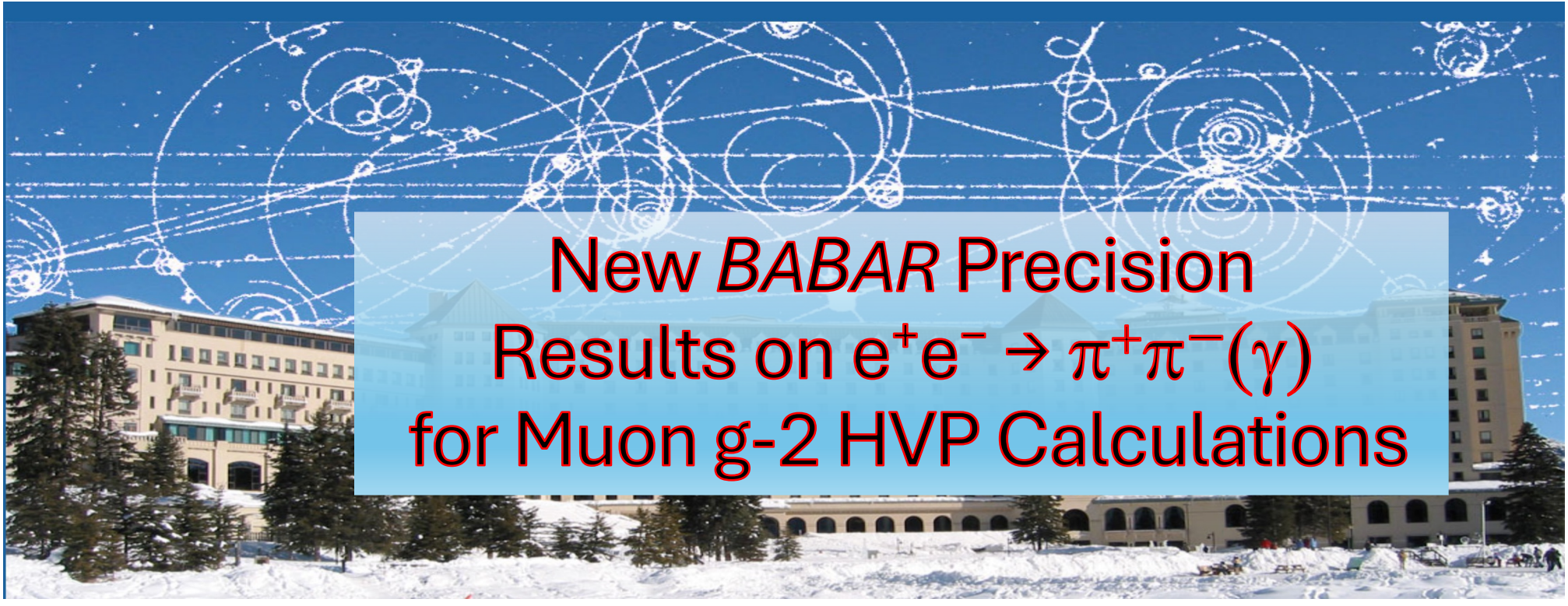

New *BABAR* Precision Results on $e^+e^- \to \pi^+\pi^-(\gamma)$ for Muon g-2 HVP Calculations

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J. Michael Roney

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On behalf of the *BABAR Collaboration*

University of Victoria

Lake Louise Winter Institute Mar 1–7, 2026

Introduction: Anomalous Magnetic Moment and Hadron Vacuum Polarization

A charged lepton with charge q_ℓ and mass m_ℓ, has a magnetic moment, $\vec{\mu}_\ell$, connected to its spin, $\vec{S}_\ell$, by a g_ℓ factor:

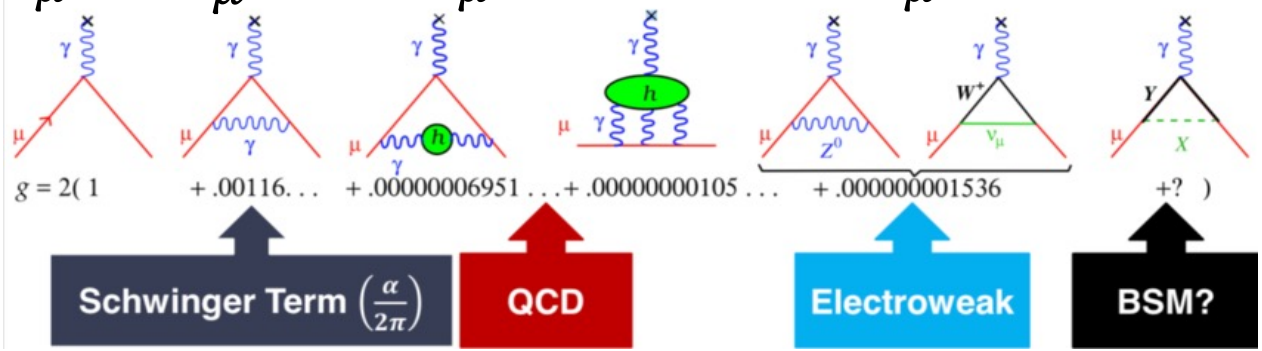
$$\vec{\mu}_\ell = g_\ell \left(\frac{q_\ell}{2m_\ell}\right) \vec{S}_\ell$$

Dirac predicted g_ℓ = 2 for electrons in 1928 [Proc. Roy. Soc. Lond. A 118, 351 (1928)]

Standard Model quantum corrections involving QED, the strong and weak interactions modify the value of g_ℓ by ~0.002

The anomalous magnetic moment, a_ℓ, quantifies deviation from 2: $a_\ell = \frac{g_\ell - 2}{2}$

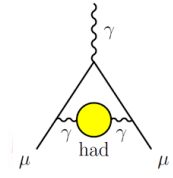
$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{hadronic} + a_\mu^{EW}$$



a_μ is one of the most precisely predicted and measured observables in particle physics

a_μ(exp)=116 592 059(22)×10^{-11} (0.19 ppm)
World Average from BNL + Fermilab
Phys. Rev. Lett. 131, 161802 (2023)

Introduction: *Anomalous Magnetic Moment and Hadron Vacuum Polarization*



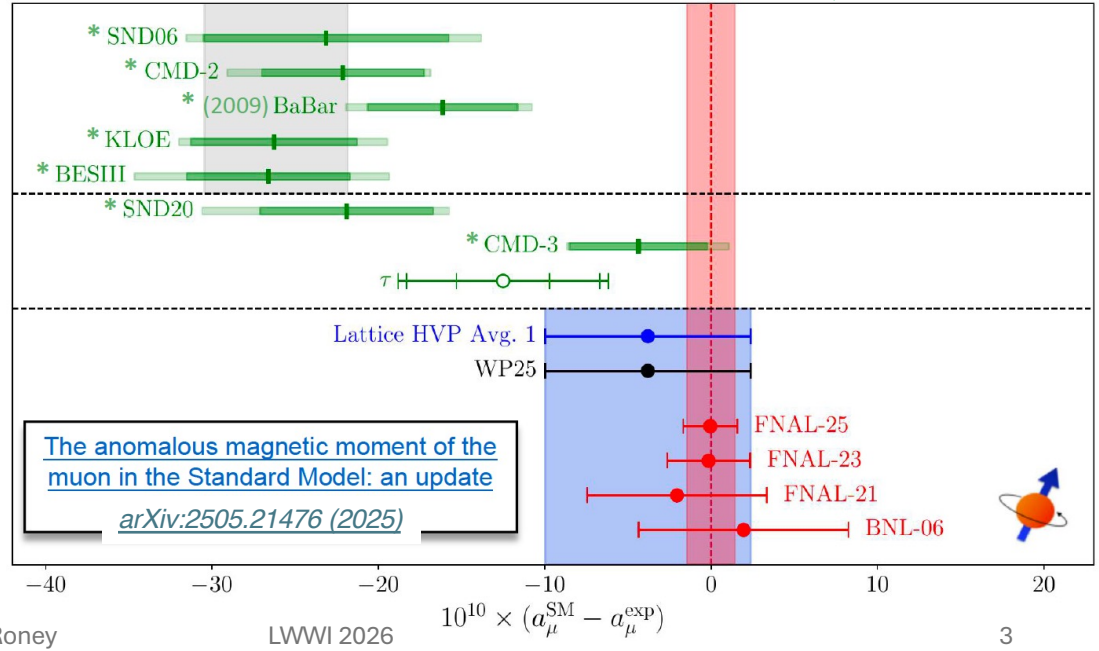
Hadronic vacuum polarization (HVP) contribution to the anomalous magnetic moment of the muon ($\mathbf{a_\mu}$) can be obtained by measuring cross sections of $e^+e^- \rightarrow$ hadrons with the largest input from **$e^+e^- \rightarrow \pi^+\pi^-$ (73%)** and also calculated with Lattice QCD

Phys. Rept. 1143 (2025) 1-158

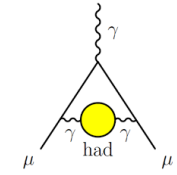
Current tensions exist between:

- predictions from **$e^+e^- \rightarrow \pi^+\pi^-$** dispersion approach and direct measurement,
- predictions from dispersion approach and lattice QCD

* ***not experimental errors***, *depict range of methods to extract a_μ (see Backup Slides)*



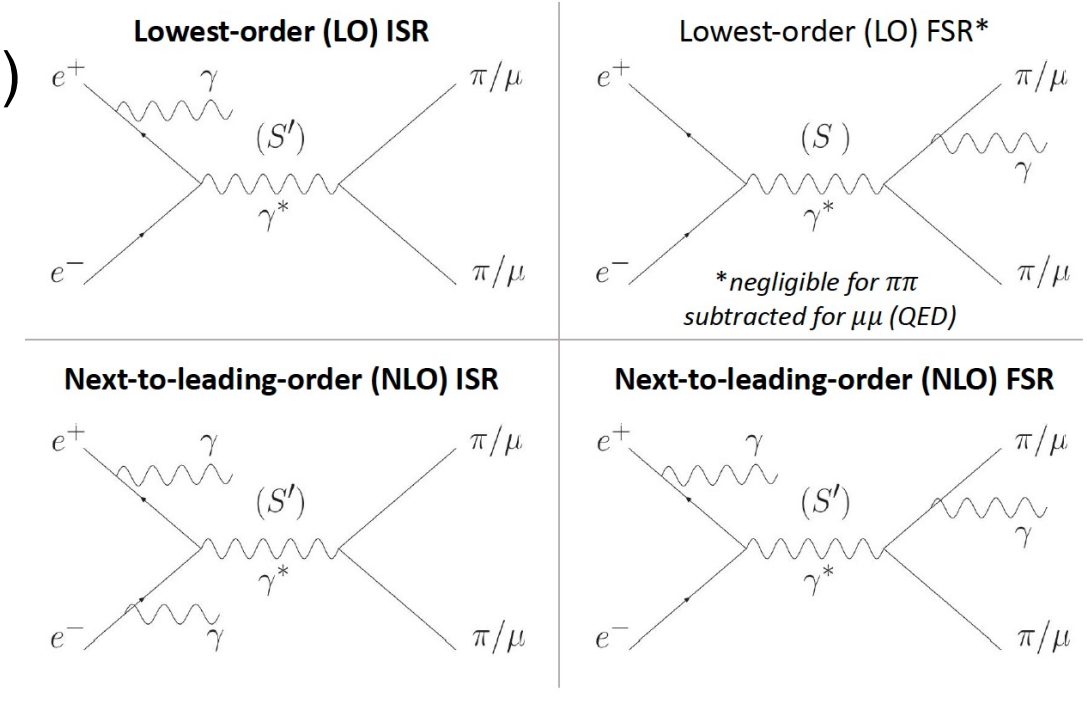
ISR Approach to Measuring the $e^+e^- \to X$ cross sections



