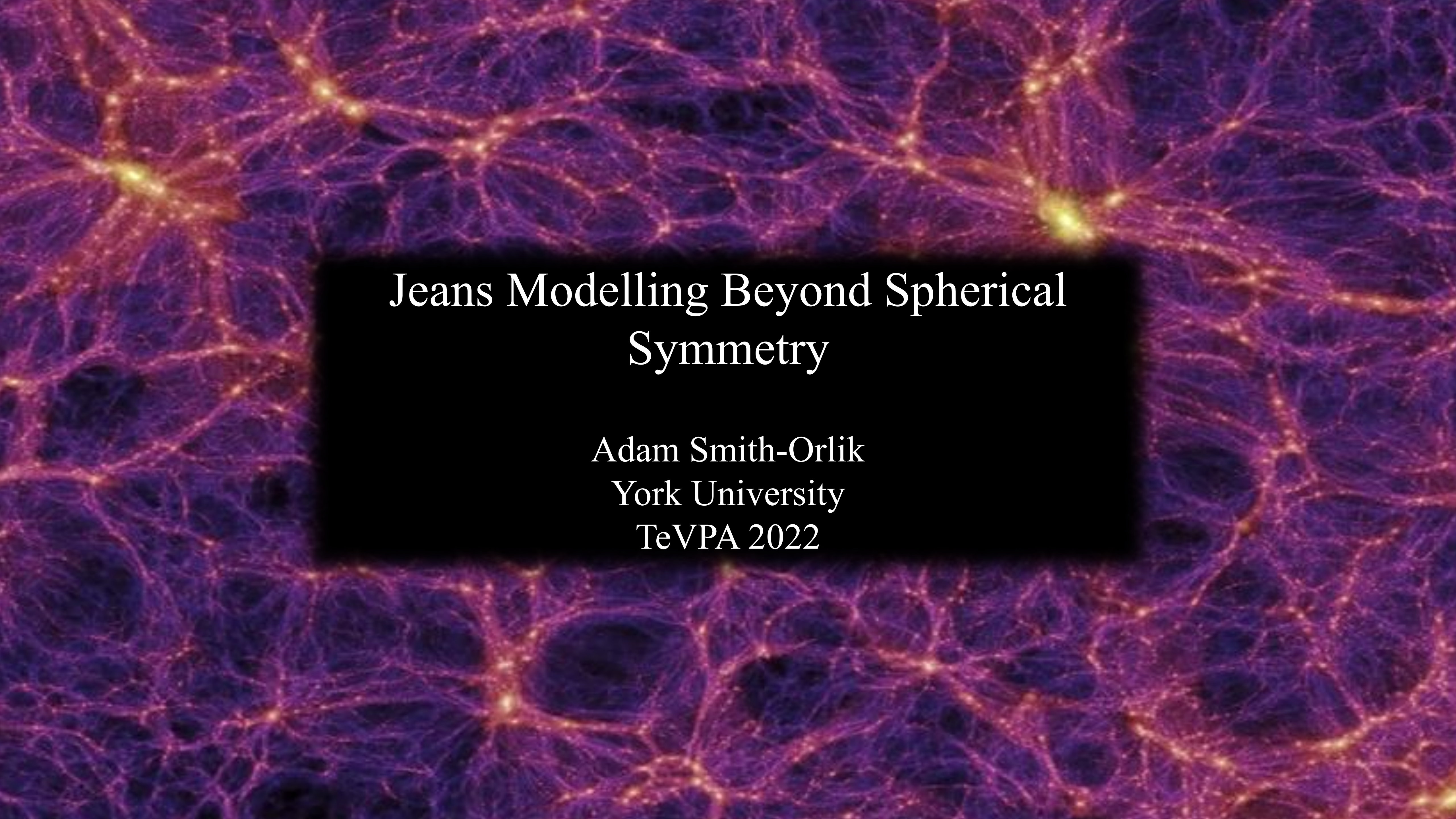


Jeans Modelling Beyond Spherical Symmetry

Adam Smith-Orlik
York University

TeVPA 2022

Team

Prof. Sean Tulin
York University

Prof. Laura Sagunski
Goethe University

Dr. Andrew Robertson
NASA JPL

Fabian Bautista
Perimeter Institute, York University

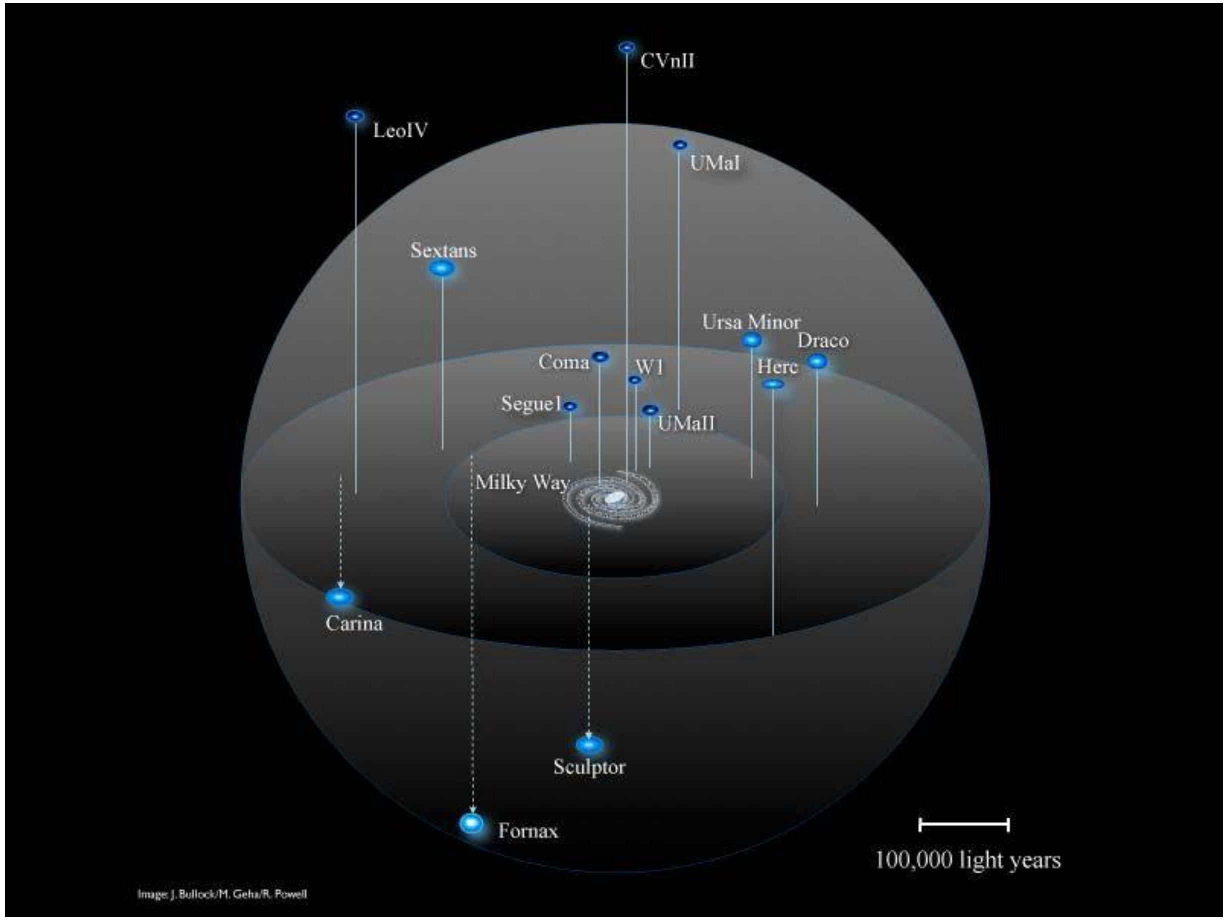
Outlook

- Orientation
- Motivation
- Jeans Model Theory
- Preliminary Results
- Next Steps

