

Hadronic Cross Section Measurements using Initial State Radiation



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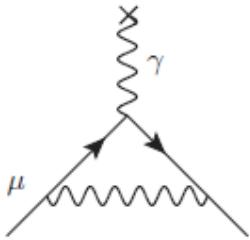
February 24, 2023

Georges Vasseur

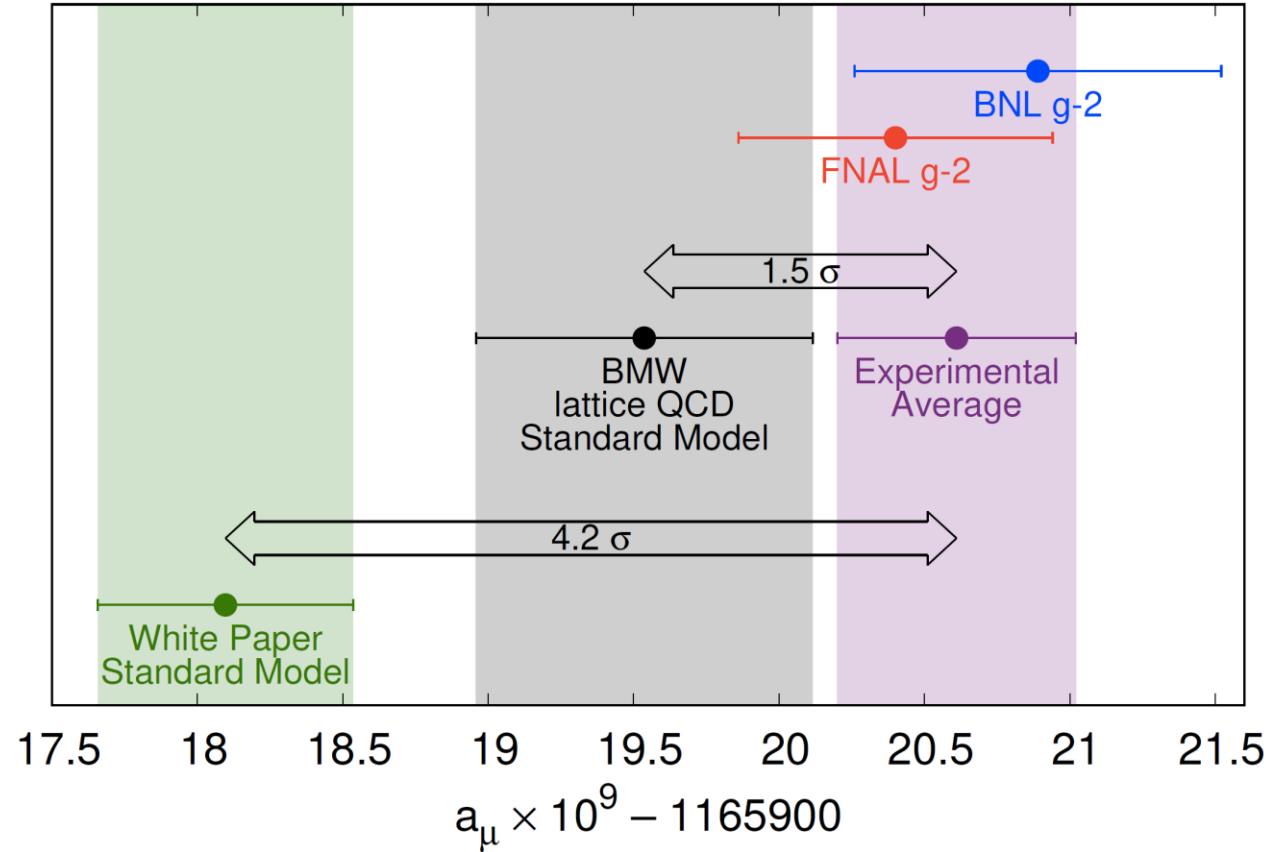
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on behalf of the BABAR Collaboration

The $g-2$ puzzle



- Lepton anomalous magnetic moment:
$$a_l = \frac{1}{2}(g - 2)_l$$
- Precise test of the Standard Model
- Long-standing discrepancy between theory and experiment for the muon ($g-2$)



Dispersive approach

- Dominant uncertainty on ($g-2$) calculation from leading order **hadronic vacuum polarization**

The diagram illustrates the equivalence between a quark loop and an annihilation process. On the left, a blue circle represents a quark loop with a wavy line labeled q entering and a wavy line labeled \bar{q} exiting. On the right, a double-headed arrow indicates equivalence to an annihilation process where an electron-positron pair (e^+ , e^-) annihilates into a quark loop (q , \bar{q}). The outgoing particles from the loop are shown as three horizontal lines.

