

Measurement of the muon anomalous magnetic moment with the Fermilab g-2 experiment

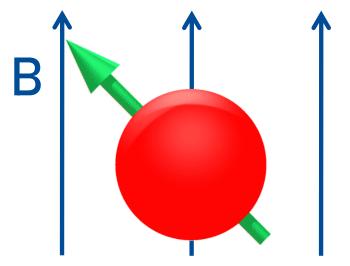
Saskia Charity, University of Liverpool Birmingham EPP Seminar, 06 December 2023

Muons & magnetic moments

g determines spin precession frequency in a magnetic field

Torque in B-field Magnetic Moment

$$\vec{\mu} \times \vec{B}$$
 $\vec{\mu} = g \frac{e}{2m} \bar{S}$

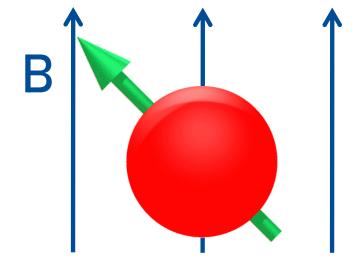


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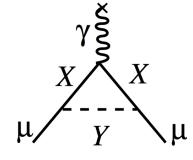
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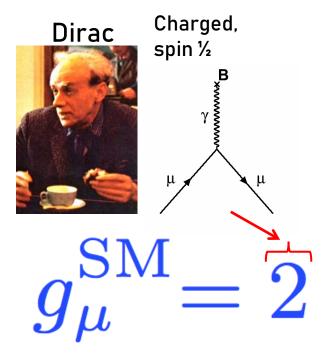


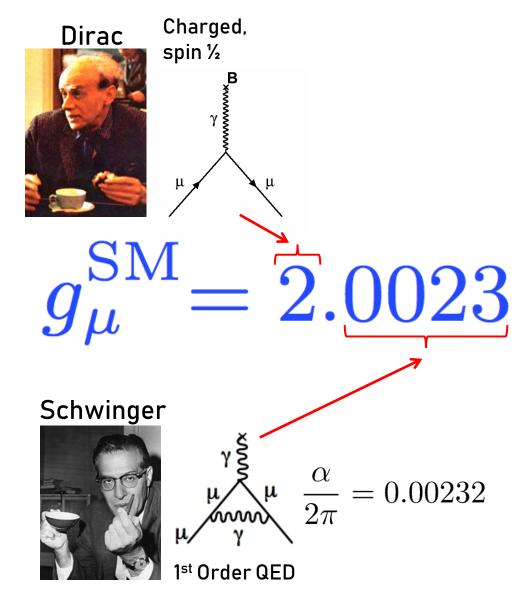
$$g=2$$
 $\gamma \xi$ μ

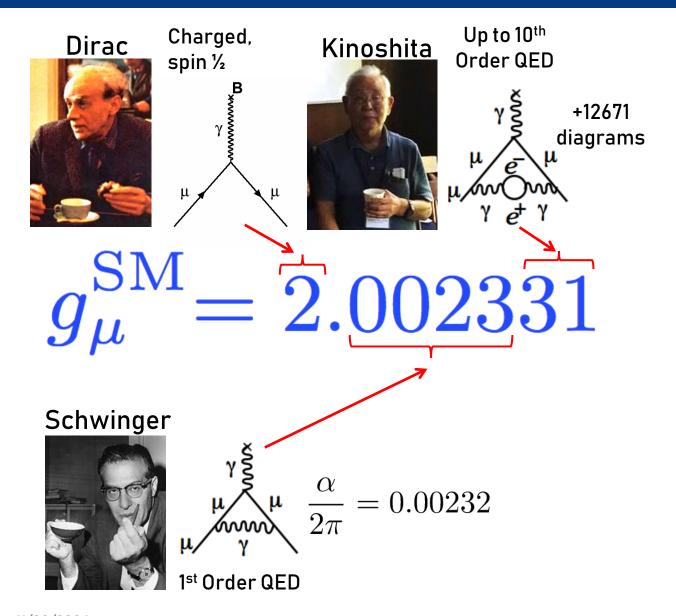
g > 2

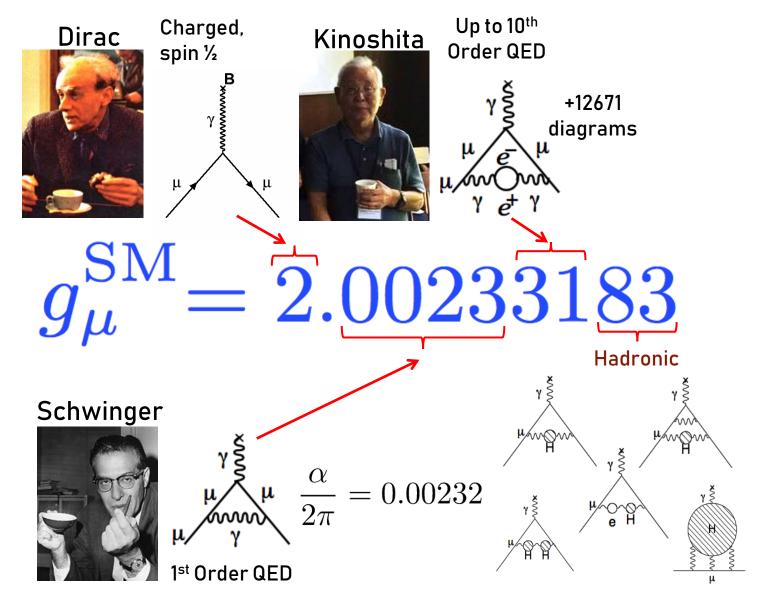


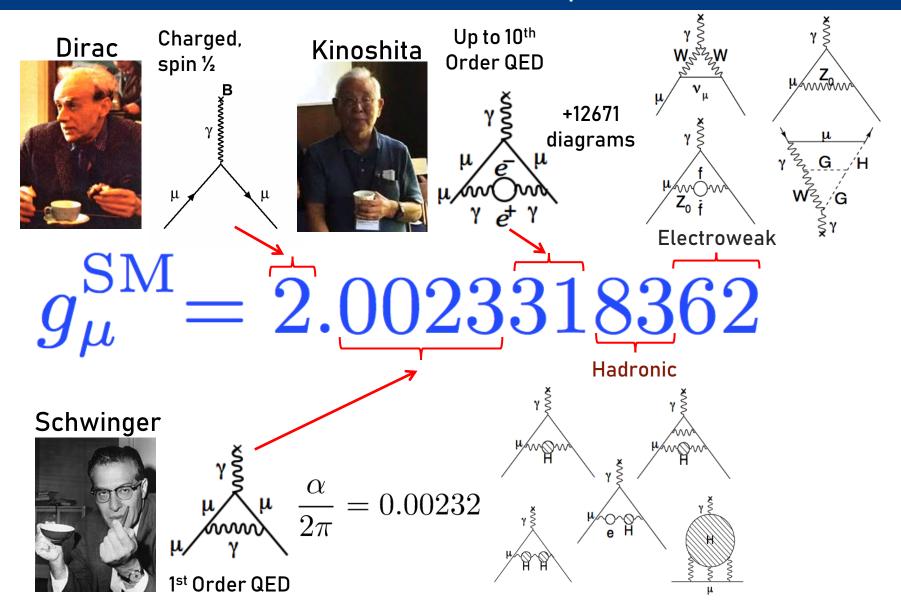
- For a pure Dirac spin-½ charged fermion, g is exactly 2
- Interactions between the muon and virtual particles alter the value: X & Y particles could be SM or new physics

