TOPOLOŠKE FAZE VIŠEG REDA

Phys. Rev. B **98**, 201114(R), arXiv: 1807.09050

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ŠTA SU TOPOLOŠKE FAZE MATERIJE?

- TOPOLOGIJA: Grana matematike koja proučava svojstva objekata koja su sačuvana usljed kontinuiranih transformacija, poput rastezanja, uvrtanja,... (preuzeto sa wiki)

$\neq$

- Objekte razlikujemo na osnovu

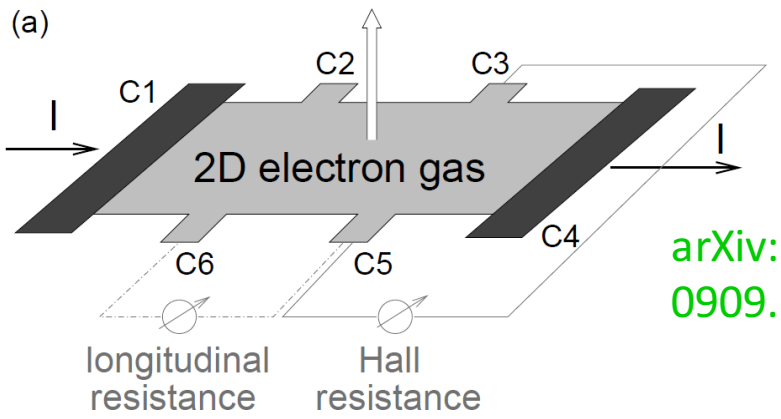
topoloških invarijanti!

U matematici, topološka invarijanta je Eulerova karakteristika χ i u slučaju zatvorene orijentabilne površine: $\chi = 2 - 2g$, gdje g naznačava genus objekta.

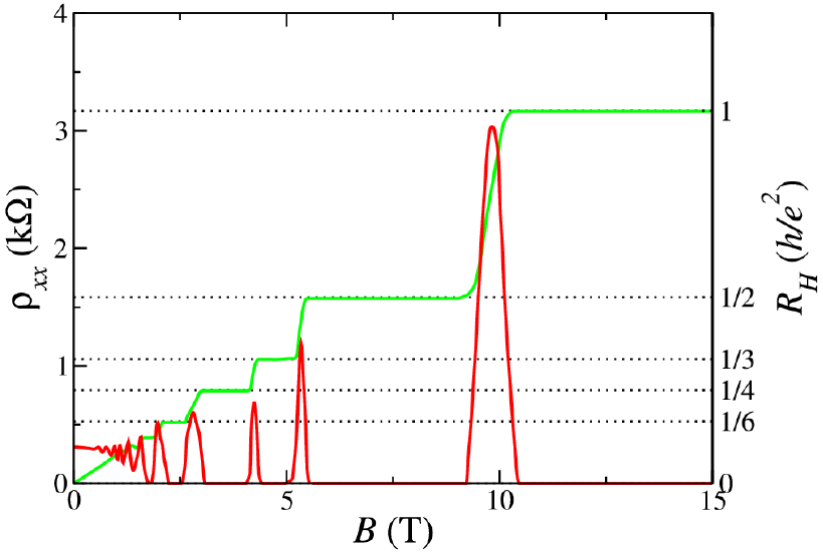
1) CJELOBROJNI KVANTNI HALLOV EFEKT

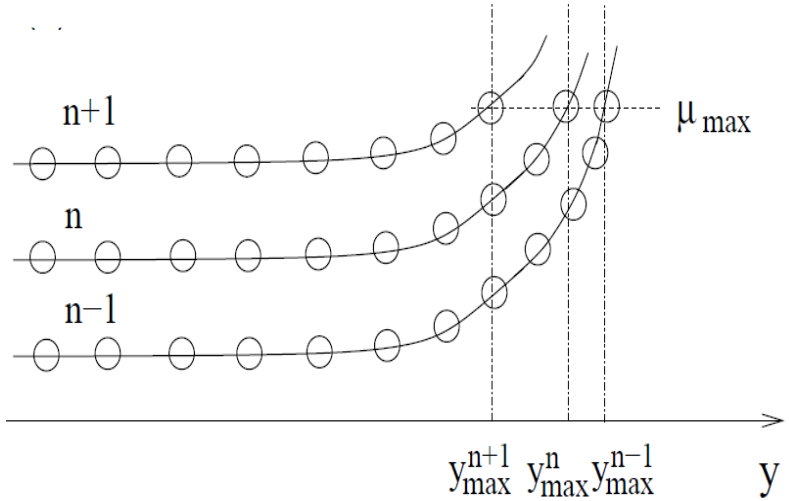
Problem: 2D elektronski gas pod djelovanjem magnetnog polja; u Landau-ovom gauge-u:

$$H = \frac{1}{2m}(\hat{\boldsymbol{p}} + e\boldsymbol{A})^2, A = (0, Bx, 0)$$

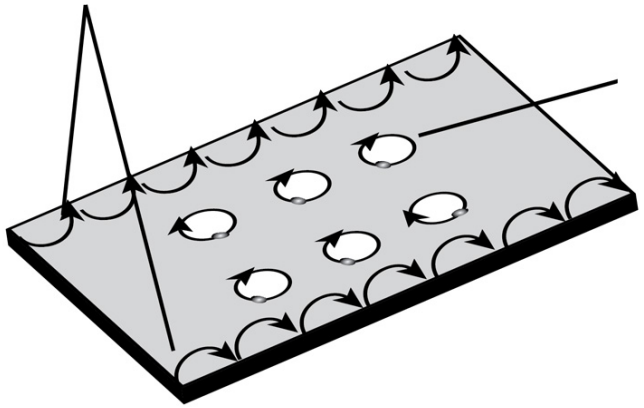


arXiv: 0909.1998





provodna ivica



- Provodnost σ je povezana sa topološkom invarijantom koja se naziva **Chern-ov broj**. (PRL 49.405)

Adijabatski teorem: $|\psi\rangle_k = e^{i\varphi}|\psi\rangle_i$, gdje φ naznačava Berry-evu fazu (za E=0). $\varphi = ?$

Iz $i\hbar \frac{\partial|\psi\rangle}{\partial t} = \mathrm{H}(\lambda(t))|\psi\rangle = 0$ i $|\psi(t)\rangle = U(t)|n(\lambda(t))\rangle$, koristeći $\langle\psi|\dot{\psi}\rangle = \cdots = 0$:

$\mathrm{U} = \exp(-\, i \int A(\lambda)\ \dot{\lambda}\, dt)$, gdje $A_i(\lambda) = -i\left\langle n\middle|\frac{\partial}{\partial\lambda^i}\middle|n\right\rangle$ je Berry-eva konekcija.

Konačno, Berry-eva zakrivljenost(curvature): $\mathcal{F}_{ij} = \frac{\partial A_i}{\partial\lambda^j} - \frac{\partial A_j}{\partial\lambda^i}$ tako da

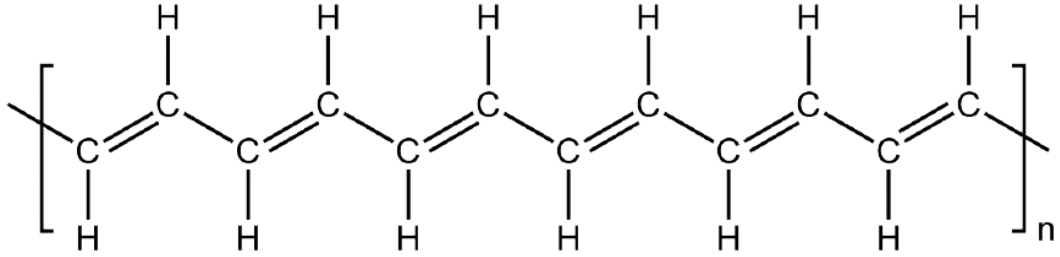
$$\sigma \propto \oint A(\lambda)\, d\lambda = \int_S \mathcal{F}_{ij}\, dS^{ij} \propto 2\pi n$$

na osnovu Gauss-Bonnet-ova teorema.