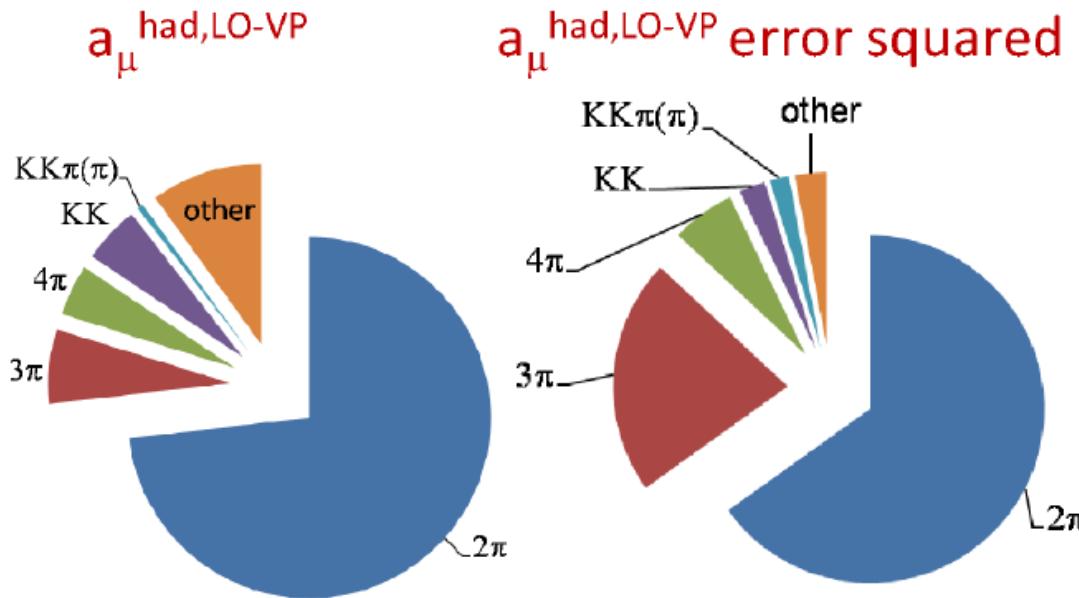
$$a_\mu^{\text{had}} \approx \frac{1}{4\pi^3} \int_{m_\pi^2}^\infty K_\mu(s) \cdot \sigma_{e^+ e^- \rightarrow \text{had}}(s) ds$$

Kernel function cross section

- Calculation needs **experimental inputs**

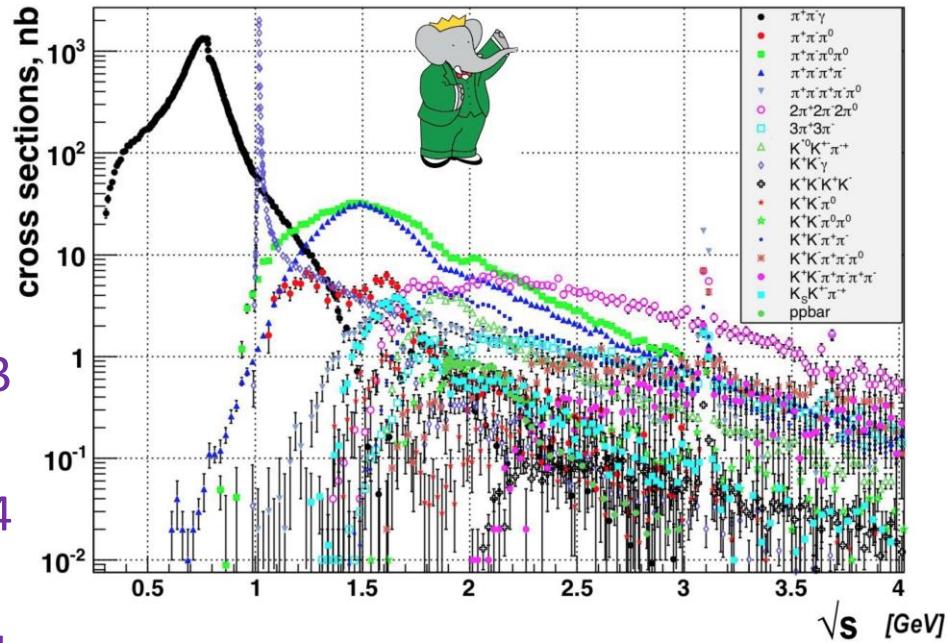
Hadronic cross sections and $g-2$

- At low energy total hadronic cross section determined from finite sum of exclusive modes
- The $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ mode contributes 7% to a_μ^{had} and 19% to its uncertainty (the second largest).

