

# Flavor Physics Review

Joachim Brod



Lake Louise Winter Institute  
February 24, 2023

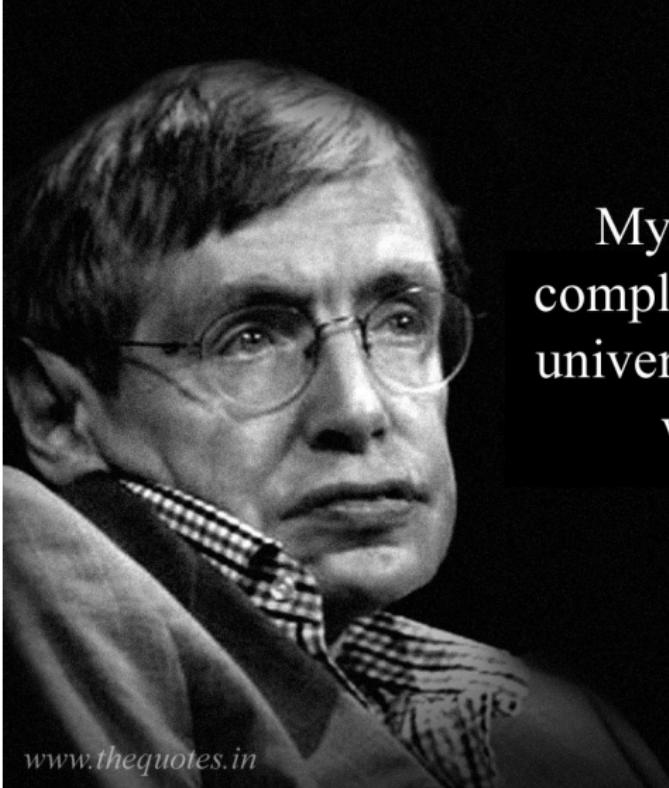
# Outline

- Theory:
  - QCD factorization
  - Flavor symmetry
  - heavy-quark expansion
  - QCD sum rules
  - ChPT
  - lattice results
  - Status of higher-order corrections
- B physics:
  - LFU ( $R_K$ ,  $R_{K^*}$ )
  - $B$  "anomalies"
  - $V_{ub}$ ,  $V_{cb}$  inclusive / exclusive tension
  - $R_D$ ,  $R_{D^*}$
  - Determination of  $\phi_3/\gamma$
  - $B \rightarrow hh$
- D physics:
  - mixing and indirect CPV
  - direct CPV,  $\Delta A_{CP}$
  - $D \rightarrow hh$
- K physics:
  - rare K decays ( $K \rightarrow \pi\nu\bar{\nu}$ )
  - other rare modes ( $K_L \rightarrow \mu\mu$ ,  $K_L \rightarrow \pi^0\ell\ell$ )
  - direct and indirect CPV ( $\epsilon_K$ ,  $\epsilon'$ )
  - First-row unitarity
- Hadrons:
  - sum rules for hadrons
  - direct CPV
  - hadron spectroscopy
  - exotic hadrons

# Outline

- Theory:
  - QCD factorization
  - Flavor symmetry
  - heavy-quark expansion
  - QCD sum rules
  - ChPT
  - lattice results
  - Status of higher-order corrections
- B physics:
  - LFU ( $R_K$ ,  $R_{K^*}$ )
  - $B$  “anomalies”
  - $V_{ub}$ ,  $V_{cb}$  inclusive / exclusive tension
  - $R_D$ ,  $R_{D^*}$
  - Determination of  $\phi_3/\gamma$
  - $B \rightarrow hh$
- D physics:
  - mixing and indirect CPV
  - direct CPV,  $\Delta A_{CP}$
  - $D \rightarrow hh$
- K physics:
  - rare K decays ( $K \rightarrow \pi\nu\bar{\nu}$ )
  - other rare modes ( $K_L \rightarrow \mu\mu$ ,  $K_L \rightarrow \pi^0\ell\ell$ )
  - direct and indirect CPV ( $\epsilon_K$ ,  $\epsilon'$ )
  - First-row unitarity
- Hadrons:
  - sum rules for hadrons
  - direct CPV
  - hadron spectroscopy
  - exotic hadrons

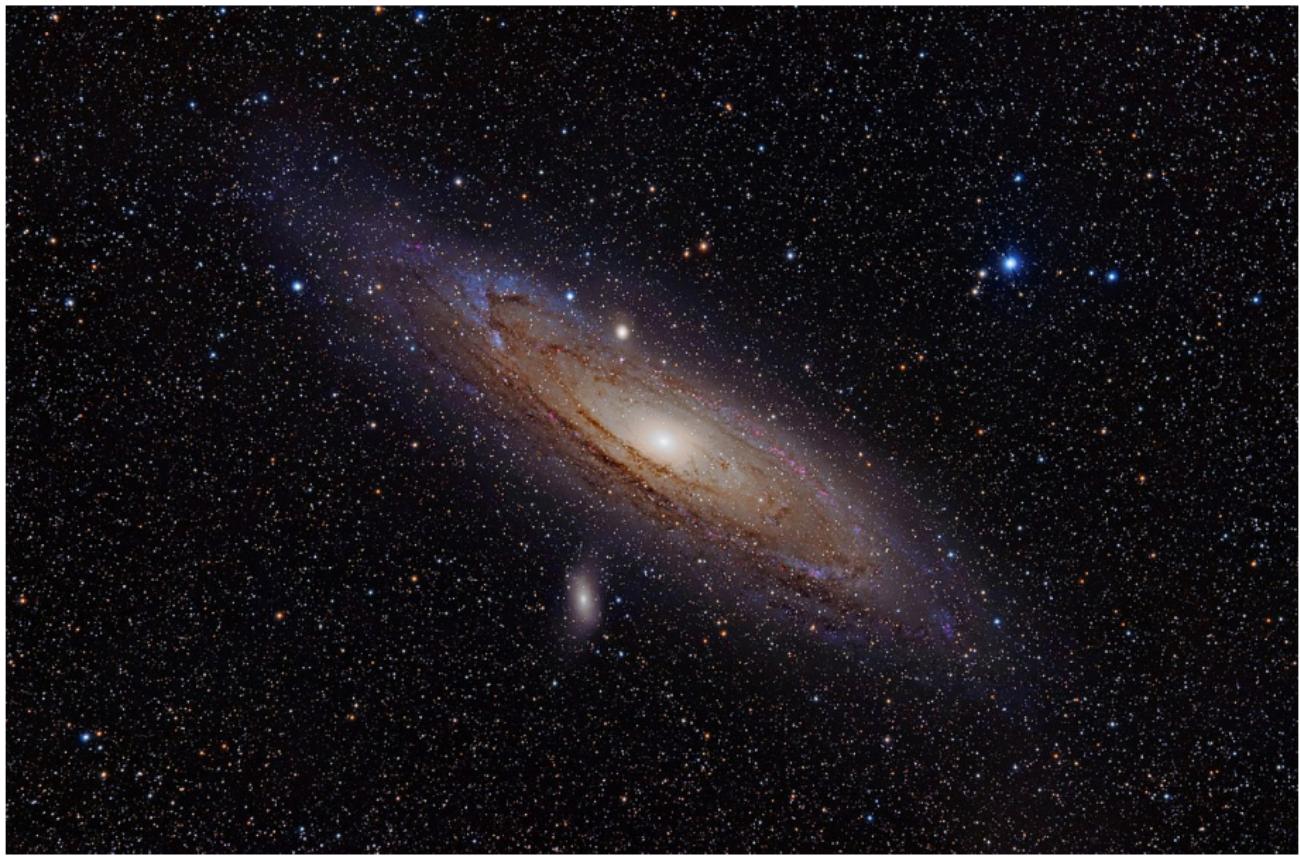
# Motivation

A black and white close-up photograph of Stephen Hawking. He is wearing round-rimmed glasses and a patterned shirt. His gaze is directed slightly to the right of the camera.

My goal is simple. It is a complete understanding of the universe, why it is as it is and why it exists at all.

*Stephen Hawking*

My Goal is even simpler:



**My Goal is even simpler:**



How come there's  
matter in the universe?