2) SSH MODEL

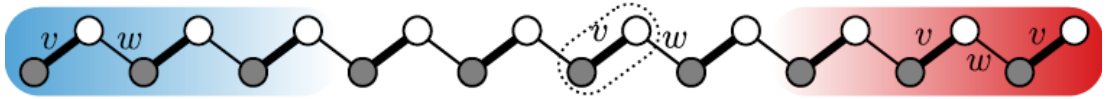
PRL **42**, 1698 (1979).

MOTIVACIJA: molekula poliacetilena

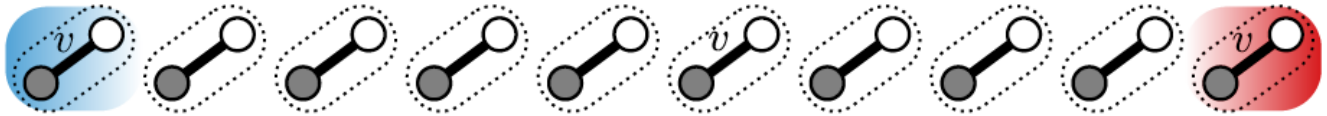


- $H = v\sum_{x=1}^{N}|x,A\rangle\langle x,B| + w\sum_{x=1}^{N-1}|x+1,A\rangle\langle x,B|$

$$H(k) = \begin{pmatrix} 0 & v+we^{-ik} \\ v+we^{ik} & 0 \end{pmatrix}$$



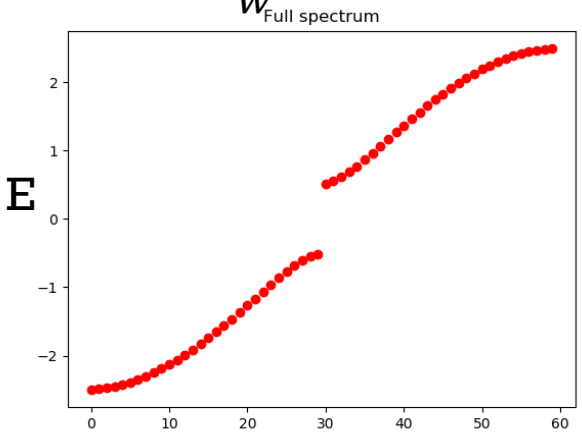
TRI:



TOP:

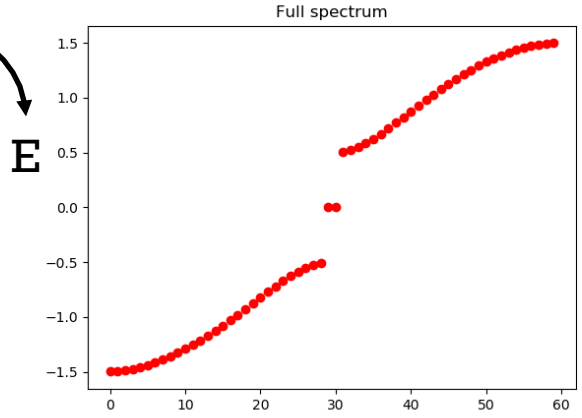
$\frac{v}{w} > 1$

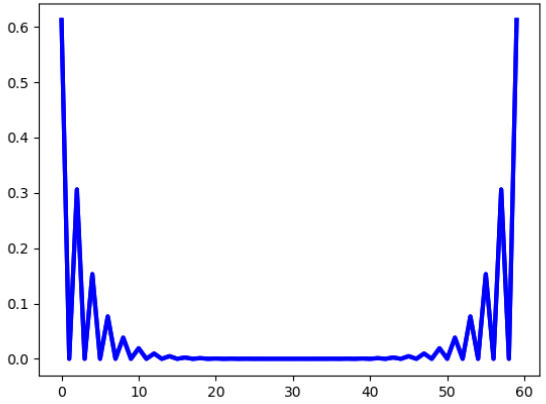
$\frac{v}{w} = 3/2$



$\frac{v}{w} < 1$

$\frac{v}{w} = 1/2$





TOPOLOŠKA INVARIJANTA - POLARIZACIJA

- Polarizacija je dobro definirana veličina u slučaju **otvorenih graničnih uslova**:

