Q-MONOPOLE BALL: A TOPOLOGICAL & NON-TOPOLOGICAL SOLITON

NICHOLAS ORLOFSKY

2111.10360: Yang Bai, Sida Lu, NO

TeVPA

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Carleton UNIVERSITY

MOTIVATION

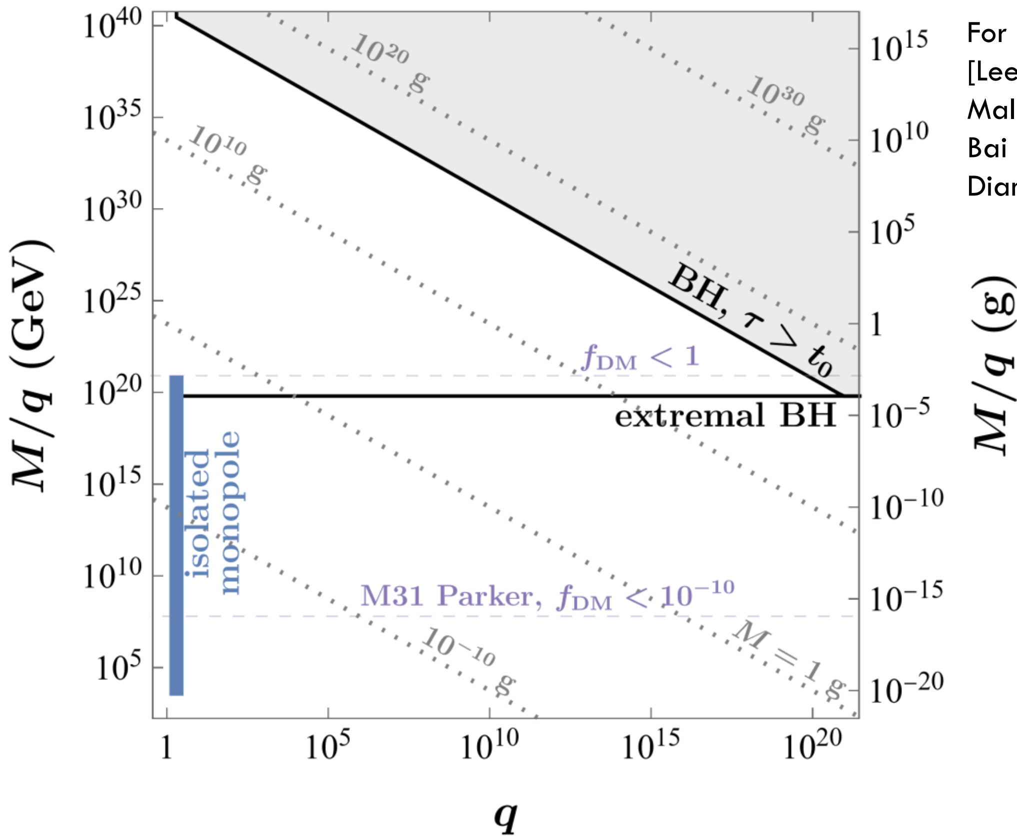
Magnetic monopoles could explain the observed quantization of electric charge, but we have not detected them.

One possibility: we are looking for the wrong type of monopole.

Larger bound states of monopoles?

MOTIVATION

Can we construct a theory which allows for a larger variety of masses and magnetic charges?



For magnetic BHs, see:
[Lee Weinberg hep-th/9406021;
Maldacena 2004.06084;
Bai Berger Korwar NO 2007.03703;
Diamond Kaplan 2103.01850]

PLAN

- Start with unit magnetic charge → spherically symmetric solutions
- Motivate Q-monopole-balls with larger magnetic charge
- A few words on formation

THEORY

$$\mathcal{L} = |\partial_\mu S|^2 + \frac{1}{2}(D_\mu \phi^a)^2 - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - V(S,\phi)\,,$$
$$V(S,\phi) = \frac{1}{8}\lambda_\phi(\phi^a\phi^a - v^2)^2 + \frac{1}{2}\lambda_{\phi S}|S|^2(\phi^a\phi^a) + \lambda_S|S|^4 + m^2_{S,0}|S|^2\,,$$

ϕ^a: SU(2) gauge triplet scalar
S: U(1) global complex scalar

Spontaneous SU(2) breaking leads to monopoles:

$$\pi_2[SU(2)/U(1)] = \mathbb{Z}$$

(All parameters are taken positive)

EQUATIONS OF MOTION

Use dimensionless rescaling of fields, with time-dependent phase for S and "hedgehog gauge" for ϕ^a:

$$\phi^a = \hat{r}^a \, v \, f(r) \, , \quad S = e^{-i\omega t} \frac{v}{\sqrt{2}} \, s(r) \, , \quad A_0 = 0 \, , \quad A_i^a = \epsilon^{aij} \frac{\hat{r}^j}{e \, r} \, a(r)$$

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Classical EOM:

$$\begin{aligned}
&a'' - \frac{1}{\bar{r}^2}\, a(1-a)(2-a) + e^2\,(1-a)\, f^2 = 0\,,\\
&f'' + \frac{2}{\bar{r}}\, f' - \frac{2}{\bar{r}^2}\,(1-a)^2\, f - \frac{1}{2}\,\lambda_\phi\, f\,(f^2-1) - \frac{1}{2}\,\lambda_{\phi S}\, s^2 f = 0\,,\\
&s'' + \frac{2}{\bar{r}}\, s' + \Omega^2\, s - \frac{1}{2}\,\lambda_{\phi S}\, f^2\, s - \lambda_S\, s^3 - \mu_0^2\, s = 0\,,
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\end{aligned}$$

Boundary conditions:

$$f(0) = 0\,,\ f(\infty) = 1\,,\ s'(0) = 0\,,\ s(\infty) = 0\,,\ a(0) = 0\,,\ a(\infty) = 1$$

Rescalings: $\bar{r} = v\, r$ $\quad\Omega = \omega / v$ $\quad\mu_0 = m_{S,0}/v$

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$$s'' + \frac{2}{\overline{r}} s' + \Omega^2 s - \frac{1}{2} \lambda_{\phi S} f^2 s - \lambda_S s^3 - \mu_0^2 s = 0 \,,$$