Orientation

- Galaxies and galaxy clusters are enveloped in dark matter (DM) “halos”

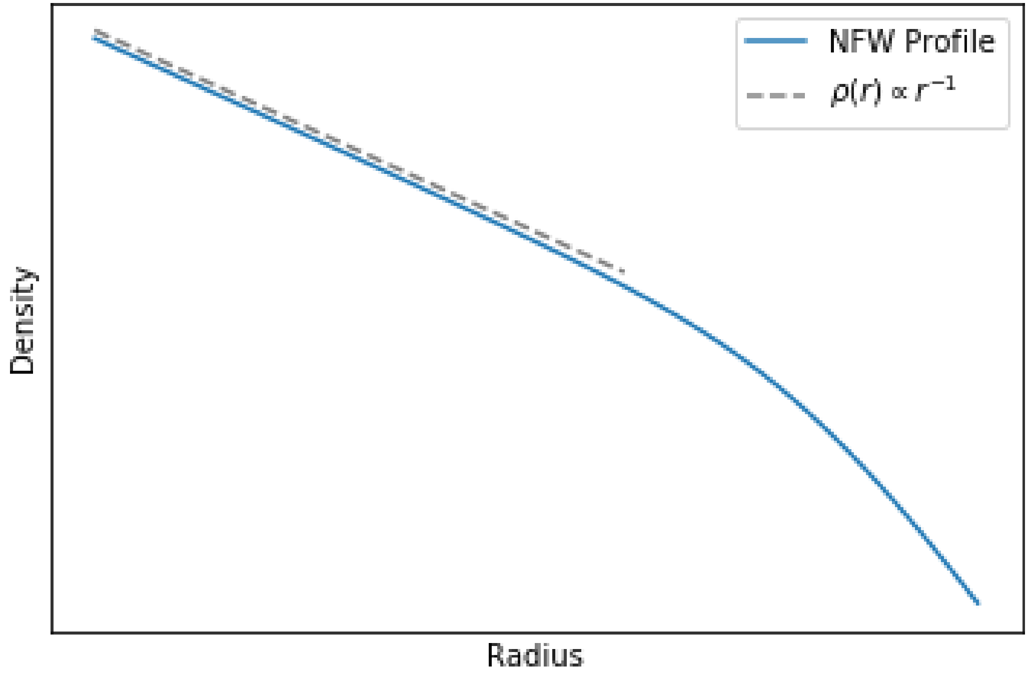
- DM halos extend far beyond the spatial extent of the baryons



J. Bullock, M. Geha, R. Powell

Orientation

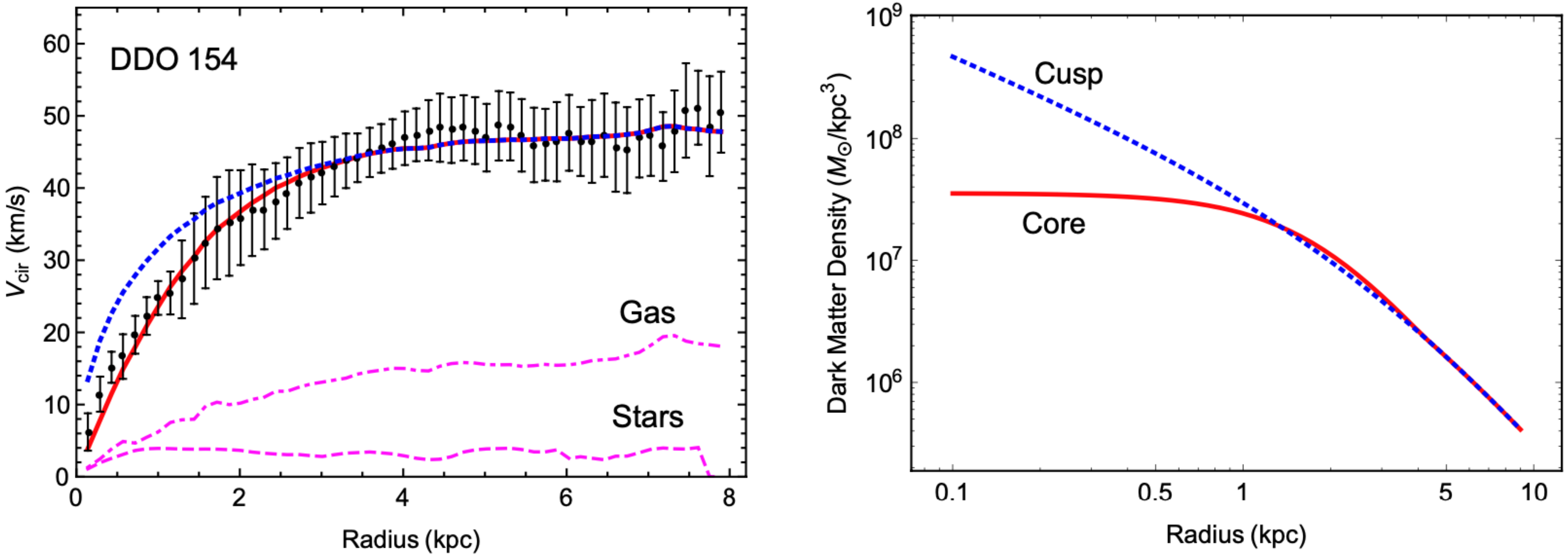
- Navarro-Frenk-White (NFW) used N-body simulations with CDM cosmologies to fit a universal DM density profile
- $\rho(r) \propto r^{\alpha}$ where $\alpha = -1$



