Discovering Composite Dark Matter with the Migdal Effect

TeVPA 2022

Javier F. Acevedo

Aug. 8th 2022

Arthur B. McDonald
Canadian Astroparticle Physics Research Institute

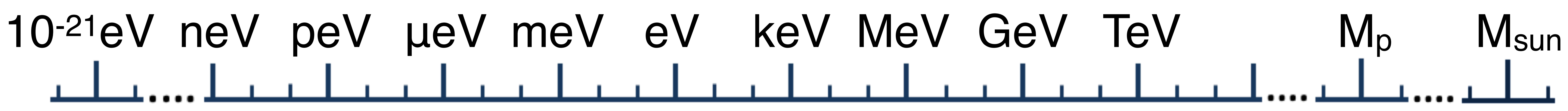
based on:

JA, Bramante & Goodman, PRD **105**, 023012

Composite Dark Matter

Composite Dark Matter

Dark matter mass



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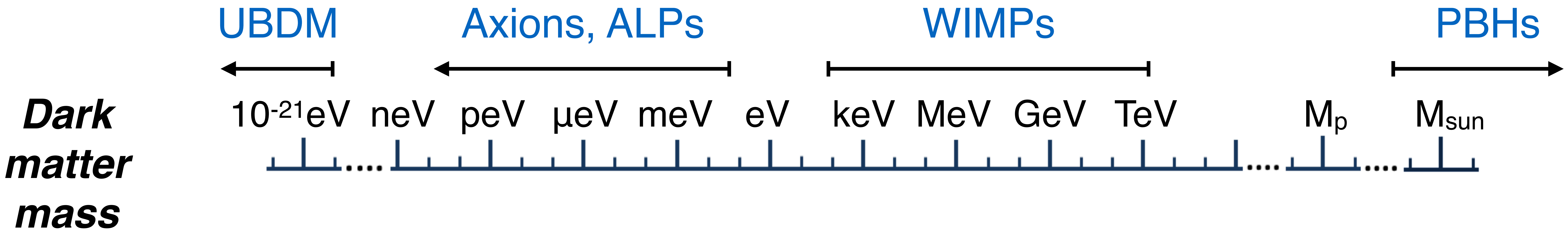

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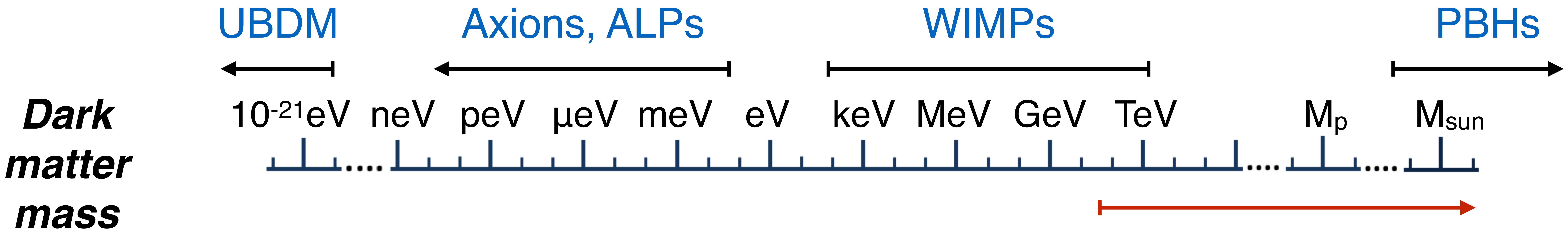
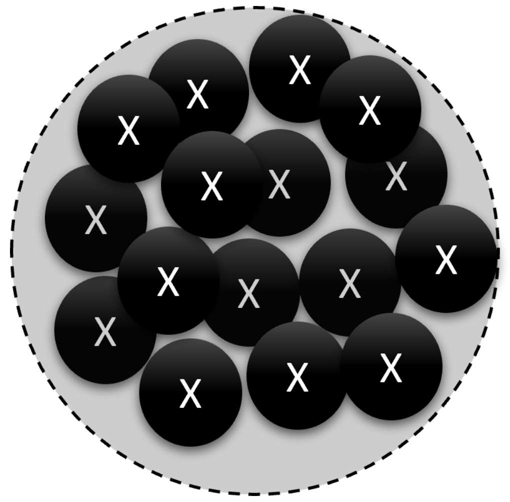


Composite DM



- *Fermionic composites*
- *Dark baryons, atoms*
- *Boson/axion stars*

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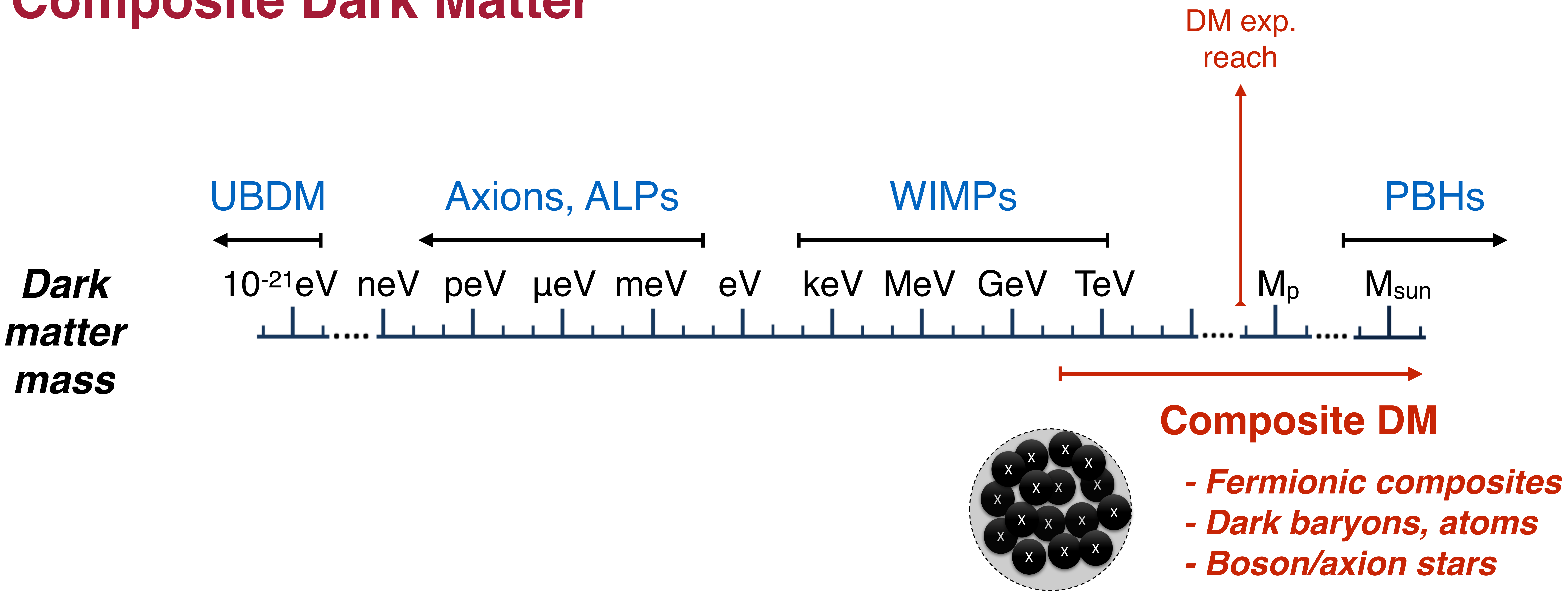

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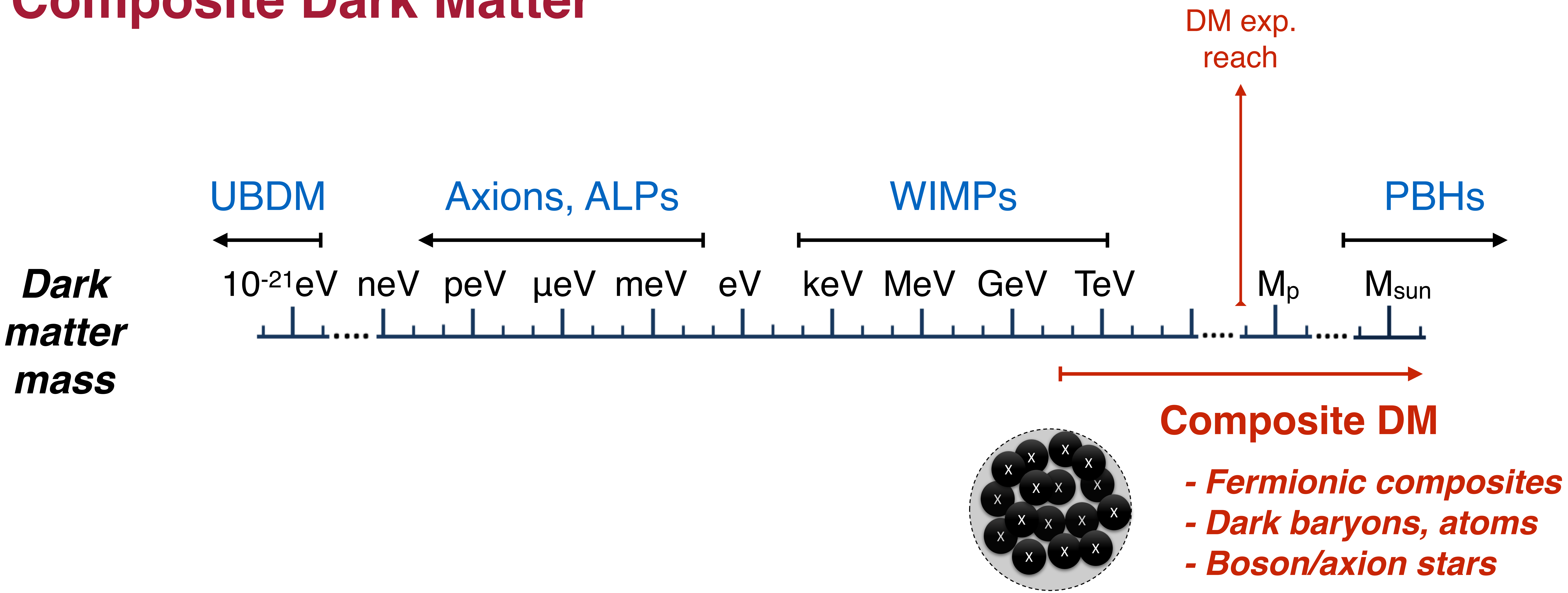


Potential issue w/ terrestrial detection:

few events at DM exp. (if any!)

Composite Dark Matter

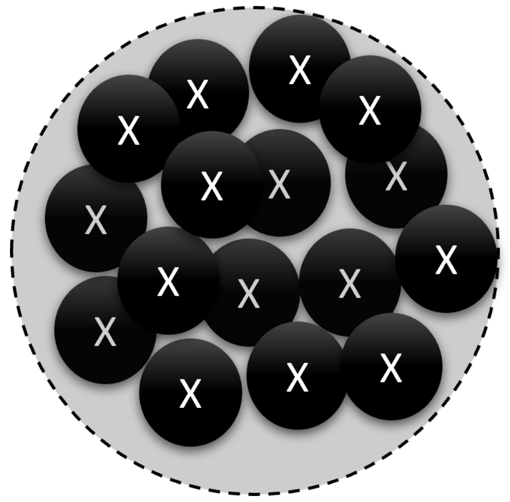
DM exp. reach

UBDM | Axions, ALPs | WIMPs | PBHs

Dark matter mass

10^{-21}eV neV peV μeV meV eV keV MeV GeV TeV M_p M_{sun}

Composite DM



- ***Fermionic composites***
- ***Dark baryons, atoms***
- ***Boson/axion stars***

Potential issue w/ terrestrial detection:

few events at DM exp. (if any!) → need large couplings to SM for detection?

A Simple Composite Model

Wise & Zhang, 1411.1772
Gresham et. al., 1707.02316
JA, Bramante & Goodman, 2012.10998

A Simple Composite Model

$$\mathscr{L}_{\rm DM} = \frac{1}{2}\partial^2\phi - \frac{1}{2}m_\phi^2\phi^2 + \bar{X}\left(i\gamma^\mu\partial_\mu - m_X\right)X + g_X\bar{X}X\phi$$

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For concreteness: $m_\phi \sim 1\ \text{MeV} - 100\ \text{MeV}$ $m_X \sim 1\ \text{GeV} - 100\ \text{TeV}$

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This model allows for bound state synthesis:

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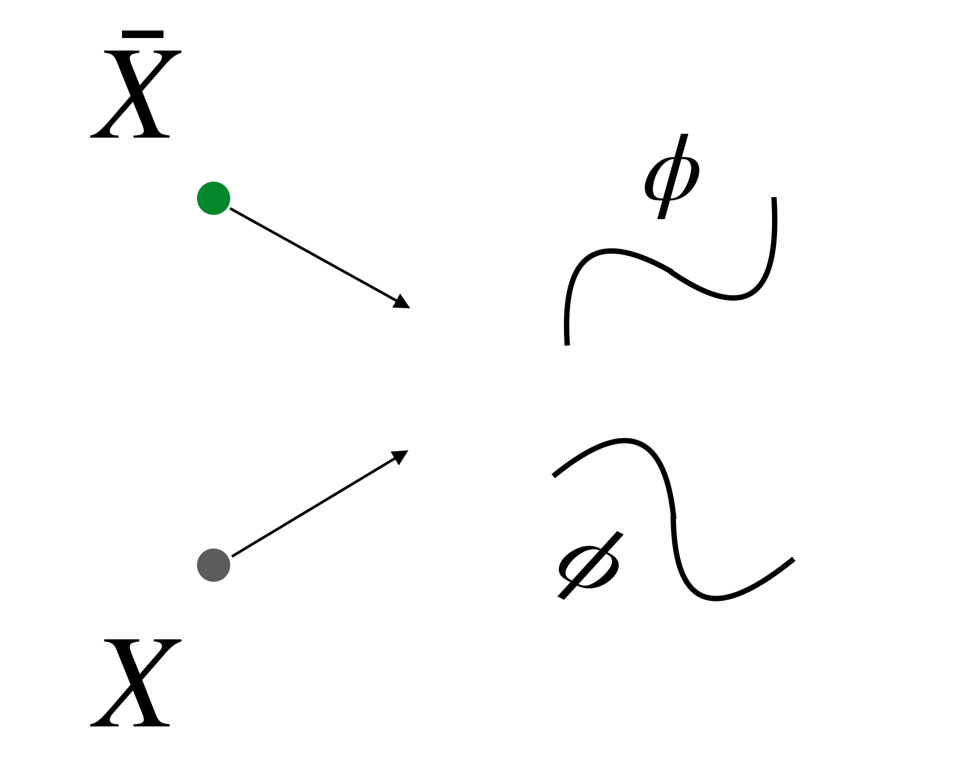

$X\bar{X}$ annihilation

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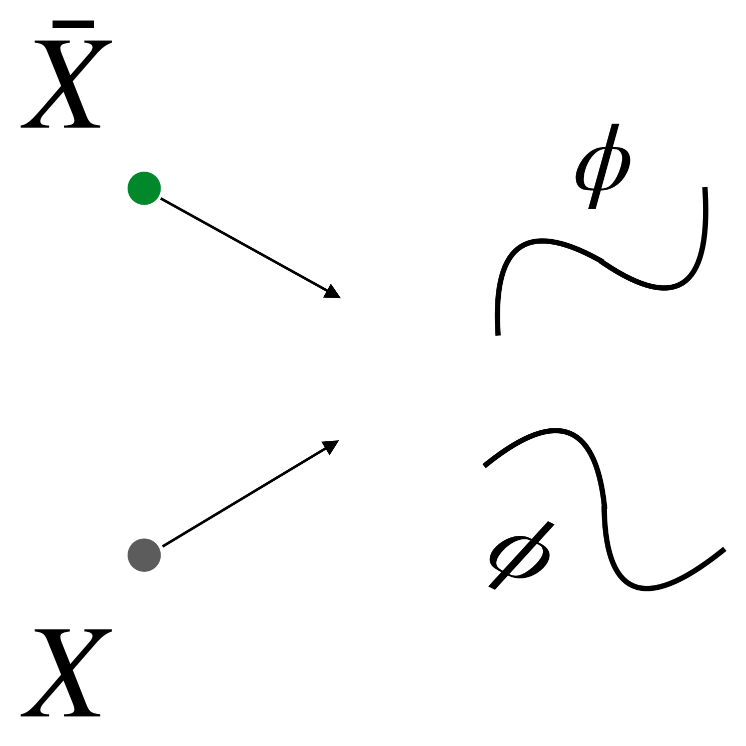