$$U_{\text{eff}} = \Omega^2 s^2/2 - V(s, f)$$

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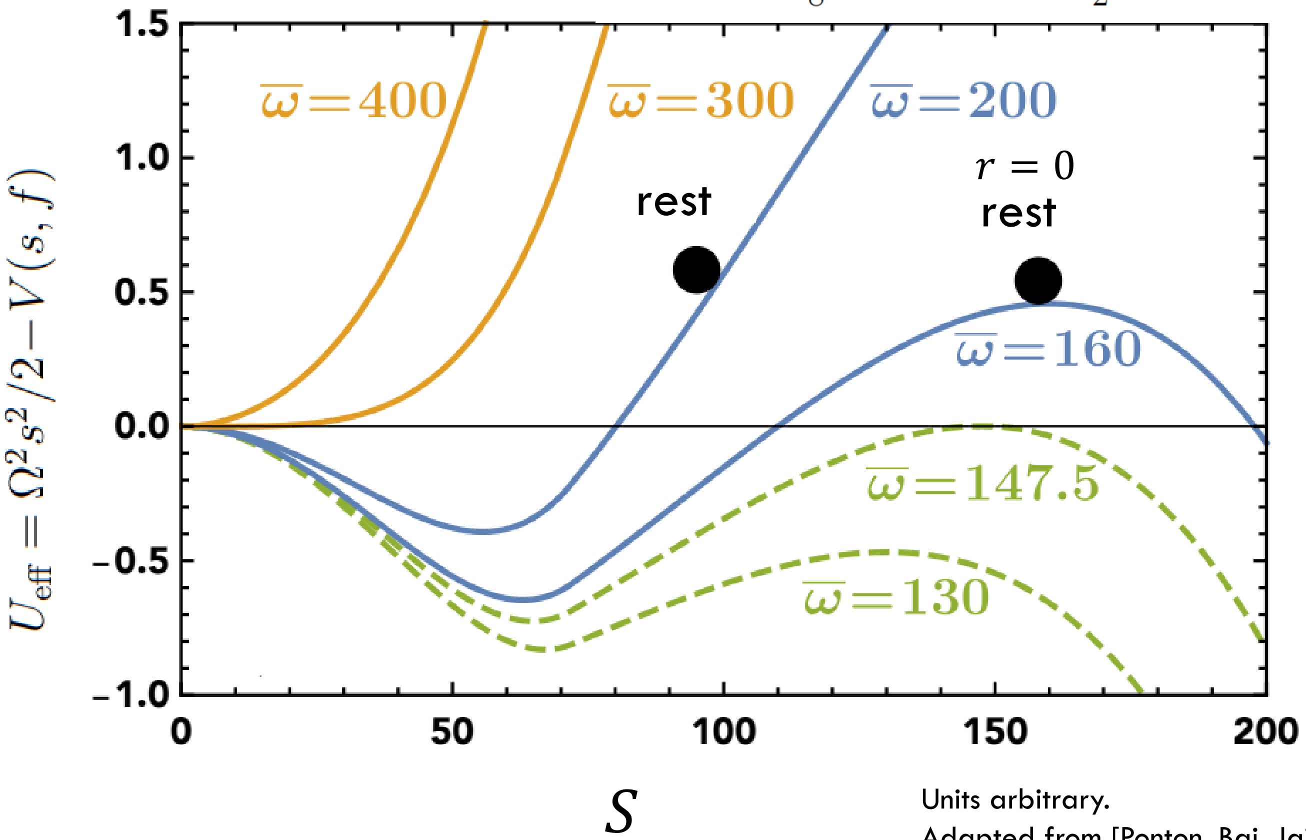
EFFECTIVE POTENTIAL

$$V(S,\phi) = \frac{1}{8}\lambda_\phi(\phi^a\phi^a - v^2)^2 + \frac{1}{2}\lambda_{\phi S}|S|^2(\phi^a\phi^a) + \lambda_S|S|^4 + m_{S,0}^2|S|^2\,,$$



Units arbitrary.
Adapted from [Ponton, Bai, Jain 1906.10739]

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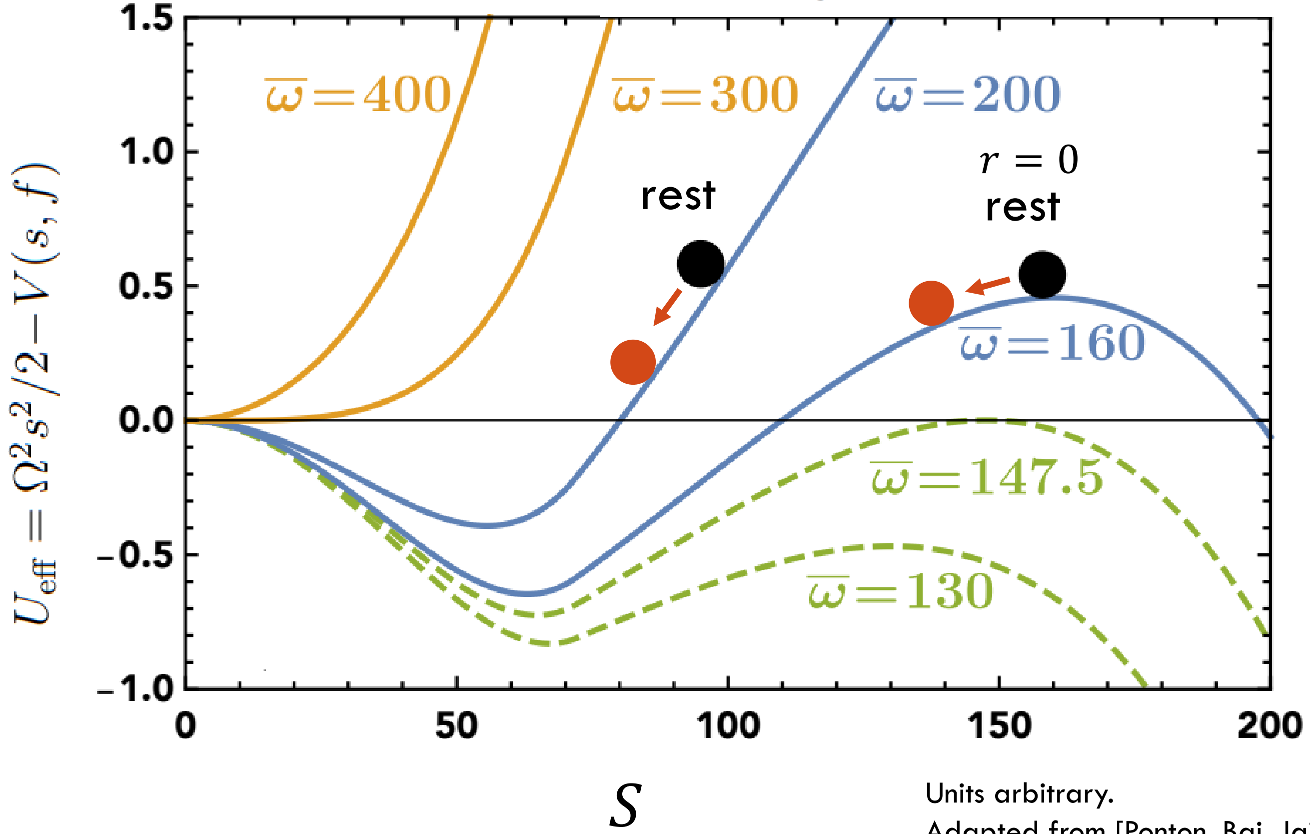

