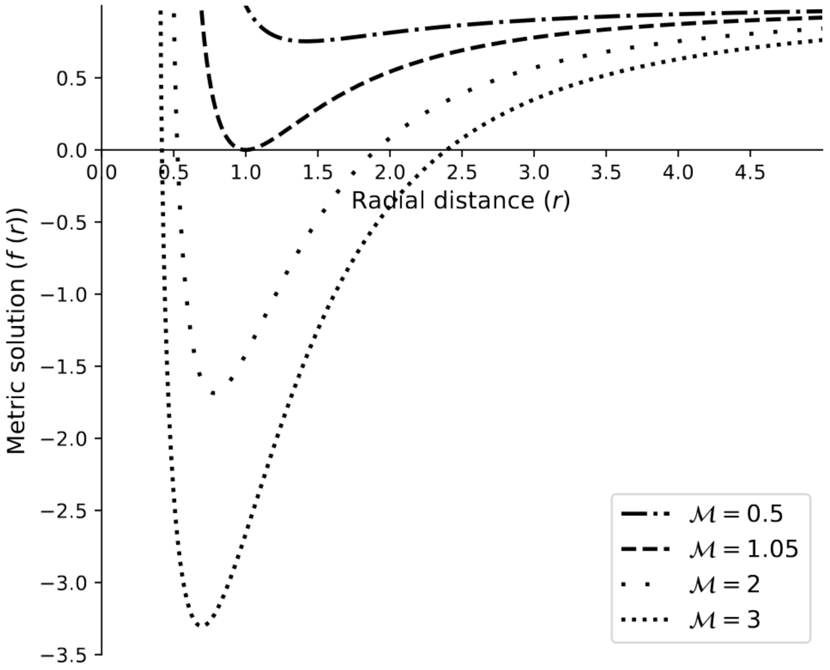
When $\mathrm{Q} \rightarrow 0$ solution returns to neutral second order solution