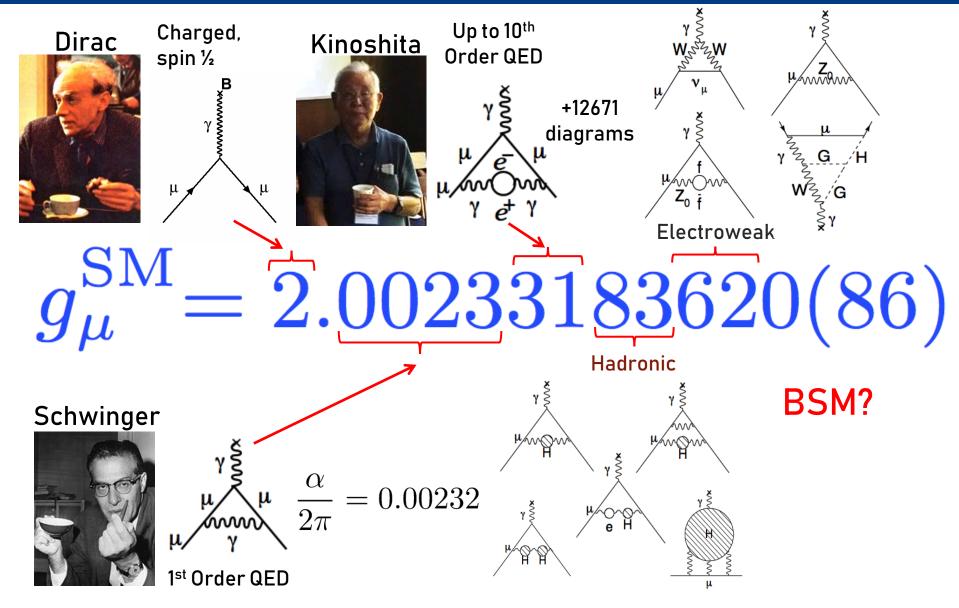




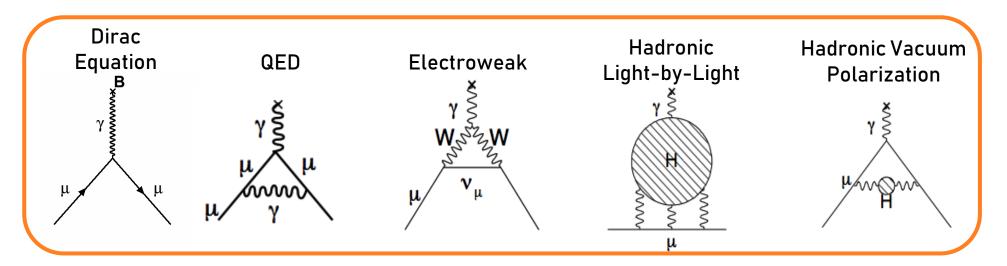








Muon g-2 Theory Initiative



SM values taken from the **Muon g-2 Theory Initiative**



TI in Bern, September 2023

- Consortium of 100+ theorists who compile the theoretical inputs and provide recommendations
- Last compilation in 2020:

White Paper: Phys. Rept. 887 (2020) 1-166 https://doi.org/10.1016/j.physrep.2020.07.006

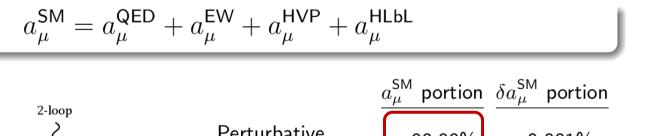
https://muon-gm2-theory.illinois.edu/

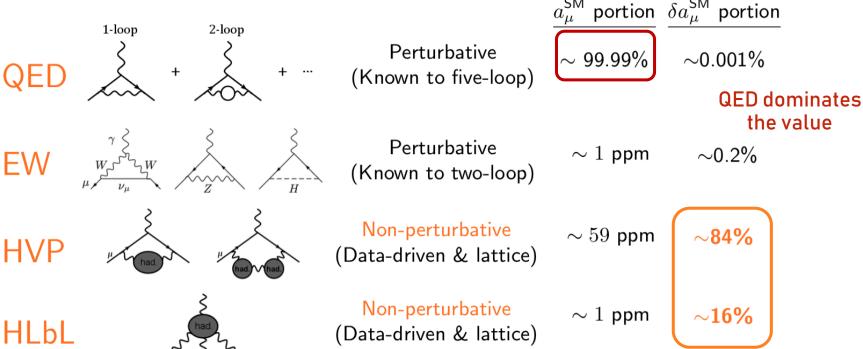
The muon anomalous magnetic moment

• Define the quantity a_{μ} : muon anomalous magnetic moment

$$a_{\mu} = \frac{g_{\mu} - 2}{2}$$

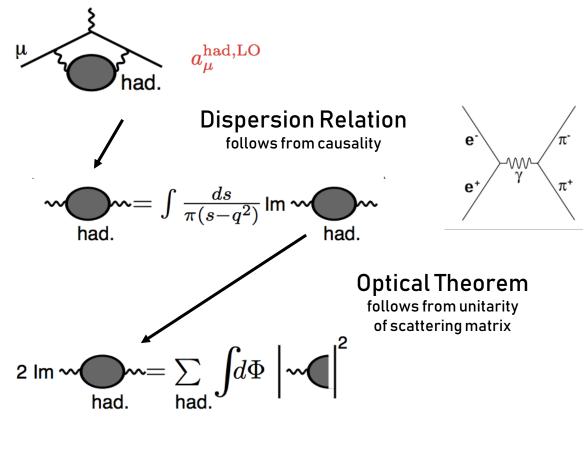
- a_{μ} arises due to higher-order interactions / loop contributions
- All SM particles contribute
- Calculate & sum all sectors of the SM





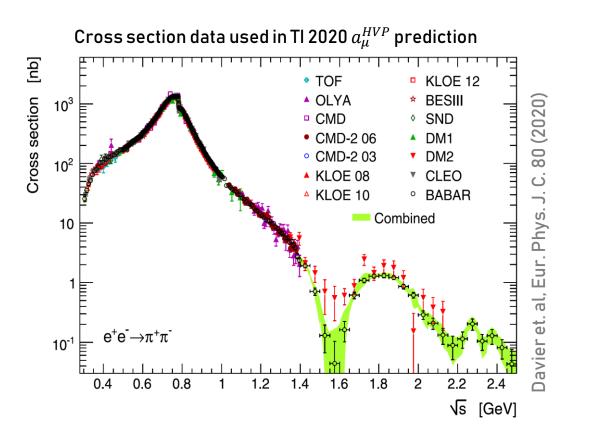
Hadronic terms dominate the uncertainty

Hadronic Vacuum Polarisation (HVP): dispersion relation



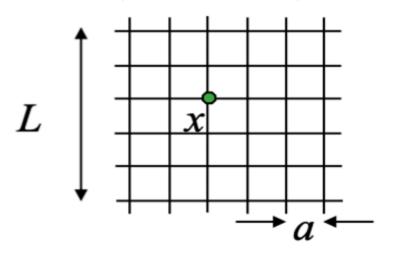
 $a_{\mu}^{
m had,LO} = rac{m_{\mu}^2}{12\pi^3} \int_{s_{
m th}}^{\infty} ds \; rac{1}{s} \hat{K}(s) \sigma_{
m had}(s)$

- 1/s weight → low energies most important
- $\pi^+\pi^-$ contributes 73% to LO
- Need to know total hadronic cross-section $\sigma_{had}(s)$

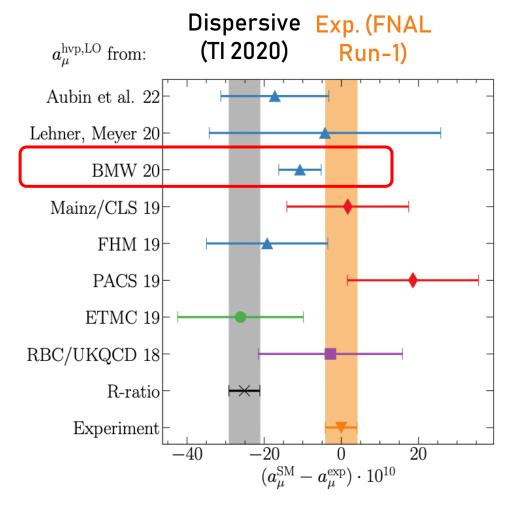


Hadronic Vacuum Polarisation (HVP): lattice approach

- First principle calculation to predict a_u
- Numerical integration on finite space-time lattice → very computationally expensive



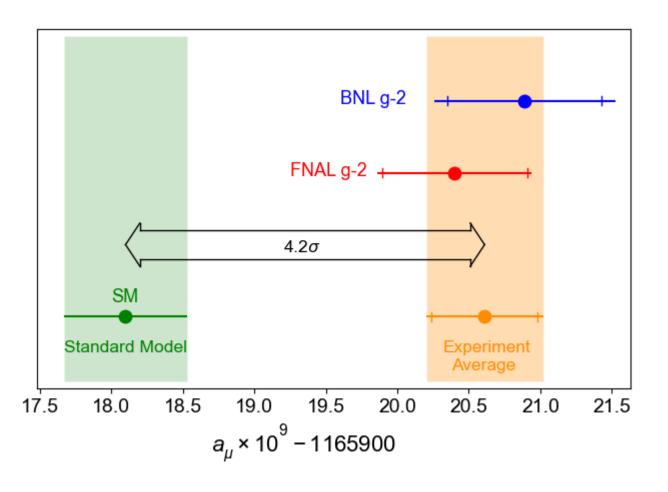
First prediction with <1% uncertainty by BMW



- Lattice results not included in TI 2020 white paper due to low precision
- So far, no other high-precision a_u from different group
- Ongoing cross-checks by many groups at different energies
- Preliminary agreement in 'intermediate energy window'