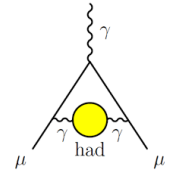
Cross section for $e^+e^- \to X$ obtained at reduced energy $\sqrt{s'}$ by measuring the radiative process:

$$e^+e^- \to X\,\gamma_{ISR}$$

- X any final state (e.g. $\mu^+\mu^-$ or $\pi^+\pi^-$)
- γ_{ISR} is from Initial State Radiation
- $\sqrt{s'} = s\,(1 - 2\,E^*_\gamma / \sqrt{s}\,)$
  - E^*_γ is the energy of γ_{ISR} in the e^+e^- centre-of-mass (CM) frame
  - $s = (p_{e-})_\mu \cdot (p_{e+})^\mu$ is the square of the CM energy of the e^+e^- system



Introduction: *Anomalous Magnetic Moment and Hadron Vacuum Polarization*



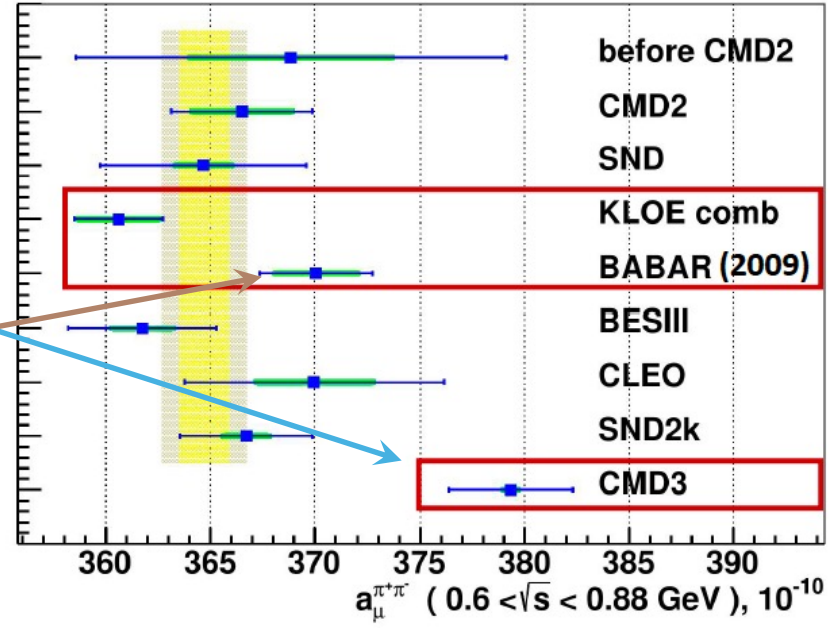
Hadronic vacuum polarization (HVP) contribution to the anomalous magnetic moment of the muon (a_μ) can be obtained by measuring cross sections of $e^+e^- \rightarrow$ hadrons with the largest input from **$e^+e^- \rightarrow \pi^+\pi^-$** (73%)

Current tensions also exist between:

- measurements from KLOE, BaBar and CMD-3
  (*KLOE vs CMD-3*: > 5σ tension at ρ peak)

CMD-3 Collaboration
Measurement of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section from threshold to 1.2 GeV with the CMD-3 detector
Phys. Rev. D 109, 112002 (2024)

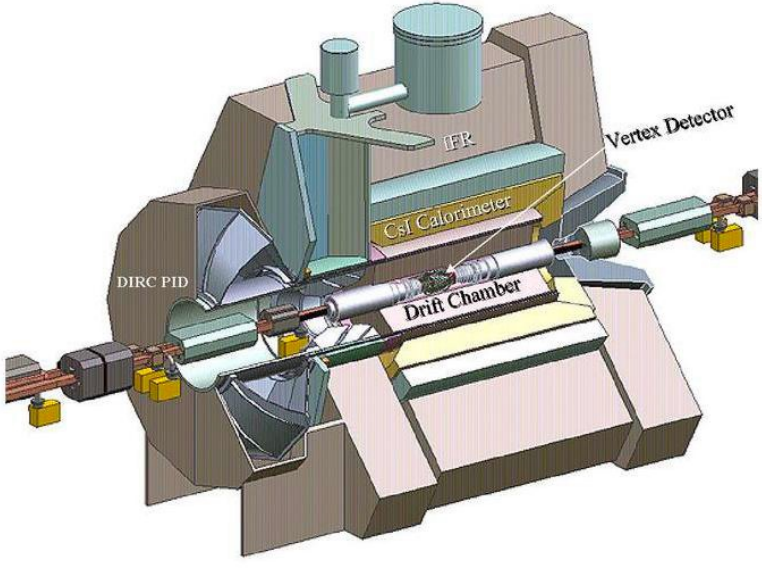
Previous 2009 BaBar result (partial data)

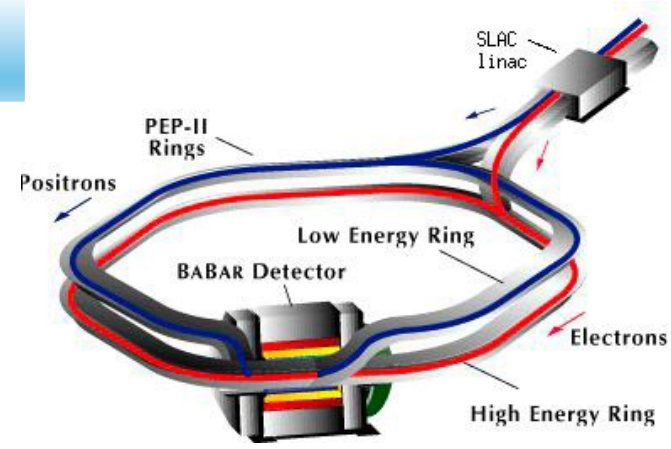


This talk presents a new preliminary *BABAR* measurement of the **$e^+e^- \rightarrow \pi^+\pi^-(\gamma)$** cross section with its FULL DATA SAMPLE plus a new & independent method aimed to improve precision.

BABAR Experiment, Data and MC Samples

PEP-II: asymmetric **e^+ (3 GeV) – e^- (9 GeV) collider** located at SLAC
Operated from 1999 to 2008 at **Υ(4S/3S/2S) resonance** energies
Collected **424.2 fb^{-1} at Υ(4S)** ($\sqrt{s}$ = 10.58 GeV) **+ 36.2 fb^{-1}** off-resonance





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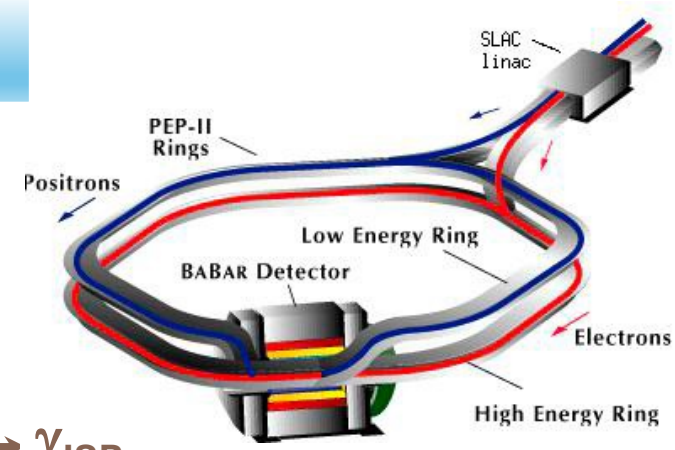
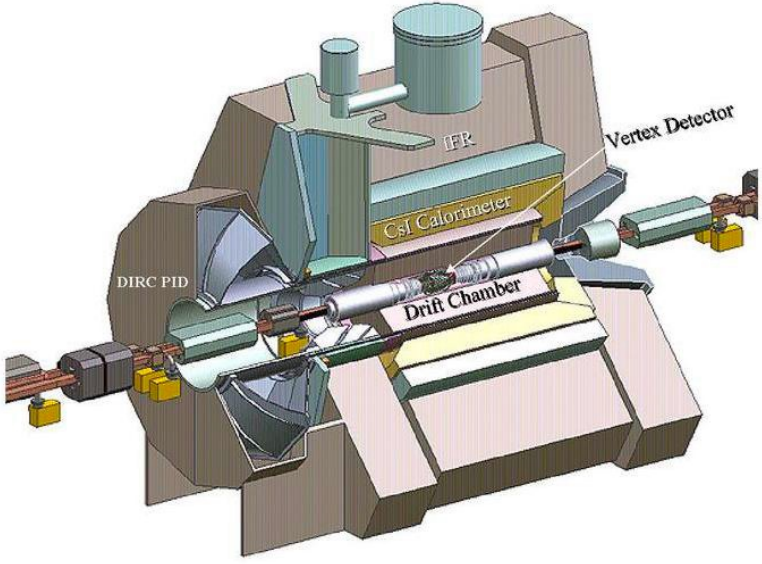
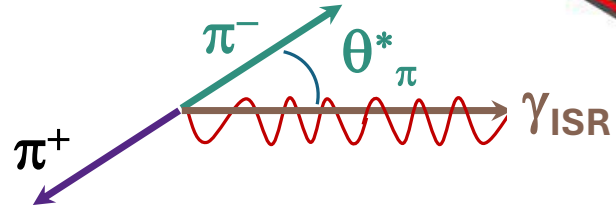
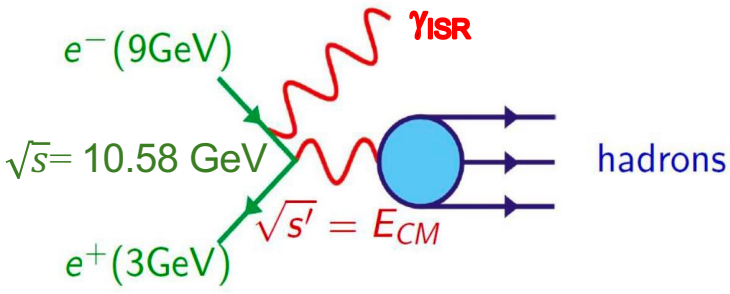








Initial state radiation (ISR) strategy: large range of center-of-mass energies probed after γ ISR emission from e^+ or e^-

Monte Carlo (MC) signals: $\pi^+\pi^-(\gamma)\,\gamma_{ISR}$ & $\mu^+\mu^-(\gamma)\,\gamma_{ISR}$ with *Phokhara9.1* generator

MC backgrounds:

- $e^+e^- \to K^+K^-(\gamma)\,\gamma_{ISR}$
- $q\bar{q}$ (q = u, d, s, c)
- $\tau^+\tau^-$
- X γ_{ISR} (X= nπ or nK + m π^0, . . .)

