

Electromagnetic Radiation from Axion Condensates in a Time Dependent Magnetic Field

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Outline of the Talk

- ▶ Axions and Axion-like Particles
- ▶ The modified Maxwell's Equations
- ▶ Axion Condensates
- ▶ Electromagnetic Radiation from Axion Condensates in
 - Static External Magnetic Field
 - Oscillating External Magnetic Field
 - Oscillating External Magnetic Field with Time Varying Frequency
- ▶ Take Home Message

Axions and Axion-like Particles

- ▶ QCD Lagrangian is not CP symmetric
- ▶ Measurements of neutron magnetic dipole moment
→ CP violation extremely small
- ▶ Peccei and Quinn showed in 1977 that introducing a pseudo scalar field $\phi(\mathbf{x}, t)$ cancels the CP-violating term dynamically
- ▶ $m_a f_a \approx m_\pi f_\pi \sim \Lambda_{\text{QCD}}^2$, $m_a < \text{eV}$

R. D. Peccei and Helen R. Quinn. “CP Conservation in the Presence of Pseudoparticles”. In: *Phys. Rev. Lett.* 38 (25 1977), pp. 1440–1443. DOI: 10.1103/PhysRevLett.38.1440. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.38.1440>

Axions and Axion-like Particles

- ▶ Particles with axion like properties are predicted by string theory
- ▶ Multiple different axions spanning many orders of magnitude
- ▶ f_a determined by string compactifications
- ▶ This motivates the search for axion-like particles with $m_a f_a \neq \Lambda_{\text{QCD}}^2$
- ▶ Axions and axion-like particles are also an excellent candidate for dark matter

The Modified Maxwell's Equations

► Axion-photon Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + J_m^\mu A_\mu + \frac{C\beta}{4\pi f_a}\phi\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) \quad (1)$$

► To first order in $\frac{C\beta}{\pi f_a}\phi$ ($\phi \lesssim f_a$)

$$\square \mathbf{A}_r(\mathbf{x}, t) = -\frac{C\beta}{\pi f_a}(\partial_t\phi(\mathbf{x}, t))\mathbf{B}_0 \equiv \mathbf{J}_a(\mathbf{x}, t) \quad (2)$$

$$\square \Phi_r(\mathbf{x}, t) = \frac{C\beta}{\pi f_a}\nabla\phi(\mathbf{x}, t) \cdot \mathbf{B}_0 \equiv \rho_a(\mathbf{x}, t) \quad (3)$$

$$(\square + m_a^2)\phi(\mathbf{x}, t) + \partial_\phi V(\phi) = -\frac{C\beta}{\pi f_a}\mathbf{E}_r(\mathbf{x}, t) \cdot \mathbf{B}_0(\mathbf{x}, t) \quad (4)$$

Axion Condensates

- ▶ Axions can form spherically symmetric, coherently oscillating lumps of Bose-Einstein condensates

$$\phi(\mathbf{x}, t) \approx \phi_0 \operatorname{sech}(r/R) \cos(\omega t).$$

- ▶ Can be dense ($m_a R \sim 1$)

- Dominated by self interactions
- $\omega \lesssim m_a$
- $\phi(\mathbf{x} = 0, t) \sim f_a$

- ▶ Or dilute ($m_a R \gg 1$)

- Dominated by gravitational interactions
- $\omega \approx m_a$
- $\phi(\mathbf{x} = 0, t) \ll f_a$

Hong Zhang. “Axion Stars”. In: *Symmetry* 12.1 (2019), p. 25.

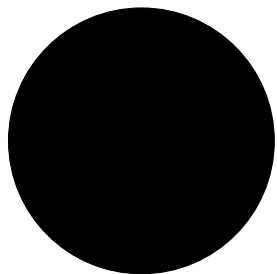
DOI: [10.3390/sym12010025](https://doi.org/10.3390/sym12010025). arXiv: [1810.11473](https://arxiv.org/abs/1810.11473) [hep-ph]

Luca Visinelli et al. “Dilute and dense axion stars”. In: *Phys. Lett. B* 777 (2018), pp. 64–72. DOI:

[10.1016/j.physletb.2017.12.010](https://doi.org/10.1016/j.physletb.2017.12.010). arXiv: [1710.08910](https://arxiv.org/abs/1710.08910)
[astro-ph.CO]

Electromagnetic Radiation from Axion Condensates in External Magnetic Field

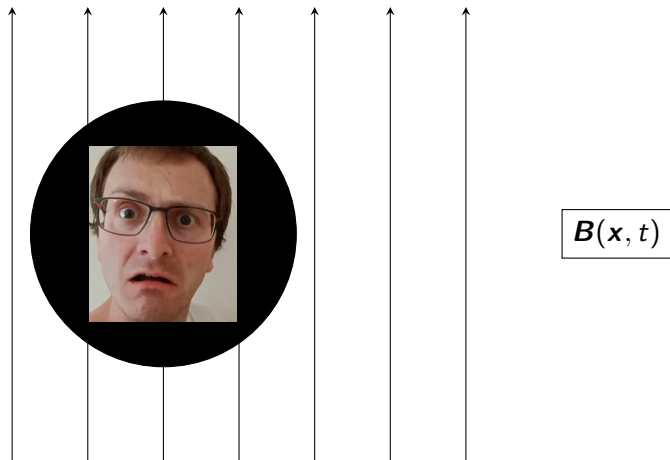
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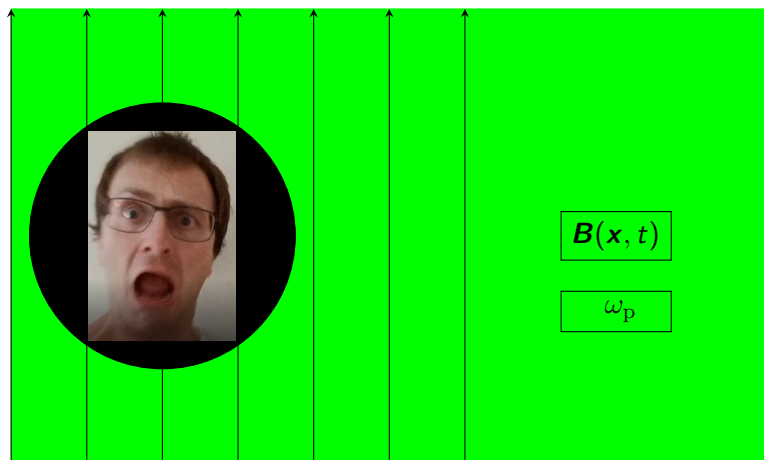
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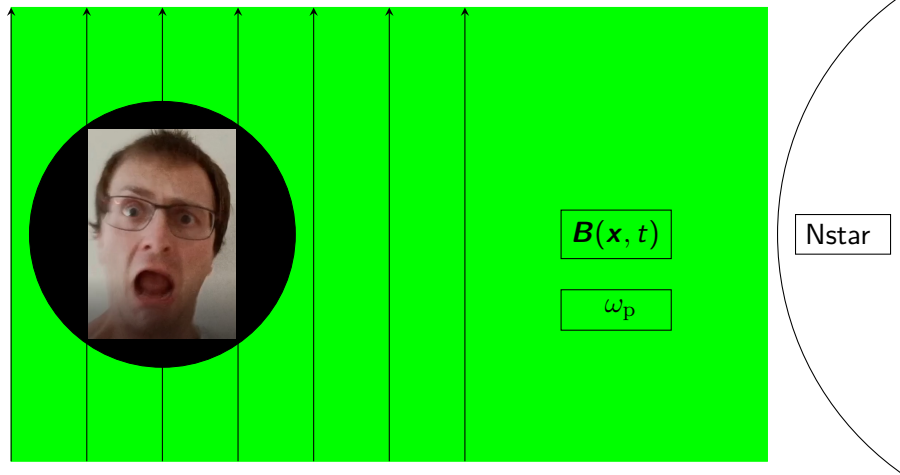
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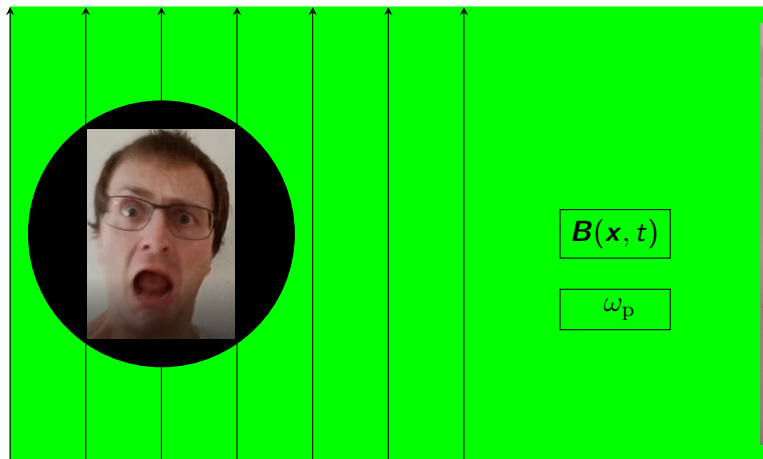
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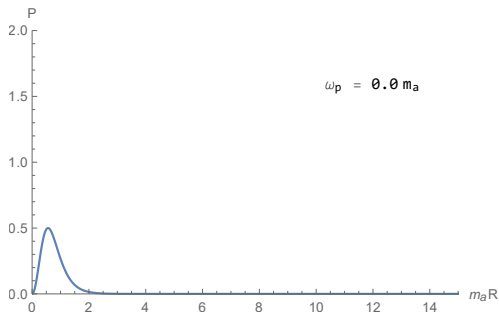
Electromagnetic Radiation from Axion Condensates in External Magnetic Field