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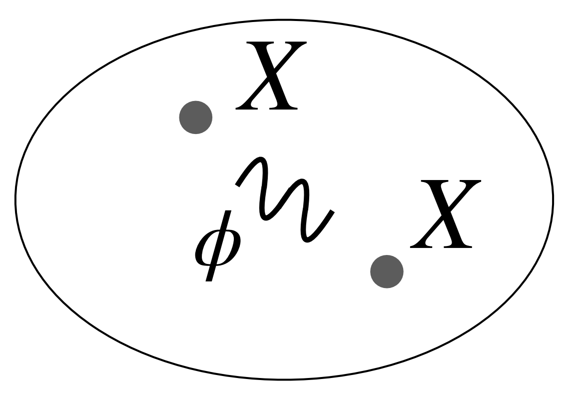

${}^2X$ formation

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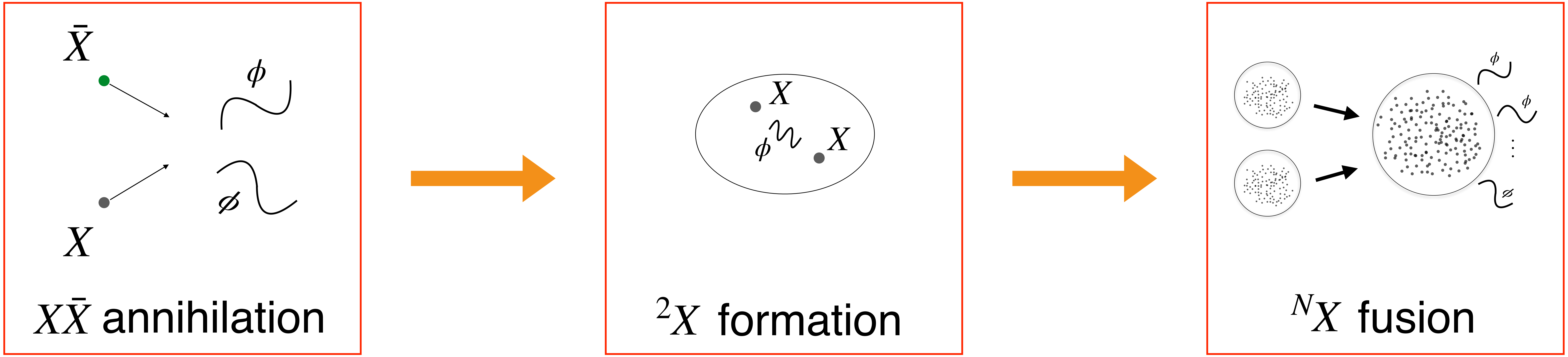
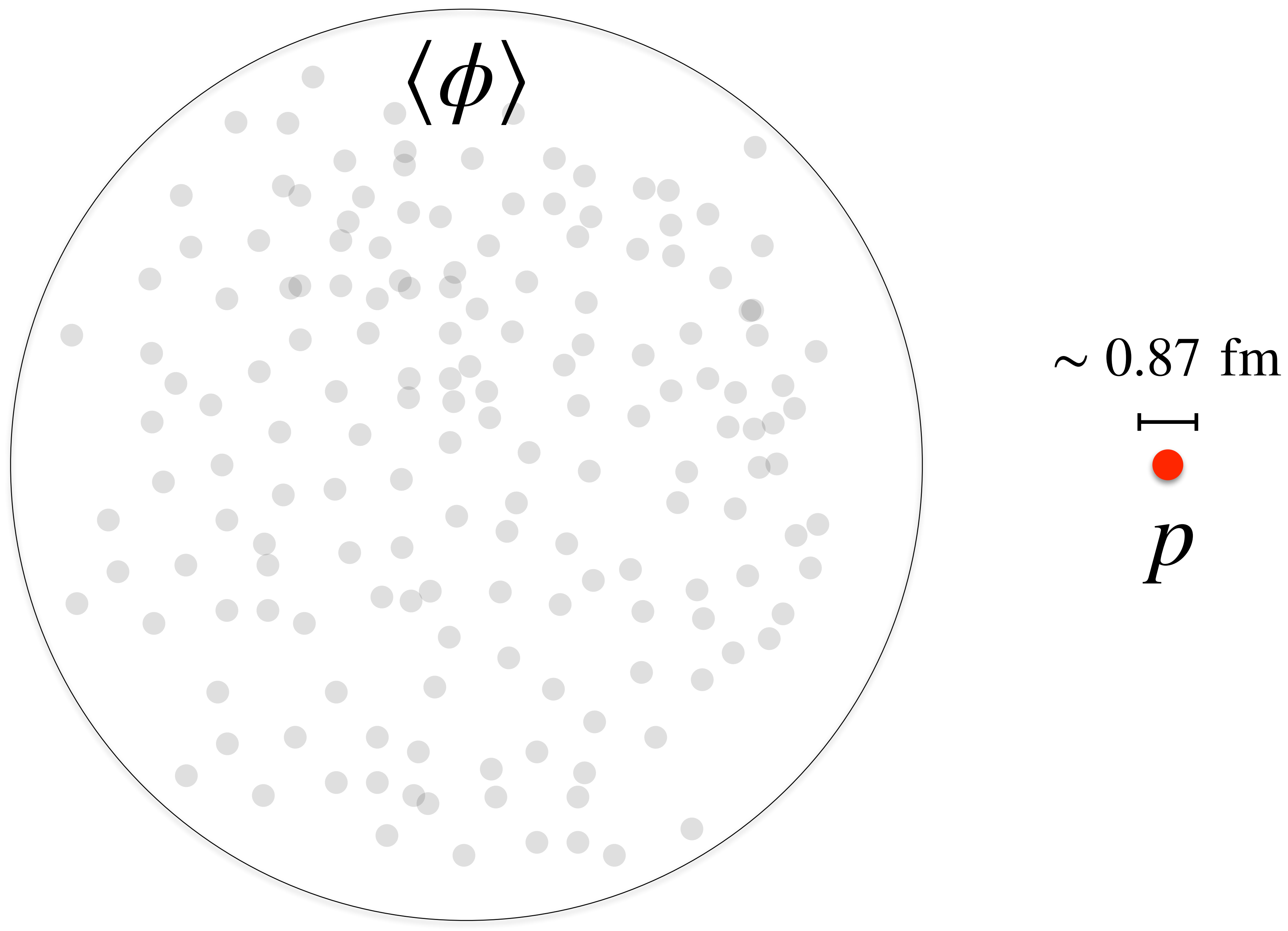


After assembly is complete:

$$10^{10}\ \text{GeV} \lesssim M_X \lesssim 10^{45}\ \text{GeV}$$

$$100\ \text{fm} \lesssim R_X \lesssim 10\ \mu\text{m}$$



^{N}X

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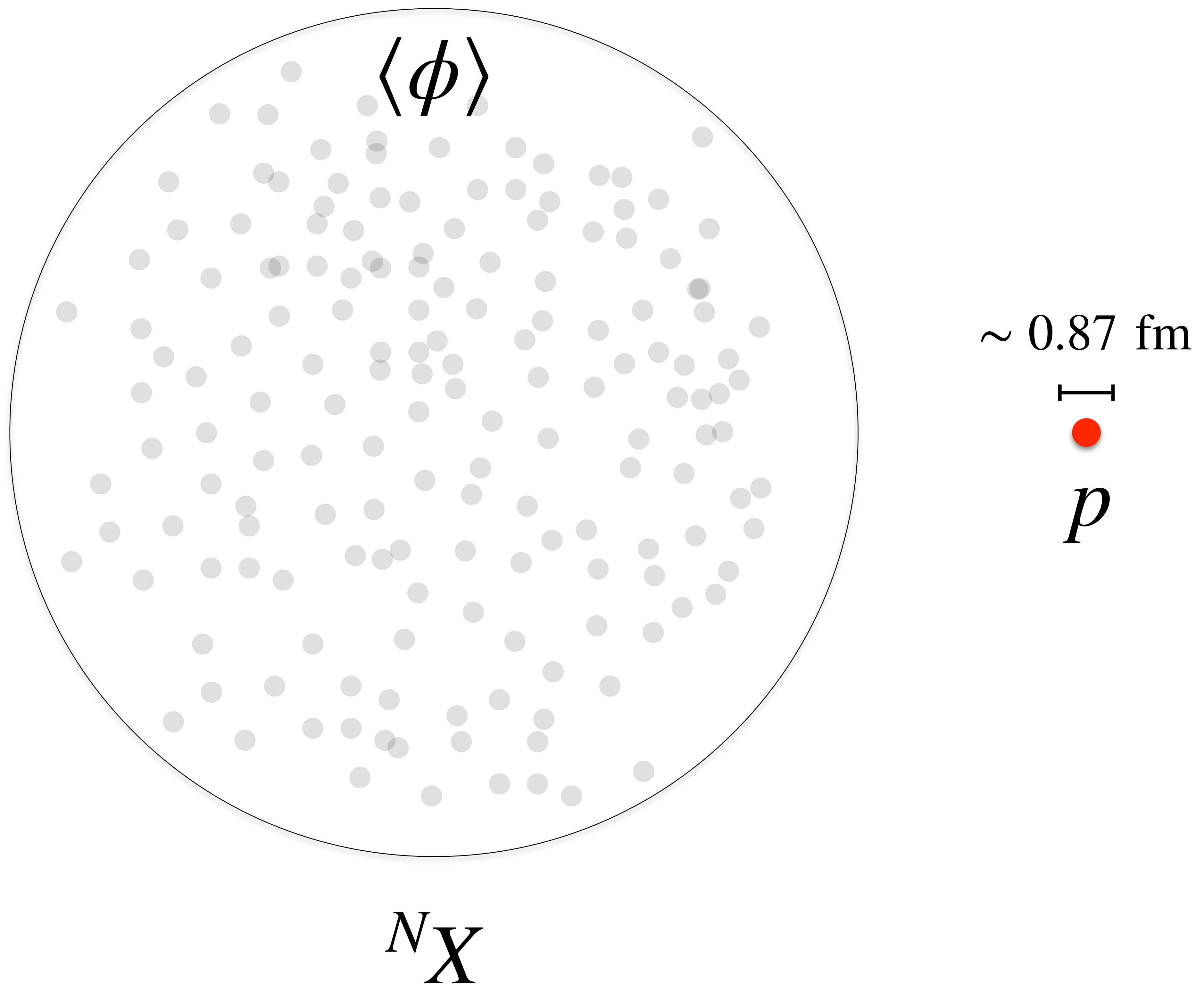
→ Highly degenerate constituents:

$$p_F \gtrsim \text{GeV}$$



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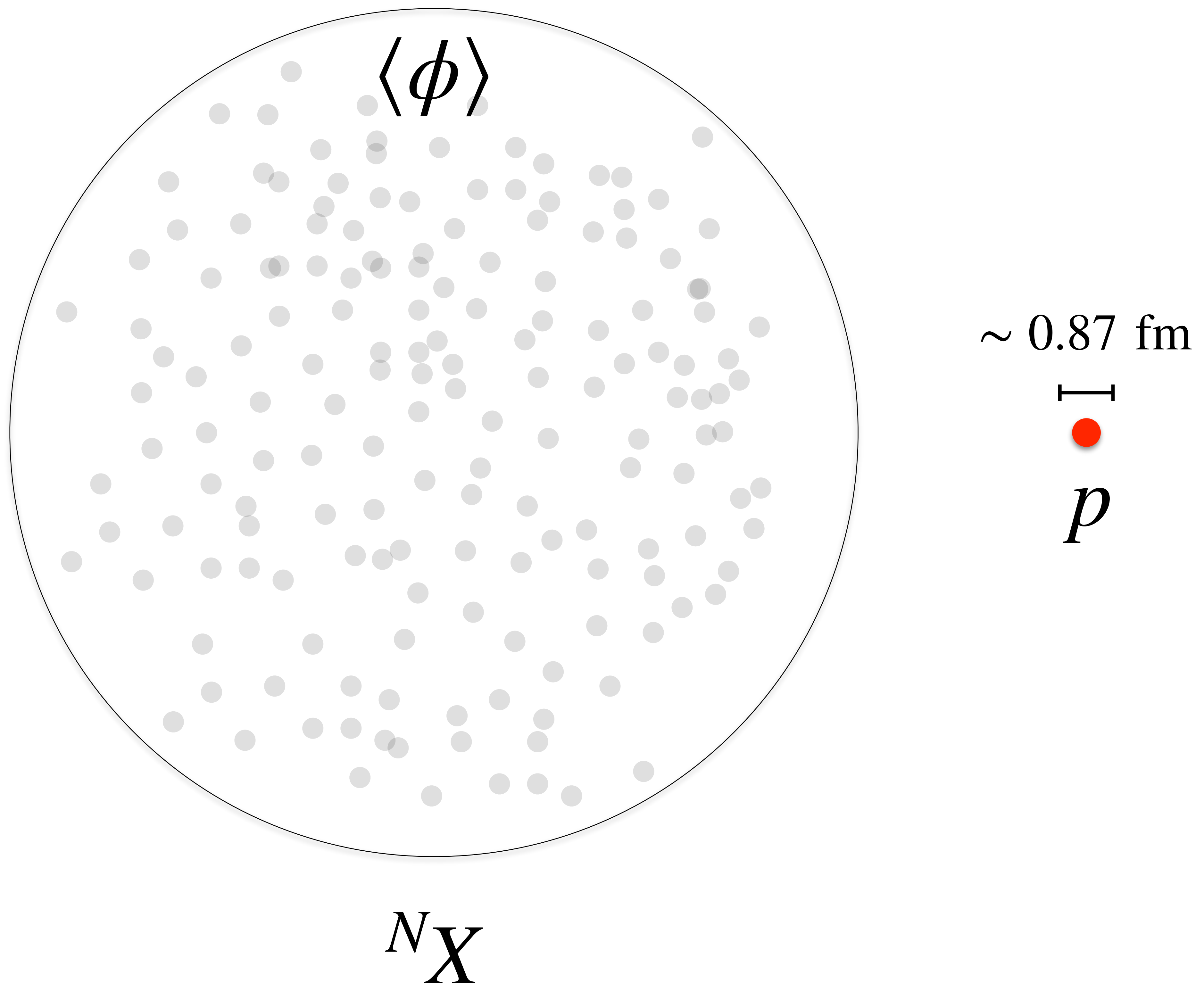
⟶ Highly degenerate constituents:

$$p_F \gtrsim \mathrm{GeV}$$

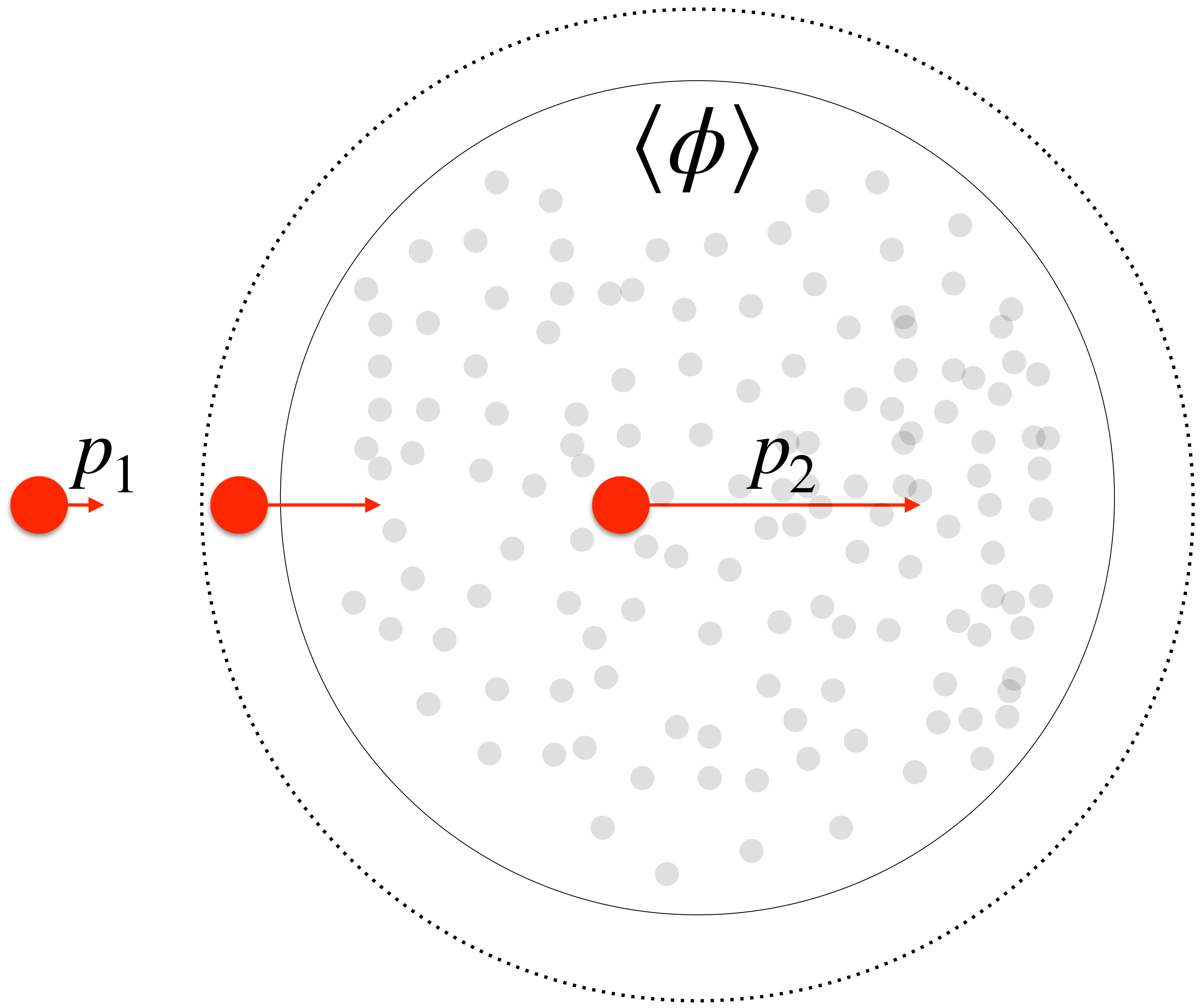
⟶ Very intense, uniform binding field:

$$\langle \phi \rangle \propto m_X \sim 1\ \mathrm{GeV} - 100\ \mathrm{TeV}$$



Adding a Nuclear Coupling

Add attractive Yukawa interaction: $\mathscr{L} = \mathscr{L}_{\rm DM} + g_n \bar{n} \phi n$