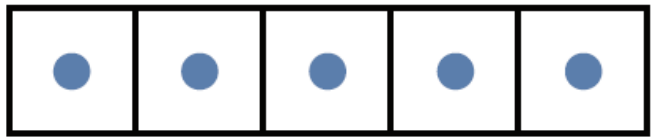
$$P = \frac{1}{V}\left[-e\ \sum_i Z_i R_i + \int d\boldsymbol{r}\ \boldsymbol{r}\ \rho(\boldsymbol{r})\right] \longrightarrow \quad P_{el} \equiv \langle \Psi | \hat{R} | \Psi \rangle$$

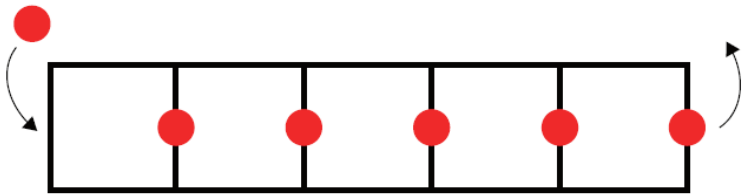
- Problem u slučaju periodičnih graničnih uslova!!!
- → U modernoj teoriji polarizacije, ova veličina je izračunata preko Berry-eve faze:

$$P_{el} = -\frac{i}{2\pi}\ Log\ Det\ \underbrace{Exp(-i\ \int_k^{k+2\,\pi} A(k))}_{\text{Wilson-ova petlja}}$$ gdje je $A(k) = -i\ \langle u_k | \partial_k\ u_k \rangle$.

- SSH model ima dvije vrste simetrija: hiralnost (chiral symmetry) $C\ H(k) C^{-1} = -H(k)$ i inverziju $I H(k) I^{-1} = H(-\ k)$ → nameče diskretnu vrijednost polarizacije

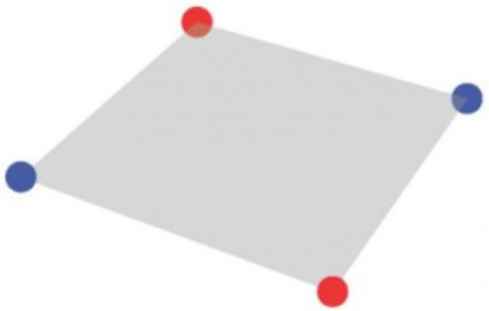
- $P = 0$
- $P = \frac{1}{2}$





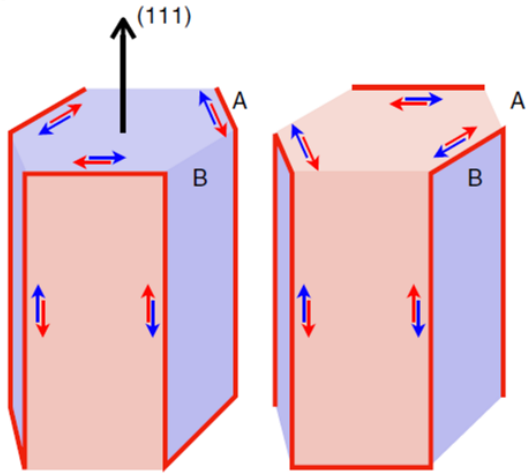
TOPOLOŠKE FAZE VIŠEG REDA

- Do 2016. godine: D-dimenzionalan sistem sa (D-1)-dimenzionalnim stanjima nulte energije lokaliziranim na rubovima sistema
- 2016-... : topološke faze višeg reda:



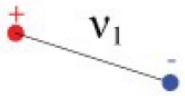
Kvantizirani električni izolatori višeg reda (Quantized electric multipole insulators)
Science 357 (6346), 61-66
Phys. Rev. B 96 (24), 245115

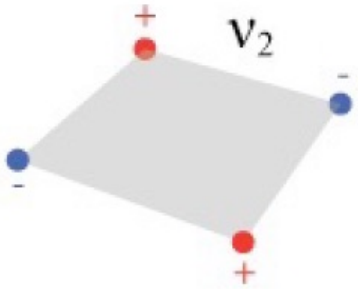
Helical HOTIs- SnTe
Science advances 4 (6), eaat0346