Static External Magnetic Field

- ▶ Already found by Amin et al. to be ($k_\omega = \sqrt{\omega^2 - \omega_p^2}$)

$$\langle P(t) \rangle_T = \left(\frac{C\beta}{\pi f_a} \right)^2 \left(\frac{\phi_0^2 B_0^2 \omega^3 R^4 \pi^5}{12 k_\omega} \right) \left(\frac{\tanh(\pi k_\omega R/2)}{\cosh(\pi k_\omega R/2)} \right)^2 \quad (5)$$

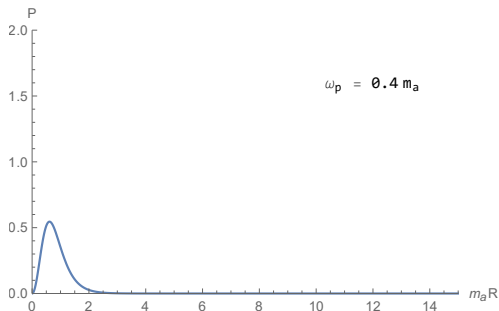


Mustafa A Amin et al. “Dipole radiation and beyond from axion stars in electromagnetic fields”. In: *Journal of High Energy Physics* 2021.6 (June 2021), p. 182

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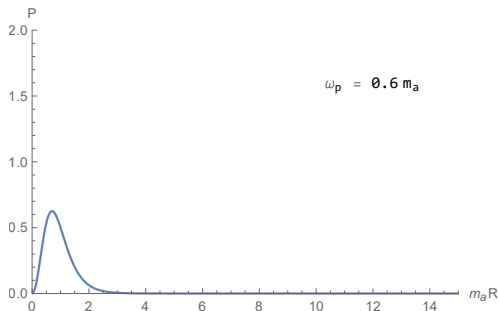


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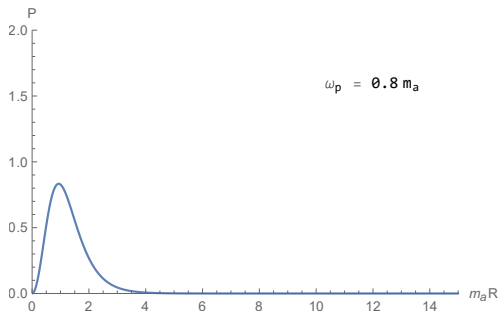


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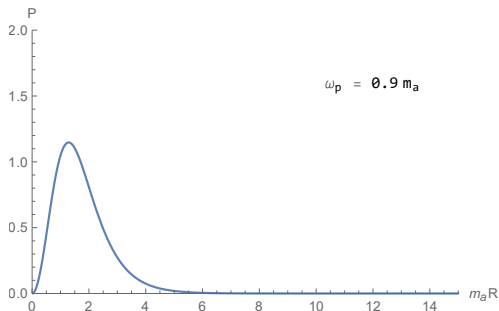


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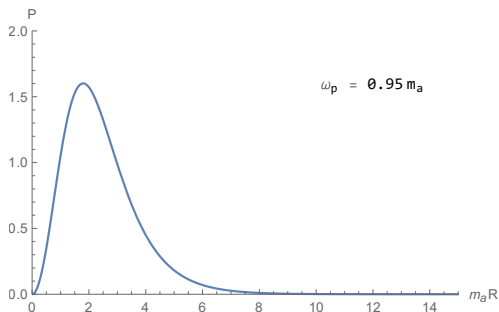


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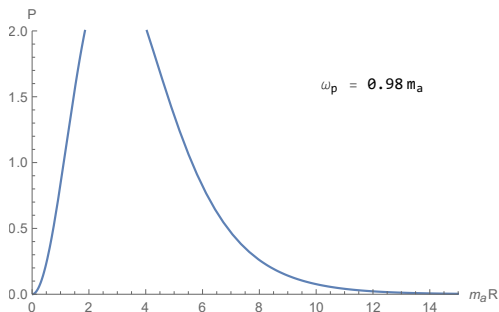


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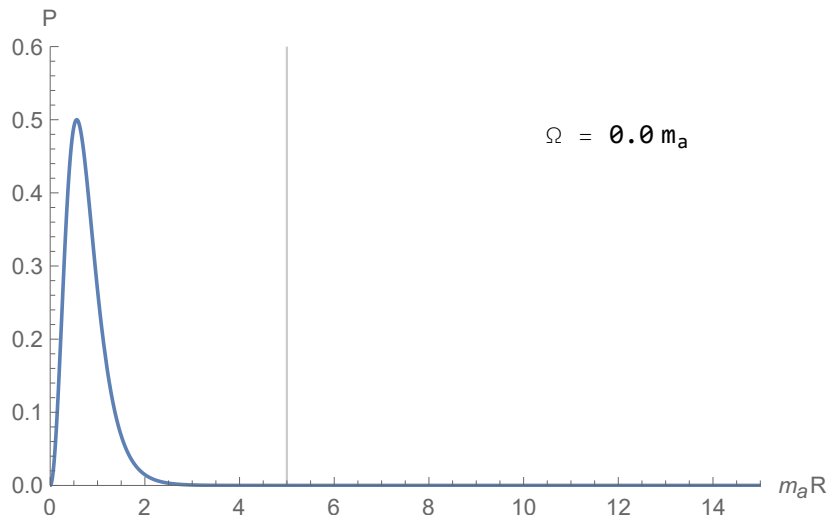
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Static External Magnetic Field, takeaways

- ▶ Radiated power vanishes exponentially with system size ωR
- ▶ Tuning plasma frequency ω_p allows larger condensates to radiate efficiently
- ▶ Want to see if we can have similar effects by making the external magnetic field oscillate

