Using Small-scale Lensing Anisotropies to Constrain Dark Matter

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With

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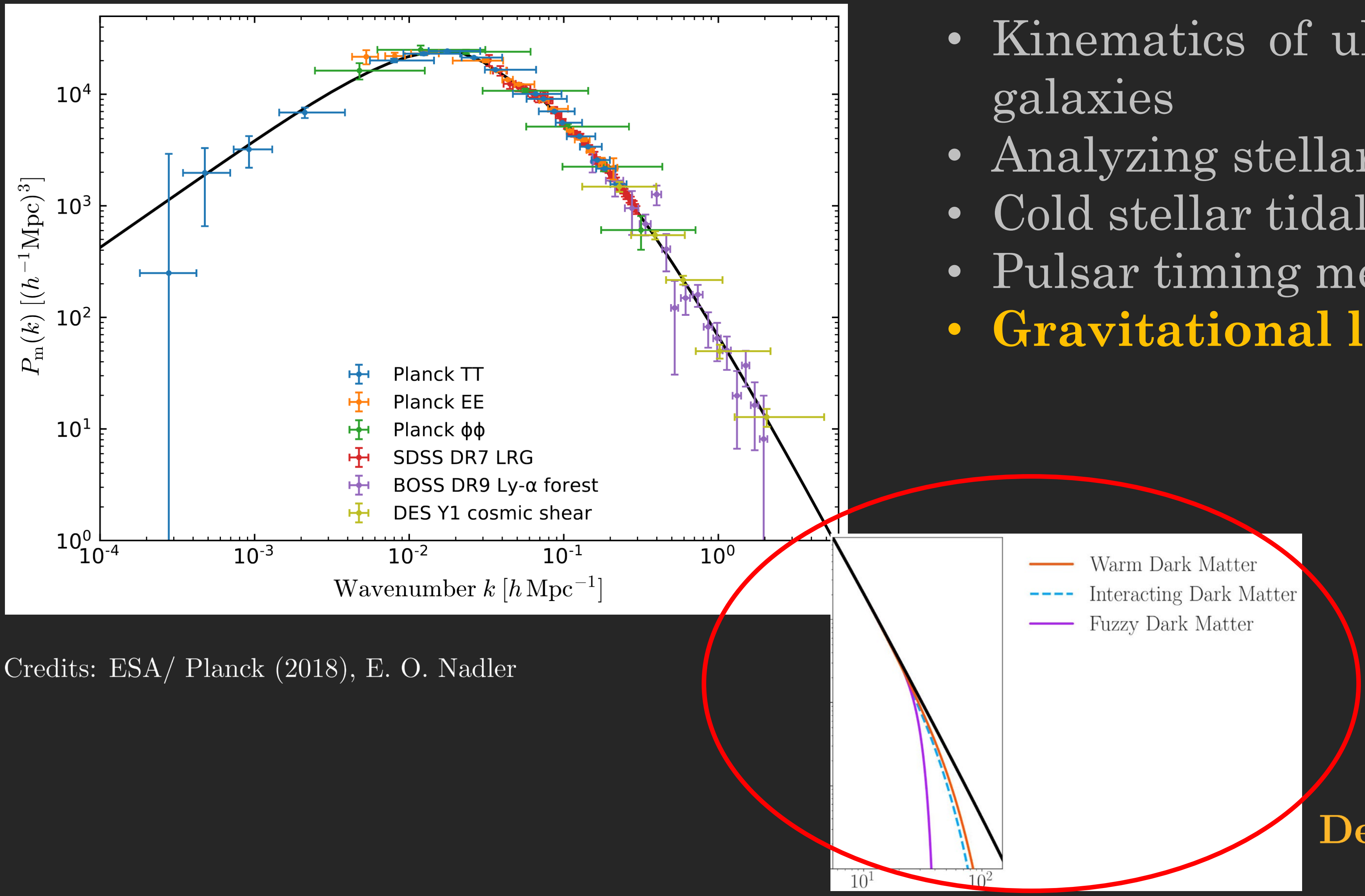
Probes of Dark Matter

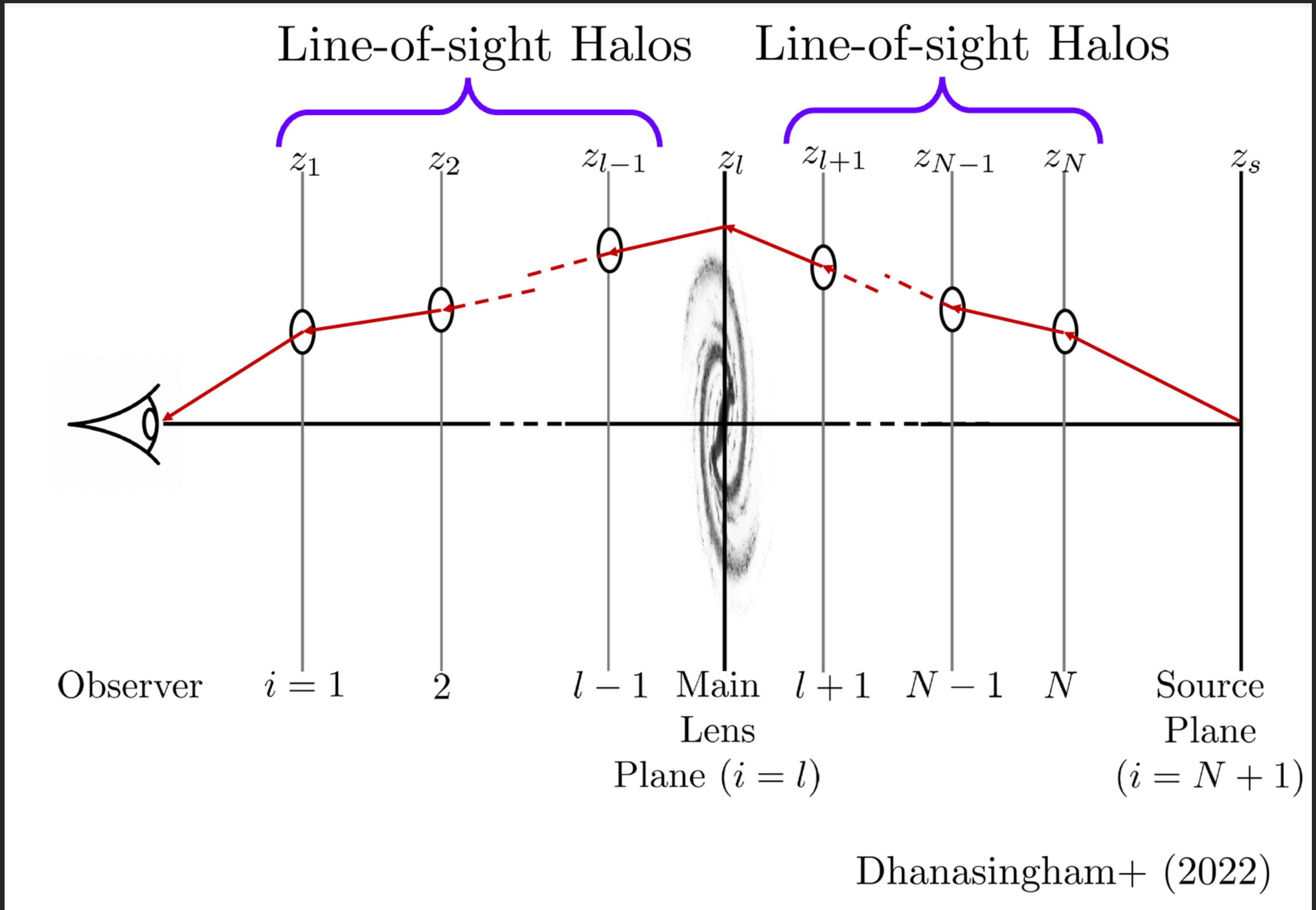


Credits: ESA/ Planck (2018), E. O. Nadler

- Kinematics of ultra-faint dwarf galaxies
- Analyzing stellar wakes
- Cold stellar tidal streams
- Pulsar timing measurements
- **Gravitational lensing**

Deviations from CDM

Multiplane Gravitational Lensing



$$\mathbf{u} = \mathbf{x}_1 - \sum_{i=1}^{N} \boldsymbol{\alpha}_i(\mathbf{x}_i)$$

Recursive multiplane lens equation:

$$\mathbf{x}_j = \mathbf{x}_1 - \sum_{i=1}^{j-1} \beta_{ij} \boldsymbol{\alpha}_i(\mathbf{x}_i)$$

where

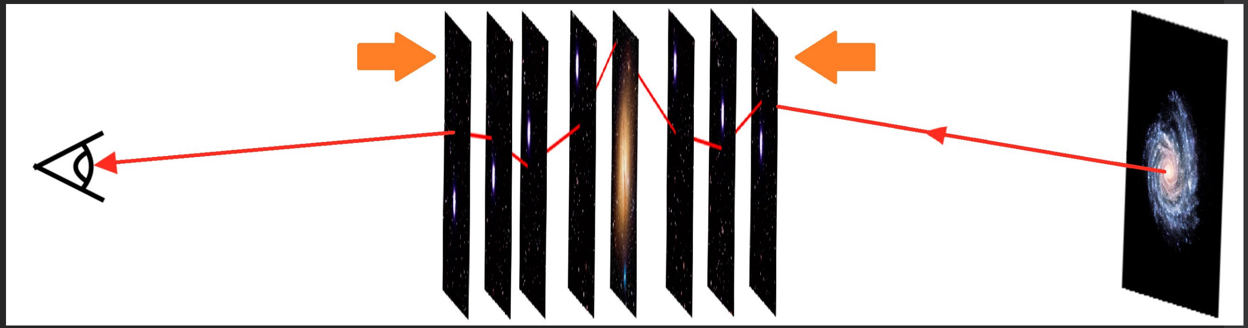
$$\beta_{ij} = \frac{D_{ij} D_s}{D_j D_{is}}$$