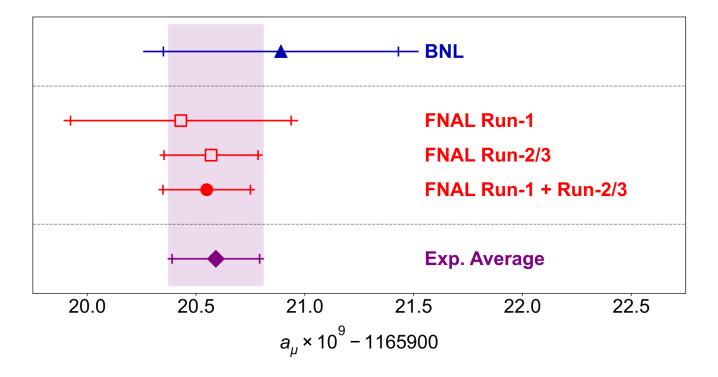
FNAL Measurement: status in 2021

- First announcement of the FNAL experiment using Run-1 (2018) data in April 2021
- Using the TI(2020) value for the SM comparison (data-driven HVP only)



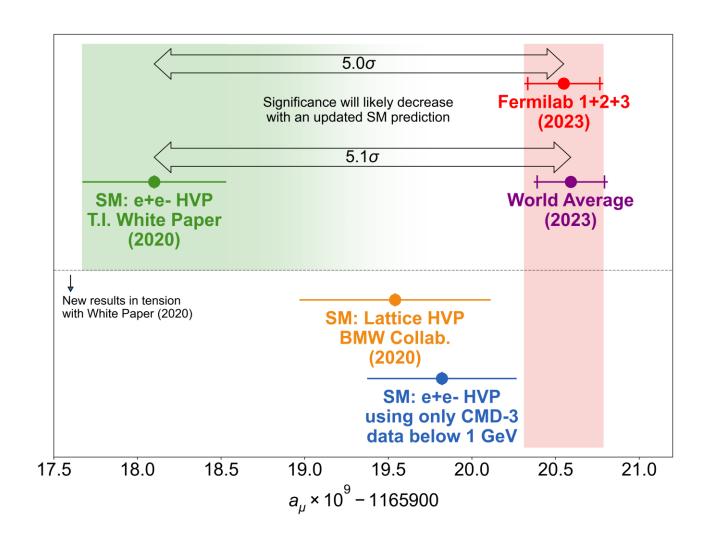
FNAL Measurement: status in 2023

- In August 2023, FNAL published updated result using Run-2 (2019) and Run-3 (2020) data
- >4x higher statistics than the Run-1 result
- Excellent agreement with Run-1 and BNL
- BNL, Run-1 and Run-2/3 statistics dominated
- Assume 100% correlated systematics
- World Average dominated by FNAL



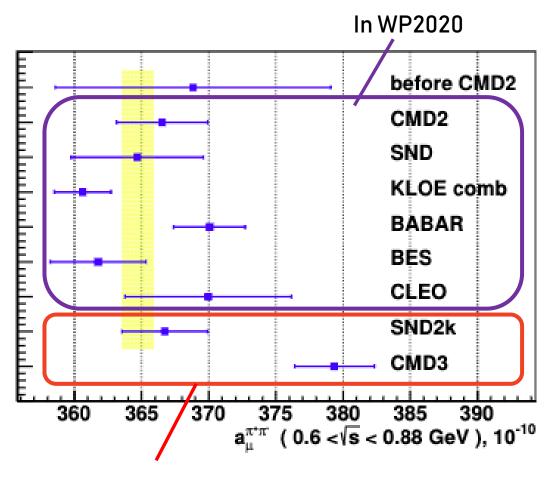
Comparison with the Standard Model

- Tension between dispersive and lattice approaches to HVP
- Lattice result needs independent confirmation
- Long-standing tension between experiment and dispersive result approaches 5σ
- New cross-section results from CMD-3 makes the dispersive situation quite puzzling
- Very dynamic situation: huge effort to reconcile differences and tensions

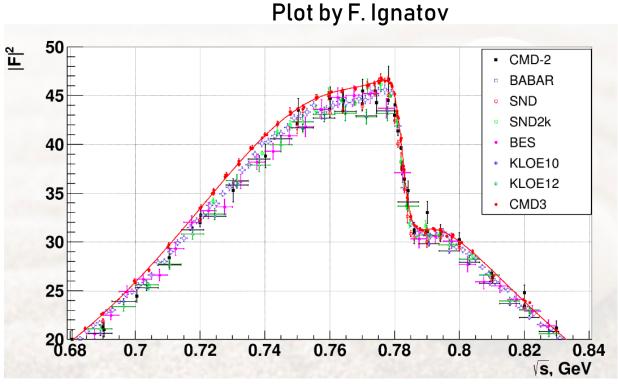


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The CMD-3 result



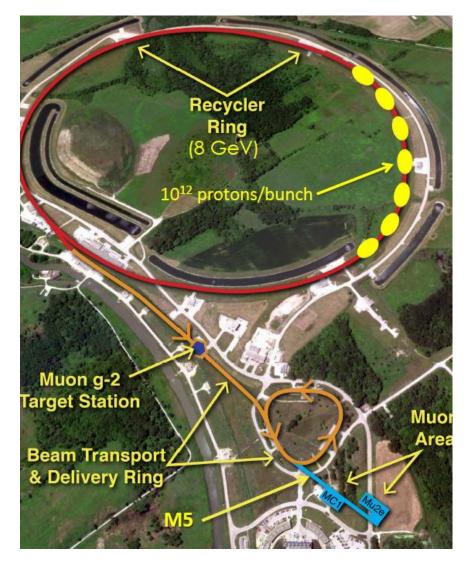
2 new HVP LO results since WP2020



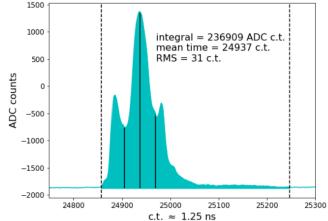
- SND2k and CMD-3 released 2 new results for $a_{\mu}^{\pi^+\pi^-}$ since the theory combination in 2020
- Detailed view of $e^+e^- \to \pi^+\pi^-$ shows **CMD-3** as outlier, but it agrees with lattice

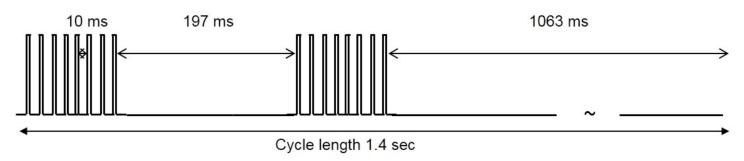
The Fermilab g-2 Experiment

The FNAL Muon Campus

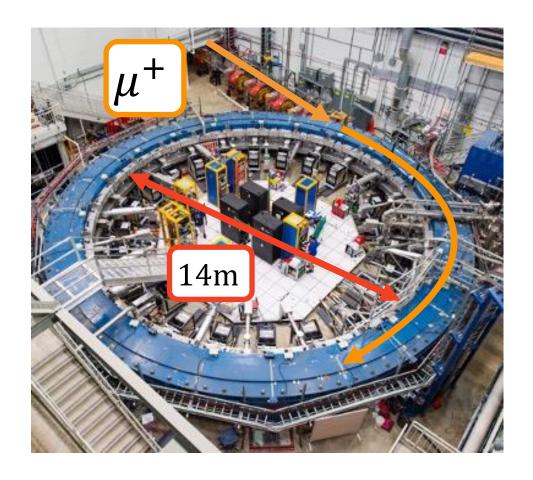


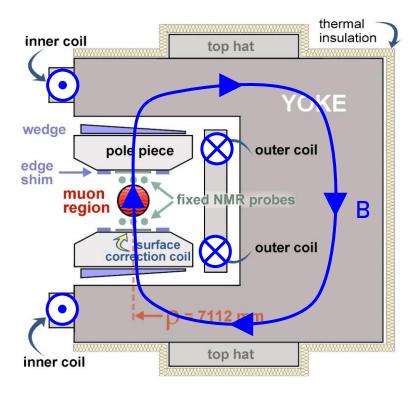
- Recycler ring: 8 GeV protons
 - 10¹² protons / bunch; 10k stored muons / bunch
- 8 GeV protons → pion production target
- Pions → delivery ring → decay to µ⁺ and momentum-selected
- 3.09 GeV $\mu^+ \rightarrow g-2$ storage ring magnet
- Beam arrives in 120 ns wide bunches
- Each bunch corresponds to a "fill" → 1 ms





The Storage Ring

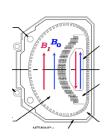


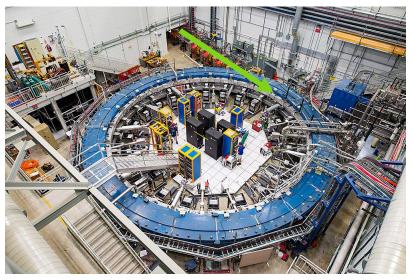


Cross-section slice of magnet 1.45 T vertical B field in muon region

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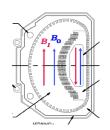
Beam injected through inflector magnet → cancel main magnetic field

