Muon Anomaly and Lattice QCD

Z. Fodor

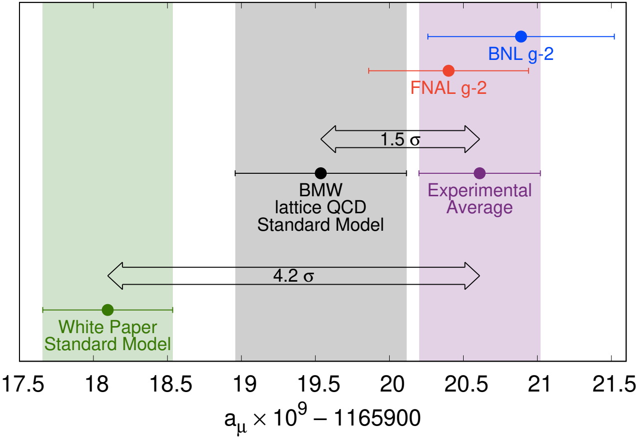
Penn State, Univ. Wuppertal, FZ Juelich, Univ. Budapest, UCSD

Budapest–Marseille–Wuppertal collaboration (BMW)

Borsanyi, Fodor, Guenther, Hoelbling, Katz, Lellouch, Lippert, Miura, Parato, Szabo, Stokes, Toth, Torok, Varnhorst

Lake Louise Winter Institute, February 21, 2023

Tensions in $(g-2)_\mu$: take-home message



[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

[Muon g-2 coll., Phys. Rev. Lett. 126, 141801 (2021)]

Lattice QCD: examples

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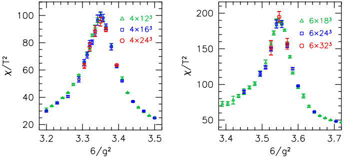

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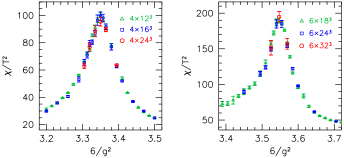

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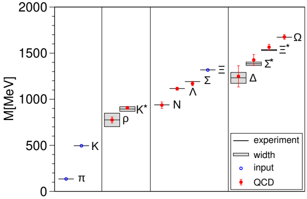

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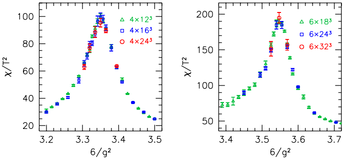

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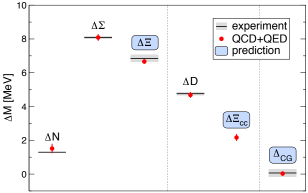

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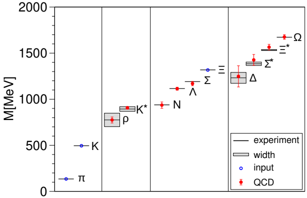

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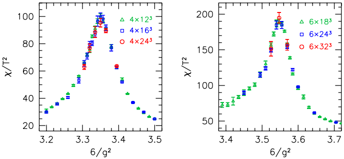

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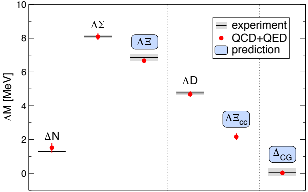

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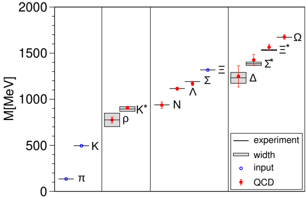
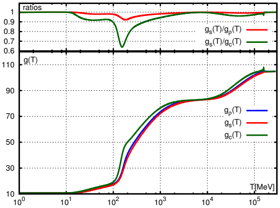


- Wuppertal–Budapest-collaboration,
  Lattice QCD for Cosmology, Nature 539 (2016) 7627, 69-71



Outline

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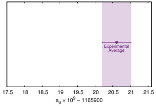
1. 

Experimental Average
17.5
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$a_\mu \times 10^9 - 1165900$

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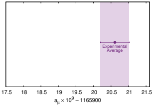
1. 
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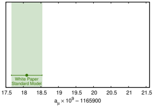
2. 
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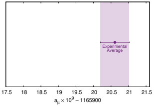
1. 
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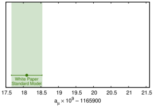
2. 
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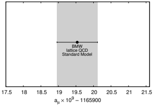
3. 
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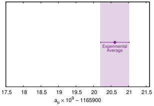
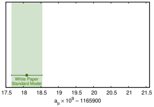
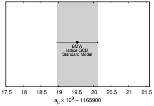
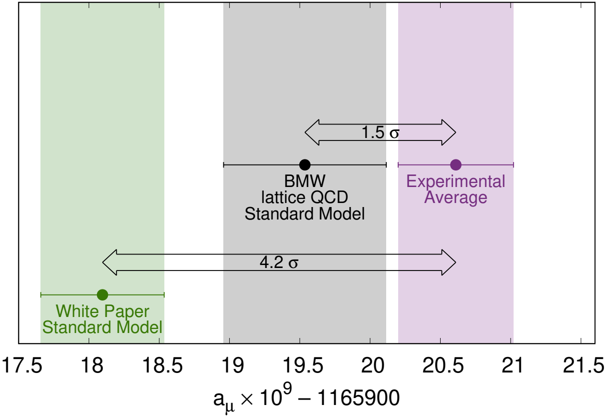
1. 

2. 

3. 

4. Summary



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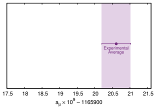
1. 
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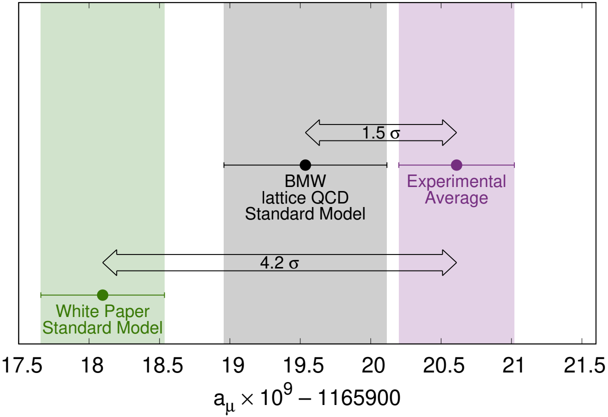
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Experimental result