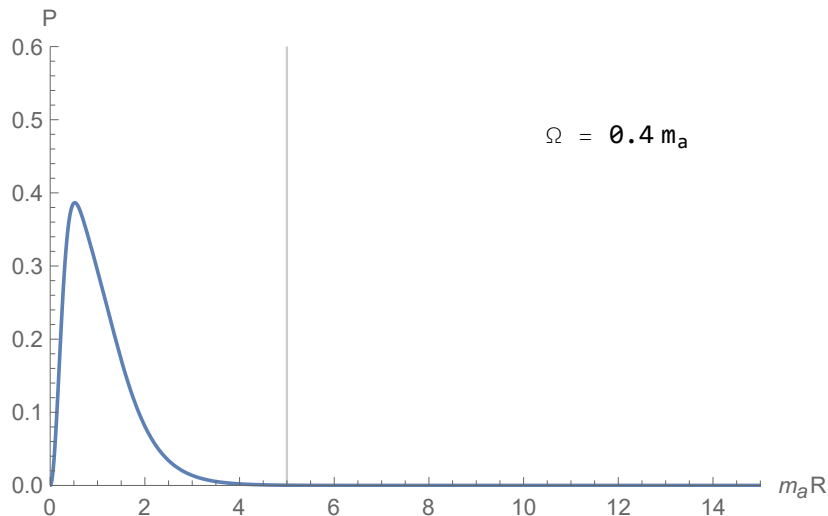
Oscillating External Magnetic Field

- ▶ When $\mathbf{B}_0 \rightarrow \mathbf{B}_0 \cos(\Omega t)$, the effective frequency splits in two $\omega \rightarrow \omega \pm \Omega$, wavenumber $k_{\pm} = |\Omega \pm \omega|$