A Simple Way: "Effective Multiplane Lensing"

$$\boldsymbol{\alpha}_{\rm eff}(\mathbf{x}) = \boldsymbol{\nabla}\phi_{\rm eff}(\mathbf{x}) + \boldsymbol{\nabla} \times \mathbf{A}_{\rm eff}(\mathbf{x})$$

$$\mathbf{u} = \mathbf{x} - \boldsymbol{\alpha}_{\rm eff}(\mathbf{x})$$

where
$\phi_{\rm eff}$ - effective scalar potential
$\mathbf{A}_{\rm eff}$ - effective vector potential



$\mathbf{A}_{\rm eff}$ points in line-of-sight direction. Therefore, introduce $\zeta_{\rm eff}(\mathbf{x})$ such that $\mathbf{A}_{\rm eff}(\mathbf{x}) = \zeta_{\rm eff}(\mathbf{x})\hat{\mathbf{z}}$

Divergence Component: $\kappa_{\rm div} \equiv \frac{1}{2}\boldsymbol{\nabla}\cdot\boldsymbol{\alpha}_{\rm eff} - \kappa_0 = \frac{1}{2}\nabla^2\phi_{\rm eff} - \kappa_0$

Curl Component: $\kappa_{\rm curl} \equiv \frac{1}{2}\boldsymbol{\nabla}\times\boldsymbol{\alpha}_{\rm eff}\cdot\hat{\mathbf{z}} = -\frac{1}{2}\nabla^2\zeta_{\rm eff}$

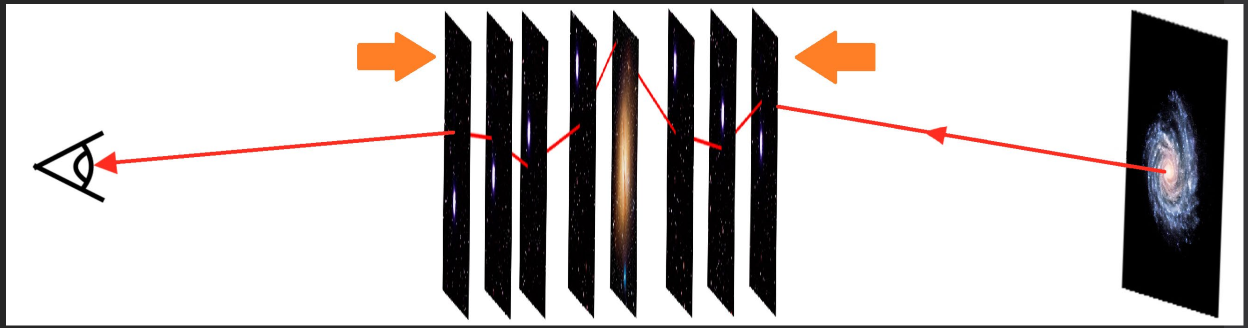
Gilman+ (2019),
Çağan Şengül+ (2020)

A Simple Way: "Effective Multiplane Lensing"

$$\boldsymbol{\alpha}_{\text{eff}}(\mathbf{x}) = \boldsymbol{\nabla}\phi_{\text{eff}}(\mathbf{x}) + \boldsymbol{\nabla} \times \mathbf{A}_{\text{eff}}(\mathbf{x})$$

$$\mathbf{u} = \mathbf{x} - \boldsymbol{\alpha}_{\text{eff}}(\mathbf{x})$$

where
ϕ_{eff} - effective scalar potential
$\mathbf{A}_{\text{eff}}$ - effective vector potential



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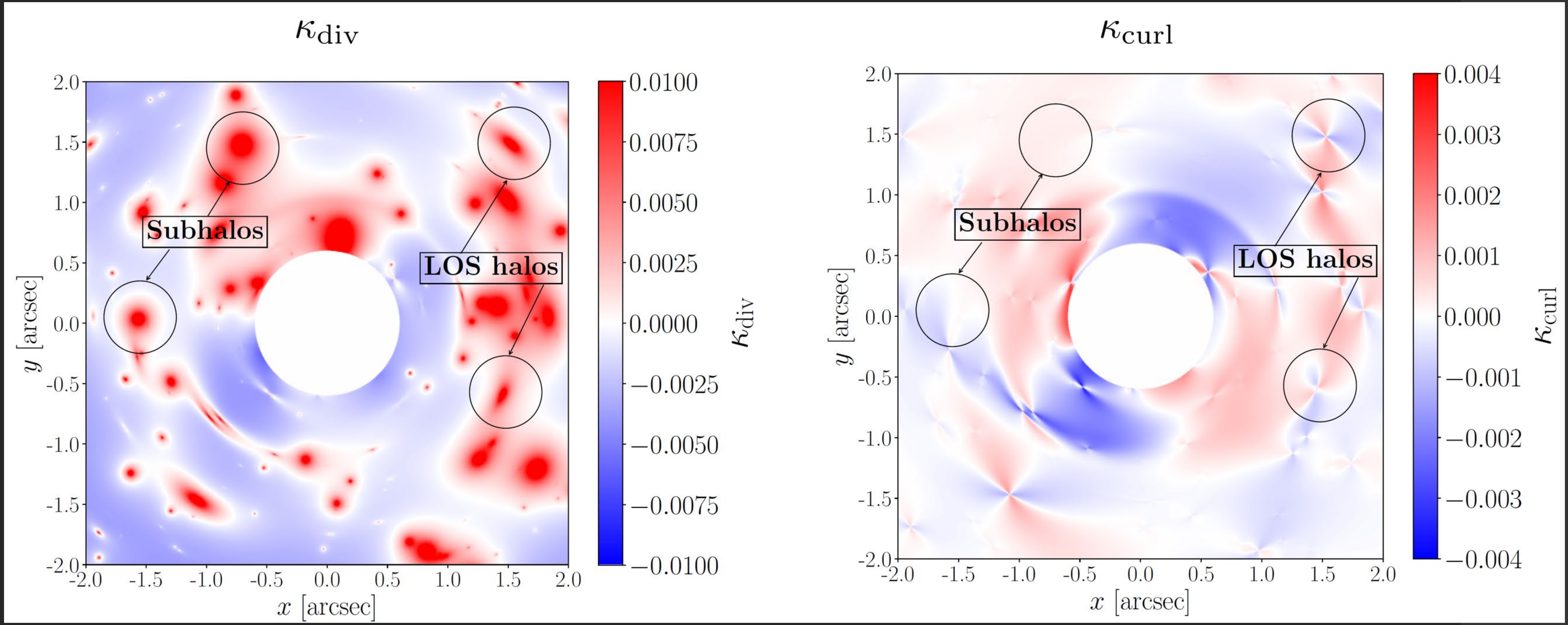
vanish for single-plane lensing

Divergence and curl of the effective deflection field



(Gilman+ (2021), Çağan Şengül+ (2020))

The Two-point Correlation Function

The correlation function

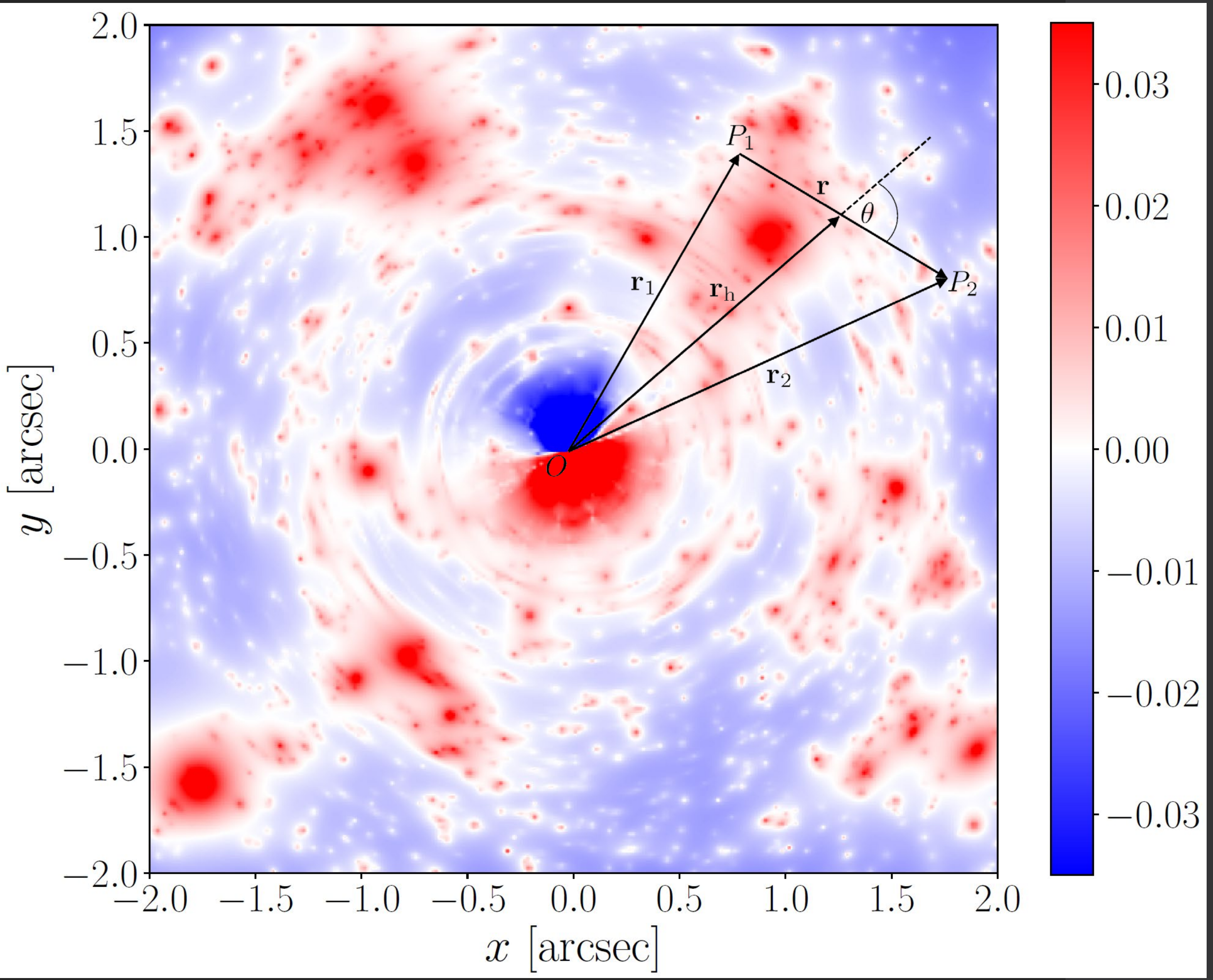
$$\xi(\mathbf{r}) = \xi(\mathbf{r}_2 - \mathbf{r}_1) = \frac{1}{A}\int_A d^2\mathbf{r}_1\ \kappa_{\rm div}(\mathbf{r}_1)\ \kappa_{\rm div}(\mathbf{r} + \mathbf{r}_1)$$

$$\xi(\mathbf{r}) = \xi(r, \mu_{\mathbf{r}}) = \sum_{\ell=0}^{\infty} \xi_\ell(r) T_\ell(\mu_{\mathbf{r}}), \text{ where } \mu_{\mathbf{r}} = \cos\theta$$