- Newly announced result at Fermilab

$$a_\mu(\mathrm{FNAL}) = 11\,659\,204.0(5.4) \cdot 10^{-10} \quad (0.46\,\mathrm{ppm})$$

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$$\begin{aligned} a_\mu(\text{BNL}) &= 11\,659\,209.1(6.3)\cdot 10^{-10} \quad (0.54\,\text{ppm}) \\ a_\mu(\text{combined}) &= 11\,659\,206.1(4.1)\cdot 10^{-10} \quad (0.35\,\text{ppm}) \end{aligned}$$

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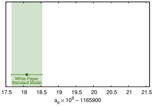
- Target uncertainty: (1.6)

Outline

2. 
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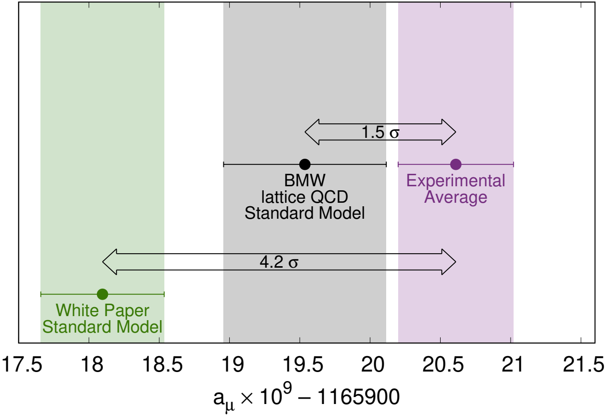
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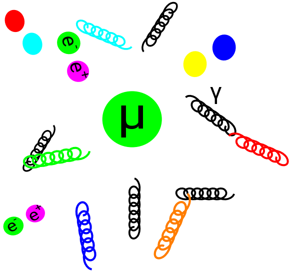
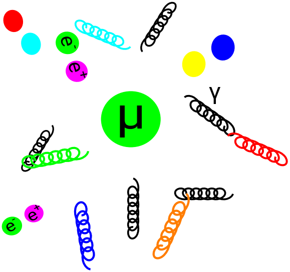
Theory: Standard Model



Sum over all known physics:

Theory: Standard Model



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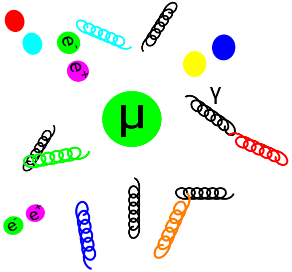

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2. electroweak (EW): W, Z, ν, Higgs

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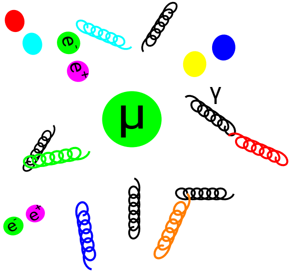

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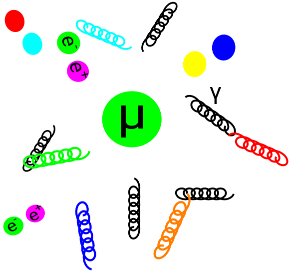

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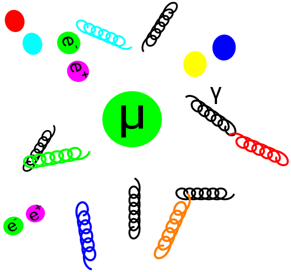

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QED	11658471.9(0.1)
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total	11659181.0(4.3)

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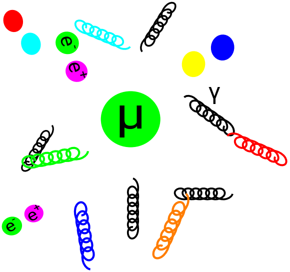

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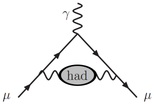
4.0 out of the 4.3 error comes from LO hadron vacuum polarisation

Hadronic contributions

- LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)



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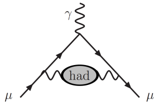

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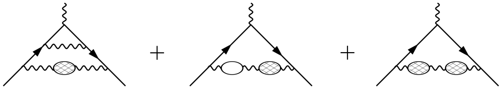

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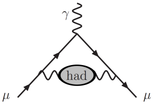

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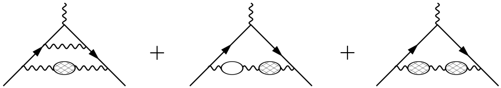
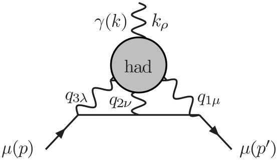


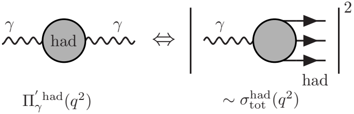
- Hadronic light-by-light (HLbL, $(\frac{\alpha}{\pi})^3$)



- pheno $a_\mu^{\text{HLbL}} = 9.2(1.9)$
  [Colangelo, Hoferichter, Kubis, Stoffer et al '15–'20]
- lattice a_μ^{HLbL}=7.9(3.1)(1.8) or 10.7(1.5)
  [RBC/UKQCD '19 and Mainz '21]

HVP from R-ratio

- Optical theorem



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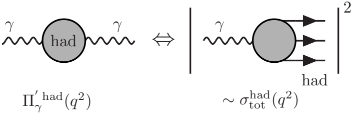

Use $e^+e^- \rightarrow$ had data of CMD, SND, BES, KLOE, BABAR, ...
systematics limited

HVP from R-ratio

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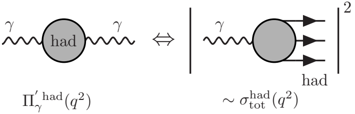

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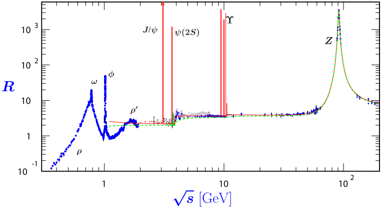
$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$



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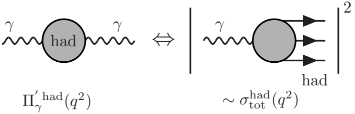

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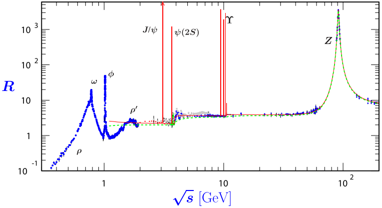

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LO	[Davier et al '19]	693.9(4.0)	0.58%
LO	[Keshavarzi et al '19]	692.78(2.42)	0.35%
LO	[Hoferichter et al '19]	692.3(3.3)	0.48%
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NLO/NNLO	[Kurz et al '14]	-9.87(0.09)/1.24(0.01)	

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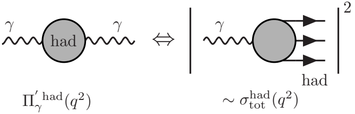

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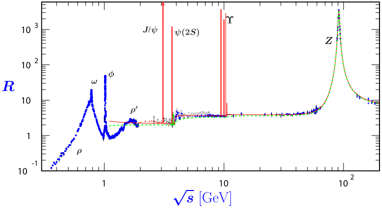

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Systematic uncertainty: ≈4 times larger than the statistical error (e.g. Davier et al.)