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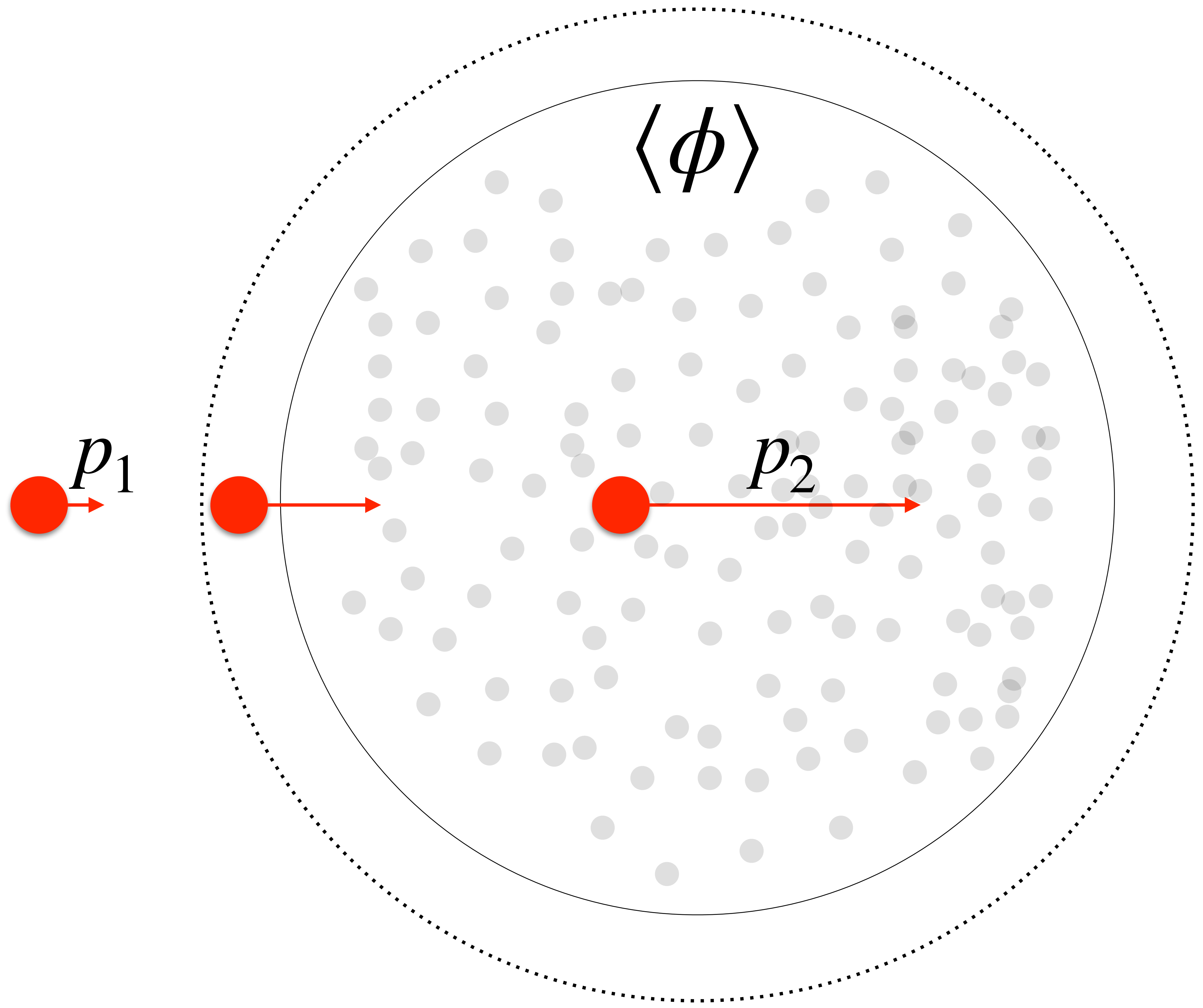

$$p_1^2 + m_N^2 = p_2^2 + (m_N - A g_n \langle\phi\rangle)^2$$

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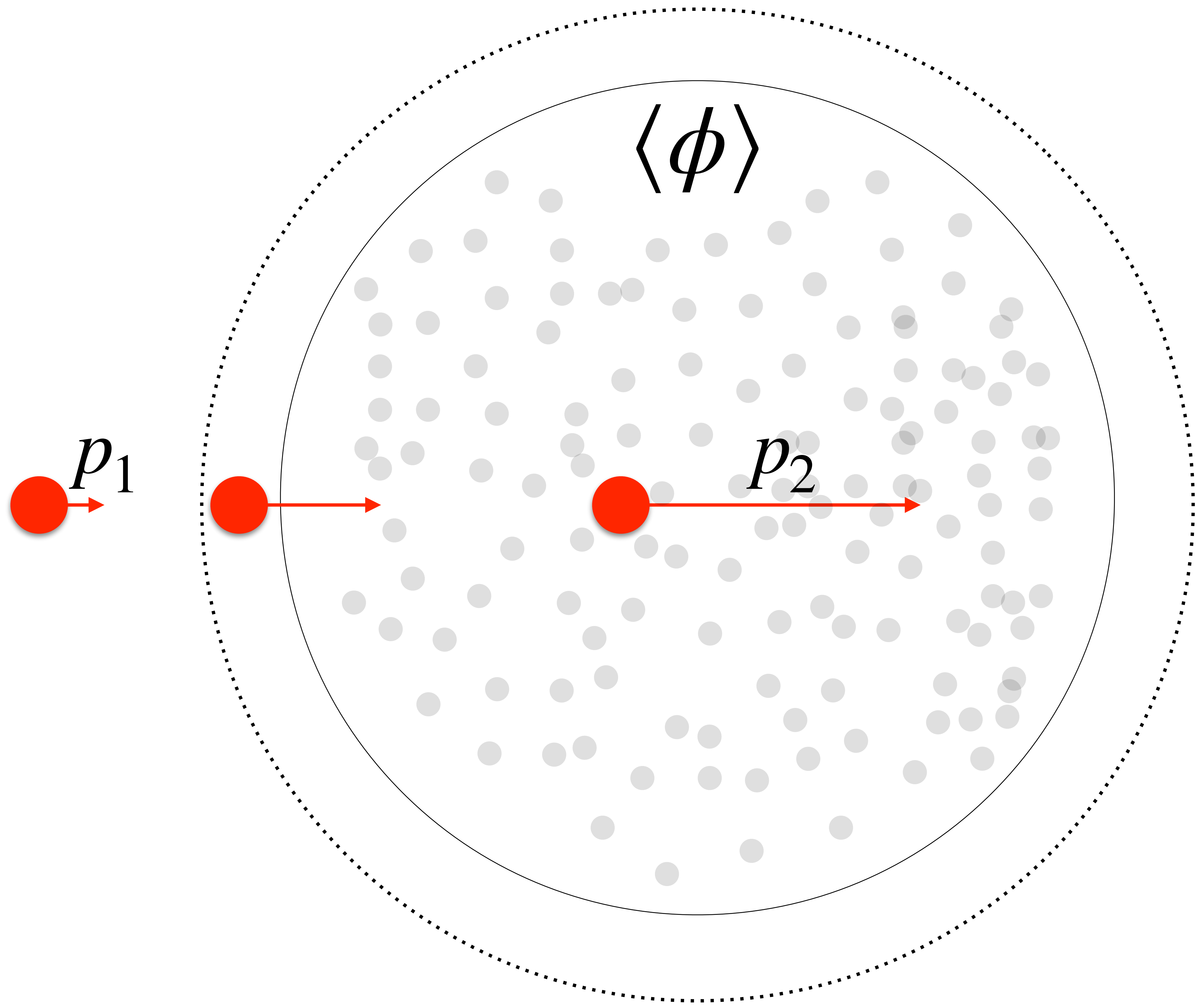

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↓ NR limit

$$\frac{p_2^2 - p_1^2}{2m_N} \propto g_n \langle \phi \rangle \propto g_n m_X$$

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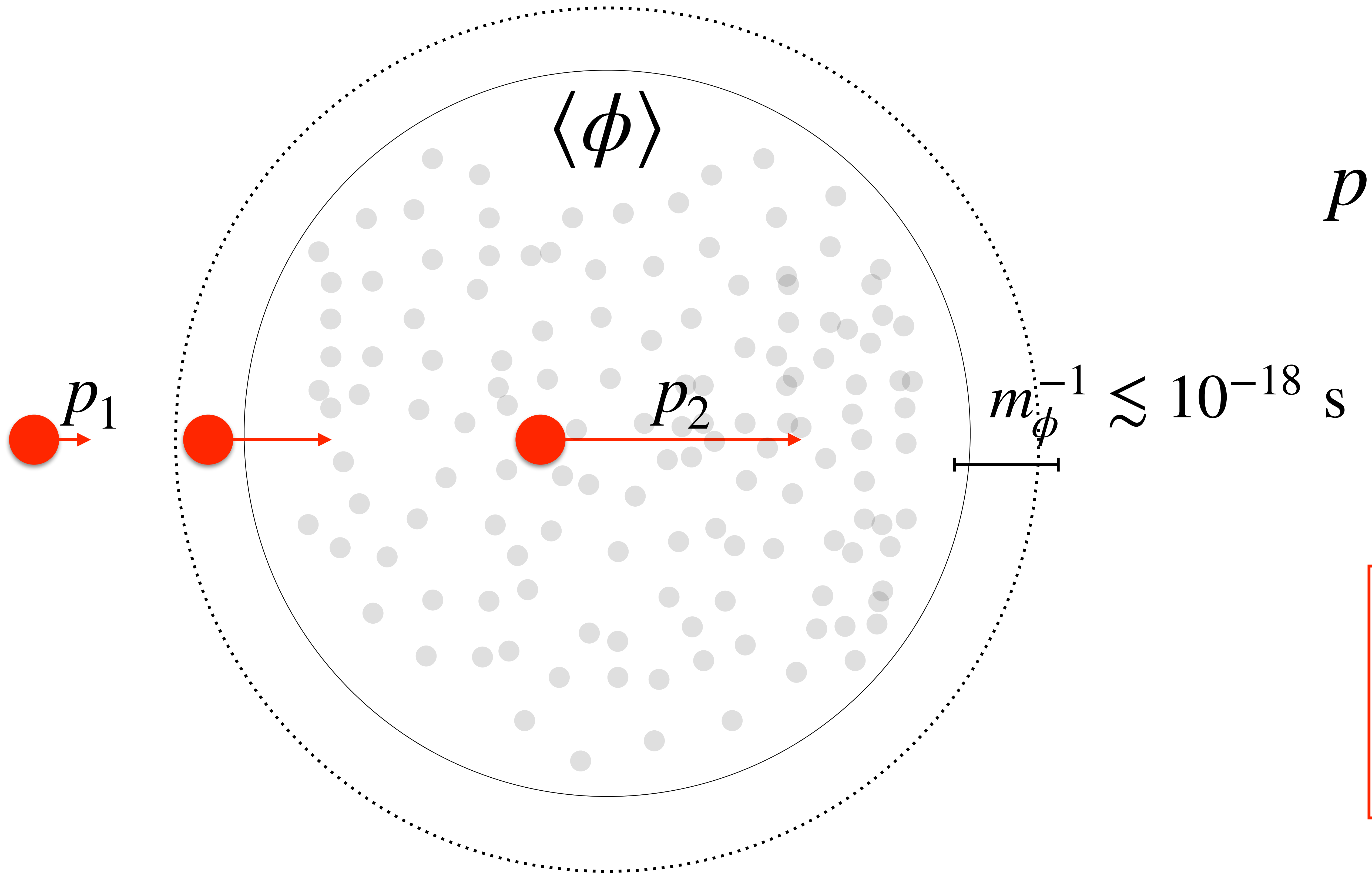