BABAR Measurements: *Differences between 2009 and 2025 Data Analyses*

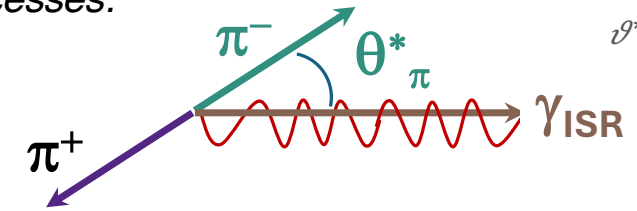
2009 analysis: *Phys. Rev. Lett. 103, 231801 – 2009* *Phys. Rev. D 86, 032013 – 2012*

- Runs 1 to 4 uses partial data sample of 232 fb^{-1}
- $\pi\pi$ /$\mu\mu$ separation using particle identification **(PID) dominant systematic**
- Momentum selection on each track: $p > 1$ GeV/c (more reliable μID),
- Signal MC uses AfkQED generator
- Normalize to $e^+e^- \rightarrow \mu^+\mu^- \gamma_{ISR}$ process
- Total relative systematic uncertainty ($0.5 - 1$ GeV/c^2) = **0.50%**

BABAR Measurements: *Differences between 2009 and 2025 Data Analyses*

$\theta^*_{\pi/\mu}$ distributions in the ISR process follow the original polar angular distributions of the corresponding Born processes:

$$\frac{d\sigma}{d\cos\theta_\pi}(e^+e^- \to \pi^+\pi^-) \sim \sin^2\theta_\pi$$



$$\frac{d\sigma}{d\cos\theta_\mu}(e^+e^- \to \mu^+\mu^-) \sim \beta_\mu[1+\cos^2\theta_\mu + (1-\beta_\mu^2)\sin^2\theta_\mu]$$

New 2025 analysis:

- Runs 1 to 6 uses full data sample of 460 fb^{-1}
- **Different method for π /μ separation:**
  Uses different polar angle distributions of π^- and μ^- in CM frame arising from π being spin 0 and μ being spin ½

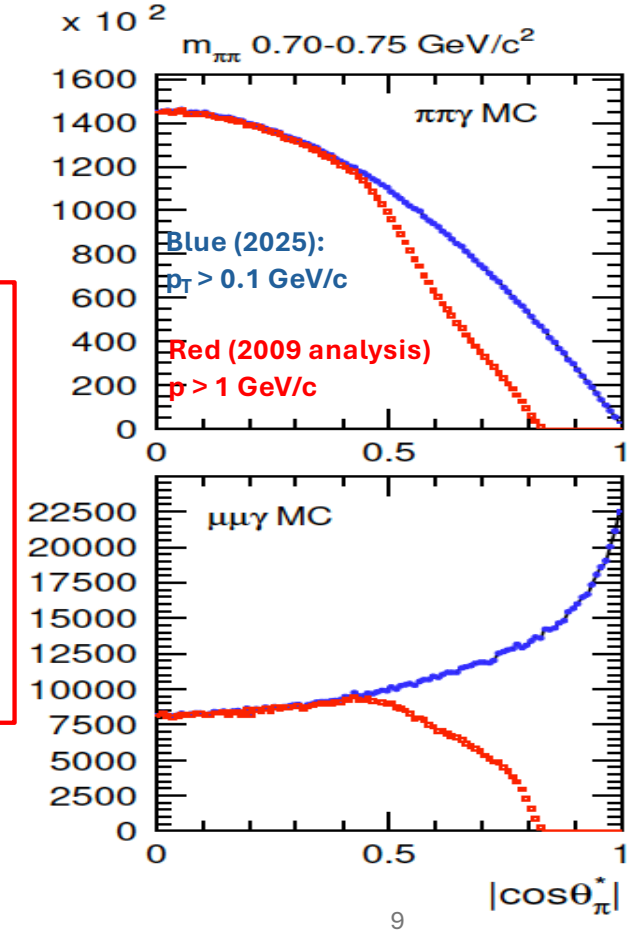
Removes explicit PID requirement and related systematic uncertainties

- Track p_T > 0.1 GeV/c (replaces p>1GeV/c requirement in 2009 analysis)
- Still normalize to $e^+e^- \to \mu^+\mu^-\gamma_{ISR}$ process
- Signal MC uses NLO Phokhara9.1 generator (+AfkQED for systematic studies)

⇒2025 analysis improved with: ×2 larger lumi, larger $p_{(T)}$ acceptance, and a different $\pi\pi/\mu\mu$ separation method

2025 analysis uses cosθ*:

ϑ^*_π = angle between π^- and γ_{ISR} in $\pi^+\pi^-$ CM



Overview of Analysis Procedure

1. **Kinematic fits** of additional radiation → 2-dimensional selection against background

2. **Angular fit** of data with data/MC corrections → $\pi\pi$ and $\mu\mu$ mass spectra
   ↪ **blinding** on absolute normalization of **tracking** corrections, **trigger** corrections and **fitted mass spectra**, independently for $\pi\pi$ and $\mu\mu$: **6 different blindings**

3. Conclusive comparison of $\mu\mu$ spectrum shape vs simulation → **unblinding of $\mu\mu$** to compare normalization to **QED prediction**

4. Successful QED test → $\mu\mu$ spectrum unfolded to $\sqrt{s'}$, i.e. centre-of-mass energy of final state including Final State Radiation (FSR) → **Initial State Radiation (ISR) luminosity**

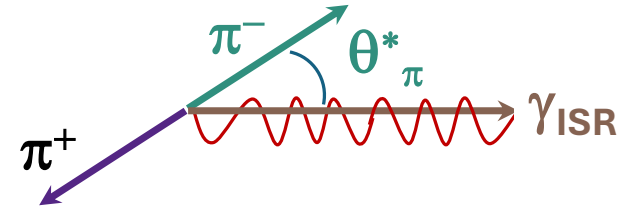
5. $\pi\pi$ spectrum unfolded → **cross section in** $\sqrt{s'}$, blinded contribution to a_μ computed to first check uncertainties

6. After extensive review and approval from the collaboration, $\pi\pi$ cross section and **contribution to a_μ** are **unblinded**

Use Two Kinematic Fits depending on γ_{ISR} scattering angle:

For both $\mu\mu\gamma$ and $\pi\pi\gamma$ processes, the event definition is enlarged to include the radiation of one γ in addition to the already required γ_{ISR}

In the $\pi^+\pi^-$ CM frame:



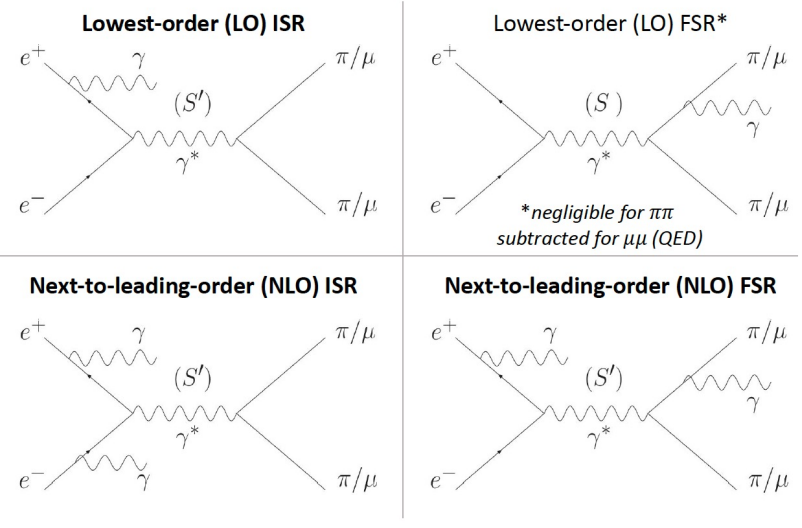
Perform Two Fits:

LARGE ANGLE (LA) Fit: The additional γ is detected in the EMC, so its energy and angles are used in the fit. The extra photon can be either from FSR or from ISR at large angle (LA) to the beams (0.35 – 2.45 radians) The threshold for the additional photon is kept low (50MeV).

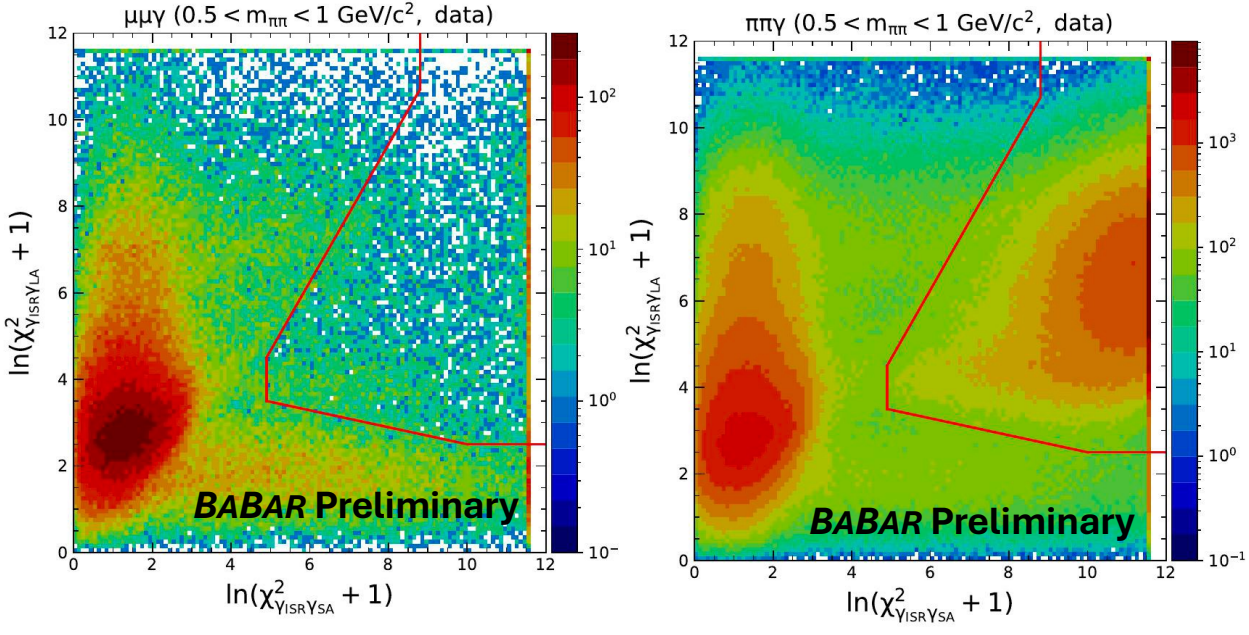
SMALL ANGLE (SA) Fit: The additional γ is assumed to be from ISR at small angle (SA) to the beams and undetected –the fit assumes this additional γ is perfectly aligned with either e+ or the e– beams. This 3-constraint fit ignores additional photons measured in the EMC: returns energy of the fitted collinear ISR γ