Tensions in the R-ratio method

CMD3 [2302.08834] $e^+e^- \to \pi^+\pi^-$ for $\sqrt{s}$: 0.60–0.88 GeV

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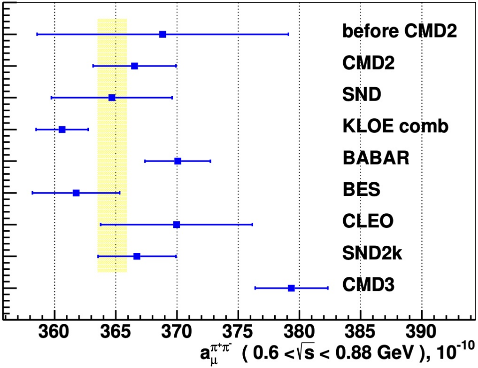

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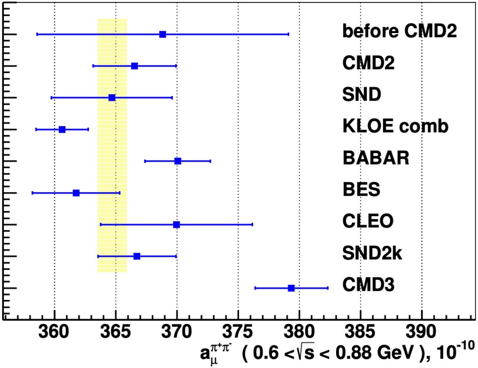

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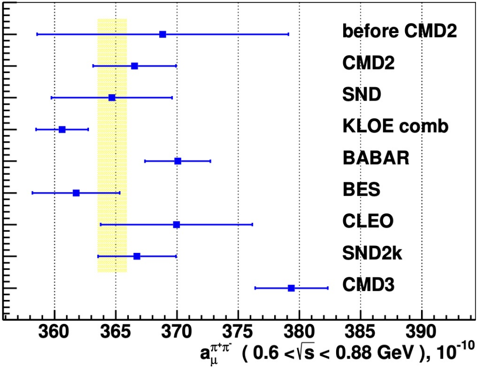

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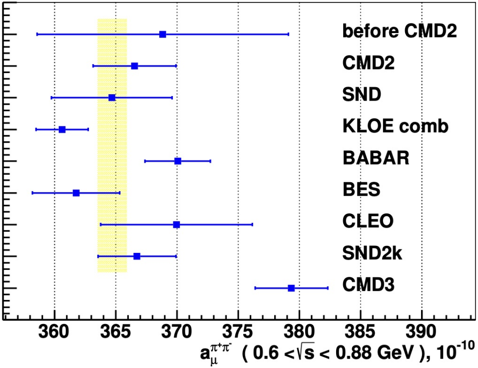

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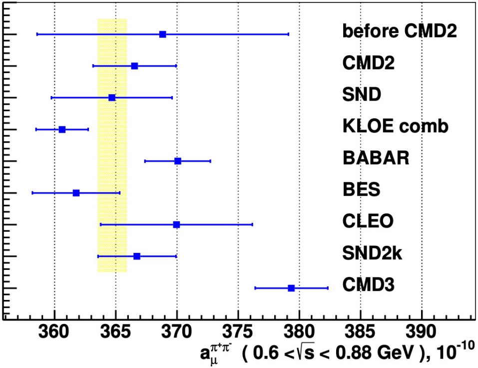

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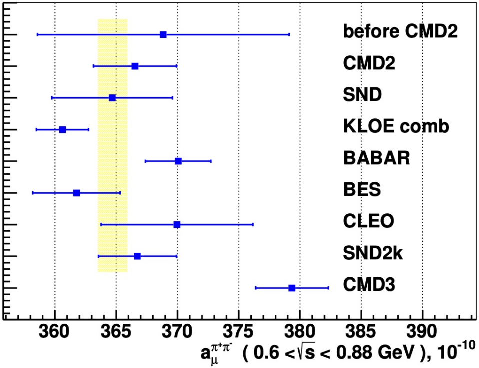

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White Paper must further inflate errors: less chance for new physics?

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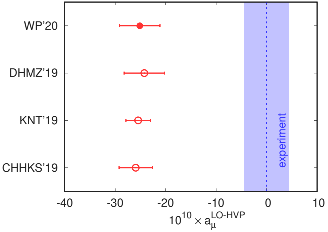

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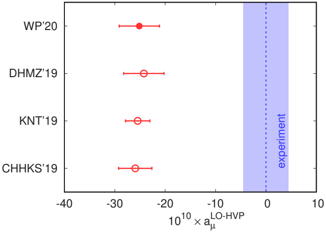

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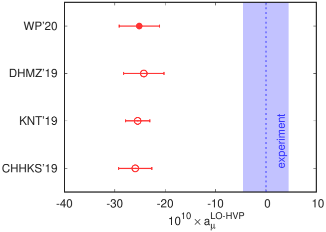

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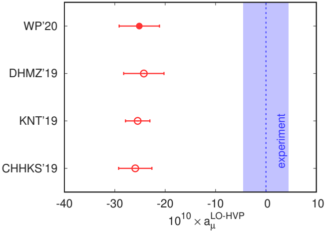

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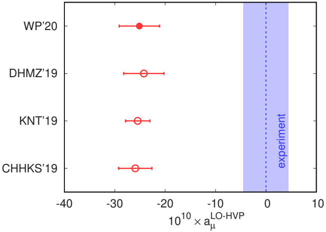