The Correlation Multipoles

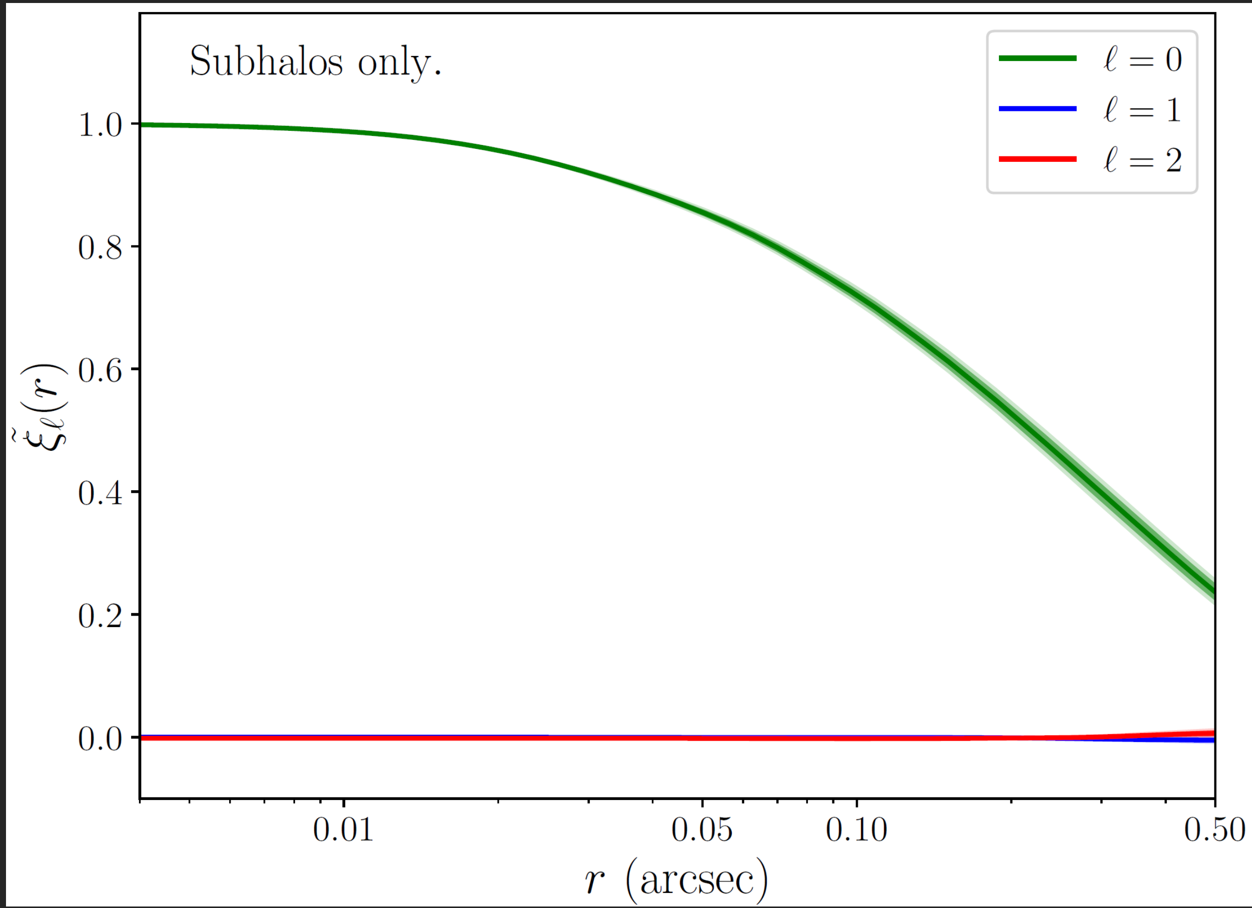
$$\xi_\ell(r) = \frac{2-\delta_{\ell 0}}{\pi}\int_{-1}^{1} d\mu_{\mathbf{r}}\ \frac{\xi(r,\mu_{\mathbf{r}})T_\ell(\mu_{\mathbf{r}})}{\sqrt{1-\mu_{\mathbf{r}}^2}},$$

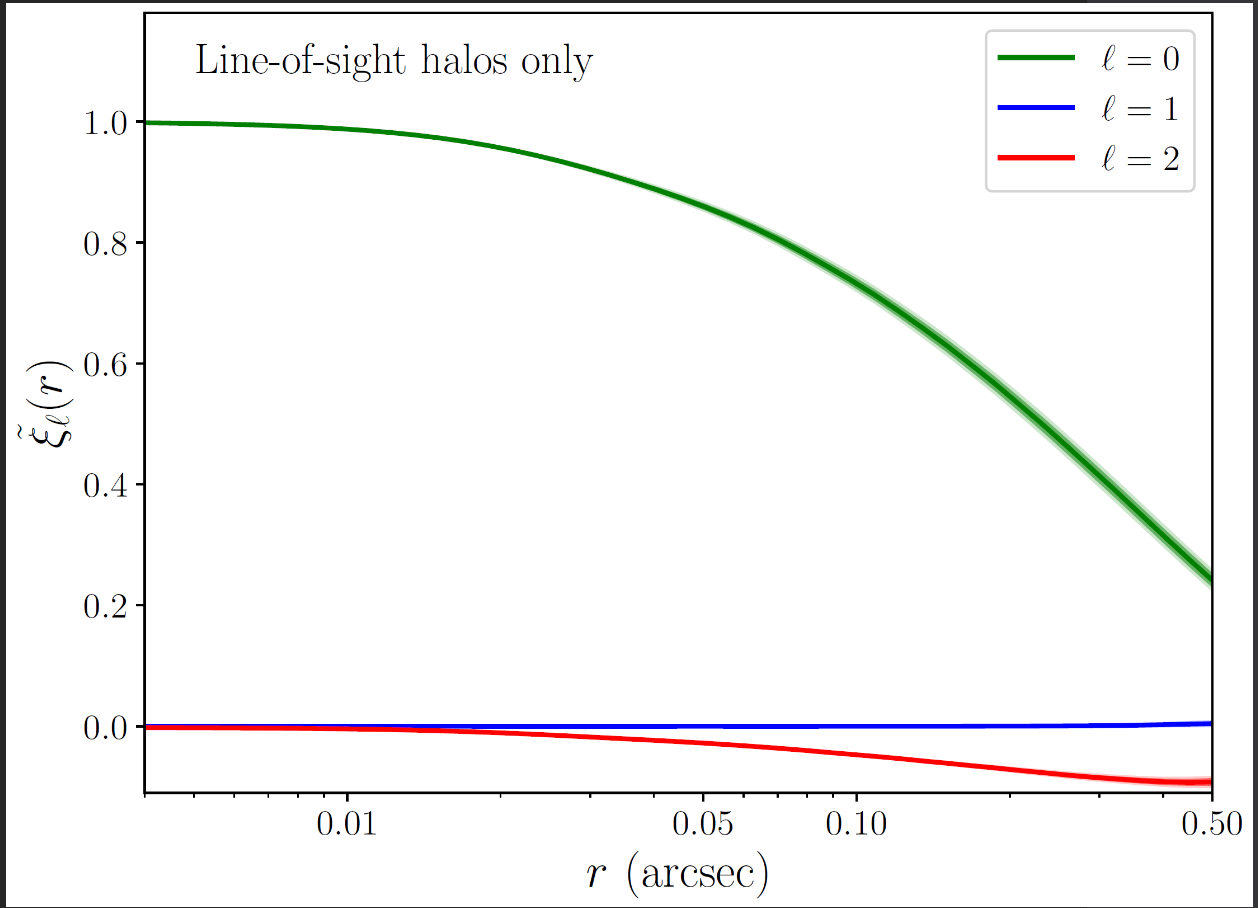
(E.g., Hezaveh+ (2016b), Brennan+ (2019),
Díaz Rivero+ (2018a, 2018b),
Cyr-Racine+ (2019), Bayer+ (2018),
Çağan Şengül+ (2020))



Dhanasingham+ (2022)