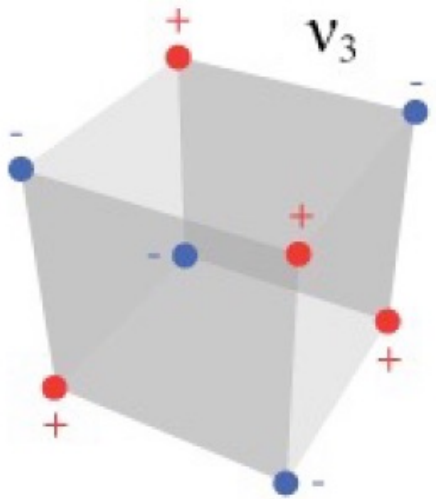


Bismuth as a HOTI
Nature Physics 14, 918–924 (2018)

KVANTIZIRANI ELEKTRIČNI IZOLATORI VIŠEG REDA

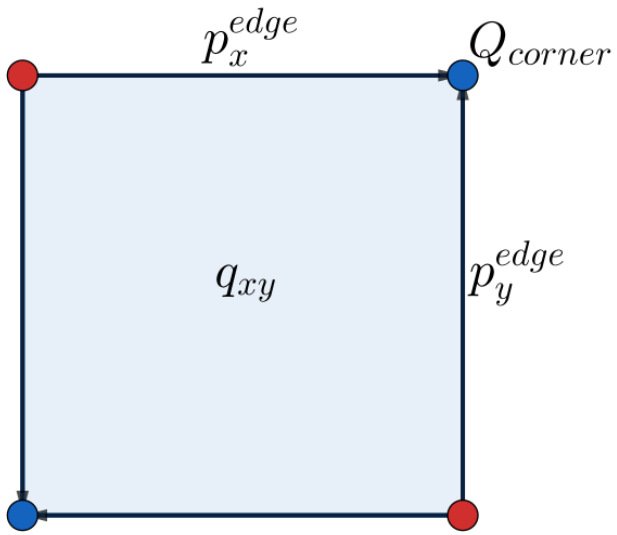
- Dipol: 

Kvadrupol: 

Oktupol: 

- Fokusirajmo se na kvadrupol, definirajuća relacija:

$$q_{xy} = |p_x^{edge}| = |p_y^{edge}| = |Q_{corner}|$$



ZAHTJEVI:

- Kvadrupol je sastavljen od dva poništavajuća dipola ⟶ paran broj okupiranih stanja!
- Potrebne simetrije: ogledalna simetrija u x-/y-smjeru

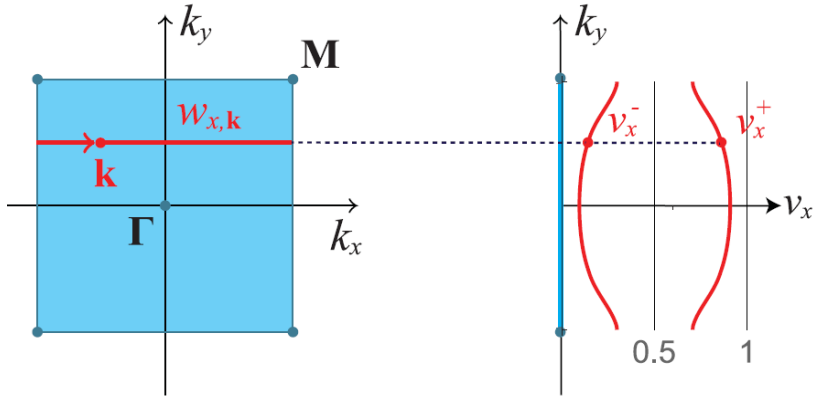
⟶ simetrija inverzije

KAKO RAČUNAMO KVADRUPOLNI MOMENT?