# Baryogenesis – where does matter come from?

- $\eta \equiv \frac{\rho_B - \rho_{\bar{B}}}{\rho_\gamma} \sim 10^{-9}$
- Three conditions to generate baryons from a symmetric initial state:  
[Sakharov 1967]
  - C / CP violation
  - Baryon number violation
  - departure from thermal equilibrium
- The SM fails due to large Higgs mass
- In addition,  $d_{CP} \sim J_{CP}/T_c^{12} \sim 10^{-19}$  [Bernreuther 2002]
  - Jarlskog invariant  $J_{CP} \sim \prod_{\text{up, down}} (m_{q_i}^2 - m_{q_j}^2) \times \text{Im}(V_{CKM}^4)$

# Wasn't this supposed to be a Flavor Review?

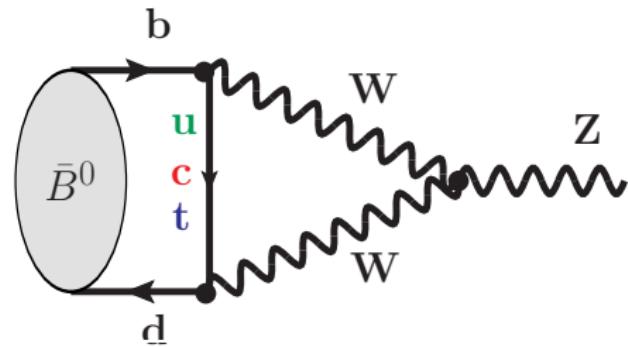
- Frequently, baryogenesis used as motivation for flavor physics
- Only models I'm aware of that use SM CP violation are Mesogenesis + variations
  - [Elor et al. 2018, Nelson et al. 2019]
    - Departure from thermal equilibrium: Late decay of scalar field to SM mesons
    - C / CP violation: SM
    - Baryon number violation: SM meson decays to dark sector
- Already strongly constrained by Belle / BaBar data
  - [2110.14086, 2302.00208 → Talk by S. Robertson]
- So, does flavor tell us anything about baryogenesis?

# What is Flavor Physics?

# What is flavor physics?

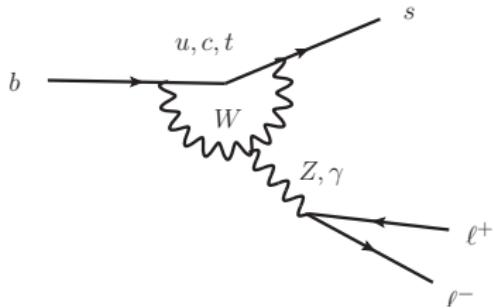
- Nothing distinguishes the three fermion generations but their masses
- Flavor is the **physics of the Yukawa couplings**
  - Yukawa: general complex  $3 \times 3$  matrices
- SM:
  - Can simultaneously diagonalize all fermion mass terms and Yukawa interactions
  - *if we introduce flavor violation in the charged current (CKM matrix)*
  - No FCNCs ("Flavor-Changing Neutral Currents") at tree level
- BSM:
  - Can have tree-level FCNCs: "standard" flavor tests of the SM
  - CP violation also in flavor-conserving processes (electric dipole moments!)
  - Effective theory (EFT): all independent
  - Concrete models: correlations (e.g. CPV entails FV...)
- So I think the motivation is still valid

# Flavor-Changing Neutral Currents

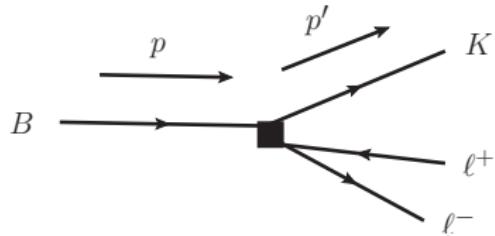


# $R_K, R_{K^*}$

$$R_{K^{(*)}} \equiv \frac{\text{BR}(B \rightarrow K^{(*)}\mu^+\mu^-)}{\text{BR}(B \rightarrow K^{(*)}e^+e^-)}$$



- In general,  $d\Gamma(B \rightarrow K l \bar{l}) \propto |\langle K l \bar{l} | H_{\text{eff}} | B \rangle|^2$  ... complicated function involving nonperturbative QCD
- For small  $q^2 = (p - p')^2$ , form factors simplify – “color transparency”



# SM Expectation

$$R_K^{\text{SM}} \equiv \frac{\Gamma_\mu^{\text{SM}}}{\Gamma_e^{\text{SM}}}$$

- SM prediction is very clean: [Hiller et al. hep-ph/0310219, Bobeth et al. 0709.4174]
- Numerically, including QED effects [Bordone et al. 1605.07633] we have

$$R_K^{\text{SM}} = 1 + \% + \text{cut-dep. QED of } \mathcal{O}(\text{few \%}) .$$

# New LHCb results for $R_K$ , $R_{K^*}$

- Most systematics cancels in double ratio

$$R_{K,K^*}^{\text{LHCb}} \equiv \frac{\text{BR}(B^{+,0} \rightarrow K^{+,*0} \mu\mu)}{\text{BR}(B^{+,0} \rightarrow K^{+,*0} ee)} \Big/ \frac{\text{BR}(B^{+,0} \rightarrow K^{+,*0} J/\psi(\mu\mu))}{\text{BR}(B^{+,0} \rightarrow K^{+,*0} J/\psi(ee))}$$

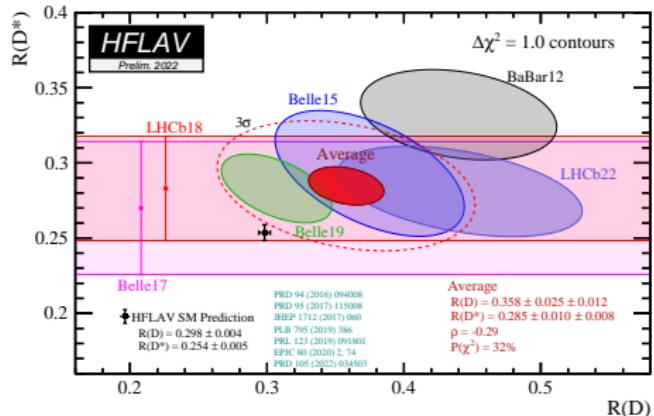
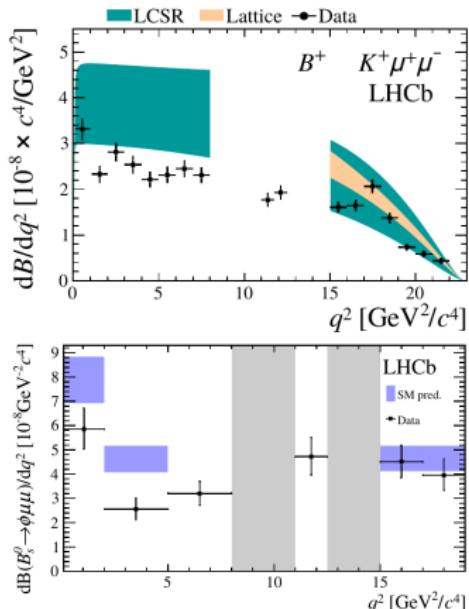
- New LHCb [2212.09152 → Talk by F. Volle]:

$$R_K^{\text{LHCb } 2022} = 0.949^{+0.042}_{-0.041} \text{ (stat.)} \quad {}^{+0.022}_{-0.022} \text{ (syst.)}$$

$$R_{K^*}^{\text{LHCb } 2022} = 1.027^{+0.072}_{-0.068} \text{ (stat.)} \quad {}^{+0.027}_{-0.026} \text{ (syst.)}$$

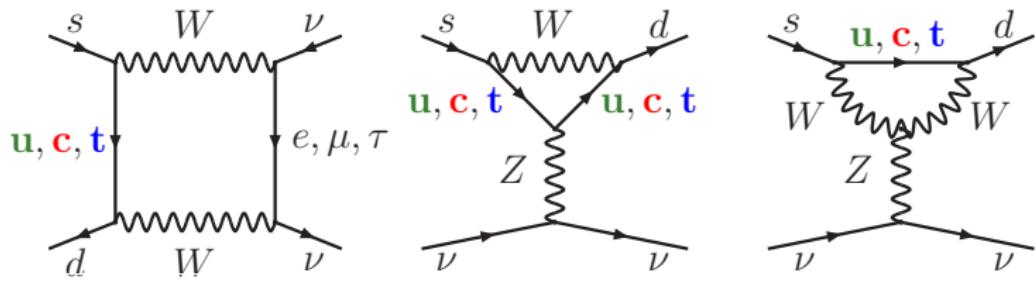
# Other $3\sigma$ “anomalies” in $b \rightarrow s\mu^+\mu^-$

- $B^+ \rightarrow K^+\mu^+\mu^-$ ,  $B_s \rightarrow \phi\mu^+\mu^-$  differential branching ratios [1403.8044, 1506.08777]
- Angular observables in  $B^0 \rightarrow K^{0*}\mu^+\mu^-$  [LHCb 2003.04831]
- Theory much more challenging – charm loops

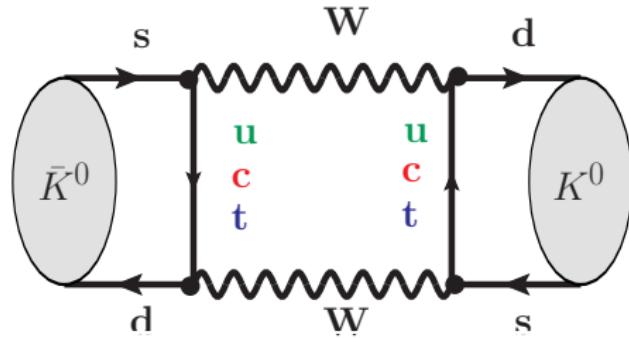


# Kaons

$$K \rightarrow \pi \nu \bar{\nu}$$



## Reminder: neutral kaons



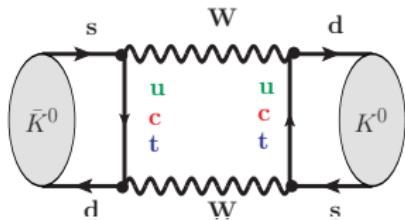
# CP Violation in the Neutral Kaon System