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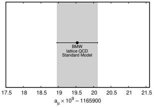
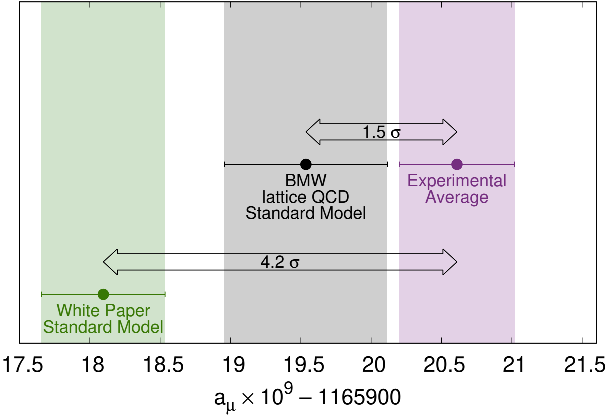
For no new physics:

- 4% larger HVP, $a_\mu^{\text{LO-HVP}} = 720.0(6.8)$
- 360% larger HLbL, $a_\mu^{\text{HLbL}} = 37.9(7.1)$

Outline

3. 
BMW
lattice QCD
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$


1.5 σ
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Experimental
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Lattice QCD

- Quantum field theory: integrate over all classical field configurations

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- 100000 years for a laptop $\longrightarrow$ 1 year for supercomputer

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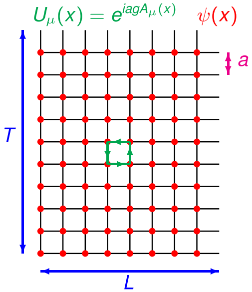

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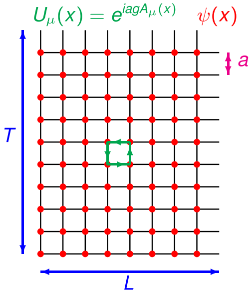

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$$\int \mathrm{d}^4 x \longrightarrow a^4 \sum_x$$

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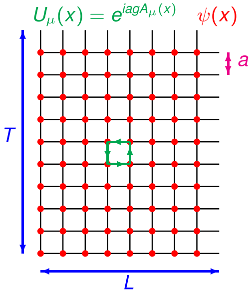

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- To get physical results, need to perform:
  1. Chiral limit ($m_{u/d} \to m_{phys}$ or use m_{phys})
  2. Infinite volume limit ($V \to \infty$) $\longrightarrow$ numerically or analytically
  3. Continuum limit ($a \to 0$) $\longrightarrow$ min. 3 different a

FLAG review of lattice results Colangelo et al. Eur.Phys.J. C71 (2011) 1695

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Collaboration	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	$m_{ud,\overline{\rm MS}(2\rm GeV)}$	$m_{s,\overline{\rm MS}(2\rm GeV)}$
PACS-CS 10	P	★	■	■	★	*a*	2.78(27)	86.7(2.3)
MILC 10A	C	●	★	★	●	–	3.19(4)(5)(16)	–
HPQCD 10	A	●	★	★	★	–	3.39(6)*	92.2(1.3)
BMW 10AB	P	★	★	★	★	*b*	3.469(47)(48)	95.5(1.1)(1.5)
RBC/UKQCD	P	●	●	★	★	*c*	3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum et al. 10	P	●	■	●	★	–	3.44(12)(22)	97.6(2.9)(5.5)

$a_\mu^{\text{LO-HVP}}$ from lattice QCD Nature 593 (2021) 7857, 51

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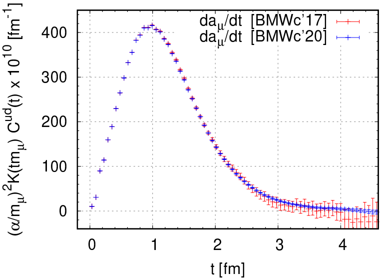
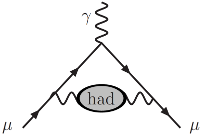


$K(t)$ describes the leptonic part of diagram



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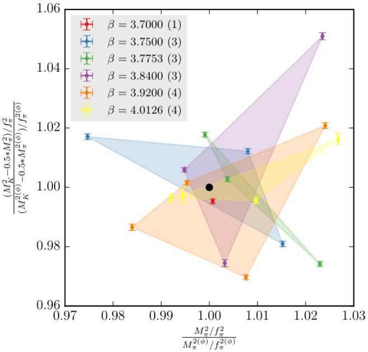

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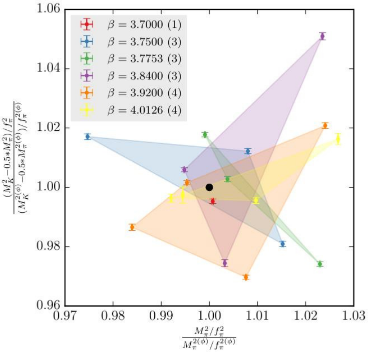

β	a[fm]	$L \times T$	#conf
3.7000	0.1315	48×64	904
3.7500	0.1191	56×96	2072
3.7753	0.1116	56×84	1907
3.8400	0.0952	64×96	3139
3.9200	0.0787	80×128	4296
4.0126	0.0640	96×144	6980

CPU demand scales as $\approx a^{-8}$:
very careful planning needed

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Depending on the action: topology is frozen for a<0.05 fm

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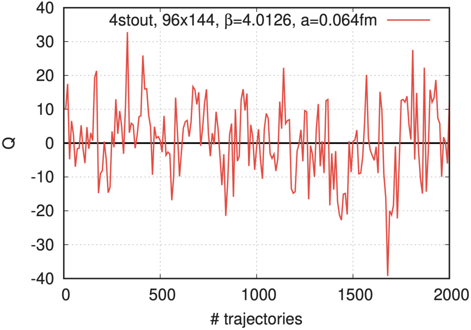

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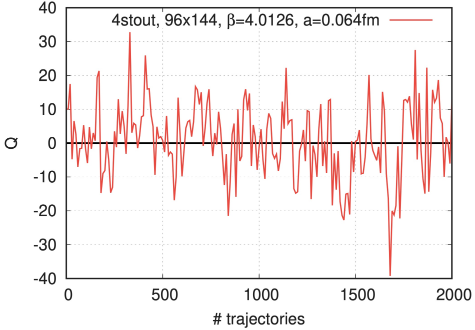
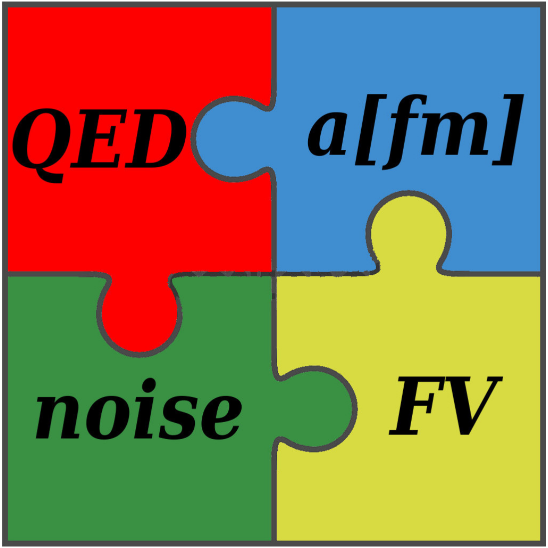


The integrated autocorrelation time of Q is 19(2) trajectories.

