

Probing the nature of electroweak symmetry breaking with Higgs boson pair-production at ATLAS

Lake Louise Winter Institute 2022

Maximilian Swiatlowski

TRIUMF



The Higgs Potential

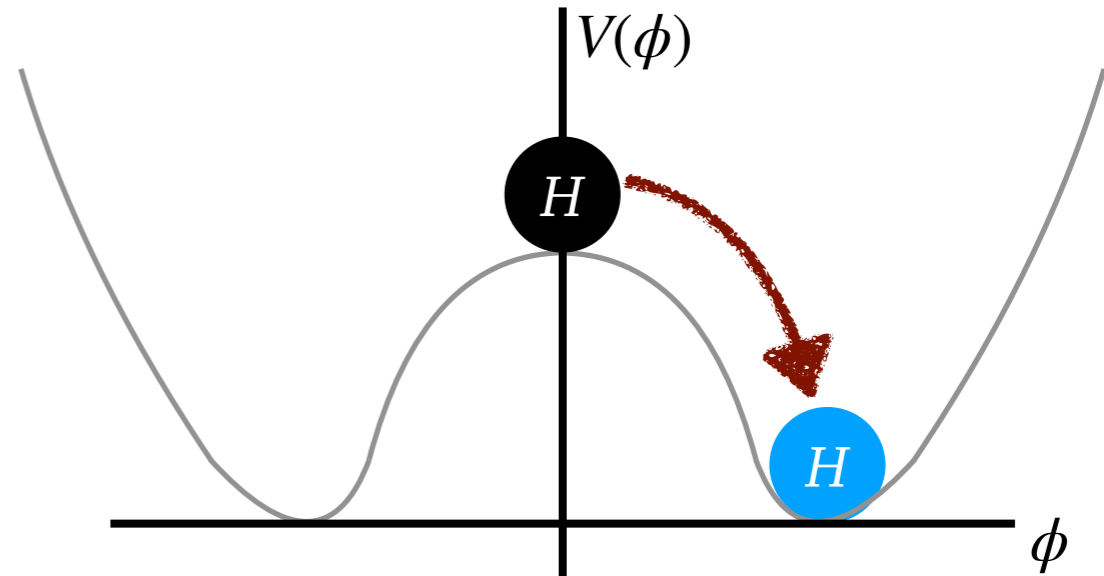


The Higgs Potential



The SM Higgs potential is:

$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4$$



The Higgs Potential



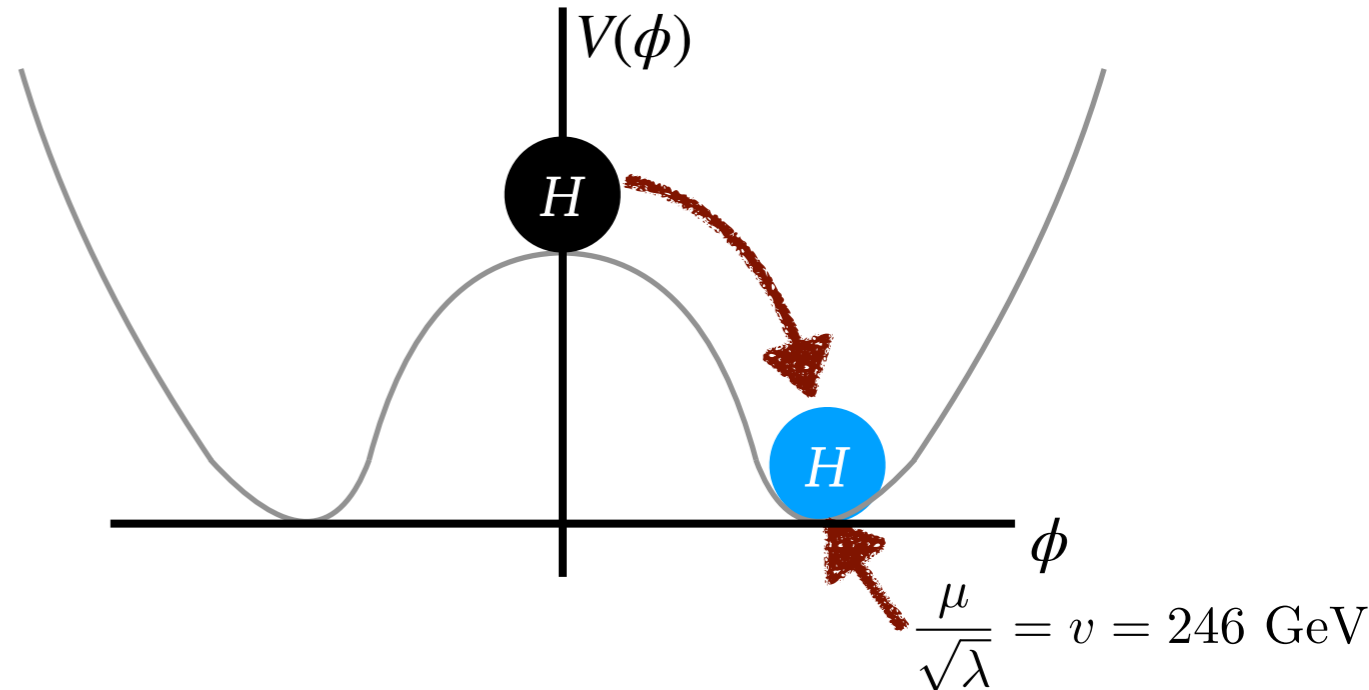
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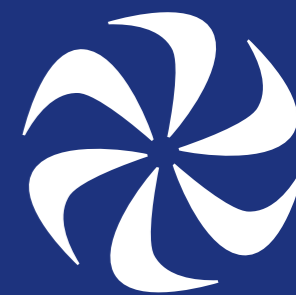
Our universe lives in the minimum:

$$V = V_0 + \lambda v^2 h^2 + \lambda v h^3 + \dots$$

$$= V_0 + \frac{1}{2}m_H^2 h^2 + \frac{m_h^2}{2v^2} v h^3 + \dots$$



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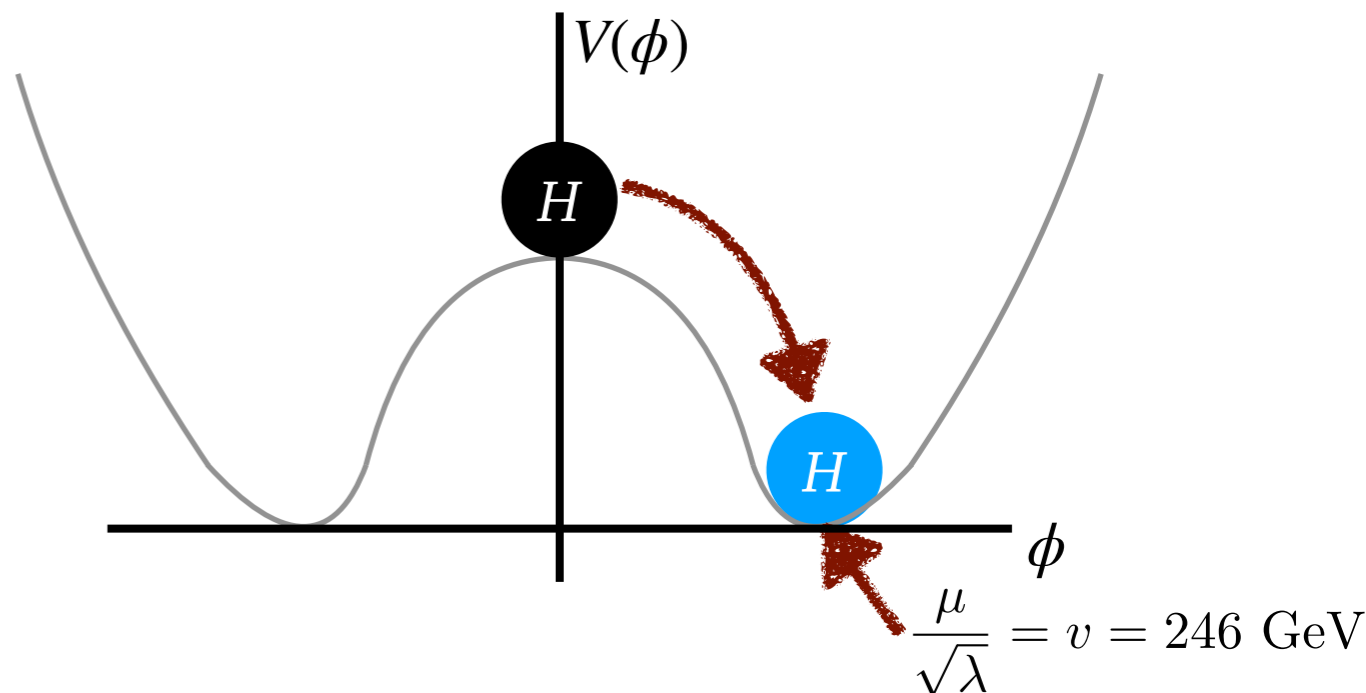
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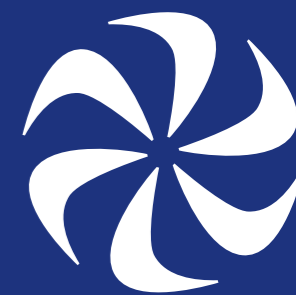
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Mass term



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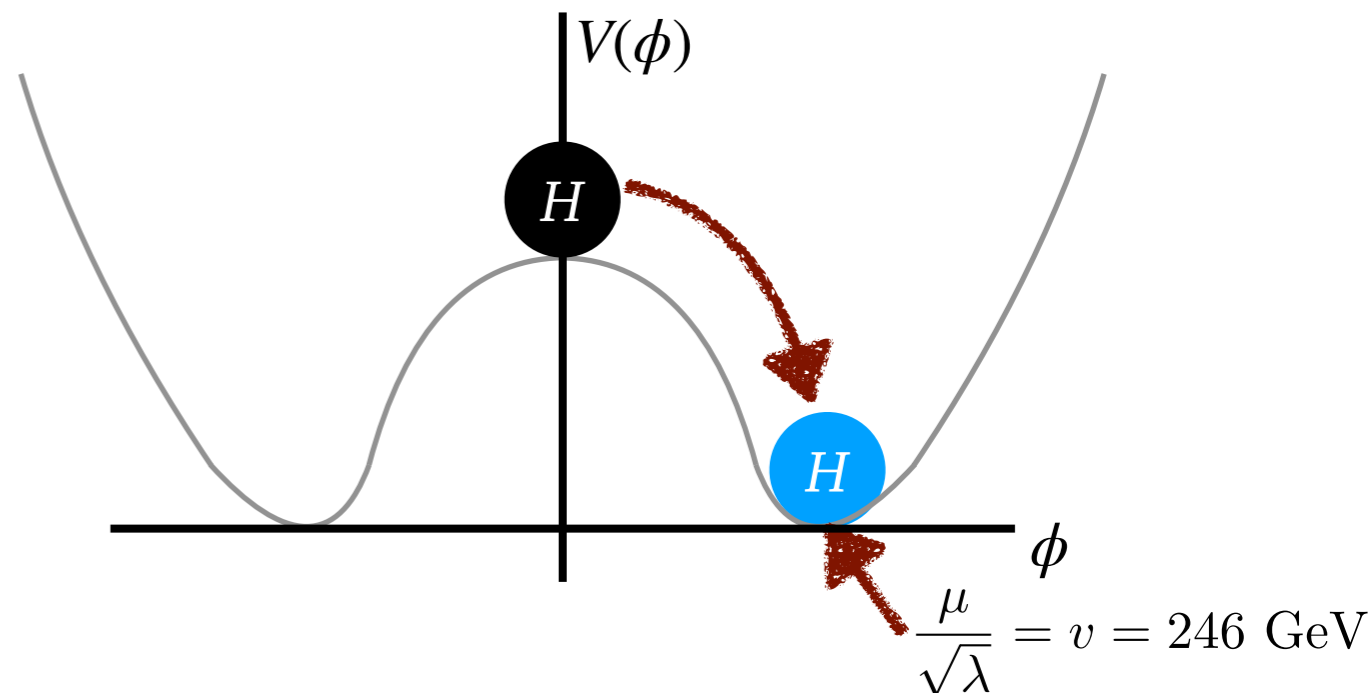
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Mass term  Self-interaction 