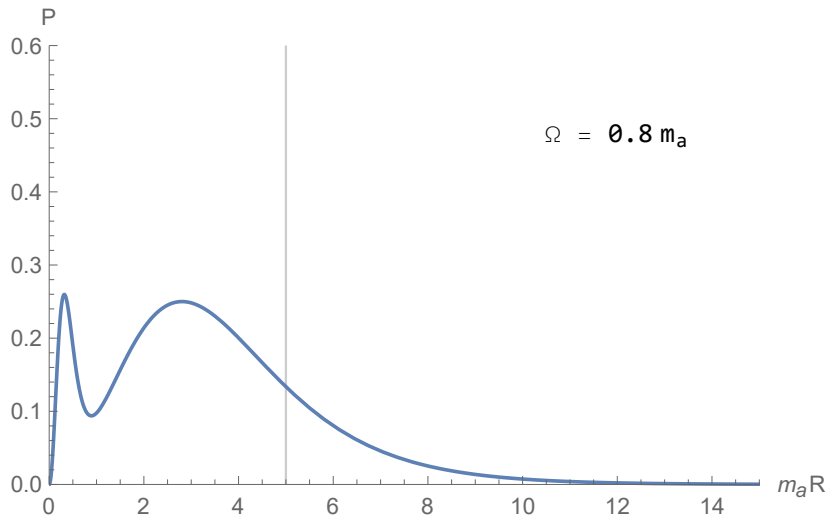
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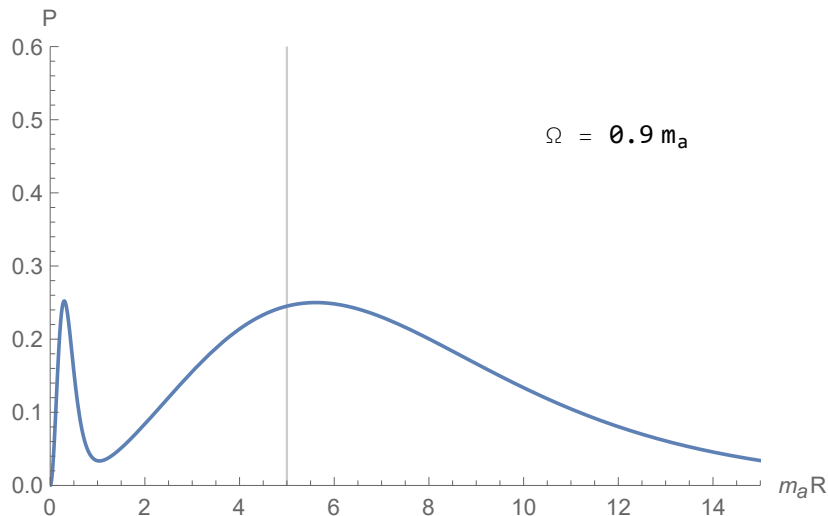
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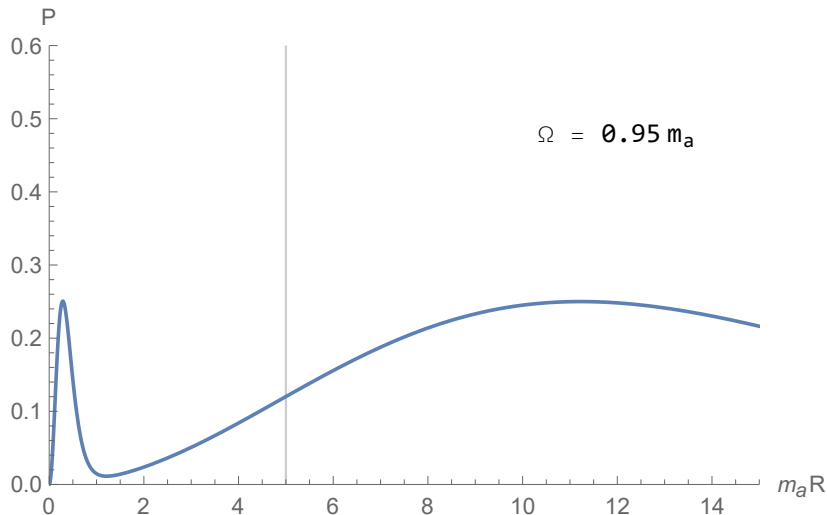
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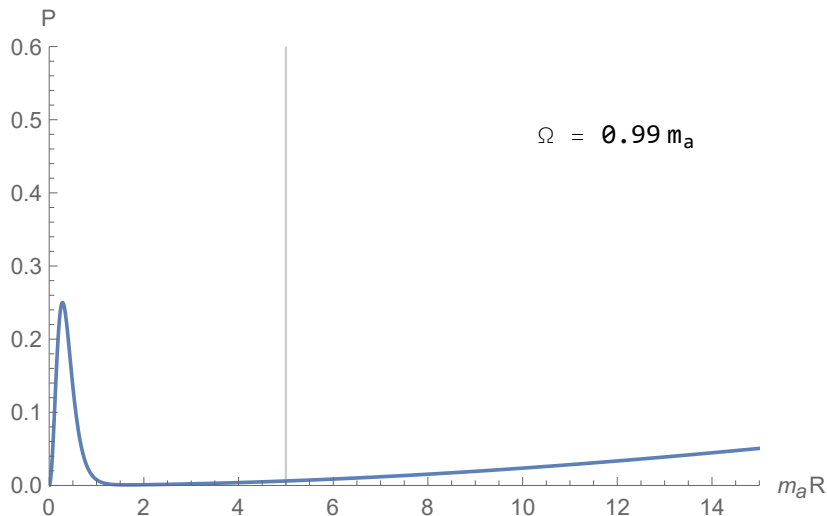
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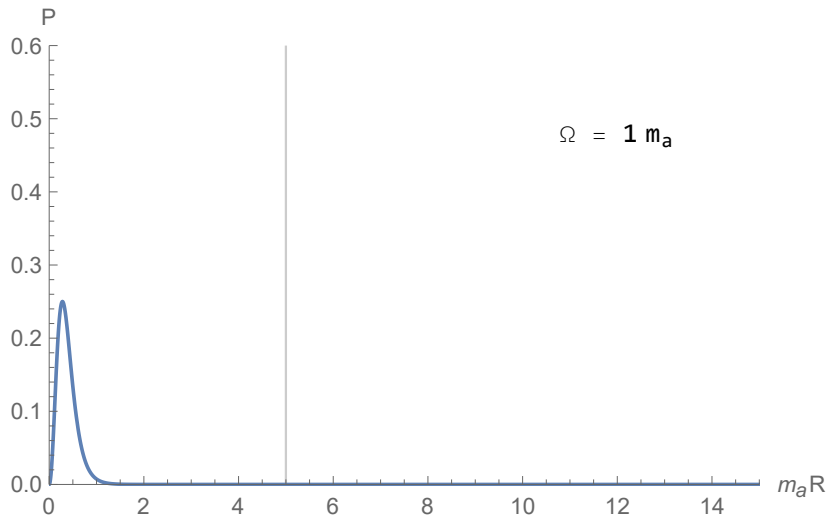
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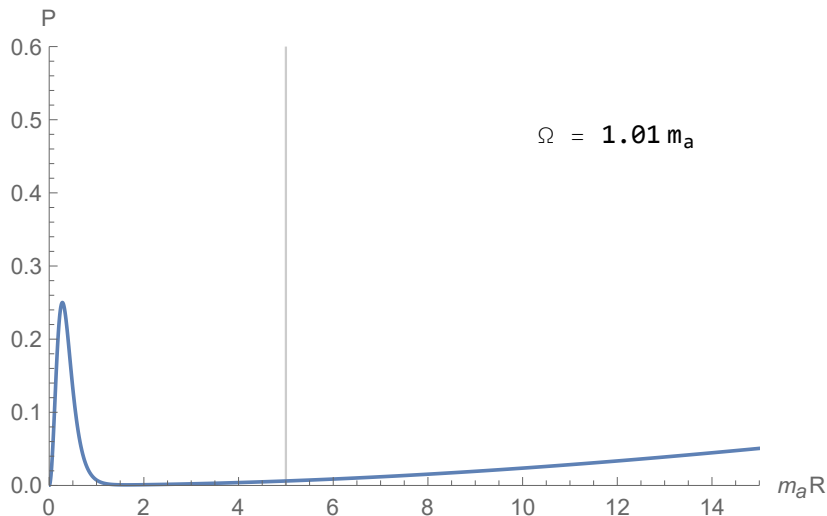
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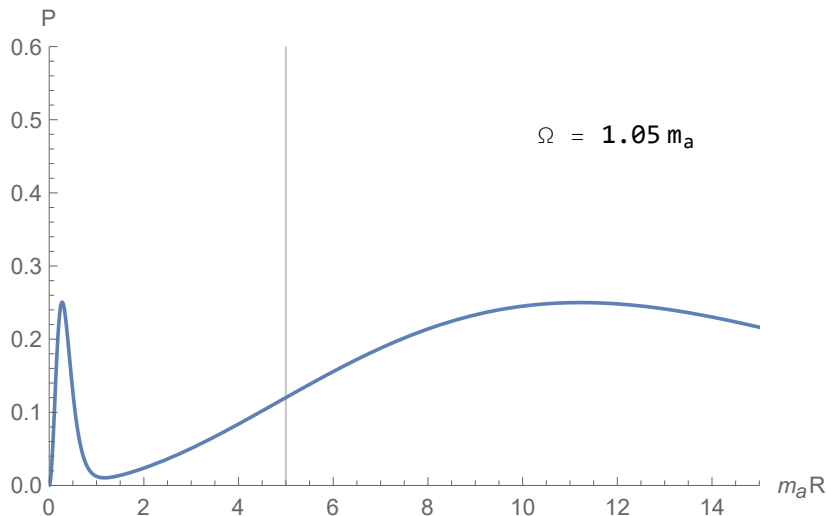
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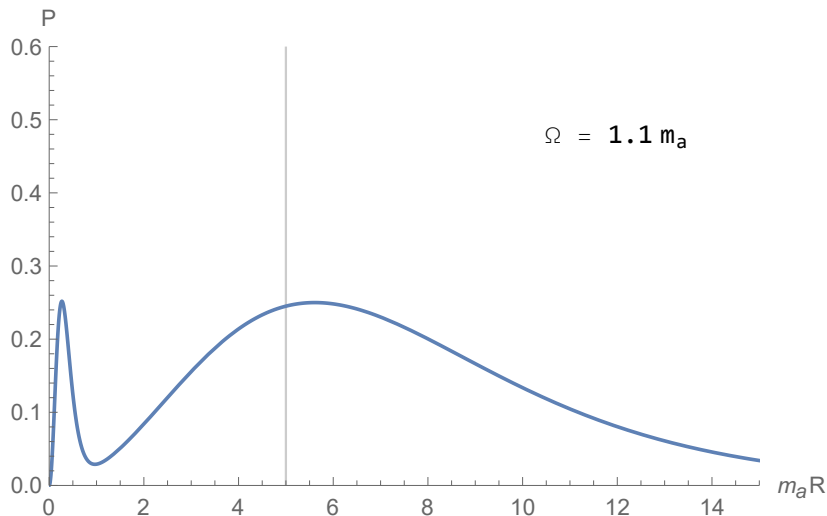
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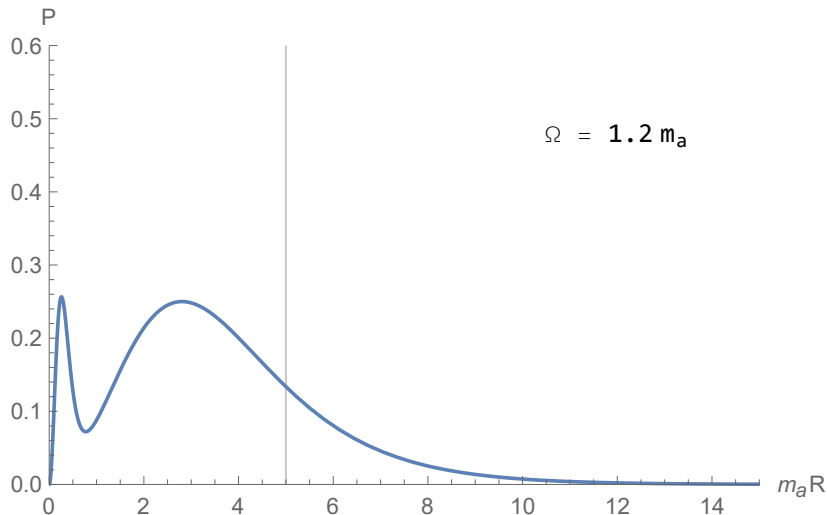
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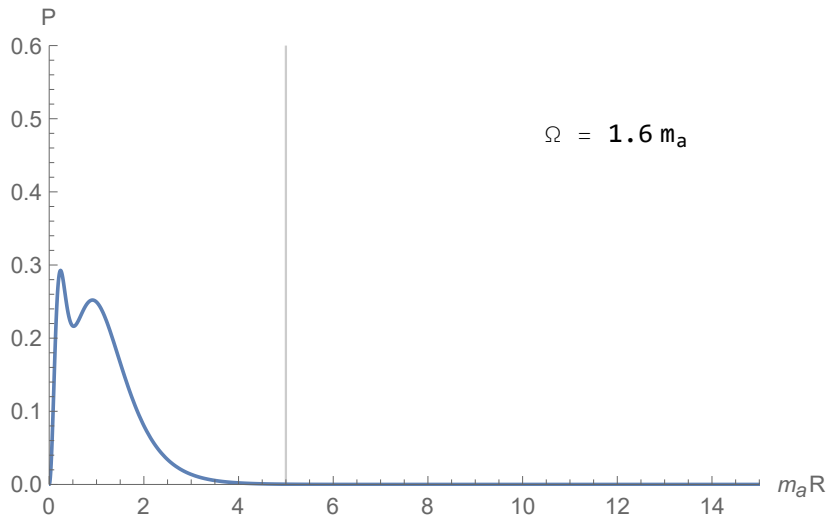
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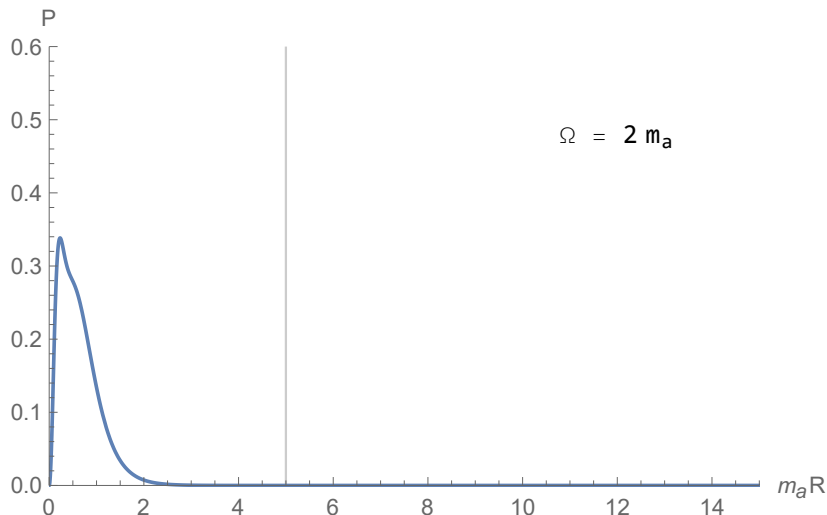
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Oscillating External Magnetic Field