New challenges



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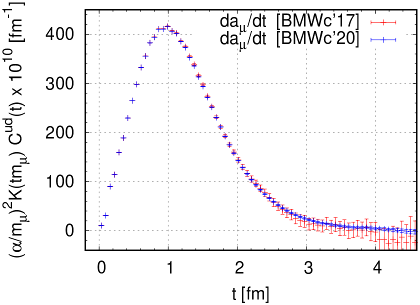
2. For separation of isospin breaking effects: w_0 scale setting
   - Moderate m_q dependence
   - Can be precisely determined on the lattice
   - No experimental value
     $\longrightarrow$ Determine value of w_0 from $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0)\rangle$ grows for large distances



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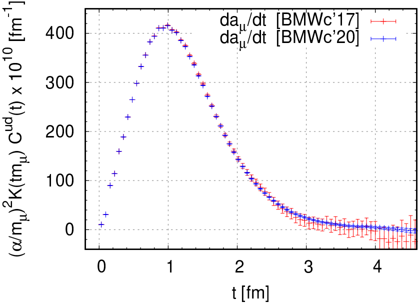

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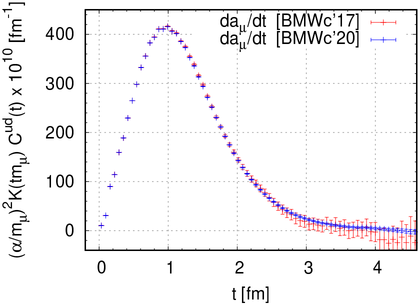

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$\longrightarrow$ few permil level accuracy on each ensemble

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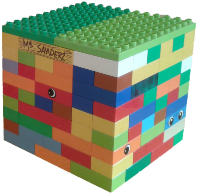

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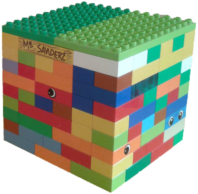

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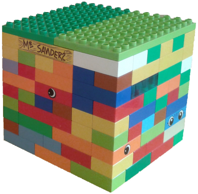
$L_{\mathrm{ref}} = 6.272\,\mathrm{fm}$   $L_{\mathrm{big}} = 10.752\,\mathrm{fm}$

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lattice	NLO XPT	NNLO XPT	MLLGS	HP	RHO
$18.1(2.0)_{\mathrm{stat}}(1.4)_{\mathrm{cont}}$	11.6	15.7	17.8	16.7	15.2

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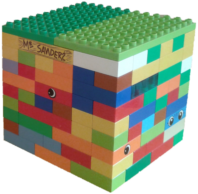


$L_{\text{big}} = 10.752\,\text{fm}$

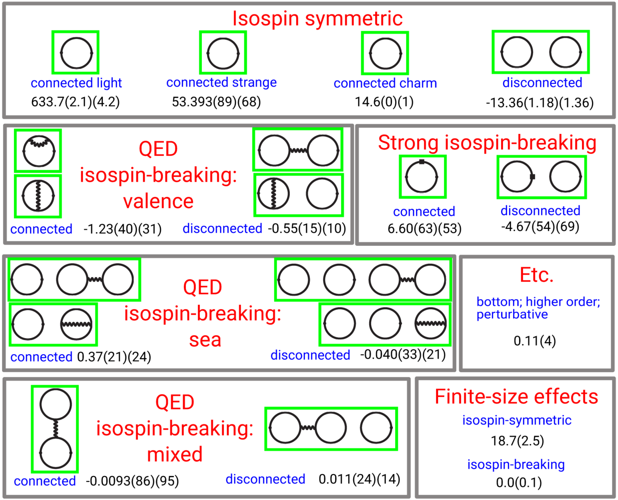
1. $a_\mu(\text{big}) - a_\mu(\text{ref})$
   - perform numerical simulations in $L_{\text{big}} = 10.752\,\text{fm}$
   - perform analytical computations to check models

lattice	NLO XPT	NNLO XPT	MLLGS	HP	RHO
$18.1(2.0)_{\text{stat}}(1.4)_{\text{cont}}$	11.6	15.7	17.8	16.7	15.2

2. $a_\mu(\infty) - a_\mu(\text{big})$
   - use models for remnant finite-size effect of “big” $\sim 0.1\%$

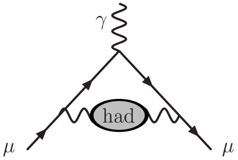
Isospin breaking effects

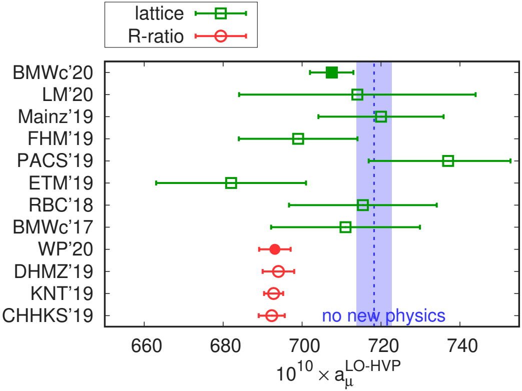
- Include leading order IB effects: $\mathcal{O}(e^2)$, $\mathcal{O}(\delta m)$



$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$

Final result for LO-HVP (hadronic vacuum polarization)





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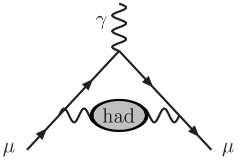