- Dipol se računa preko Wilsonove petlje: $P_{el} = -\frac{i}{2\pi}\, Log\, Det\, W = -\frac{i}{2\pi}\, Log\, Det\, Exp(-i \int_{k}^{k+2\pi} A(k))$

⇒ **Metod ukliještene Wilsonove petlje**:

1. Diagonaliziramo Hamiltonijan zadan u recipročnom prostoru $H(k)$
2. Od vlastitih vektora $|u_k\rangle$ izračunamo Wilsonovu petlju u samo jednom smjeru momenta i diagonaliziramo je
3. Pomoću njenih vlastitih vrijednosti provjerimo da dipolni moment nestaje, a vlastiti vektori služe za definiranje Wannier-ovih stanja pomoću kojih definiramo Wannier-ovu polarizaciju sektora:



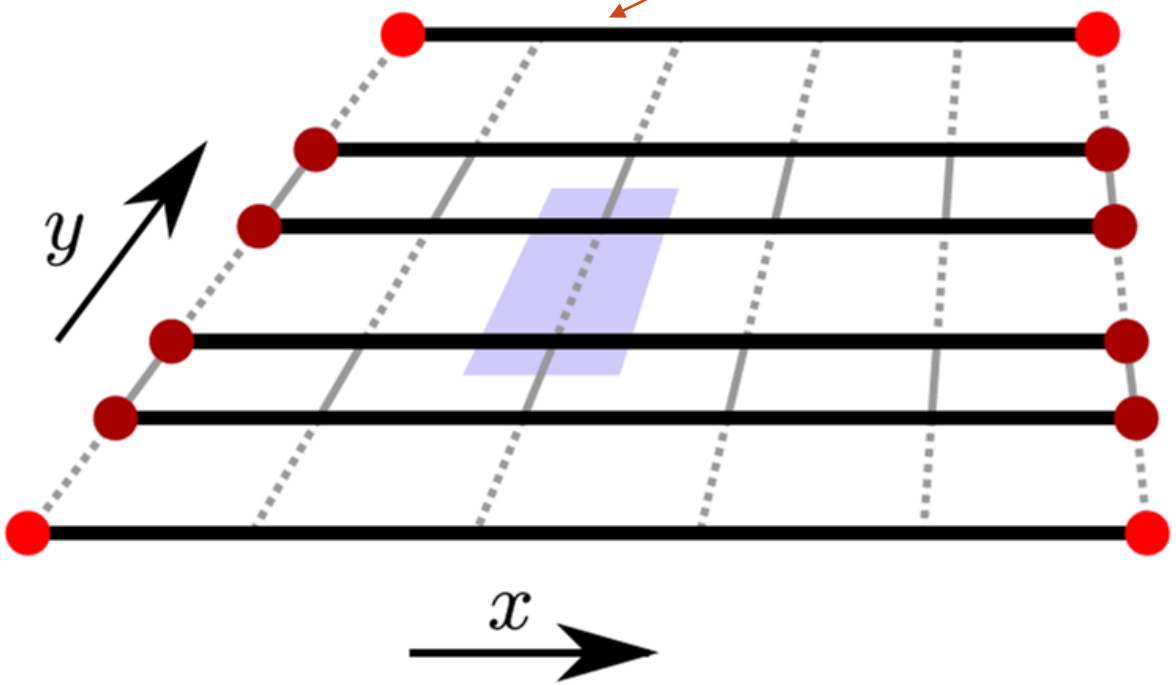
$$p_y^{\nu_x^s} = -\frac{1}{(2\pi)^2}\int_{BZ} Tr[\tilde{A}^s_{y,\boldsymbol{k}}]\, d^2\boldsymbol{k}$$