Motivation

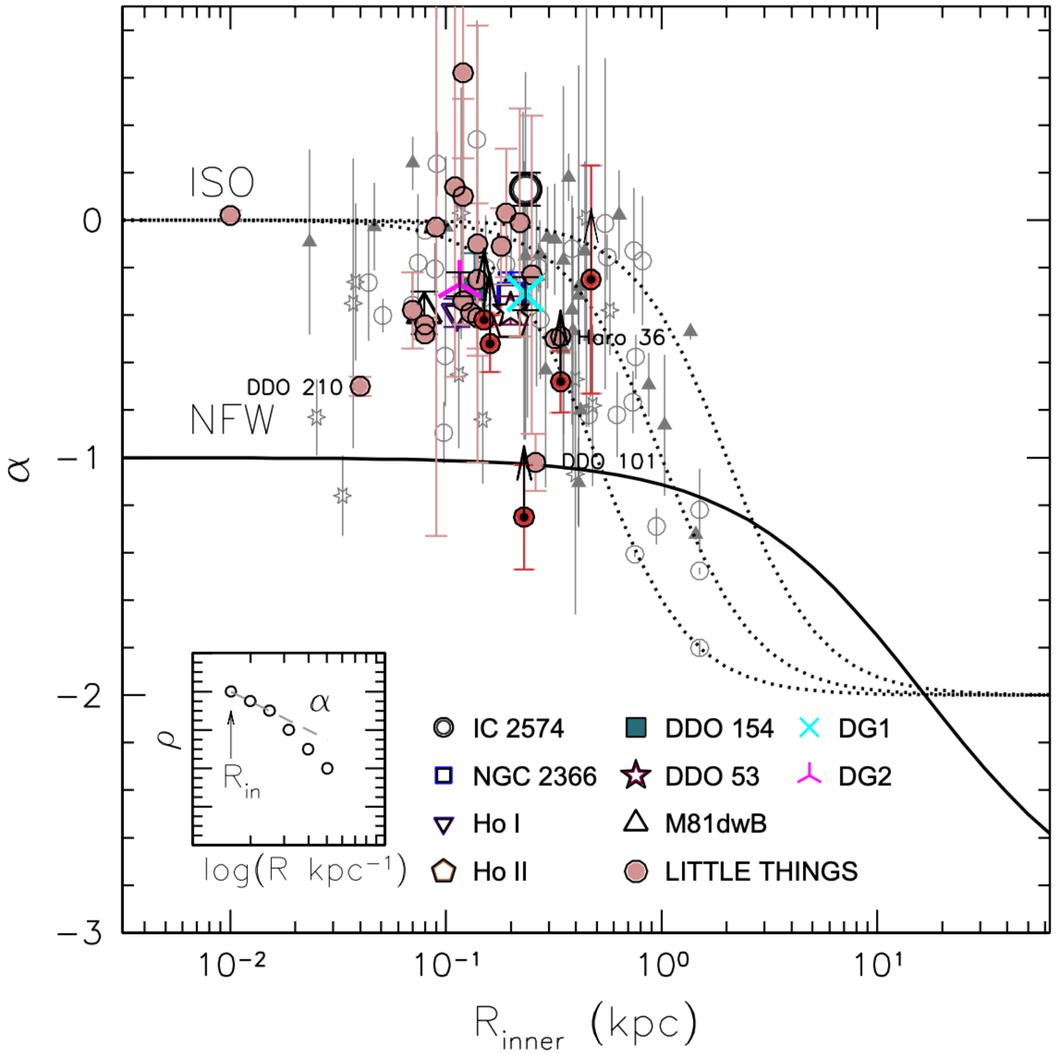
- CDM paradigm describes remarkably well the large-scale structure and evolution of the universe
- Small-scales (< 1 Mpc) CDM predictions are in tension with observations
- Core-cusp, diversity in rotation curves, missing satellites, too-big-to-fail, etc.

Motivation



Se-Heon Oh et al. (2015)

Motivation



Se-Heon Oh et al. (2015)
Survey of LITTLE THINGS data

Motivation

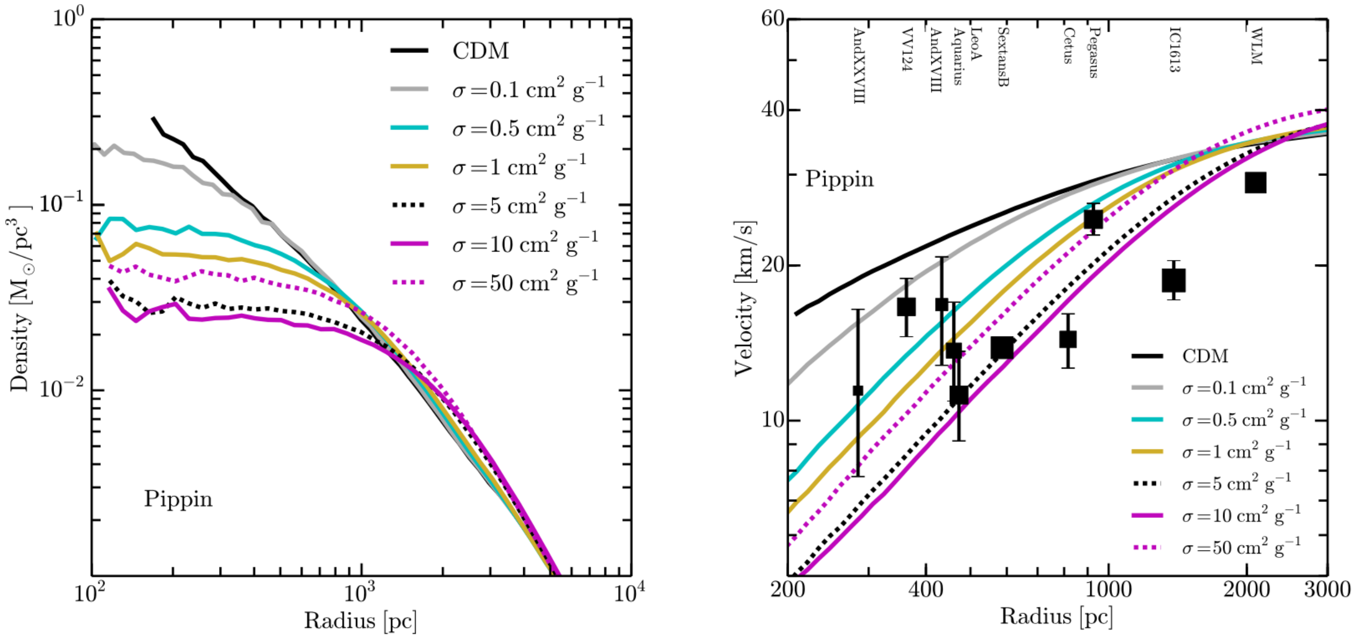
- Baryonic feedback processes -- star formation, AGNs, and supernovae
- Hydrodynamical solutions do not yet fully resolve small scale issues
- One economical solution as proposed by Spergel and Steinhardt (2000) is self interacting dark matter (SIDM), which has the potential to solve some small-scale issues while retaining the success of CDM on large-scales

Observational evidence for self-interacting cold dark matter

David N. Spergel and Paul J. Steinhardt
Princeton University, Princeton NJ 08544 USA

Motivation

- Allowing DM to scatter leads to predictions that better match the observations



Oliver D. Elbert et al. (2014)

Motivation

- N-body simulations have been used extensively to test the predictions of SIDM though suffer from being computationally intensive and unable to reproduce observed systems

- A semi-analytical alternative to N-body simulations is the Jeans model which is derived from the **collisional Boltzmann equation**

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} f - \boldsymbol{\nabla}\Phi_{\rm tot} \cdot \frac{\partial f}{\partial \mathbf{v}} = \mathscr{C}[f]$$

Motivation

- N-body simulations have been used extensively to test the predictions of SIDM though suffer from being computationally intensive and unable to reproduce observed systems

- A semi-analytical alternative to N-body simulations is the Jeans model which is derived from the **collisional Boltzmann equation**

$$\boldsymbol{\nabla}(\sigma_0^2 \rho_{\rm dm}) = -\rho_{\rm dm} \boldsymbol{\nabla} \Phi_{\rm tot}$$

Motivation

- The 1D isothermal Jeans model works well!