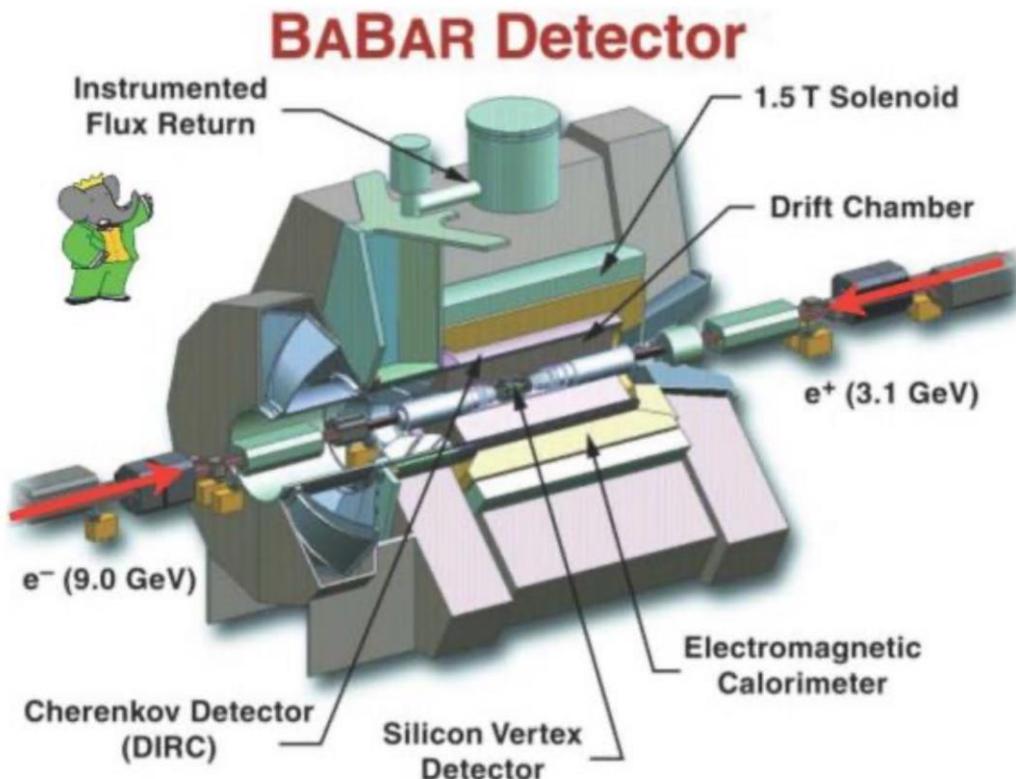


Cross sections from BABAR

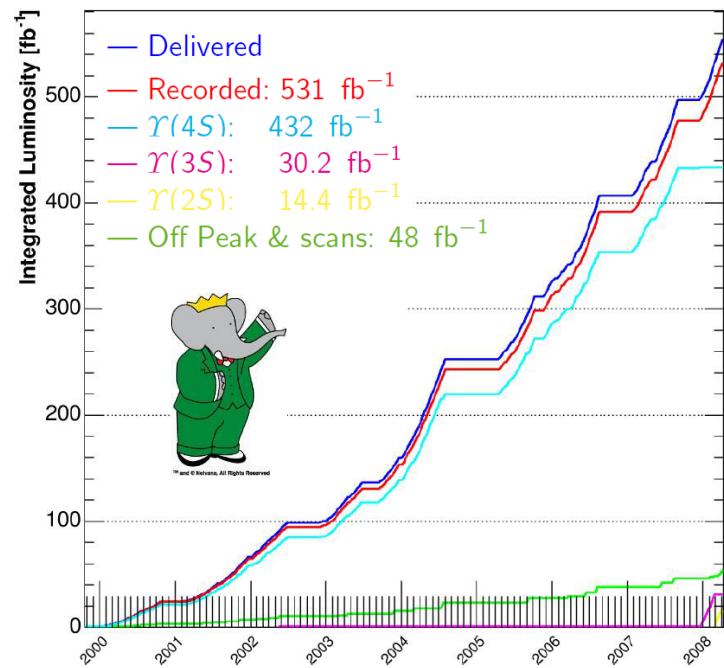
- Comprehensive program of cross section measurements in BABAR
- Most recent results
 - $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$
 - Phys. Rev. D 104 (2021), 112003
 - $e^+ e^- \rightarrow \pi^+ \pi^- 4\pi^0$
 - Phys. Rev. D 104 (2021), 112004
 - $e^+ e^- \rightarrow 2(\pi^+ \pi^-) 3\pi^0$
 - Phys. Rev. D 103 (2021), 092001
 - $e^+ e^- \rightarrow K K \pi \pi \pi$
 - arXiv:2207.10340 (2022)



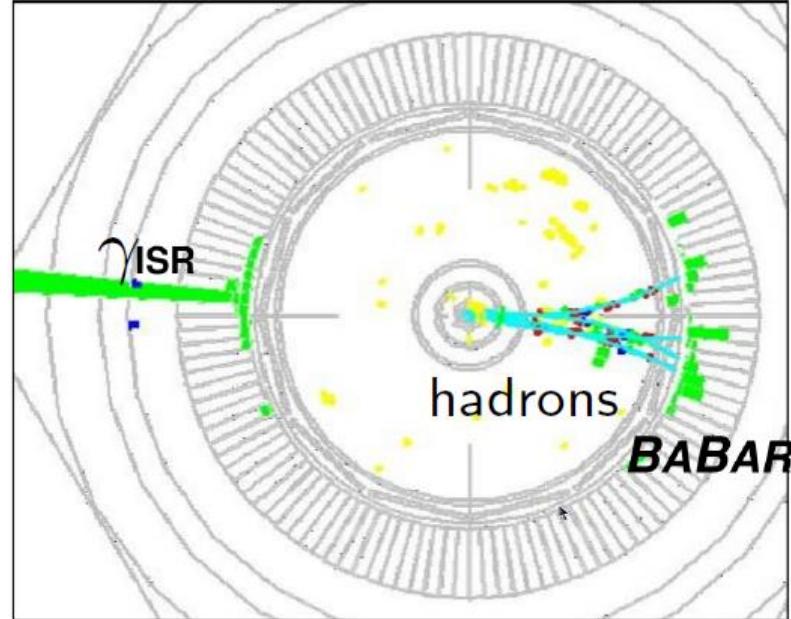
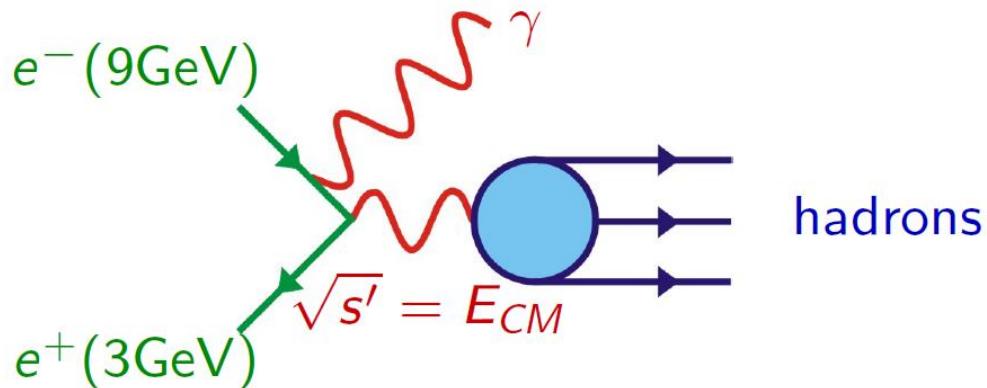
Detector and data sample