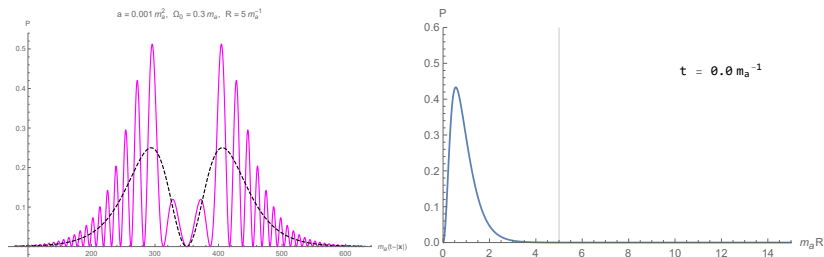
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Oscillating Magnetic Background with Time Dependent frequency

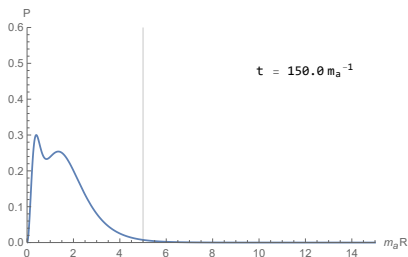
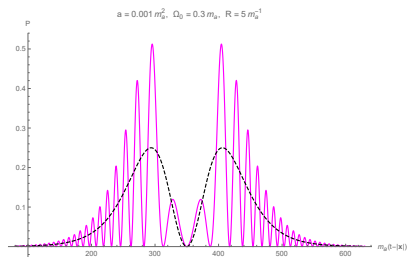
- ▶ Axion condensates radiate efficiently when $k_\omega R \sim 1$
- ▶ No reason why ω, ω_p, Ω should combine to make this true
- ▶ Let $\Omega \rightarrow \Omega(t) = \Omega_0 + at \rightarrow$ can scan over a set of axion masses and condensate sizes that radiates significantly

Oscillating Magnetic Background with Time Dependent Frequency: Radiated Power



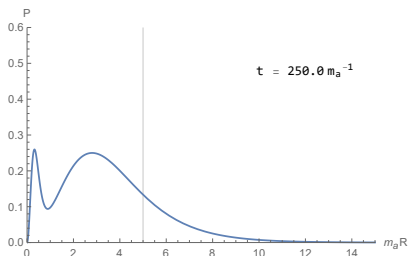
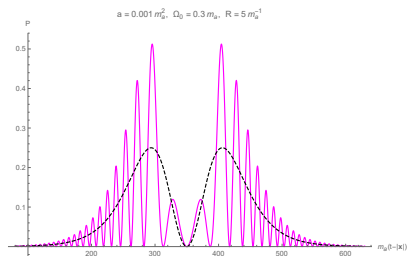
► Time in resonant region $\Delta t \sim \frac{1}{aR}$