- CP transformation ( $|K^0\rangle \sim |\bar{s}d\rangle$ ,  $|\overline{K^0}\rangle \sim |\bar{d}s\rangle$ )
  - $CP|K^0\rangle = -|\overline{K^0}\rangle$
  - $CP|\overline{K^0}\rangle = -|K^0\rangle$
- CP eigenstates
  - $|K_1\rangle \equiv (|K^0\rangle - |\overline{K^0}\rangle)/\sqrt{2}$  (CP even)
  - $|K_2\rangle \equiv (|K^0\rangle + |\overline{K^0}\rangle)/\sqrt{2}$  (CP odd)
- $K_2^0 \rightarrow \pi\pi$  is forbidden by CP

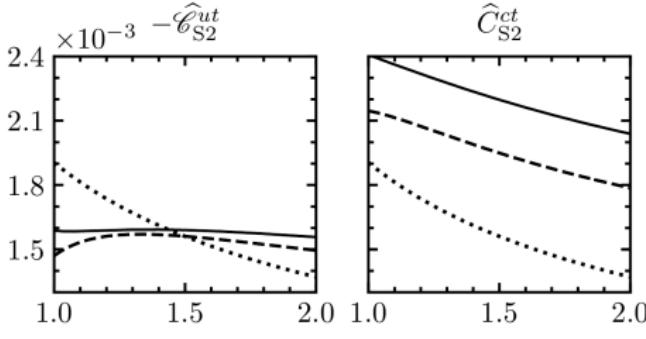
# Progress in $\epsilon_K$

# Definition of $\epsilon_K$

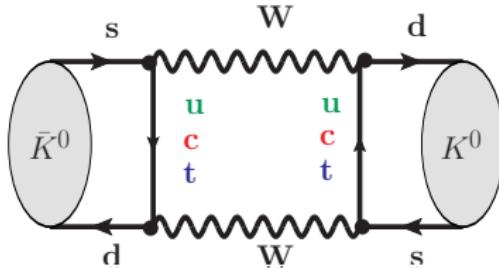
$$\epsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} \stackrel{\text{exp}}{=} 2.228(11) \times 10^{-3}$$



- Calculation split into two parts:
  - “Short-distance” (perturbative)
  - “Long-distance” (lattice, ChPT)
- “Simple” rearrangement of effective Hamiltonian has reduced perturbative uncertainty from order 30% to order 1% [Brod et al. 1911.06822]



# c-t vs. u-t Unitarity



$$\lambda_u = V_{us} V_{ud}^*, \quad \lambda_c = V_{cs} V_{cd}^*, \quad \lambda_t = V_{ts} V_{td}^*, \quad \lambda_u + \lambda_c + \lambda_t = 0$$

c-t unitarity

	Im	Re
$\lambda_t^2$	$\sim \lambda^{10}$	$\sim \lambda^{10}$
$\lambda_c \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$
$\lambda_c^2$	$\sim \lambda^6$	$\sim \lambda^2$

u-t unitarity

	Im	Re
$\lambda_t^2$	$\sim \lambda^{10}$	$\sim \lambda^{10}$
$\lambda_u \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$
$\lambda_u^2$	0	$\sim \lambda^2$

$$\text{Im}(M_{12}) \rightarrow \epsilon_K$$

$$\text{Re}(M_{12}) \rightarrow \Delta M_K$$

$$K \rightarrow \mu^+ \mu^-$$

# A new golden mode?

- Assume that  $K_L$  and  $K_S$  are CP eigenstates:

	$(\mu^+ \mu^-)_{\ell=0}$	$(\mu^+ \mu^-)_{\ell=1}$
$K_L$	CP conserving	(CP violating)
$K_S$	CP violating	CP conserving

- However, experiment cannot distinguish  $\ell = 0$  and  $\ell = 1$

- Notation: write

- $A_{\ell}^L = A[K_L \rightarrow (\mu^+ \mu^-)_{\ell}]$
- $A_{\ell}^S = A[K_S \rightarrow (\mu^+ \mu^-)_{\ell}]$

# Time-dependent decay rate of neutral kaons

$$\frac{d\Gamma(K(t) \rightarrow \mu^+ \mu^-)}{dt} \propto C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2[C_{\sin} \sin(\Delta M t) + C_{\cos} \cos(\Delta M t)] e^{-\Gamma t}$$

- $C_i$  given in terms of amplitudes, e.g.
  - $C_L = |A_{\ell=0}^L|^2 + |A_{\ell=1}^L|^2$
  - $C_S = |A_{\ell=0}^S|^2 + |A_{\ell=1}^S|^2$
- Interference terms  $C_{\text{int}}^2 = C_{\cos}^2 + C_{\sin}^2$  sensitive to short-distance component!  
[D'Ambrosio, Kitahara 1707.06999]

# A useful relation

- Single assumption:  $A_{\ell=1}^L = 0$
- It follows immediately (“in two lines!”)

$$\text{Br}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} = \text{Br}(K_L \rightarrow \mu^+ \mu^-) \times \frac{\tau_S}{\tau_L} \times \left( \frac{C_{\text{int}}}{C_L} \right)^2$$

[A. Dery et al. 2104.06427]

# You don't believe it?

$$\frac{\text{Br}(K_S \rightarrow (\mu\mu)_{\ell=0})}{\text{Br}(K_L \rightarrow \mu\mu)}$$

# You don't believe it?

$$\frac{\text{Br}(K_S \rightarrow (\mu\mu)_{\ell=0})}{\text{Br}(K_L \rightarrow \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S}$$

# You don't believe it?

$$\frac{\text{Br}(K_S \rightarrow (\mu\mu)_{\ell=0})}{\text{Br}(K_L \rightarrow \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2 |A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

# You don't believe it?

$$\frac{\text{Br}(K_S \rightarrow (\mu\mu)_{\ell=0})}{\text{Br}(K_L \rightarrow \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2 |A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

$$C_{\text{int}}^2 = |(A_0^S)^* A_0^L|^2 = |A_0^S|^2 |A_0^L|^2 ,$$

# You don't believe it?

$$\frac{\text{Br}(K_S \rightarrow (\mu\mu)_{\ell=0})}{\text{Br}(K_L \rightarrow \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2 |A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

$$C_{\text{int}}^2 = |(A_0^S)^* A_0^L|^2 = |A_0^S|^2 |A_0^L|^2, \quad C_L^2 = |A_0^L|^4$$

# You don't believe it?

$$\frac{\text{Br}(K_S \rightarrow (\mu\mu)_{\ell=0})}{\text{Br}(K_L \rightarrow \mu\mu)} = \frac{|A_0^S|^2}{|A_0^L|^2} \times \frac{\tau_L}{\tau_S} = \frac{|A_0^S|^2 |A_0^L|^2}{|A_0^L|^4} \times \frac{\tau_L}{\tau_S}$$

$$C_{\text{int}}^2 = |(A_0^S)^* A_0^L|^2 = |A_0^S|^2 |A_0^L|^2, \quad C_L^2 = |A_0^L|^4$$

$$\Rightarrow \text{Br}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} = \text{Br}(K_L \rightarrow \mu^+ \mu^-) \times \frac{\tau_S}{\tau_L} \times \left( \frac{C_{\text{int}}}{C_L} \right)^2$$

# A useful relation

- Single assumption:  $A_{\ell=1}^L = 0$
- It follows immediately (“in two lines!”)

$$\text{Br}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} = \text{Br}(K_L \rightarrow \mu^+ \mu^-) \times \frac{\tau_S}{\tau_L} \times \left( \frac{C_{\text{int}}}{C_L} \right)^2$$

[A. Dery et al. 2104.06427]

- Recall  $K_S \rightarrow (\mu^+ \mu^-)_{\ell=0}$  is CPV  $\Rightarrow$  sensitive to UV physics!
- Corrections to  $A_{\ell=1}^L = 0$  expected to be order  $\epsilon_K \sim 10^{-3}$

# Compare to SM

- Experiment will provide:

$$\text{Br}^{\text{exp}}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} = \text{Br}(K_L \rightarrow \mu^+ \mu^-) \times \frac{\tau_S}{\tau_L} \times \left( \frac{C_{\text{int}}}{C_L} \right)^2$$

- We want to compare to the SM prediction!
- Three parts:
  - Hadronic matrix element is just  $f_K$  – kaon decay constant
  - SD contribution – can calculate
  - Effect of indirect CP violation – estimate from data!