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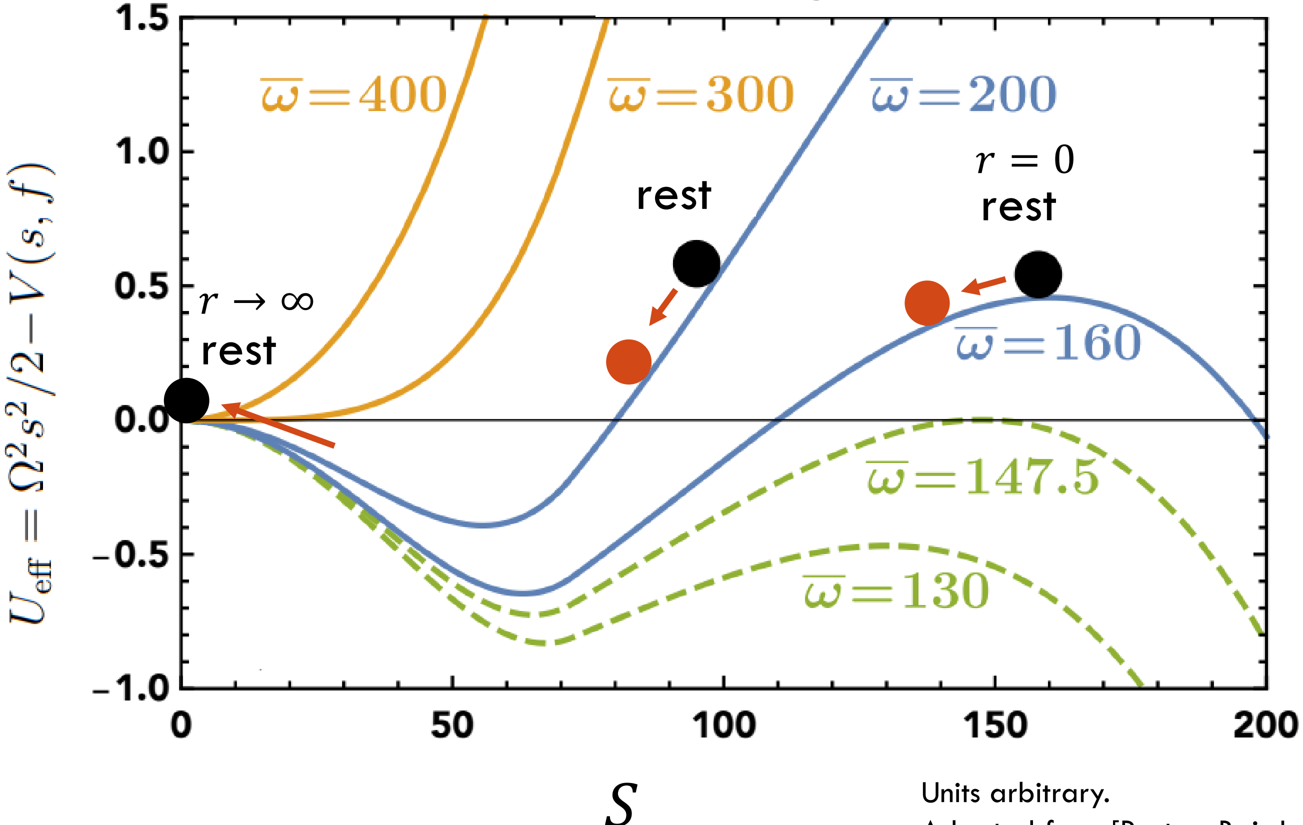

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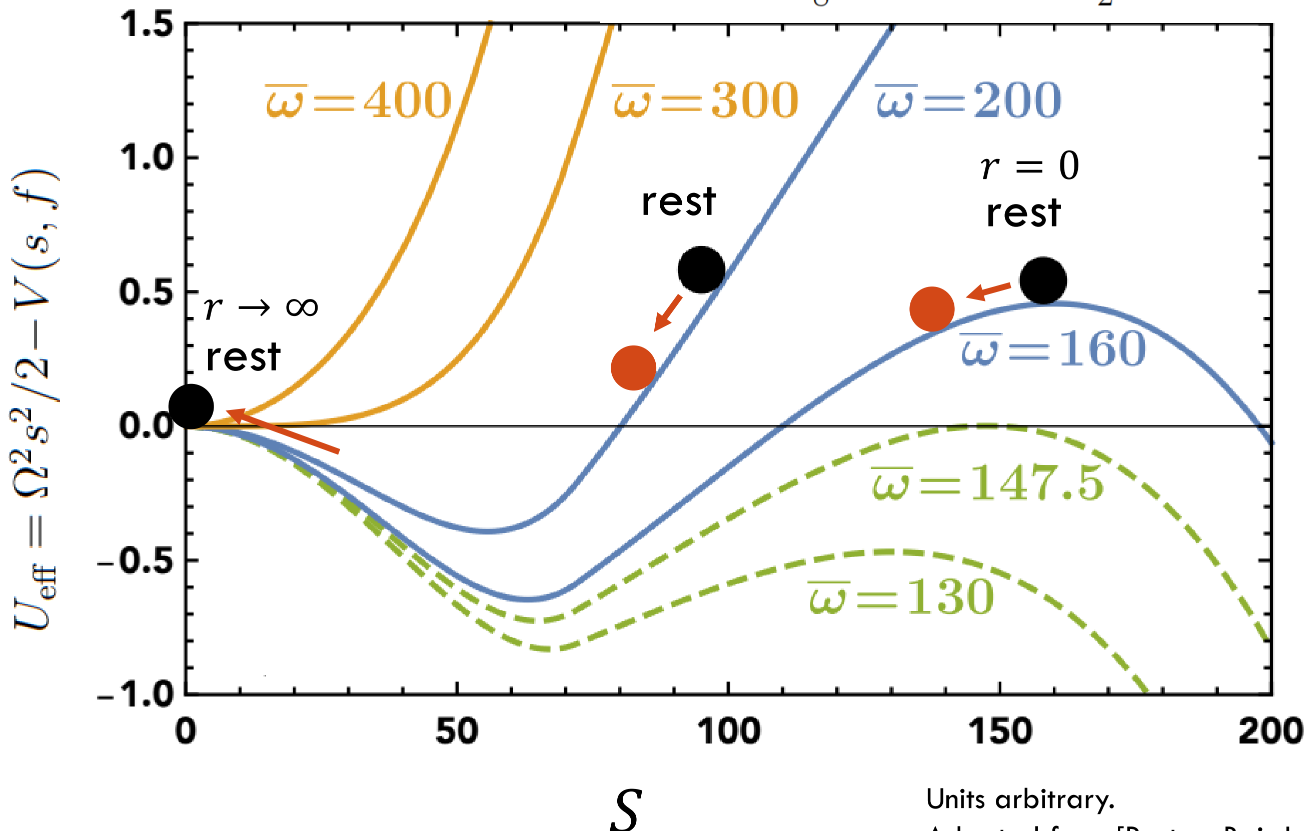

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Conditions for field to roll and come to rest at $S = 0$:

$$\overline{\Omega}^2 \equiv \Omega^2 - \mu_0^2 < \frac{\lambda_{\phi S}}{2}$$

$$\overline{\Omega}^4 > \overline{\Omega}_c^4 \equiv \frac{1}{2}\lambda_S\,\lambda_\phi$$

MASS & CHARGE

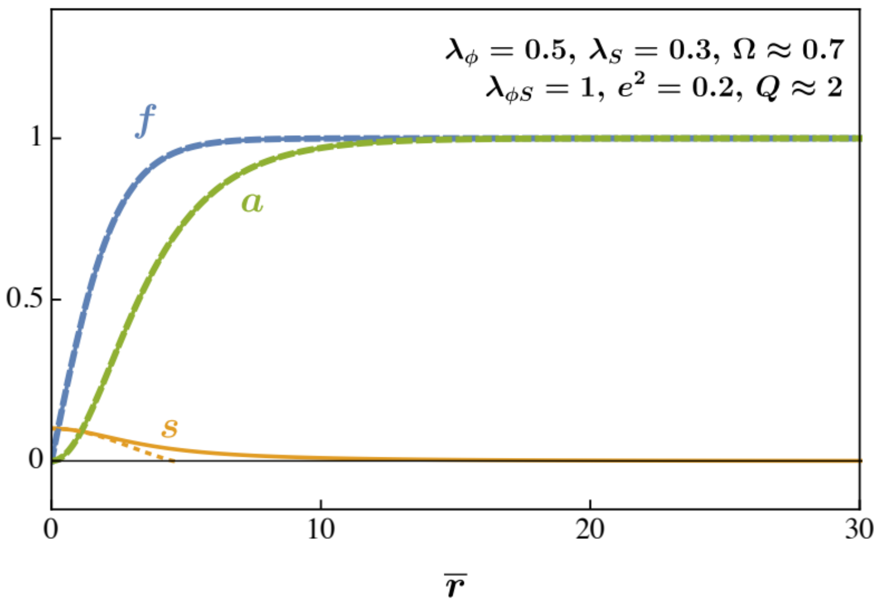
Once a solution is determined, its mass and charge are calculated by:

$$Q = i\int d^3x \,\left(S^\dagger\,\partial_t\,S - S\,\partial_t\,S^\dagger\right) = 4\pi\,\Omega\int_0^\infty d\overline{r}\,\overline{r}^2 s^2$$

$$M = 4\pi v\int_0^\infty d\overline{r}\left\{\frac{1}{2e^2}\left[2a'^2 + \frac{1}{\overline{r}^2}\left(2a - a^2\right)^2\right] + (1-a)^2 f^2 + \overline{r}^2\left[\frac{1}{2}f'^2 + \frac{1}{8}\lambda_\phi (f^2-1)^2\right]\right.$$
$$\left. + \overline{r}^2\left[\frac{1}{2}s'^2 + \frac{1}{2}\Omega^2 s^2 + \frac{1}{4}\lambda_{\phi S} f^2 s^2 + \frac{1}{4}\lambda_S s^4 + \frac{1}{2}\mu_0^2 s^2\right]\right\}.$$

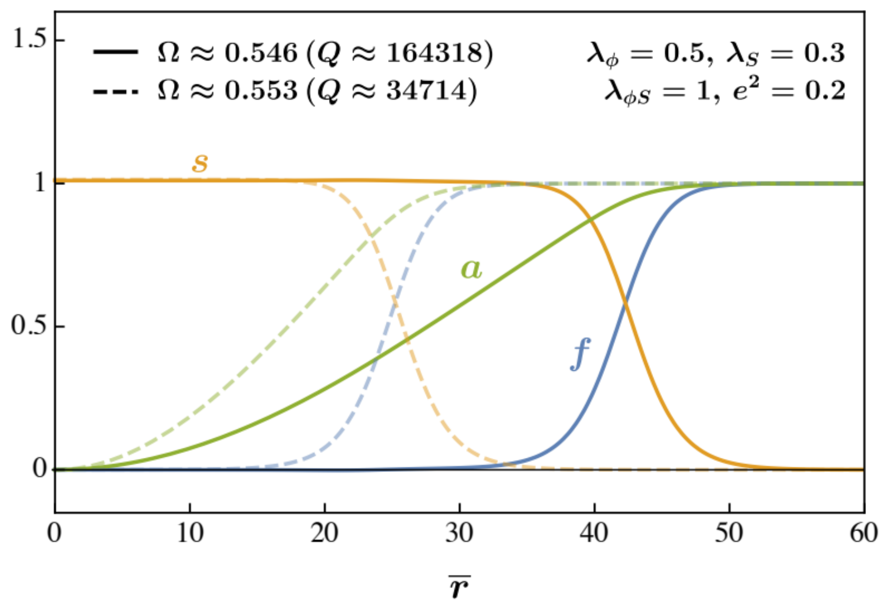
FIELD PROFILES

Small Q



"S particles bound in a monopole"

Large Q



"Monopole bound in a Q-ball"

LARGE Q

Ansatz:

$$f(\bar{r}) \approx \begin{cases} 0\,, & \bar{r} < \bar{r}_b \\ 1\,, & \bar{r} > \bar{r}_b \end{cases}\,, \quad s(\bar{r}) \approx \begin{cases} s_0\,, & \bar{r} < \bar{r}_b \\ 0\,, & \bar{r} > \bar{r}_b \end{cases}\,, \quad a(\bar{r}) \approx \begin{cases} \bar{r}^2/\bar{r}_b^2\,, & \bar{r} < \bar{r}_b \\ 1\,, & \bar{r} > \bar{r}_b \end{cases}\,.$$

Mass ($\mu_0 = 0$):

$$M_{(2,Q)} \approx \underbrace{\frac{304\,\pi\,v}{35\,e^2\,\bar{r}_b}}_{\int d^3x B^2/2} + \underbrace{\frac{4\pi}{3}\bar{r}_b^3 v \left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)}_{\text{Vacuum energy}}$$

Global charge Q:

$$Q \approx \frac{4\pi}{3}\,\bar{r}_b^3\,\Omega\,s_0^2$$

LARGE Q

Ansatz:

$$f(\overline{r}) \approx \begin{cases} 0\,, & \overline{r} < \overline{r}_b \\ 1\,, & \overline{r} > \overline{r}_b \end{cases} \,, \quad s(\overline{r}) \approx \begin{cases} s_0\,, & \overline{r} < \overline{r}_b \\ 0\,, & \overline{r} > \overline{r}_b \end{cases} \,, \quad a(\overline{r}) \approx \begin{cases} \overline{r}^2/\overline{r}_b^2\,, & \overline{r} < \overline{r}_b \\ 1\,, & \overline{r} > \overline{r}_b \end{cases} \,.$$

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Eliminate Ω for Q, minimize w.r.t. $\bar{r}_b, s_0$:

$$\begin{aligned} \overline{r}_b &\approx \frac{(3/\pi)^{1/3}}{2^{5/12}}\,\frac{\lambda_S^{1/12}}{\lambda_\phi^{1/4}}Q^{1/3}\,, \\ M_{(2,Q)} \approx M_{(0,Q)} &\approx Q\,\Omega_c\,v\,, \\ s_0 &\approx \left(\frac{\lambda_\phi}{2\,\lambda_S}\right)^{1/4}\,, \\ \Omega &\approx \Omega_c\,. \end{aligned}$$

$$\overline{\Omega}^4 > \overline{\Omega}_c^4 \equiv \frac{1}{2}\lambda_S\,\lambda_\phi$$

LARGE Q STABILITY

"Monopole bound in a Q-ball":

$$\Delta M = M_{(2,0)} + M_{(0,Q)} - M_{(2,Q)} \approx \frac{4\pi v}{e} Y - \frac{304 \times 2^{5/12}\,\pi^{4/3}}{35 \times 3^{1/3}} \frac{\lambda_\phi^{1/4}}{e^2\,\lambda_S^{1/12}} Q^{-1/3}\, v$$