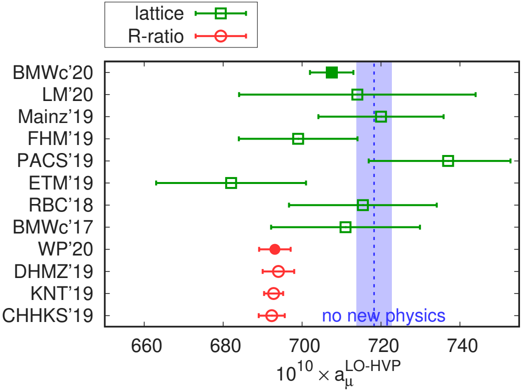

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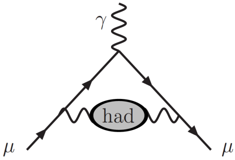

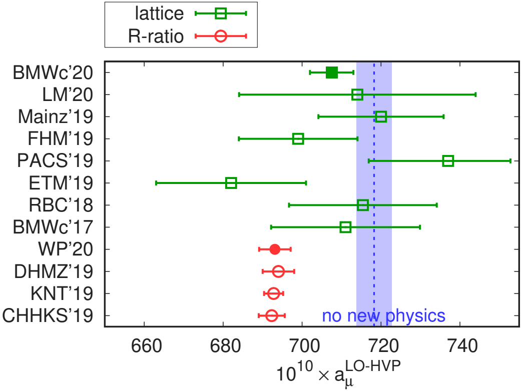

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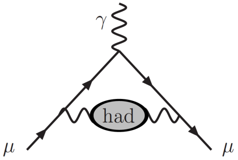

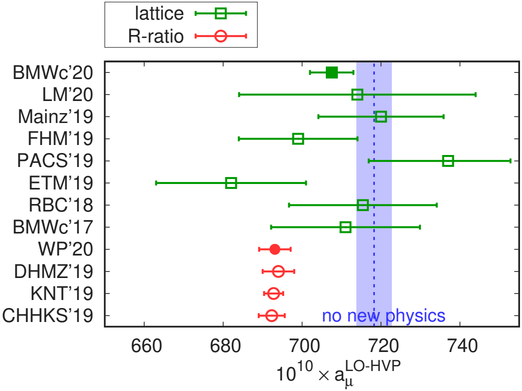

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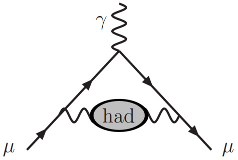

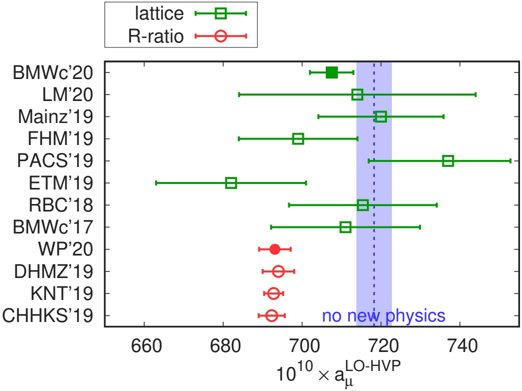

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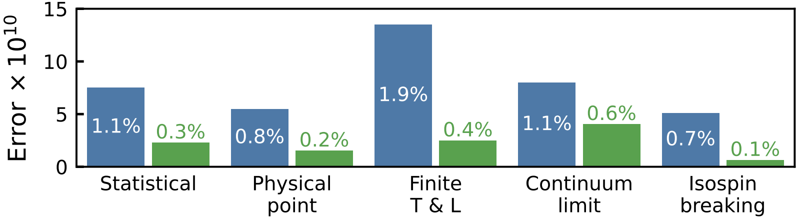
- CMD3 is also 15 units larger than the White Paper: spot on

Improvements on the errors from 2017–2020



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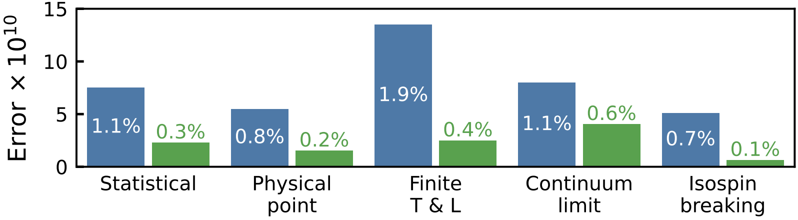

2017: (blue columns) the dominant error was finite size error

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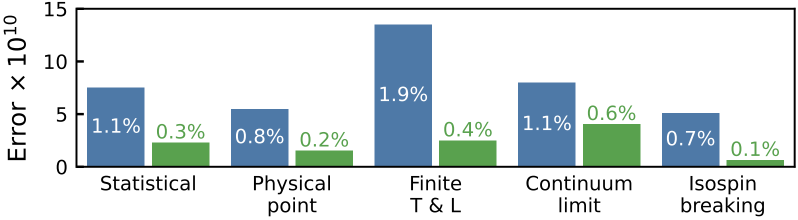

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still the second largest error

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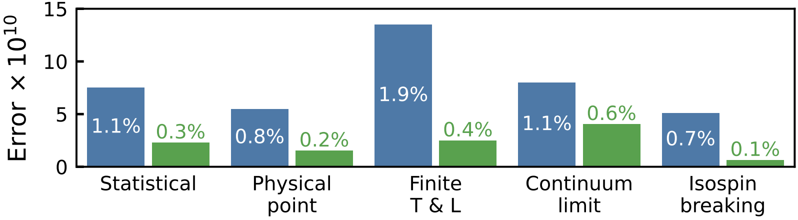

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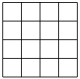
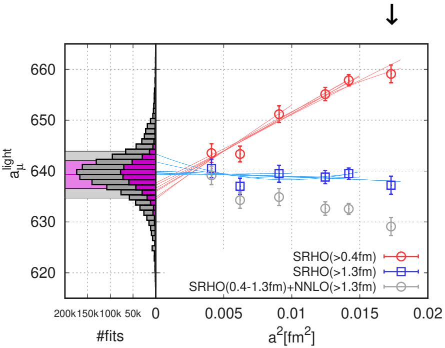
Today: largest uncertainty is the continuum extrapolation
best way to reduce: get closer to the continuum limit, reduce ”a”
presently running a=0.046 fm lattice (CPU grows as a^{-8})