The surprising accuracy of isothermal Jeans modelling of self-interacting dark matter density profiles

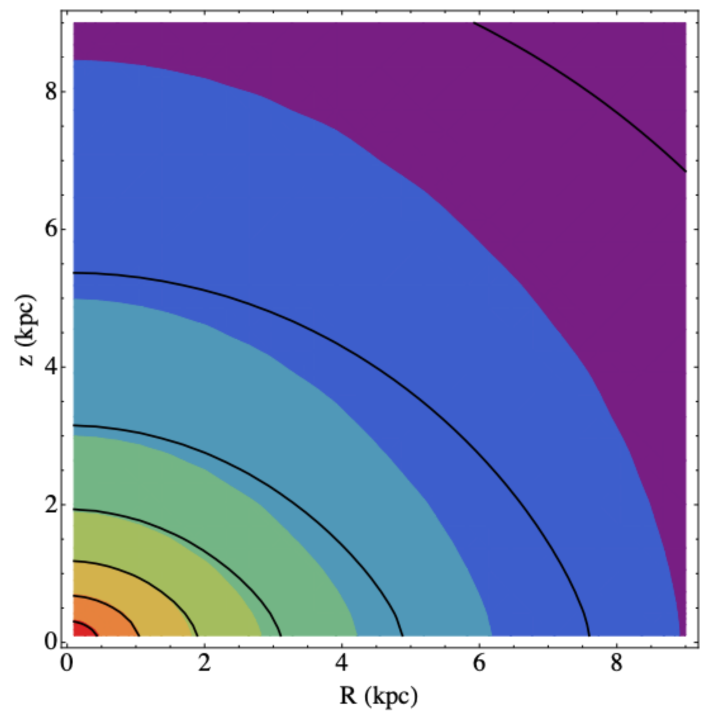
Andrew Robertson*[1], Richard Massey[1], Vincent Eke[1], Joop Schaye[2] and Tom Theuns[1]

[1] *Institute for Computational Cosmology, Durham University, South Road, Durham DH1 3LE, UK*
[2] *Leiden Observatory, Leiden University, PO Box 9513, NL-2300 RA Leiden, the Netherlands*

- While computationally cheaper and able to match observables of systems, the Jeans model has only been explored in 1D assuming spherical symmetry or 2D assuming azimuthal symmetry

Motivation

- Departures from spherical symmetry in DM halos with large baryon components (like the MW) are known to be important



M. Kaplinghat et al. (2014)

To Do List:

✓ Extend Jeans model to higher dimensions and allow for asymmetry

✓ Validate model using mock data from simulations

✓ Confront model predictions with simulations and observations

Jeans Model Theory

- Following M. Kaplinghat et al. (2016) we define a characteristic radius r_1 by the rate equation

$$\rho_{\rm dm}(r_1)\frac{\langle \sigma v_{\rm rel}\rangle}{m}t_{\rm age} = 1$$

$$\rho_{\rm SIDM}(r) = \begin{cases} \rho_{\rm iso}(r) & r < r_1 \\ \rho_{\rm NFW}(r) & r > r_1 \end{cases}$$

- Where inside r_1 we use the Jeans model to determine the isothermal density profile and outside r_1 we fit an NFW (CDM) profile

Jeans Model Theory

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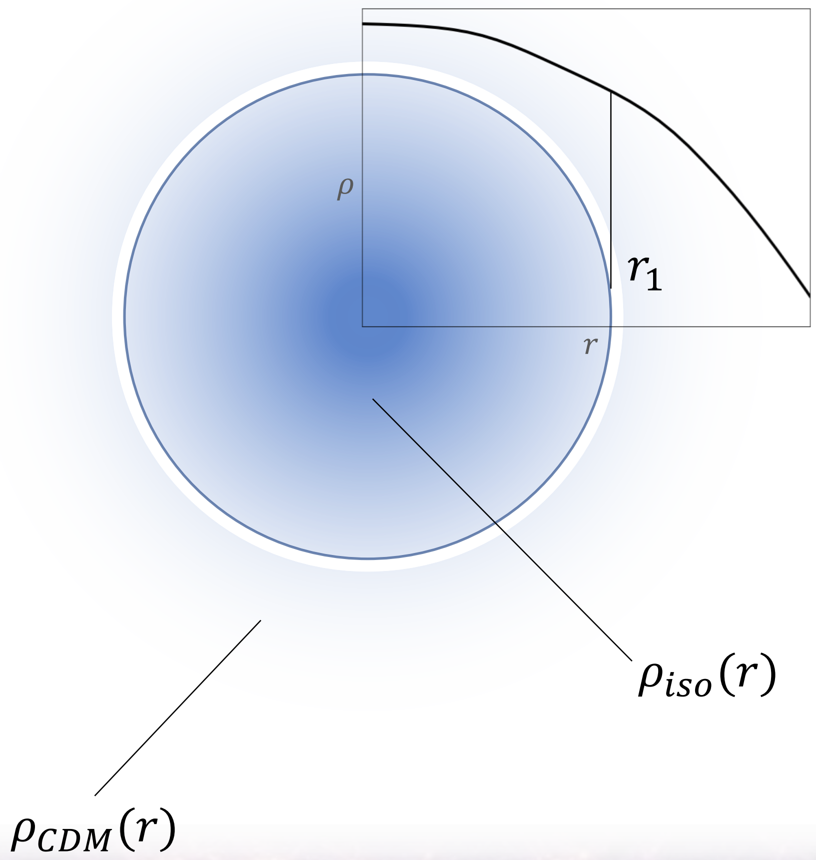


Jeans Model Theory

- For the inner profile, the Jeans equation is

$$\boldsymbol{\nabla}(\sigma_0^2 \rho_{\rm dm}) = -\rho_{\rm dm} \boldsymbol{\nabla} \Phi_{\rm tot}$$

- To which the solution is

$$\rho_{\rm dm} = \rho_0 \exp\left(-\frac{\Phi(\mathbf{r}) - \Phi_0}{\sigma_0^2} \right)$$

- Redefining the potentials in terms of dimensionless variables we get

$$\rho_{\rm dm} = \rho_0 e^{-\phi_{\rm dm}(\mathbf{r}) - \phi_b(\mathbf{r})}$$

Jeans Model Theory

- To solve for $\rho_{\rm dm}$ we need to first solve for the potential $\phi_{\rm dm}$ using Poisson's equation

$$\nabla^2 \phi_{\rm dm}(\mathbf{r}) = \left(\frac{4\pi G \rho_0}{\sigma_0^2}\right) e^{-\phi_{\rm dm}(\mathbf{r})} e^{-\phi_b(\mathbf{r})}$$