Oscillating Magnetic Background with Time Dependent Frequency: Radiated Power



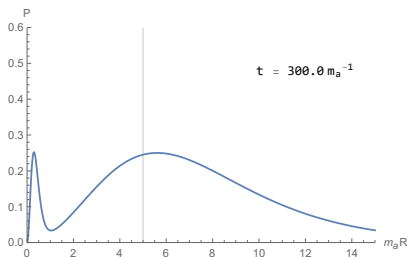
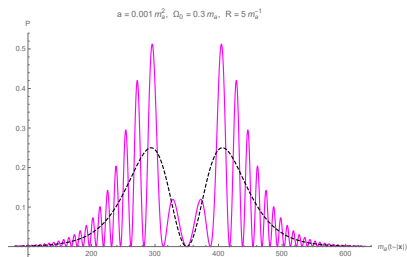
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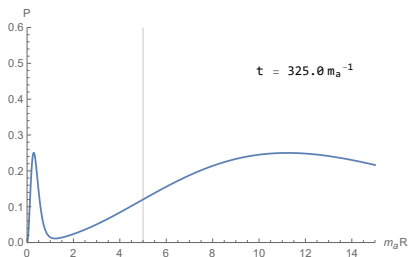
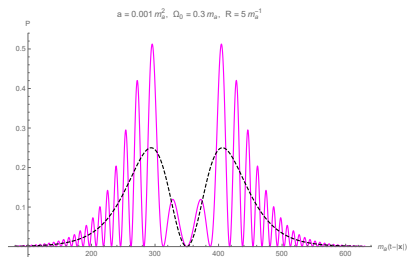
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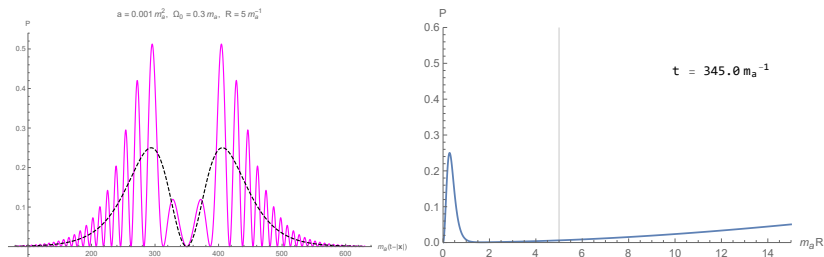
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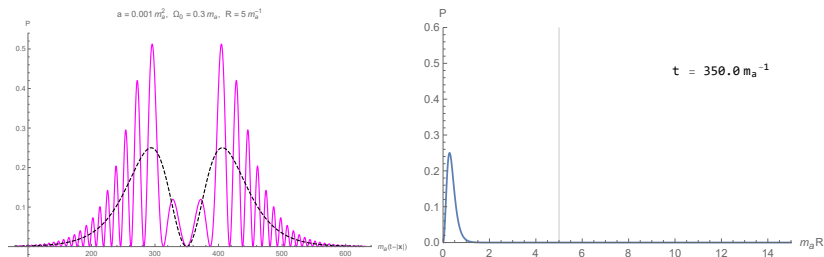
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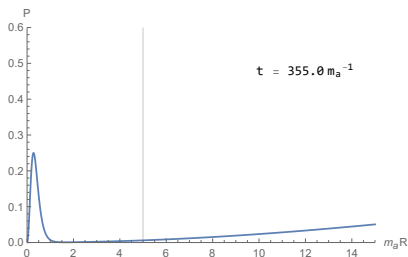
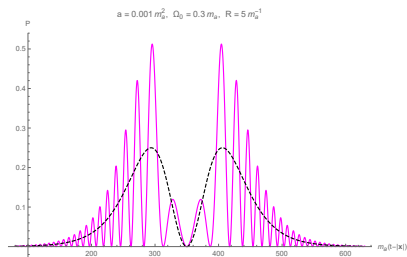
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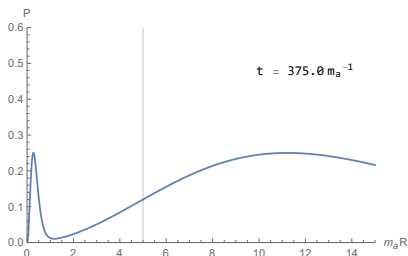
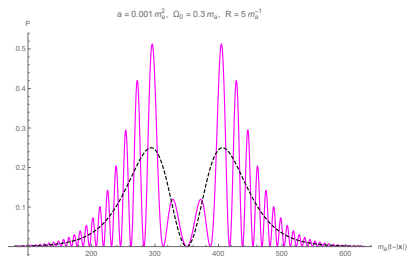
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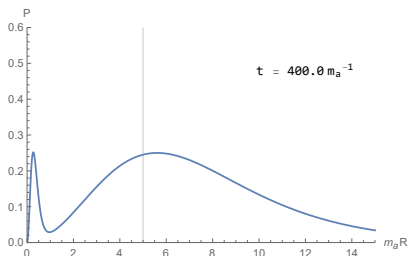
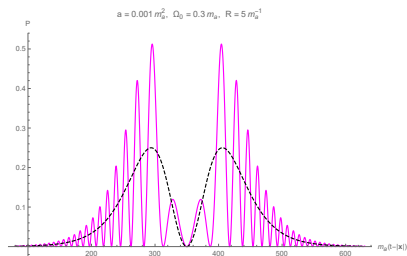
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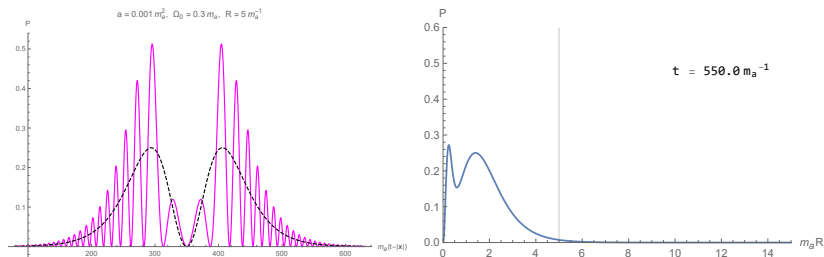
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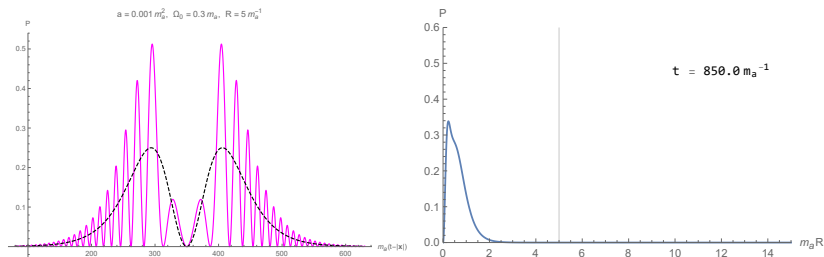
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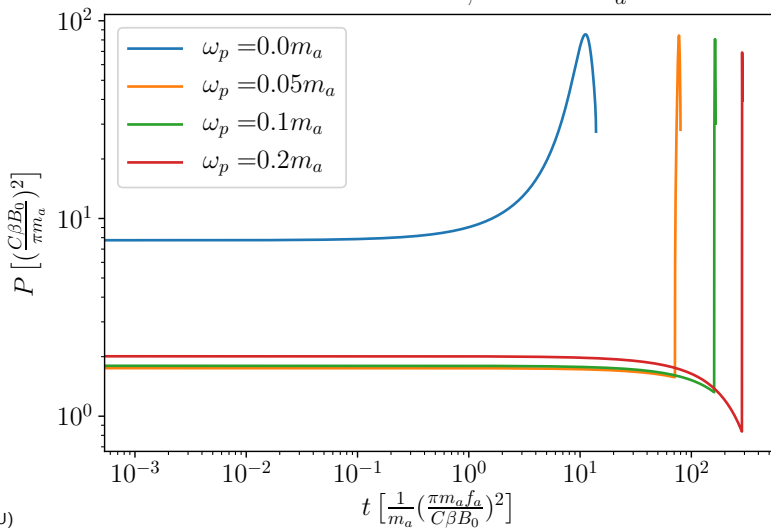


► Time in resonant region $\Delta t \sim \frac{1}{aR}$

Including Backreaction

Preliminary

Potential = \tanh , $\Omega = 0.55m_a$



Take Away Message

- ▶ Axion BEC start to give off electromagnetic radiation when subject to an external magnetic field
- ▶ There is a resonance that happens if the plasma frequency is similar to the axion mass
- ▶ A similar resonance occurs for oscillating magnetic fields when the magnetic field frequency is close to the axion mass
- ▶ A time dependent magnetic frequency allows for scanning a larger parameter space where condensates radiate efficiently
- ▶ Including spatial dependence for ω_p and \mathbf{B}_0 can allow BEC that normally do not radiate, to radiate efficiently as they pass through

Thanks for listening!

The next slides are extra

- ▶ Modified Maxwell equations

$$\begin{aligned}\nabla \times \mathbf{B}(\mathbf{x}, t) - \partial_t \mathbf{E}(\mathbf{x}, t) - \mathbf{J}_m(\mathbf{x}, t) \\ = -\frac{C\beta}{\pi f_a} \left[(\partial_t \phi(\mathbf{x}, t)) \mathbf{B}(\mathbf{x}, t) + \nabla \phi(\mathbf{x}, t) \times \mathbf{E}(\mathbf{x}, t) \right] \quad (12)\end{aligned}$$

$$\nabla \times \mathbf{E}(\mathbf{x}, t) = -\partial_t \mathbf{B}(\mathbf{x}, t) \quad (13)$$

$$\nabla \cdot \mathbf{E}(\mathbf{x}, t) = \rho_m(\mathbf{x}, t) + \frac{C\beta}{\pi f_a} \nabla \phi(\mathbf{x}, t) \cdot \mathbf{B}(\mathbf{x}, t) \quad (14)$$

$$\nabla \cdot \mathbf{B}(\mathbf{x}, t) = 0 \quad (15)$$

- ▶ Axion Equation of motion

$$(\square + m_a^2)\phi(\mathbf{x}, t) + \partial_\phi V(\phi) = -\frac{C\beta}{\pi f_a} \mathbf{E}(\mathbf{x}, t) \cdot \mathbf{B}(\mathbf{x}, t) \quad (16)$$

Estimation of Energy Radiated

- ▶ Energy stored in condensates $R \sim m_a^{-1}$

$$E_\phi \sim m_a^2 \phi_0^2 R^3 \sim m_a^2 f_a^2 R^3 \sim \frac{f_a^2}{m_a} \quad (17)$$

- ▶ A few orders of magnitude above or below a solar mass depending on the axion mass
- ▶ Energy stored in condensates $R \gg m_a^{-1}$

$$E_\phi \sim m_a^2 \phi_0^2 R^3 \sim \frac{m_{\text{P}}^2}{m_a} \frac{1}{(m_a R)} \quad (18)$$

Estimation of Energy Radiated

- ▶ For $m_a R \sim 100$, E_ϕ is about the same as in the case $R \sim m_a^{-1}$
- ▶ Dense and Large condensates can radiate with similar efficiency
- ▶ For a time dependent frequency we need to estimate Δt
- ▶ Define resonant conversion as at least a fraction $\epsilon = 0.1$ of the maximum

$$\left(\frac{\tanh(\pi R k_-^{(2)}/2)}{\cosh(\pi R k_-^{(2)})} \right)^2 \geq \epsilon \left(\frac{\tanh(\log(\sqrt{2} + 1))}{\cosh(\log(\sqrt{2} + 1))} \right)^2 \quad (19)$$