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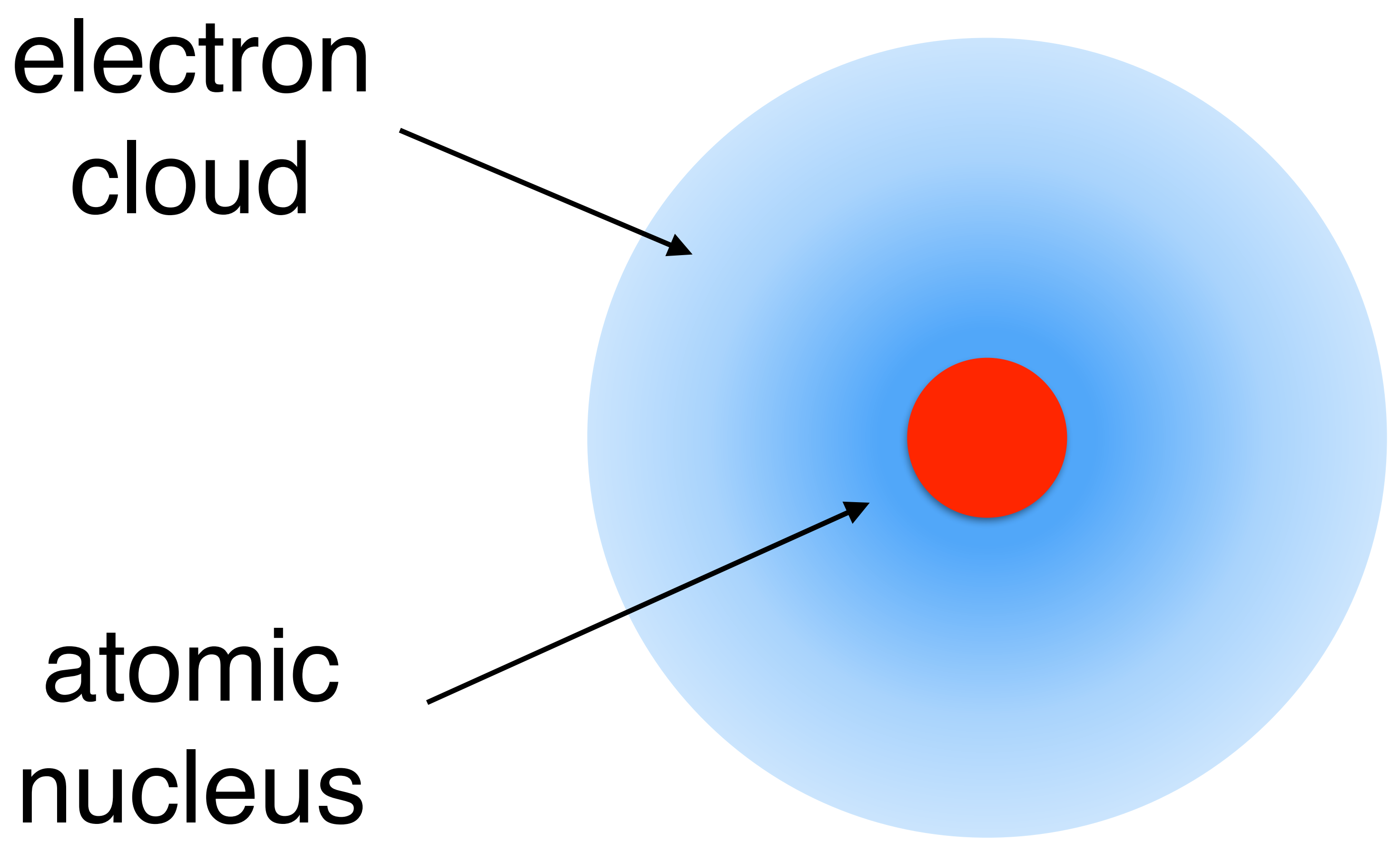
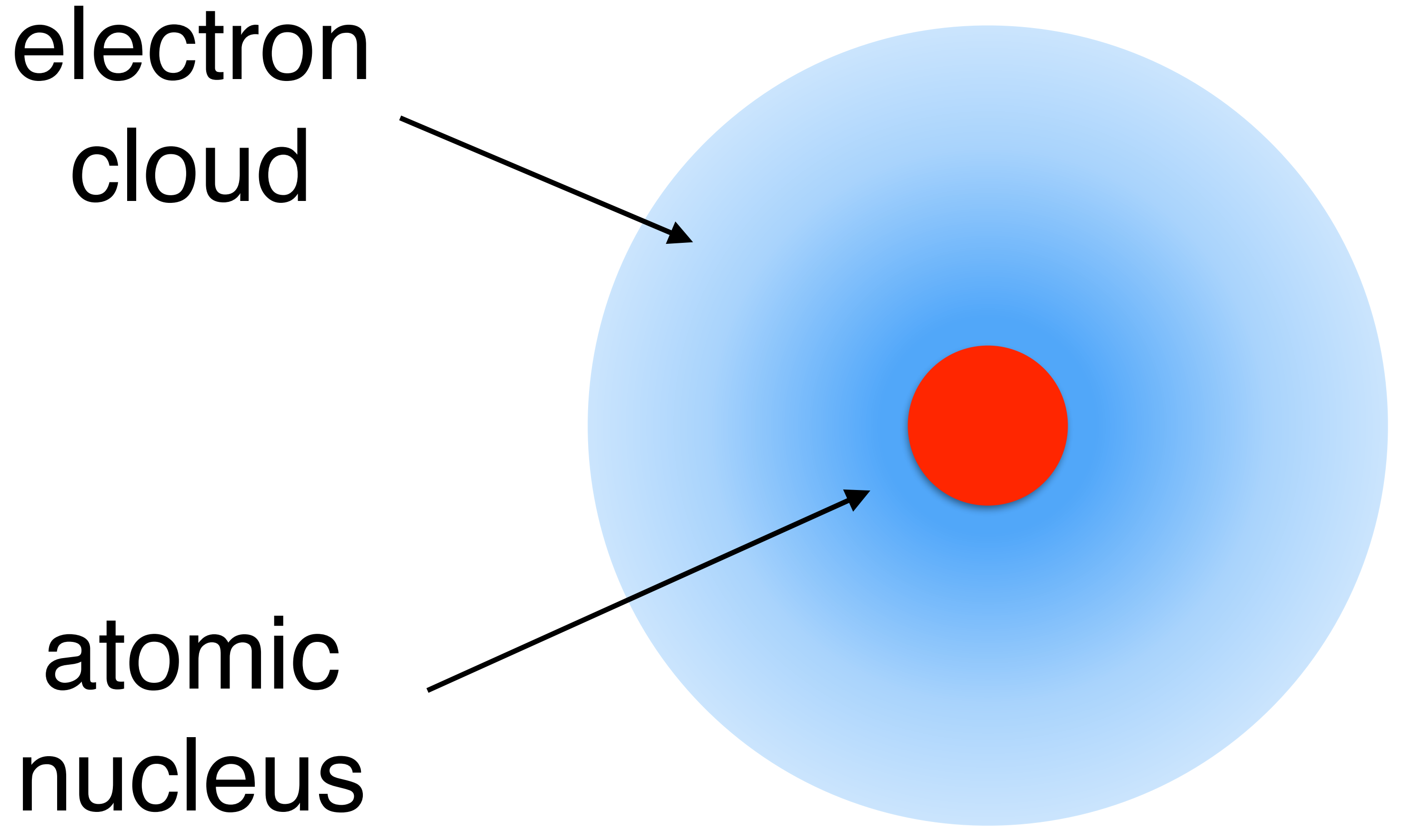
The Migdal Effect



$$|\psi_0\rangle$$

$$\langle\psi_k|\psi_0\rangle = 0$$

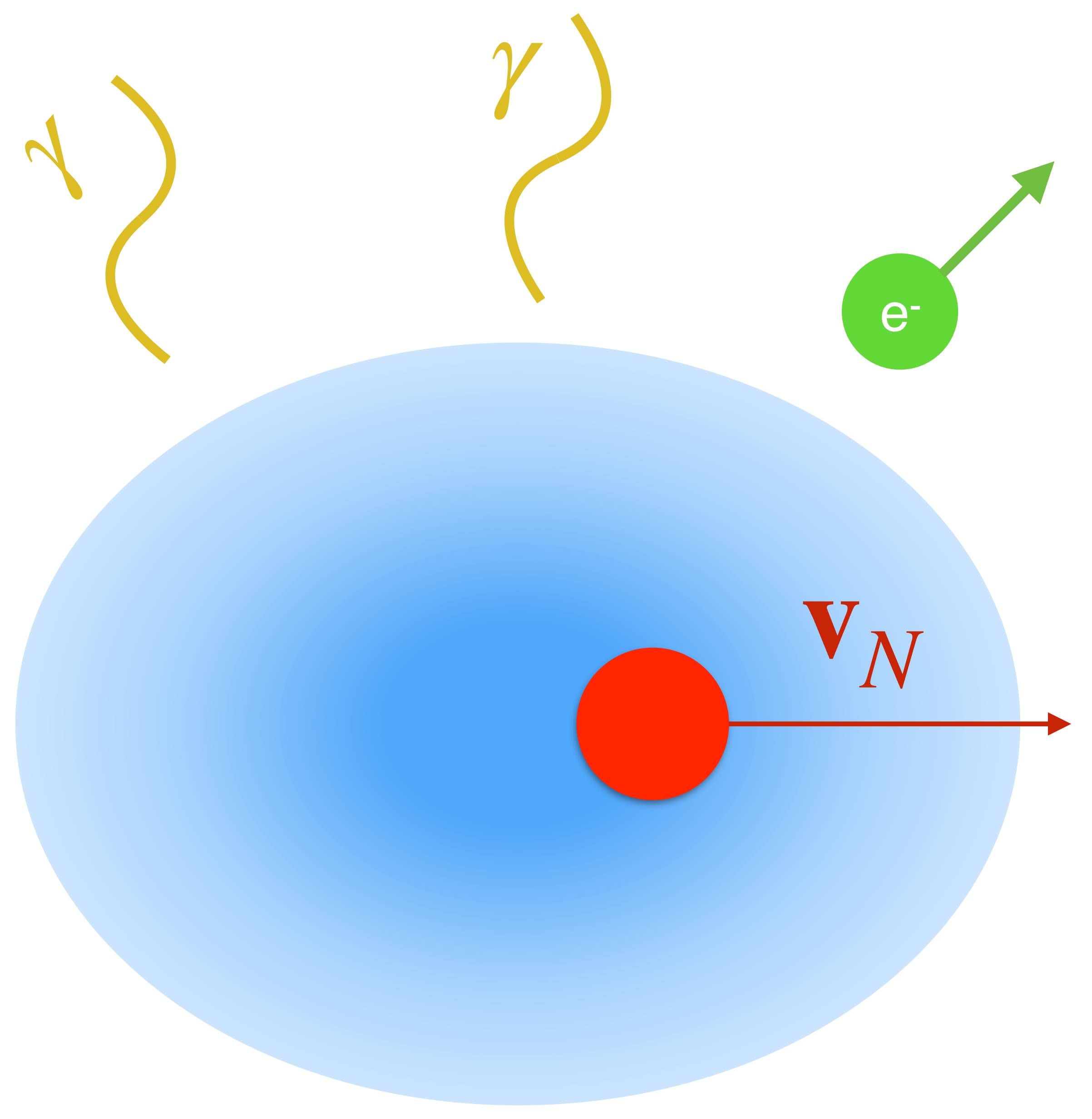
The Migdal Effect



sudden nuclear recoil

e.g. $\alpha, \beta^{\pm}$ decay

DM scattering?



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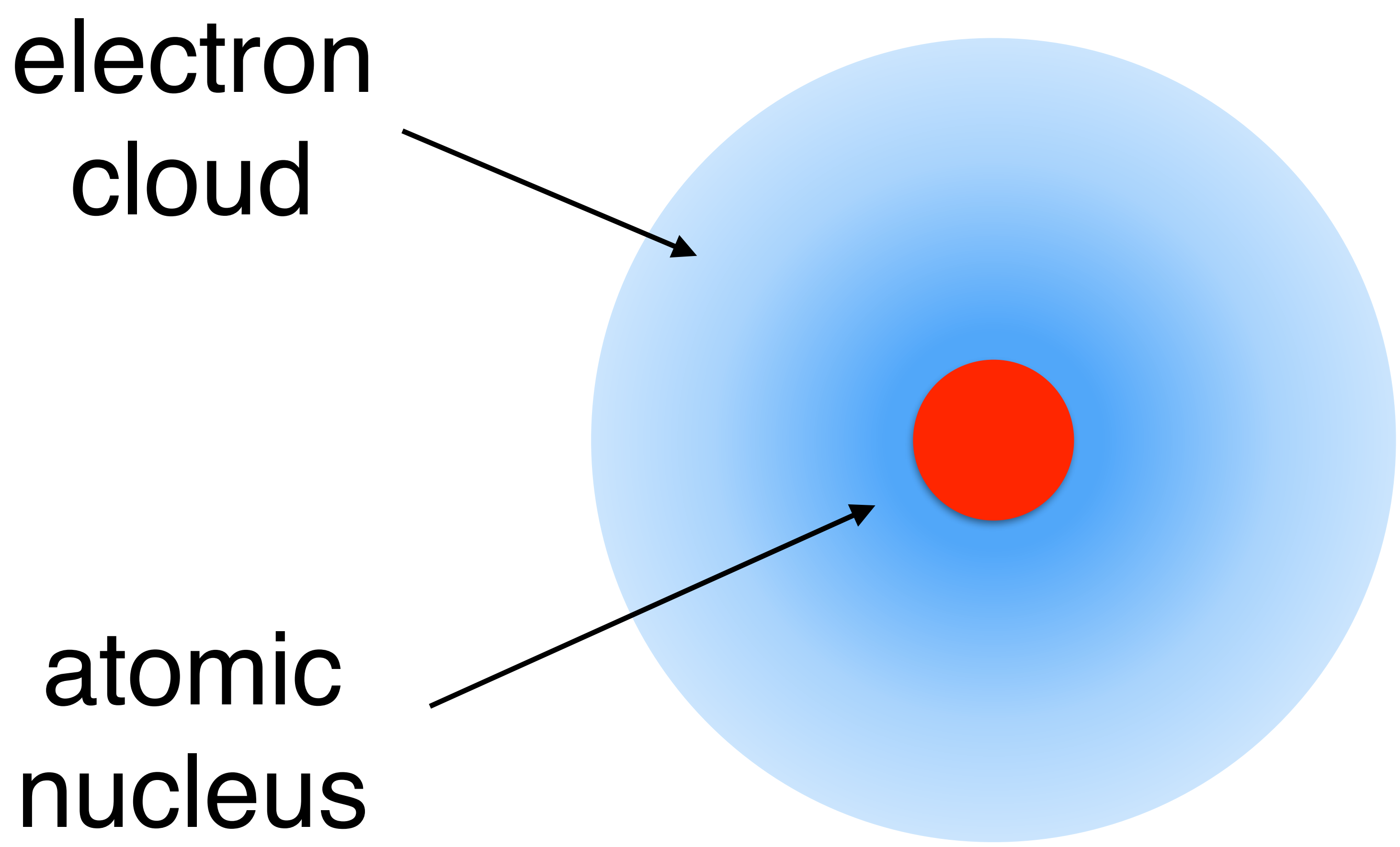
$$\langle\psi_k|\psi_0\rangle = 0$$

$$|\psi\rangle \simeq e^{\left(-im_e \sum_j \mathbf{v_N}\cdot\hat{\mathbf{x}}_j\right)}|\psi_0\rangle$$

$$\langle\psi_k|\psi\rangle \neq 0$$

The Migdal Effect



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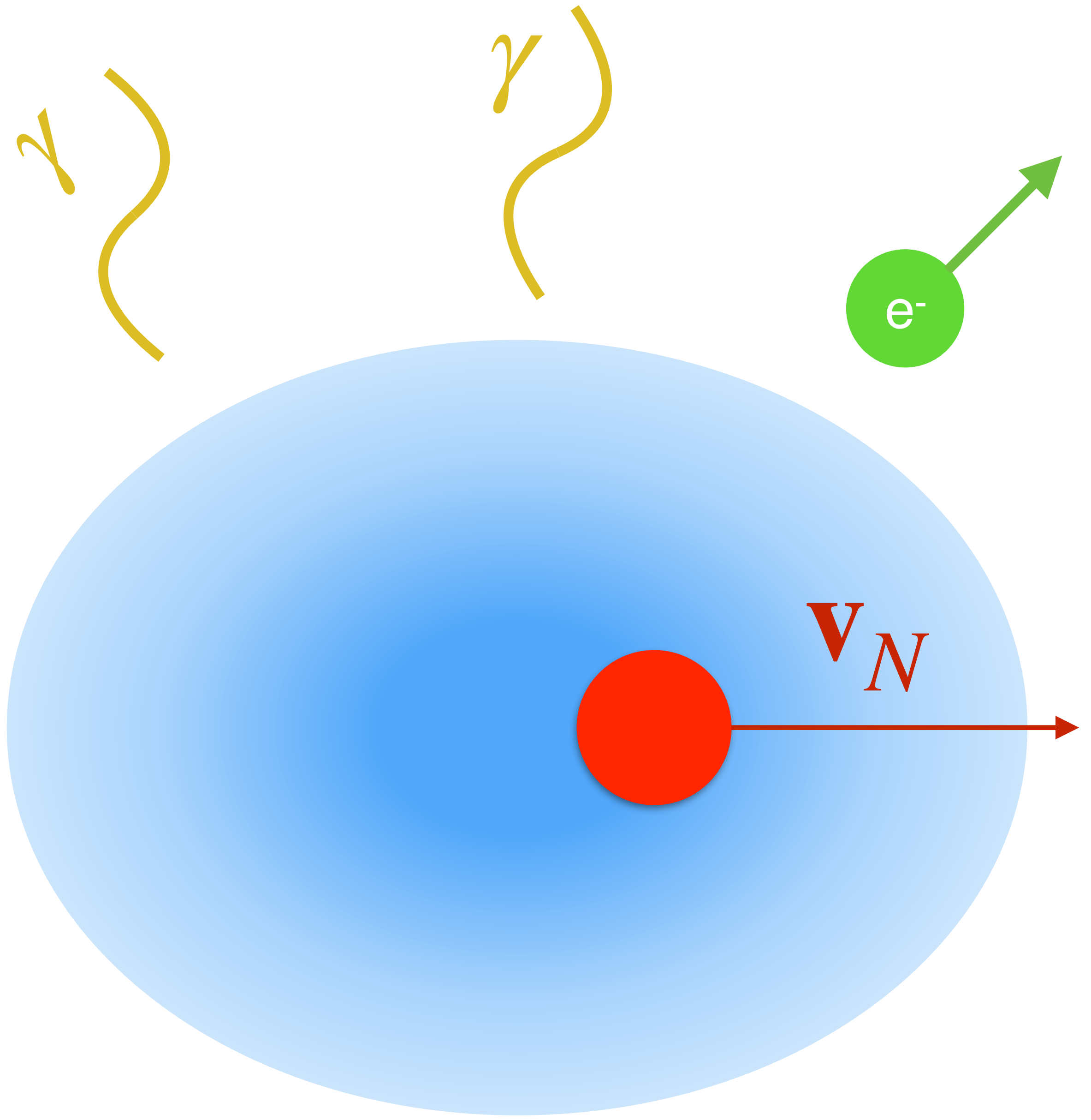

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How sudden? $\Delta t_{\mathrm{recoil}} \ll 10^{-17}\ \mathrm{s}$ Migdal approximation

(e.g. Xe, Ar)

Excitation & Ionization Probabilities

Xe ($q_e = m_e \times 10^{-3}$)

(n,ℓ)	$\mathcal{P}_{\to 4f}$	$\mathcal{P}_{\to 5d}$	$\mathcal{P}_{\to 6s}$	$\mathcal{P}_{\to 6p}$	$E_{n\ell}$ [eV]	$\frac{1}{2\pi}\int dE_e \frac{dp^c}{dE_e}$
1s	–	–	–	7.3×10^{-10}	3.5×10^{4}	4.9×10^{-6}
2s	–	–	–	1.8×10^{-8}	5.4×10^{3}	3.0×10^{-5}
2p	–	3.0×10^{-8}	6.5×10^{-9}	–	4.9×10^{3}	1.3×10^{-4}
3s	–	–	–	2.7×10^{-7}	1.1×10^{3}	1.1×10^{-4}
3p	–	3.4×10^{-7}	4.0×10^{-7}	–	9.3×10^{2}	6.0×10^{-4}
3d	2.3×10^{-9}	–	–	4.3×10^{-7}	6.6×10^{2}	3.6×10^{-3}
4s	–	–	–	3.1×10^{-6}	2.0×10^{2}	3.6×10^{-4}
4p	–	4.1×10^{-8}	3.0×10^{-5}	–	1.4×10^{2}	1.5×10^{-3}
4d	7.0×10^{-7}	–	–	1.5×10^{-4}	6.1×10	3.6×10^{-2}
5s	–	–	–	1.2×10^{-4}	2.1×10	4.7×10^{-4}
5p	–	3.6×10^{-2}	2.1×10^{-2}	–	9.8	7.8×10^{-2}

(n,ℓ)	$4f$	$5d$	$6s$	$6p$
$E_{n\ell}$[eV]	0.85	1.6	3.3	2.2

Excitation & Ionization Probabilities

initial level →

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Ibe et. al., 1707.07258

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initial level

ionization prob.