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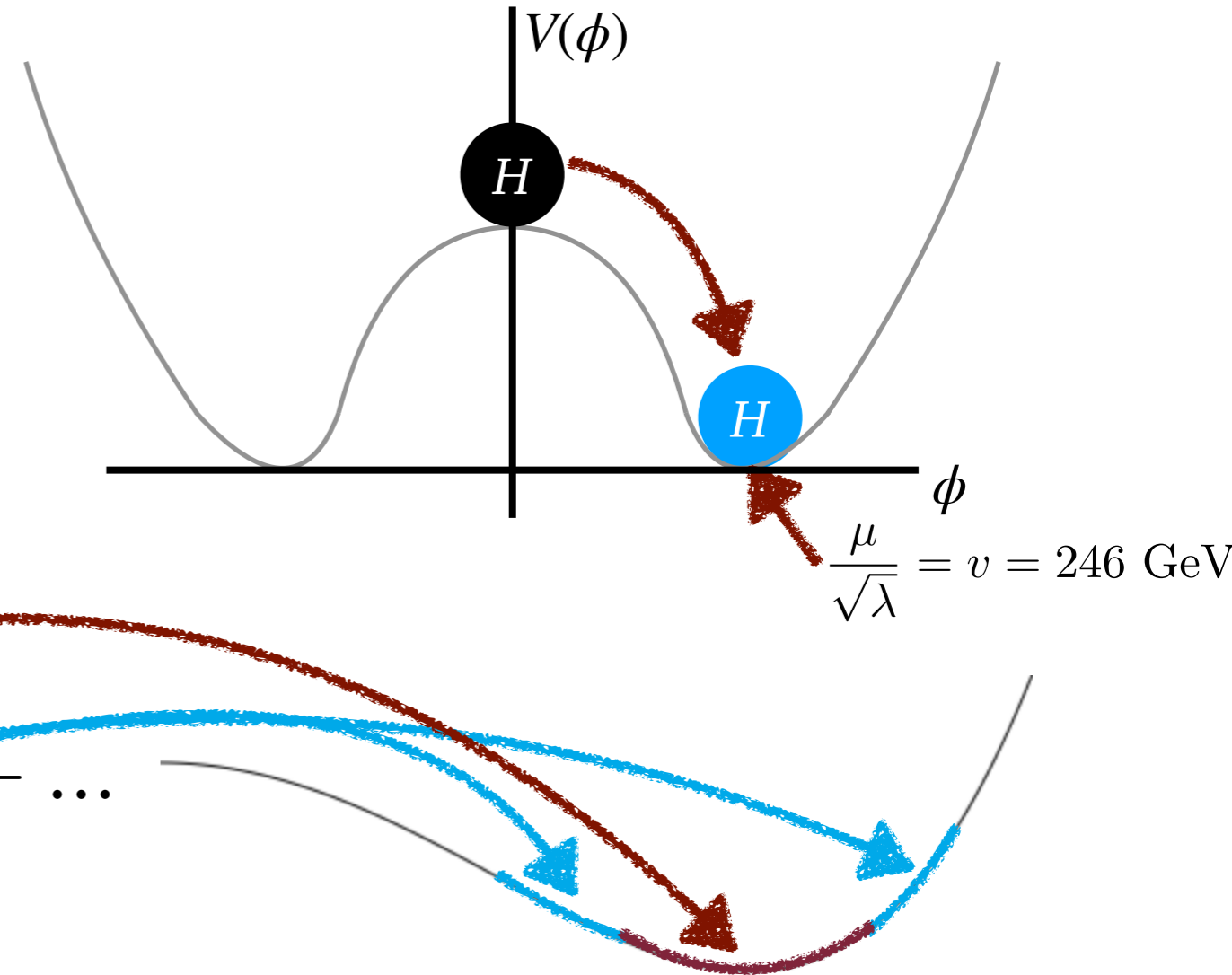
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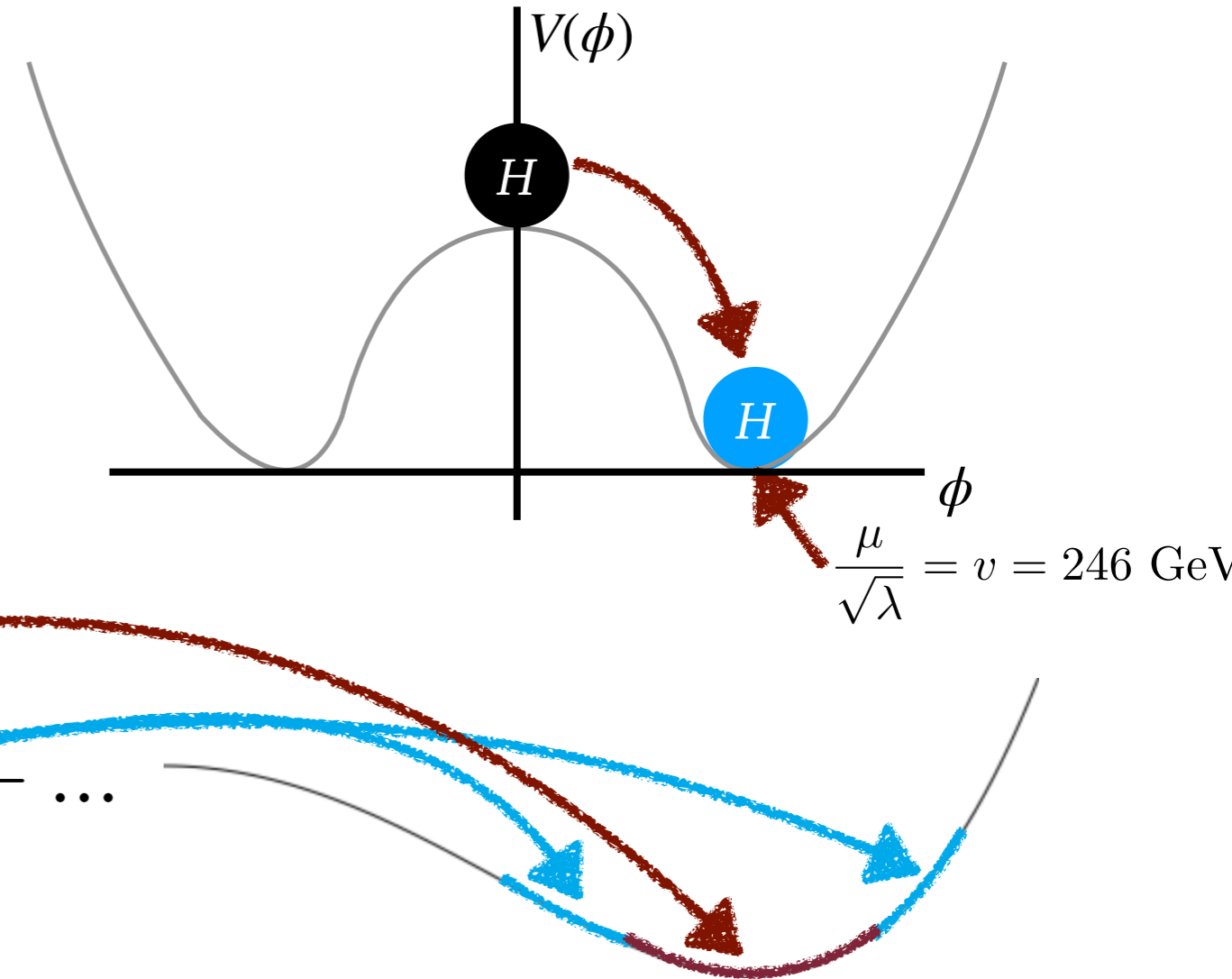
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Mass term Self-interaction



$$\lambda_{HHH}^{SM} = \frac{m_h^2}{2v^2}$$

$$\kappa_\lambda = \frac{\lambda_{HHH}}{\lambda_{HHH}^{SM}}$$

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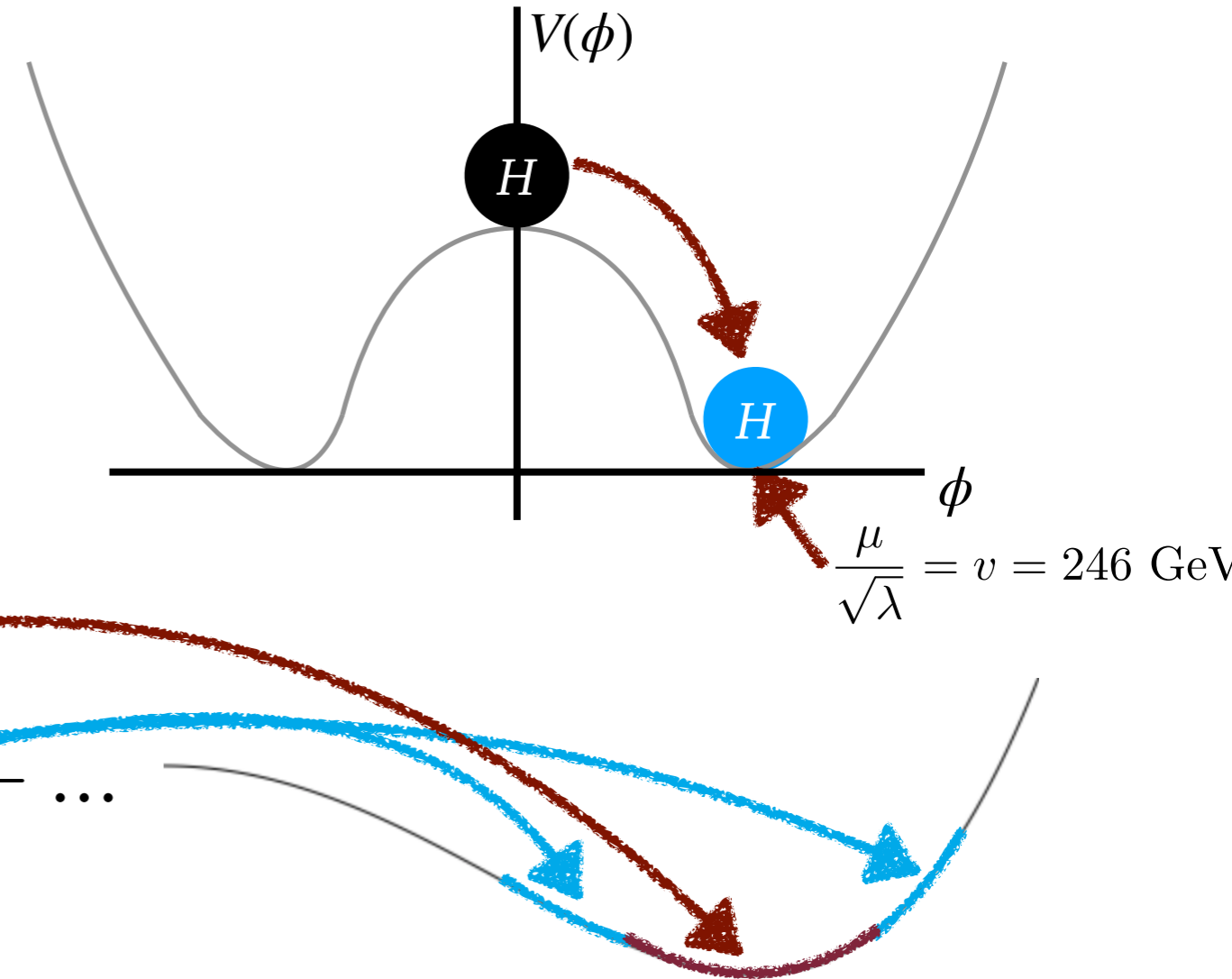
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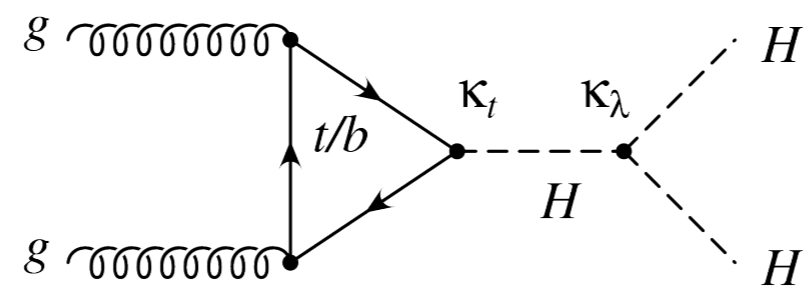
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Mass term Self-interaction