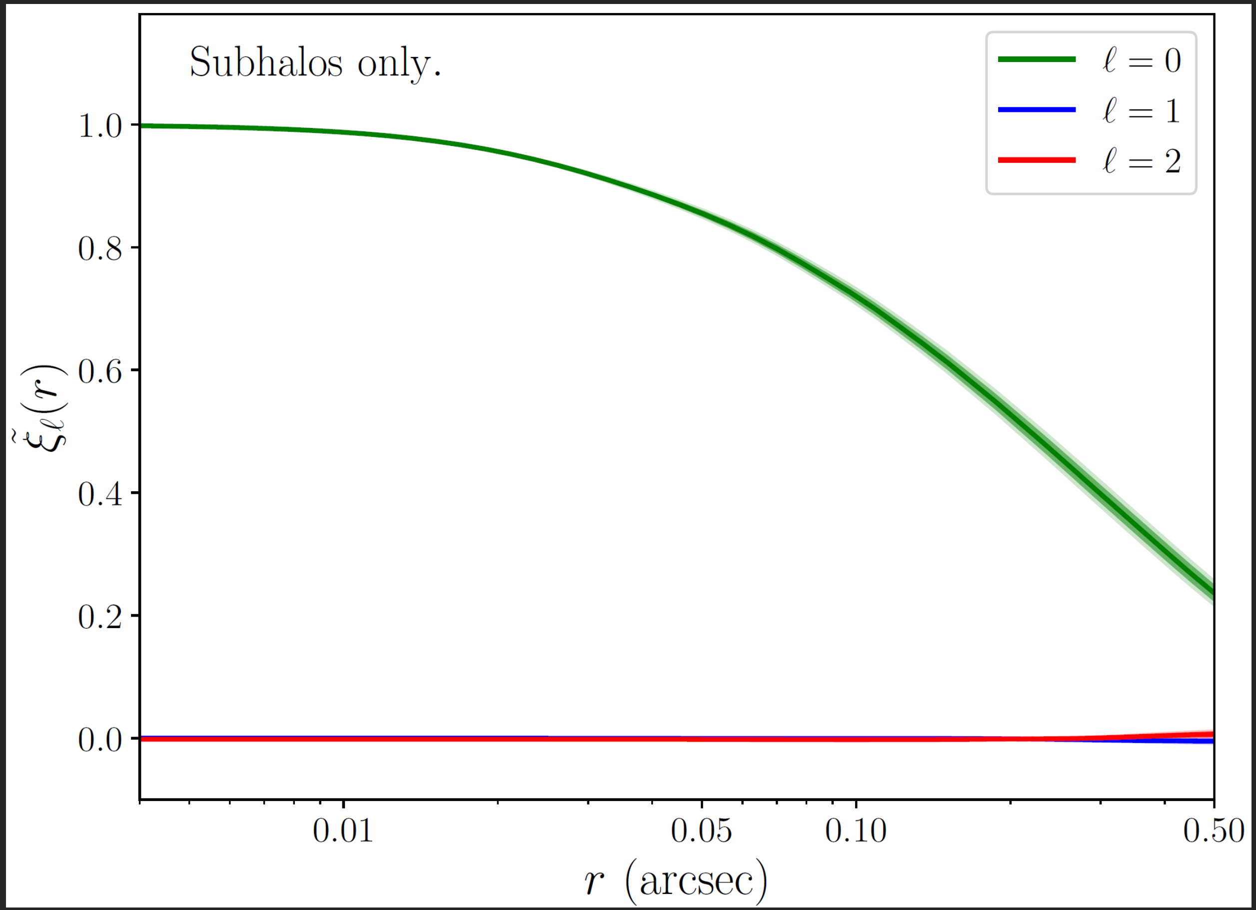
Masked-normalized Correlation Function Multipoles (ξ_ℓ) of κ_{div} Fields

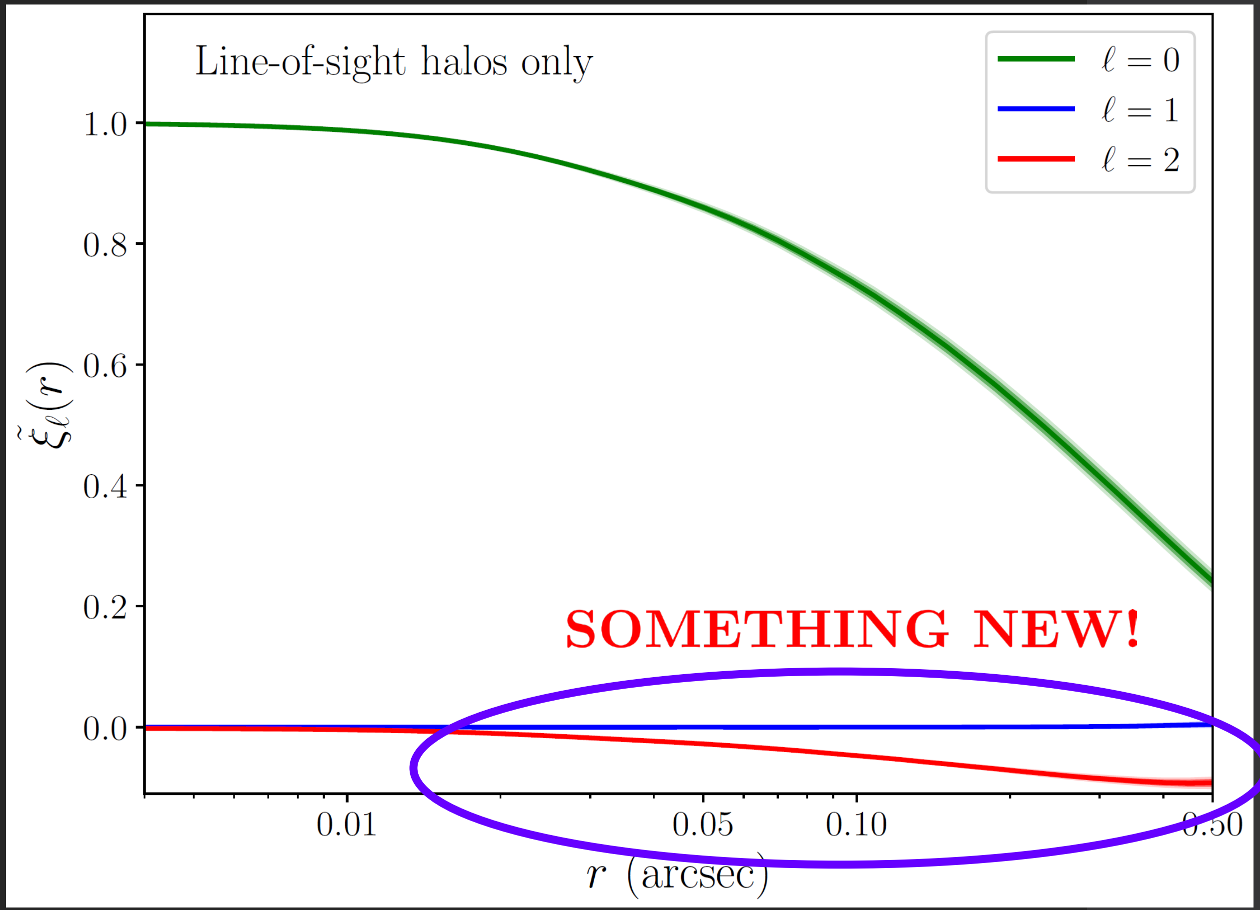




Dhanasingham+ (2022)

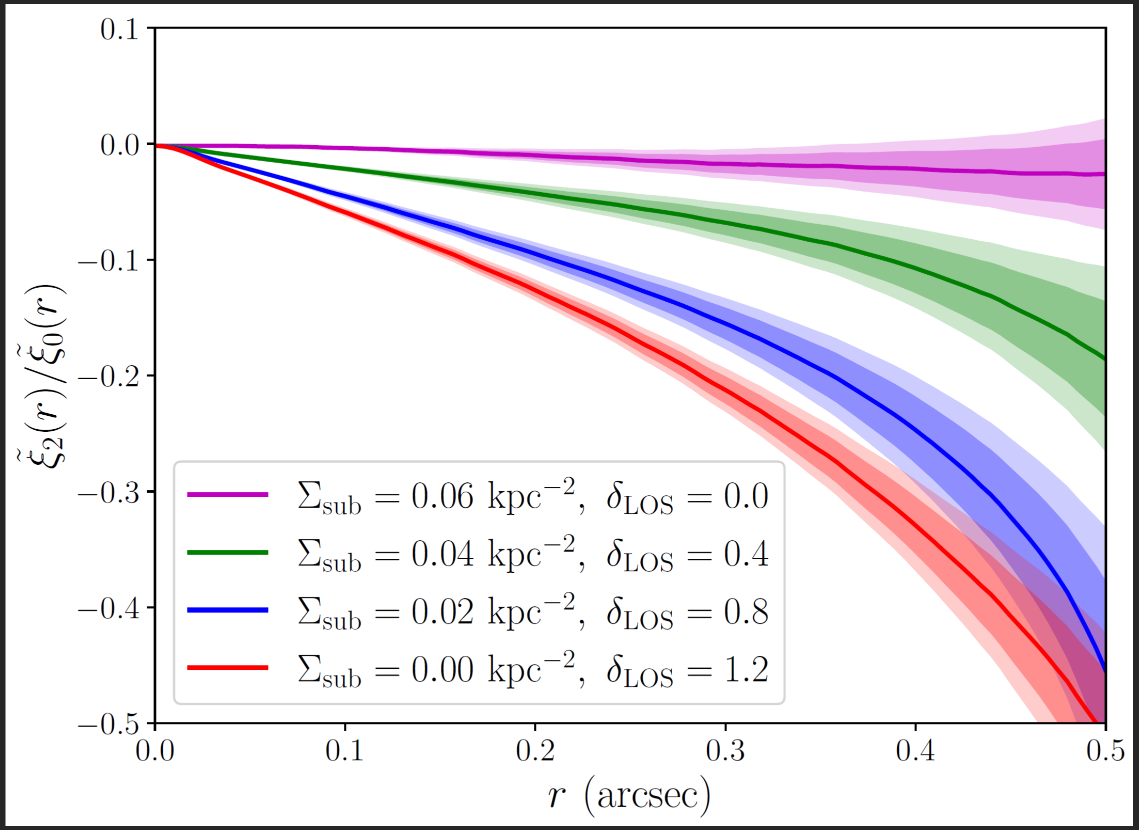
Masked-normalized Correlation Function Multipoles (ξ_ℓ) of $\kappa_{\rm div}$ Fields