- To allow for the departure from symmetry we expand $\phi_{\rm dm}$ in real-valued combination of spherical harmonics called *Tesseral harmonics* Z_{lm} (using spherical coordinates)

$$\phi_{\rm dm}(r,\theta,\varphi) = \sum_{\ell,m} \phi_{\ell m}(r) Z_{\ell m}(\theta,\varphi)$$

Jeans Model Theory

- Substituting the expansion in we get the non-linear partial differential equation

$$\frac{\partial^2 \phi_{\ell m}}{\partial r^2} + \frac{2}{r}\frac{\partial \phi_{\ell m}}{\partial r} - \frac{\ell(\ell+1)}{r^2}\phi_{\ell m} = \left(\frac{4\pi G \rho_0}{\sigma_0^2}\right)\int d\Omega\, Z_{\ell m}(\theta,\varphi) e^{-\phi_b(\mathbf{r})}\, e^{-\phi_{\rm dm}(\mathbf{r})}$$

- Casting into a system of coupled first-order equations gives

$$\frac{\partial \phi_{\ell m}(r)}{\partial r} = \frac{\mu_{\ell m}(r)}{r^2}$$

$$\frac{\partial \mu_{\ell m}(r)}{\partial r} = \ell(\ell+1)\phi_{\ell m} + \frac{r^2}{r_0^2}\int d\Omega\, Z_{\ell m}(\theta,\varphi) e^{-\phi_b(\mathbf{r})}\, e^{-\phi_{\rm dm}(\mathbf{r})}$$

Jeans Model Theory

- Solve our system of equation using an iterative method called the **relaxation method**

Steps:

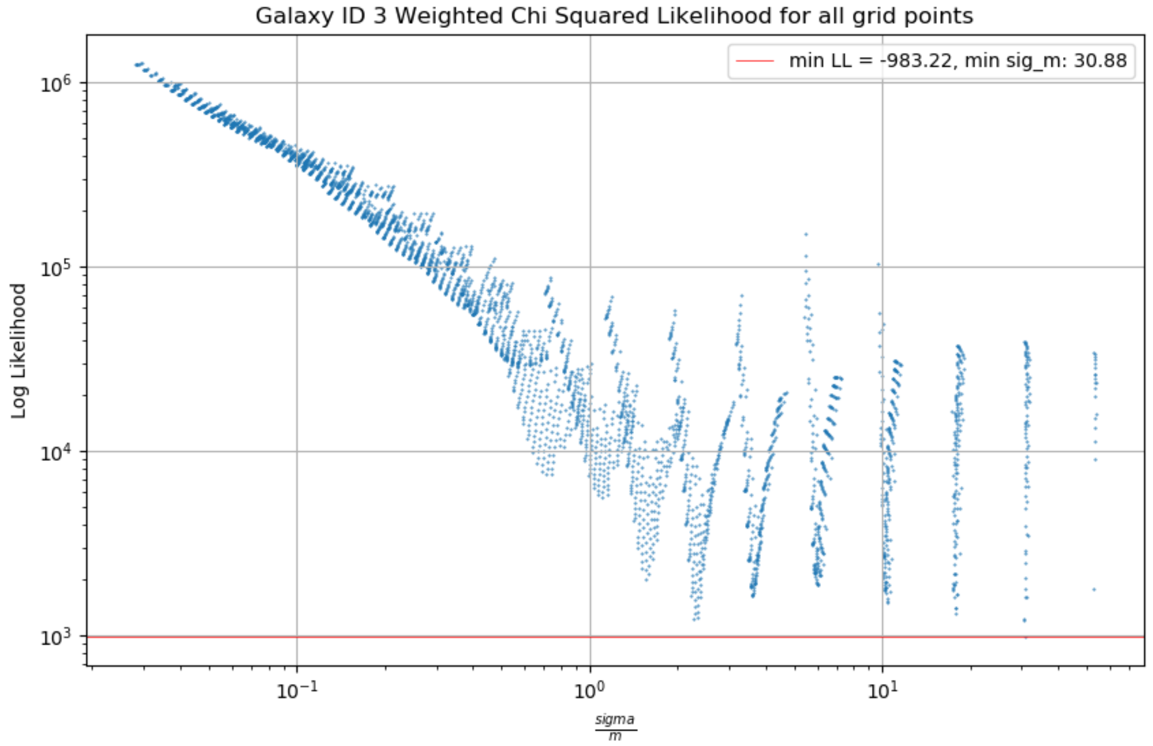
1. Guess solution
2. Calculate shift
3. Linearize in the shifts and solve for new shifts
4. Shift solutions by new shifts
5. Repeat until convergence condition is met

Results

- To validate the Jeans model we compare results to $\sigma/m = 1\mathrm{cm}^2/\mathrm{g}$ versions of the EAGLE SIDM-plus-baryons simulations

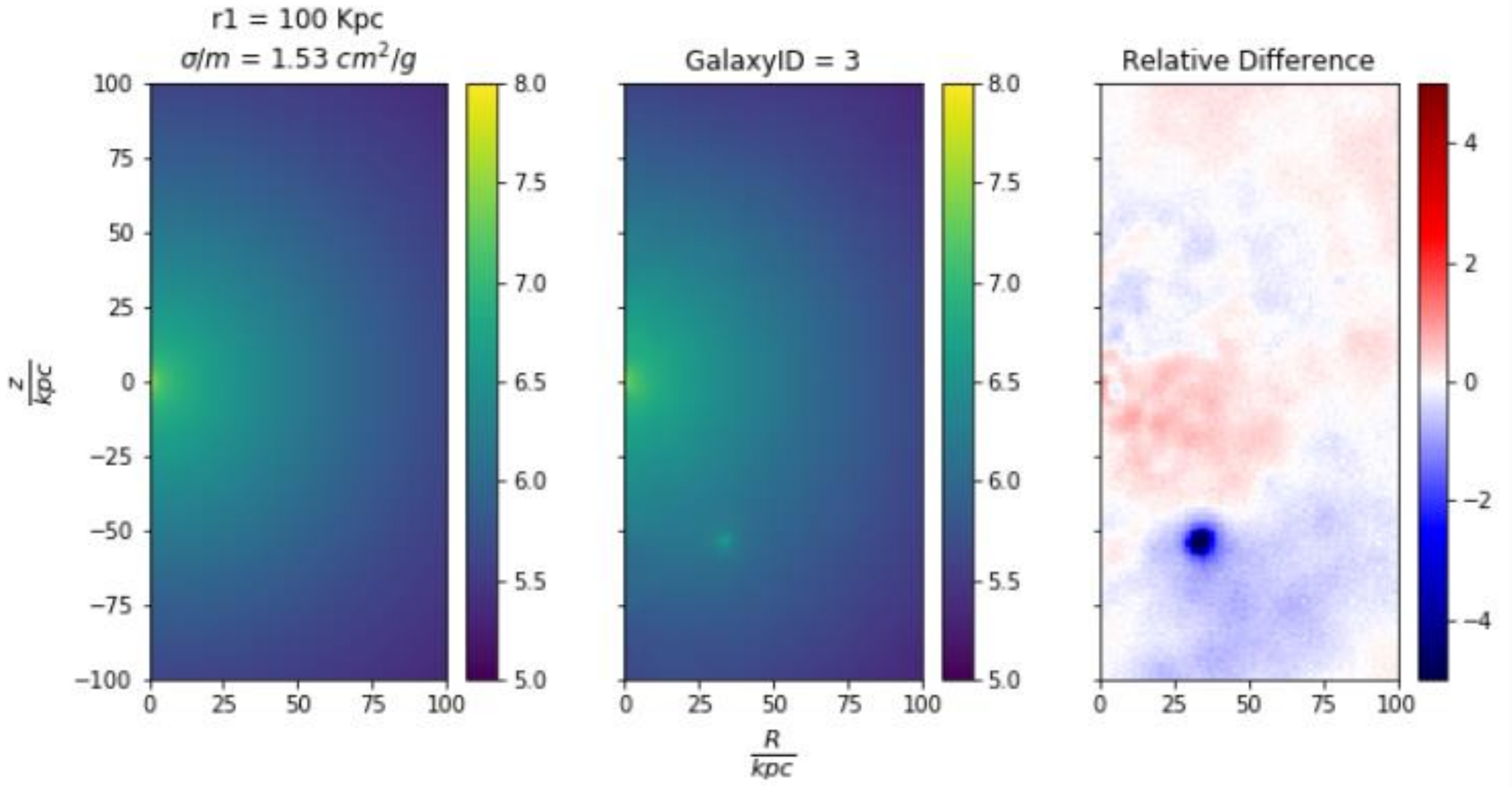
- Ran a grid in $r_1\left(\frac{\sigma}{m}\right), M200, c$