Excitation & Ionization Probabilities

e.g. ~ eV nuclear recoil for Xe:

$$P\left(5p \to \text{free}\right) \sim \left(\frac{\text{eV}}{10^{-3} \cdot m_e}\right)^2 \int \left(\frac{dp_q}{dE_e}\right)_{q=10^{-3}\cdot m_e} dE_e \sim 10^{-7}$$

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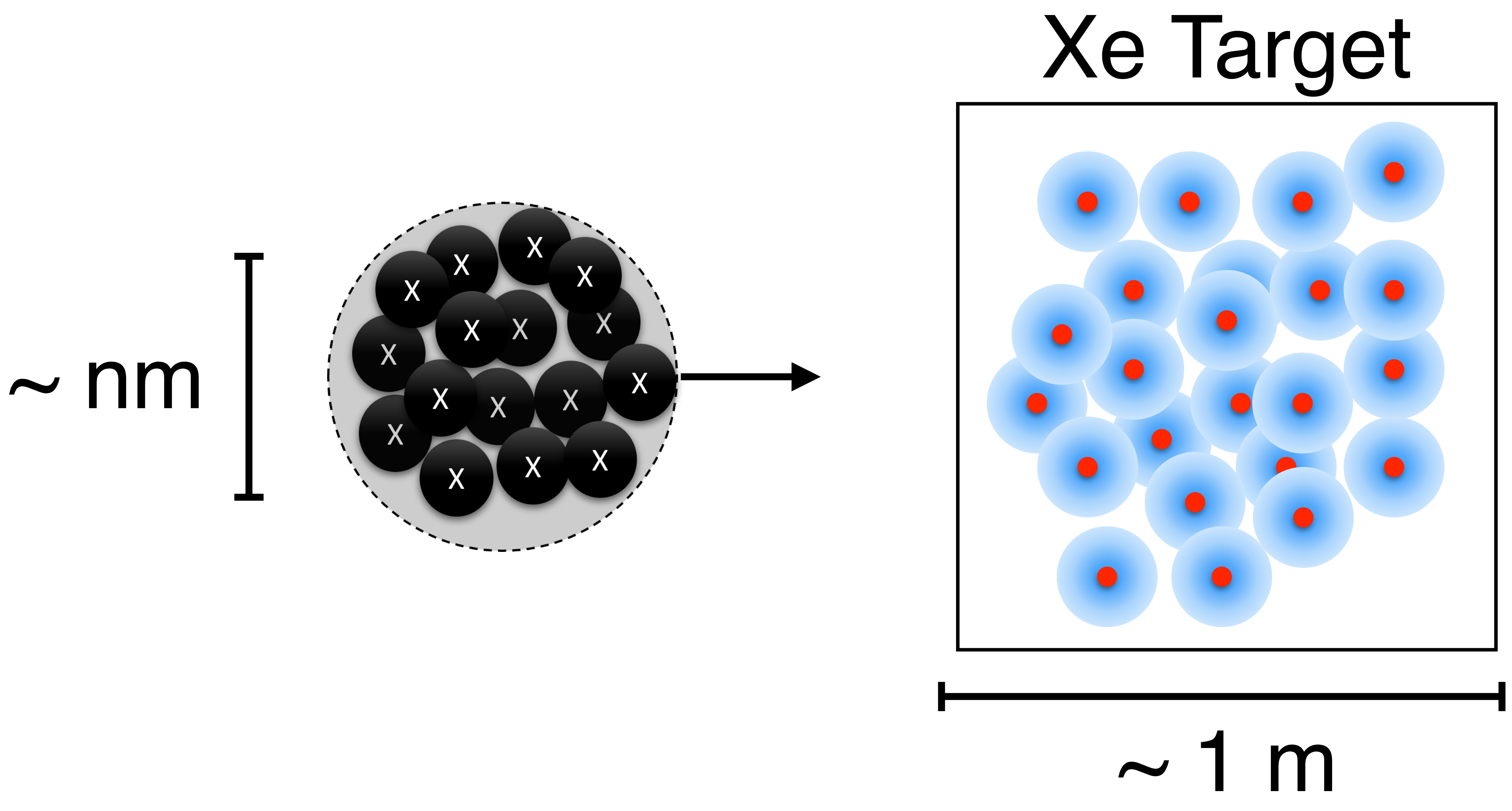

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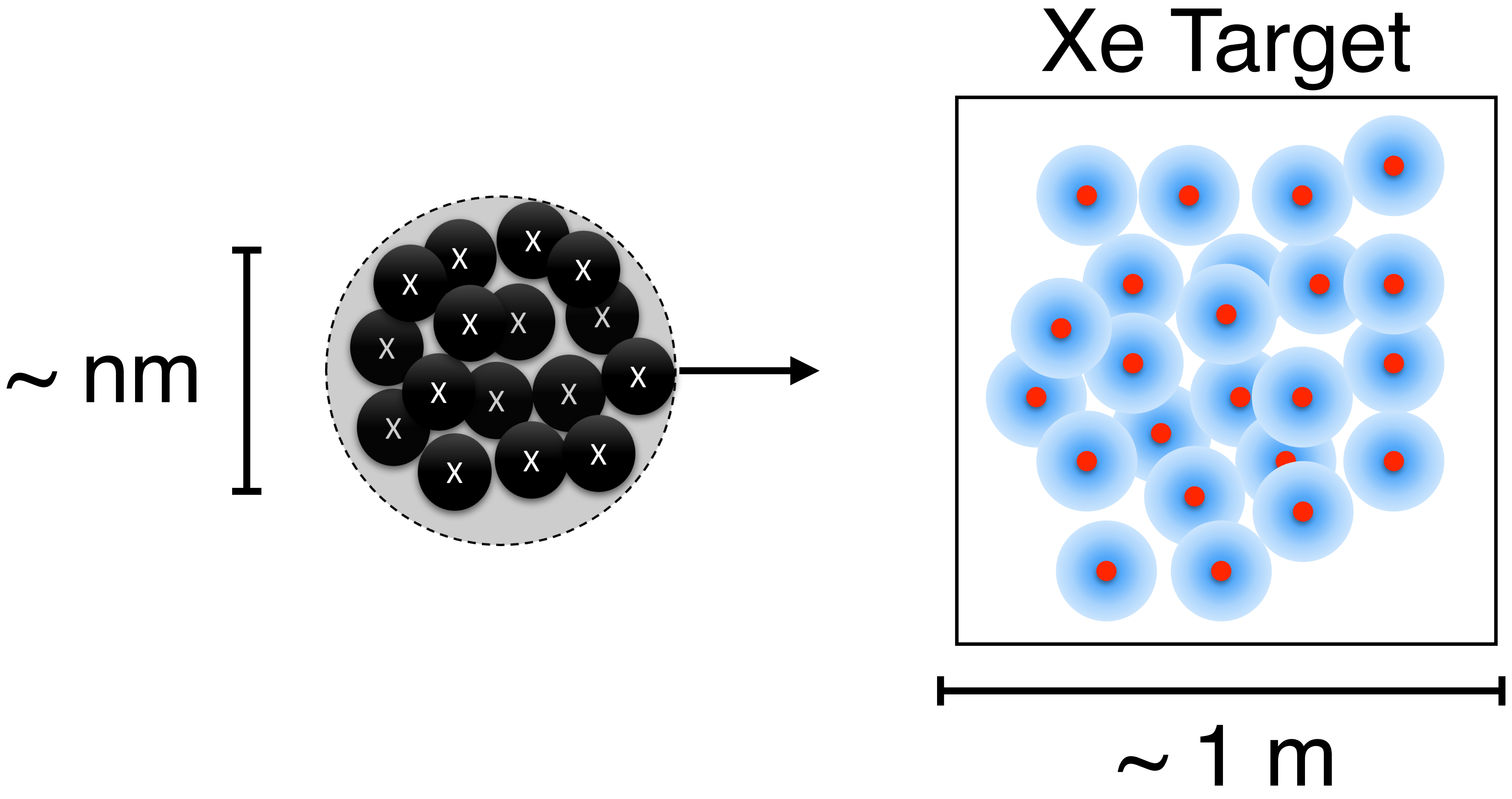
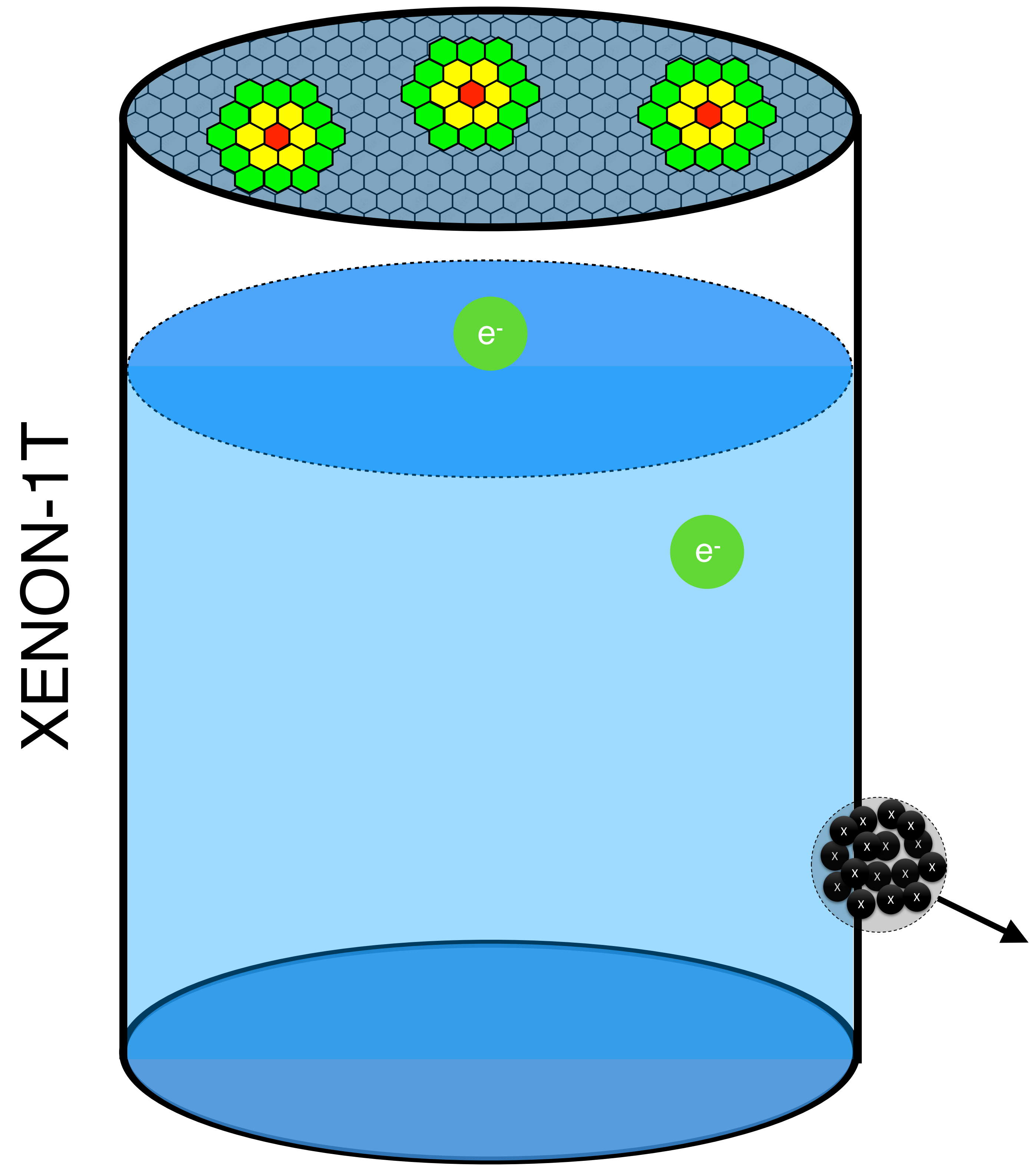


$$\sigma_X n_{\text{Xe}} L_{\text{path}} \times P\left(5p \to \text{free}\right)$$

$$\sim 10^2 \text{ ionized e}^-$$

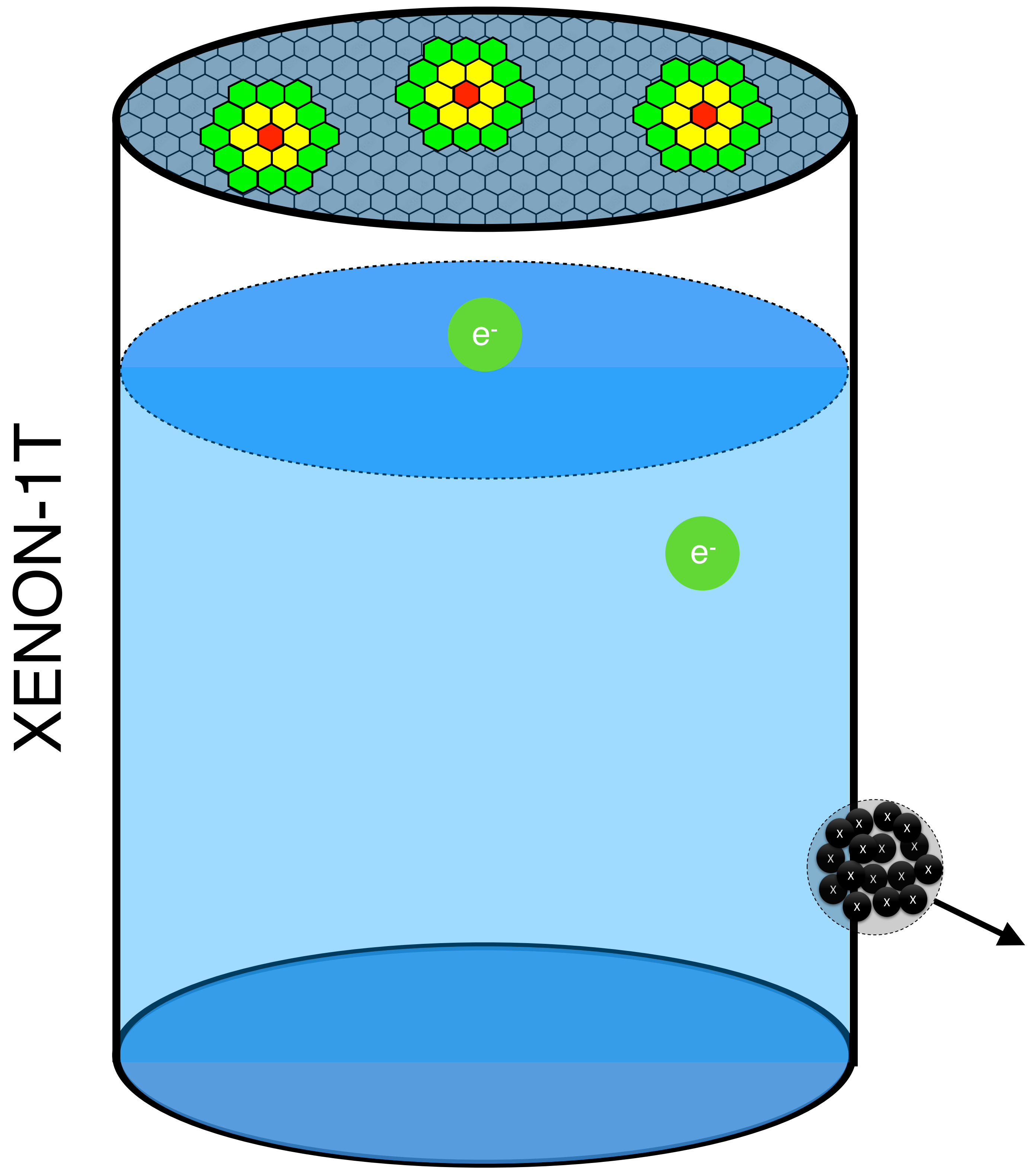
Ionization Signal



Event Rate:

$$R_{ion} \propto 4\pi R_X^2 \cdot \left[\int_{v>v^{(\min)}} dv \; n_X v \; g(v) \right] \cdot \left[\frac{1}{2\pi} \sum_{n,l} \int dE_e \; \varepsilon(E_{em}) \frac{dp_q}{dE_e}(n, l \to E_e) \right]$$