0.5 ab⁻¹ over 10 years



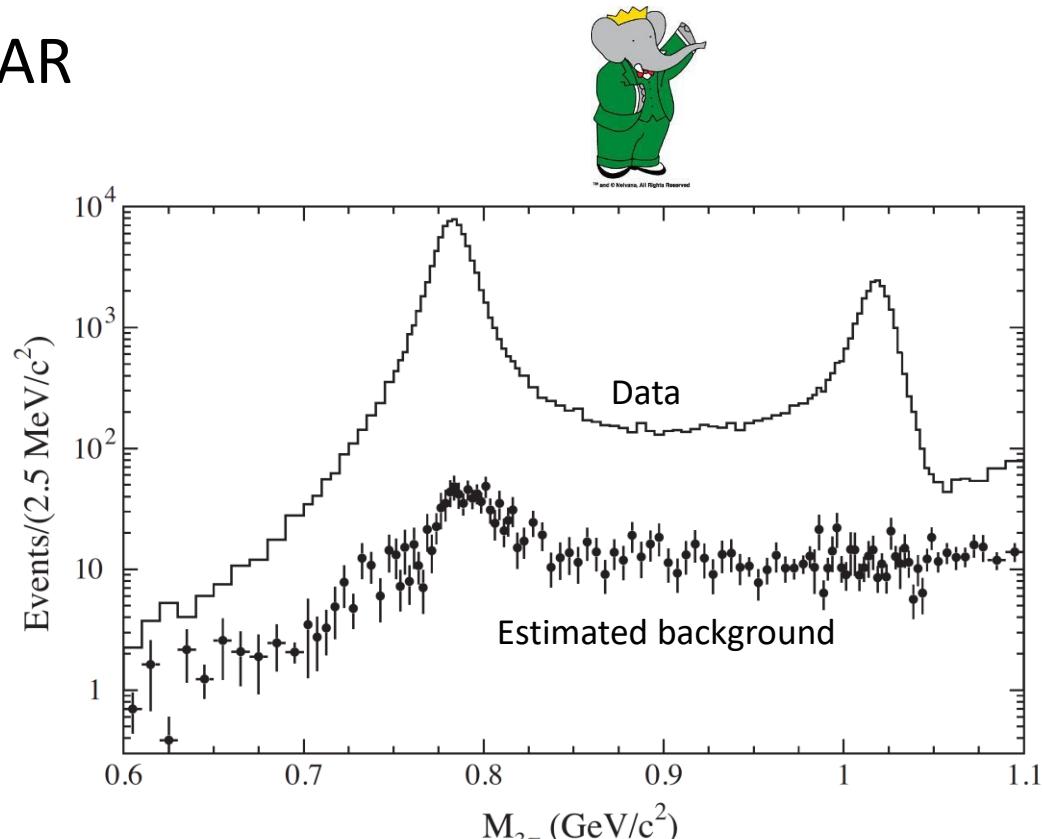
ISR method



- Photon emitted from e^+ or e^- as Initial State Radiation (ISR).
 - allows to measure cross sections at low energy.
- Hadronic system boosted and back to back with photon.
 - Good detection even at threshold.
 - In detector acceptance: fully reconstructed.

$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ selection

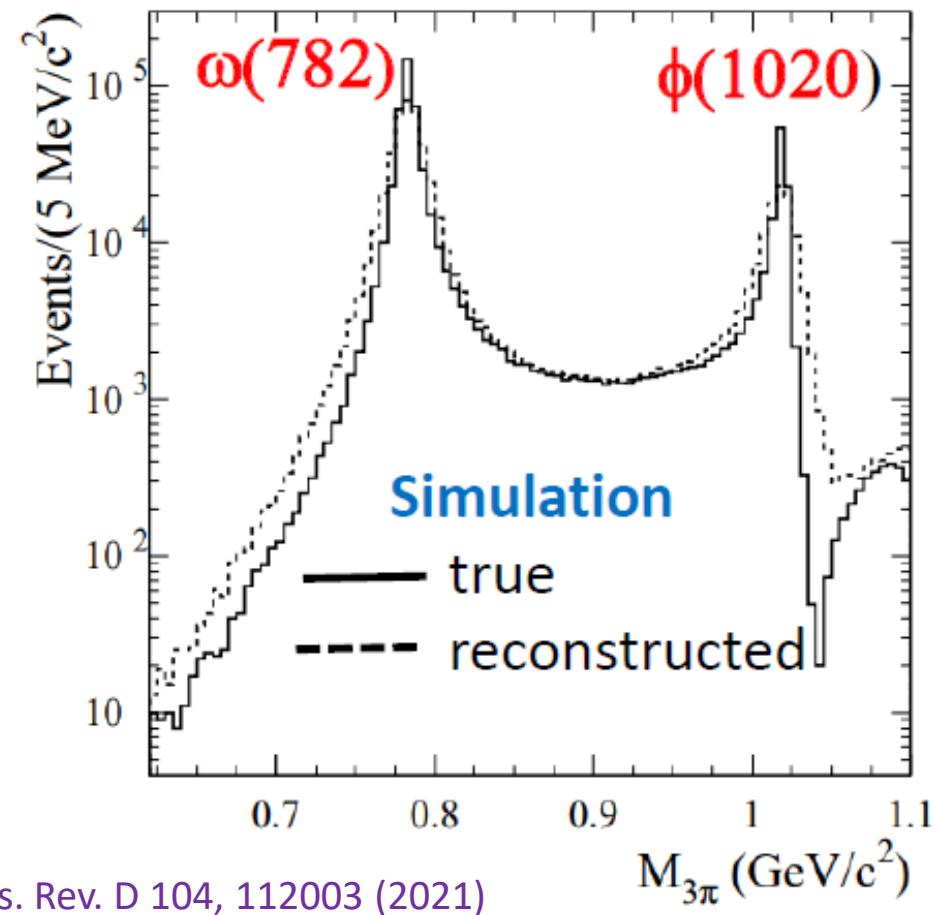
- Analysis on the full BABAR dataset (469 fb^{-1}).
- ISR γ : γ with highest energy over 3 GeV.
- Exactly 2 “good” tracks (opposite sign).
- At least 2 additional photons.
- $0.10 < m_{\gamma\gamma} < 0.17 \text{ GeV}$
- Kinematic fit.
- Further cuts to reduce backgrounds.