$$\lambda_{HHH}^{SM} = \frac{m_h^2}{2v^2}$$

$$\kappa_\lambda = \frac{\lambda_{HHH}}{\lambda_{HHH}^{SM}}$$



SM predicts HH production, and gives measures potential

The Higgs Potential



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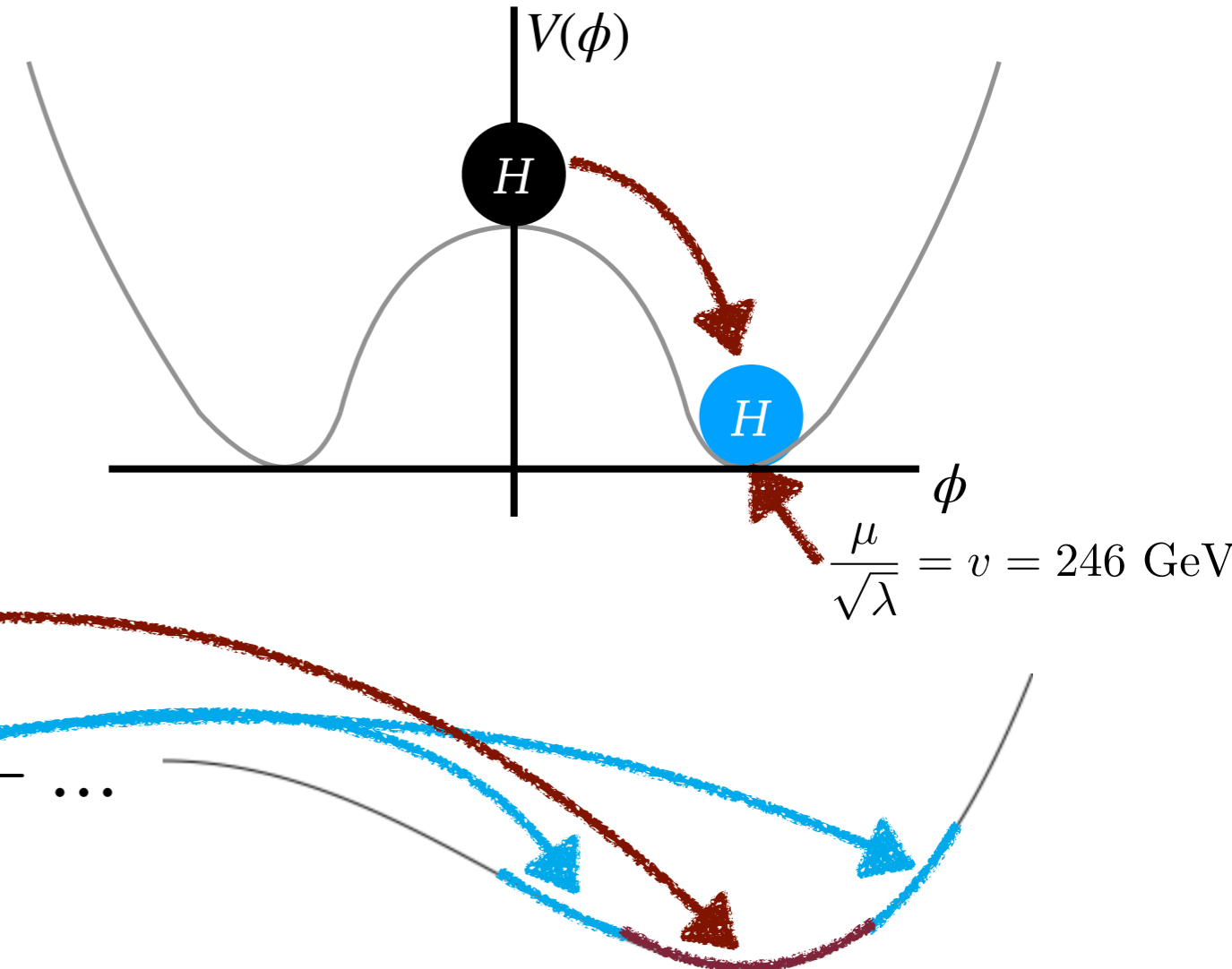
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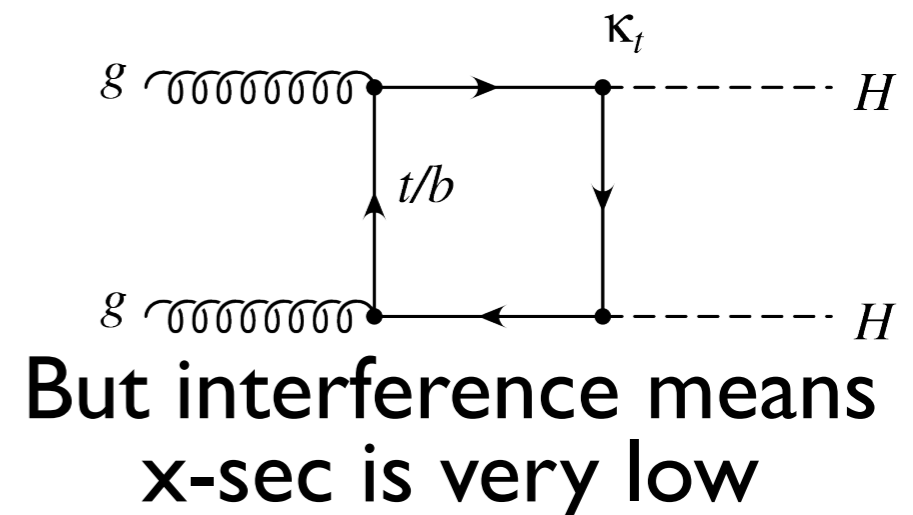
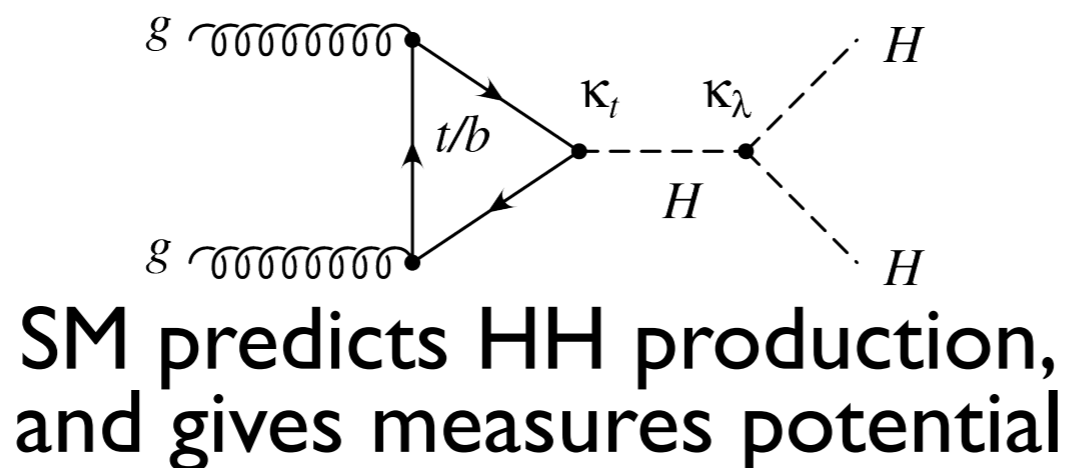
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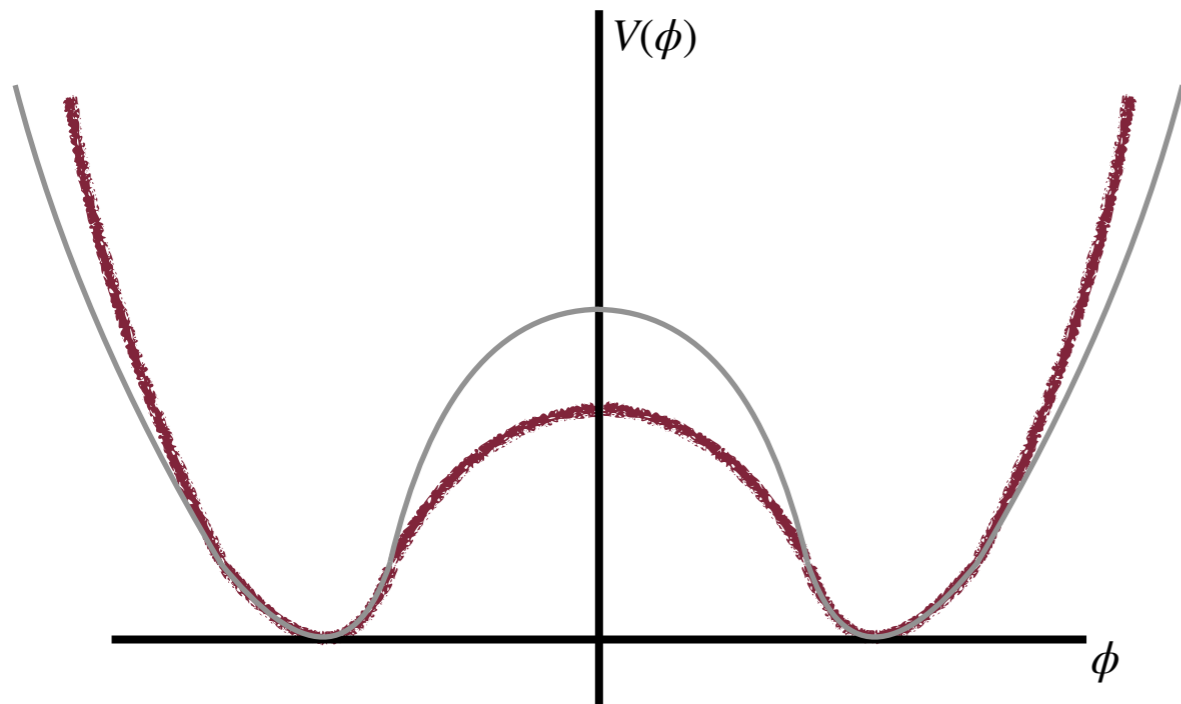
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Potential Higgs Potentials