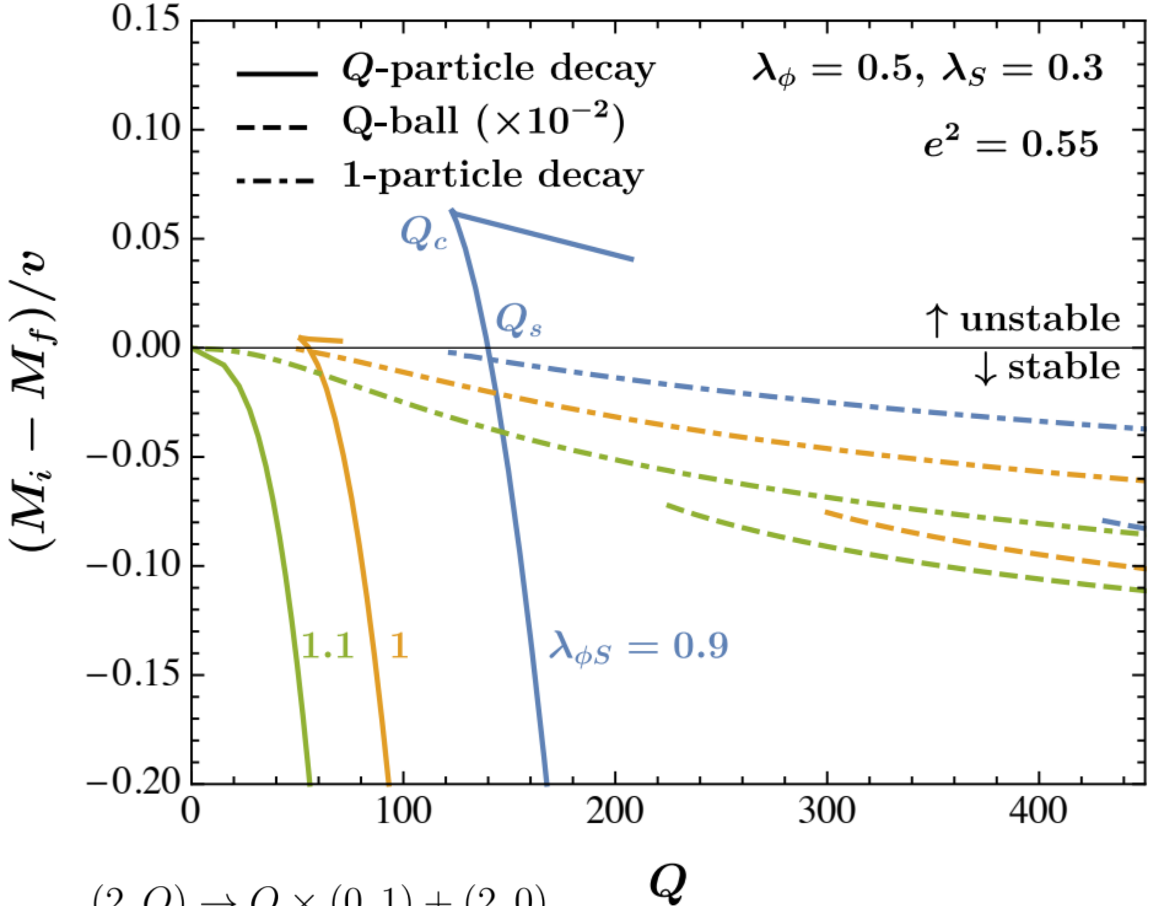
Evaporation into free S particles stable if:

$$(M_{(2,Q)} - M_{(2,0)})/Q \; < \; m_S \; = \; v\sqrt{\lambda_{\phi S}/2},$$

$$\left[\begin{array}{c} M_{(2,Q)} \approx M_{(0,Q)} \;\; \approx \;\; Q\,\Omega_c\, v\,, \\ \overline{\Omega}^2 \equiv \Omega^2 - \mu_0^2 < \dfrac{\lambda_{\phi S}}{2} \end{array} \right]$$

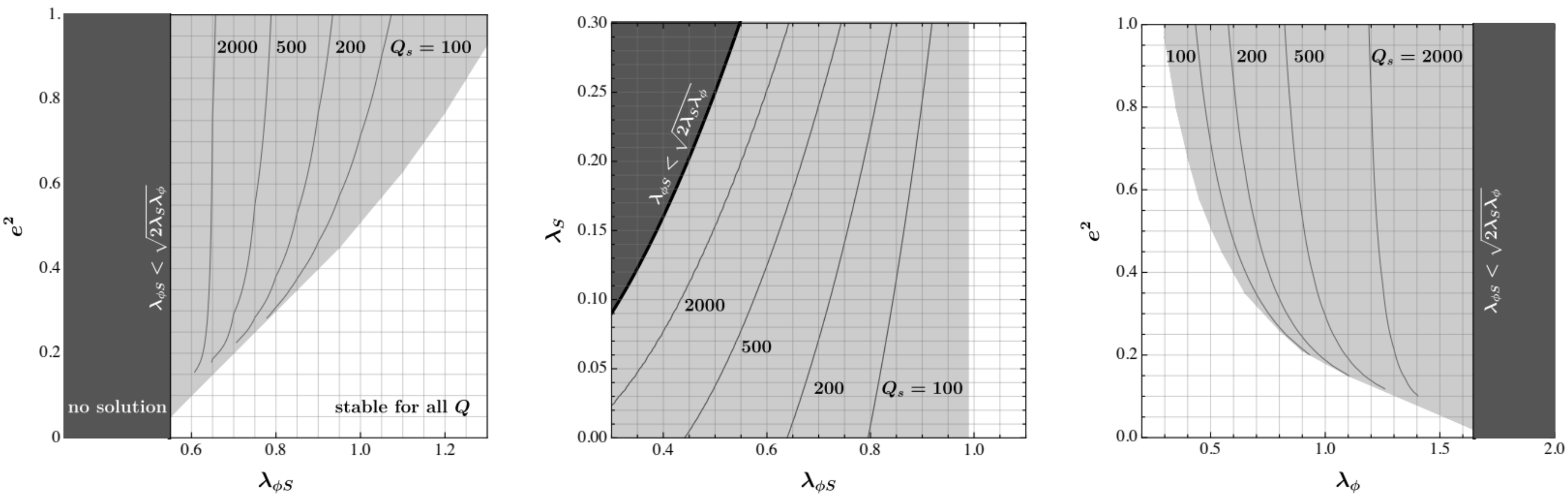
<u>Always stable at sufficiently large Q.</u>

MASS AND STABILITY



$$
\begin{aligned}
Q\text{-particle decay}: &\quad (2, Q) \to Q \times (0, 1) + (2, 0)\,, \\
\text{Q-ball decay}: &\quad (2, Q) \to (0, Q) + (2, 0)\,, \\
\text{1-particle decay}: &\quad (2, Q) \to (2, Q-1) + (0, 1)\,.
\end{aligned}
$$

STABILITY



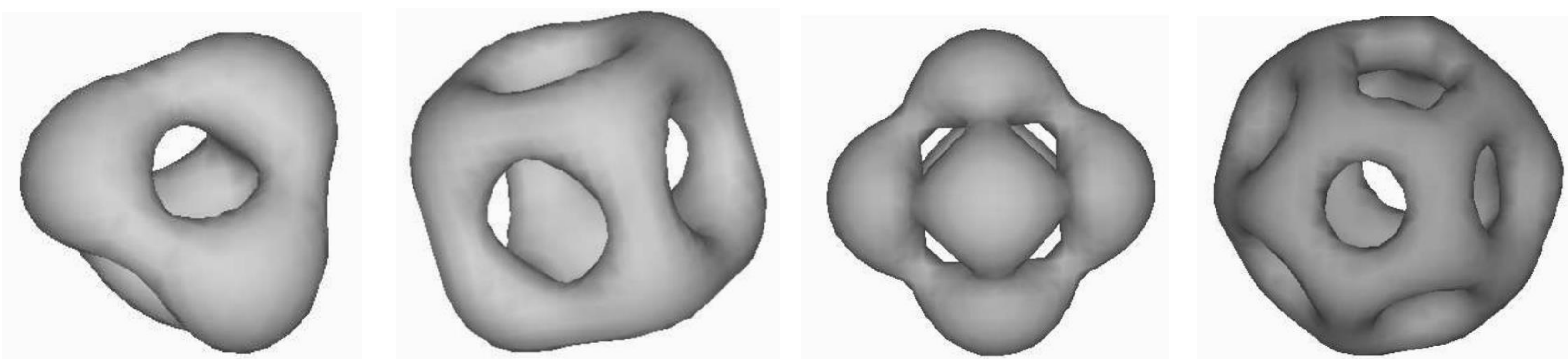
When not being varied:

$$\lambda_\phi = 0.5,\ e^2 = 0.5,\ \lambda_S = 0.3,\ \lambda_{\phi S} = 1, \text{ and } m_{S,0} = 0.$$

LARGER CHARGE

Like-charged monopoles repel and <u>only form bound states in the "BPS limit"</u> (attractive Yukawa interaction cancels the repulsive magnetic force)