Phys. Rev. D 104, 112003 (2021)

Expected mass spectrum

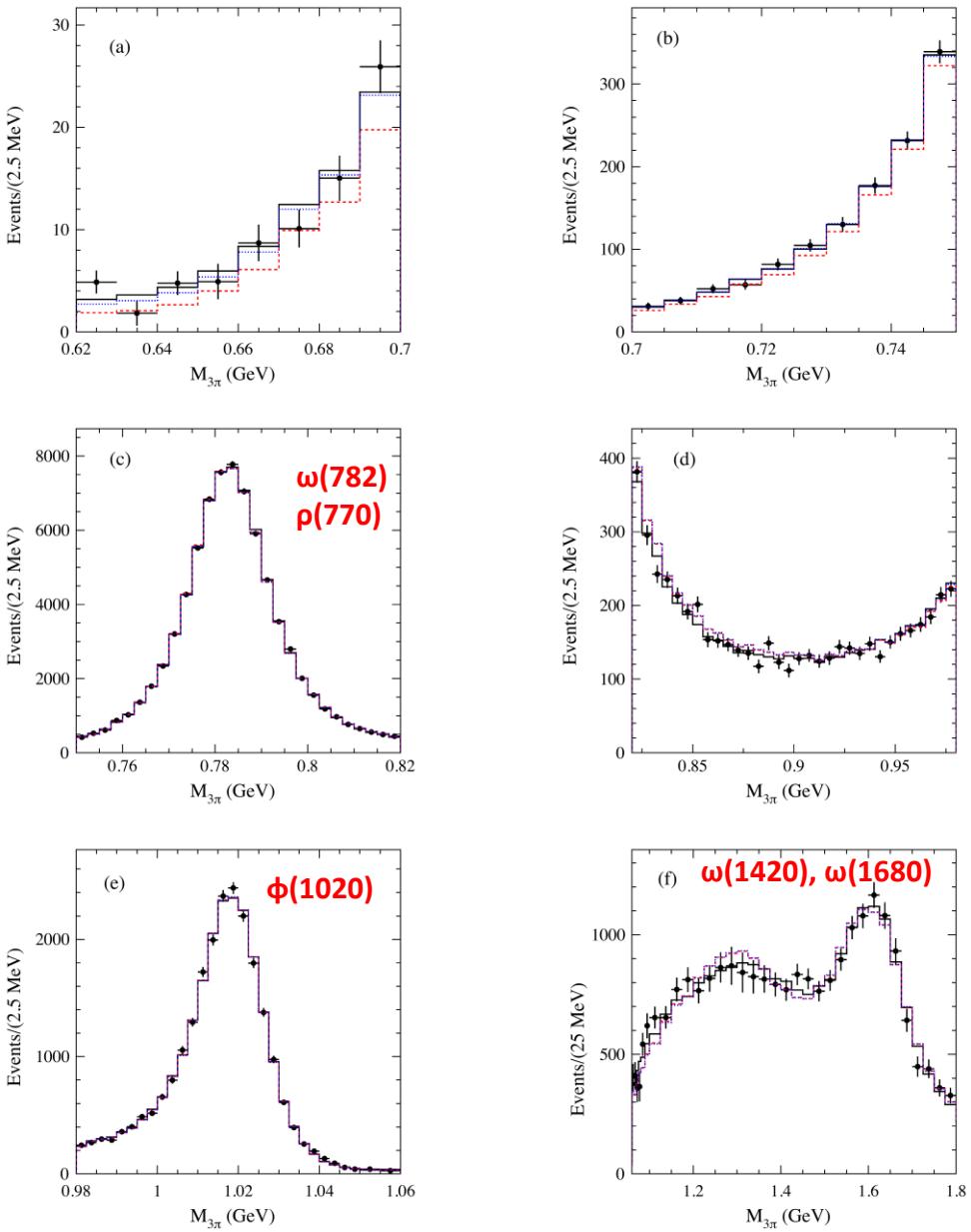
- Sharp structure of the mass spectrum below 1.1 GeV.
 - over 4 orders of magnitude.
- Unfolding the detector resolution effects is essential.



Phys. Rev. D 104, 112003 (2021)

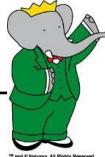
Measured mass spectrum

- Fit using Vector Dominance Model
 - including $\rho(770) + \omega(782) + \phi(1020)$
+ $\omega(1420) + \omega(1680)$
 - overall fit shown in different mass ranges
- Best fit (in black) including $\rho(770)$
- Alternate fit (in red) with no $\rho(770)$



Phys. Rev. D 104, 112003 (2021)