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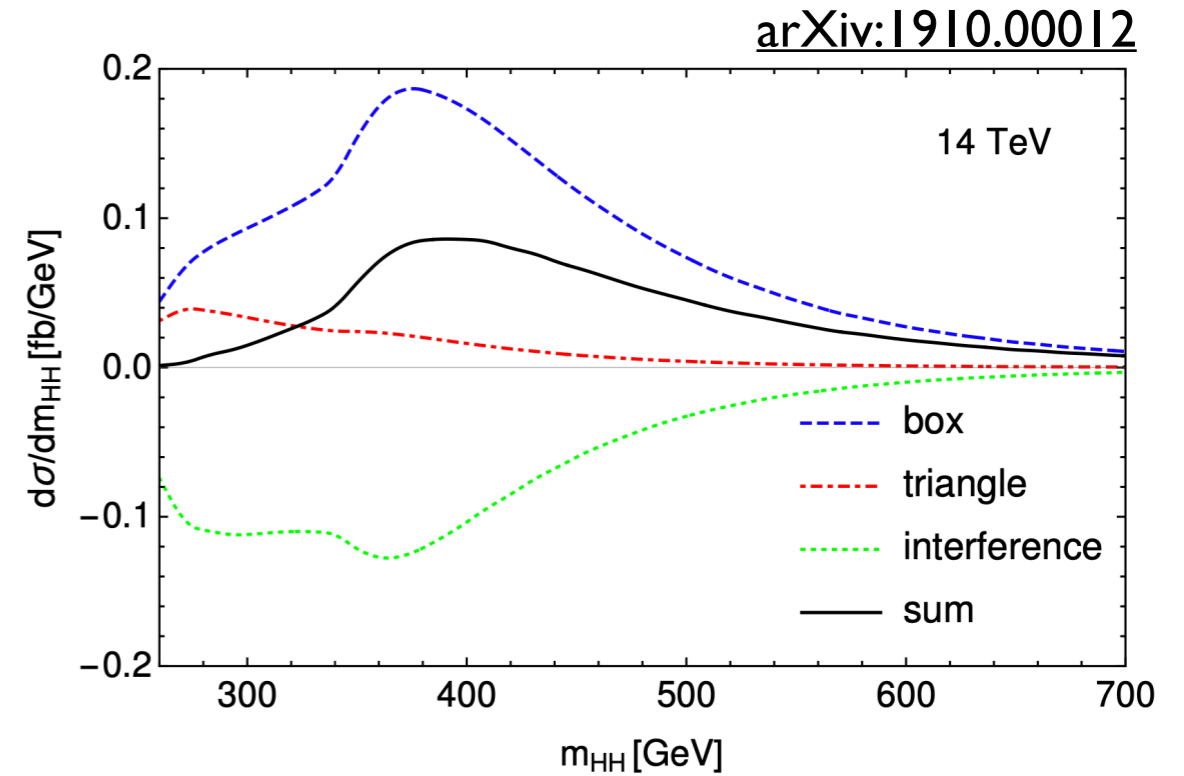
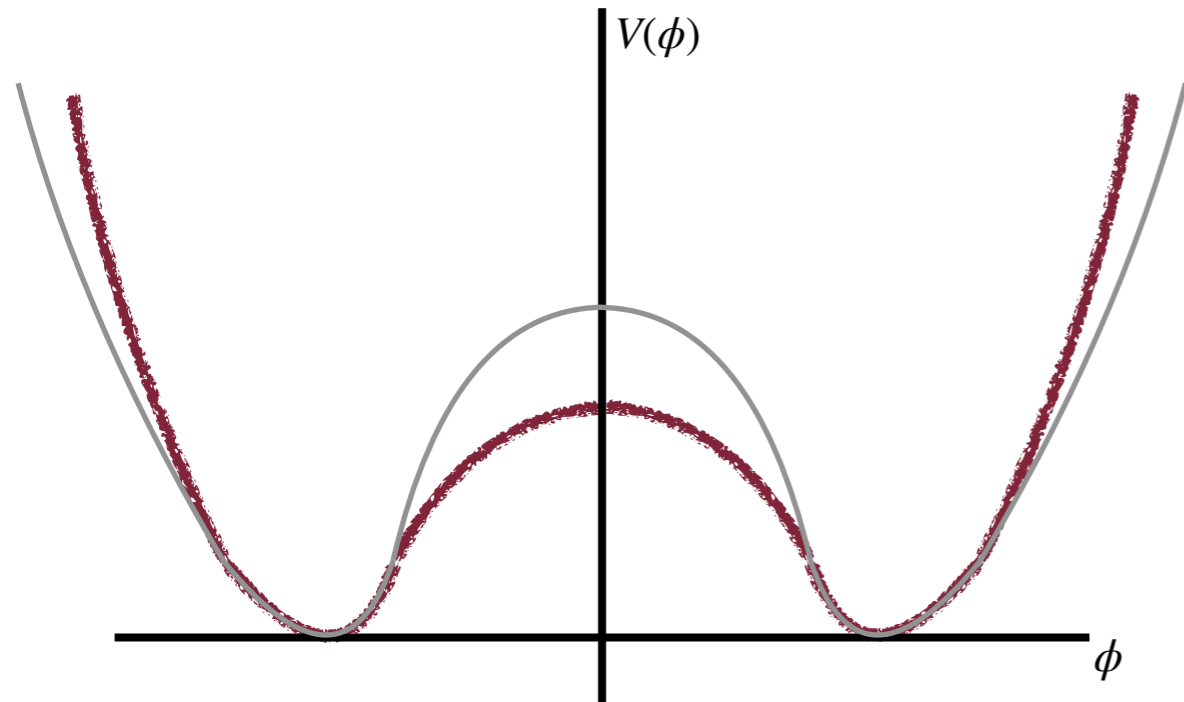


We have a prediction for the shape
from the SM...

But **other shapes** of the potential
still allow for
Electroweak Symmetry Breaking

Other shapes could reveal evidence
for *Electroweak Baryogenesis*, or hints
to vacuum stability

Potential Higgs Potentials



We have a prediction for the shape from the SM...

But **other shapes** of the potential still allow for Electroweak Symmetry Breaking

Other shapes could reveal evidence for *Electroweak Baryogenesis*, or hints to vacuum stability

Signal distribution strongly depends on κ_λ

Increasing κ_λ leads the 'triangle diagram' to dominate: signal peak shifts to lower m_{HH}

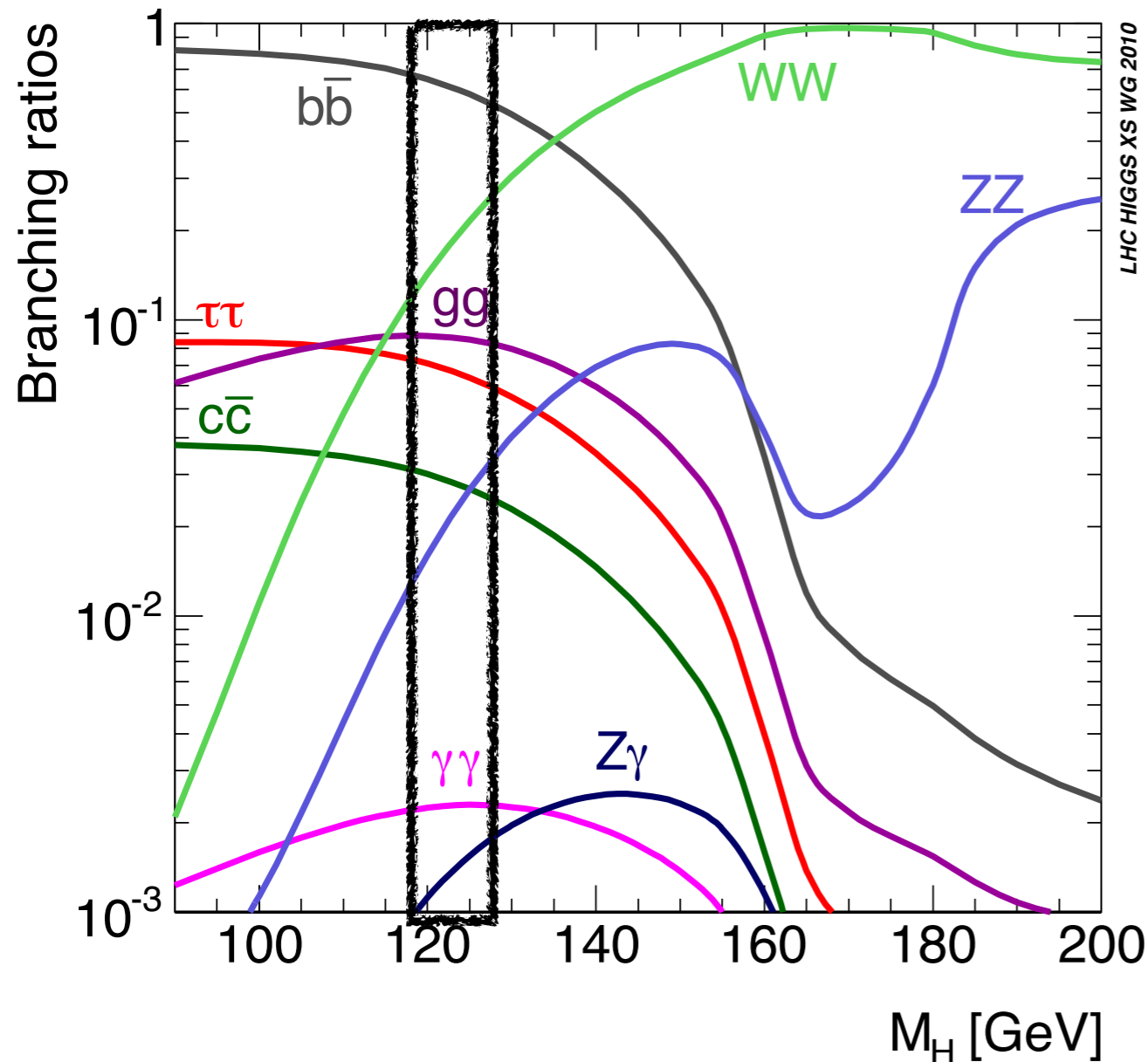
What Does This Look Like?



What Does This Look Like?