- Run for a subset of 250 systems shows degenerate solutions



Results

- 2D pixel maps for the Jeans model (left) compared with the SIDM1b simulations (middle)



Results

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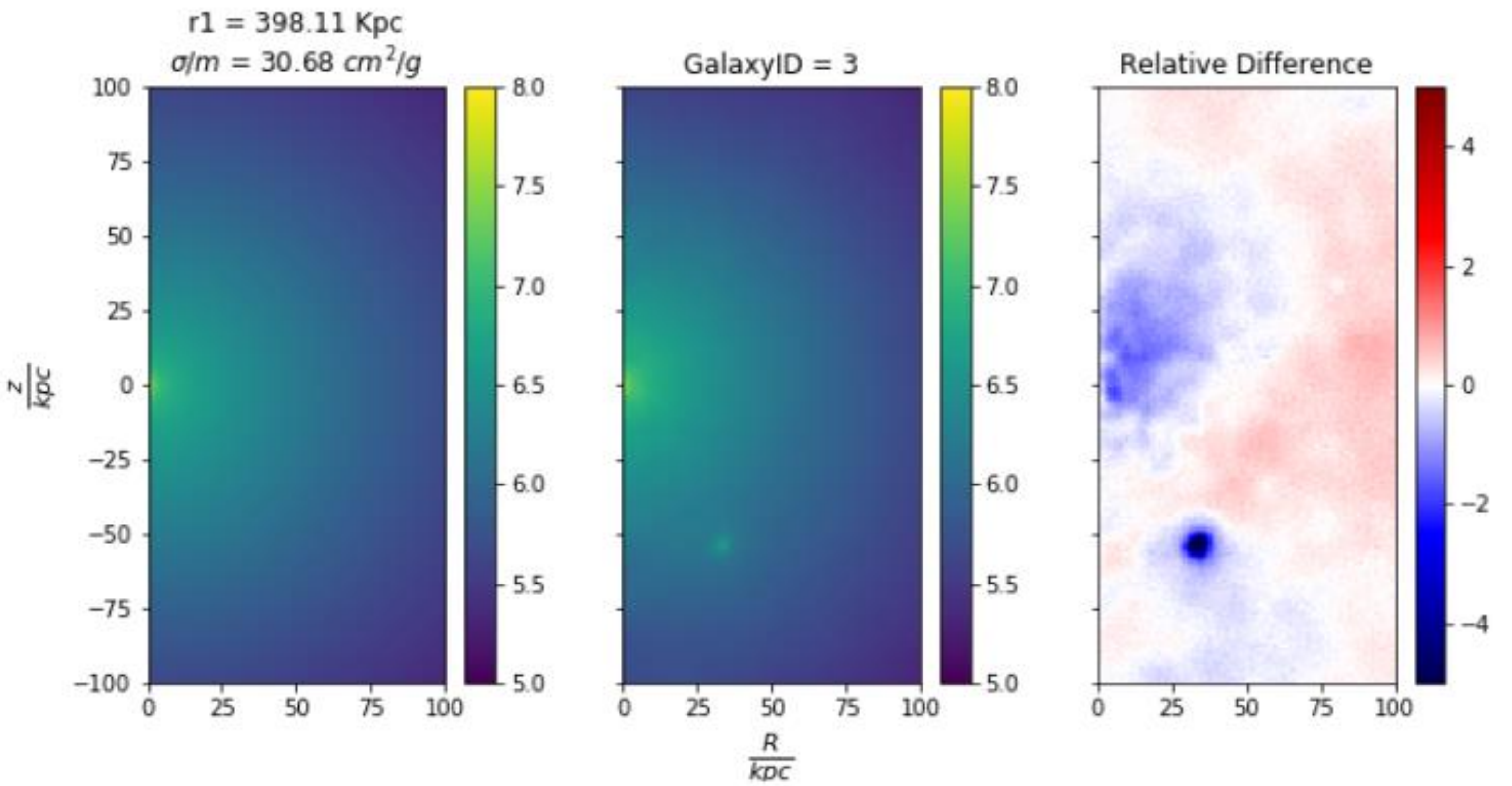
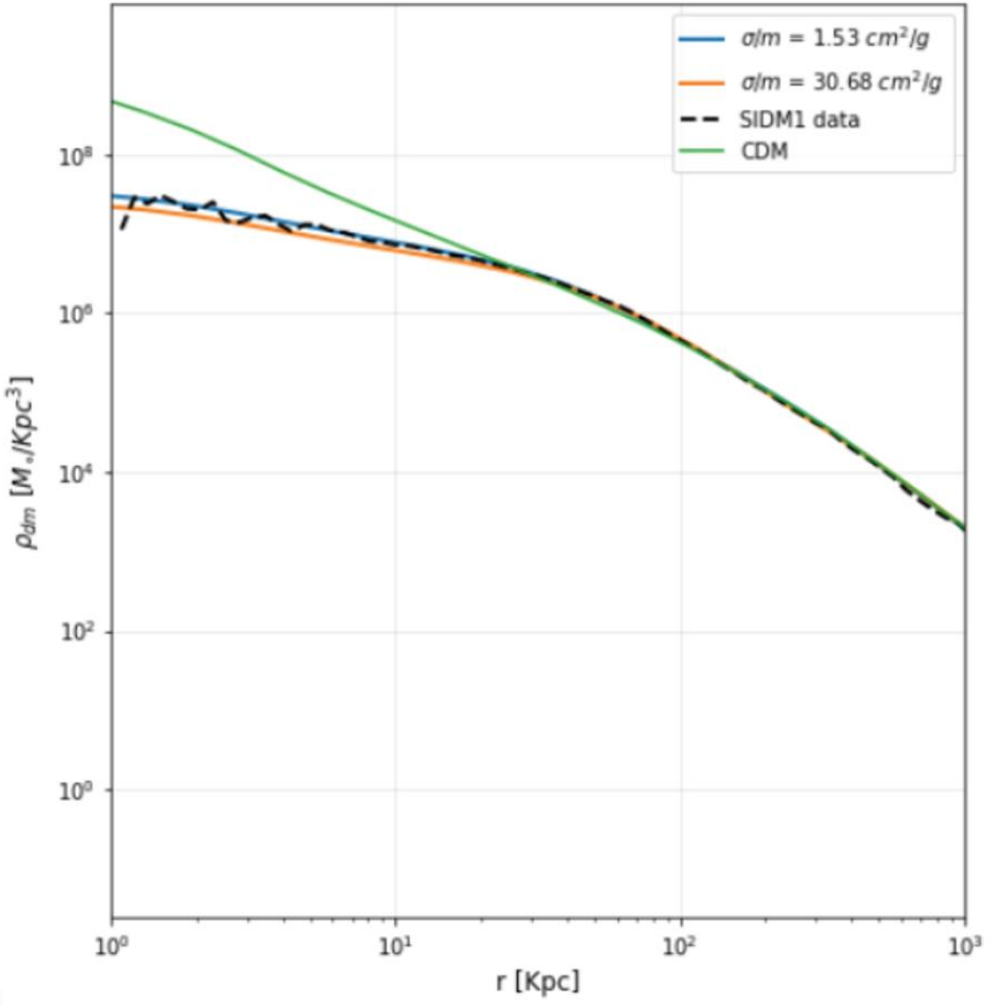