Fit results

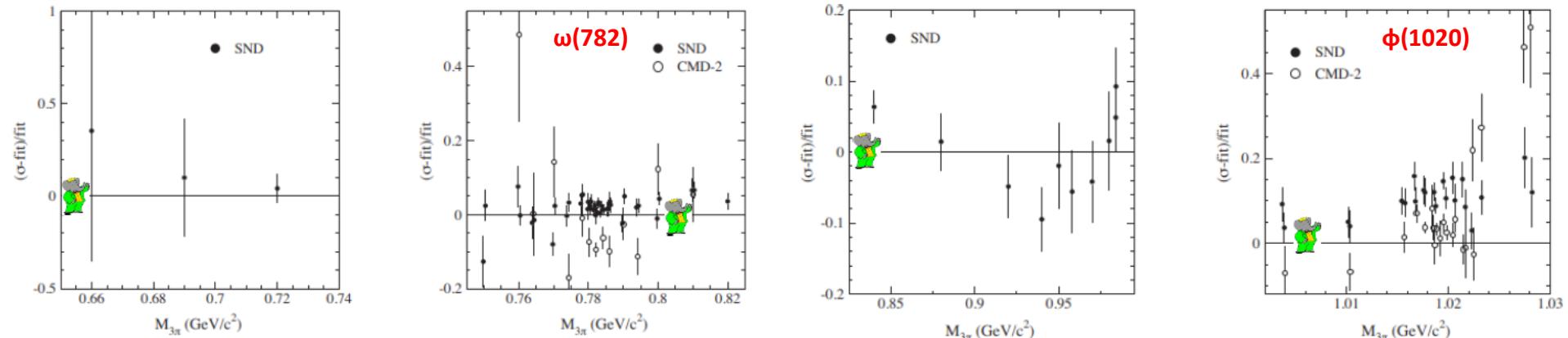


	BABAR	Previous measurement
$\Gamma(\omega \rightarrow e^+ e^-) \times B(\omega \rightarrow \pi^+ \pi^- \pi^0)$	$0.5698 \pm 0.0031 \pm 0.0082$ keV	0.557 ± 0.011 keV (PDG)
$\Gamma(\phi \rightarrow e^+ e^-) \times B(\phi \rightarrow \pi^+ \pi^- \pi^0)$	$0.1841 \pm 0.0021 \pm 0.0080$ keV	0.1925 ± 0.0043 keV (PDG)
$B(\rho \rightarrow \pi^+ \pi^- \pi^0)$	$(0.88 \pm 0.23 \pm 0.30) \times 10^{-4}$	$(1.01^{+0.54}_{-0.34} \pm 0.34) \times 10^{-4}$ (SND)
$(\Phi_\rho - \Phi_\omega)$	$-(99 \pm 9 \pm 15)^\circ$	$-(135^{+17}_{-13} \pm 9)^\circ$ (SND)

Phys. Rev. D 104, 11203 (2021)

- BABAR results in agreement with previous values.
- Rare decay $\rho \rightarrow \pi^+ \pi^- \pi^0$ observed with significance greater than 6σ .

Relative cross section differences of $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ below 1.1 GeV



Good Agreement

$$\begin{aligned}\Delta(\text{SND-BaBar}) &= 2\% \\ \Delta(\text{CMD-2-BaBar}) &= 7\%\end{aligned}$$

Good agreement

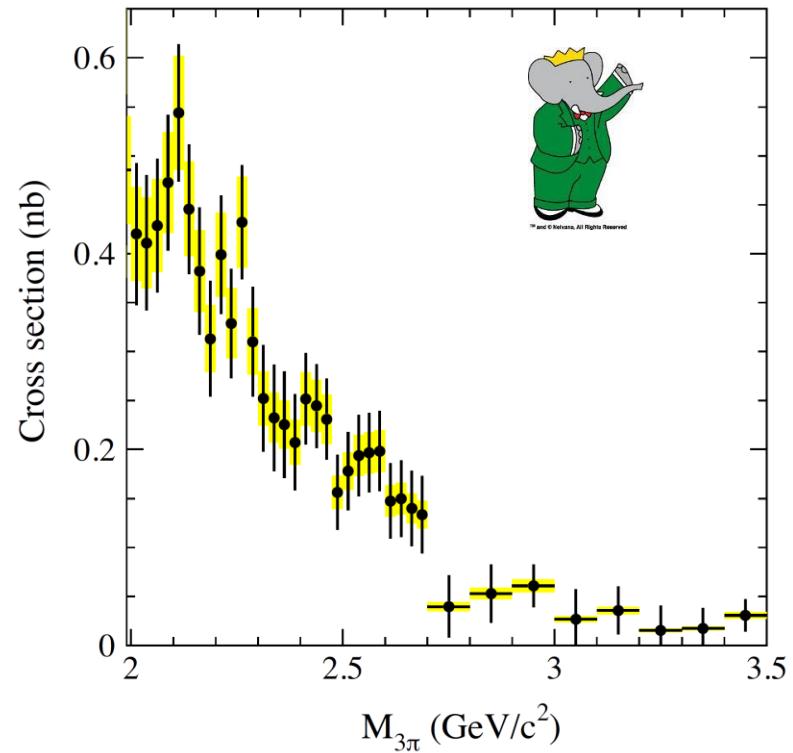
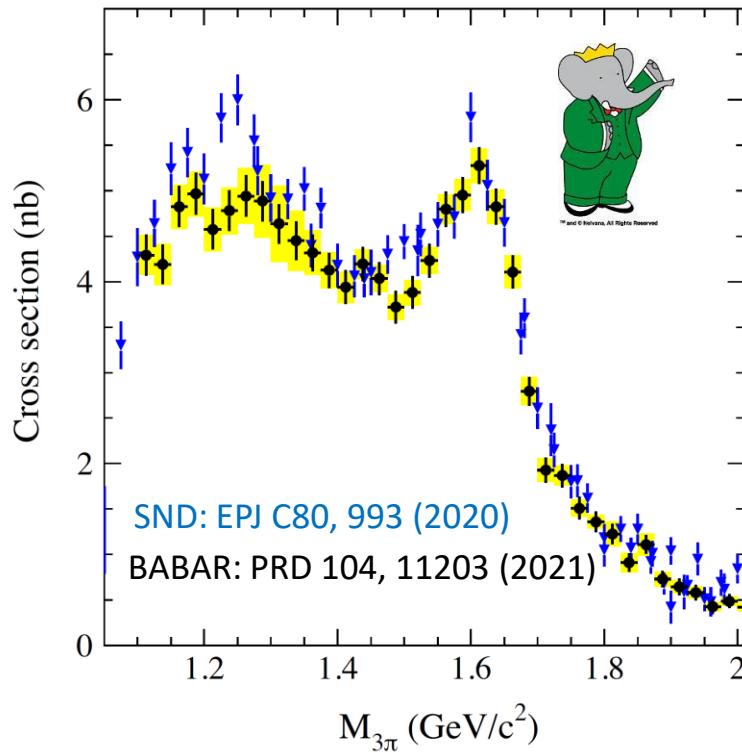
$$\begin{aligned}\Delta(\text{SND-BaBar}) &= 11\% \\ \Delta(\text{CMD-2-BaBar}) &= 3\%\end{aligned}$$