# SM short-distance contribution

- CPV – imaginary part of weak Hamiltonian
  - $\Rightarrow$  Only top-quark contribution relevant
- Including three-loop QCD and two-loop electroweak: [Bobeth et al., 1311.0903]

$$\text{Br}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}^{\text{pert.}} = 1.70(02)_{\text{QCD/EW}}(01)_{f_K}(19)_{\text{param.}} \times 10^{-13}$$

# Impact of indirect CP violation

- $K_S = K_1 + \epsilon_K K_2$  (mainly CP even)
- $K_L = K_2 + \epsilon_K K_1$  (mainly CP odd)
- Can then show:  $A_0^S = A_0^S|_{\epsilon_K=0} + \epsilon_K A_0^L$
- Squaring to get decay rate [Brod, Stamou 2209.07445]

$$\begin{aligned}\text{Br}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} &= \text{Br}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}^{\text{pert.}} \\ &\times \left( 1 + \sqrt{2} |\epsilon_K| \frac{|A_0^L|}{|A_0^S|} (\cos \phi_0 - \sin \phi_0) \right),\end{aligned}$$

- From branching ratios, we know  $|A_0^L| \gg |A_0^S|$
- Obtain  $\phi_0$  from  $K_L \rightarrow \mu^+ \mu^-$  and  $K_L \rightarrow \gamma\gamma$ , up to four-(two-)fold ambiguity  
[A. Dery et al., 2211.03804]
- Additional  $\pm 2\%$  or  $\pm 3\%$  correction to  $\text{Br}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}$

# Flavor Symmetry

## Example: Neutral $D$ -meson decays

$$D^0(\bar{D}^0) \rightarrow K^+K^- , \quad \pi^+K^- , \quad K^+\pi^- , \quad \pi^+\pi^-$$

## Example: Neutral $D$ -meson decays

$$D^0(\bar{D}^0) \rightarrow K^+K^- , \quad \pi^+K^- , \quad K^+\pi^- , \quad \pi^+\pi^-$$

U-spin symmetry:  $s \leftrightarrow d$

$$\pi \leftrightarrow K$$

Expected corrections  $f_K/f_\pi - 1 \sim 20\%$

## A few branching ratios

$$\text{BR}(D^0 \rightarrow \pi^+ K^-) = 3.89(4) \times 10^{-2}$$

$$\text{BR}(D^0 \rightarrow K^+ K^-) = 3.97(7) \times 10^{-3}$$

$$\text{BR}(D^0 \rightarrow \pi^+ \pi^-) = 1.407(25) \times 10^{-3}$$

$$\text{BR}(D^0 \rightarrow K^+ \pi^-) = 1.48(7) \times 10^{-4}$$

# Normalized amplitudes

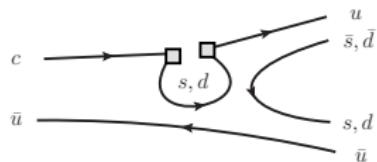
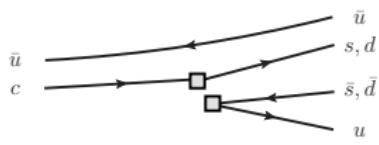
$$|A(D^0 \rightarrow \pi^+ K^-)| = 2.09 \text{ GeV}$$

$$|A(D^0 \rightarrow K^+ K^-)| = 3.22 \text{ GeV}$$

$$|A(D^0 \rightarrow \pi^+ \pi^-)| = 1.73 \text{ GeV}$$

$$|A(D^0 \rightarrow K^+ \pi^-)| = 2.56 \text{ GeV}$$

# Most general decomposition of amplitudes



$$A(K^+ \pi^-) = A_{K^+ \pi^-}^T$$

$$A(\pi^+ \pi^-) = A_{\pi^+ \pi^-}^T (1 + r_f e^{i(\delta_{\pi^+ \pi^-} - \phi_{\pi^+ \pi^-})})$$

$$A(K^+ K^-) = A_{K^+ K^-}^T (1 + r_f e^{i(\delta_{K^+ K^-} - \phi_{K^+ K^-})})$$

$$A(\pi^+ K^-) = A_{\pi^+ K^-}^T$$

- Total of 4+2 independent amplitudes

# Three amplitude relations

$$\frac{|A(D^0 \rightarrow K^+ K^-)|}{|A(D^0 \rightarrow \pi^+ \pi^-)|} - 1 = (82 \pm 2)\%$$

$$\frac{|A(D^0 \rightarrow K^- \pi^+)|}{|A(D^0 \rightarrow K^+ \pi^-)|} - 1 = (15 \pm 3)\%$$

$$\frac{|A(D^0 \rightarrow K^+ K^-)| + |A(D^0 \rightarrow \pi^+ \pi^-)|}{|A(D^0 \rightarrow K^+ \pi^-)| + |A(D^0 \rightarrow K^- \pi^+)|} - 1 = (4.0 \pm 1.6)\%$$

# *U*-spin decomposition for *D* decays

- Initial state:  $D^0 \sim [c\bar{u}]$  is *U*-spin singlet
- Final states: singlet  $(\langle K^+K^-| + \langle \pi^+\pi^-|)/\sqrt{2} \sim [\bar{s}u][\bar{u}s] + [\bar{d}u][\bar{u}d]$

$$\text{triplet} \begin{pmatrix} \frac{1}{\sqrt{2}}(\langle K^+K^-| - \langle \pi^+\pi^-|) & \sim & [\bar{s}u][\bar{u}d] \\ \langle K^-\pi^+| & \sim & [\bar{s}u][\bar{u}s] - [\bar{d}u][\bar{u}d] \\ & & [\bar{u}s][\bar{d}u] \end{pmatrix}$$

- Hamiltonian: singlet  $(Q_i^{\bar{s}s} + Q_i^{\bar{d}d})/\sqrt{2} \sim (\bar{c}s)(\bar{s}u) + (\bar{c}d)(\bar{d}u)$

$$\text{triplet} \begin{pmatrix} Q_i^{\bar{d}s} & \sim & (\bar{c}s)(\bar{d}u) \\ \frac{1}{\sqrt{2}}(Q_i^{\bar{s}s} - Q_i^{\bar{d}d}) & \sim & (\bar{c}s)(\bar{s}u) - (\bar{c}d)(\bar{d}u) \\ Q_i^{\bar{s}d} & \sim & (\bar{c}d)(\bar{s}u) \end{pmatrix}$$

# Flavor symmetry breaking

- Two independent singlet amplitudes:

$$t_0 \propto \langle f_1 | H_1 | \bar{D} \rangle, \quad p_0 \propto \langle f_0 | H_0 | \bar{D} \rangle.$$

- Flavor symmetry is not exact!  $\epsilon \sim f_K/f_\pi - 1 \sim 20\%$
- Include U-spin breaking due to QED and quark-mass effects
  - We find three amplitudes linear in  $\epsilon$
  - We find three amplitudes quadratic in  $\epsilon$
  - Plus more at higher order in U-spin breaking

# Fitting the pieces together

- $\epsilon \sim (f_K/f_\pi - 1) \sim 0.2$ ,  $\xi_{\text{CKM}} = |V_{cb}^* V_{ub} / V_{cs} V_{us}^*| \sim 6 \times 10^{-4}$

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = t_0 - \epsilon t_1 + \epsilon^2 t'_2 + \dots$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = t_0 + \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}} (p_0 - \epsilon p_1 + \epsilon^2 p_2) + \dots$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = t_0 - \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}} (p_0 + \epsilon p_1 + \epsilon^2 p_2) + \dots$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = t_0 + \epsilon t_1 + \epsilon^2 t'_2 + \dots$$

- Only six amplitudes are independent, two can be absorbed by redefinition
- Also all higher terms  $\propto \epsilon^3, \epsilon^4, \dots$  can be reabsorbed

# Fitting the pieces together

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = t_0 - \epsilon t_1$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = t_0 + \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 - \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = t_0 - \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 + \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = t_0 + \epsilon t_1$$

# U-spin limit

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = \textcolor{orange}{t_0} - \epsilon t_1$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = \textcolor{orange}{t_0} + \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 - \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = \textcolor{orange}{t_0} - \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 + \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = \textcolor{orange}{t_0} + \epsilon t_1$$

# U-spin limit

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = t_0$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = t_0$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = t_0$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = t_0$$

- All amplitudes are equal in the U-spin limit
- Clearly not a good approximation to data

# CF vs. DCS

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = t_0 - \epsilon t_1$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = t_0 + \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 - \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = t_0 - \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 + \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = t_0 + \epsilon t_1$$

- Keep terms linear in  $\epsilon$

# CF vs. DCS

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = t_0 - \epsilon t_1$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = t_0 + \epsilon t_1$$

- Explains rate difference between CF and DCS decay modes with nominal U-spin breaking  $\epsilon \sim 20\%$

$$\frac{|A(D^0 \rightarrow K^- \pi^+)|}{|A(D^0 \rightarrow K^+ \pi^-)|} - 1 = (15 \pm 3)\%$$

# Large penguins

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = t_0 - \epsilon t_1$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = \textcolor{orange}{t_0 + \epsilon s_1} + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 - \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = \textcolor{orange}{t_0 - \epsilon s_1} + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 + \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = t_0 + \epsilon t_1$$

- Keep terms linear in  $\epsilon$ ; neglect terms  $\propto \xi_{\text{CKM}} \sim 10^{-4}$