Continuum limit





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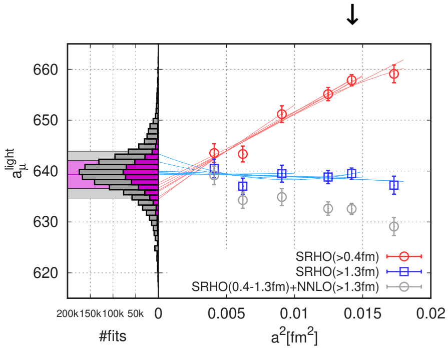

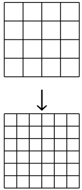

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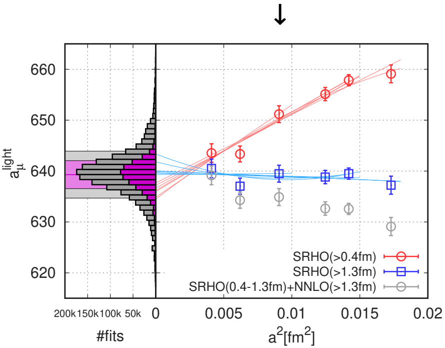

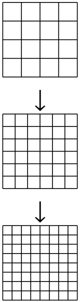

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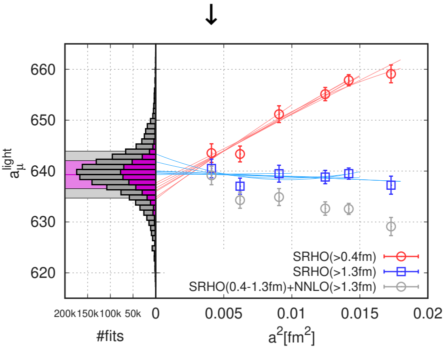

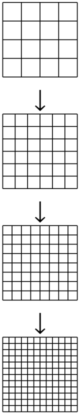

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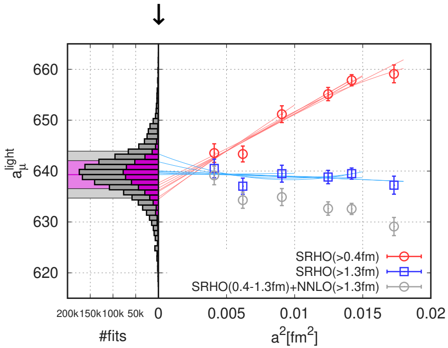
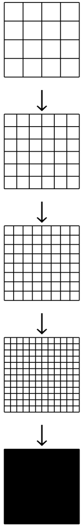
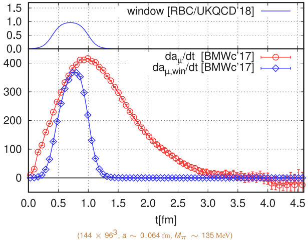




Window observable

- Restrict correlator to window between $t_1 = 0.4\,\mathrm{fm}$ and $t_2 = 1.0\,\mathrm{fm}$

[RBC/UKQCD'18]



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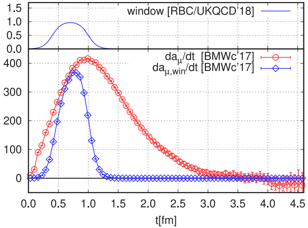

(144×96^3, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

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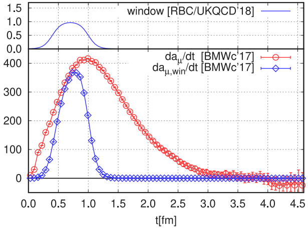

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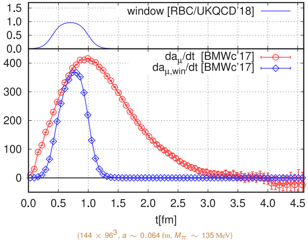

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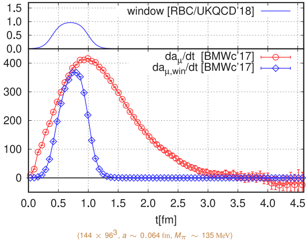

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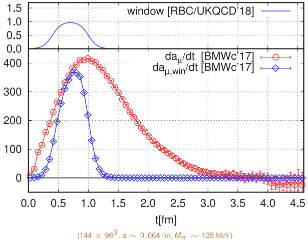

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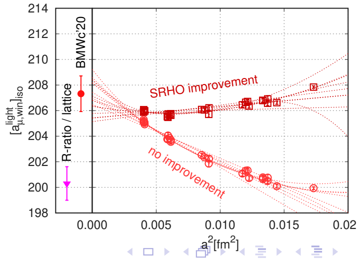
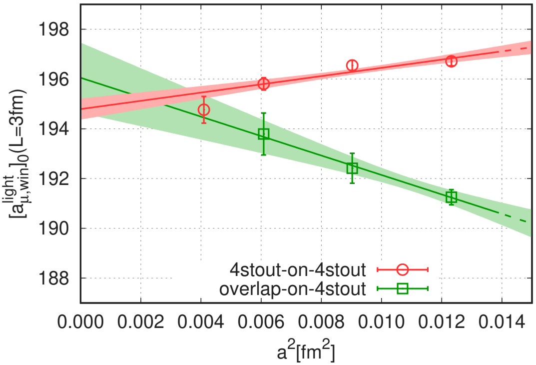


Crosscheck – overlap



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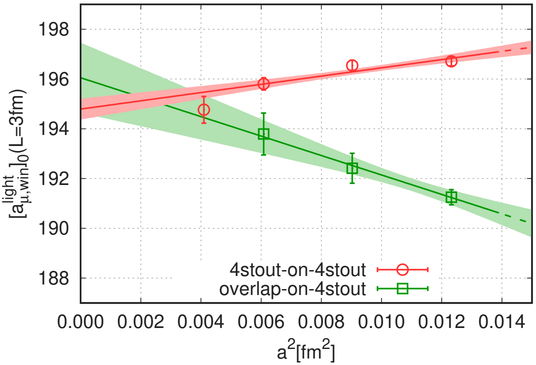
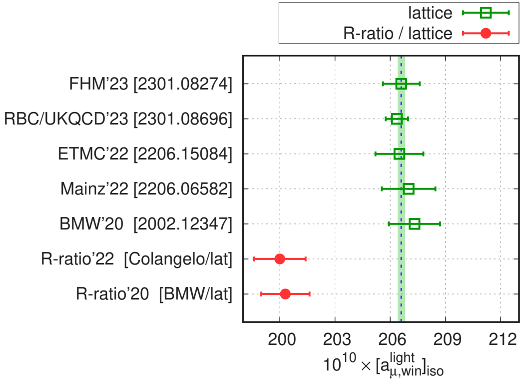