Coupling Constant Variation



Charge Variation

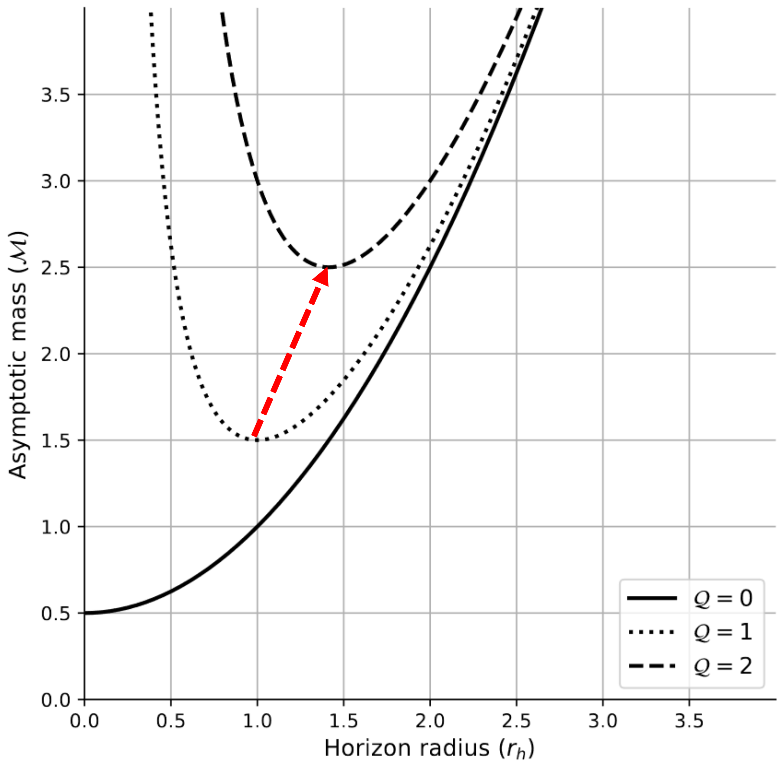


Mass Variation

$\uparrow \tilde{\alpha}_2 \rightarrow \uparrow r_{ch}$

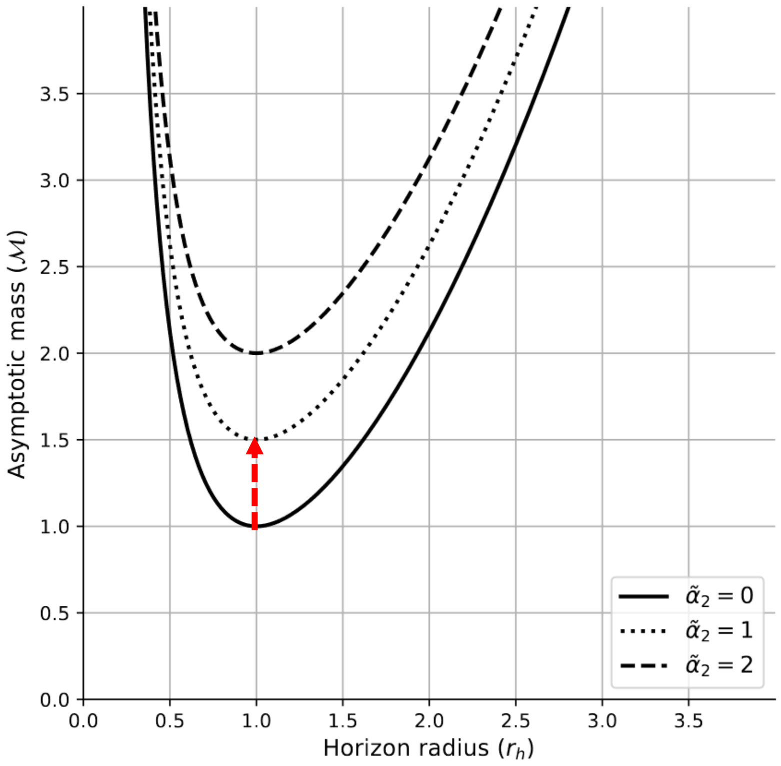
Metric Solution (Charged Second Order Black Hole)[11]

Bigger charge -> bigger extremal mass, bigger extremal radius



Charge Variation

Bigger coupling constant -> bigger extremal mass, same extremal radius



Coupling Constant Variation

Conclusion

1

- Four Metric Solutions of Lovelock black holes in five-dimensional spacetime were reformulated

2

- Neutral black holes only has an event horizon, charged black holes has an event horizon and a Cauchy horizon

3

- Event horizon radius is proportional to black hole Mass, but is inversely proportional to Black hole charge and gravity coupling constant $\widetilde{\alpha}_2$
- Cauchy horizon radius is proportional to black hole Charge and gravity coupling constant but inversely proportional to black hole mass

4

- To maintain consistencies with general relativity $\widetilde{\alpha}_2$ should be sufficiently small

Future Works

1

- **Works on non-static objects** [12]

2

- **Works on the stability of black hole solutions found in this study** [3]

3

- **Works on orbit stability of particles around black hole solutions found in this study** [7]

[12] B. Kleihaus, J. Kunz, E. Radu, and B. Subagyo, “Spinning black strings in five-dimensional Einstein–Gauss–Bonnet gravity,” *Phys. Lett. B*, vol. 713, no. 2, pp. 110–116, 2012.

Thank You!

SNOLAB

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APPENDIX

Stress-Energy Tensor Components ($T_{\mu}{}^{\nu}$)

$$T_{\mu}{}^{\nu} = \begin{bmatrix} \frac{1.5Q^2}{r^6} & 0 & 0 & 0 & 0 \\ 0 & \frac{1.5Q^2}{r^6} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1.5Q^2}{r^6} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1.5Q^2}{r^6} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1.5Q^2}{r^6} \end{bmatrix}$$