The Higgs decays instantly, to a range of particle types

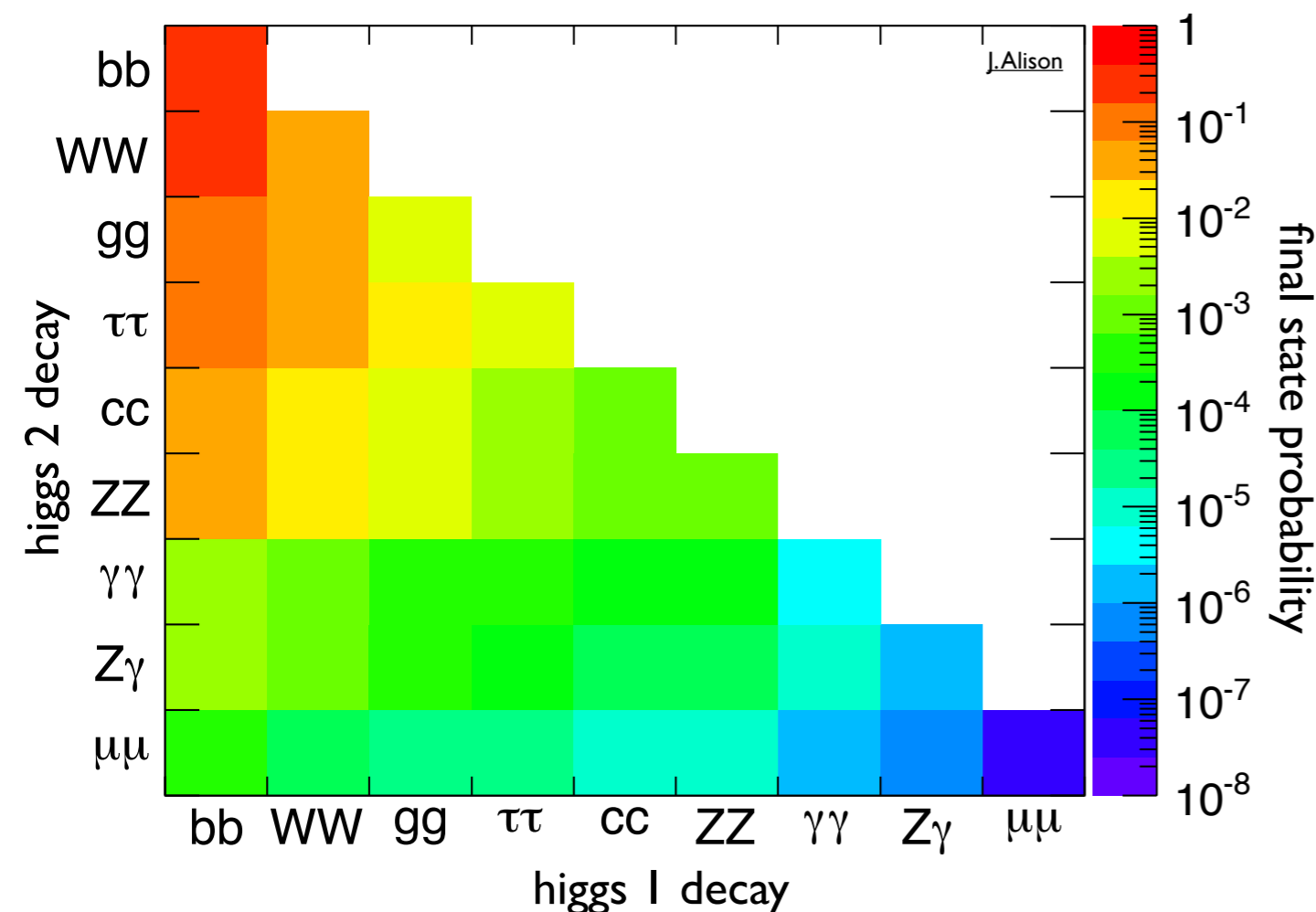


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The Higgs decays instantly, to a range of particle types

Higgs pairs are rare, and have a hugely rich structure of final states

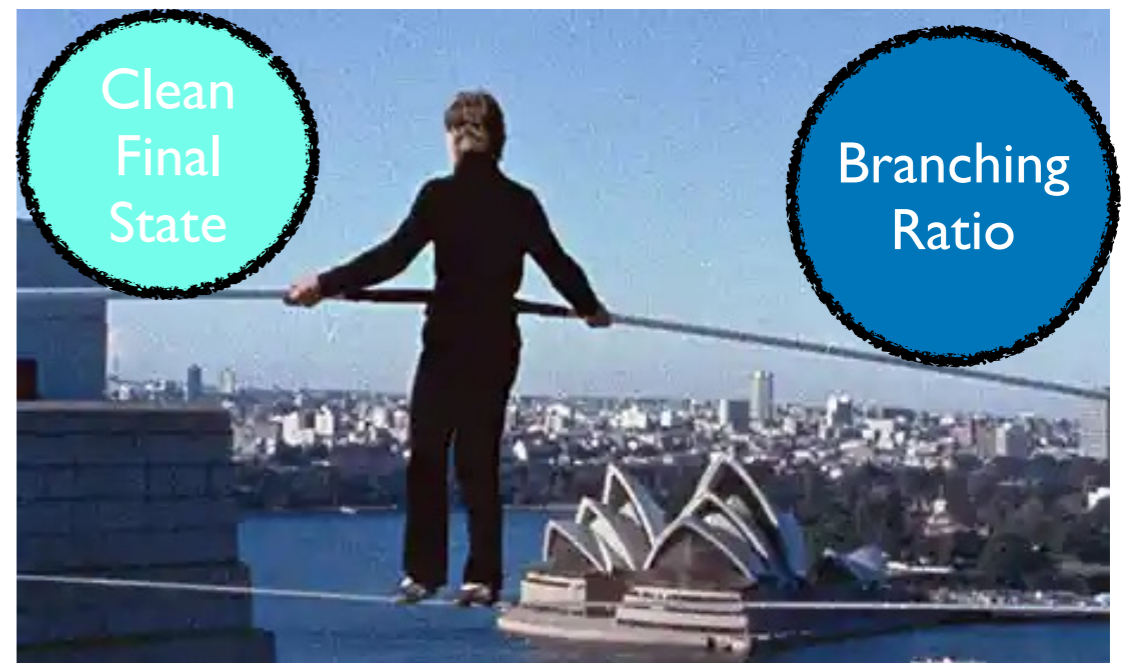
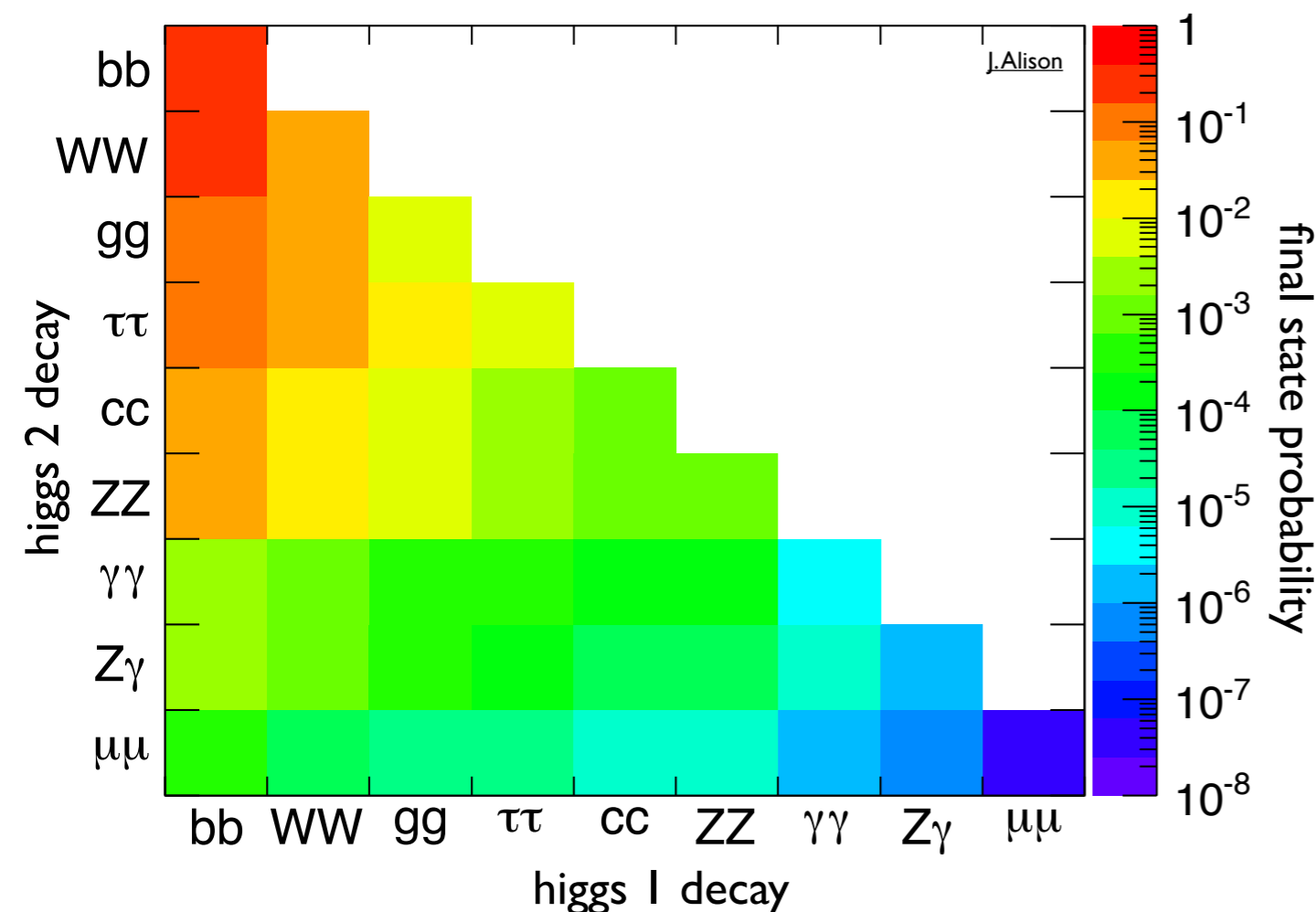