# Large penguins

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = t_0 - \epsilon t_1$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = t_0 + \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 - \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = t_0 - \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 + \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = t_0 + \epsilon t_1$$

$$\frac{|A(D^0 \rightarrow K^+ K^-)|}{|A(D^0 \rightarrow \pi^+ \pi^-)|} - 1 = (82 \pm 2)\%$$

- Data tells us that  $s_1 \epsilon \gg t_1 \epsilon$ 
  - Unexplained hierarchy in U-spin breaking??
  - Large penguins:  $s_1 \gg t_1$ !
- Explains rate difference between SCS decay modes with nominal U-spin breaking  $\epsilon \sim 20\%$

# Quadratic U-spin sum rule

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = t_0 - \epsilon t_1$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = t_0 + \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 - \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = t_0 - \epsilon s_1 + \epsilon^2 t_2 - \xi_{\text{CKM}}(p_0 + \epsilon p_1)$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = t_0 + \epsilon t_1$$

- Keep terms linear in  $\epsilon$ ; neglect terms  $\propto \xi_{\text{CKM}} \sim 10^{-4}$

# Quadratic U-spin sum rule

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = t_0 - \epsilon t_1$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = t_0 + \epsilon s_1$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = t_0 - \epsilon s_1$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = t_0 + \epsilon t_1$$

- The following relation (“sum rule”) is valid to quadratic order in U-spin breaking:
- $A(\bar{D}^0 \rightarrow K^+ \pi^-) + A(\bar{D}^0 \rightarrow \pi^+ K^-) = A(\bar{D}^0 \rightarrow \pi^+ \pi^-) + A(\bar{D}^0 \rightarrow K^+ K^-)$
- This is borne out by the experimental relation

$$\frac{|A(D^0 \rightarrow K^+ K^-)| + |A(D^0 \rightarrow \pi^+ \pi^-)|}{|A(D^0 \rightarrow K^+ \pi^-)| + |A(D^0 \rightarrow K^- \pi^+)|} - 1 = (4.0 \pm 1.6)\%$$

# Three types of $CP$ violation

I  $|\bar{A}_f/A_f| \neq 1$  ( $CP$  violation in decay)

$$a_f^d := \frac{\Gamma(D \rightarrow f) - \Gamma(\bar{D} \rightarrow \bar{f})}{\Gamma(D \rightarrow f) + \Gamma(\bar{D} \rightarrow \bar{f})}$$

II  $|q/p| \neq 1$  ( $CP$  violation in mixing)

$$a_{sl} := \frac{\Gamma(\bar{D}^0(t) \rightarrow \ell^+ X) - \Gamma(D^0(t) \rightarrow \ell^- X)}{\Gamma(\bar{D}^0(t) \rightarrow \ell^+ X) + \Gamma(D^0(t) \rightarrow \ell^- X)}$$

III  $\text{Im}(\lambda_f) \equiv \text{Im}\left(\frac{q}{p} \frac{\bar{A}_f}{A_f}\right) \neq 0$  (interference-type  $CP$  violation)

$$a_{f_{CP}} := \frac{\Gamma(\bar{D}^0(t) \rightarrow f_{CP}) - \Gamma(D^0(t) \rightarrow f_{CP})}{\Gamma(\bar{D}^0(t) \rightarrow f_{CP}) + \Gamma(D^0(t) \rightarrow f_{CP})}$$

## Example: $\Delta A_{CP}$

- Experiments measure

$$\Delta A_{CP} \equiv A_{CP}(K^+ K^-) - A_{CP}(\pi^+ \pi^-)$$

- Theory “predicts”

$$\Delta A_{CP} = 2\text{Im}(\xi_{\text{CKM}}) \left| \frac{p_0}{t_0} \right| \sin \delta_{\text{strong}}$$

- $\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}$  [LHCb combi 1903.08726]  
gives  $|p_0/t_0| = -0.65$  [Grossman, Schacht 1903.10952]
- Consistent picture assuming nominal  $U$ -spin breaking  $\epsilon_U \approx 20\%$   
[Brod et al. 1203.6659]

# A consistent picture emerges...

$$\Delta A_{CP} \approx -0.15\%$$

$$\frac{|A(D^0 \rightarrow K^+ K^-)|}{|A(D^0 \rightarrow \pi^+ \pi^-)|} - 1 = (82 \pm 2)\%$$

$$\frac{|A(D^0 \rightarrow K^- \pi^+)|}{|A(D^0 \rightarrow K^+ \pi^-)|} - 1 = (15 \pm 3)\%$$

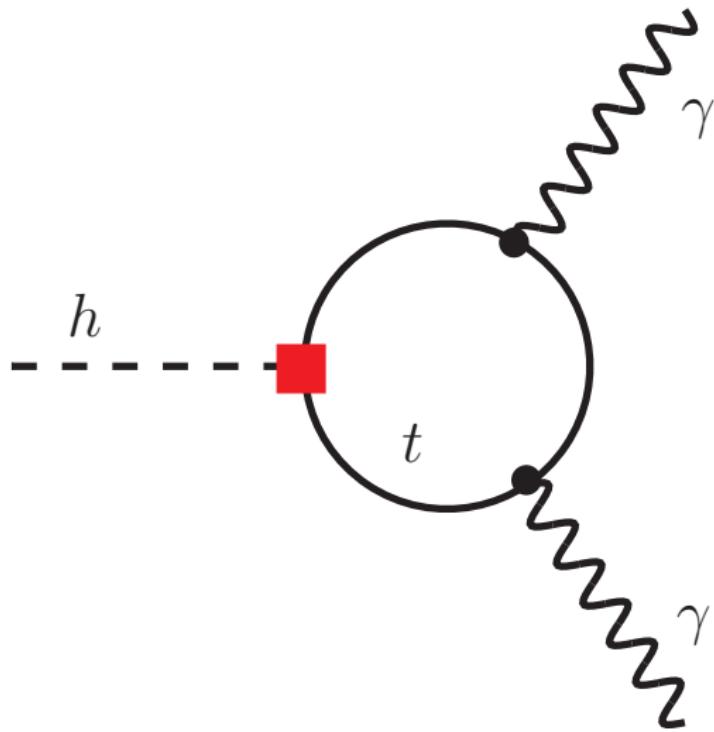
$$\frac{|A(D^0 \rightarrow K^+ K^-)| + |A(D^0 \rightarrow \pi^+ \pi^-)|}{|A(D^0 \rightarrow K^+ \pi^-)| + |A(D^0 \rightarrow K^- \pi^+)|} - 1 = (4.0 \pm 1.6)\%$$