Tensor Components ($E_\mu^{\ \nu}$) with constant ψ

$$E_\mu^\nu = \begin{bmatrix} \tilde{\alpha}_1\psi & 0 & 0 & 0 & 0 \\ 0 & \tilde{\alpha}_1\psi & 0 & 0 & 0 \\ 0 & 0 & \tilde{\alpha}_1\psi & 0 & 0 \\ 0 & 0 & 0 & \tilde{\alpha}_1\psi & 0 \\ 0 & 0 & 0 & 0 & \tilde{\alpha}_1\psi \end{bmatrix}$$

$$\tilde{\alpha}_1 = \frac{1}{8\pi G} \equiv 1 \longrightarrow P[\psi] \equiv \psi$$

$$E_\mu^\nu = \begin{bmatrix} (\tilde{\alpha}_1\psi + \tilde{\alpha}_2\psi^2) & 0 & 0 & 0 & 0 \\ 0 & (\tilde{\alpha}_1\psi + \tilde{\alpha}_2\psi^2) & 0 & 0 & 0 \\ 0 & 0 & (\tilde{\alpha}_1\psi + \tilde{\alpha}_2\psi^2) & 0 & 0 \\ 0 & 0 & 0 & (\tilde{\alpha}_1\psi + \tilde{\alpha}_2\psi^2) & 0 \\ 0 & 0 & 0 & 0 & (\tilde{\alpha}_1\psi + \tilde{\alpha}_2\psi^2) \end{bmatrix}$$

$$P[\psi] = \tilde{\alpha}_1\psi + \tilde{\alpha}_2\psi^2 \longrightarrow P[\psi] = \psi + \tilde{\alpha}_2\psi^2$$

First Order Lovelock Tensor Components $E_\mu^{\ \nu}$

$$E_\mu^\nu = \begin{bmatrix} -\frac{3\tilde{\alpha}_1\left(r\frac{d}{dr}\psi(r)+4\psi(r)\right)}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{3\tilde{\alpha}_1\left(r\frac{d}{dr}\psi(r)+4\psi(r)\right)}{2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{\tilde{\alpha}_1\left(r^2\frac{d^2}{dr^2}\psi(r)+8r\frac{d}{dr}\psi(r)+12\psi(r)\right)}{2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{\tilde{\alpha}_1\left(r^2\frac{d^2}{dr^2}\psi(r)+8r\frac{d}{dr}\psi(r)+12\psi(r)\right)}{2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\tilde{\alpha}_1\left(r^2\frac{d^2}{dr^2}\psi(r)+8r\frac{d}{dr}\psi(r)+12\psi(r)\right)}{2} \end{bmatrix}$$

$$E_\mu^\nu = T_\mu^\nu$$

Vacuum: $$P[\psi(r)] = \frac{2\mathcal{M}}{r^4}$$

Charged: $$P[\psi(r)] = \frac{2\mathcal{M}}{r^4} - \frac{Q}{r^6}$$

Second Order Lovelock Tensor Components $E_\mu^{\ \nu}$

$$E_\mu^\nu = \begin{bmatrix} E_0^0 & 0 & 0 & 0 & 0 \\ 0 & E_1^1 & 0 & 0 & 0 \\ 0 & 0 & E_2^2 & 0 & 0 \\ 0 & 0 & 0 & E_3^3 & 0 \\ 0 & 0 & 0 & 0 & E_4^4 \end{bmatrix}$$

$$E_0^0 = -1.5\tilde{\alpha}_1 r \frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_1\psi(r) - 3.0\tilde{\alpha}_2 r\psi(r)\frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_2\psi^2(r)$$