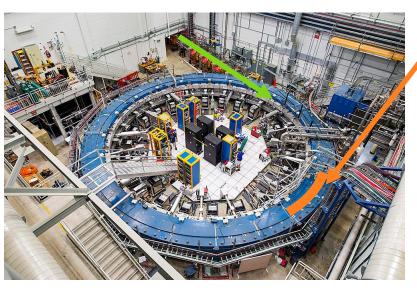


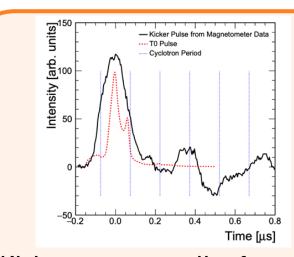


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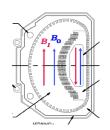


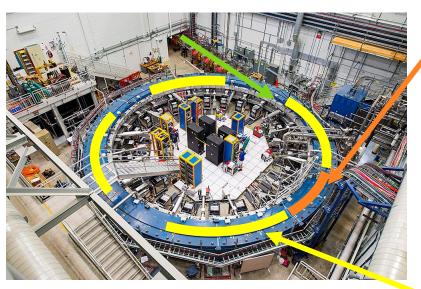


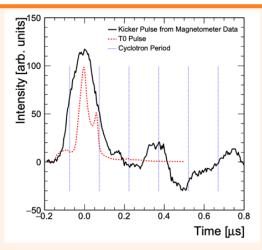


Kicker magnet applies fast radial magnetic field on first turn

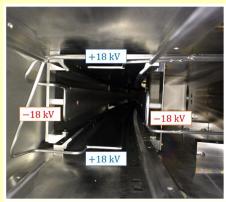
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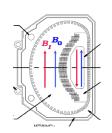


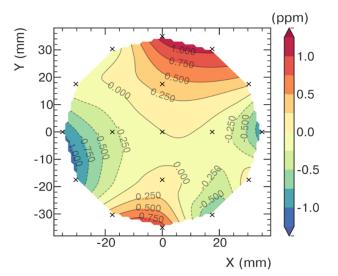
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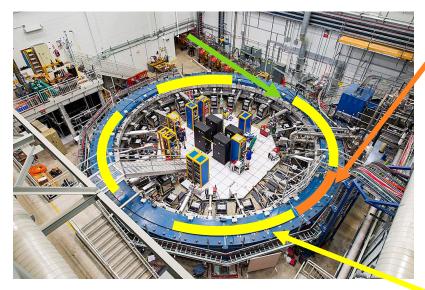
Quads pulse throughout muon fill -> vertical focusing

Beam injected through inflector magnet → cancel main magnetic field





Magnetic field map measured every 3 days, changes monitored continuously

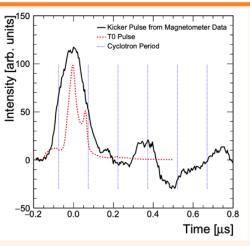


run 29709 :: Dipole

RMS: 17.4 ppm, peak-to-peak: 114.6 ppm, avg: 51.036 kHz

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0 50 100 150 200 250 300 350

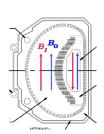


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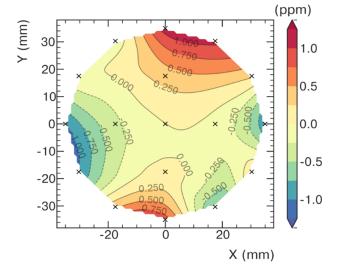


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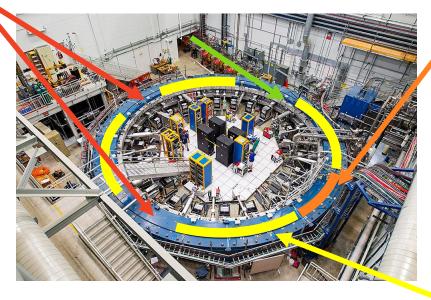
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Straw trackers measure beam profile at 180° and 270°



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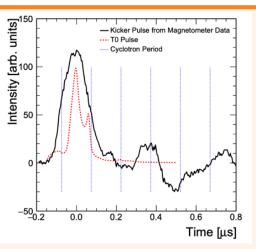


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PE 52
0 50 100 150 200 250 300 350

azimuth [deg]



Kicker magnet applies fast radial magnetic field on first turn

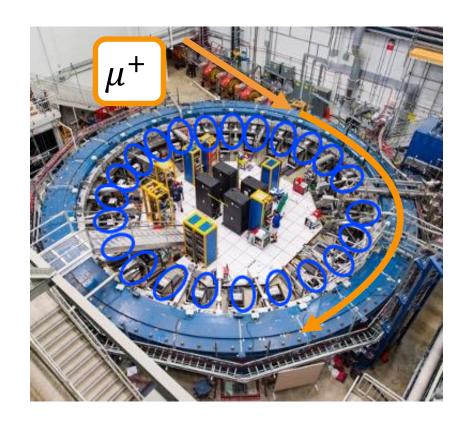


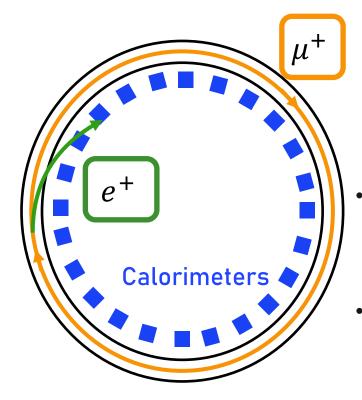
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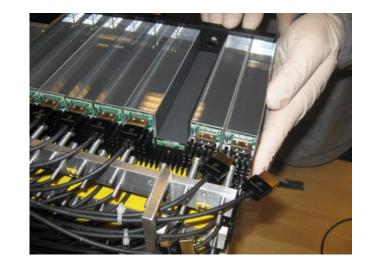
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Calorimeter detectors

- Positive muons, 3.09 GeV/c, injected into Storage Ring in 125 ns bunches
- Muons orbit inside storage ring, spin precesses about vertical B field
- Decay to positrons → lower momentum → spiral into centre of storage ring



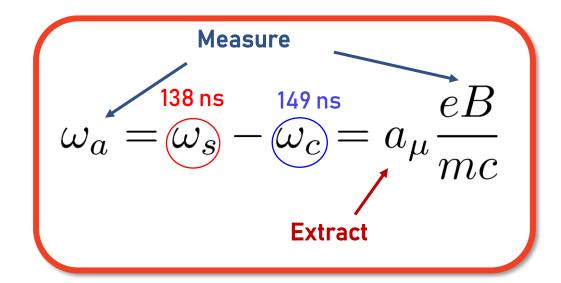


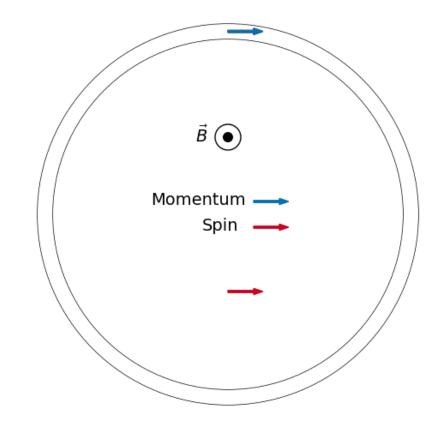


- Each calo is a 6x9 array of PbF₂ crystals
 - 2.5 cm x 2.5 cm x 14 cm $(15X_0)$
- Readout by SiPMs to 800 MHz WFDs (1296 channels in total)

Spin Precession in the Storage Ring

- Muons orbit the ring with cyclotron frequency ω_c
- Spin precesses with frequency ω_s
- Both spin and cyclotron frequencies are proportional to B
- Spin rotates ahead of momentum as the muon orbits the ring
- Difference frequency ω_a is proportional to a_u and B

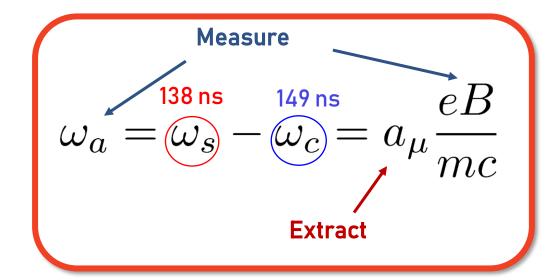


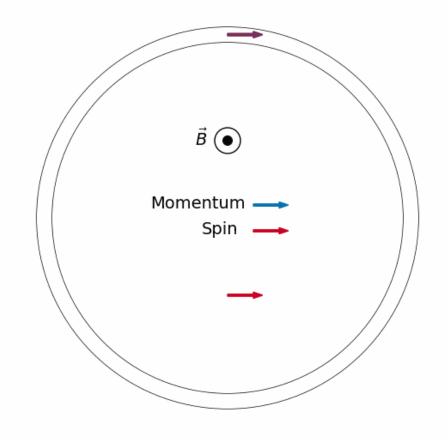


• If g were exactly 2, $\omega_s = \omega_c$ and $\omega_a = 0$

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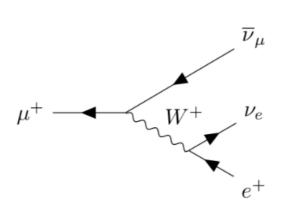