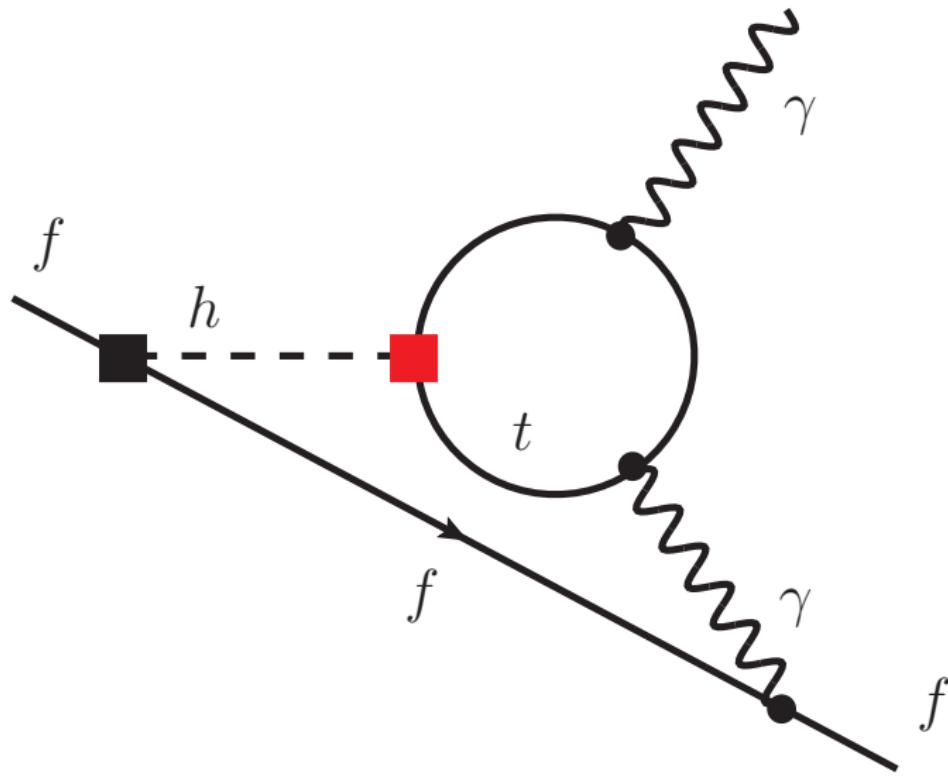
**...but is it the SM?**

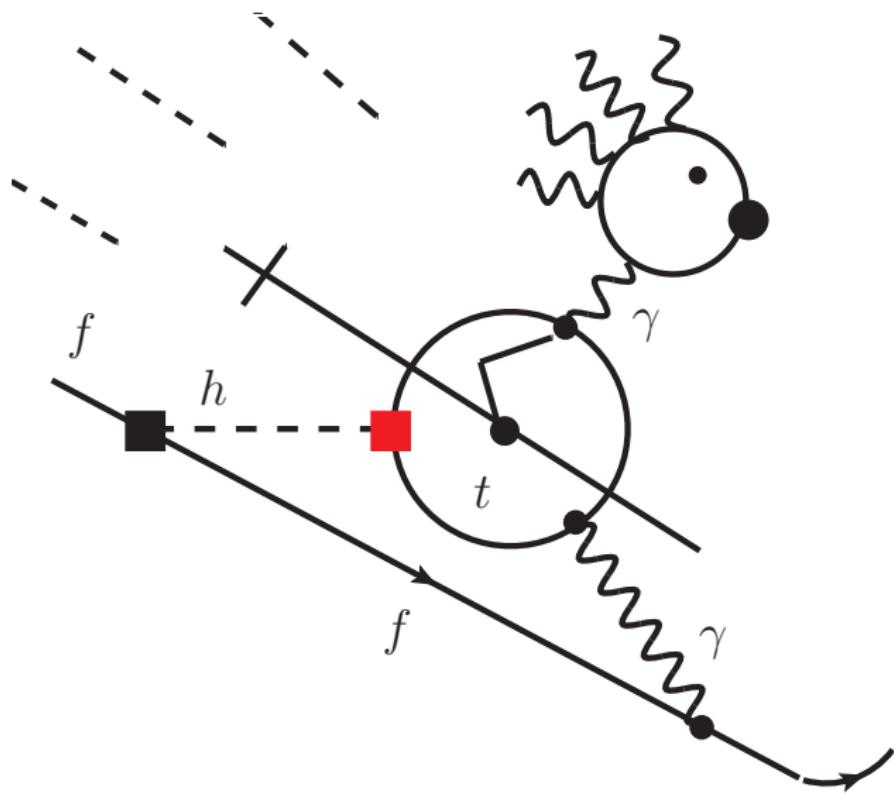
# An application in $B$ decays

- $B_s \rightarrow K^0 \bar{K}^0$  is FCNC decay
- $\text{Br}(B_s^0 \rightarrow K^0 \bar{K}^0) = (1.76 \pm 0.31) \times 10^{-5}$   
[LHCb 2002.08229, Belle 1512.02145]
- Related to  $B_s \rightarrow K^+ \bar{K}^-$ ,  $B^0 \rightarrow K^0 \bar{K}^0$ ,  $B^0 \rightarrow K^+ \pi^-$  by flavor symmetry
- Three puzzles: [Amhis et al. 2212.03874]
  - $\frac{|V_{td}^2|}{|V_{ts}^2|} \frac{\Gamma(B_s \rightarrow K^0 \bar{K}^0)}{\Gamma(B^0 \rightarrow K^0 \bar{K}^0)} = 0.61(13) \ll 1$       (Rescattering?)
  - $\frac{\Gamma(B_s \rightarrow K^0 \bar{K}^0)}{\Gamma(B_s \rightarrow K^+ \bar{K}^-)} = 0.66(13) \ll 1.06(2)$       (Large e/w penguins?)
  - $\frac{\Gamma(B_s \rightarrow K^0 \bar{K}^0)}{\Gamma(B_s \rightarrow K^+ \bar{K}^-)} \frac{\Gamma(B^0 \rightarrow \pi^- \bar{K}^+)}{\Gamma(B^+ \rightarrow \pi^+ K^0)} = 0.59(12) \ll 1$       (Large U-spin breaking?)

# Flavor Conserved – EDMs







# Modified Higgs Yukawas

- In SM, can always rotate fields such that  $m_f = y_f v / \sqrt{2}$
- Flavor and CP violation only in charged currents (CKM matrix)
- BSM: no longer true (e.g. vector-like top quark, 2HDM, ...)
- Frequently parameterize as (“ $\kappa$  framework”)

$$\mathcal{L} \supset -\frac{y_f^{\text{SM}}}{\sqrt{2}} \kappa_f \bar{f} (\cos \phi_f + i \gamma_5 \sin \phi_f) f h$$

- SM:  $\kappa_f = 1$  and  $\phi_f = 0$
- THIS IS NOT A CONSISTENT QUANTUM FIELD THEORY
- Calculations / results may be wrong (e.g. gauge dependent)

[Brod et al. 1811.05480, Altmannshofer et al. 2009.01258]

# Effective theory framework

- Better to use EFT
  - (Best, in my opinion, to commit to a model...)
- Closest to  $\kappa$  framework: HEFT (dim.4 in unitarity gauge)
- More popular: SMEFT

$$\begin{aligned}\mathcal{L}_{\text{Yukawa+SMEFT}} = & - \bar{Q}_L \tilde{H} Y_u u_R + \frac{1}{\Lambda^2} (H^\dagger H) \bar{Q}_L \tilde{H} C'_{uH} u_R \\ & - \bar{Q}_L H Y_d d_R + \frac{1}{\Lambda^2} (H^\dagger H) \bar{Q}_L H C'_{dH} d_R \\ & - \bar{L}_L H Y_{\ell\ell} \ell_R + \frac{1}{\Lambda^2} (H^\dagger H) \bar{L}_L H C'_{\ell H} \ell_R + \text{h.c.}.\end{aligned}$$

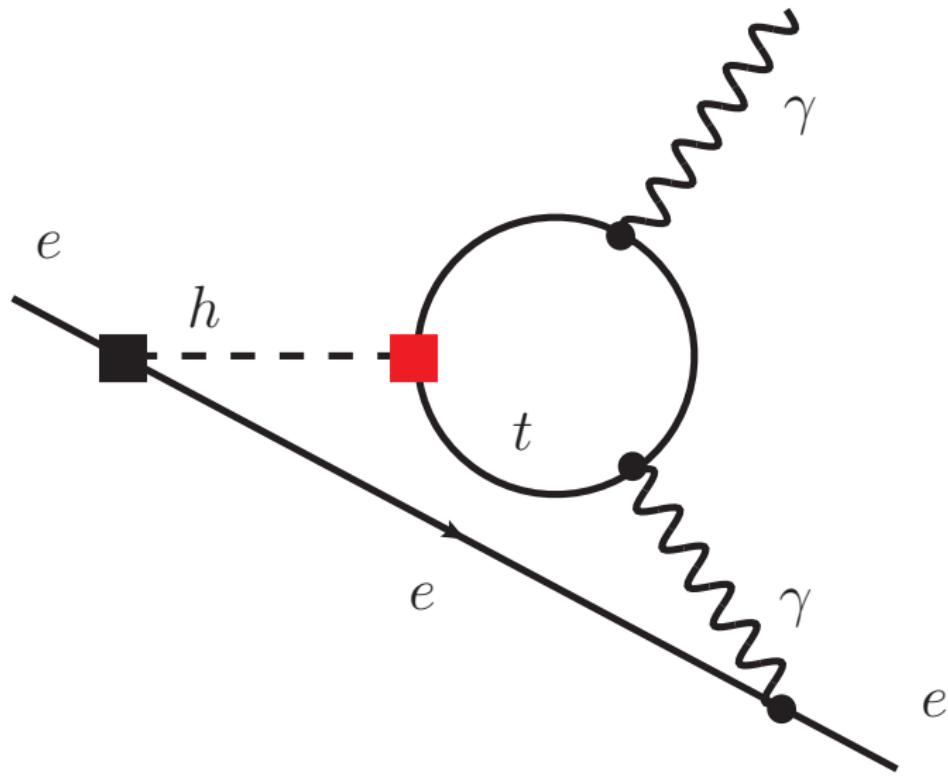
- Primes indicate that not all entries are physical