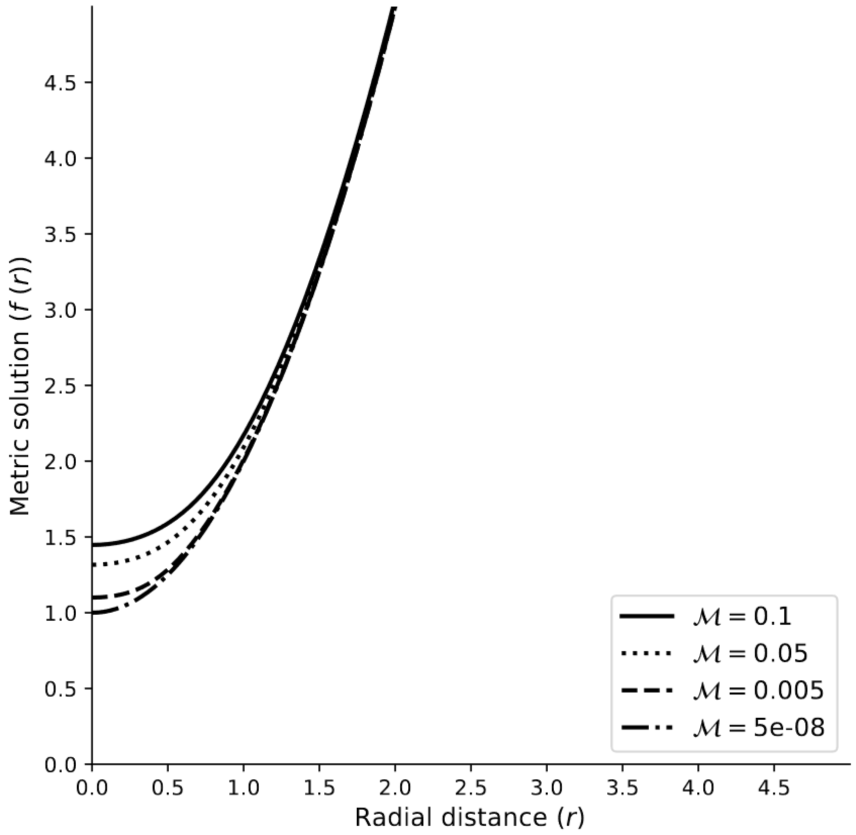
$$E_1^1 = -1.5\tilde{\alpha}_1 r \frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_1\psi(r) - 3.0\tilde{\alpha}_2 r\psi(r)\frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_2\psi^2(r)$$

$$E_2^2 = -0.5\tilde{\alpha}_1 r^2 \frac{d^2}{dr^2}\psi(r) - 4.0\tilde{\alpha}_1 r\frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_1\psi(r) - 1.0\tilde{\alpha}_2 r^2\psi(r)\frac{d^2}{dr^2}\psi(r) - 1.0\tilde{\alpha}_2 r^2\left(\frac{d}{dr}\psi(r)\right)^2 - 8.0\tilde{\alpha}_2 r\psi(r)\frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_2\psi^2(r)$$