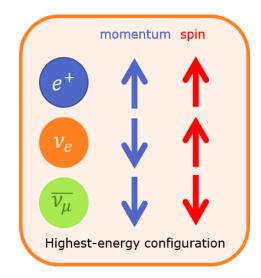


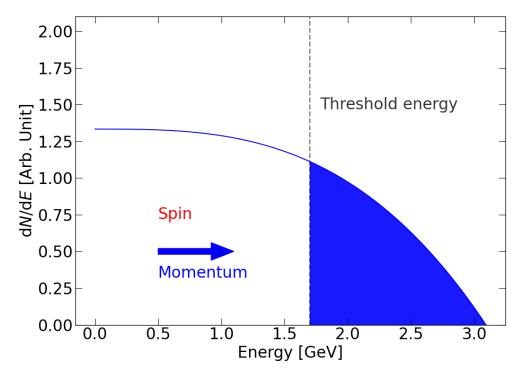
• If g were exactly 2, $\omega_s = \omega_c$ and $\omega_a = 0$

Measuring ω_a

- Highest energy e⁺ preferentially emitted in direction of muon spin
- Oscillation in number of emitted decay e⁺ vs time directly proportional to decay asymmetry
- Count N(t) = number of e⁺ above an energy threshold over time.
- Higher fraction detected above threshold when e⁺ emitted parallel to muon spin
- N(t) depends on anomalous precession frequency ω_a



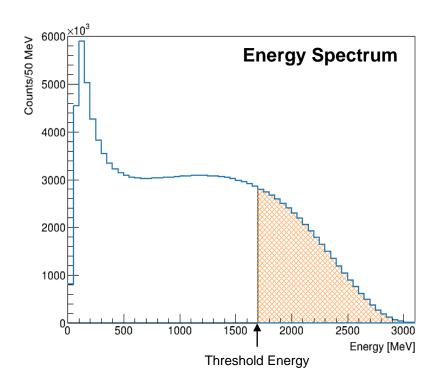


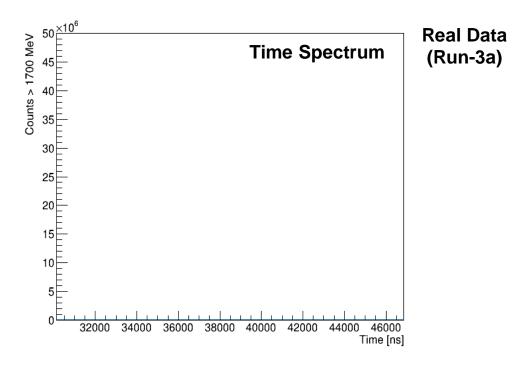


Frequency of oscillation = ω_a

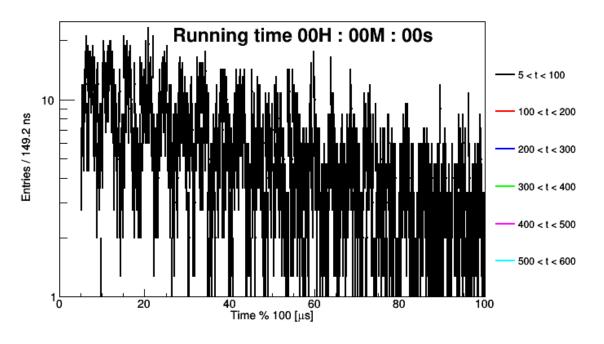
Measuring muon spin precession

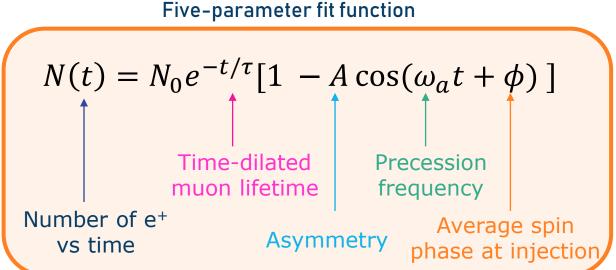
- Number of high energy positrons oscillates as a function of time as muon spin points towards/away from detector
- Count positrons above an energy threshold
- Counts oscillate at frequency $\omega_a \rightarrow \text{extract from time spectrum}$



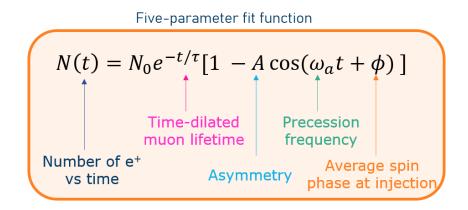


Extracting ω_a : 5-parameter fit function

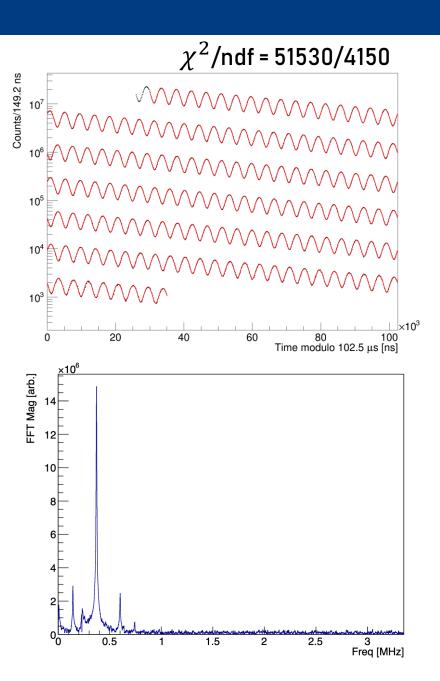




Fitting the wiggle

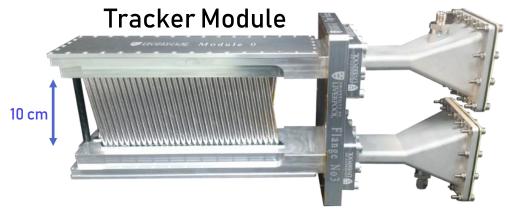


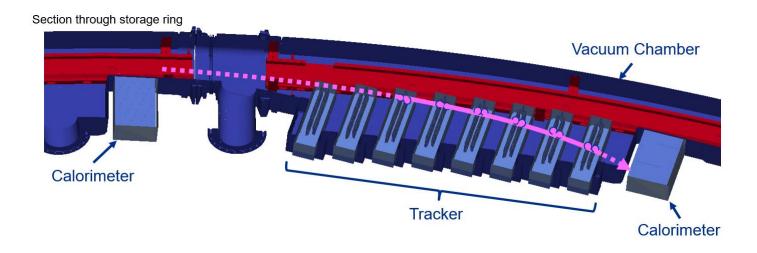
- Simplest 5-parameter fit function captures exponential decay and the g-2 oscillation only
- Not sufficient → large spikes in residuals FFT
- Need to measure and account for additional terms in the fit



Tracking detectors: measuring the muon beam

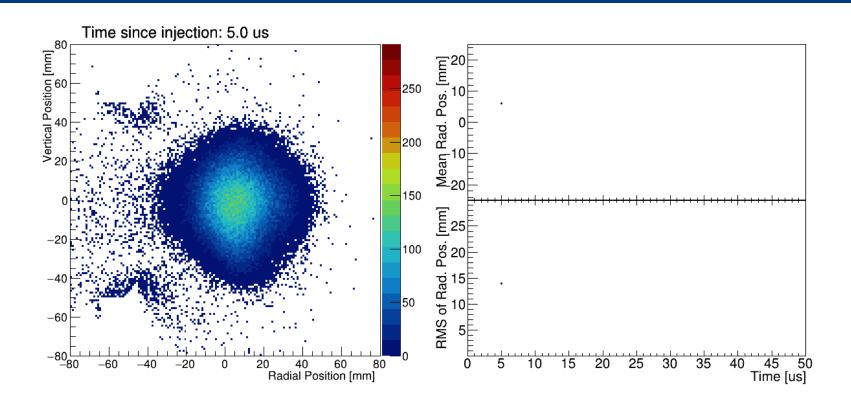
- > Straw trackers: two stations located at 180° and 270°
- > One station = 8 modules
- > One module = 128 Argon-Ethane gas-filled straws
- > Fit tracks and extrapolate to muon decay point







Tracker measurement of the muon beam

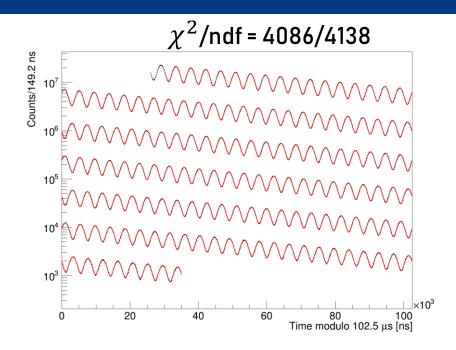


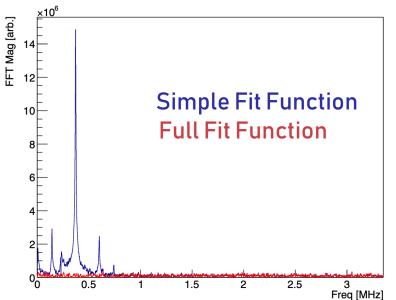
- The mean and width of the muon beam distribution varies as a function of time
- Average radial and vertical position vary around the ring \rightarrow measure at two locations
- Frequency of beam oscillations must be measured \rightarrow affects ω_a measurement