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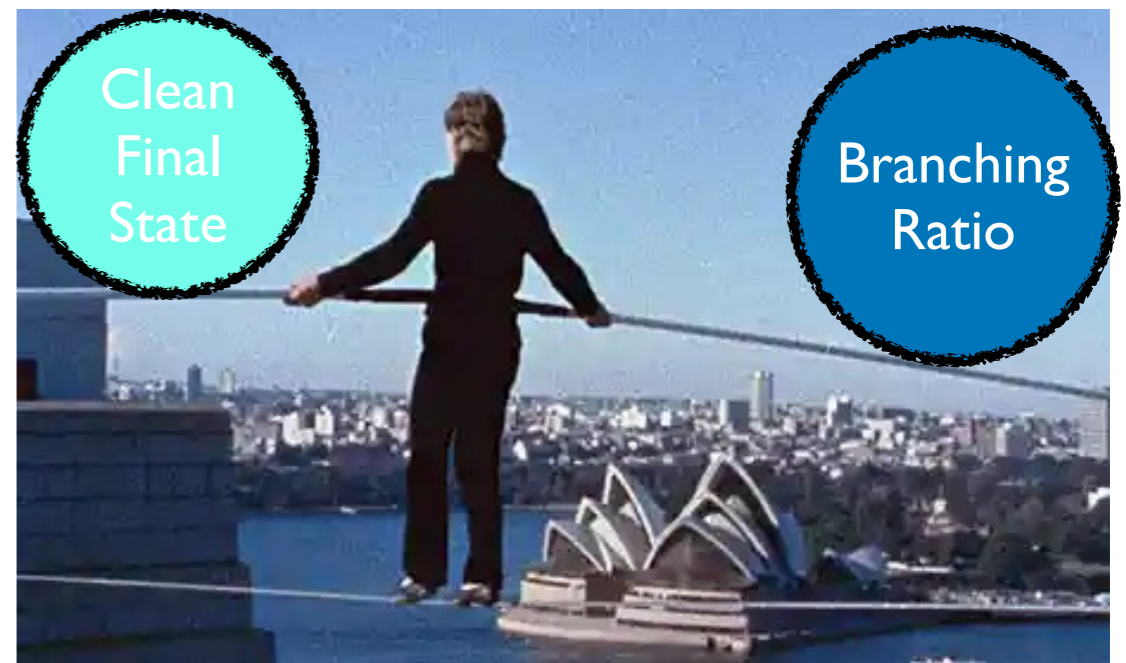
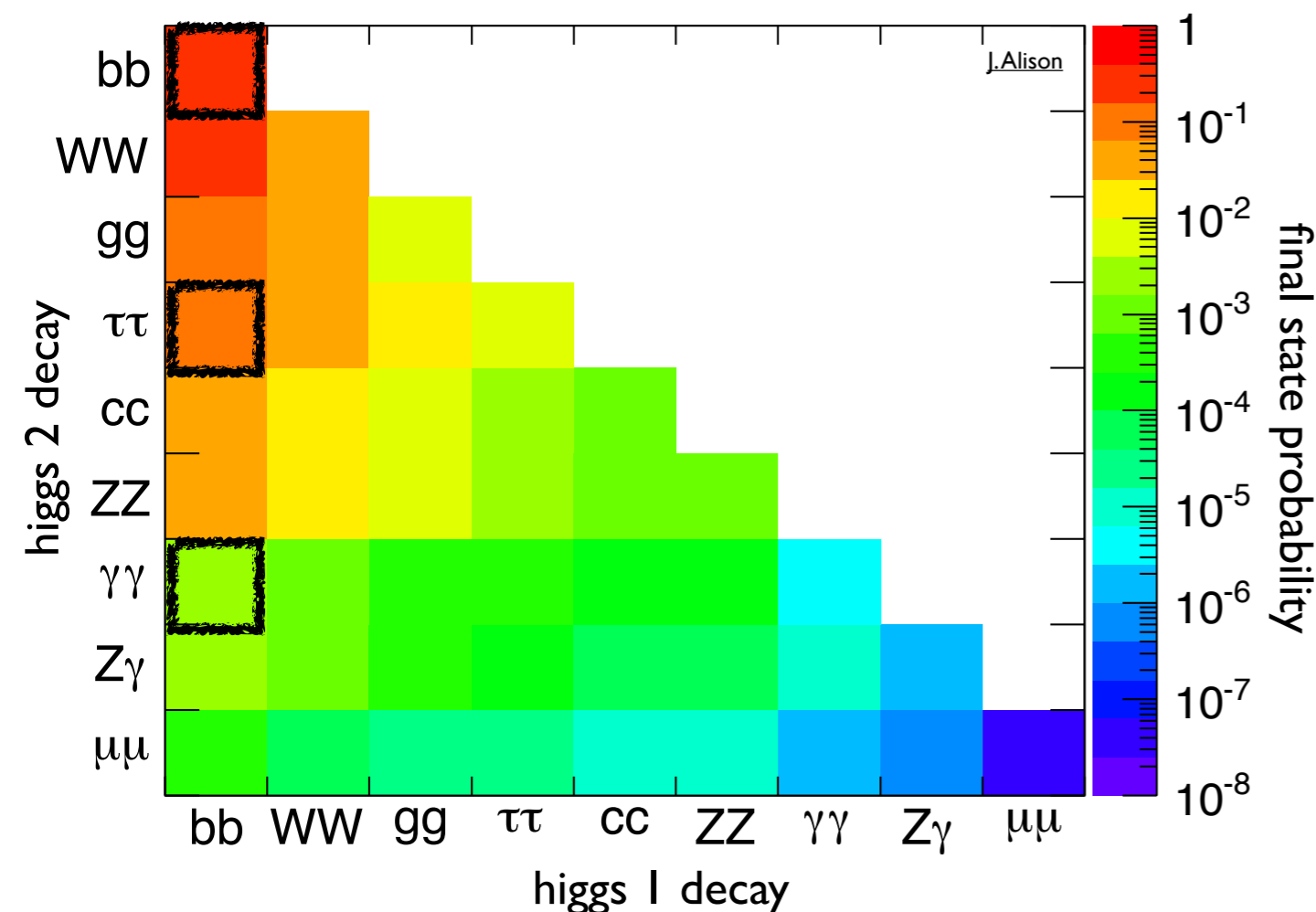
Man on Wire, Guardian

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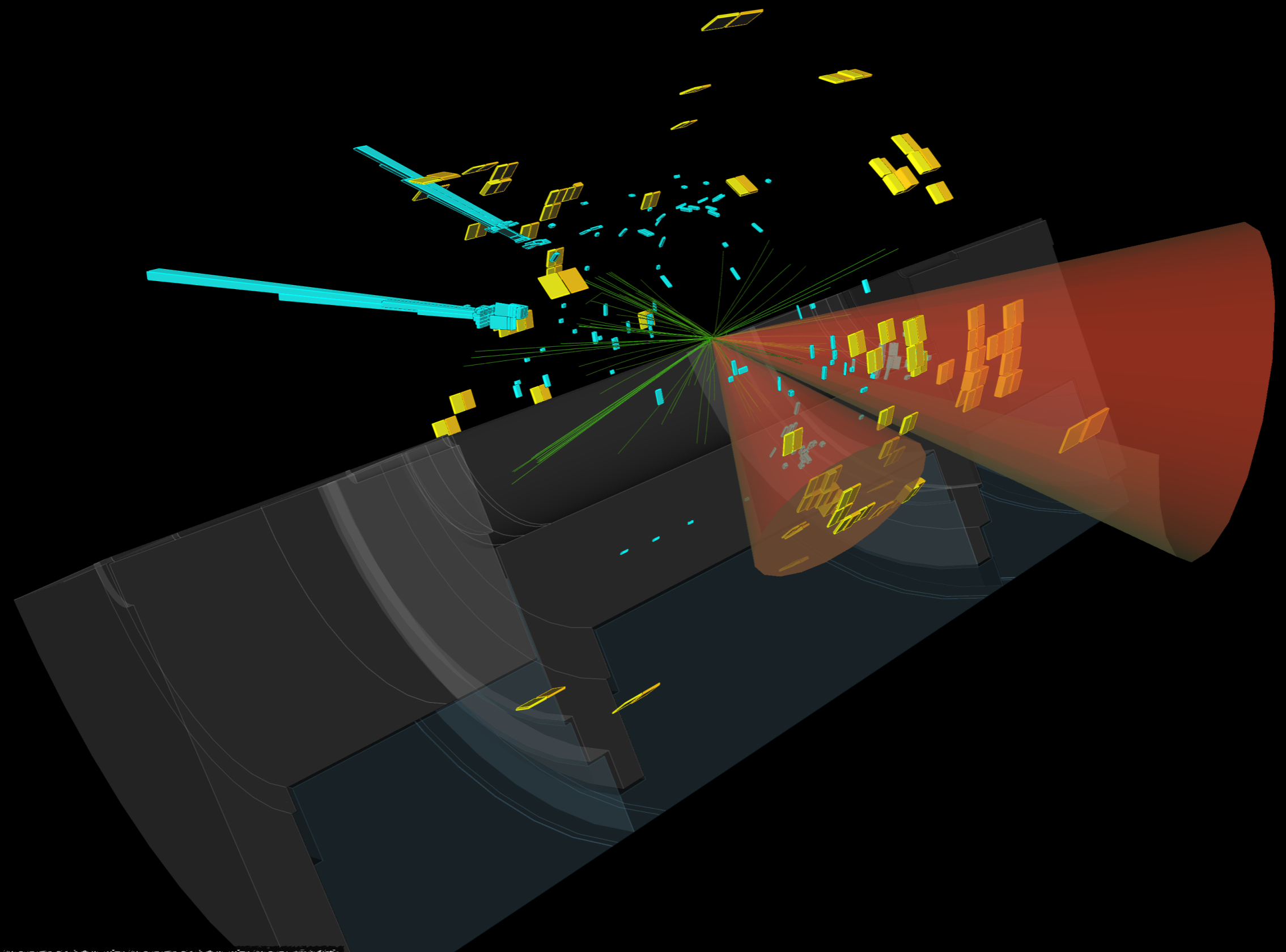
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Man on Wire, Guardian

$4b$, $b\bar{b}\tau\bar{\tau}$, and $b\bar{b}\gamma\gamma$ are the most powerful

$$HH \rightarrow b\bar{b}\gamma\gamma$$



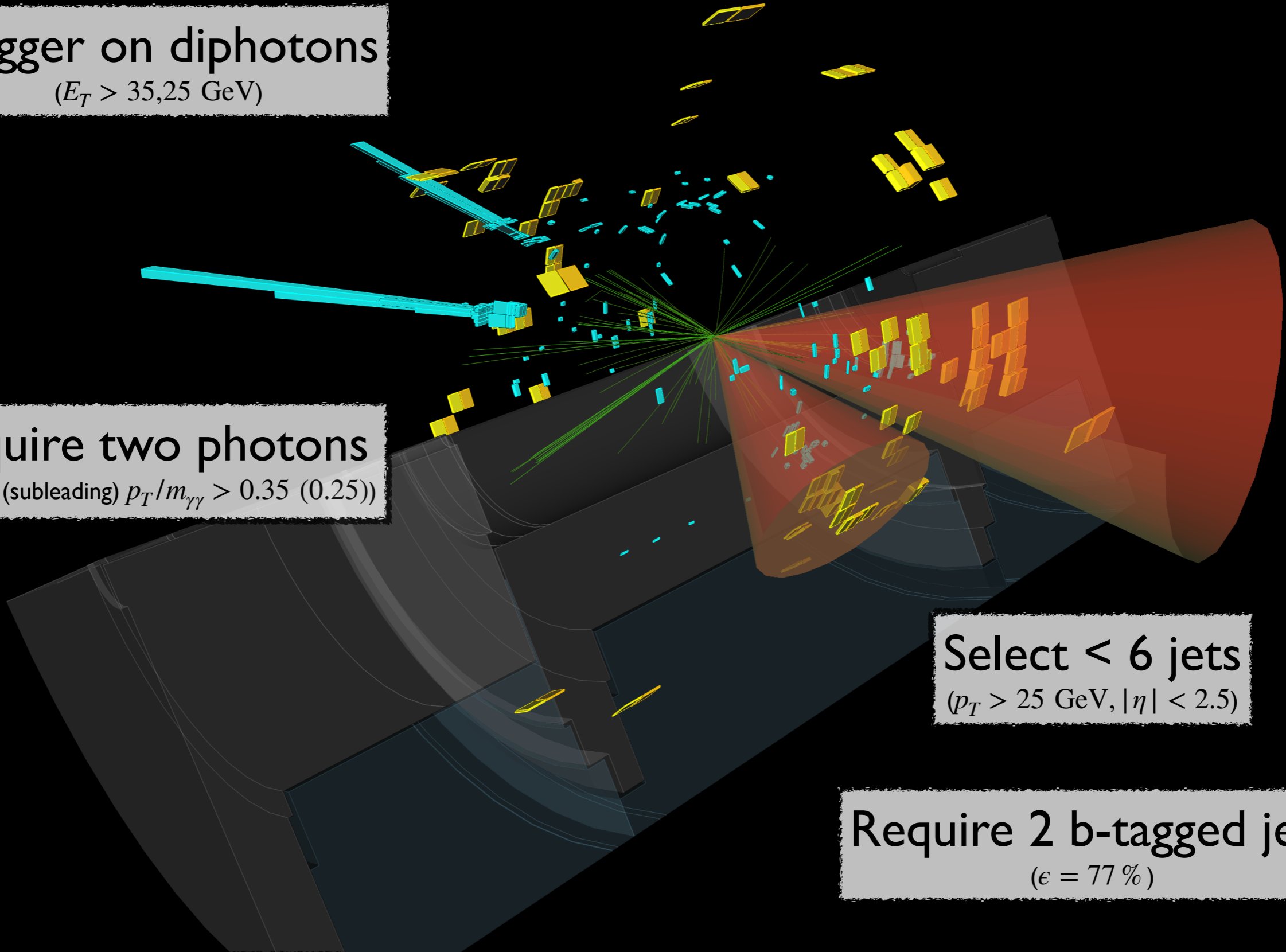
$$HH \rightarrow b\bar{b}\gamma\gamma$$

Trigger on diphotons
($E_T > 35,25$ GeV)