Larger-charged monopoles cannot be spherically symmetric [Weinberg Guth 76]



[Houghton Sutcliffe hep-th/9601146, hep-th/9601147]

Or larger "magnetic bags" [Bolognesi hep-th/0512133; Lee Weinberg 0810.4962]

LARGE MONOPOLE CHARGE $q > 2$

$$M_{(2,Q)} \approx \frac{304\,\pi\,v}{35\,e^2\,\overline{r}_b} + \frac{4\pi}{3}\overline{r}_b^3 v\left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)$$

$$\Downarrow$$

$$M_{(q,Q)} \sim \frac{304\pi v(q/2)^2}{35e^2\overline{r}_b} + \frac{4\pi}{3}\overline{r}_b^3 v\left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)$$

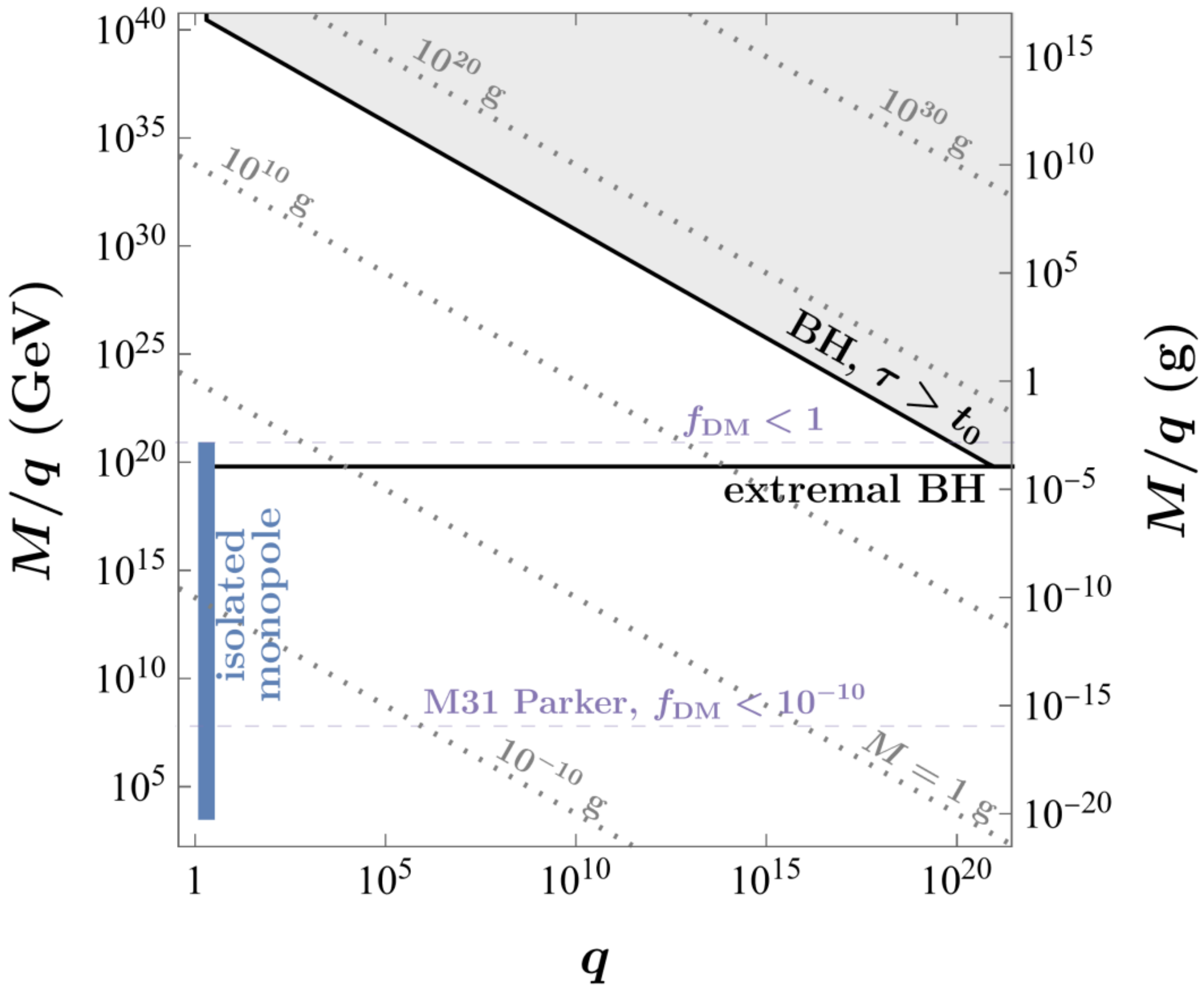
Stability against $(q, Q) \rightarrow (q-2, Q) + (2{,}0)$:

$$q \lesssim 1 + \frac{35}{76}\,e\,\overline{r}_b\,Y \approx \frac{35(3/\pi)^{1/3}}{76\times 2^{5/12}}\,e\,Y\frac{\lambda_S^{1/12}}{\lambda_\phi^{1/4}}Q^{1/3}$$

Minimum stable mass for stability:

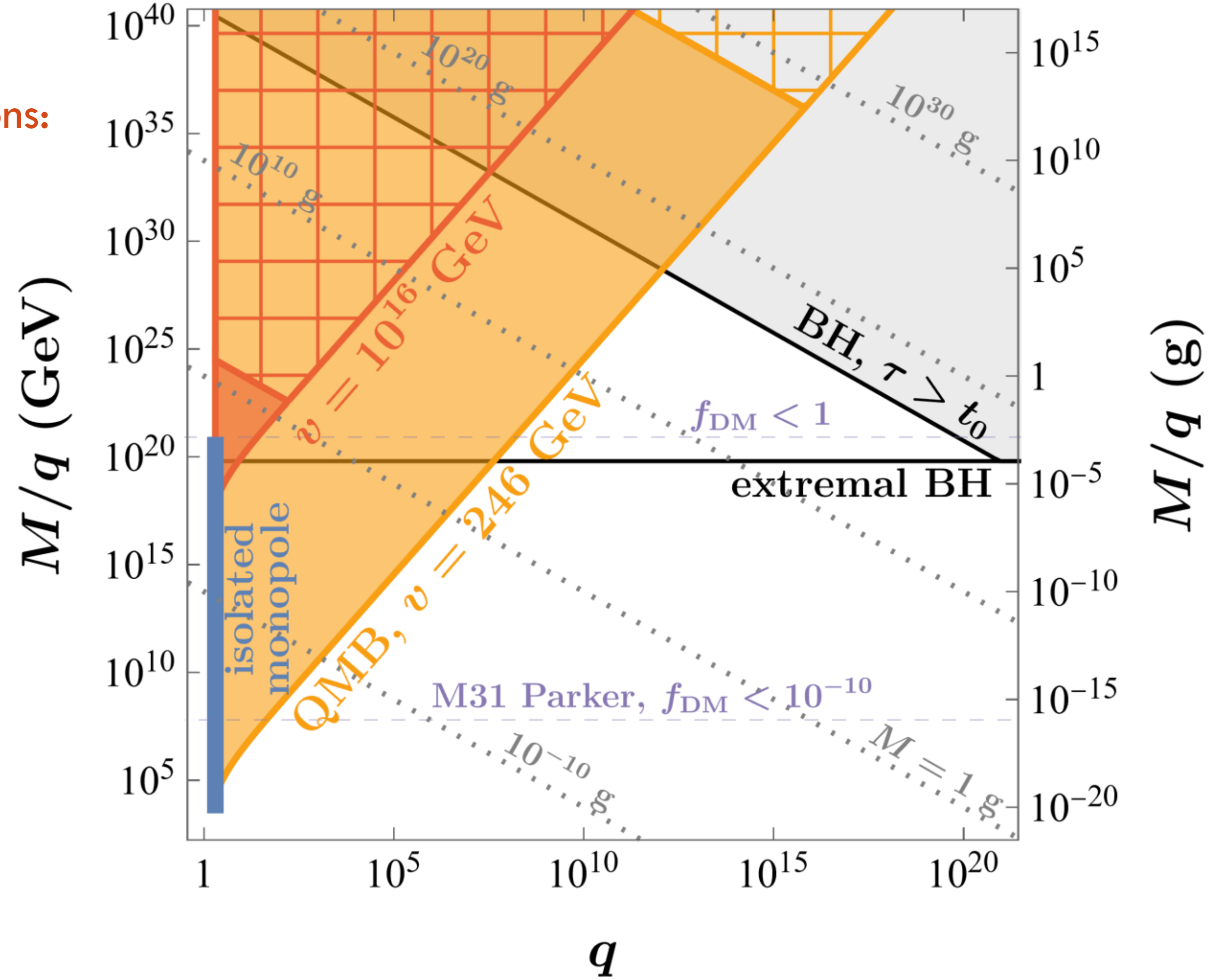
$$M_{(q,Q)} \gtrsim 21\frac{\lambda_\phi}{e^3Y^3}q^3 v$$