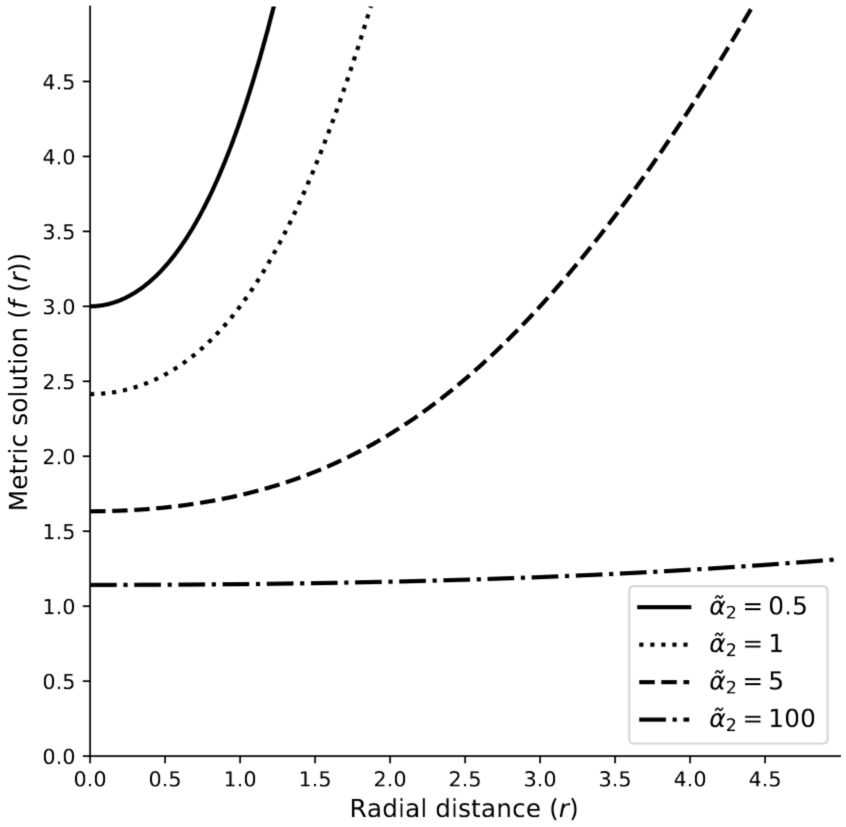
$$E_3^3 = -0.5\tilde{\alpha}_1 r^2 \frac{d^2}{dr^2}\psi(r) - 4.0\tilde{\alpha}_1 r\frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_1\psi(r) - 1.0\tilde{\alpha}_2 r^2\psi(r)\frac{d^2}{dr^2}\psi(r) - 1.0\tilde{\alpha}_2 r^2\left(\frac{d}{dr}\psi(r)\right)^2 - 8.0\tilde{\alpha}_2 r\psi(r)\frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_2\psi^2(r)$$

$$E_4^4 = -0.5\tilde{\alpha}_1 r^2 \frac{d^2}{dr^2}\psi(r) - 4.0\tilde{\alpha}_1 r\frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_1\psi(r) - 1.0\tilde{\alpha}_2 r^2\psi(r)\frac{d^2}{dr^2}\psi(r) - 1.0\tilde{\alpha}_2 r^2\left(\frac{d}{dr}\psi(r)\right)^2 - 8.0\tilde{\alpha}_2 r\psi(r)\frac{d}{dr}\psi(r) - 6.0\tilde{\alpha}_2\psi^2(r)$$