Ionization Signal



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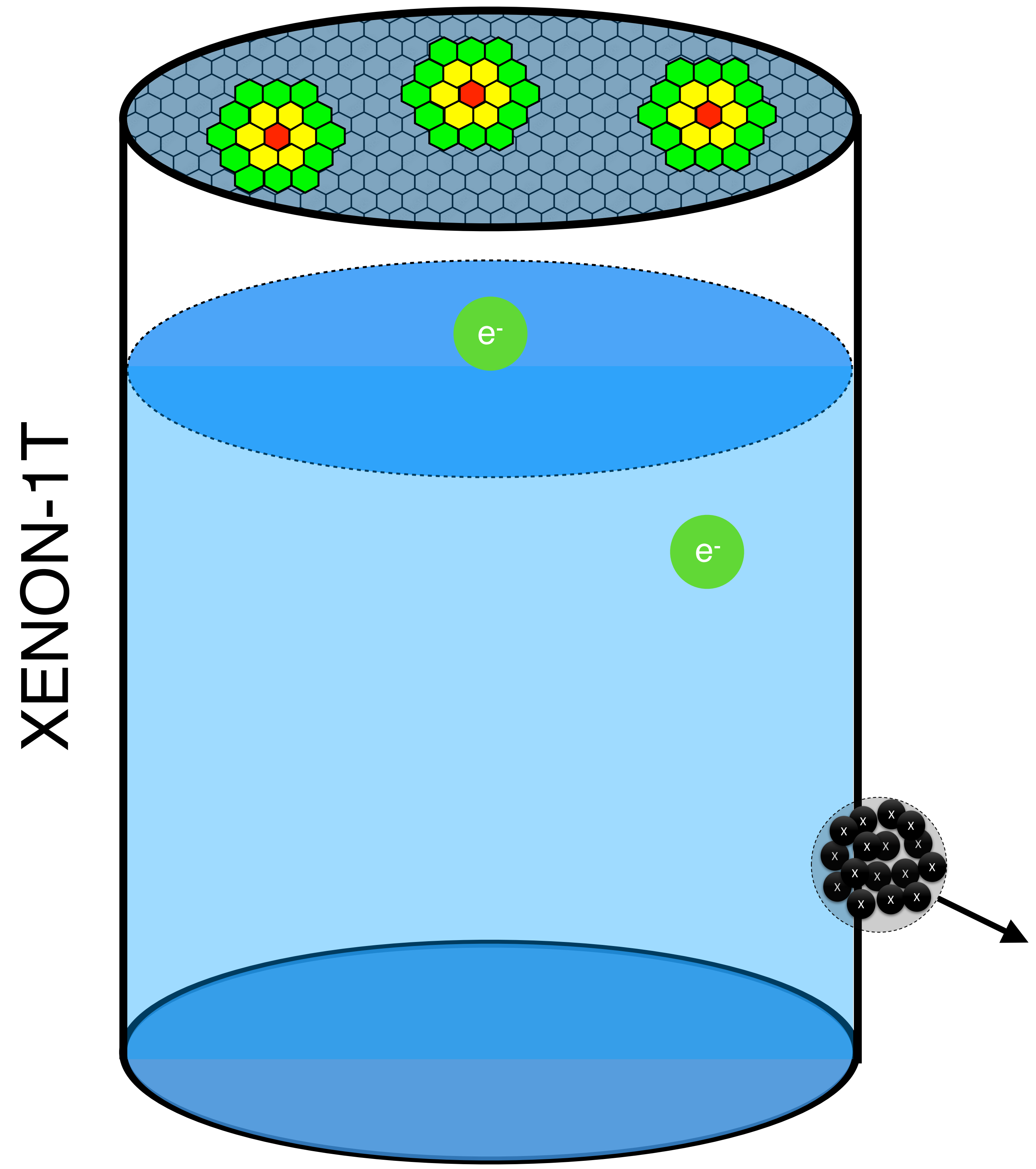
geom.
cross-section

Ionization Signal



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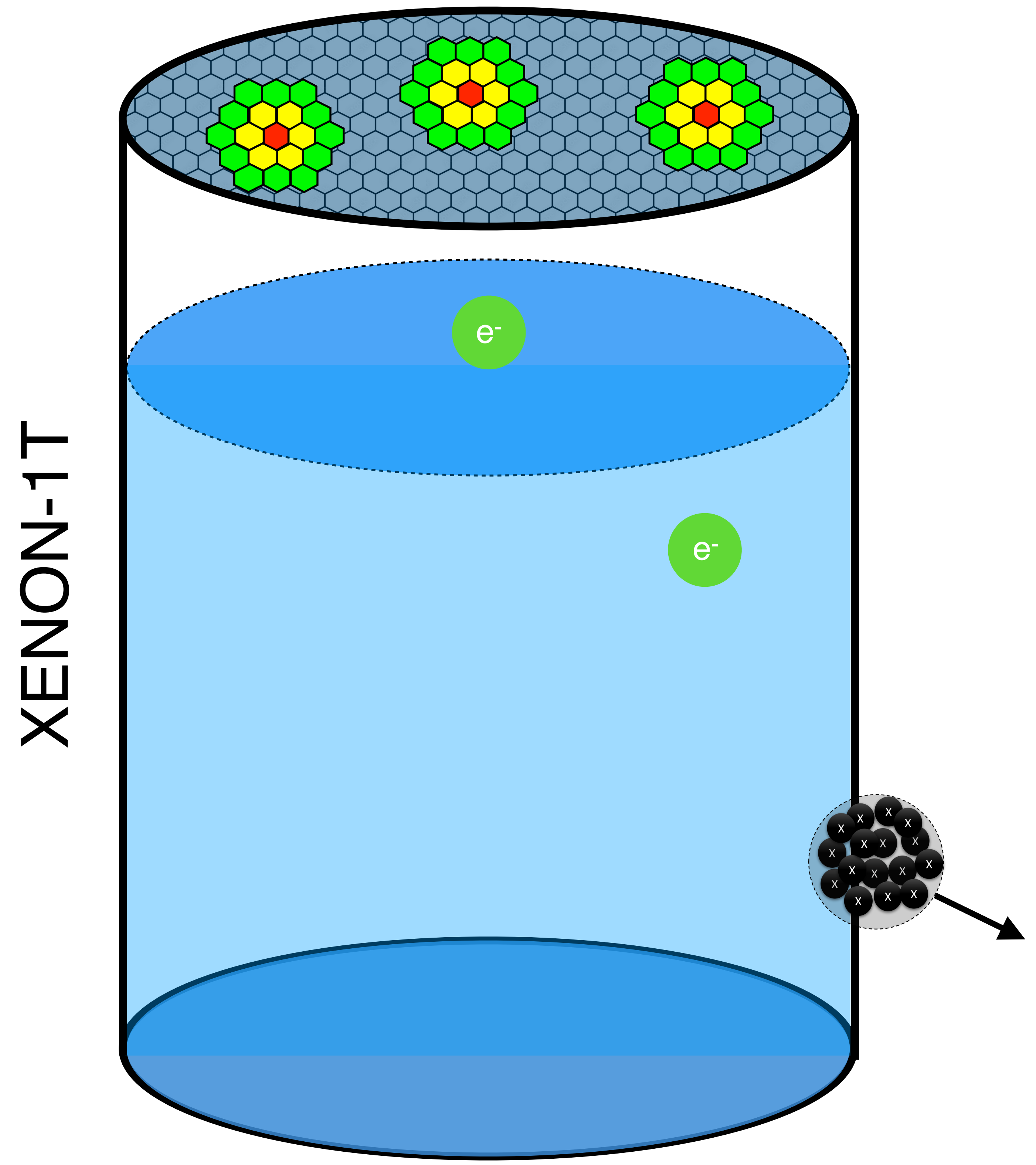
geom. cross-section

dark matter flux

Ionization Signal



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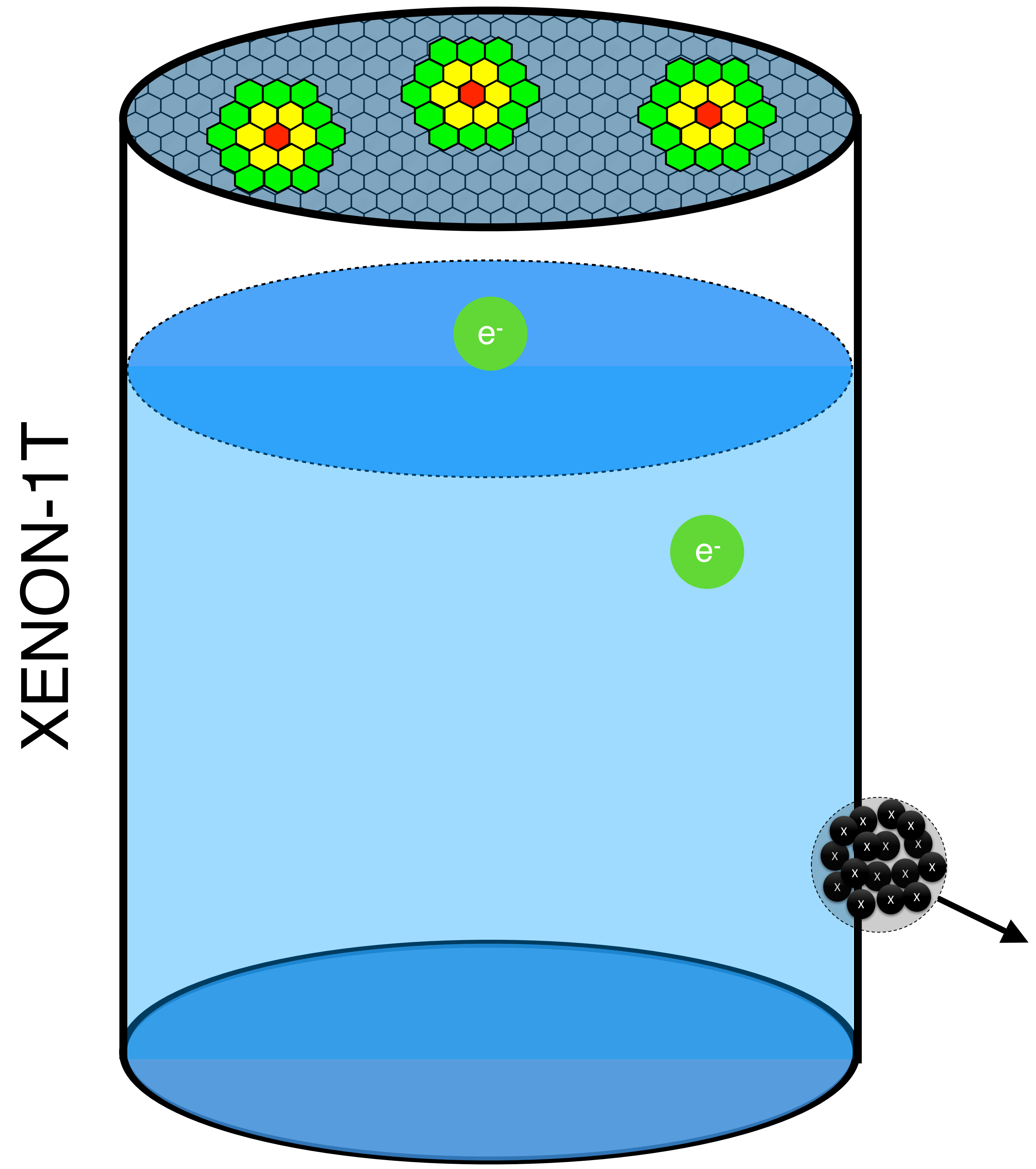
geom. cross-section

dark matter flux

Migdal probabilities

Ionization Signal



Event Rate:

$$R_{ion} \propto 4\pi R_X^2 \cdot \left[\int_{v>v^{(\min)}} dv \; n_X v \; g(v)\right] \cdot \left[\frac{1}{2\pi}\sum_{n,l}\int dE_e \; \varepsilon(E_{em})\frac{dp_q}{dE_e}(n,l \to E_e)\right]$$