Each event characterized by two χ^2 values: χ^2_{LA} and χ^2_{SA}
We report $\ln(\chi^2 + 1)$ to better visualize the long χ^2 tails
Events without extra measured photons only have χ^2_{SA} and are plotted separately.
In the case of several extra detected photons, FSR fits are performed using each photon in turn and the fit with the best χ^2_{SA} is retained.
(The pion mass is assumed for the two charged particles)

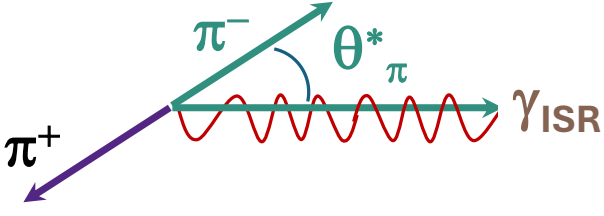
Use Two Kinematic Fits depending on γ_{ISR} scattering angle:



Sample plots of $\ln(\chi^2_{LA} - 1)$ vs $\ln(\chi^2_{SA} - 1)$



In the $\pi^+\pi^-$ CM frame:

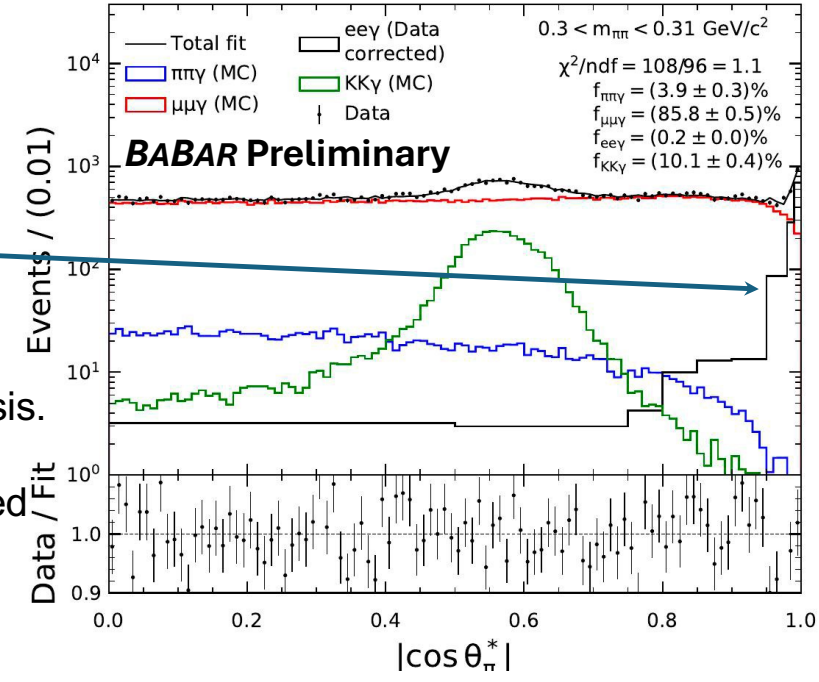


Larger background in $\pi\pi\gamma$ process are suppressed with optimized BDT-based **2D-χ^2 selection** (98-99% signal efficiency).

Methodology of the Angular Fits

$|\cos\theta^*|$ fits on background-subtracted data distributions with:

- templates of $\mu\mu\gamma$ and $\pi\pi\gamma$ and $KK\gamma$ from MC simulation
  + small data/MC corrections studied separately
- **data-driven** templates of $ee\gamma$ process (no reliable simulation) using cut-based and BDT selections



Fits performed twice: in pion mass ($m_{\pi\pi}$) and muon mass ($m_{\mu\mu}$) charged track hypotheses to get both $\pi\pi\gamma$ and $\mu\mu\gamma$ spectra in their respective basis.

Data $|\cos\theta^*|$ distribution adjusted by a linear combination of normalized templates in > 300 mass bins: 2 MeV/c^2 bins between 0.5 - 1 GeV/c^2, 10 MeV/c^2 elsewhere.

3-step strategy for the fits:

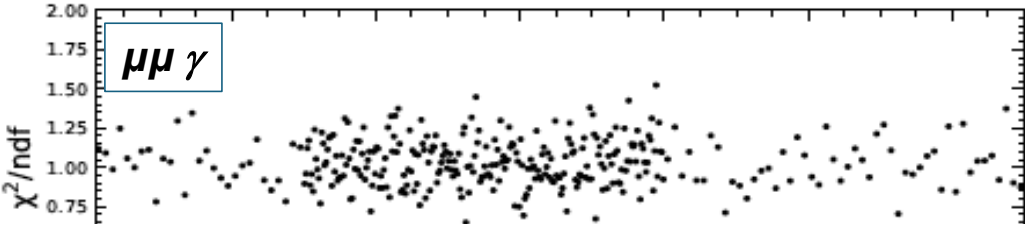
- **1st fit** between $0.9 < |\cos\theta^*| < 1$ to obtain the $ee\gamma$ normalization
- **2nd fit** between $0 < |\cos\theta^*| < 0.9$ after subtracting $ee\gamma$ from data
- **Extrapolation** of results to $|\cos\theta^*| = 1$.

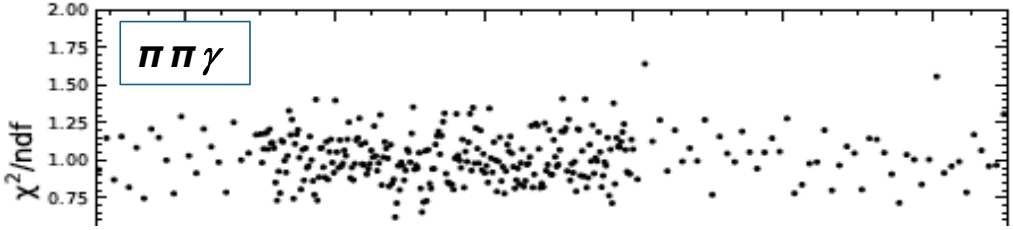
⇒ reduces sensitivity to systematic errors on data-driven $ee\gamma$ determination.

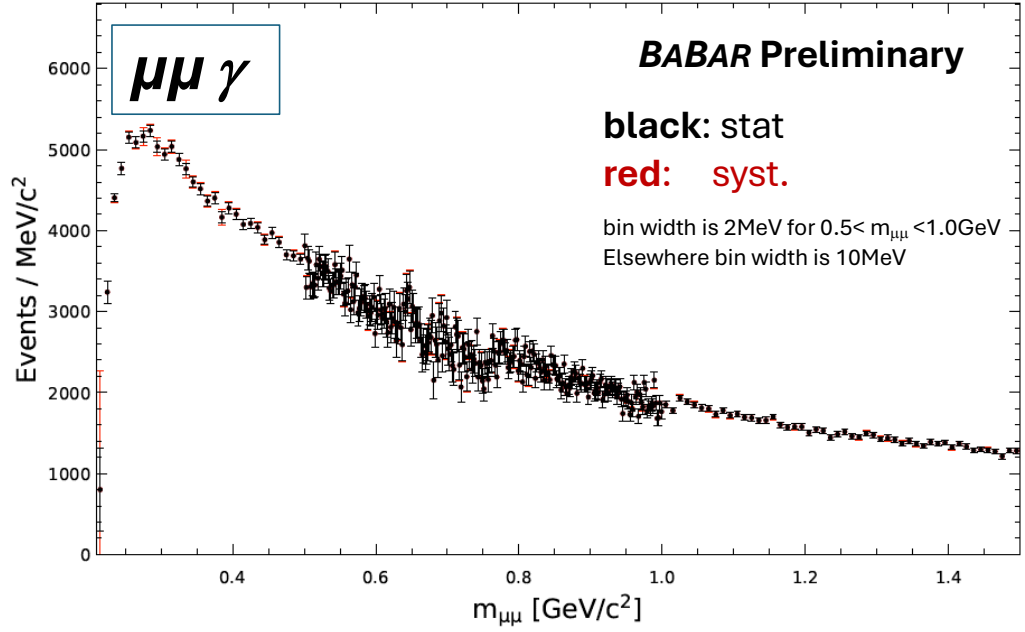
⇒ Closure test of accuracy of the fit on simulation show resulting spectra are consistent with initial inputs

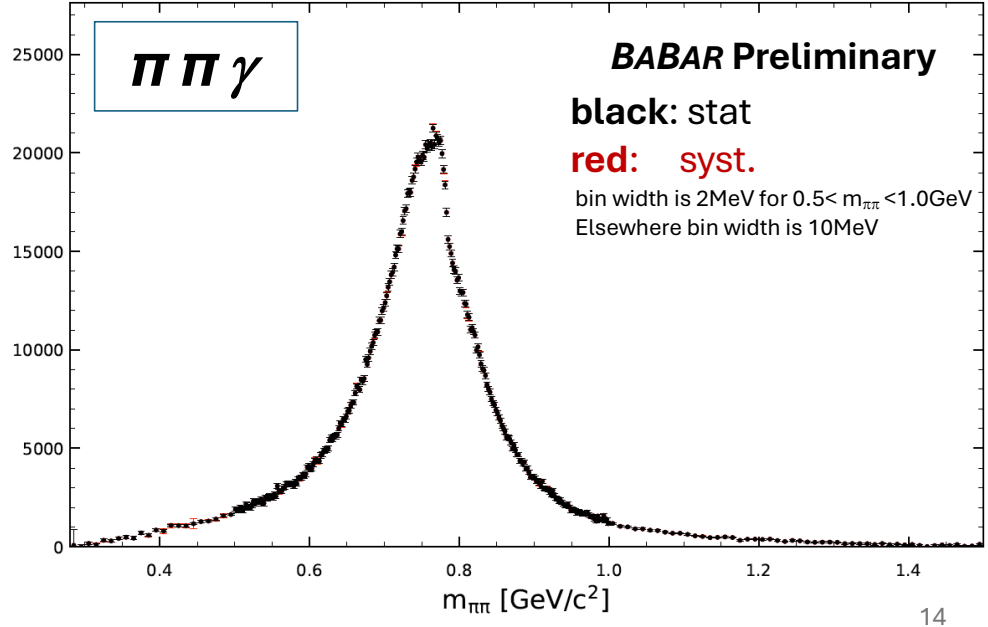
$\mu^+\mu^-\gamma$ and $\pi^+\pi^-\gamma$ Mass Spectra

Mass spectra obtained from angular fit with corrected templates (masses corrected for mass-dependent efficiencies)
Normalization of each spectrum ***initially blinded*** with a multiplicative factor. Here shown after unblinding
Statistical errors computed with bootstrap method









$\mu^+\mu^-\gamma$ Spectrum Comparison to QED Prediction

Compare the $\mu\mu\gamma$ data spectrum with QED prediction obtained by correcting the simulated *Phokhara* spectrum for:

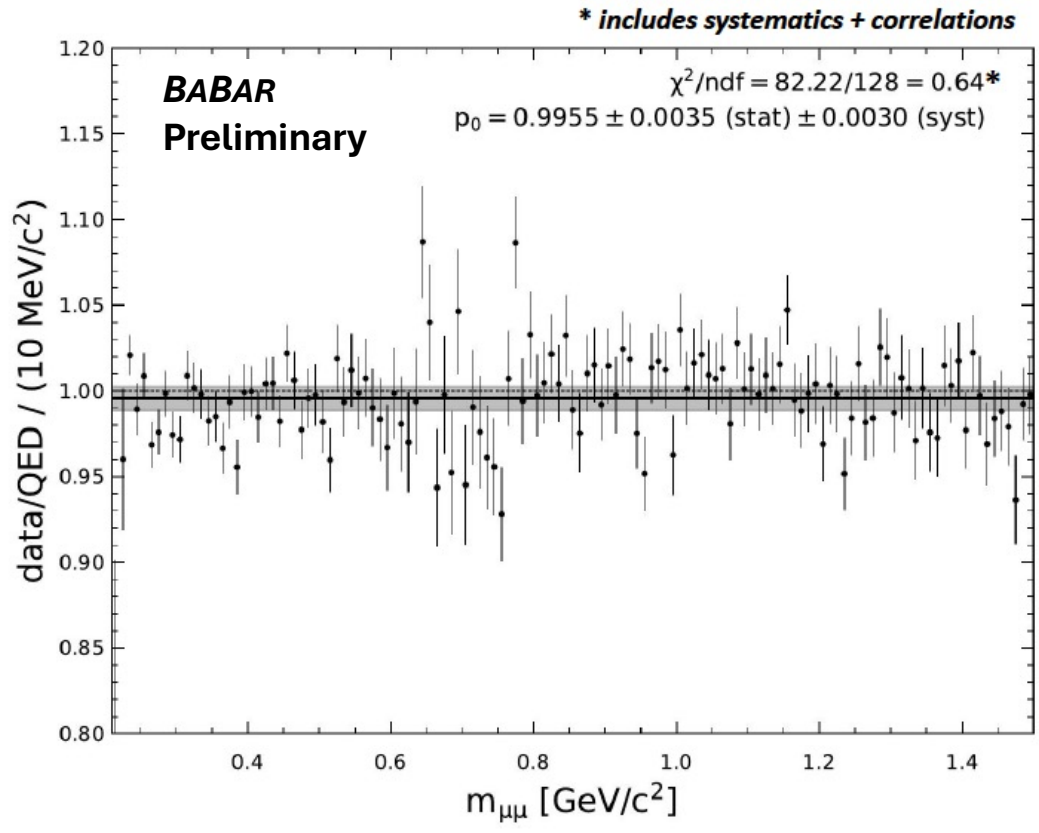
- ISR photon efficiency difference with data,
- Shortcomings** related to overestimation of ISR "NLO" and absence of NNLO in *Phokhara*, and
- Imprecise description of vacuum polarization effects.

The data/QED ratio is fitted with a constant:

$$0.9955 \pm \underbrace{0.0035_{\text{stat}}}_{\text{data + stat errors on corrections}} \pm \underbrace{0.0030_{\text{fit}}}_{\text{syst errors on corrections}} \pm \underbrace{0.0033_{\gamma\ \text{ISR}}}_{\text{ISR photon data/MC efficiency}} \pm \underbrace{0.0043_{\text{lumi}\ ee}}_{\text{error on } e^+e^- \text{ luminosity}}$$

Compatible with unity within 0.71% precision

⇒Validates the $\mu\mu/\pi\pi$ separation procedure



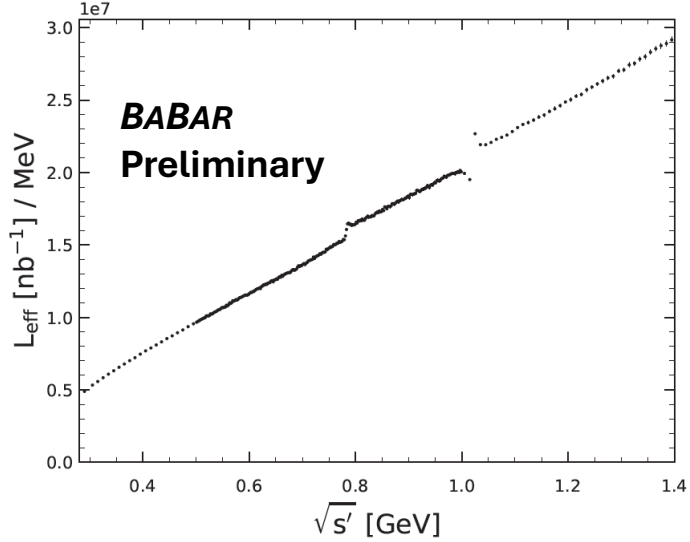
** *BABAR* Paper: *Phys. Rev. D* ***108****, L111103 (2023)*

Determining the ISR Effective Luminosity

Effective ISR luminosity obtained from unfolded $\mu\mu\gamma$ data spectrum $dN_{\mu\mu}^{\mathrm{ISR}}/d\sqrt{s'}$ as:

$$\frac{dL_{\mathrm{ISR}}^{\mathrm{eff}}}{d\sqrt{s'}} = \frac{dN_{\mu\mu}^{\mathrm{ISR}}/d\sqrt{s'}}{\epsilon_{\mu\mu}(\sqrt{s'})\ \sigma_{\mu\mu}^{0}(\sqrt{s'})}$$

where $\epsilon_{\mu\mu}$ is the acceptance of the selection (total efficiency) for muons obtained with Phokhara and $\sigma_{\mu\mu}^{0}$ is the $\mu\mu$ bare cross section (without vacuum polarization)



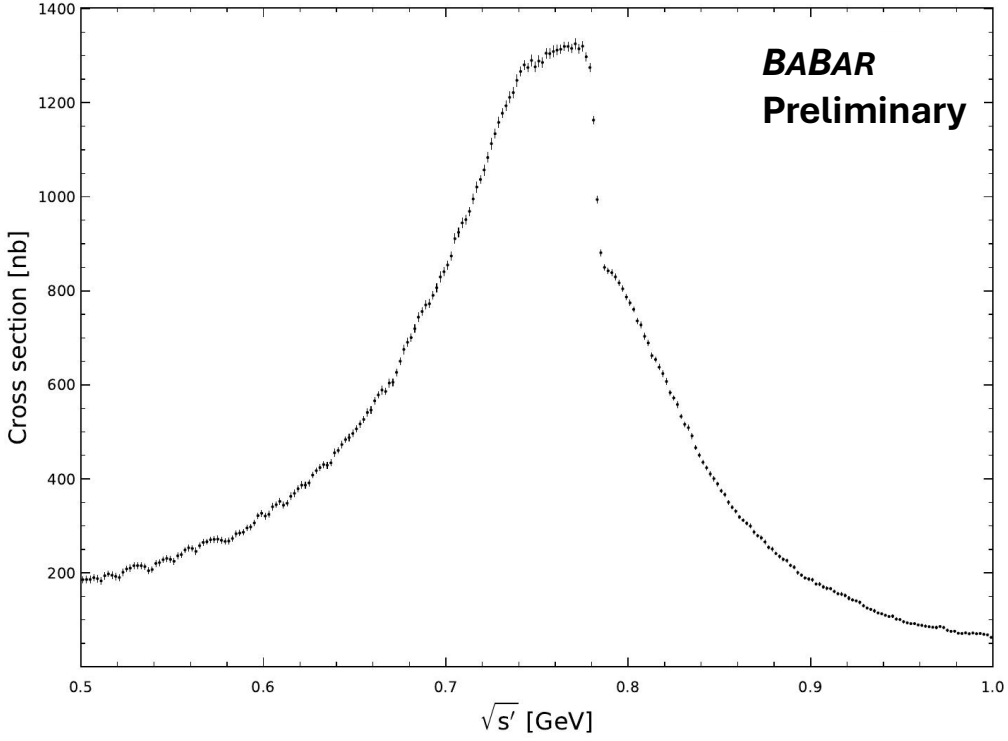
Two discontinuities at ω and ϕ associated with the vacuum polarization contributions being included in the unfolded data energy spectrum but not in the bare cross section

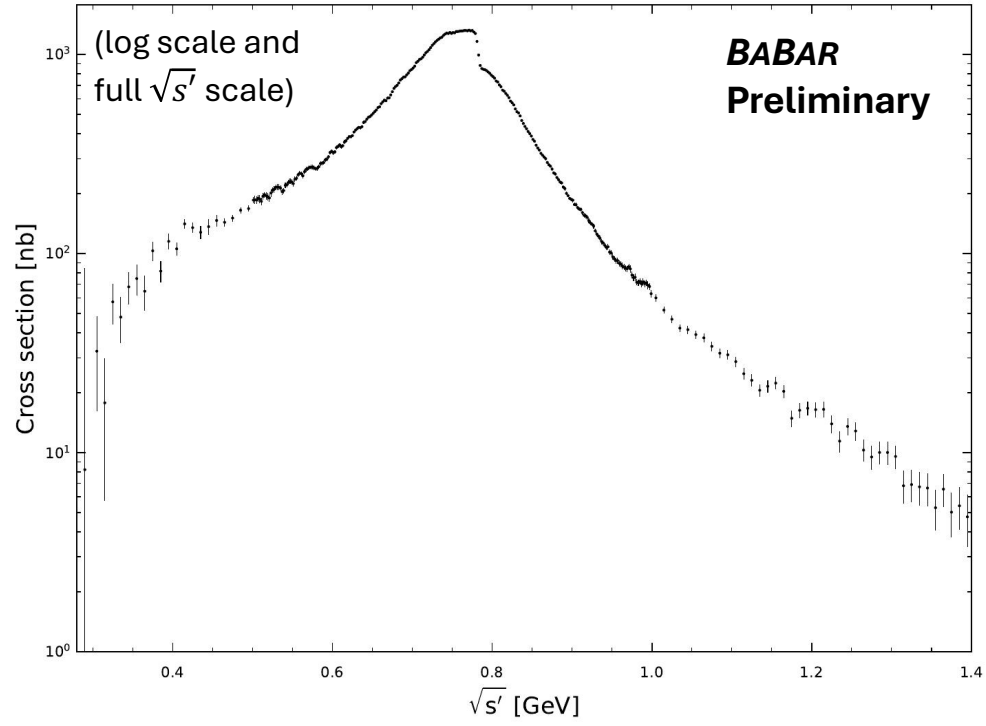
Measurement of the $e^+e^- \to \pi^+\pi^- (\gamma)\gamma_{ISR}$ Cross Section

Bare cross section for $\pi^+\pi^-$ process obtained from unfolded data spectrum using: $\sigma^0_{\pi\pi}(\sqrt{s'}) = \dfrac{dN_{\pi\pi}/d\sqrt{s'}}{\epsilon_{\pi\pi}(\sqrt{s'})\ dL^{\rm eff}_{\rm ISR}/d\sqrt{s'}}$
where $\epsilon_{\pi\pi}(\sqrt{s'})$ is the acceptance of the selection (total efficiency) for pions.