Results

- Spherically averaged density plot for low and high cross section solutions



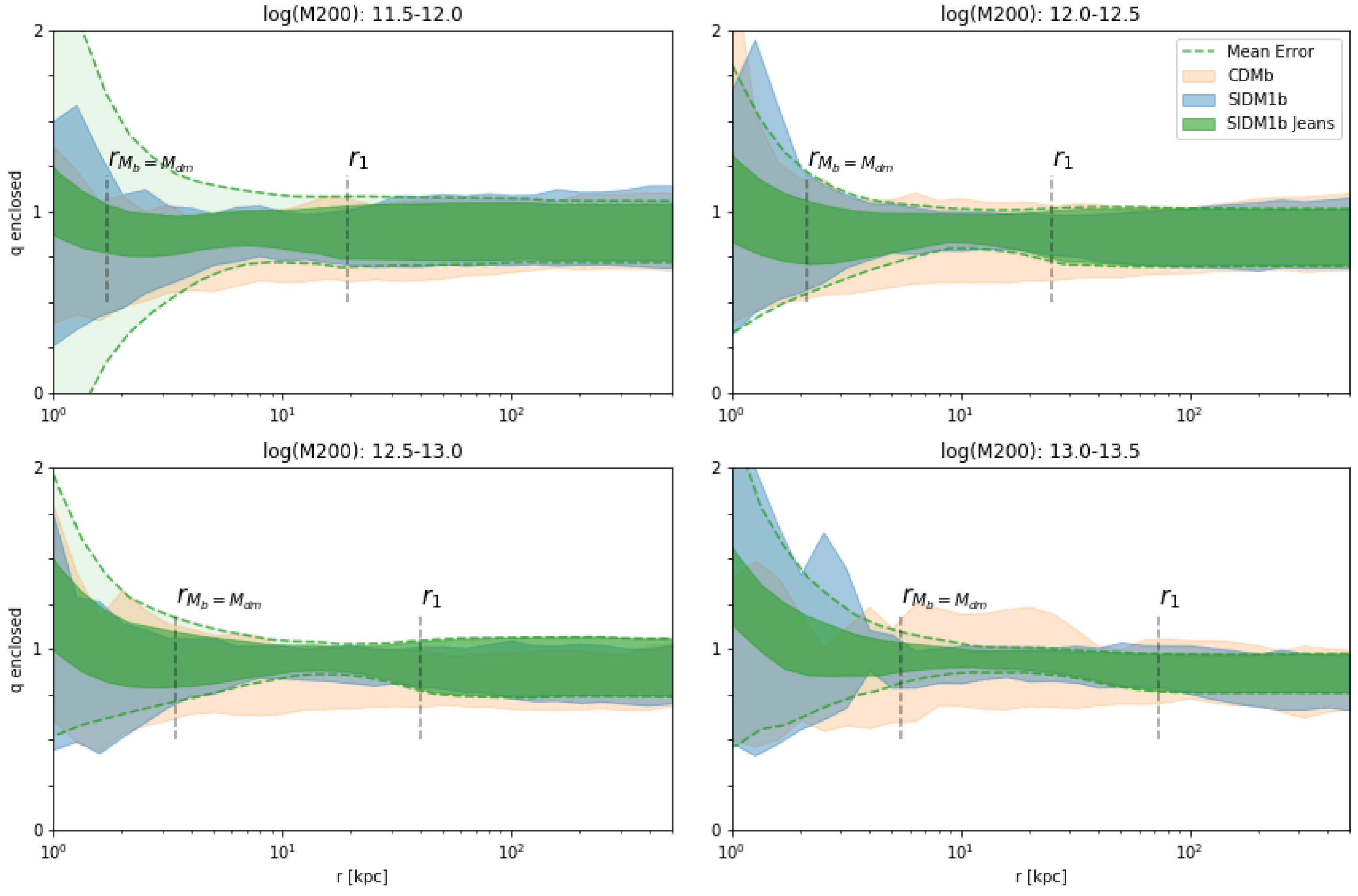
Results

- To distinguish between high and low cross-sections we looked for a halo shape observable to compare with the simulations

- We define $q_{enclosed}(r)$ by computing the ratio of the moment of inertia tensor in the R and z coordinates, i.e. $q^2 = I_{ZZ}/I_{RR}$, in analogue to the major-to-minor axis ratio $q = c/a$