Fitting the wiggle

$$N(t) = Ne^{-t/\tau_{\mu}} \left[1 + A \cdot \cos(\omega_{a}t - \phi + \phi_{BO}(t)) \right] \cdot \left(1 + A_{CBO} \cos(\omega_{CBO}t - \phi_{CBO})e^{-\frac{t}{\tau_{CBO}}} \right) \cdot \left(1 + A_{VW} \cos(\omega_{VW}t - \phi_{VW})e^{-\frac{t}{\tau_{VW}}} \right) \cdot \left(1 + A_{2CBO} \cos(\omega_{2CBO}t - \phi_{2CBO})e^{-\frac{t}{\tau_{2CBO}}} \right) \cdot \left(1 + A_{y} \cos(\omega_{y}t - \phi_{y})e^{-\frac{t}{\tau_{y}}} \right) \cdot \left(1 - k_{LM} \int_{0}^{t} L(t')e^{t'/\tau_{\mu}}dt' \right) \cdot \left(1 + \left[A_{+} \cos(\omega_{+}(t)t - \phi_{+}) + A_{-} \cos(\omega_{-}(t)t - \phi_{-}) \right] e^{-\frac{t}{\tau_{CBOVW}}} \right)$$

- Accounting fully for all the beam oscillations shifts ω_a by 1.6 ppm
- Final fit function has 27 fit parameters
- 8 different analysis teams, 19 separate analyses





Extracting a_{μ}

Relativistic expression for ω_a

$$\vec{\omega}_a = -\frac{q}{m} \left[a_{\mu} \vec{B} - \left(\frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(a_{\mu} - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{\mathcal{E}}}{c} \right]$$

Non-relativistic limit

Motion nonperpendicular to B-field

Cyclotron motion assumes motion perpendicular to B-field

Relativistic motional magnetic field

Proportional to electric field

Disappears for "magic" γ = 29.3 \rightarrow magic momentum 3.09 GeV/c

Not all muons are at the 'magic' momentum of 3.1 GeV

Vertical momentum component aligned with B field

E-field correction

$$C_E = \frac{\Delta \omega_a}{\omega_a}$$

Pitch correction

$$C_p = \frac{\Delta \omega_a}{\omega_a}$$

"Never measure anything but frequency..."

$$\vec{\omega}_a = -\frac{q}{m} \left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{\mathcal{E}}}{c} \right]$$

10.5 ppb Exact 22 ppb 0.28 ppt

$$a_{\mu} = \left[\frac{\omega_{a}}{\widetilde{\omega}'_{p}(T_{r})} \right] \frac{\mu'_{p}(T_{r})}{\mu_{e}(H)} \frac{\mu_{e}(H)}{\mu_{e}} \frac{m_{\mu}}{m_{e}} \frac{g_{e}}{2}$$

- We measure the ratio of two frequencies: ω_a and ω_p
 - ω_a : anomalous spin precession frequency
 - ω_p : muon-weighted magnetic field expressed in terms of Larmor frequency of free proton

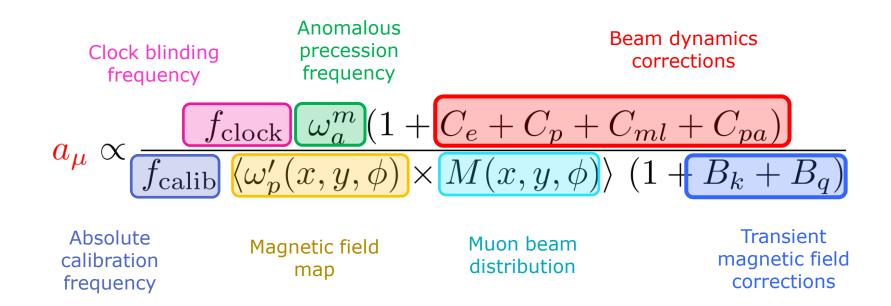
Phys. Rev. A 83, 052122 (2011)

Metrologia 13, 179 (1977)

Rev. Mod. Phys. 88 035009 (2016)

Phys. Rev. Lett. 82, 711 (1999)

"... and corrections"



"... and corrections"

$$a_{\mu} \propto \frac{f_{\text{clock}} \ \omega_{a}^{m} \left(1 + C_{e} + C_{p} + C_{ml} + C_{pa}\right)}{f_{\text{calib}} \left\langle \omega_{p}'(x, y, \phi) \times M(x, y, \phi) \right\rangle \left(1 + B_{k} + B_{q}\right)}$$

Absolute calibration frequency

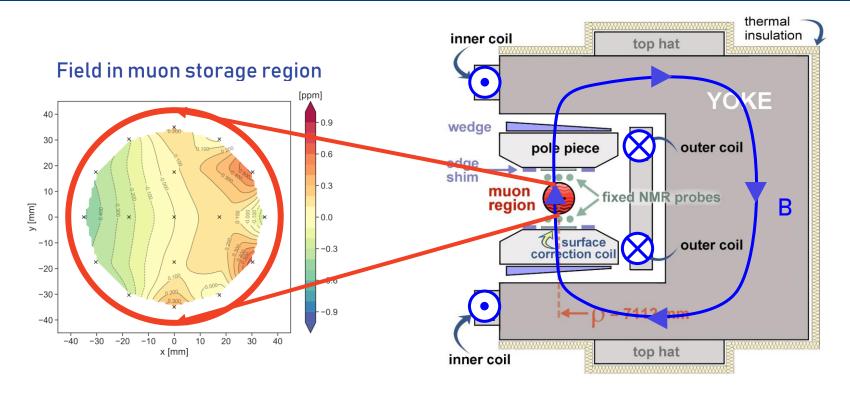
Magnetic field map

Muon beam distribution

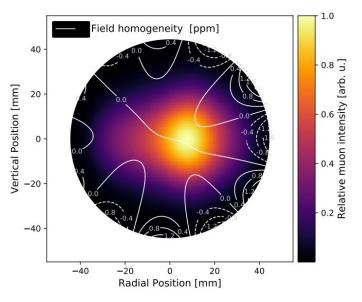
Transient magnetic field corrections

Measuring the magnetic field

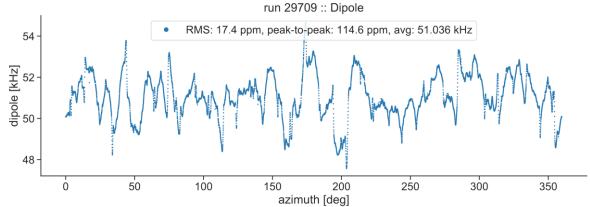
The g-2 Storage Ring magnet



Weight measured field by the muon beam distribution



- Dipole field has ppm-level uniformity (<20 ppm RMS across the full azimuth)
- Shimming devices (active and passive) minimise gradients and keep field uniform



Measuring the magnetic field

Trolley containing 17 NMR probes maps full azimuth every few days (muons not present)

378 fixed probes monitor between trolley runs (during muon data collection)

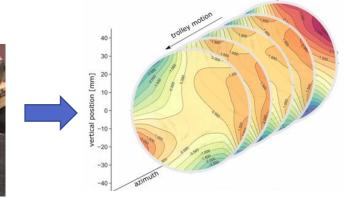
Field map is interpolated between trolley runs using fixed probe information

Fold with muon beam distribution

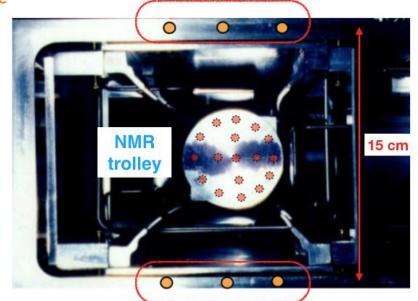


RF coil

NMR



Sequence 2D field slices as trolley moves



Absolute Calibration

- Trolley and fixed NMR probes use petroleum jelly as the proton sample low volatility
- Need to measure protons in H_20 (measurement standard) \rightarrow calibration

• Trolley and cylindrical H_20 calibration probe switch places to repeatedly measure the same field

in the same place. Calibration performed ~once per year.