BABAR: Phys. Rev. D 104, 112003 (2021)

Systematic uncertainty $\sim 1.3\%$ on ω and ϕ
dominated by detection efficiency

SND: PRD 63, 72002 (2001), PRD 68, 52006 (2003)
CMD-2: PLB 578, 285 (2004), PLB 642, 203 (2006)

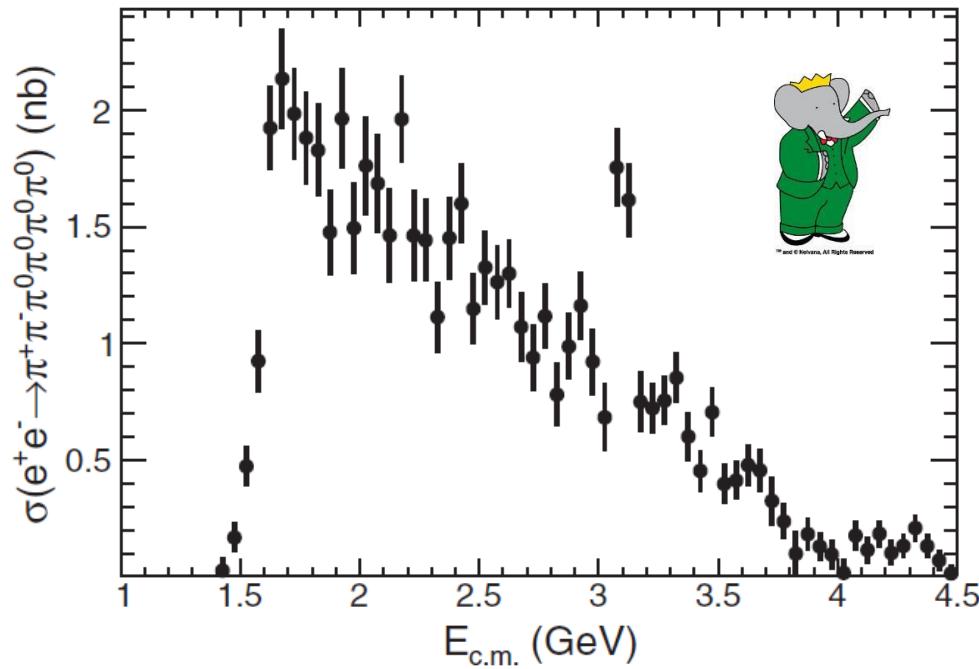
$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ cross section above 1.1 GeV



- Systematic uncertainty 4-15%, dominated by background.
- Good agreement between SND and BABAR with localized differences around 1.25 GeV and 1.5 GeV.

$e^+ e^- \rightarrow \pi^+ \pi^- 4\pi^0$ cross section

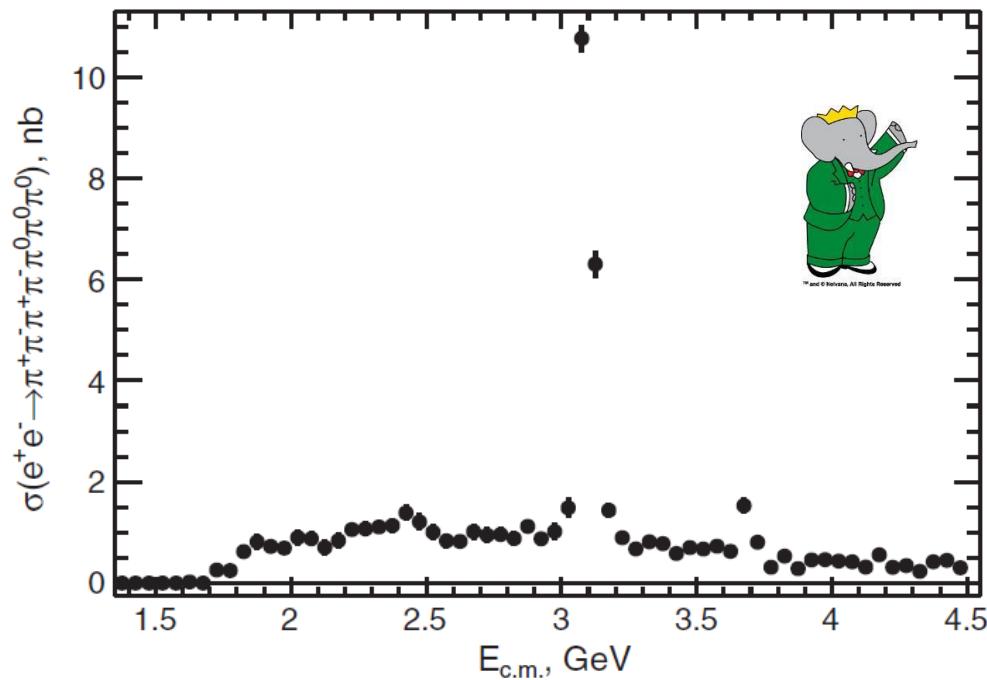
- Measured for the first time
- Similar technique to $\pi^+ \pi^- \pi^0$
- Same integrated luminosity (469 fb^{-1})
- Also measured
 - Intermediate states:
 - $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \eta (\rightarrow 3\pi^0)$
 - $e^+ e^- \rightarrow \omega (\rightarrow \pi^+ \pi^- \pi^0) 3\pi^0$
 - $e^+ e^- \rightarrow \omega (\rightarrow \pi^+ \pi^- \pi^0) \eta (\rightarrow 3\pi^0)$
 - Related state:
 - $e^+ e^- \rightarrow \pi^+ \pi^- 3\pi^0 \eta (\rightarrow \gamma\gamma)$