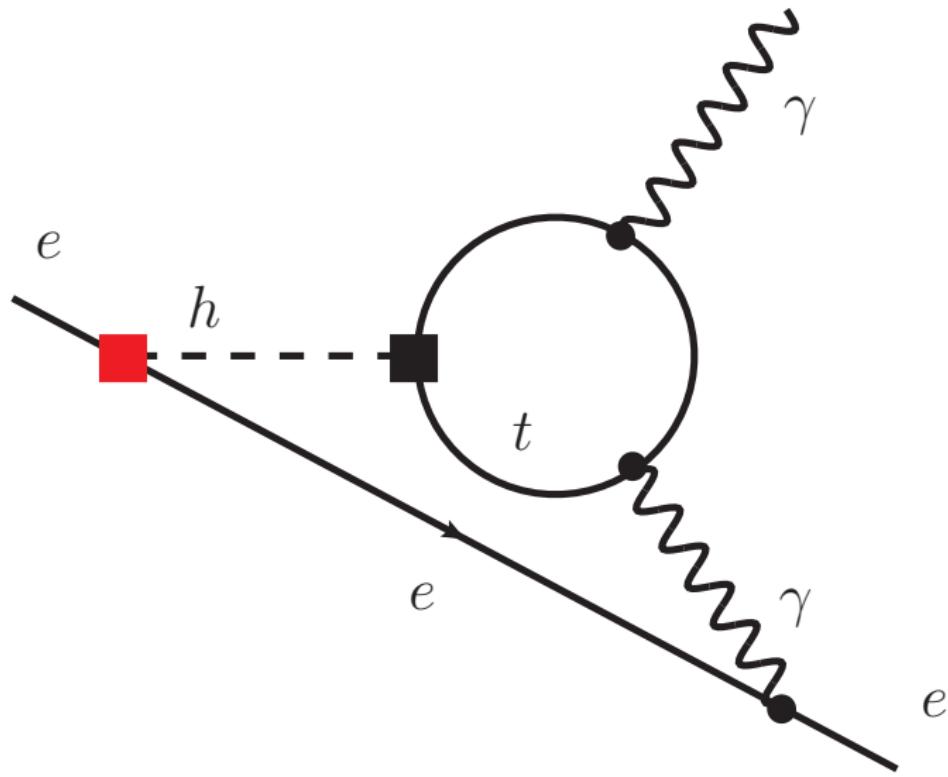
# Effective theory framework

- Express as many parameters as possible in terms of **observables**:
  - fermion masses
  - CKM matrix elements
- Split rotated WC into real and imaginary,  $C_{fH} \equiv C_{fH+} + iC_{fH-}$
- Connection to  $\kappa$  framework:

$$\kappa_f \cos \phi_f \stackrel{\circ}{=} 1 - \frac{v}{\sqrt{2}m_f} \frac{v^2}{\Lambda^2} C_{fH+}, \quad \kappa_f \sin \phi_f \stackrel{\circ}{=} -\frac{v}{\sqrt{2}m_f} \frac{v^2}{\Lambda^2} C_{fH-}.$$

- Correlation CPV and FV [Alonso-González et al. 2103.16569 , 2109.07490]



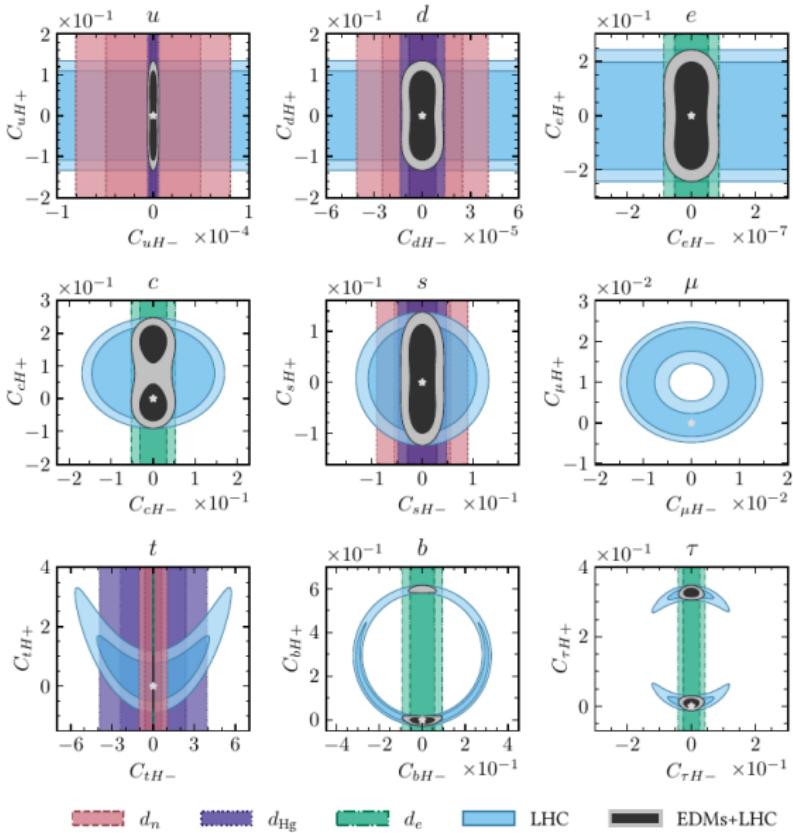


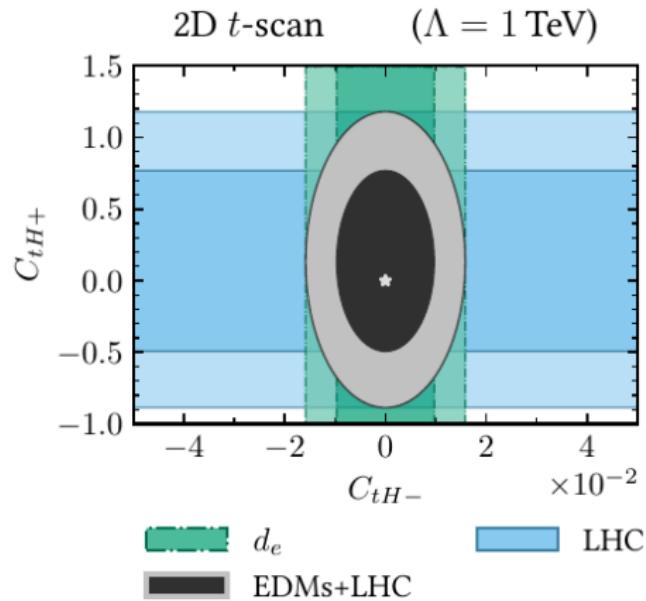
# A first global analysis

- Scanning up to six parameters (Wilson coefficients) [Brod et al. JHEP 08 (2022) 294]

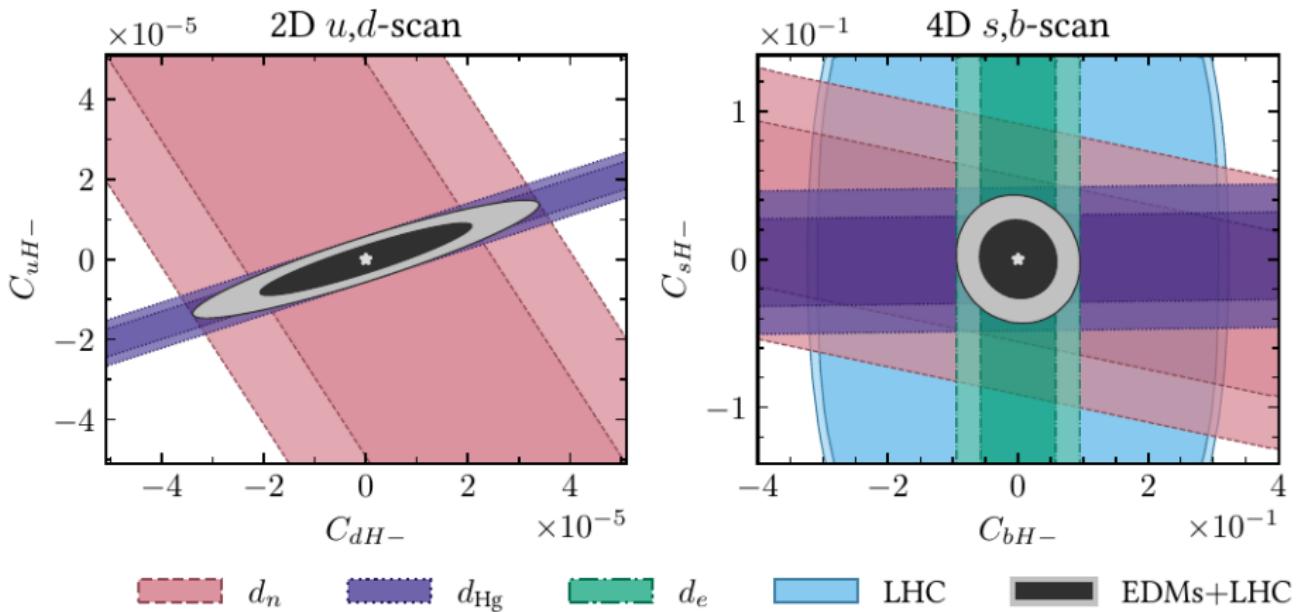
- Electron, neutron, mercury EDM (one- and two-loop; one-loop RG)
- $gg \rightarrow h$ ,  $h \rightarrow \gamma\gamma$  (one-loop);  $h \rightarrow b\bar{b}, c\bar{c}, \tau^+\tau^-, \mu^+\mu^-$  decay rates
- total Higgs decay width
- $h \rightarrow \tau^+\tau^-$  angular analysis [CMS 2110.04836 → Talk by M. Sessini]
- HiggsSignals\_2.5.0 [Bechtle et al. 2012.09197]
- HiggsBounds\_5.8.0 [Bechtle et al. 2006.06007 ]
- GAMBIT [1705.07908, 1705.07919]