geom. cross-section

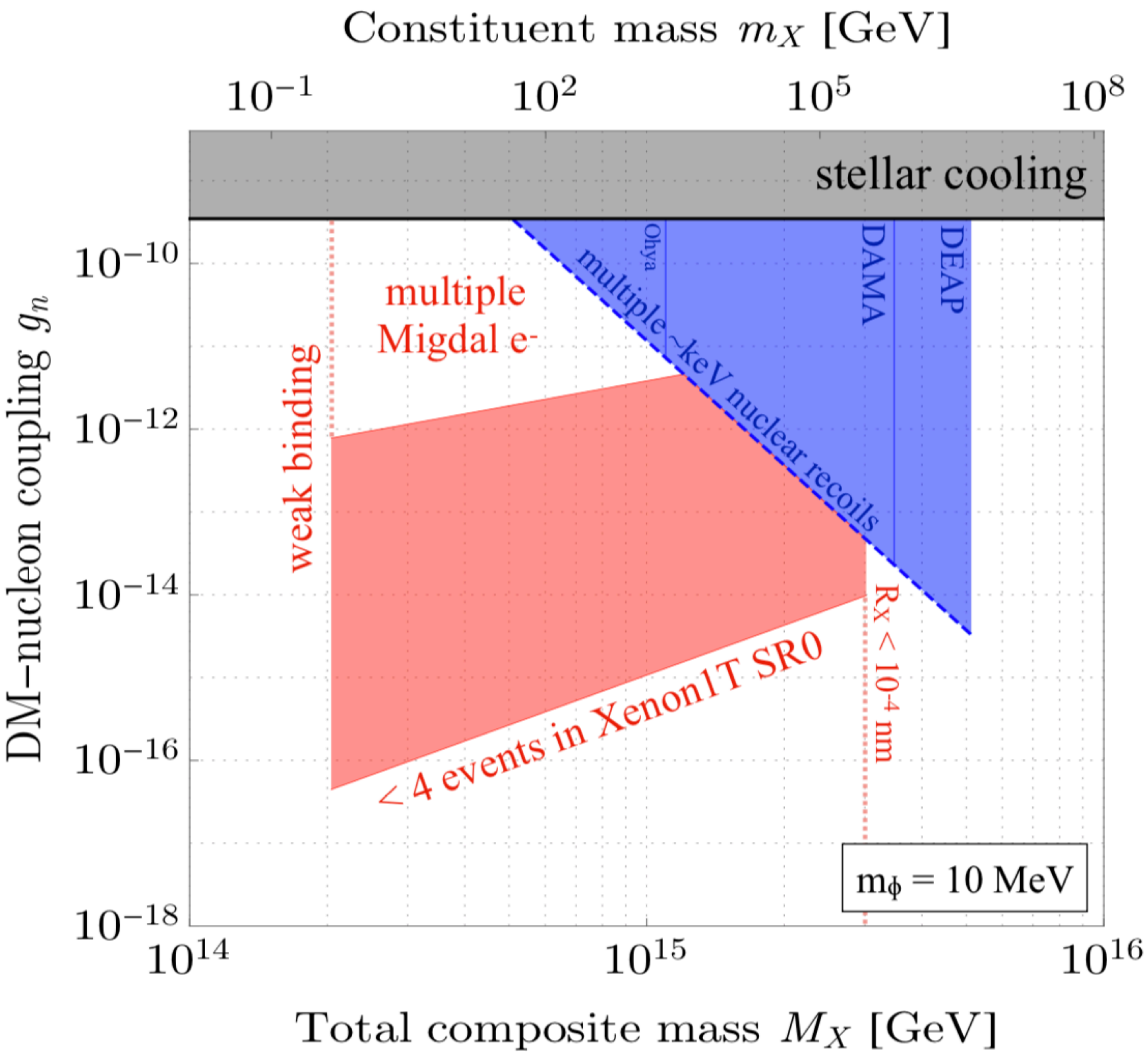
dark matter flux

Migdal probabilities

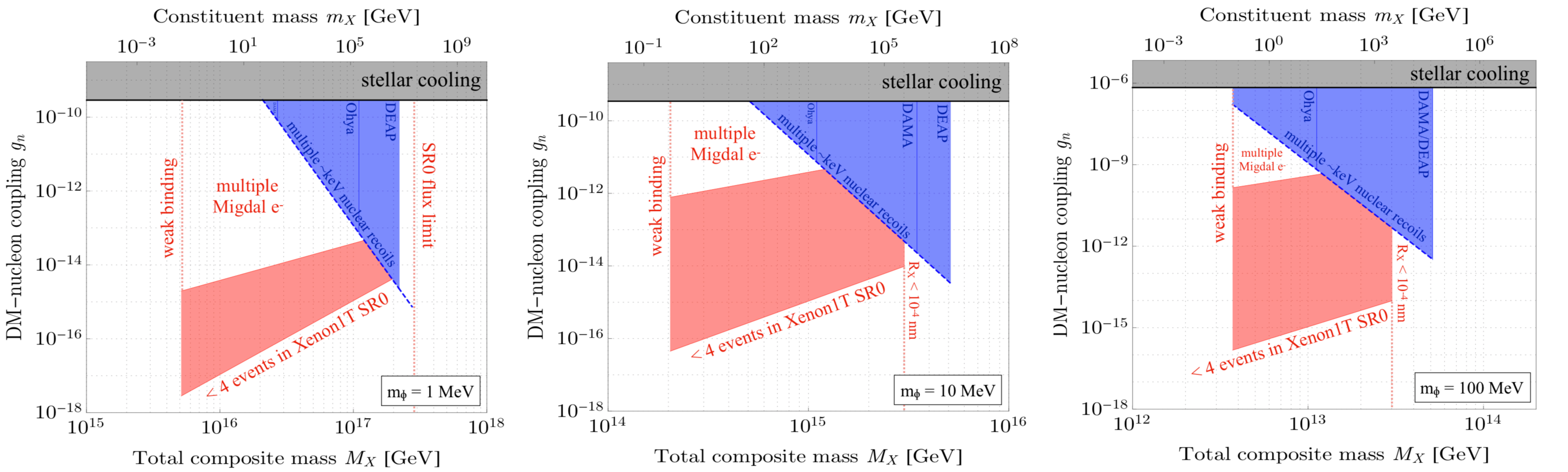
XENON-1T 1st DM search:

$$N_{ion} \simeq (98 \text{ kg yr}) \; R_{ion} \simeq 10 \left(\frac{m_X}{\text{TeV}}\right)^{-\frac{2}{5}} \left(\frac{m_\phi}{\text{MeV}}\right)^{-\frac{4}{5}} \left(\frac{g_n}{10^{-17}}\right) \left(\frac{\alpha_X}{0.3}\right)^{-\frac{1}{10}}$$

Xenon-1T Constraints



Xenon-1T Constraints



Migdal effect covers wide range of masses & couplings!

Some Final Remarks

- The Migdal effect is a promising venue to search for weakly-coupled composite dark matter in experiments like XENON-1T, DEAP-3600, LZ.

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Some Final Remarks

- The Migdal effect is a promising venue to search for weakly-coupled composite dark matter in experiments like XENON-1T, DEAP-3600, LZ.

- Main requirements are large geometric cross-section + sudden nuclear recoils induced by the composite transit.

- Other models are candidates for a similar study, e.g. dark quark nuggets, bosonic blobs, composites w/ additional fields.

Thank you!

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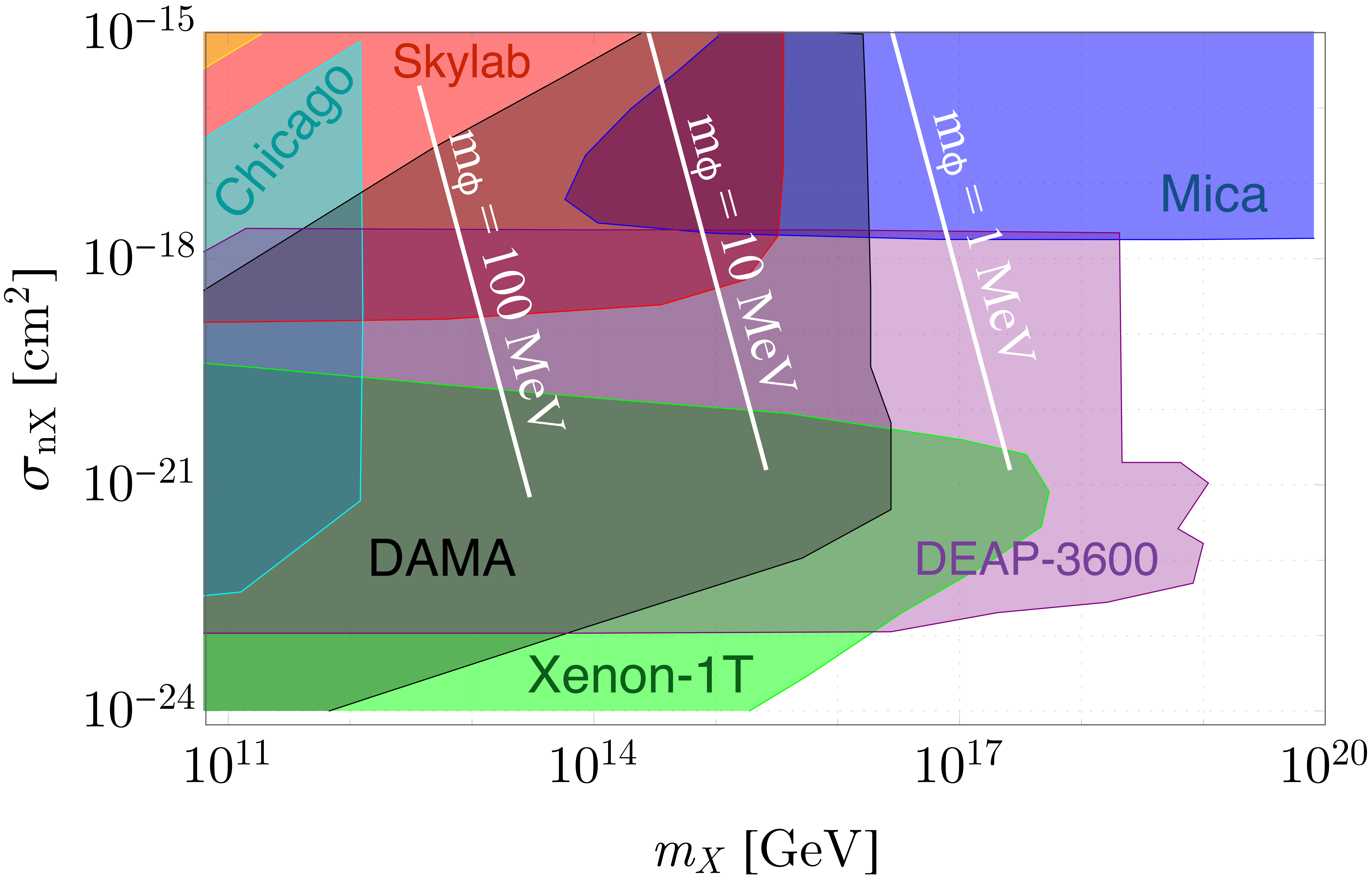
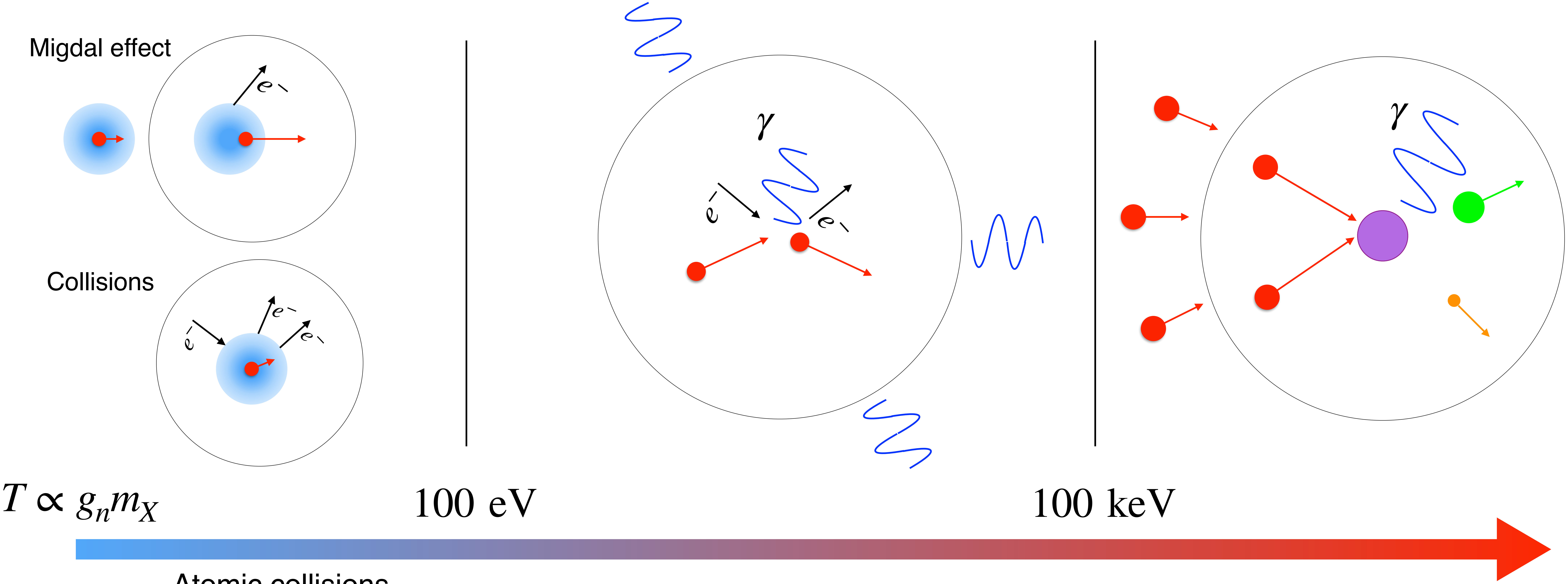
Bhoonah et. al., 2012.13406
JA, Bramante & Goodman, 2105.06473
DEAP collaboration, 2108.09405

Backup Slide: Experimental Landscape



Backup Slide: Composite Pheno Summary



Atomic collisions
Migdal effect
Recombination
Line emission

Full ionization
Thermal bremsstrahlung

Nuclear fusion

Backup slide: Composite Equations I

Scalar only:

i) $$\frac{\partial \varepsilon}{\partial \langle \phi \rangle} = 0 \longrightarrow 3C_\phi^2 \left(\frac{m_*}{m_X}\right) \int_0^{\frac{p_F}{m_X}} \frac{x^2 dx}{\sqrt{x^2 + \left(m_*/m_X\right)^2}} = 1 - \frac{m_*}{m_X}$$

ii) $$p = 0 \longrightarrow \int_0^{\frac{p_F}{m_X}} \frac{x^4 dx}{\sqrt{x^4 + \left(m_*/m_X\right)^2}} = \frac{1}{2C_\phi^2} \left(1 - \frac{m_*}{m_X}\right)^2$$

iii) $$C_\phi^2 = \frac{4\alpha_\phi}{3\pi} \frac{m_X^2}{m_\phi^2}$$

Backup slide: Composite Equations II

Add vector field:

i) $$\frac{\partial \varepsilon}{\partial \langle \phi \rangle} = 0 \longrightarrow 3C_\phi^2 \left(\frac{m_*}{m_X}\right) \int_0^{\frac{p_F}{m_X}} \frac{x^2 dx}{\sqrt{x^2 + (m_*/m_X)^2}} = 1 - \frac{m_*}{m_X}$$

ii) $$p = 0 \longrightarrow \int_0^{\frac{p_F}{m_X}} \frac{x^4 dx}{\sqrt{x^4 + (m_*/m_X)^2}} = \frac{1}{2C_\phi^2}\left(1 - \frac{m_*}{m_X}\right)^2 - \frac{C_V^2}{2}\left(\frac{p_F}{m_X}\right)^6$$

iii) $$C_\phi^2 = \frac{4\alpha_\phi}{3\pi}\frac{m_X^2}{m_\phi^2} \qquad C_V^2 = \frac{4\alpha_V}{3\pi}\frac{m_X^2}{m_V^2}$$

Backup slide: Composite Equations III

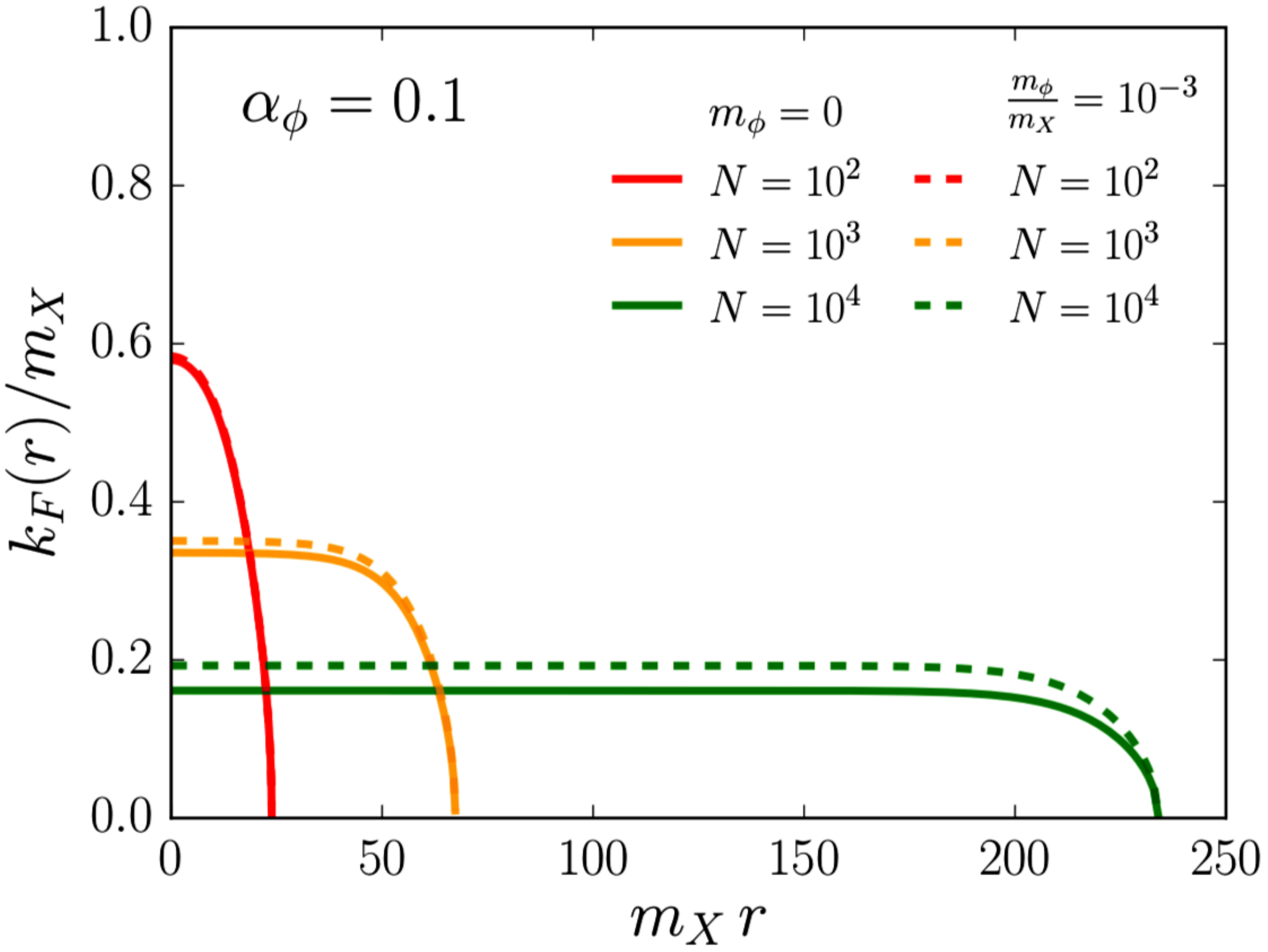
Add $V(\phi) \sim \lambda\phi^4$ potential:

i) $$\frac{\partial \varepsilon}{\partial \langle \phi \rangle} = 0 \longrightarrow 3C_\phi^2 \left(\frac{m_*}{m_X}\right) \int_0^{\frac{p_F}{m_X}} \frac{x^2 dx}{\sqrt{x^2 + (m_*/m_X)^2}} = 1 - \frac{m_*}{m_X} + C_\phi^2 \lambda \left(1 - \frac{m_*}{m_X}\right)^3$$