Phys. Rev. D 104, 112004 (2021)

$e^+ e^- \rightarrow 2(\pi^+ \pi^-) 3\pi^0$ cross section

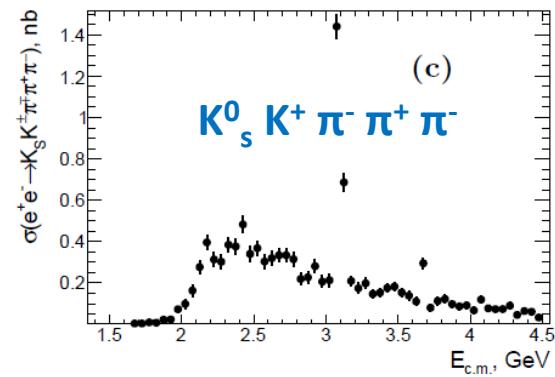
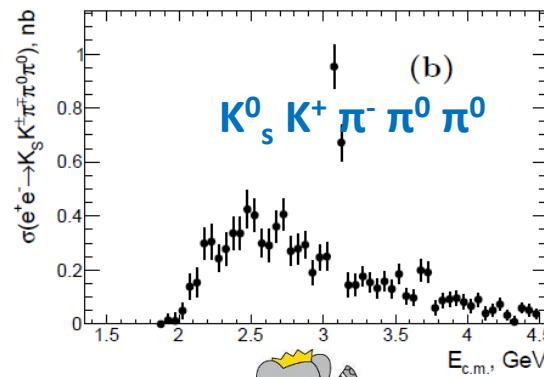
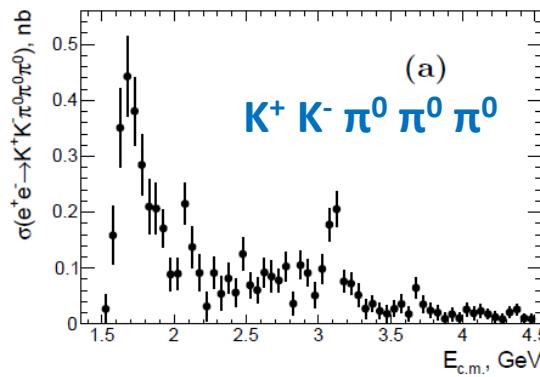
- Measured for the first time
- Similar technique to $\pi^+ \pi^- \pi^0$
- Same integrated luminosity (469 fb^{-1})
- Also measured
 - Intermediate states:
 - $e^+ e^- \rightarrow 2(\pi^+ \pi^-) \eta (\rightarrow 3\pi^0)$
 - $e^+ e^- \rightarrow \eta (\rightarrow \pi^+ \pi^- \pi^0) \pi^+ \pi^- 2\pi^0$
 - $e^+ e^- \rightarrow \omega (\rightarrow \pi^+ \pi^- \pi^0) \pi^+ \pi^- 2\pi^0$
 - Related state:
 - $e^+ e^- \rightarrow 2(\pi^+ \pi^-) 2\pi^0 \eta (\rightarrow \gamma\gamma)$



Phys. Rev. D 103, 092001 (2021)

$e^+ e^- \rightarrow K K \pi \pi \pi$ cross sections

- Measured for the first time
- Similar technique to $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$
- Same integrated luminosity (469 fb^{-1})
- Also measured: modes with intermediate resonances η, ρ, K^*

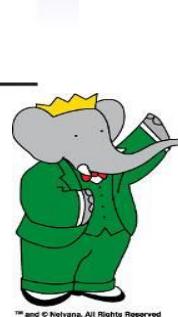


arXiv:220710340 (2022)

Impact on g-2

- $e^+ e^- \rightarrow \pi^+ \pi^- 3\pi^0, 2(\pi^+ \pi^-) 3\pi^0, K K \pi \pi \pi$
 - Measured for the first time
 - Contribute < 0.5% to a_μ^{had} to and <1.5% to its uncertainty
- $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$
 - Second largest contributor to uncertainty on a_μ^{had}
 - **Uncertainty on its contribution reduced by a factor 2**

$M_{3\pi}$ GeV/ c^2	$a_\mu^{3\pi} \times 10^{10}$
0.62–1.10	$42.91 \pm 0.14 \pm 0.55 \pm 0.09$
1.10–2.00	$2.95 \pm 0.03 \pm 0.16$
< 2.00	$45.86 \pm 0.14 \pm 0.58$
< 1.80 [A]	$46.21 \pm 0.40 \pm 1.40$
< 1.97 [B]	46.74 ± 0.94
< 2 [C]	44.32 ± 1.48



Summary

- Using the ISR technique BABAR does precision studies of low energy e^+e^- annihilations.
- Long history of cross section measurements.
- Most recent results
 - precision measurement in $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ mode,
 - first measurements in several high multiplicity modes.
- Contribute to improve ($g-2$) calculation.