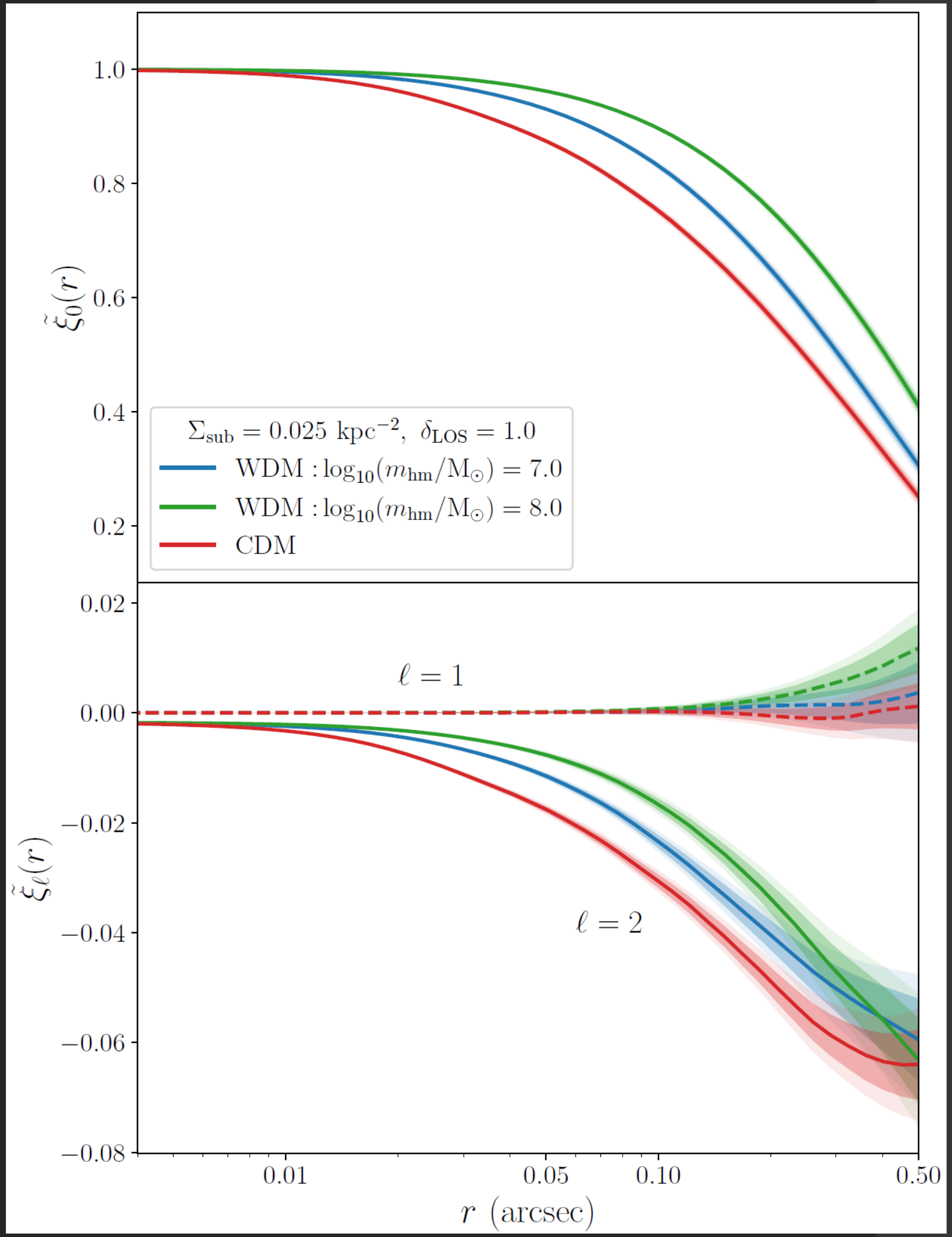


Dhanasingham+ (2022)

Multipoles probe dark matter microphysics



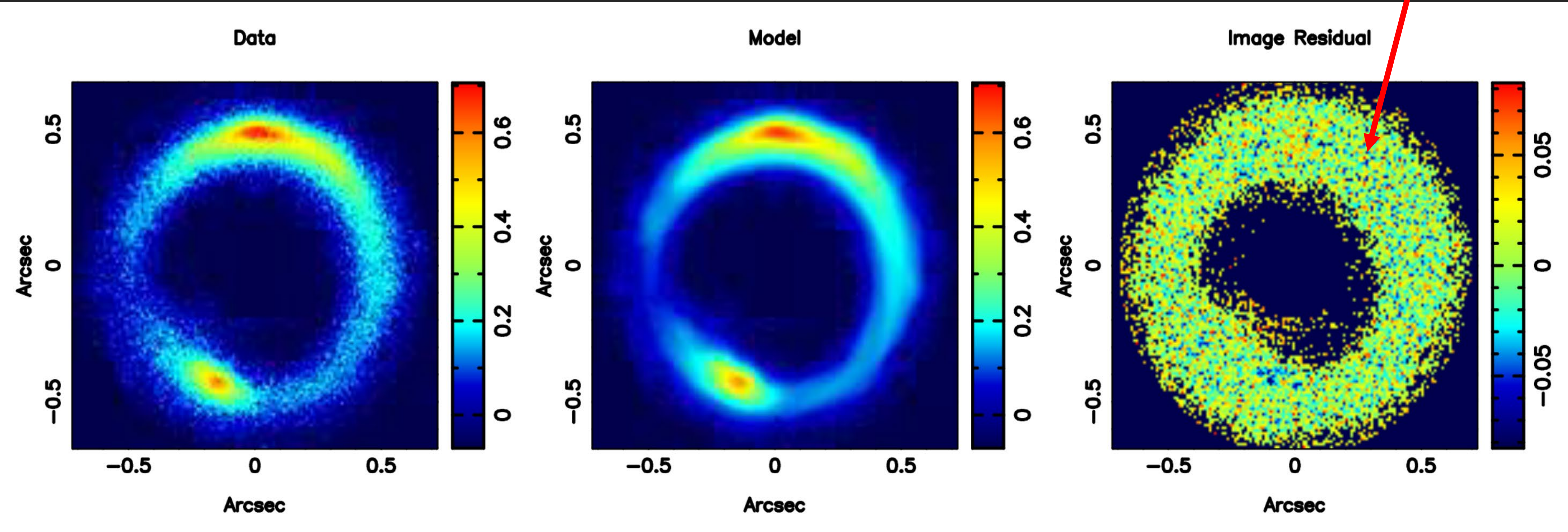
Dhanasingham+ (2022)



Dhanasingham & Cyr-Racine (in prep.)

Lens Modeling & Residuals

Noise + (Sub+LOS) Signal +
Source & Macrolens Mismodelling



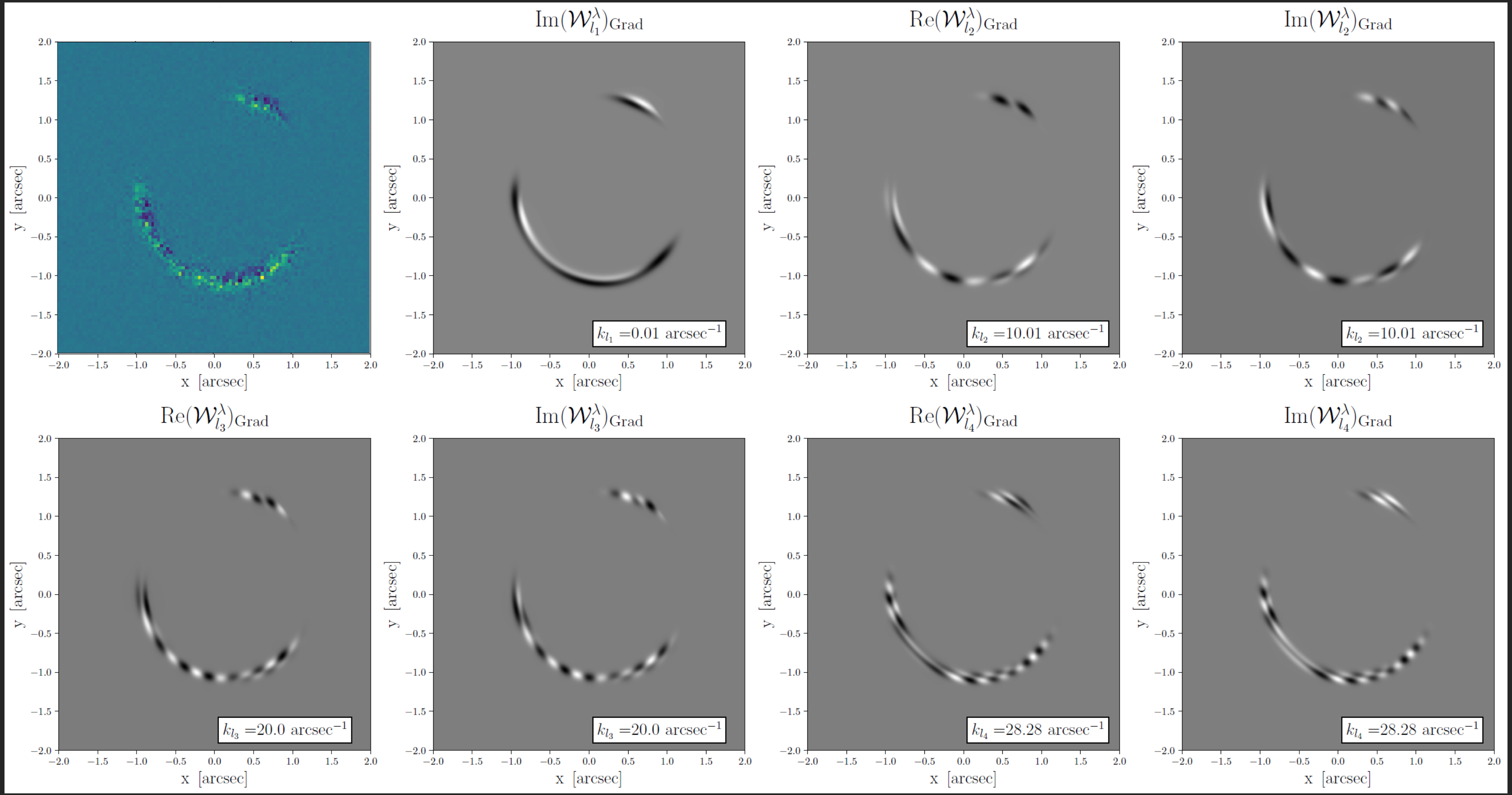
Vegetti+ (2012)

$$\delta O_{\mathrm{obs}} \approx -\nabla S_{\mathrm{rec}}|_{\mathbf{x}-\boldsymbol{\alpha}_{\mathrm{bf}}(\mathbf{x})} \cdot \delta\boldsymbol{\alpha}(\mathbf{x}) + \delta S[\mathbf{x}-\boldsymbol{\alpha}_{\mathrm{bf}}(\mathbf{x})] + N(\mathbf{x})$$