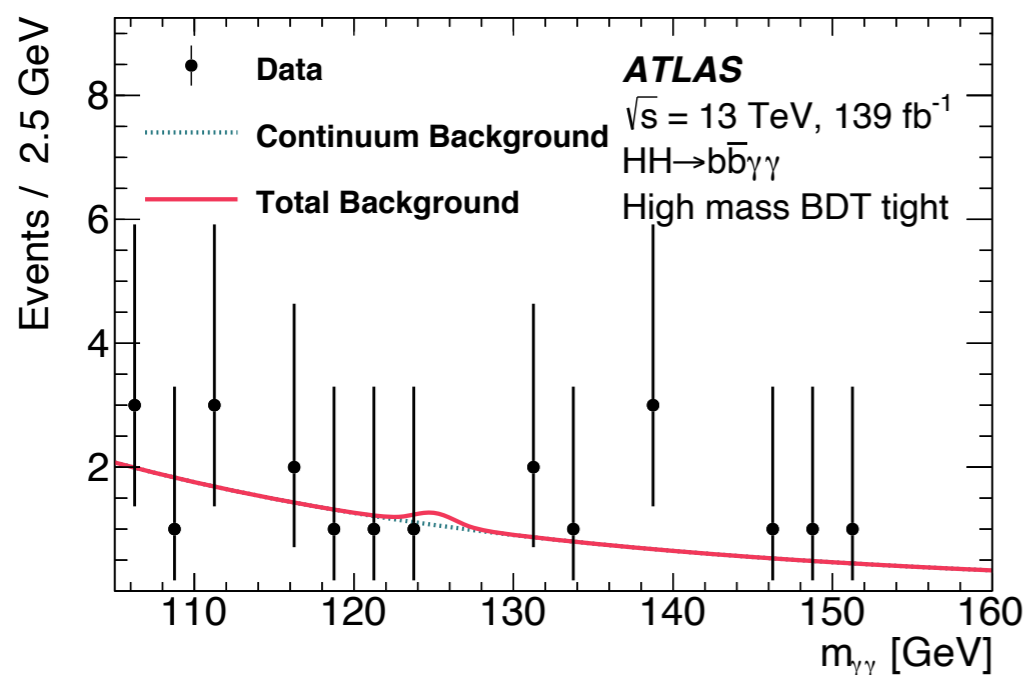
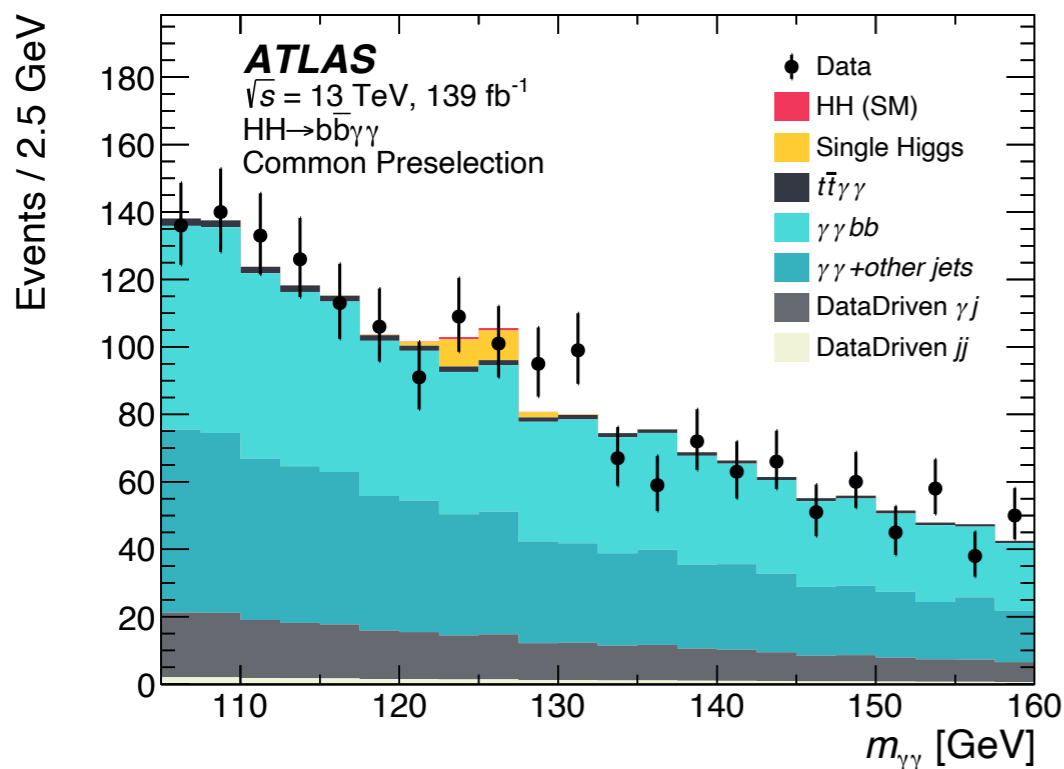
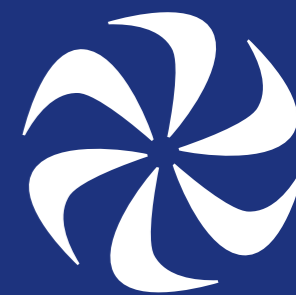
Require two photons
(Leading (subleading) $p_T/m_{\gamma\gamma} > 0.35$ (0.25))

Select < 6 jets
($p_T > 25$ GeV, $|\eta| < 2.5$)

Require 2 b-tagged jets
($\epsilon = 77\%$)



$b\bar{b}\gamma\gamma$ Background Estimate



Background estimate formed on fit to $m_{\gamma\gamma}$ in different signal regions

Shape of background function determined from simulation, norm determined from data ‘sidebands’

Contributions from fake γ estimated using data-driven method

Single Higgs background determined from simulation

Largest systematic from “spurious signal”: fit signal + background on background-only MC template

$b\bar{b}\gamma\gamma$ Analysis Strategy

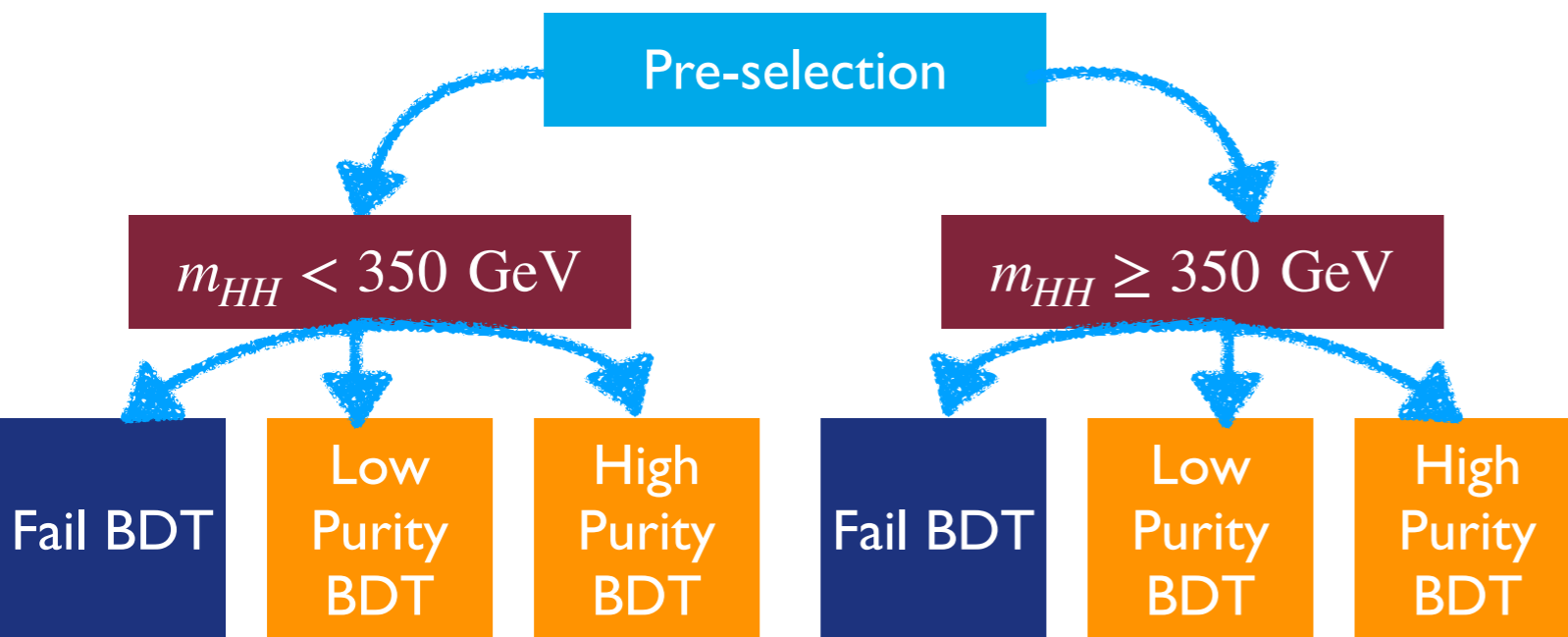
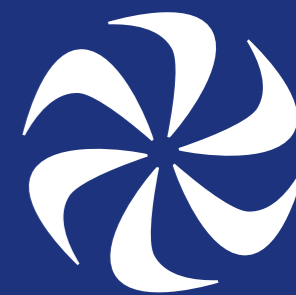