Calibration region calibration region Uncertainty 17 ppb holder for 17 NMR probes multi-/duplexer & preamplifier RF amplifier enclosure NMR electronics enclosure 3D translation support rail structure cable clamp barcode reader

Trolley probes

11/03/2024

Cylindrical H₂0 probe



Spherical H₂0



Spherical ³He

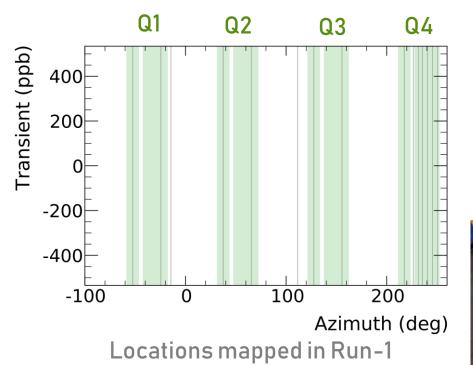


Cross-check with spherical probes
Uncertainty: 9 ppb

Liv.INNO Seminar 44

Transient fields from pulsed systems: Quads

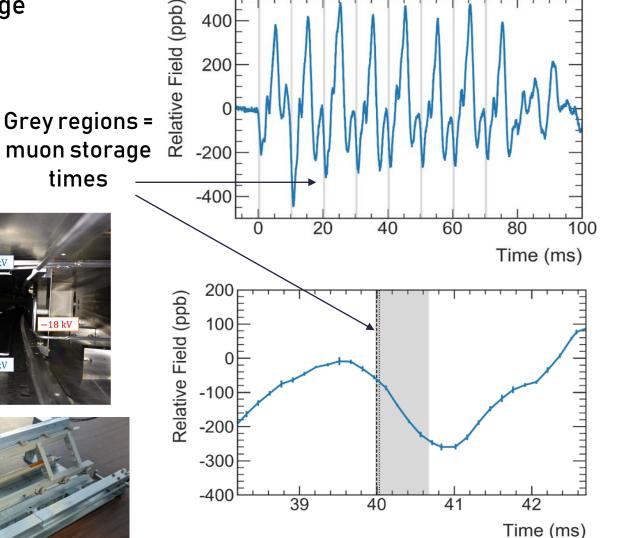
- Electrostatic quadrupoles pulse during muon storage
- No pulsing during trolley runs
- Quad plates vibrate \rightarrow induces a field
- Muons experience a field change which the fixed probes do not see (shielded by Al)
- Must be measured using dedicated apparatus





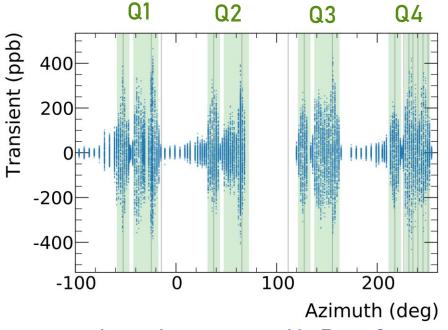
times



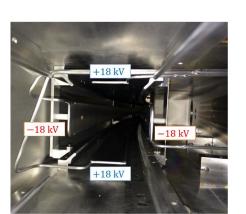


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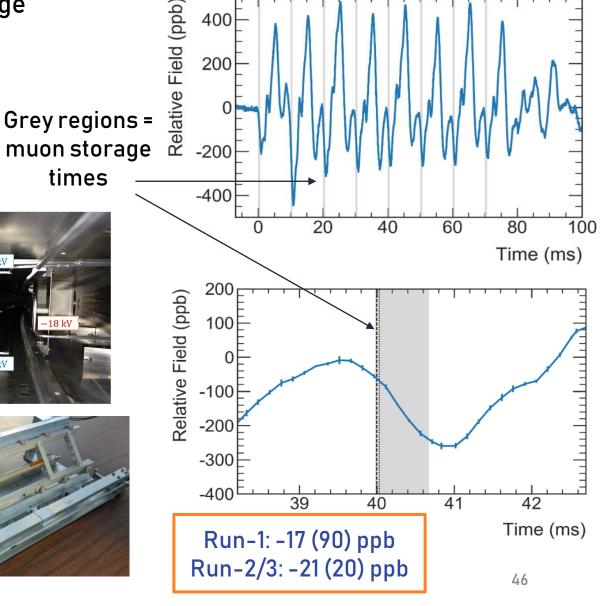






times

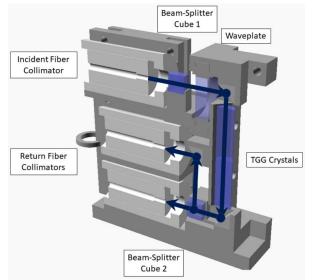


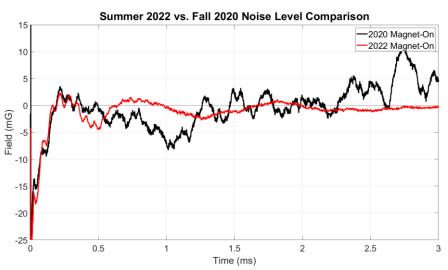


Transient fields from pulsed systems: Kicker

- Kicker pulse of 22 mT for 150 ns just after muon injection
- Field change caused by residual field (eddy currents) after kicker pulse
- Muons present from $30\mu s$ to $700\mu s$ after the kick (fit region)
- Measure eddy current using Faraday magnetometer
- Run-2/3: improved hardware setup reduced vibrations

Correction Run-1: -27 (37) ppb Run-2/3: -21 (13) ppb





units]

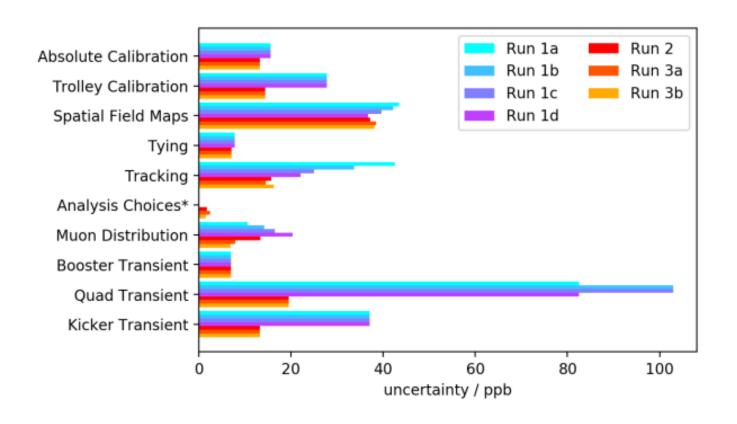
Intensity [arb. u

···· T0 Pulse Cyclotron Period

0.2

Time [µs]

Magnetic field systematics in Run-2/3

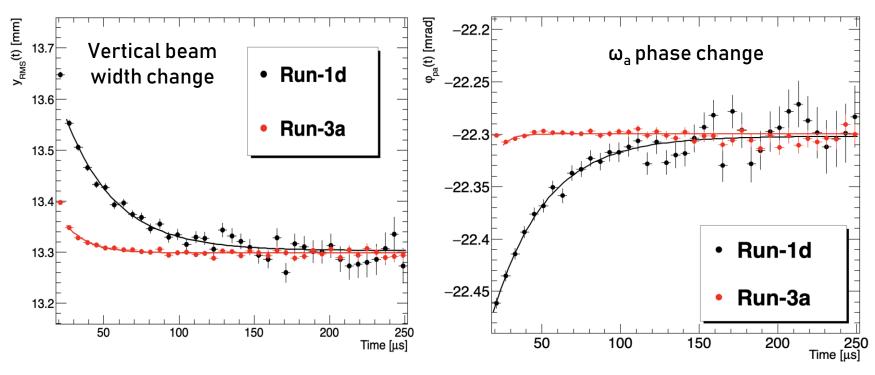


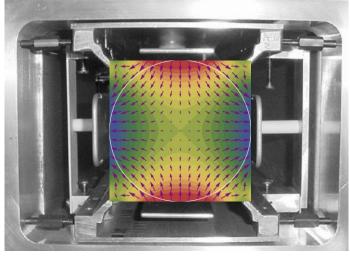
- Main reduction in uncertainty comes from better understanding of transient field effects
- Interpolation uncertainty has also reduced with number of trolley runs
- Uncertainty already at TDR goal

Improvements between Run-1 and Run-2/3

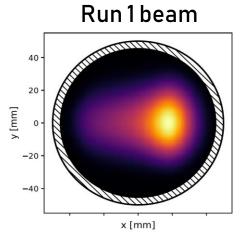
Damaged resistors

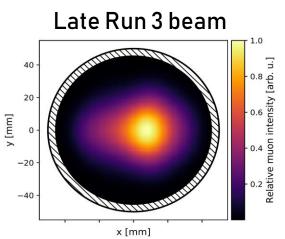
- In Run-1, 2 out of 32 resistors damaged in quad plates
 → unstable beam storage
- Redesigned and replaced before Run-2
- Reduces phase acceptance (C_{pa}) uncertainties 75ppb \rightarrow 13 ppb
- Beam oscillation frequencies also become more stable