Negative Branch of Neutral Second Order Solution

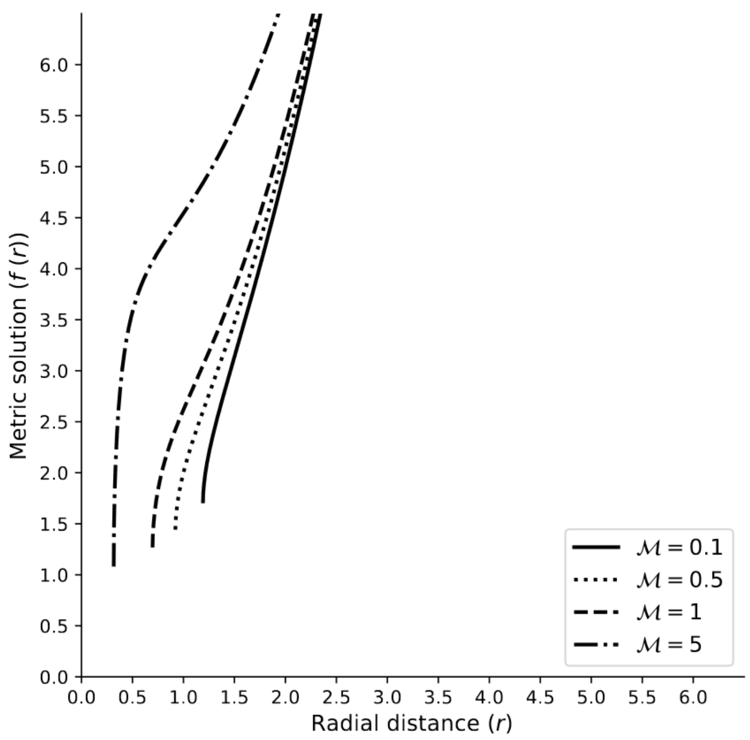


Mass Variation

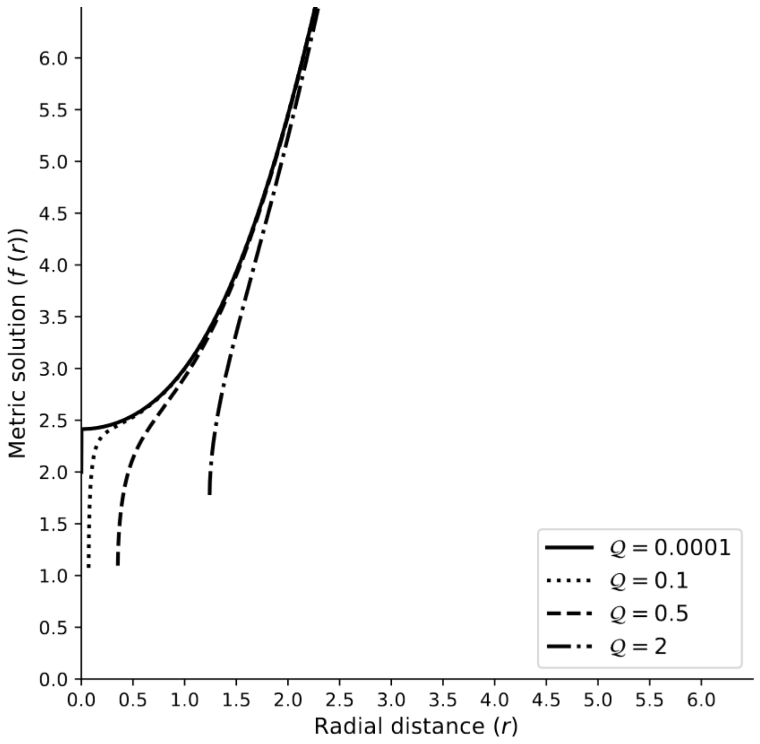


Coupling Constant Variation

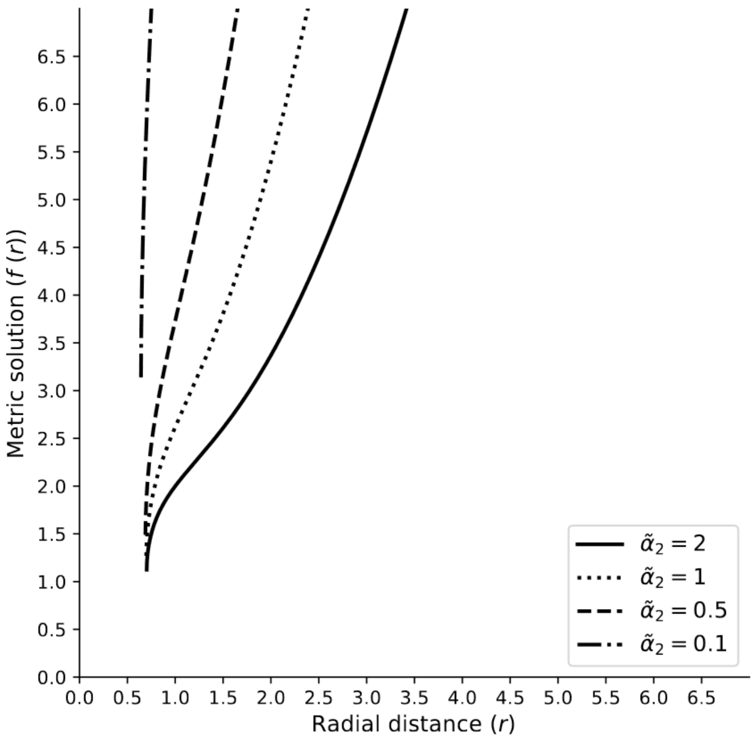
Negative Branch of Charged Second Order Solution



Mass Variation



Charge Variation



Coupling Constant Variation