⇒ equivalent to: $\sigma^0_{\pi\pi}(\sqrt{s'}) = \sigma^0_{\mu\mu}(\sqrt{s'}) \dfrac{dN_{\pi\pi}/d\sqrt{s'}}{dN^{\rm MC\ gen}_{\mu\mu}/d\sqrt{s'}\ (1 - f_{\rm LO\ FSR})\ f_{\mu\mu}(\sqrt{s'})} \dfrac{1}{\epsilon_{\pi\pi}(\sqrt{s'})}$

Resulting in reduced systematic uncertainties from cancellations of common error sources or corrections

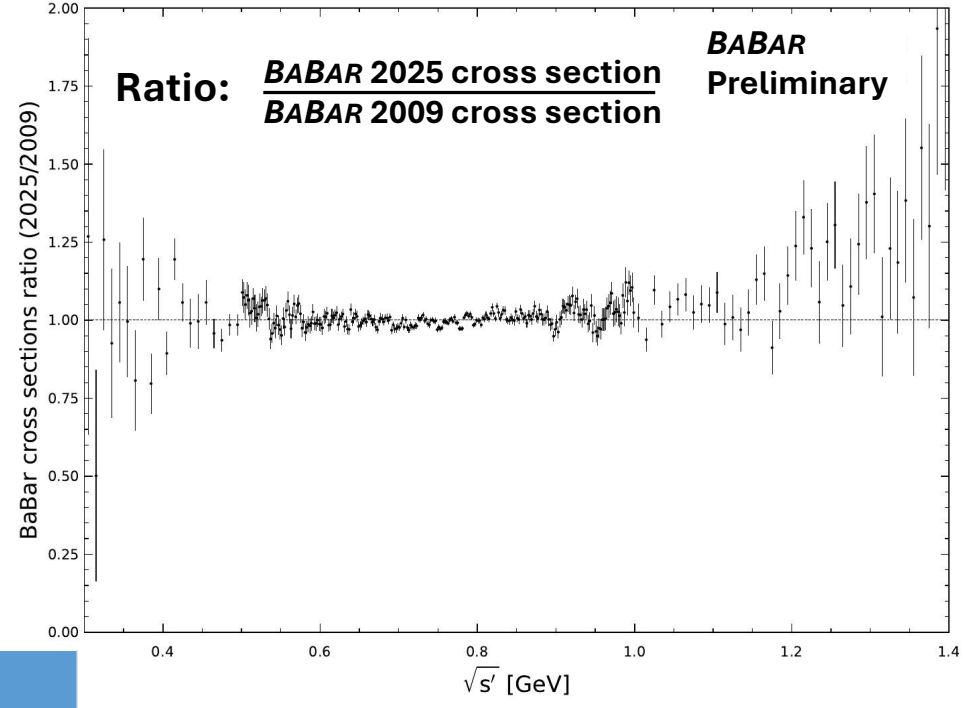




Comparison with 2009 *BABAR* Result and $\pi\pi$ Contribution to a_μ

Overall, the 2025 cross section is in good agreement with our previous 2009 measurement, except at large masses.

Not as precise at low & high masses because muons dominate these regions resulting in less precise determination of pion fraction in the angular fit.



$\pi\pi$ Contribution to a_μ

Energy range [GeV]	2025 $a_\mu^{2\pi}$ ± stat ± syst [10^{-10}]	2009 $a_\mu^{2\pi}$ ± stat ± syst [10^{-10}]
0.3 - 0.5	58.0 ± 5.5 ± 1.7	57.6 ± 0.6 ± 0.6
0.5 - 1.4	456.2 ± 2.2 ± 1.7	455.6 ± 2.1 ± 2.6

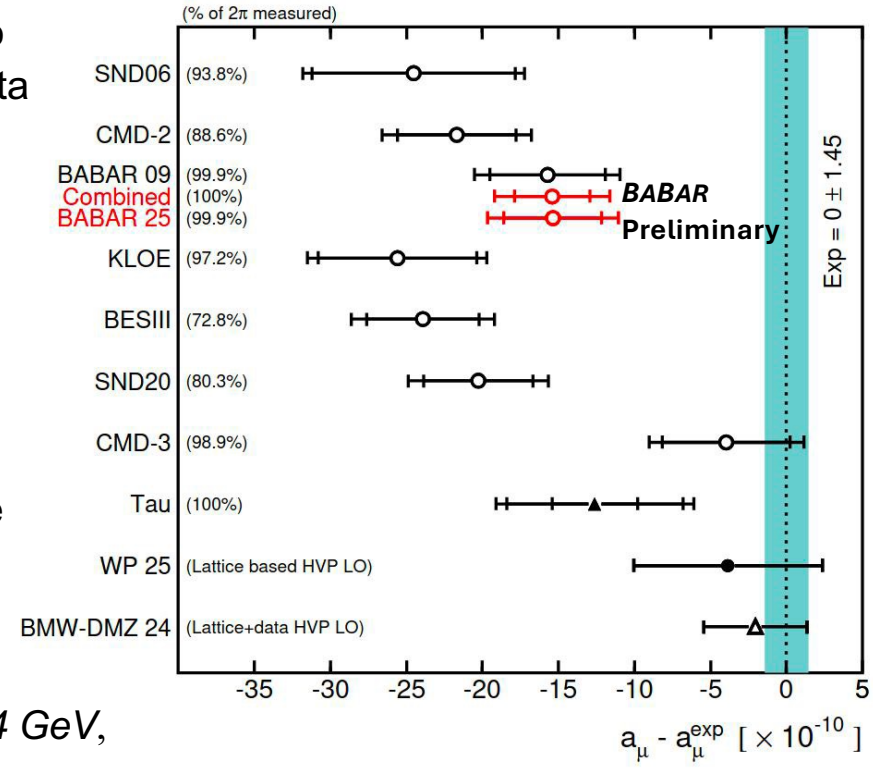
Energy range [GeV]	2025-2009 average (preliminary) $a_\mu^{2\pi}$ [10^{-10}]
Below 0.5	58.2 ± 0.8
0.5 - 1.4	455.9 ± 2.1
Below 1.4	514.1 ± 2.5
Below 1.8 (1.4 - 1.8 from 2009)	514.4 ± 2.5

⇒ **2025 and 2009 results are compatible and in excellent agreement**

⇒ **the averages yield the most precise $a_\mu^{2\pi}$ measurement from a single experiment.**

Summary and Conclusions

Report 2025 measurement of $a_\mu^{2\pi}$ (the contribution of $\pi\pi$ channel to a_μ) measured by *BABAR* via ISR in new blind analysis using full data statistics 460 fb^{-1}. Independent of method from the previous 2009 measurement, which used 232 fb^{-1}



•Separation of $\pi\pi$ and $\mu\mu$ final states in data carried out with fits of angular distributions. PID requirements removed in this work (dominant systematics in the 2009 study).
•Unblinded $\mu\mu\gamma$ spectrum compared to QED prediction, compatible with unity within uncertainties.
•Unblinded $\pi\pi$ cross section found to be in good agreement with the 2009 measurement, leading to the values of:

$a_\mu^{2\pi}$ (2025) = (58.0 ± 5. 5 (stat.) ±1.7 (syst.) × 10^{-10}) *below 0.5 GeV*,

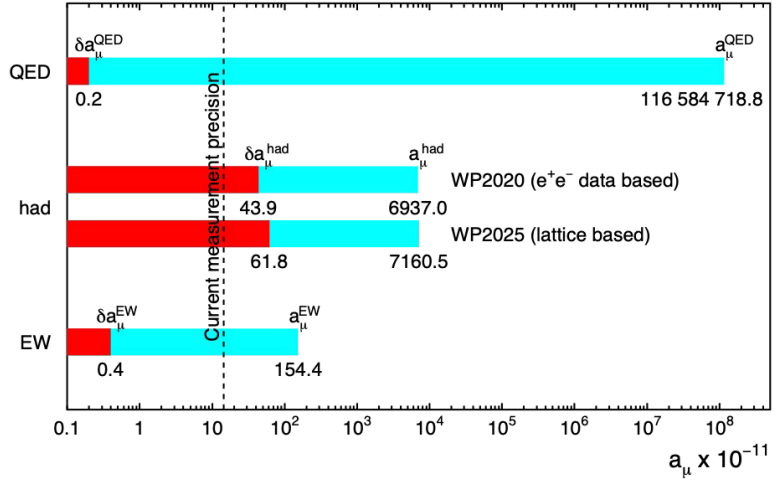
$a_\mu^{2\pi}$ (2025) = (456.2 ± 2. 2 (stat.) ± 1.7 (syst.) × 10^{-10}) *between 0.5 - 1.4 GeV*,
both consistent with the previous 2009 results.
Comparable statistical errors and reduced systematics between 0.5 - 1.4 GeV (*still preliminary*).

⇒ This consistency obtained with an independent and fully blinded procedure shows the robustness of both analyses, which combined provide the most precise measurement of $a_\mu^{2\pi}$ from a single experiment.

Backup Slides

Introduction: Anomalous Magnetic Moment and Hadron Vacuum Polarization

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{EW}}$$



WP2020: Phys. Rept. 887 (2020) 1
WP2025: arXiv:2505.21476
WP stands for the white paper of the muon g-2 theory initiative

Figure 40 in Phys. Rept. 1143 (2025) 1-158

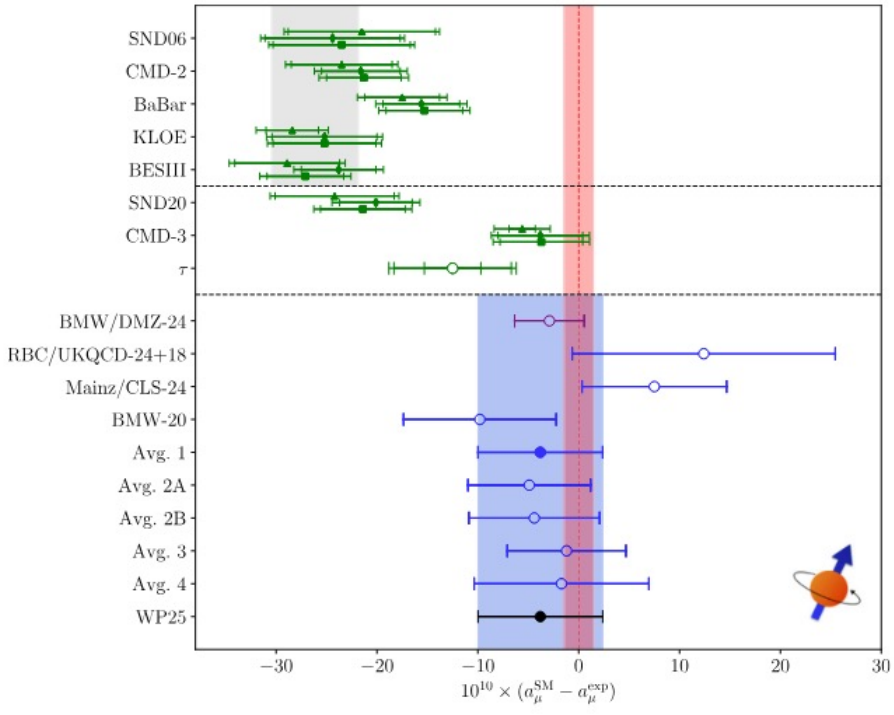


Fig. 40. Final summary of various determinations of $a_\mu^{\rm HVP,\ LO}$ discussed in Sections 2 and 3, propagated to $a_\mu^{\rm SM}$. The first two panels refer to data-driven determinations, where the three points for each e^+e^- experiment reflect the "CHKLS", "DHMZ", and "KNTW" methods, see Figs. 26 and 27 for more details. The gray band indicates the WP20 result, based on the e^+e^- experiments above the first dashed line. The τ point corresponds to Eq. (2.23). The last panel summarizes lattice-QCD determinations, including the hybrid evaluation [24], the three individual lattice-QCD calculations shown in Fig. 36, and the five lattice HVP averages from Fig. 37. The blue band refers to the final WP25 result, which coincides with "Avg. 1". In all cases, except for the gray WP20 band, the remaining contributions to $a_\mu^{\rm SM}$ beyond $a_\mu^{\rm HVP,\ LO}$ are taken from WP25, as given in Table 1. The red band denotes the experimental world average, which has been updated including the final results from the Fermilab experiment.