- ▶ $\rightarrow \Delta t \approx \frac{4.71}{\pi a R} \sim \frac{1}{aR}$

Estimation of Energy Radiated

- ▶ If we want to exhaust the condensate, we need

$$\Delta t \sim \frac{1}{aR} \geq \tau \sim \frac{E_\phi}{\langle P(t) \rangle} \quad (20)$$

$$\rightarrow a \leq m_a^2 \left(\frac{C\beta B_0}{\pi m_a f_a} \right)^2 \quad (21)$$

- ▶ For ultralight axion in $B_0 \sim 10^{15} \text{G}$

$$a \leq 10^0 - 10^4 \text{s}^{-2} \quad (22)$$

- ▶ Comparable to that of inspiraling neutron stars

Axion Equations of Motion: Axion Condensates

- ▶ Interested in localized solutions of the source free equation

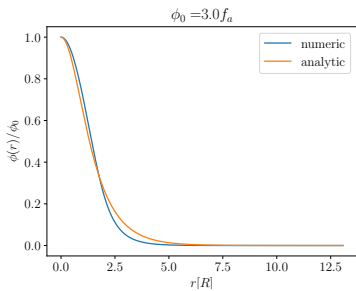
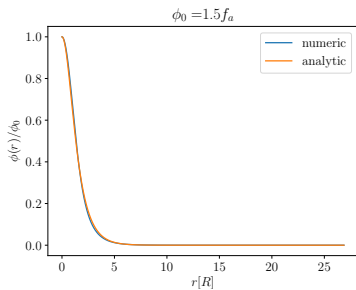
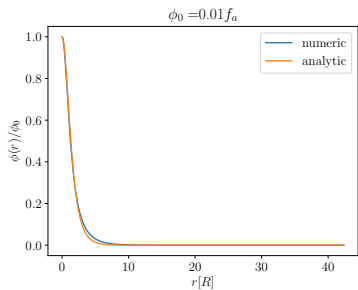
$$(\square + m_a^2)\phi(\mathbf{x}, t) + \partial_\phi V(\phi) = 0 \quad (23)$$

- ▶ Good approximation

$$\phi(\mathbf{x}, t) = \phi_0 \operatorname{sech}(r/R) \cos(\omega t) \quad (24)$$

- ▶ $\omega \sim m_a$

- ▶ R is a free parameter



Axion Equations of Motion: External magnetic field

- ▶ We write $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_r$ and $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_r$
- ▶ Assume \mathbf{E}_0 and \mathbf{B}_0 satisfy usual Maxwell equations

$$\nabla \times \mathbf{B}_0(\mathbf{x}, t) - \partial_t \mathbf{E}_0(\mathbf{x}, t) - \mathbf{J}_m(\mathbf{x}, t) = 0 \quad (25)$$

$$\nabla \times \mathbf{E}_0(\mathbf{x}, t) = -\partial_t \mathbf{B}_0(\mathbf{x}, t) \quad (26)$$

$$\nabla \cdot \mathbf{E}_0(\mathbf{x}, t) = \rho_m(\mathbf{x}, t) \quad (27)$$

$$\nabla \cdot \mathbf{B}_0(\mathbf{x}, t) = 0 \quad (28)$$

- ▶ Set $\mathbf{E}_0 = 0$ and $\mathbf{B}_0 = B_0 \hat{z}$
- ▶ Assume free space and expand in $\frac{c\beta}{\pi f_a} \phi$

- ▶ To leading order in $\frac{C\beta}{\pi f_a}\phi$

$$\nabla \times \mathbf{B}_r(\mathbf{x}, t) - \partial_t \mathbf{E}_r(\mathbf{x}, t) = -\frac{C\beta}{\pi f_a} (\partial_t \phi(\mathbf{x}, t)) \mathbf{B}_0 \quad (29)$$

$$\nabla \times \mathbf{E}_r(\mathbf{x}, t) = -\partial_t \mathbf{B}_r(\mathbf{x}, t) \quad (30)$$

$$\nabla \cdot \mathbf{E}_r(\mathbf{x}, t) = \frac{C\beta}{\pi f_a} \nabla \phi(\mathbf{x}, t) \cdot \mathbf{B}_0 \quad (31)$$

$$\nabla \cdot \mathbf{B}_r(\mathbf{x}, t) = 0 \quad (32)$$

$$(\square + m_a^2)\phi(\mathbf{x}, t) + \partial_\phi V(\phi) = -\frac{C\beta}{\pi f_a} \mathbf{E}_r(\mathbf{x}, t) \cdot \mathbf{B}_0 \quad (33)$$

- ▶ In particular, ϕ only coupled to \mathbf{B}_0

Axion Equations of Motion: Gauge Field

- ▶ Focus on

$$\nabla \times \mathbf{B}_r(\mathbf{x}, t) - \partial_t \mathbf{E}_r(\mathbf{x}, t) = -\frac{C\beta}{\pi f_a} (\partial_t \phi(\mathbf{x}, t)) \mathbf{B}_0 \quad (34)$$

$$\nabla \cdot \mathbf{E}_r(\mathbf{x}, t) = \frac{C\beta}{\pi f_a} \nabla \phi(\mathbf{x}, t) \cdot \mathbf{B}_0 \quad (35)$$

- ▶ Insert the gauge field

$$\mathbf{B}_r(\mathbf{x}, t) = \nabla \times \mathbf{A}(\mathbf{x}, t) \quad (36)$$

$$\mathbf{E}_r(\mathbf{x}, t) = -\nabla \Phi(\mathbf{x}, t) - \partial_t \mathbf{A}(\mathbf{x}, t) \quad (37)$$

$$(38)$$

- ▶ Choosing the Lorenz gauge

$$\nabla \cdot \mathbf{A}(\mathbf{x}, t) + \partial_t \Phi(\mathbf{x}, t) = 0 \quad (39)$$

Axion Equations of Motion: Gauge Field

- ▶ Equations simplify to

$$\square \mathbf{A}(\mathbf{x}, t) = -\frac{C\beta}{\pi f_a} (\partial_t \phi(\mathbf{x}, t)) \mathbf{B}_0 \equiv \mathbf{J}_a(\mathbf{x}, t) \quad (40)$$

$$\square \Phi(\mathbf{x}, t) = \frac{C\beta}{\pi f_a} \nabla \phi(\mathbf{x}, t) \cdot \mathbf{B}_0 \equiv \rho_a(\mathbf{x}, t) \quad (41)$$

- ▶ Using the standard free retarded Green's function

$$G(\mathbf{x}, t; \mathbf{x}', t') = -\frac{\delta(t - t' - |\mathbf{x} - \mathbf{x}'|)}{4\pi |\mathbf{x} - \mathbf{x}'|} \quad (42)$$

- ▶ we can solve this

$$\mathbf{A}(\mathbf{x}, t) = \int d^4x' G(\mathbf{x}, t; \mathbf{x}', t') \mathbf{J}_a(\mathbf{x}', t') \quad (43)$$

$$\Phi(\mathbf{x}, t) = \int d^4x' G(\mathbf{x}, t; \mathbf{x}', t') \rho(\mathbf{x}', t') \quad (44)$$

Equations of Motion: Gauge Field

- ▶ Insert ansatz for $\phi(\mathbf{x}, t)$

$$\mathbf{J}_a(\mathbf{x}, t) = \omega B_0 \left(\frac{C\beta}{\pi f_a} \right) \sin(\omega t) \operatorname{sech}(r/R) \hat{r}_z \quad (45)$$

$$\rho_a(\mathbf{x}, t) = -\frac{B_0}{R} \left(\frac{C\beta}{\pi f_a} \right) \left(\frac{\tanh(r/R)}{\cosh(r/R)} \right) \cos(\omega t) \cos \theta \quad (46)$$

- ▶ Note that far away from the source

$$\nabla \Phi(\mathbf{x}, t) \sim \hat{r} \quad (47)$$

- ▶ Can ignore $\Phi(\mathbf{x}, t)$ when discussing radiation since

$$\frac{dP}{d\Omega_s} = r^2 \hat{r} \cdot (\mathbf{E}_r(\mathbf{x}, t) \times \mathbf{B}_r(\mathbf{x}, t)) \quad (48)$$