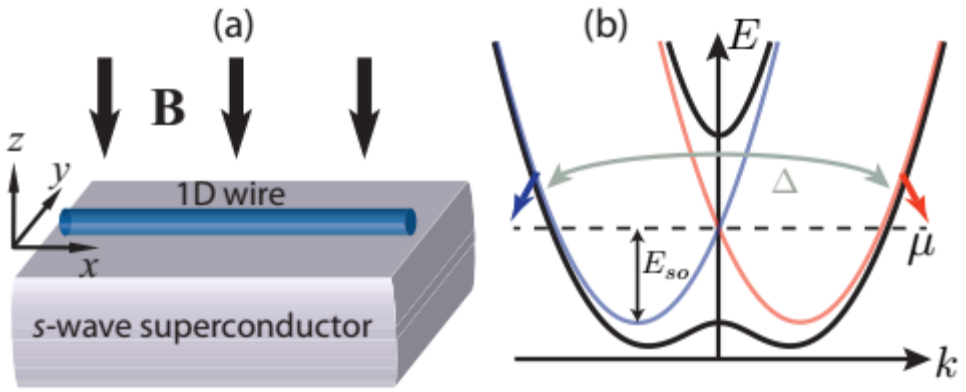
$$\tilde{A}^{s,r,r'}_{y,\boldsymbol{k}} = -i\left\langle \omega^{s,r}_{x,\boldsymbol{k}} \middle| \partial_{k_y} \middle| \omega^{s,r'}_{x,\boldsymbol{k}} \right\rangle \quad i \quad |\omega^{s,r}_{x,\boldsymbol{k}}\rangle = \sum_{n=1}^{N_{occ}} |u^n_{\boldsymbol{k}}\rangle\, \nu^{s,r,n}_{x,\boldsymbol{k}}$$

Konačno: $$q_{xy} = \sum_{s=+,-} p_y^{\nu_x^s} p_x^{\nu_y^s}$$

NAŠ MODEL:

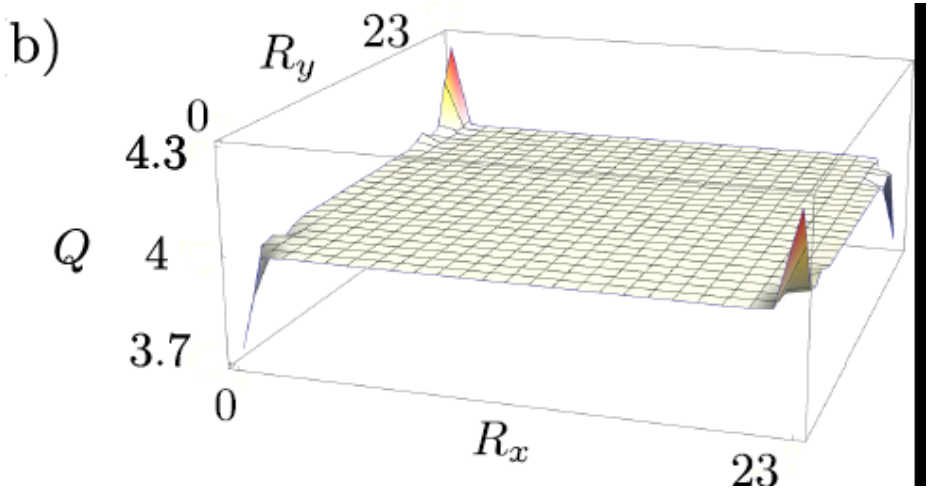
Majorana nanožica





$$h(\mathbf{k}) = [2t_x(1-\cos k_x) - \mu]\tau_z\sigma_0\eta_0 + V_z\tau_0\sigma_z\eta_0 + \Delta\tau_x\sigma_0\eta_0 + \alpha\sin k_x\tau_z\sigma_y\eta_0 - \beta_1\tau_z\sigma_x\eta_y - \beta_2\sin k_y\tau_z\sigma_x\eta_x + \beta_2\cos k_y\tau_z\sigma_x\eta_y$$

SA OTVORENIM GRANIČNIM UVJETIMA:



$$q_{xy} = \sum_{r=1}^{\frac{N_{occ}}{2}} p_y^{\nu_x^{+,r}} \; p_x^{\nu_y^{+,r}} + p_y^{\nu_x^{-,r}} \; p_x^{\nu_y^{-,r}}$$