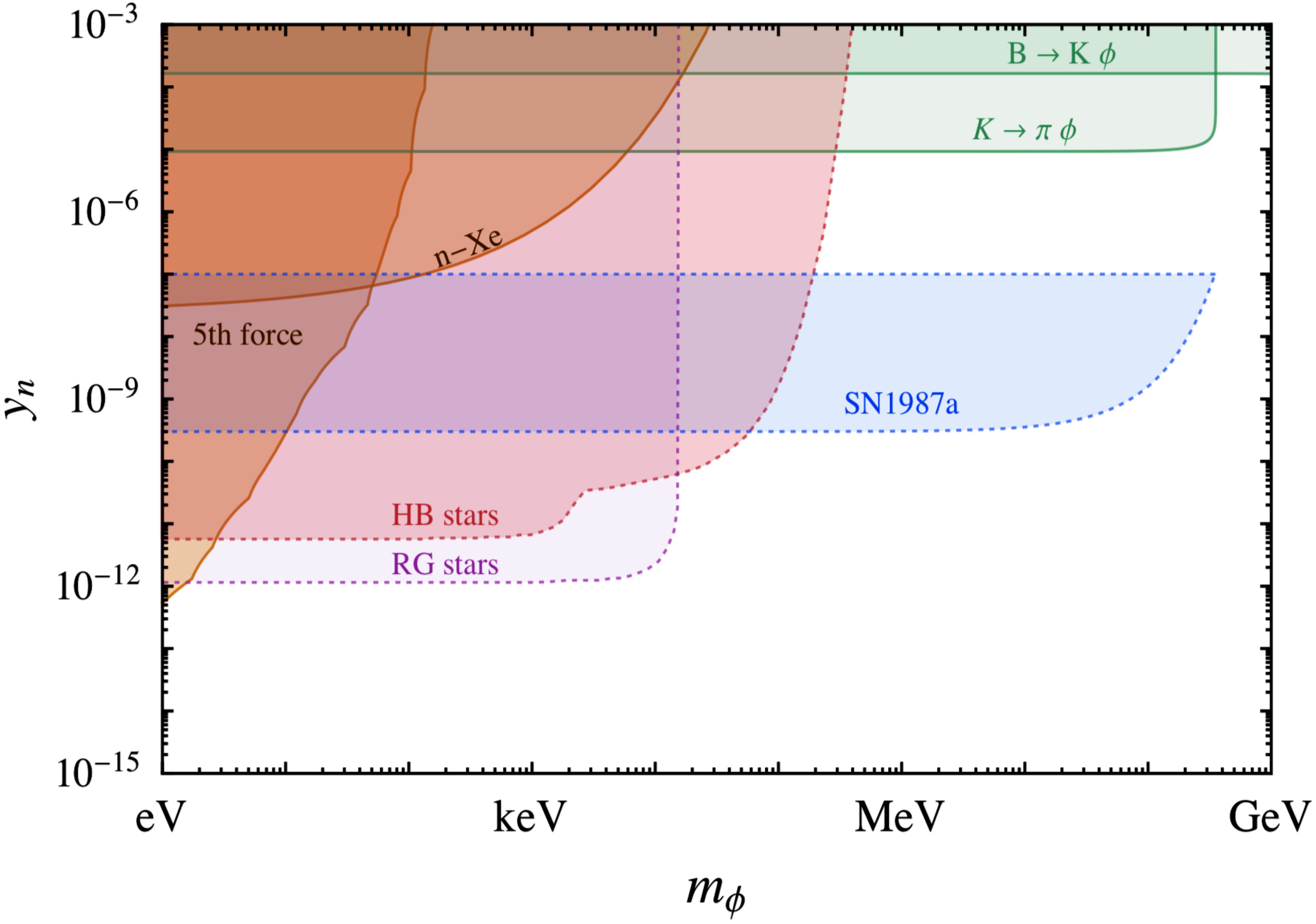
ii) $$p = 0 \longrightarrow \int_0^{\frac{p_F}{m_X}} \frac{x^4 dx}{\sqrt{x^4 + (m_*/m_X)^2}} = \frac{1}{2C_\phi^2} \left(1 - \frac{m_*}{m_X}\right)^2 + \frac{\lambda}{4} \left(1 - \frac{m_*}{m_X}\right)^4$$

iii) $$C_\phi^2 = \frac{4\alpha_\phi}{3\pi} \frac{m_X^2}{m_\phi^2}$$

Backup slide: Composite Profiles



Backup slide: Stellar Cooling Bounds on g_n



limits energy to:

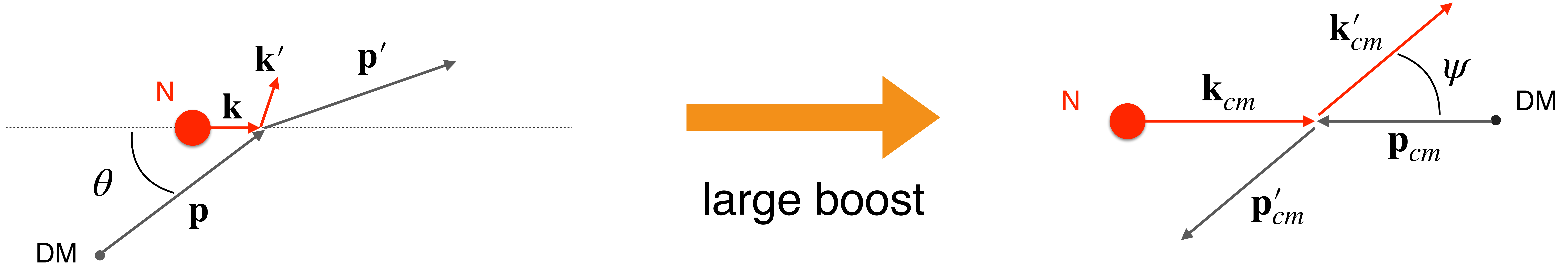
$$\Delta E \simeq A g_n \left(\frac{m_X}{g_\phi}\right)$$

$$\lesssim \text{keV} \left(\frac{g_n}{10^{-10}}\right) \left(\frac{m_X}{\text{TeV}}\right) \left(\frac{1}{g_\phi}\right) \left(\frac{A}{10}\right)$$

Joglekar et. al., 1911.13293
2004.09539

Backup slide: DM-Nucleus Scattering I

Composite frame: CM frame:



Moller velocity

$$\Gamma_{NX} = n_X \int_0^{p_F} \frac{dp\, p^2}{V_F} \int d\varphi\, d(\cos\theta) \int d\alpha\, d(\cos\psi) \left(\frac{d\sigma}{d\Omega}\right)_{(CM)} \tilde{v}\, \Theta(\Delta E + p - p_F)$$

Pauli-blocking

integrate over target phase space (composite rest frame)

relativistic kinematics (centre-of-momentum frame)

Backup slide: DM-Nucleus Scattering II

$$\tilde{v} \simeq 1 - v_N \cos\theta$$ Moller velocity

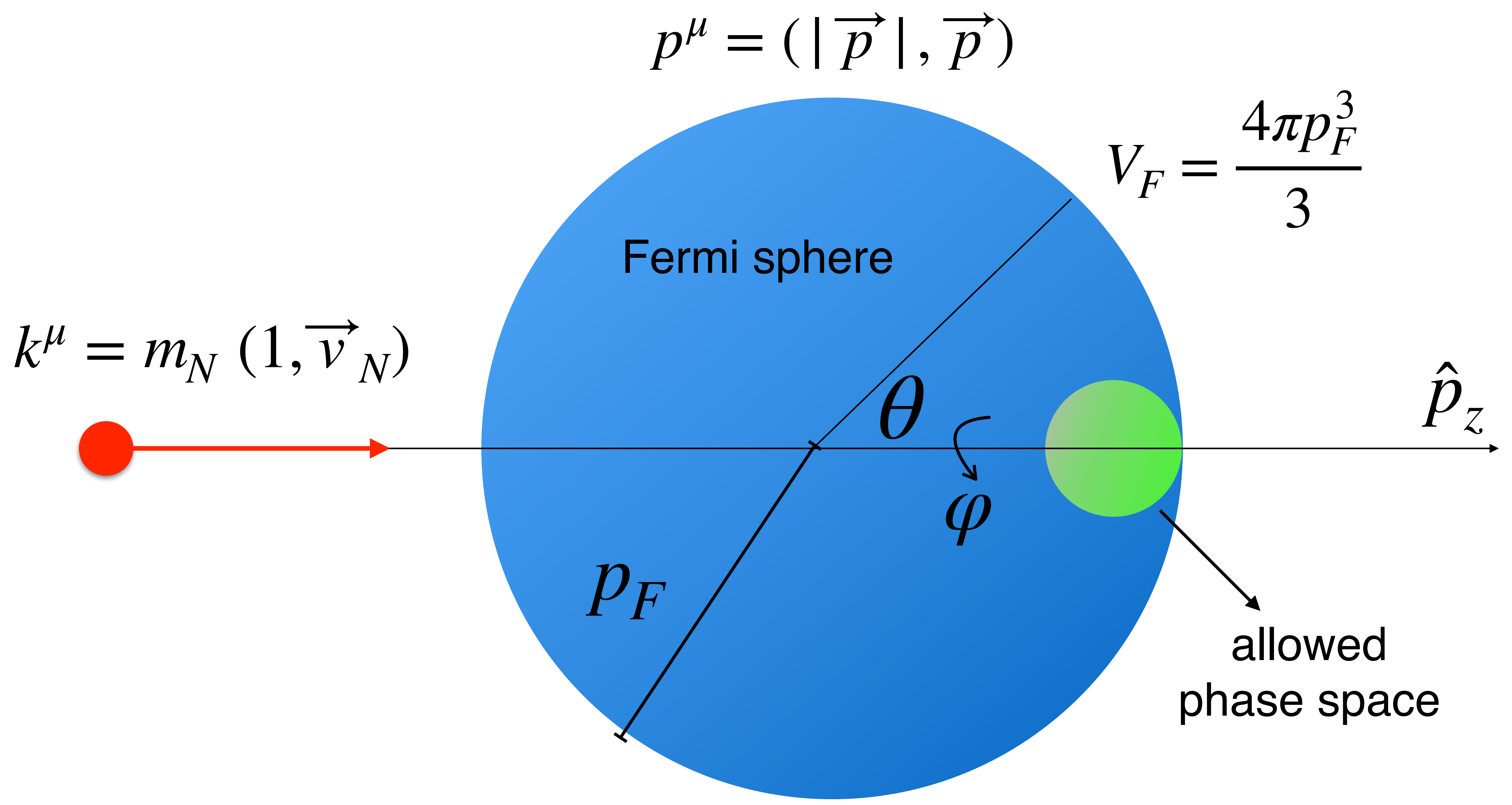
$$E_{cm}^2 \simeq m_N^2 + 2m_N p(1 - v_N \cos\theta)$$ CM energy

$$\Delta E_{max} \simeq \frac{1}{2} m_N v_N^2 \sin^2\theta$$ Max energy transfer

$$k_{cm}^2 \simeq \frac{m_N p^2}{m_N + 2p} - \frac{2m_N p^2 (m_N + p) v_N \cos\theta}{(m_N + 2p)^2}$$ CM momentum

$$\beta \simeq \frac{p}{m_N + p} + \frac{m_N v_N \cos\theta}{m_N + p}$$ Boost parameter

$$\psi_{max} \simeq \frac{\left(m_N(m_N + 2p)\right)^{1/2} v_N \cos\alpha}{p}$$ Max scattering angle



$$\Gamma_{NX} \sim \frac{A^2 g_n^2 g_X^2 m_N^4 (m_N + 2p_F) v_N^6}{p_F^4}$$

scattering rate

Backup slide: Coherent Composite-Nucleus Scattering

$$\left(\frac{d\sigma}{dq}\right)_{XN\to XN} = A^2 N_X^2 f^2(\Lambda)\bar{\sigma}_0 \left(\frac{q}{2m_N^2 v_X^2}\right) |F_X(qR_X)|^2 \; |F_a(qr_N)|^2$$

diff. cross section

$$F_X(qR_X) = \frac{3j_1(qR_X)}{qR_X}$$

composite substructure

$$\bar{\sigma}_0 = \frac{g_n^2 g_X^2 m_N^2}{4\pi \tilde{m}_\phi^4}$$

ref. cross section

$$F_a(qr_N) = \frac{3j_1(qr_N)}{qr_N} e^{-q^2 r_N^2}$$

nuclear substructure

$$f(\Lambda) = \min\left[1, \left(\frac{\Lambda}{R_X}\right)^3\right]$$

scatterer wavefunction overlap

Backup slide: Collective Excitations - Surface Modes

$$\left(\frac{d\sigma}{dq}\right)_{0\to 1_l} \simeq A^2 N_X^2 f^2(\Lambda)\bar{\sigma}_0 \left(\frac{q}{2m_N^2 v_X^2}\right) |F_{\rm surf}^{(l)}(qR_X)|^2$$

diff. cross section

$$F_{\rm surf}^{(l)}(qR_X) = \epsilon_l \ (2l+1)^{1/2} j_l(qR_X)$$

surface mode form factor

$$\epsilon_l \propto m_X^{-1/4} \bar{m}_X^{-3/2} R_X^{-7/4} \simeq 10^{-14} \left(\frac{m_X}{\rm TeV}\right)^{-\frac{1}{4}} \left(\frac{\bar{m}_X}{5\ \rm GeV}\right)^{-\frac{3}{2}} \left(\frac{R_X}{\rm nm}\right)^{-\frac{7}{4}}$$

mode amplitude

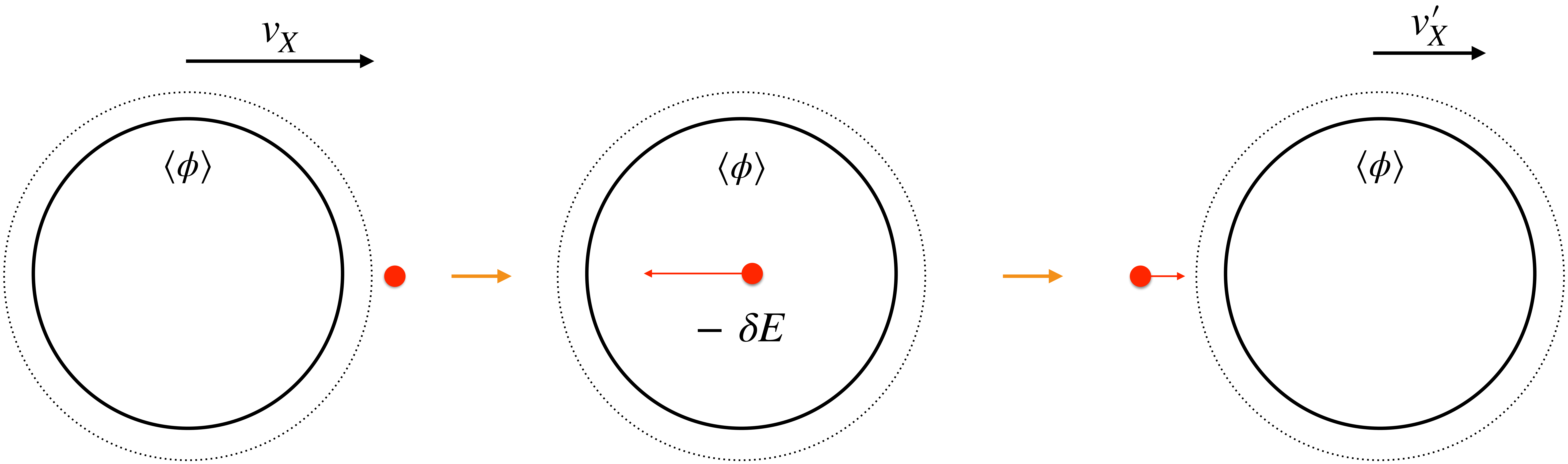
$$\bar{\sigma}_0 = \frac{g_n^2 g_X^2 m_N^2}{4\pi \tilde{m}_\phi^4}$$

reference cross section

$$f(\Lambda) = \min\left[1, \left(\frac{\Lambda}{R_X}\right)^3\right]$$

scatterer wavefunction overlap

Backup slide: Composite Stopping



nucleus interacts with composite

nucleus loses energy

$$\frac{1}{2} M_X \left(v_X' - v_X \right) \simeq \delta E$$