ALLOWED MASS AND CHARGE



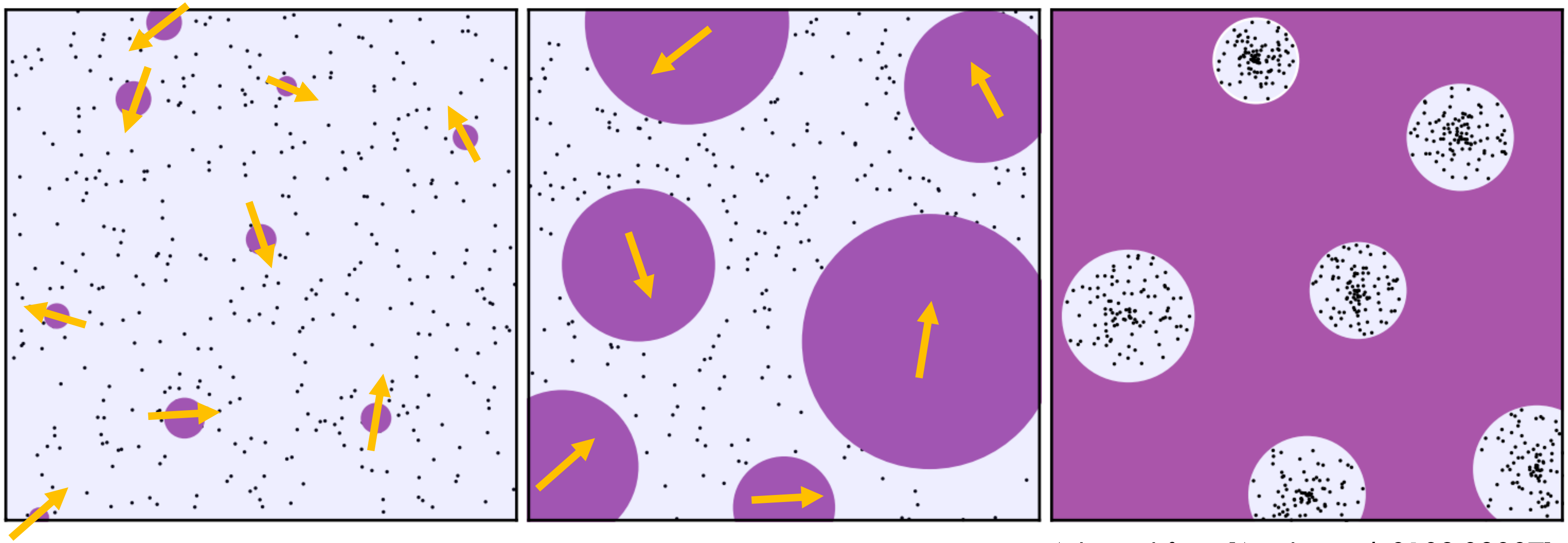
ALLOWED MASS AND CHARGE

Hatched regions:
$R > r_s$



FORMATION

In a first order phase transition, monopoles and Q-balls tend to form in the same places.



Adapted from [Asadi, *et. al.* 2103.09827]

Q-monopole-balls have smaller Q_S than Q-balls.

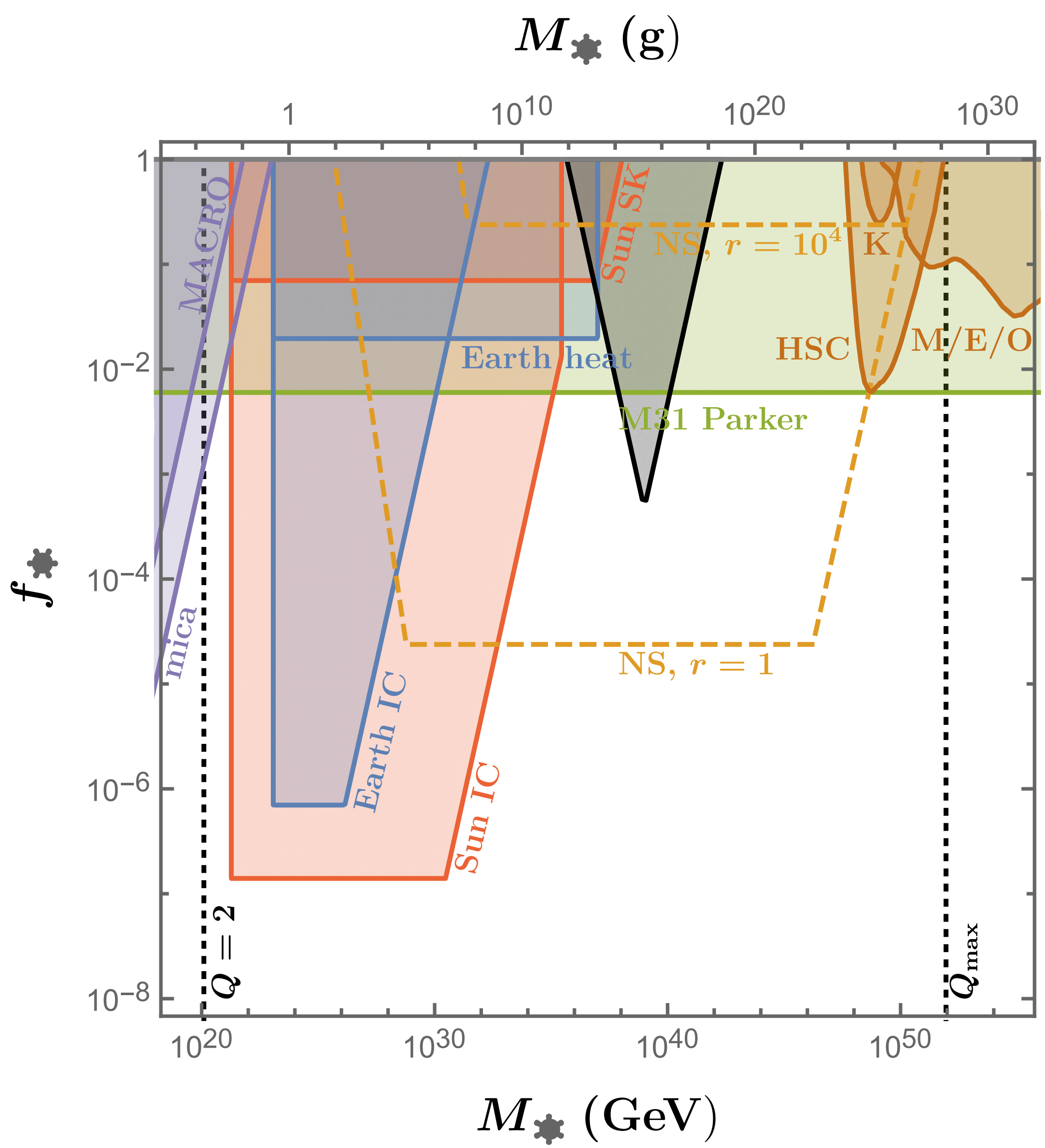
$$\langle Q_{\rm QMB}\rangle \sim \max\left[(3\times 10^{16})\,v_3^{-3/2}\left(\frac{\lambda_{\phi S}}{3}\right)^7,\quad \eta_Q\,(1\times 10^{33})\,v_3^{-3}\left(\frac{\lambda_{\phi S}}{3}\right)^{14}\right]$$