Residuals And Decomposition

$$\delta O_{\rm obs}(\mathbf{x}) = \delta O_{\rm div}(\mathbf{x}) + \delta O_{\rm curl}(\mathbf{x}) + N(\mathbf{x}) \approx \underbrace{\sum_{m=1}^{N_{\rm modes}} \mathcal{B}_m \mathcal{W}_m}_{\delta O_{\rm div}} + N(\mathbf{x})$$

where $\mathcal{W}_m(\mathbf{x}) = -\nabla_{\mathbf{u}} S(\mathbf{u})|_{\mathbf{u}=\mathbf{x}-\nabla\phi_o(\mathbf{x})} \cdot \nabla\phi_m(\mathbf{x})$.

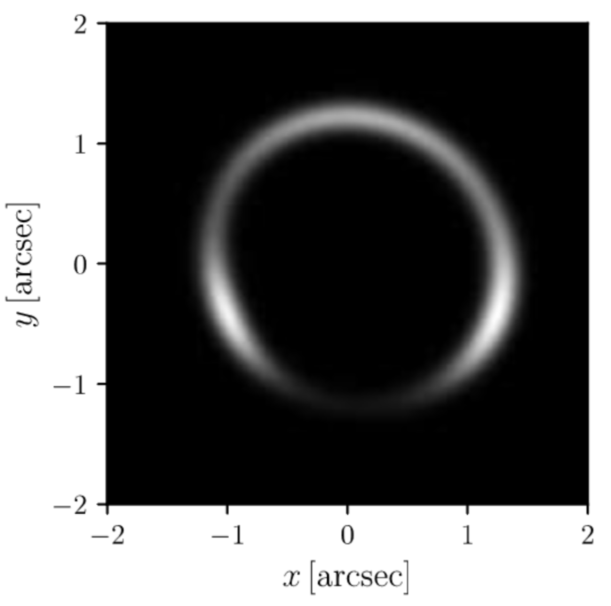


(Cyr-Racine+ (2019), Hezaveh+ (2016b))

Test Sensitivity Using Mock Observations

- **Two instruments**

	HST-like	ELT-like
Resolution (arcsec/pixel)	0.05	0.01
FWHM_{PSF} (arcsec)	0.07	0.15
T_{exp} (hours)	1.0	1.0
σ_{bkg} (counts/s/pixel)	0.005	0.02
N_{obs}	70	4



Dhanasingham+ (2022)

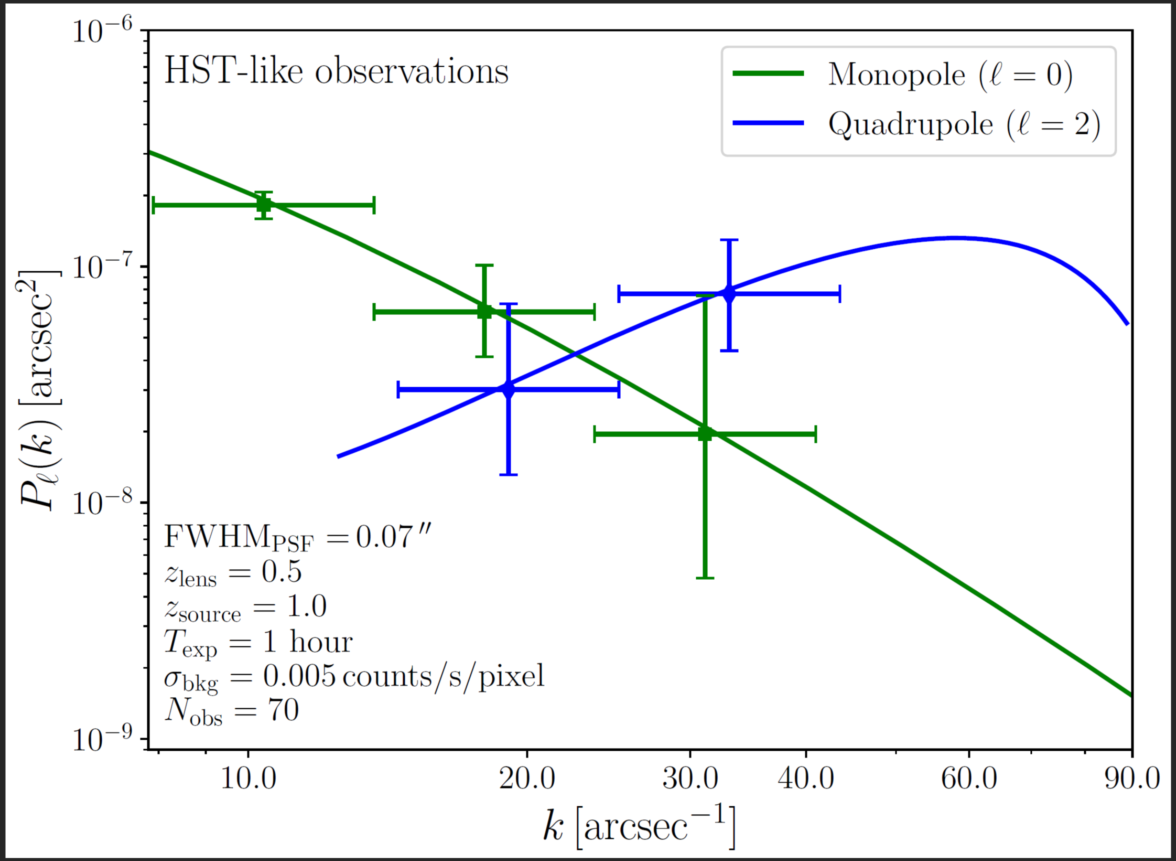
- **Likelihood**

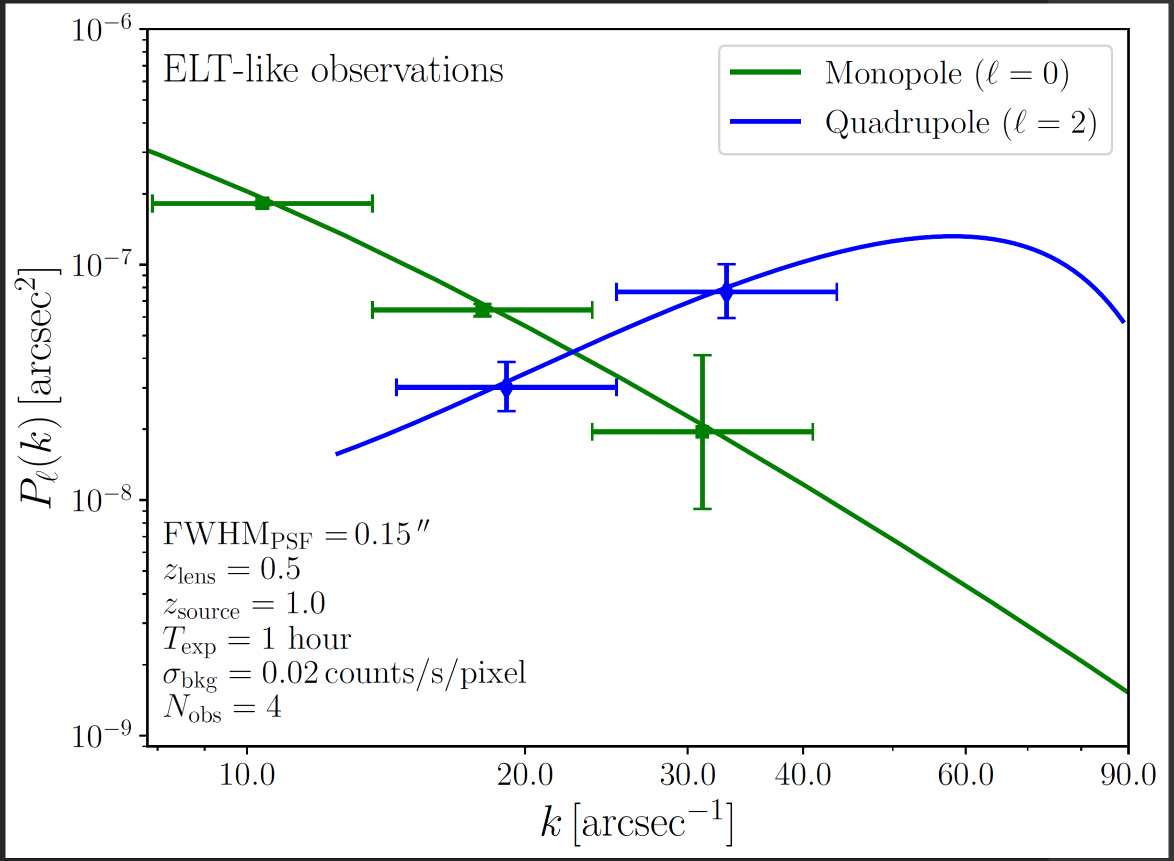
$\mathcal{L} \propto \frac{e^{-\frac{1}{2}\delta\mathbf{O}_{\text{obs}}^{\text{T}}\mathbf{C}^{-1}\delta\mathbf{O}_{\text{obs}}}}{\sqrt{|\mathbf{C}|}}$ where the covariance matrix is given by $\mathbf{C} \equiv \mathbf{C}_S + \mathbf{C}_N$