Figure 83 in Phys. Rept. 1143 (2025) 1-158

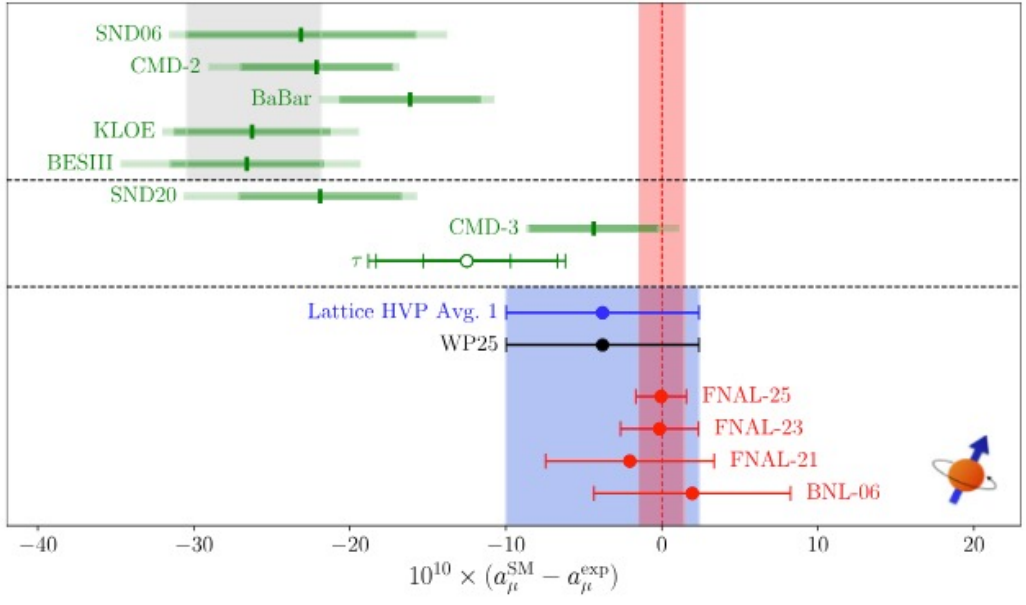


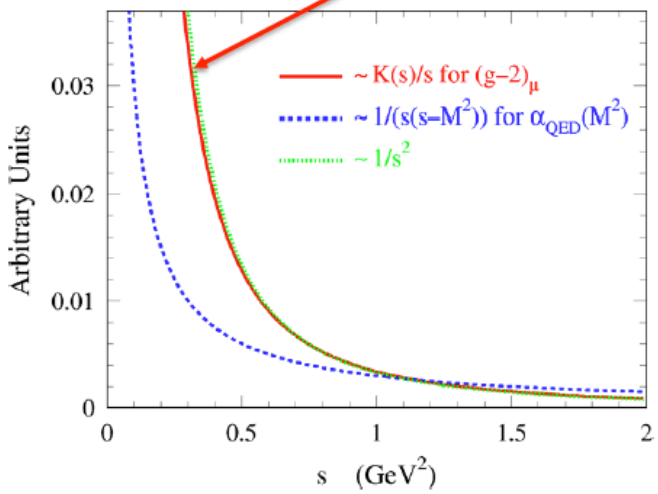
Fig. 83. Summary of the current SM prediction for a_μ in comparison to experiment (red band and data points). The final WP25 prediction is denoted in black and via the blue band, it derives from the LO HVP result defined by the lattice-QCD "Avg. 1" shown in blue, see Eq. (3.37). The gray band indicates the WP20 result, based on the e^+e^- experiments above the first dashed line. These experimental ranges, as well as the ones for SND20 and CMD-3 that appeared after WP20, are produced as in Fig. 27; they are meant to illustrate the current situation, but cannot be interpreted as uncertainties with a proper statistical meaning. The τ point refers to Eq. (2.23), the numerical results are collected in Table 5. In all cases except for the gray WP20 band the LO HVP results are combined with WP25 values for the remaining contributions, as summarized in Table 1. The figure has been updated after the announcement of the final results from the Fermilab experiment, including the corrections to the previous experimental points as detailed in Ref. [8].

e^+e^- Data-Based HVP Calculation

From Zhiqing Zhang Presentation at Lepton Photon 2025

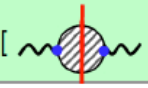
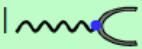
Based on analyticity and unitarity, the LO HVP contribution can be calculated using the dispersion relation [1] over $e^+e^- \rightarrow$ hadrons cross sections

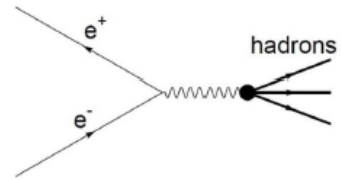
$$a_\mu^{\text{HVP LO}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$



Bare: $\sigma^{(0)}(s) = \sigma(s)(\alpha/\alpha(s))^2$

$$12\pi\, \text{Im}\Pi_\gamma(s) = \frac{\sigma^0\,[e^+e^- \rightarrow \text{hadrons}\,(\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

Im[] ∝ |  hadrons |²



The QED kernel K(s) [2] has such an *s* dependence that low energy data contribute most:

→ The precision is driven by that of $e^+e^- \rightarrow$ hadrons

[1] Bouchiat and Michel, 1961
[2] Brodsky, de Rafael, 1968

cos θ* distributions for different mass ranges

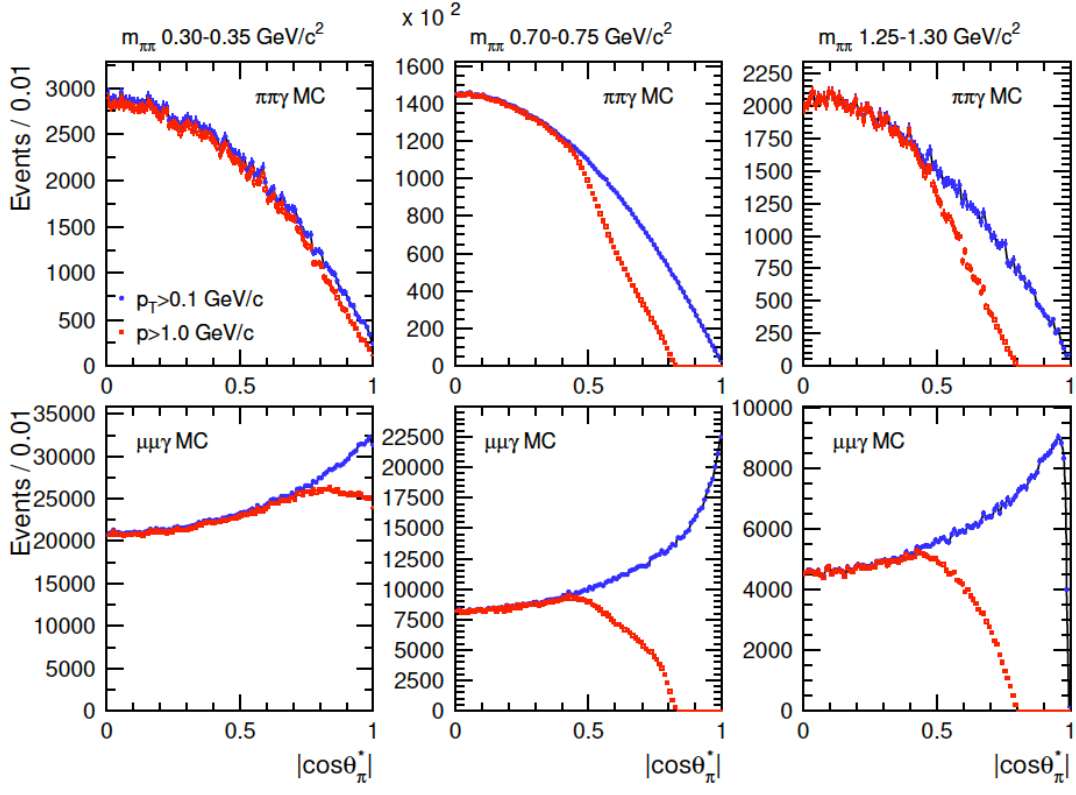


Figure 3: The $|\cos\theta^*_\pi|$ distributions in three mass intervals for $\pi\pi\gamma$ (top) and $\mu\mu\gamma$ (bottom) MC events within the geometrical acceptance selections, without momentum selection (black histogram), and different threshold values: $p > 1.0\,\mathrm{GeV/c}$ (red, 2009 analysis) and $p_T > 0.1\,\mathrm{GeV/c}$ (blue, this analysis).

Signal Event Selection

- ISR photon: $0.35 < \theta^{\gamma}_{Lab} < 2.40$ rad, $E\gamma > 3$ GeV, $E^*_{\gamma} > 4$ GeV
- 2 opposite-charge high quality tracks:
  - $p_T > 0.1$ GeV, 0.4-2.45 rad
  - at least 15 drift chamber hits,
  - xy distance of closest approach to interaction point (IP) (doca) < 5mm and ldzl < 6cm
- Allow more photons and other quality tracks per event
- $V_{xy} < 0.5$ cm. Distance from vertex of two tracks and IP in transverse plane this suppresses electron backgrounds from photon conversions generating displaced two-track vertices
- Veto electron contamination
- Two NLO radiation fits performed per event give goodness of fits input to BDT selection to further suppress backgrounds

Hadronic vacuum polarization (HVP) contribution to the anomalous magnetic moment of the muon (a_μ) can be obtained by measuring cross sections of $e^+e^- \rightarrow$ hadrons with the largest input from $e^+e^- \rightarrow \pi^+\pi^-$ (73%)

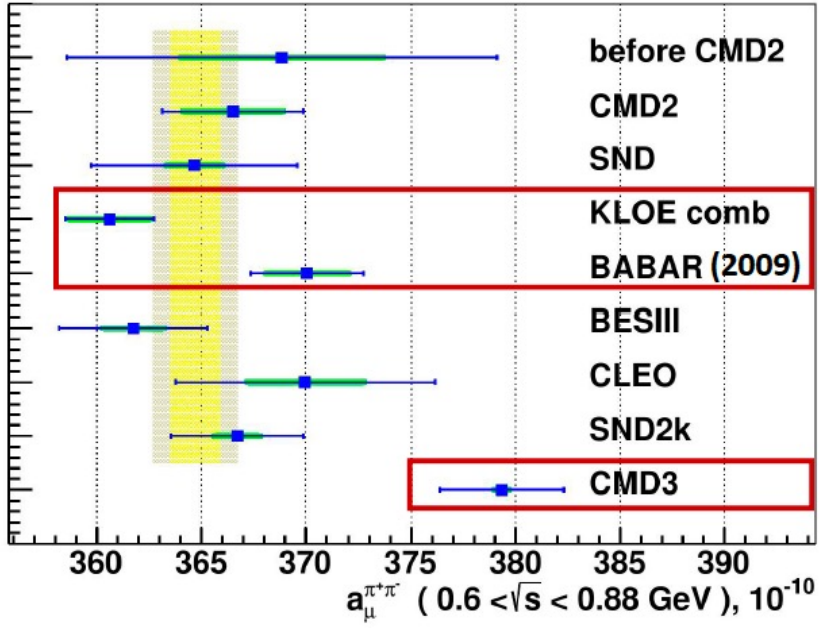
Current tensions also exist between:

- measurements from KLOE, BaBar and CMD-3
  (*KLOE vs CMD-3*: > 5σ tension at ρ peak)

CMD-3 Collaboration
Measurement of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section from threshold to 1.2 GeV with the CMD-3 detector
Phys. Rev. D 109, 112002 (2024)

Previous BaBar result from 2009 (partial data)

This talk presents a new preliminary *BABAR* measurement of the $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ cross section with its FULL DATA SAMPLE plus a new & independent method aimed to improve precision.