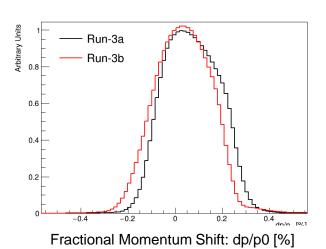


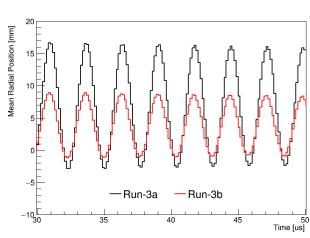


Kicker strength upgrade



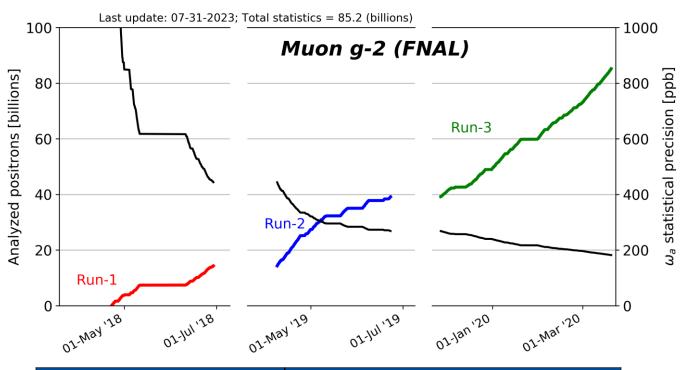






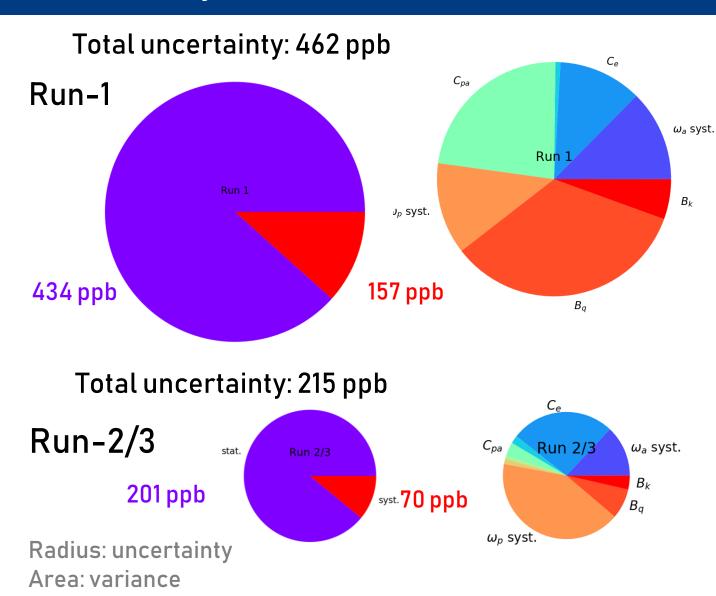
- Upgraded kicker cables while Run-3 so kicker can run at nominal strength
- Muon distribution more centered
 - All systematics related to asymmetric beam shape reduced
- Muon momentum distribution better centered on magic momentum
 - E field correction reduced
- Phase space matching improved
 - Smaller beam oscillations

Statistical improvement: Run-2/3 vs Run-1



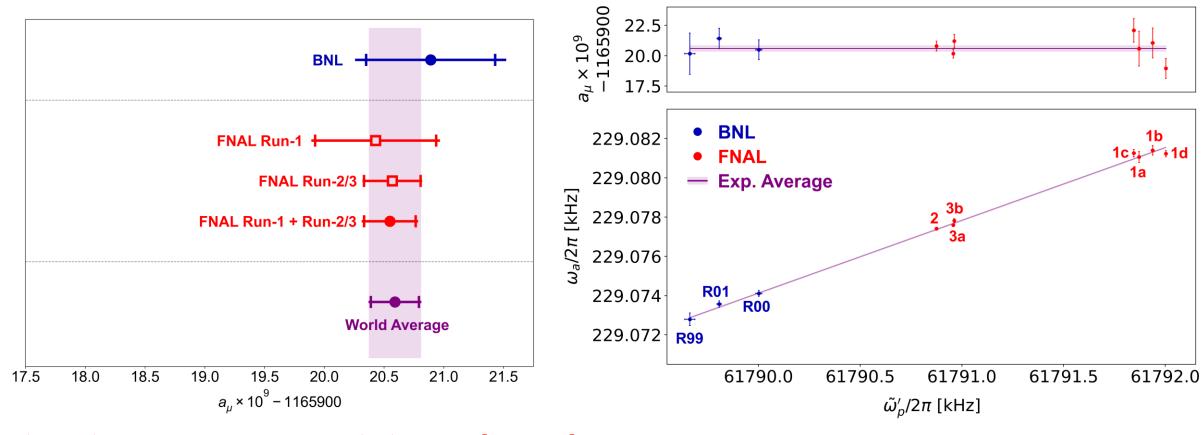
Dataset	Statistical Error [ppb]
Run-1	434
Run-2/3	201
Run-1+ Run-2/3	185

Total uncertainty



- Uncertainty reduced by factor >2
- Statistic and systematic uncertainty reduced by similar amount
- Systematic uncertainty below TDR goal
- Still statistics dominated

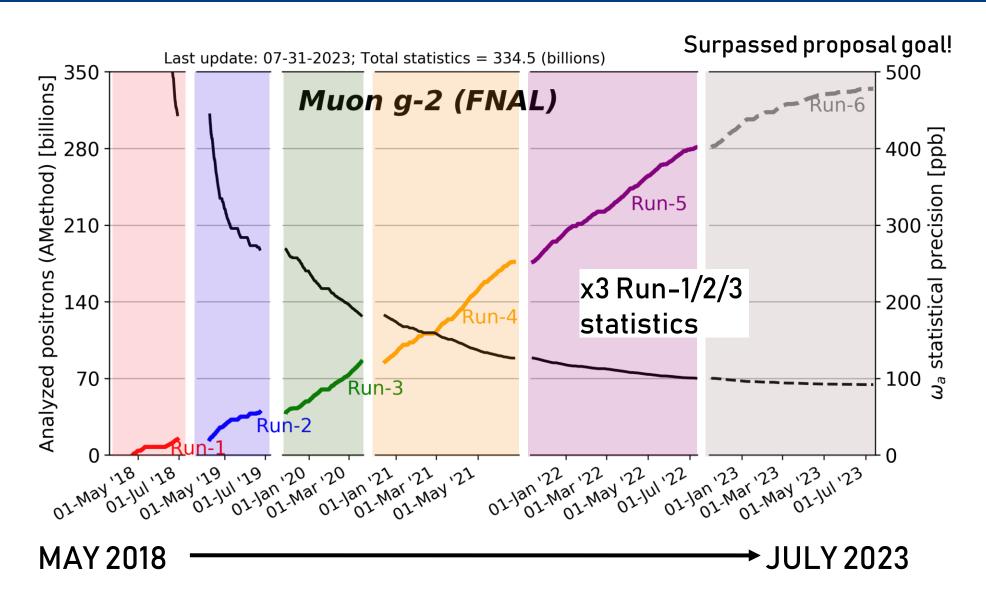
Unblinded result & cross-checks



- $a_{\mu}(Run-1) =$ $a_{\mu}(Run-2/3) =$ $a_{\mu}(FNAL) =$ $a_{\mu}(Exp) =$
- 0.00 116 592 040(54) 0.00 116 592 057(25) 0.00 116 592 055(24) 0.00 116 592 059(22)

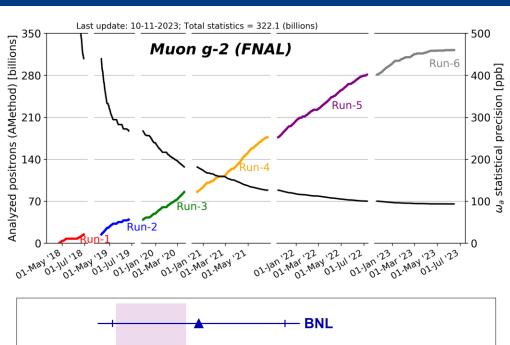
- [463 ppb] [215 ppb] [203 ppb] [190 ppb]
- Different runs performed at different magnet setpoints – a useful consistency check
- Also divide datasets by other variables to check consistency (e.g. day/night, temperature, etc)

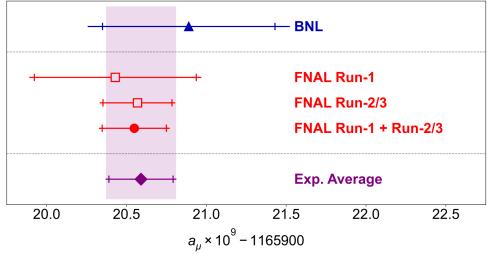
Beyond Run-2/3: Total statistics



Conclusions

- High precision measurements of Muon g-2 stringent test on SM theory
- Run-2/3 data consistent with Run-1 and BNL
- Factor > 2 in statistical and systematic uncertainty
- Surpassed TDR goals in statistics and systematics
- First time a three-way comparison of a_{μ} is possible (dispersive-approach, lattice approach, experiment)
- Another reduction by factor of 2 in statistical uncertainty from Run-4/5/6
- Expect final result in 2025





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