- compute $a_{\mu,\mathrm{win}}$ with overlap valence
- local current instead of conserved $\longrightarrow$ had to compute Z_V
- cont. limit in $L = 3$ fm box consistent with staggered valence

Tension in the window observables



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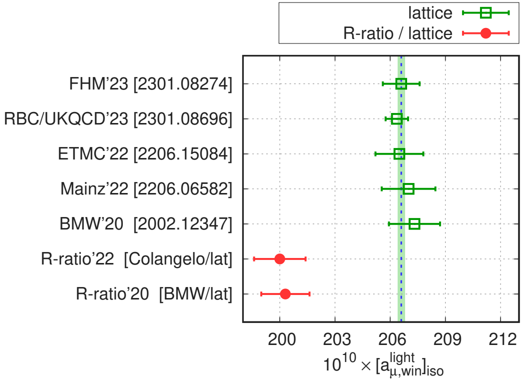

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most of them: blinded(*)
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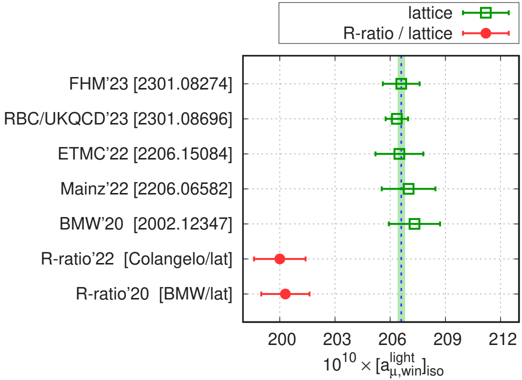

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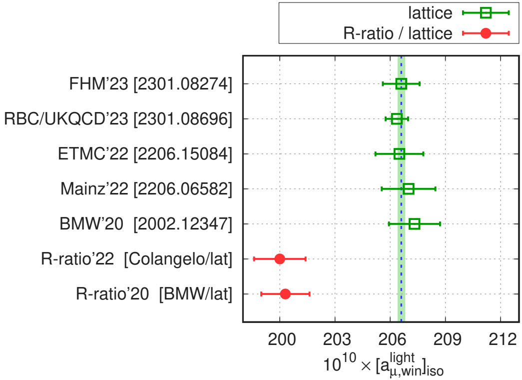

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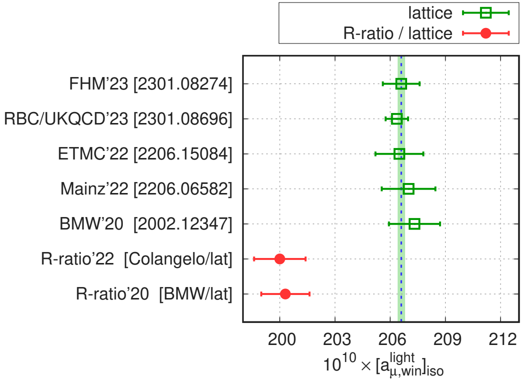

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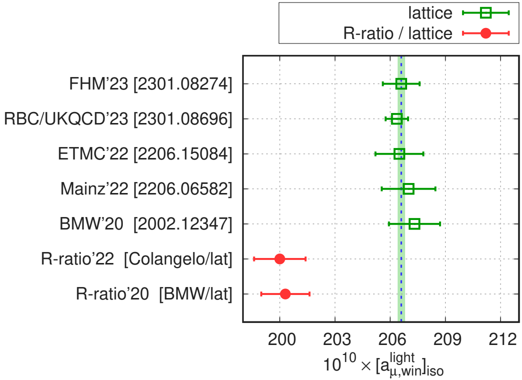

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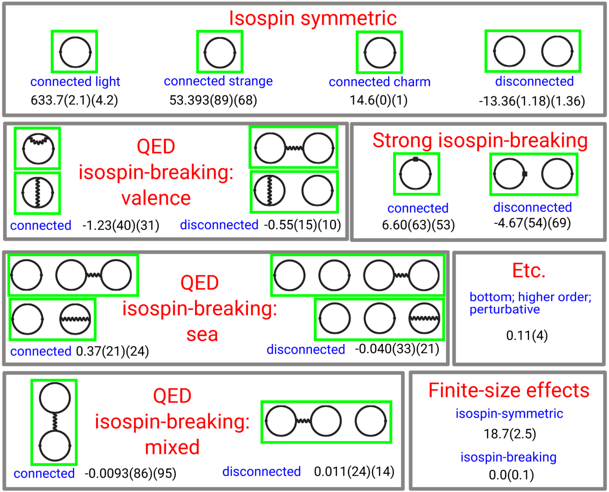
either new physics
or underestimated errors

Outline

5. Summary

Final result



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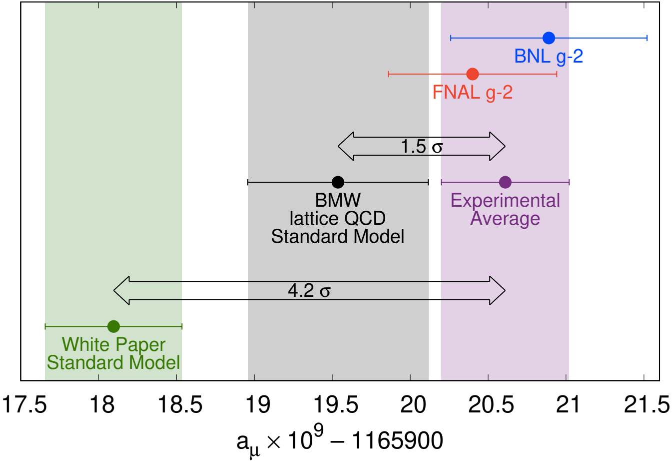
Tension: take-home message #1 full g-2

Systematic/statistical error ratios: lattice ≈2; R-ratio ≈4



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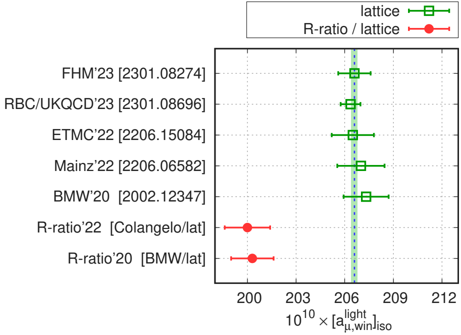

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