- **Fisher Matrix**

$$F_{ij} \equiv -\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \ln P_{\ell_1,i}\, \partial \ln P_{\ell_2,j}} \right\rangle$$

Fisher Forecast Results





Dhanasingham+ (2022)

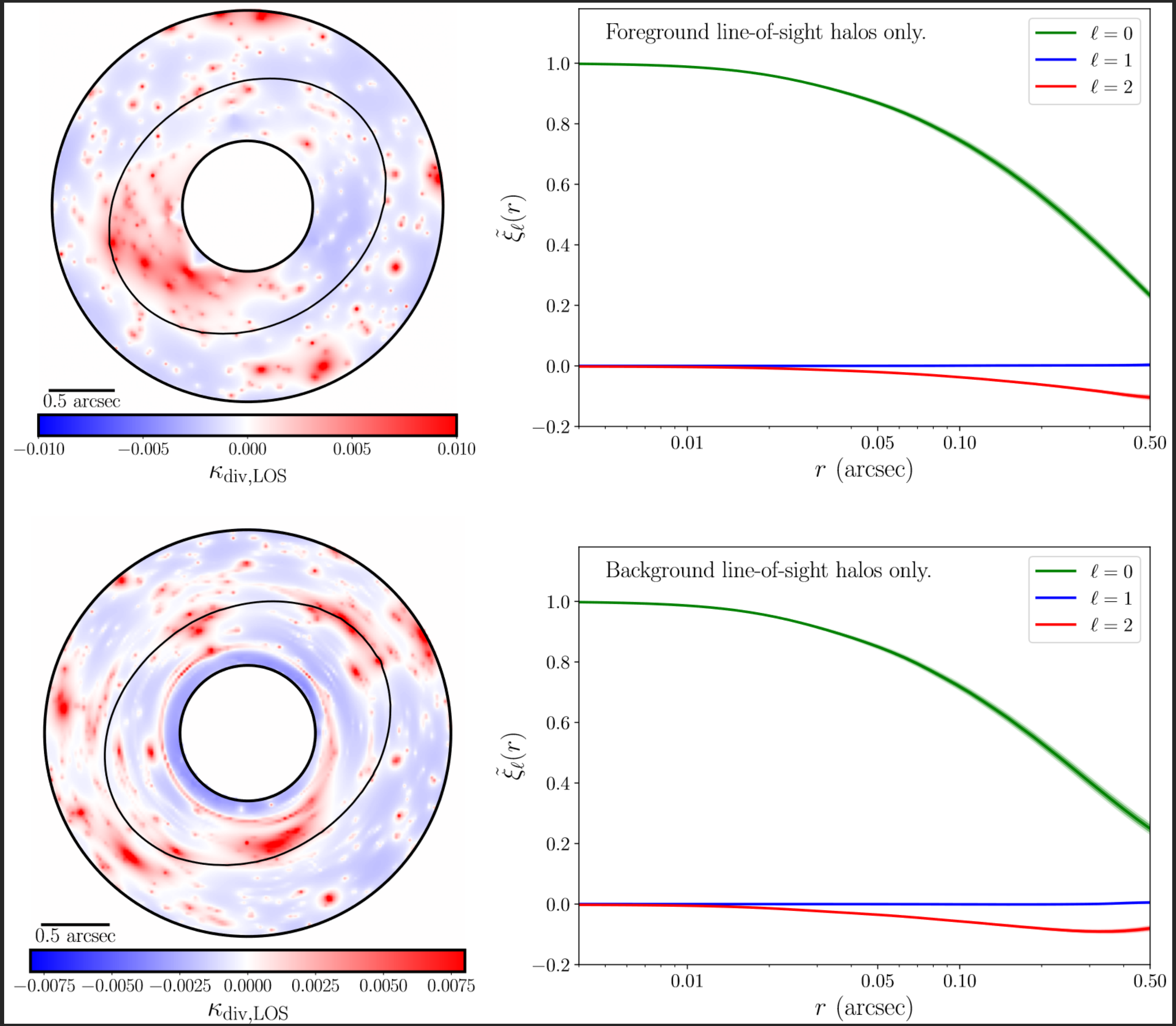
Takeaway Message

The line-of-sight halos in a strong lens system create anisotropic signatures in κ_{div} maps and add non-zero quadrupole moment to the 2-point correlation function multipoles.

This detectable non-zero quadrupole can be used to study dark matter microphysics.

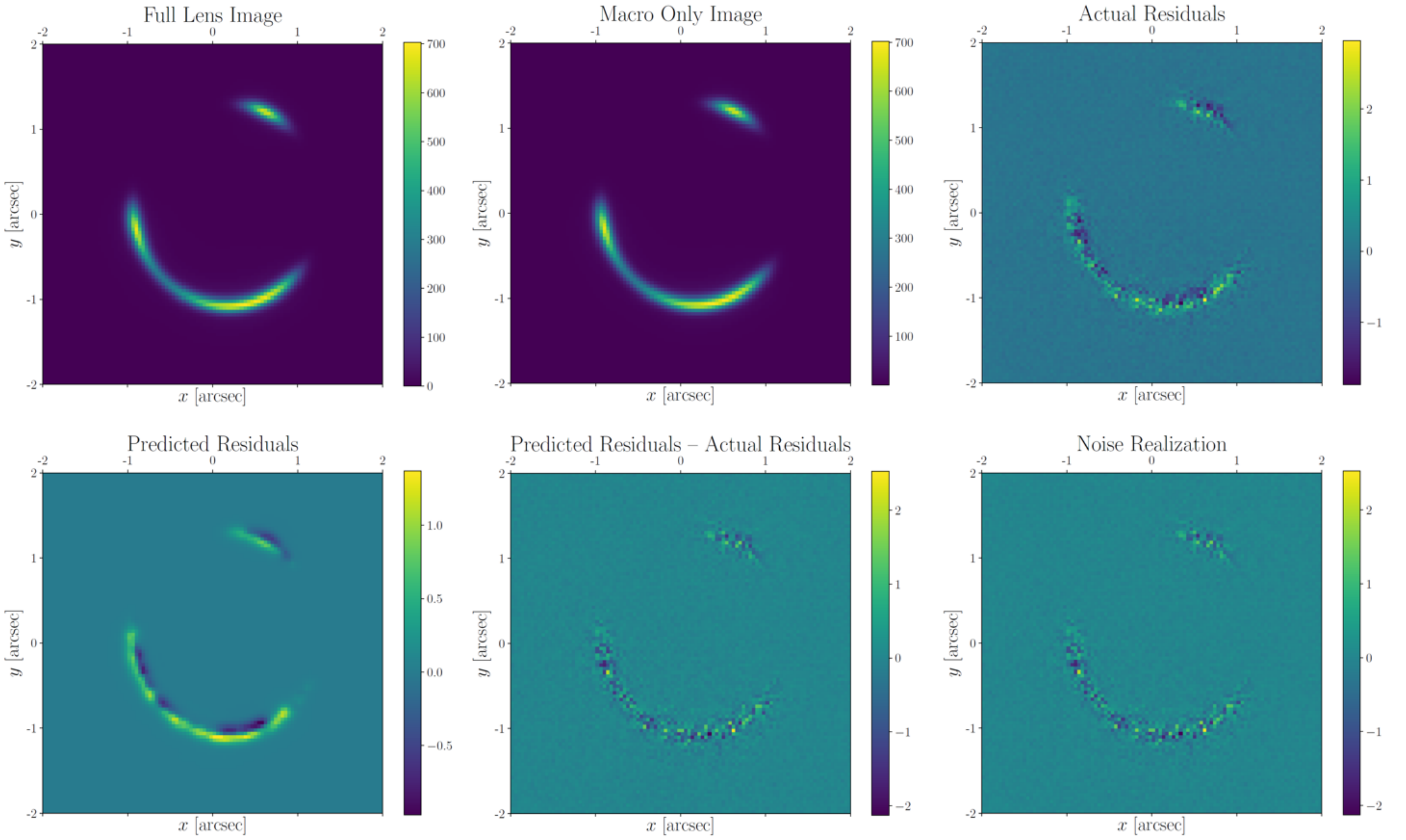
Thank you!