BABAR Measurements: *Differences between 2009 and 2025 Data Analyses*

2009 analysis: *Phys. Rev. Lett. 103, 231801 – 2009* *Phys. Rev. D 86, 032013 – 2012*

- Runs 1 to 4 uses partial data sample of 232 fb^{-1}
- $\pi\pi$ /$\mu\mu$ separation using particle identification **(PID) dominant systematic**
- Momentum selection on each track: $p > 1$ GeV/c (more reliable μID),
- Signal MC uses AfkQED generator
- Normalize to $e^+e^- \rightarrow \mu^+\mu^- \gamma_{ISR}$ process
- Total relative systematic uncertainty ($0.5 - 1$ GeV/c^2) = **0.50%**

New 2025 analysis:

- Runs 1 to 6 uses full data sample of 460 fb^{-1}
- **Different method for π /μ separation:**
  Uses different polar angle distributions of π^- and μ^- in CM frame arising from π being spin 0 and μ being spin ½

Removes explicit PID requirement and related systematic uncertainties

- Track $p_T > 0.1$ GeV/c (replaces p>1GeV/c requirement in 2009 analysis)
- Still normalize to $e^+e^- \rightarrow \mu^+\mu^- \gamma_{ISR}$ process
- Signal MC uses NLO Phokhara9.1 generator (+AfkQED for systematic studies)

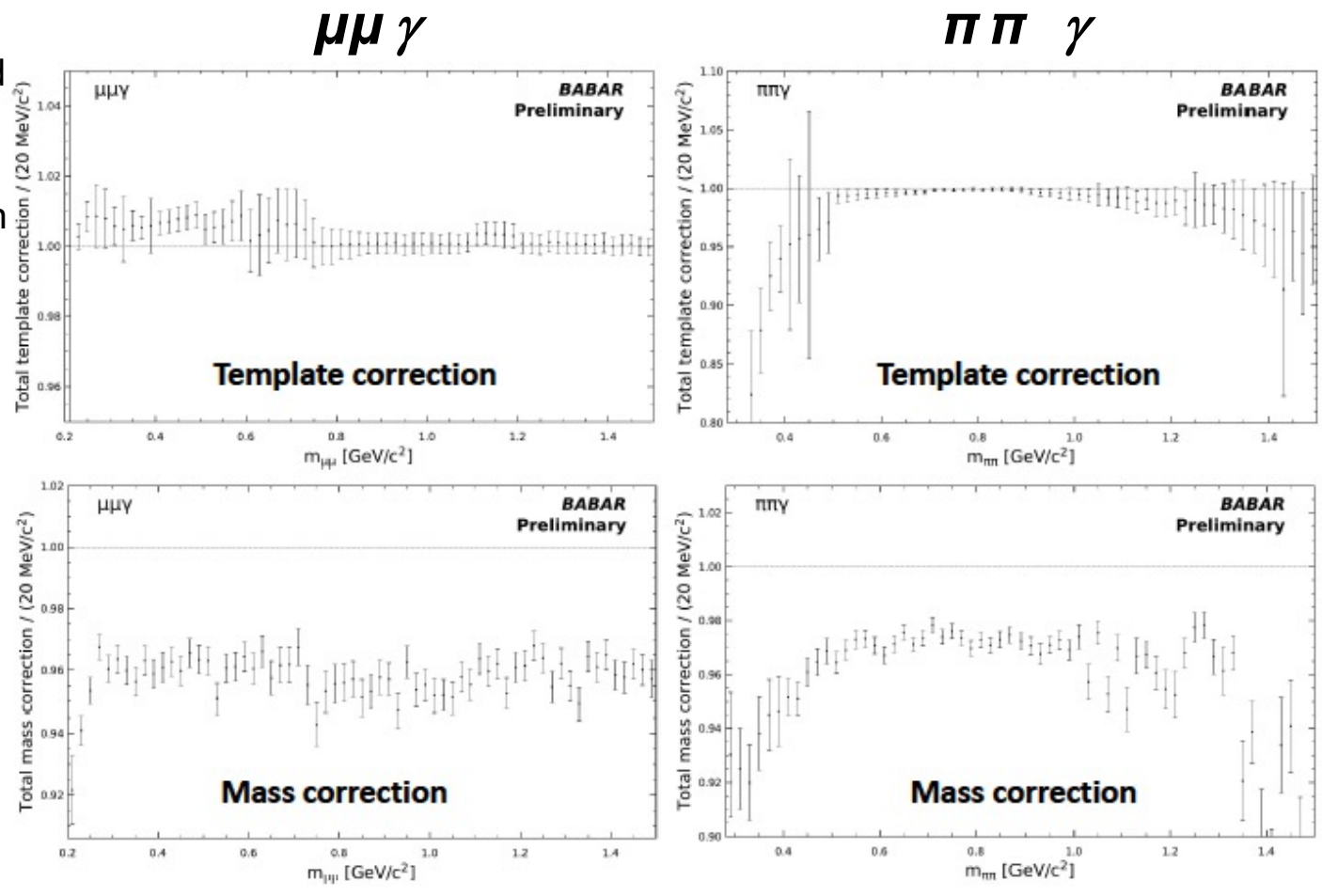
⇒2025 analysis improved with: ×2 larger lumi, larger $p_{(T)}$ acceptance, and a different $\pi\pi/\mu\mu$ separation method

Corrections to the MC Simulation

Efficiency corrections determined for $|\cos\theta^*|$ templates and fitted masses due to data/MC differences, e.g. in 2D-χ^2 selection or trigger and tracking efficiencies (***corrections initially blinded***).

Large effect of corrections on mass spectra (at most ±5% difference to uncorrected spectra), however they tend to cancel when combined

Overall, total template and mass corrections are rather flat (larger errors in ρ region from templates in muon mass).



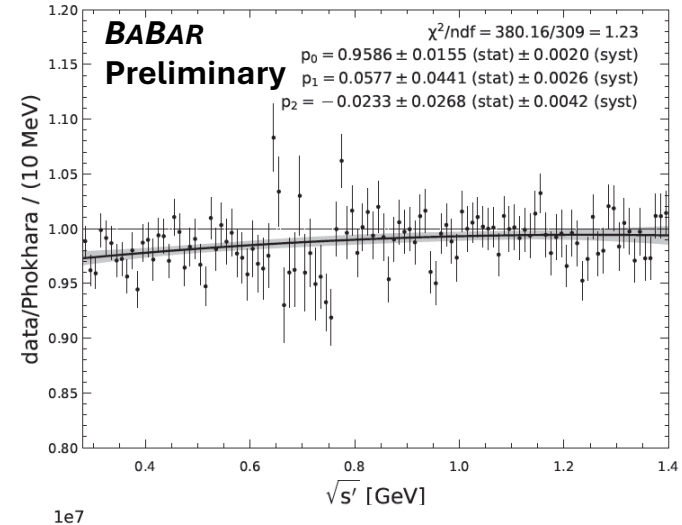
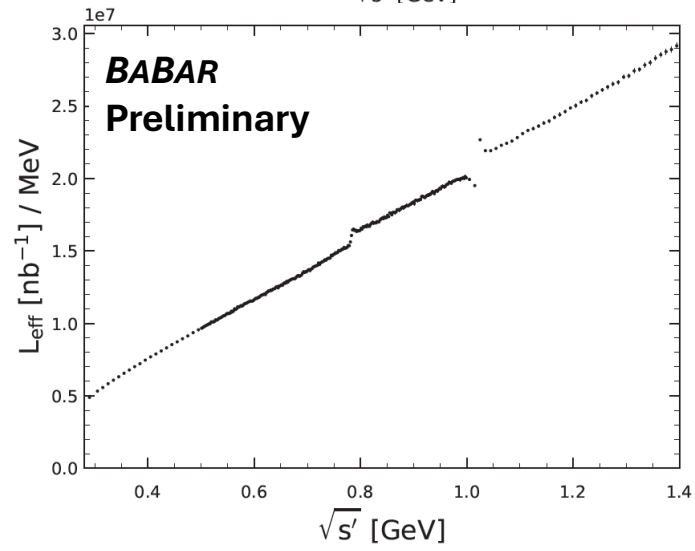
Determining the ISR Effective Luminosity

Effective ISR luminosity obtained from unfolded $\mu\mu\gamma$ data spectrum $dN^{\mathrm{ISR}}_{\mu\mu}/d\sqrt{s'}$ as:

$$\frac{dL^{\mathrm{eff}}_{\mathrm{ISR}}}{d\sqrt{s'}} = \frac{dN^{\mathrm{ISR}}_{\mu\mu}/d\sqrt{s'}}{\epsilon_{\mu\mu}(\sqrt{s'})\ \sigma^{0}_{\mu\mu}(\sqrt{s'})}$$

where $\epsilon_{\mu\mu}$ is the acceptance of the selection (total efficiency) for muons and $\sigma^{0}_{\mu\mu}$ is the $\mu\mu$ bare cross section (without vacuum polarization)

To mitigate the large statistical fluctuations in the unfolded spectrum, $\left(dN^{\mathrm{ISR}}_{\mu\mu}/d\sqrt{s'}\right)/\epsilon_{\mu\mu}$ is replaced by:

$$\underbrace{\frac{dN^{\mathrm{MC\ gen}}_{\mu\mu}}{d\sqrt{s'}}}_{\substack{\textit{Phokhara}\\ \textit{spectrum at}\\ \textit{generation level}}} \times \underbrace{(1 - f_{\mathrm{LO\ FSR}})}_{\substack{\textit{removes the}\\ \textit{LO FSR}\\ \textit{contribution}}} \times \underbrace{f_{\mu\mu}(\sqrt{s'})}_{\substack{\textit{2}^{\textit{nd}}\ \textit{order}\\ \textit{polynomial fit}\\ \textit{to data/MC}}}$$

⇒ **Justified by the successful QED test and the slow/smooth variation of global template/mass corrections**