Results

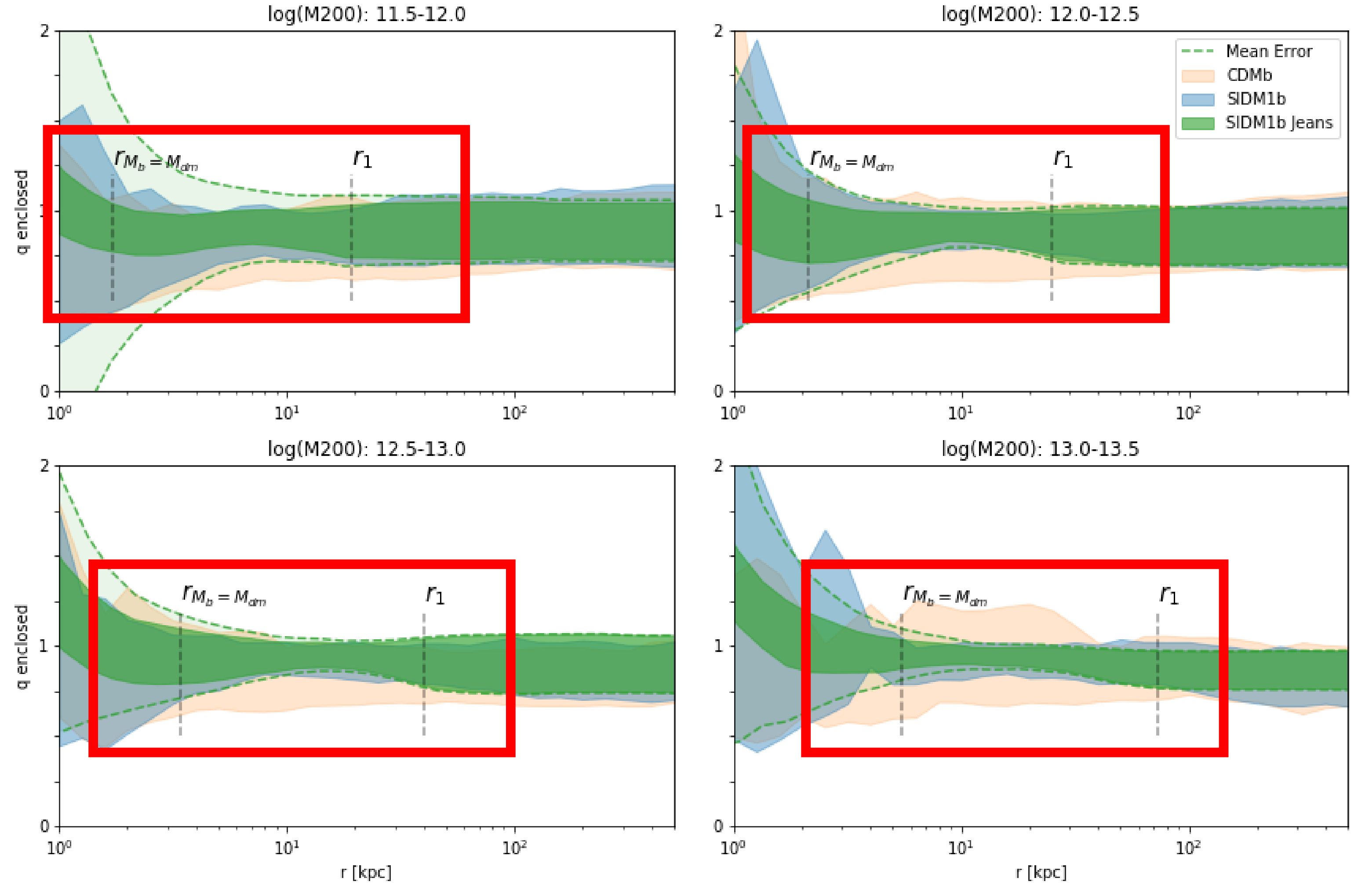
PRELIMINARY



Results

PRELIMINARY

- Intermediate regions where SIDM dominates is driven to sphericity for all mass scales



Next Steps

- Further work to quantify the “tightening” effect and to extend our model to 3D
- Compare with observational probes from gravitational lensing and x-ray surface brightness to constrain the cross section as in A.H.G. Peter et al. (2012)
- Reassess constraints from x-ray emitting elliptical galaxies
- Include Jeans model SIDM halos directly into a strong lensing models

Thank you!

Backup

1. System of ODEs

$$
\begin{gathered}
y_1'(x) + F_1(y_1, \dots, y_n; x; p) = 0 \\
y_2'(x) + F_2(y_1, \dots, y_n; x; p) = 0 \\
\vdots \\
y_n'(x) + F_n(y_1, y_2, \dots, y_n; x; p) = 0
\end{gathered}
$$

2. Finite difference method and define a new vector

$$\vec{E}_i(\vec{y}_{i+1}, \vec{y}_i, p) = \vec{y}_{i+1} - \vec{y}_i + (x_{i+1} - x_i)\vec{F}\left(\frac{\vec{y}_{i+1} + \vec{y}_i}{2}; \frac{x_{i+1} + x_i}{2}; p\right)$$

3. Input initial guess and compute shift towards solution

$$\vec{y}_i \to \vec{y}_i^{\,\text{new}} = \vec{y}_i + \Delta\vec{y}_i \,, \qquad p \to p^{\text{new}} = p + \Delta p$$

4. Linearize in the shifts

$$
\begin{aligned}
E_{ai}(\vec{y}_{i+1}^{\,\text{new}}, \vec{y}_i^{\,\text{new}}, p^{\text{new}}) &\approx E_{ai}(\vec{y}_{i+1}, \vec{y}_i, p) + \sum_{b,j} \frac{\partial E_{ai}}{\partial y_{bj}} \Delta y_{bj} + \frac{\partial E_{ai}}{\partial p} \Delta p \\
&\approx E_{ai}(\vec{y}_{i+1}, \vec{y}_i, p) + \Delta y_{a,i+1} - \Delta y_{ai} \\
&\quad + (x_{i+1} - x_i)\left(\sum_{b=1}^{n} \frac{\partial F_a}{\partial y_b} \frac{\Delta y_{b,i+1} + \Delta y_{bi}}{2} + \frac{\partial F_a}{\partial p} \Delta p\right)
\end{aligned}
$$

5. Define

$$\boldsymbol{\Delta}\vec{\mathbf{y}} = \begin{pmatrix} \Delta\vec{y}_0 \\ \Delta\vec{y}_1 \\ \vdots \\ \Delta\vec{y}_N \\ \Delta p \end{pmatrix}$$

$$(\mathbf{A}_i)_{ab} = \frac{1}{2}(x_{i+1} - x_i) \left.\frac{\partial F_a}{\partial y_b}\right|_{\vec{y} = \frac{1}{2}(\vec{y}_{i+1} + \vec{y}_i)}$$

$$\vec{B}_i = (x_{i+1} - x_i) \left.\frac{\partial F_a}{\partial p}\right|_{\vec{y} = \frac{1}{2}(\vec{y}_{i+1} + \vec{y}_i)}$$

Backup

6. Define a matrix equation

$$\begin{pmatrix} -\mathbf{1}+\mathbf{A}_0 & +\mathbf{1}+\mathbf{A}_0 & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \vec{B}_0 \\ \mathbf{0} & -\mathbf{1}+\mathbf{A}_1 & +\mathbf{1}+\mathbf{A}_1 & \mathbf{0} & \dots & \mathbf{0} & \vec{B}_1 \\ & & \ddots & \ddots & & & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & -\mathbf{1}+\mathbf{A}_{N-1} & +\mathbf{1}+\mathbf{A}_{N-1} & \vec{B}_{N-1} \end{pmatrix} \begin{pmatrix} \Delta\vec{y}_0 \\ \Delta\vec{y}_1 \\ \vdots \\ \Delta\vec{y}_N \\ \Delta p \end{pmatrix} = - \begin{pmatrix} \vec{E}_0 \\ \vec{E}_1 \\ \vdots \\ \vec{E}_{N-1} \end{pmatrix}$$

7. Solve for the shifts and repeat until solution is found

$$\mathbf{\Delta\vec{y}} = -\mathbf{M}^{-1}\vec{\mathbf{E}}$$