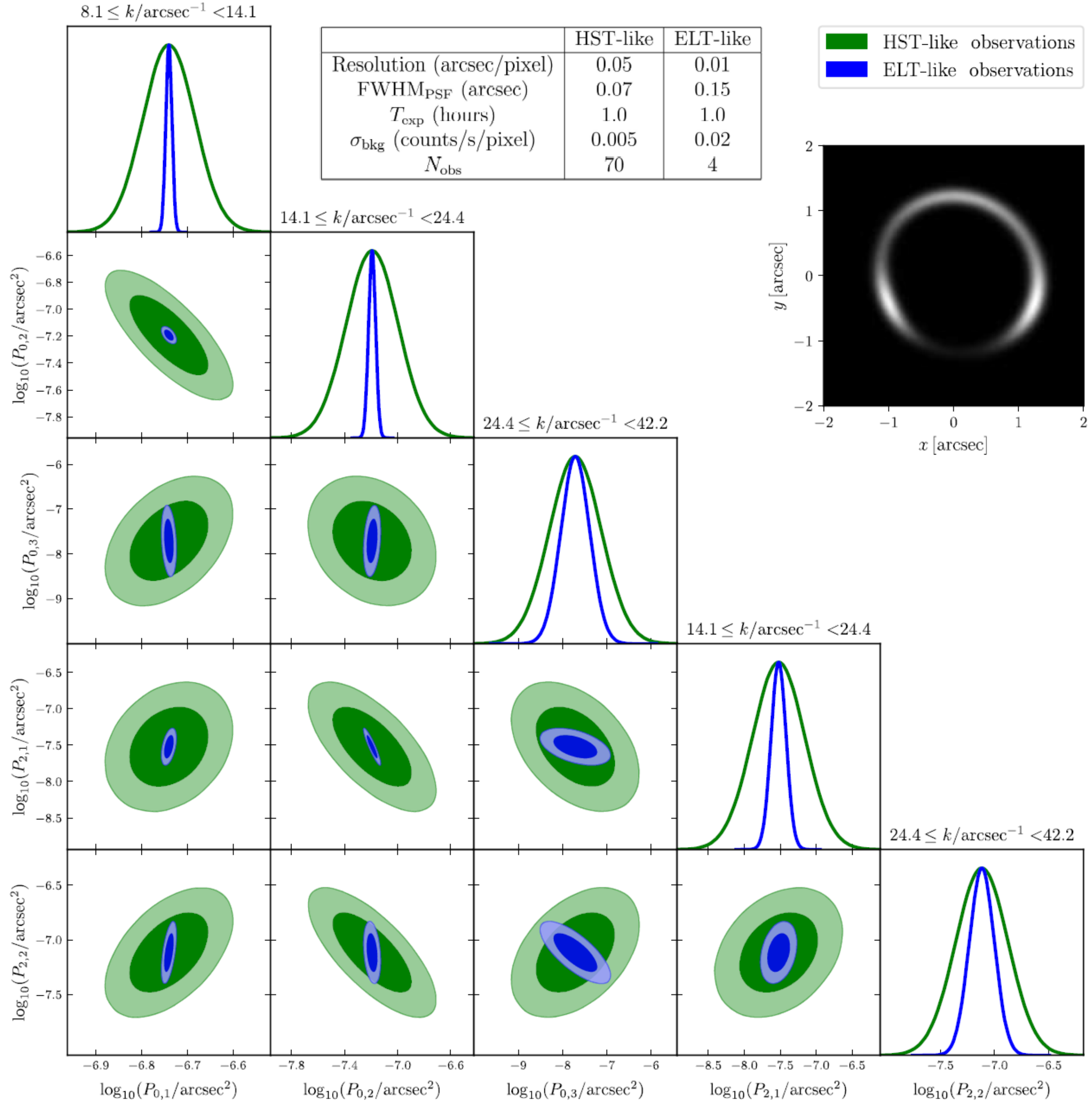
Back-up Slides


0.5 arcsec
−0.010 −0.005 0.000 0.005 0.010
$\kappa_{\mathrm{div,LOS}}$
Foreground line-of-sight halos only.
$\ell = 0$
$\ell = 1$
$\ell = 2$
$\tilde{\xi}_\ell(r)$
1.0 0.8 0.6 0.4 0.2 0.0 −0.2
0.01 0.05 0.10 0.50
r (arcsec)
0.5 arcsec
−0.0075 −0.0050 −0.0025 0.0000 0.0025 0.0050 0.0075
$\kappa_{\mathrm{div,LOS}}$
Background line-of-sight halos only.
$\ell = 0$
$\ell = 1$
$\ell = 2$
$\tilde{\xi}_\ell(r)$
1.0 0.8 0.6 0.4 0.2 0.0 −0.2
0.01 0.05 0.10 0.50
r (arcsec)

UNDERSTANDING RESIDUALS

$$\delta O_{\text{obs}} \approx -\nabla S_{\text{rec}}|_{\mathbf{x}-\boldsymbol{\alpha}_{\text{bf}}(\mathbf{x})} \cdot \delta\boldsymbol{\alpha}(\mathbf{x}) + \delta S[\mathbf{x} - \boldsymbol{\alpha}_{\text{bf}}(\mathbf{x})] + N(\mathbf{x})$$





	HST-like	ELT-like
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σ_{bkg} (counts/s/pixel)	0.005	0.02
N_{obs}	70	4

UNDERSTANDING RESIDUALS

- The true source

$$S_{\text{true}} = S_{\text{rec}} + \delta S \qquad \text{where } S_{\text{rec}} - \text{ reconstructed source}$$

- The true deflection field

$$\boldsymbol{\alpha}_{\text{true}}(\mathbf{x}) = \boldsymbol{\alpha}_{\text{bf}}(\mathbf{x}) + \delta\boldsymbol{\alpha}(\mathbf{x}) \qquad \text{where } \boldsymbol{\alpha}_{\text{bf}} - \text{ best-fit deflection field}$$

- Image residuals between the best-fit image and the true lensed image

$$\delta O_{\text{obs}} = S_{\text{true}}[\mathbf{x} - \boldsymbol{\alpha}_{\text{true}}(\mathbf{x})] - S_{\text{rec}}[\mathbf{x} - \boldsymbol{\alpha}_{\text{bf}}(\mathbf{x})] + N(\mathbf{x})$$

$$\boxed{\delta O_{\text{obs}} \approx -\boldsymbol{\nabla} S_{\text{rec}}|_{\mathbf{x}-\boldsymbol{\alpha}_{\text{bf}}(\mathbf{x})} \cdot \delta\boldsymbol{\alpha}(\mathbf{x}) + \delta S[\mathbf{x} - \boldsymbol{\alpha}_{\text{bf}}(\mathbf{x})] + N(\mathbf{x})}$$

where

$$\delta\boldsymbol{\alpha} = \boldsymbol{\alpha}_{\text{true}} - \boldsymbol{\alpha}_{\text{bf}} = \boldsymbol{\nabla}\phi_{\text{eff}} + \boldsymbol{\nabla} \times \mathbf{A}_{\text{eff}} - \boldsymbol{\nabla}\phi_{\text{eff,bf}} = \boldsymbol{\nabla}\delta\phi_{\text{eff}} + \boldsymbol{\nabla} \times \mathbf{A}_{\text{eff}}$$

RESIDUALS AND DECOMPOSITION

$$\delta O_{\mathrm{obs}} \approx -\boldsymbol{\nabla} S_{\mathrm{true}}|_{\mathbf{x}-\boldsymbol{\alpha}_{\mathrm{o}}(\mathbf{x})} \cdot \delta\boldsymbol{\alpha}(\mathbf{x}) + N(\mathbf{x}) = \delta O_{\mathrm{div}} + \delta O_{\mathrm{curl}} + N(\mathbf{x})$$

where $\delta\boldsymbol{\alpha} = \boldsymbol{\nabla}\phi_{\mathrm{eff,sub+LOS}} + \boldsymbol{\nabla} \times \mathbf{A}_{\mathrm{eff,LOS}}$

- **Divergence part of the deflection field (Fourier Bases)**

$$\boldsymbol{\nabla}\phi_{\mathrm{div}}(\mathbf{x}) = \boldsymbol{\nabla}\phi_{\mathrm{eff,sub+LOS}}(\mathbf{x}) = \sum_{m=1}^{N_{\mathrm{modes}}} \mathcal{B}_m \boldsymbol{\nabla}\phi_m(\mathbf{x})$$

where

$$\mathcal{B}_m = \frac{1}{A}\int_A d^2\mathbf{x}\ \boldsymbol{\nabla}\varphi_m^*(\mathbf{x}) \cdot \boldsymbol{\nabla}\phi_{\mathrm{div}}(\mathbf{x}).$$

$A-$ Area of the image

- **Residuals**

$$\delta O_{\mathrm{obs}}(\mathbf{x}) = \delta O_{\mathrm{div}}(\mathbf{x}) + \delta O_{\mathrm{curl}}(\mathbf{x}) + N(\mathbf{x}) \approx \underbrace{\sum_{m=1}^{N_{\mathrm{modes}}} \mathcal{B}_m \mathcal{W}_m}_{\text{Divergence Contribution}} + \underbrace{N(\mathbf{x})}_{\text{Noise}}$$

where

$$\mathcal{W}_m(\mathbf{x}) = -\boldsymbol{\nabla}_{\mathbf{u}} S(\mathbf{u})|_{\mathbf{u}=\mathbf{x}-\boldsymbol{\nabla}\phi_{\mathrm{o}}(\mathbf{x})} \cdot \boldsymbol{\nabla}\phi_m(\mathbf{x}).$$

- **Residuals**

Divergence Contribution

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Noise

where

$$\mathcal{W}_m(\mathbf{x}) = -\boldsymbol{\nabla}_{\mathbf{u}} S(\mathbf{u})|_{\mathbf{u}=\mathbf{x}-\boldsymbol{\nabla}\phi_o(\mathbf{x})} \cdot \boldsymbol{\nabla}\varphi_m(\mathbf{x}).$$

- **Likelihood**

$$\mathcal{L} \propto \frac{e^{-\frac{1}{2}\delta\mathbf{O}^{\rm T}_{\rm obs}\mathbf{C}^{-1}\delta\mathbf{O}_{\rm obs}}}{\sqrt{|\mathbf{C}|}}$$ where the covariance matrix is given by $\mathbf{C} \equiv \mathbf{C}_S + \mathbf{C}_N$

$$\mathbf{C}_{S,ij} \equiv \langle \delta\mathbf{O}_{{\rm div},i}\ \delta\mathbf{O}^{\dagger}_{{\rm div},j} \rangle$$

- **Fisher Matrix**

$$F_{ij} \equiv -\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \ln P_{\ell_1,i}\ \partial \ln P_{\ell_2,j}} \right\rangle$$