SUMMARY

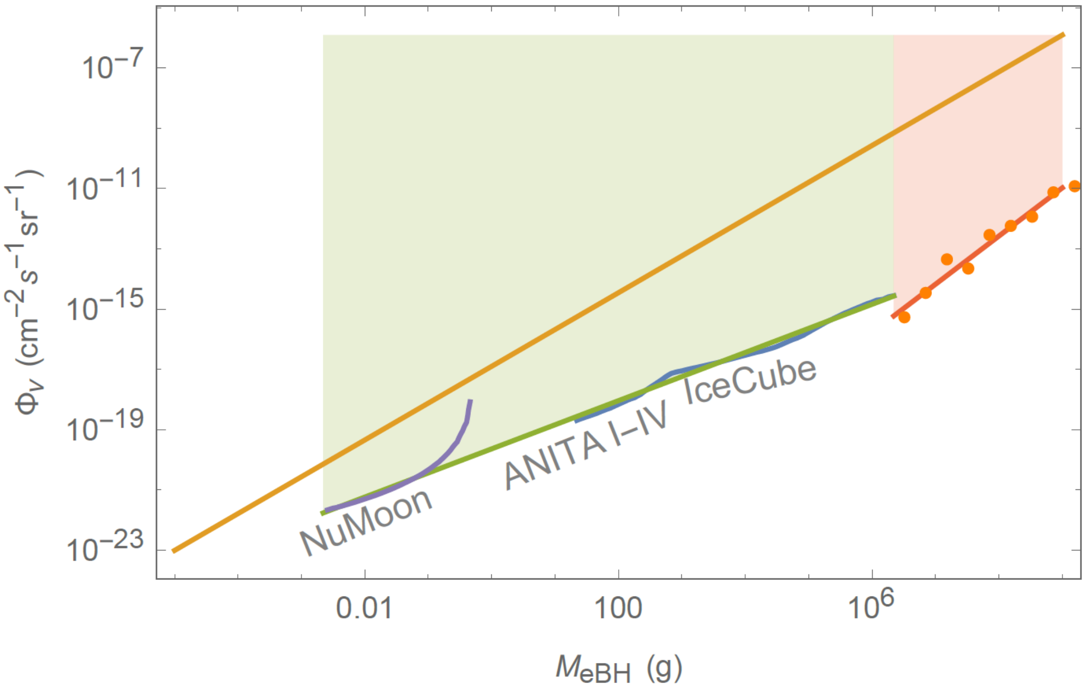
- Q-monopole balls are generally expected when monopoles exist with a “Higgs-portal type” coupling to a scalar
- They could greatly expand the possible charge and mass for relic monopoles

BACKUP

BOUNDS ON EXTREMAL MAGNETIC BHS



BINARY MERGERS OF EXTREMAL BHS



Simplifying assumptions:

- No binary disruptions
- Monochromatic mass function
- Only primary particles, no cascade decays

Expect binary disruptions to alleviate these constraints. [Raidal, *et. al.* 1812.01930]

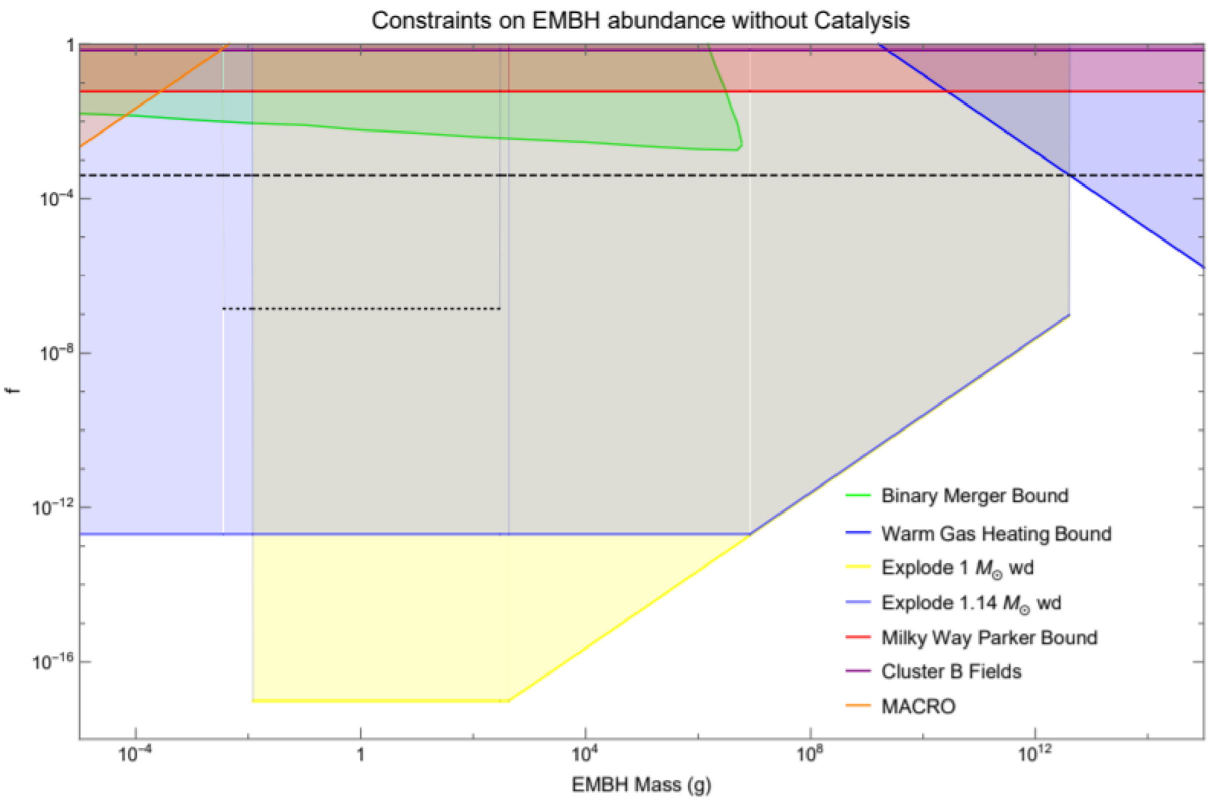
N-body simulations required.

[Bai, **NO** 1906.04858]

BOUNDS ON MAGNETIC BHS

Other bounds pointed out in [Diamond, Kaplan 2103.01850]

- Gas cloud heating
- White dwarf capture ⇒ destruction



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