Wannier-ove vrpce u unutrašnjosti

Wannier-ov spektar u polu-beskonačnoj geometriji

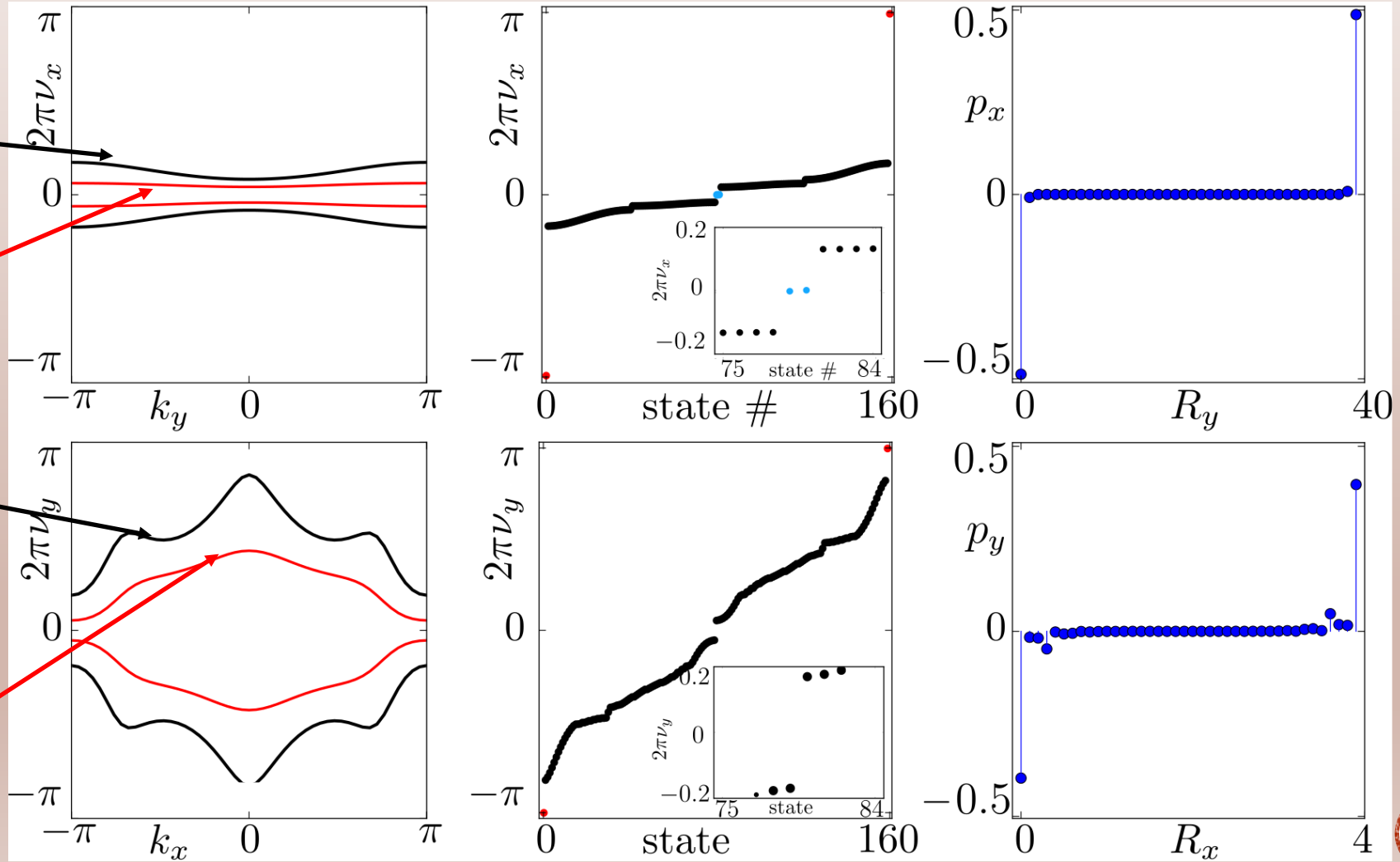
Polarizacija na rubu sistema

$$p_y^{\upsilon_x^{+,2}} = \frac{1}{2}$$

$$p_y^{\upsilon_x^{+,1}} = \frac{1}{2}$$

$$p_x^{\upsilon_y^{+,2}} = \frac{1}{2}$$

$$p_x^{\upsilon_y^{+,1}} = 0$$



HVALA NA PAŽNJI! ☺