Constant Magnetic Background: Radiated Fields

- ▶ To leading order in r^{-1}

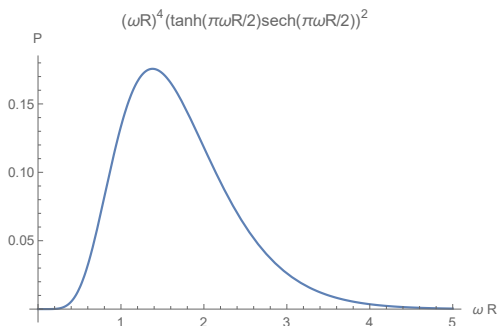
$$\begin{aligned} \mathbf{E}_r(\mathbf{x}, t) &= \left(\frac{C\beta}{\pi f_a} \right) \left(\frac{\phi_0 B_0 \pi^2 \omega R^2}{4r} \right) \\ &\times \left(\frac{\tanh(\pi\omega R/2)}{\cosh(\pi\omega R/2)} \right) \cos(\omega t - \omega r) \hat{r}_z + (\text{terms} \sim \hat{r}) \end{aligned} \quad (49)$$

$$\begin{aligned} \mathbf{B}_r(\mathbf{x}, t) &= \left(\frac{C\beta}{\pi f_a} \right) \left(\frac{\phi_0 B_0 \pi^2 \omega R^2}{4r} \right) \\ &\times \left(\frac{\tanh(\pi\omega R/2)}{\cosh(\pi\omega R/2)} \right) \cos(\omega t - \omega r) (\hat{r} \times \hat{r}_z) \end{aligned} \quad (50)$$

Constant Magnetic Background: Radiated Power

- ▶ Radiated power is given by

$$\begin{aligned}\langle P \rangle_T &= \left\langle \int d\Omega_s \hat{r} \cdot (\mathbf{E}_r(\mathbf{x}, t) \times \mathbf{B}_r(\mathbf{x}, t)) \right\rangle_T \\ &= \left(\frac{C\beta}{\pi f_a} \right)^2 \left(\frac{\phi_0^2 B_0^2}{12\omega^2} \right) (\omega R)^4 \left(\frac{\tanh(\pi\omega R/2)}{\cosh(\pi\omega R/2)} \right)^2\end{aligned}\quad (51)$$



Oscillating Magnetic Background: Average Power Radiated

- ▶ When $\omega \ll \Omega$ we can average over a time $\frac{2\pi}{\Omega}$

$$\langle P(t) \rangle_T = \left(\frac{C\beta}{\pi f_a} \right)^2 \left(\frac{B_0^2 \phi_0^2 \omega^2 R^4 \pi^5}{12} \right) \left(\frac{\tanh(\pi\Omega R/2)}{\cosh(\pi\Omega R/2)} \right)^2 \quad (52)$$

- ▶ Maximum for smaller radii R

- ▶ When $\omega \gg \Omega$ we average over $T = \frac{2\pi}{\omega}$

$$\langle P(t) \rangle_T = \left(\frac{C\beta}{\pi f_a} \right)^2 \left(\frac{B_0^2 \phi_0^2 \omega^2 R^4 \pi^5}{12} \right) \left(\frac{\tanh(\pi\omega R/2)}{\cosh(\pi\omega R/2)} \right)^2 \quad (53)$$

- ▶ Back where we started

Oscillating Magnetic Background: Average Power Radiated

- ▶ Most interesting when $\omega \sim \Omega$
- ▶ If we consider pulsars, $\omega \approx m_a \sim 10^{-12} \text{eV}$
- ▶ Power radiated averaged over a time $T \gg \frac{2\pi}{\omega}$ ($\gg \frac{2\pi}{\Omega}$)

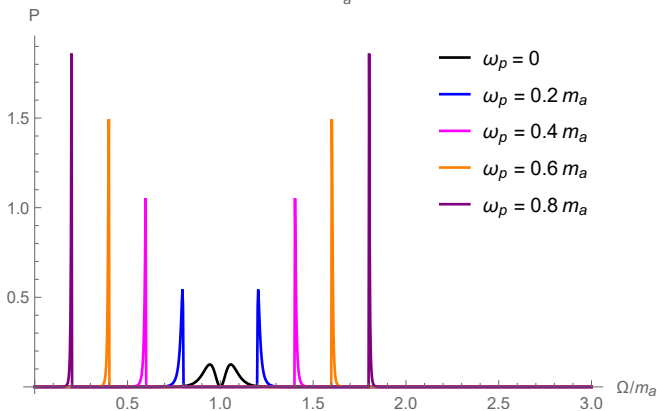
$$\begin{aligned} \langle P(t) \rangle_T = & \left(\frac{C\beta}{\pi f_a} \right)^2 \left(\frac{B_0^2 \phi_0^2 \omega^2 R^4 \pi^5}{24} \right) \\ & \times \left[\left(\frac{\tanh(\pi\omega_+ R/2)}{\cosh(\pi\omega_+ R/2)} \right)^2 + \left(\frac{\tanh(\pi\omega_- R/2)}{\cosh(\pi\omega_- R/2)} \right)^2 \right] \end{aligned} \quad (54)$$

- ▶ Second term allows us to realize enhancement for larger R

Oscillating Magnetic Background with Plasma

- Power averaged over 500 periods of the axion condensate measured in units $\left(\frac{C\beta}{\pi f_a}\right)^2 \left(\frac{\phi_0^2 B_0^2 \omega^2 R^4 \pi^5}{48}\right)$

$$R = 10 m_a^{-1}$$



Oscillating Magnetic Background with Time Dependent Frequency

- ▶ Assume linear growth in frequency

$$\Omega(t) = \Omega_0 + at \quad (55)$$

- ▶ For now ignore plasma
- ▶ Axion current is on the same form

$$\mathbf{J}(\mathbf{x}, t) = \frac{\omega B_0}{2} \left(\frac{C\beta}{\pi f_a} \right) \text{sech}(r/R) \left[\sin(\omega_- t) + \sin(\omega_+ t) \right] \hat{r}_z \quad (56)$$

$$\omega_- = \omega - \Omega_0 - at, \quad \omega_+ = \omega + \Omega_0 + at \quad (57)$$

Oscillating Magnetic Background with Time Dependent Frequency: Radiated Power

- ▶ Define

$$k_{\pm}^{(1)} = \omega \pm [\Omega_0 + a(t - r)] \quad (58)$$

$$k_{\pm}^{(2)} = \omega \pm [\Omega_0 + 2a(t - r)] \quad (59)$$

- ▶ Assume late time $t - r \gg R$

$$\begin{aligned}
P(t) = & \left[\left(\frac{C\beta}{\pi f_a} \right)^2 \left(\frac{\phi_0^2 B_0^2 \omega^2 R^4 \pi^5}{24(k_-^{(2)})^2} \right) \operatorname{sech} \left[\frac{\pi R k_-^{(2)}}{2} \right] \right. \\
& \times \left\{ a \pi R k_-^{(2)} \operatorname{sech}^2 \left[\frac{\pi R k_-^{(2)}}{2} \right] \sin \left[(t-r) k_-^{(1)} \right] \right. \\
& \quad \left. - (k_-^{(2)})^2 \tanh \left[\frac{\pi R k_-^{(2)}}{2} \right] \cos \left[(t-r) k_-^{(1)} \right] \right\} \\
& + 2a \tanh \left[\frac{\pi R k_-^{(2)}}{2} \right] \sin \left[(t-r) k_-^{(1)} \right] \\
& \left. - a \pi R k_-^{(2)} \tanh^2 \left[\frac{\pi R k_-^{(2)}}{2} \right] \sin \left[(t-r) k_-^{(1)} \right] \right\} \\
& \left. + (a \leftrightarrow -a, \Omega_0 \leftrightarrow -\Omega_0) \right]^2 \tag{60}
\end{aligned}$$