2D single-flavour scans       $\Lambda = 1 \text{ TeV}$

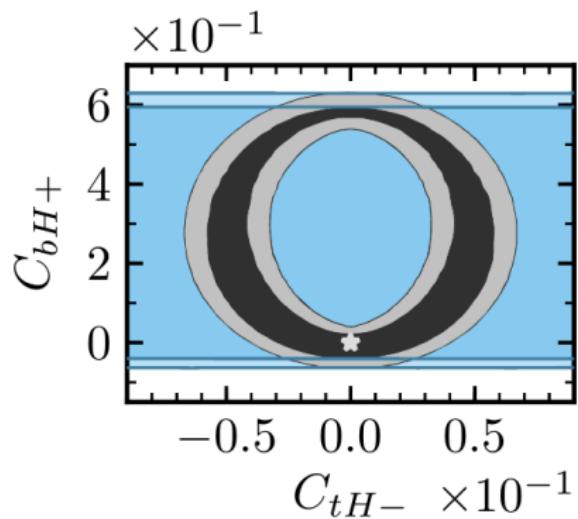
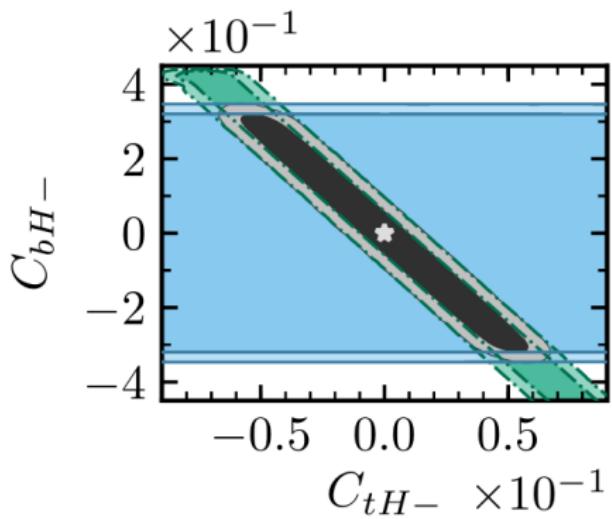




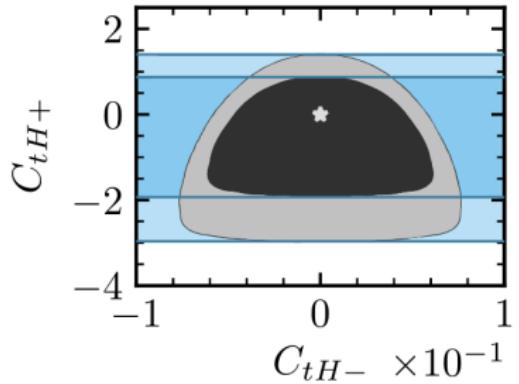
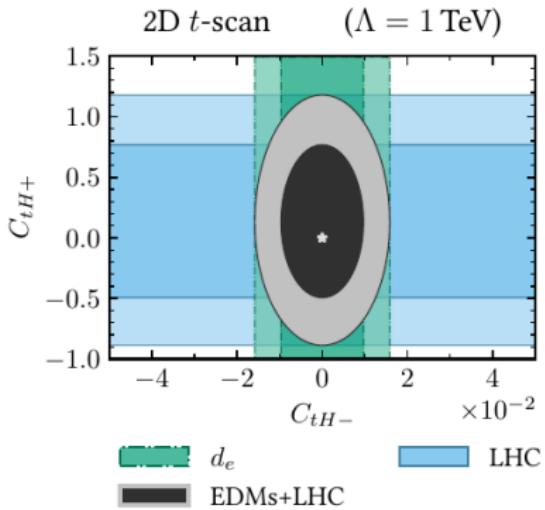
$$\Lambda = 1 \text{ TeV}$$



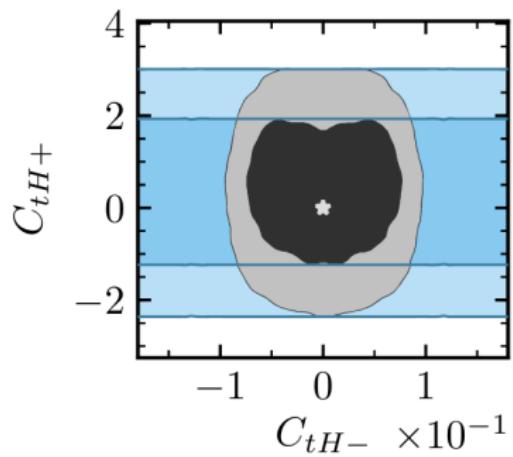
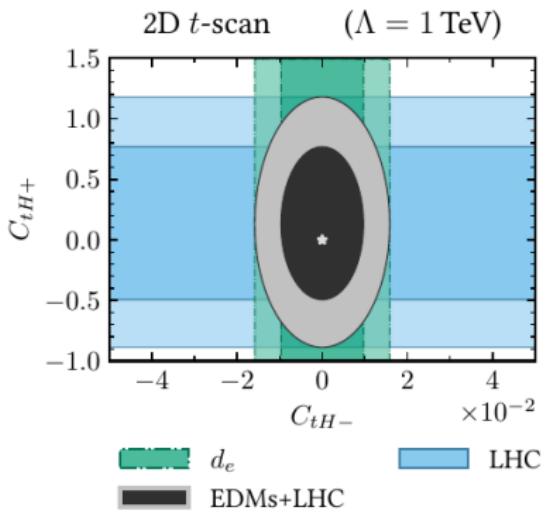
## 4D scans – $C_{tH+}$ , $C_{tH-}$ , $C_{bH+}$ , $C_{bH-}$



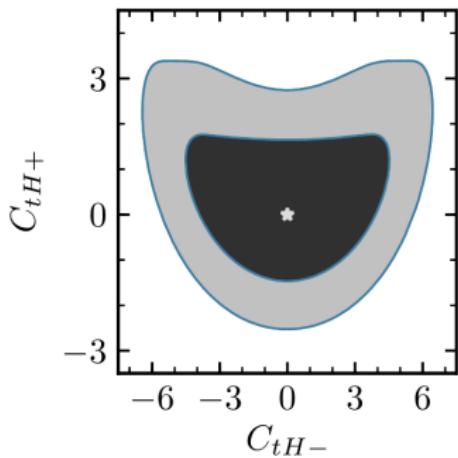
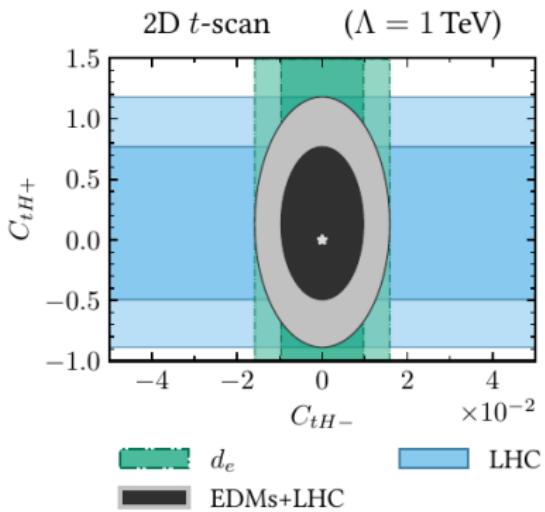
## 2D vs. 4D scans – top quark



## 2D vs. 6D scans – top quark



# Beyond 6 parameters – 2D vs. 4D top quark



# Summary

- Status of  $B$  anomalies
- Two “new” precision observables in kaon physics:
  - Indirect CPV ( $\epsilon_K$ )
  - $\text{Br}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}$
- Flavor symmetries help understand flavor data
- EDMs constrain flavor-diagonal CPV

# Appendix

# In terms of explicit matrix elements

$$P \equiv \langle K^+ K^- | C_i Q_i^{\bar{d}d} | \bar{D}^0 \rangle = \langle \pi^+ \pi^- | C_i Q_i^{\bar{s}s} | \bar{D}^0 \rangle$$

$$T \equiv -\frac{1}{2} \left( \langle K^+ K^- | C_i (Q_i^{\bar{d}d} - Q_i^{\bar{s}s}) | \bar{D}^0 \rangle - \langle \pi^+ \pi^- | C_i (Q_i^{\bar{d}d} - Q_i^{\bar{s}s}) | \bar{D}^0 \rangle \right)$$

$$= \langle K^+ \pi^- | C_i Q_i^{\bar{d}s} | \bar{D}^0 \rangle = \langle \pi^+ K^- | C_i Q_i^{\bar{s}d} | \bar{D}^0 \rangle$$

$$P_{\text{break}} \equiv \frac{1}{2} \left( \langle K^+ K^- | C_i (Q_i^{\bar{d}d} - Q_i^{\bar{s}s}) | \bar{D}^0 \rangle + \langle \pi^+ \pi^- | C_i (Q_i^{\bar{d}d} - Q_i^{\bar{s}s}) | \bar{D}^0 \rangle \right)$$

- $p_0 = P + \frac{1}{2} T$
- $p_1 = P, t_0 = T, s_1 = P_{\text{break}} + \frac{1}{2} T$
- $t_1 = T, t_2 = -P_{\text{break}}$