$b\bar{b}\gamma\gamma$ Analysis Strategy



After **pre-selection**, split into **high-mass** and **low-mass** selections

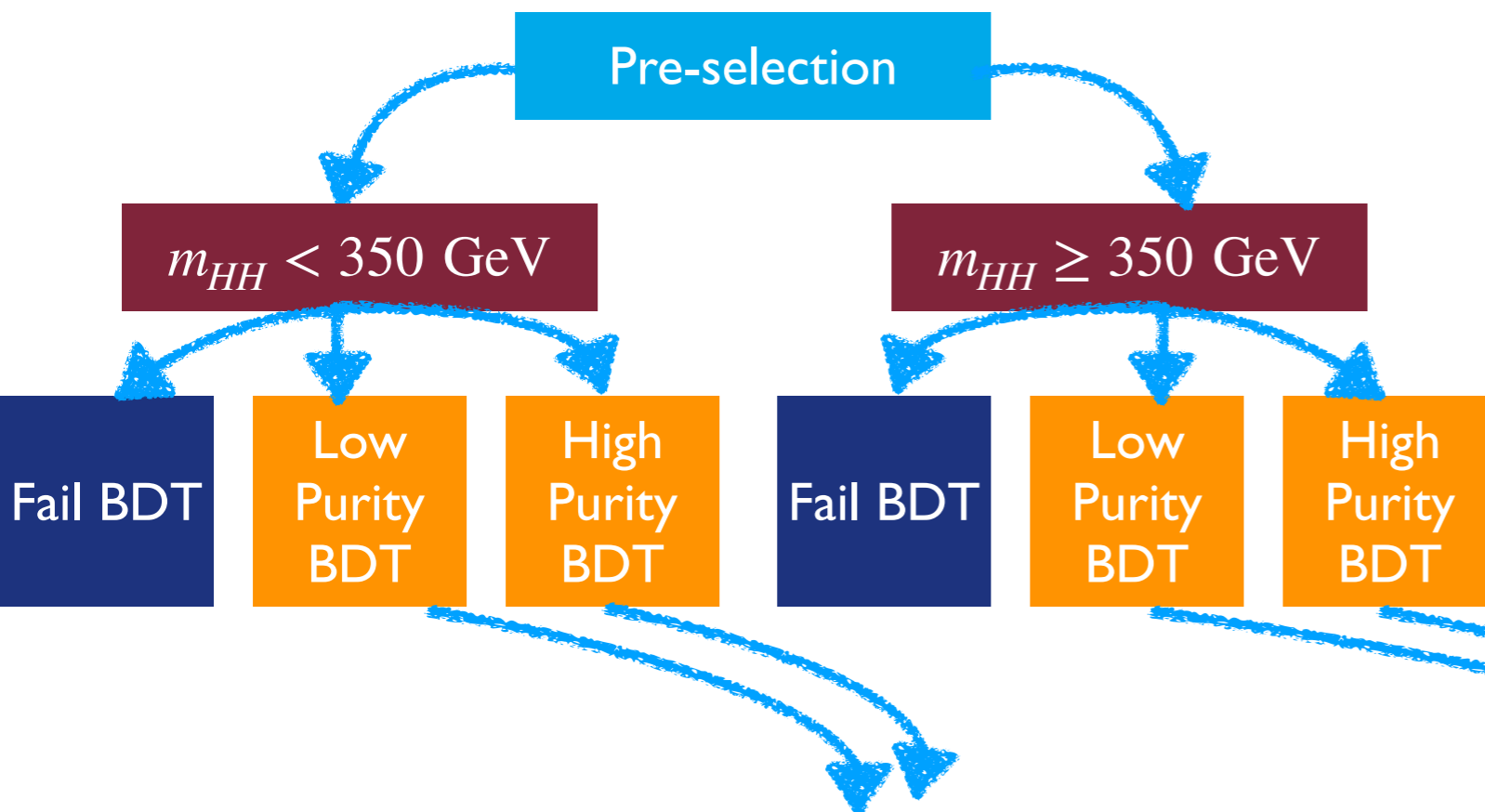
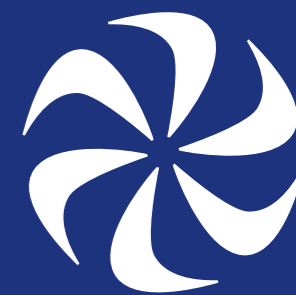
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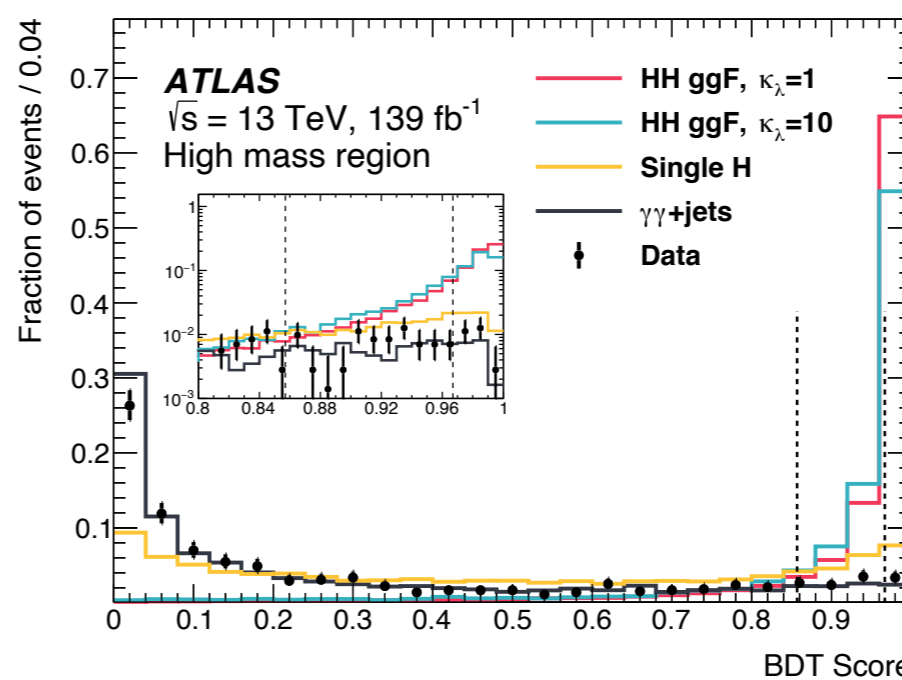
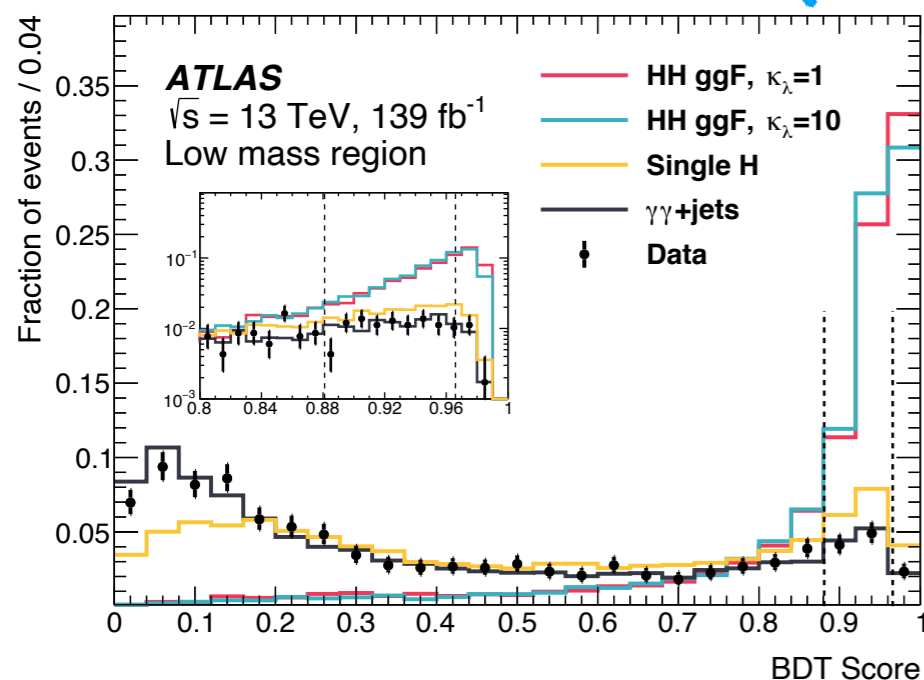
BDT trained in each region: select low- and high-purity **signal regions** with BDT

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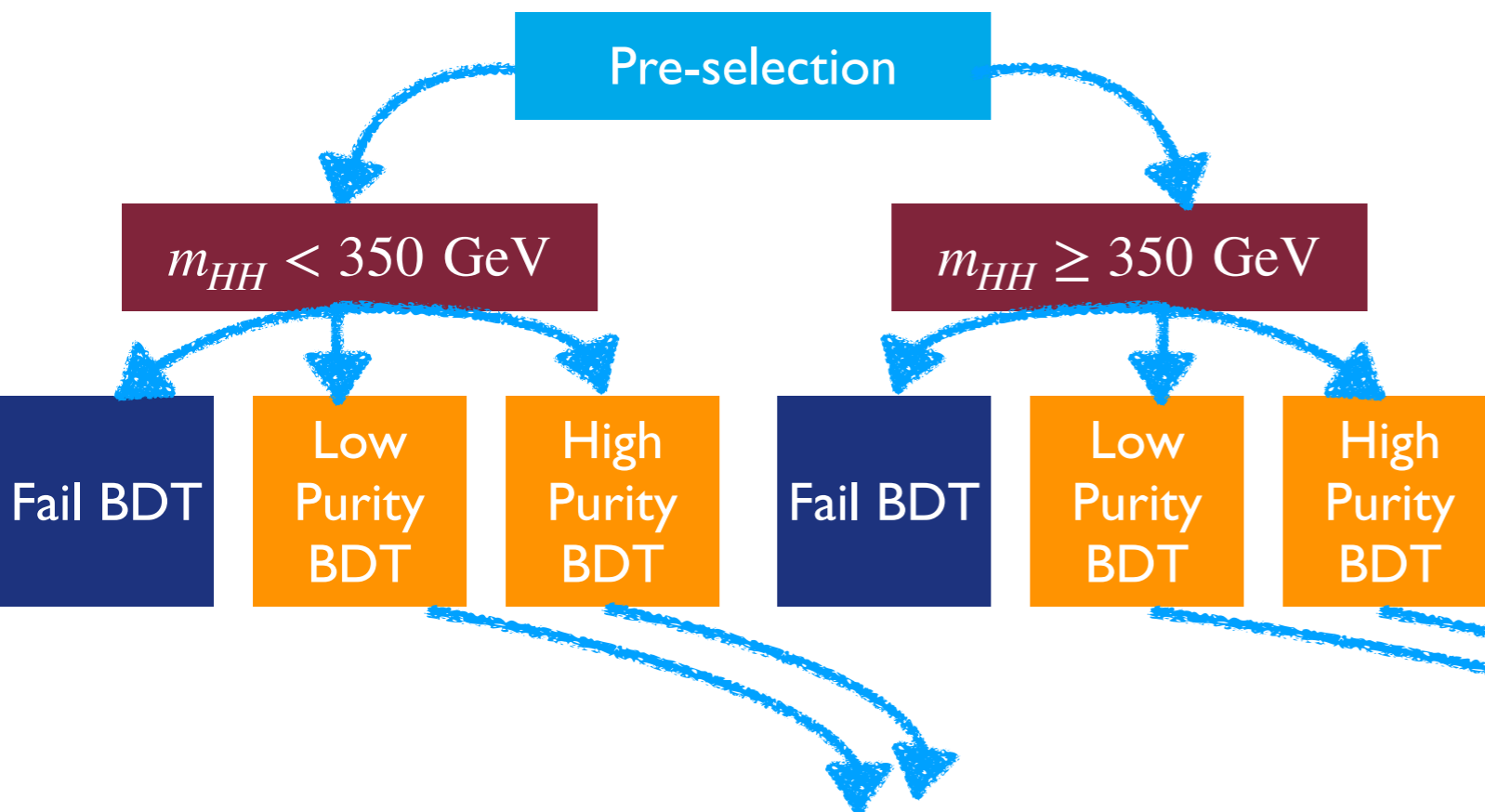
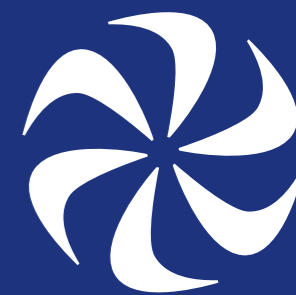
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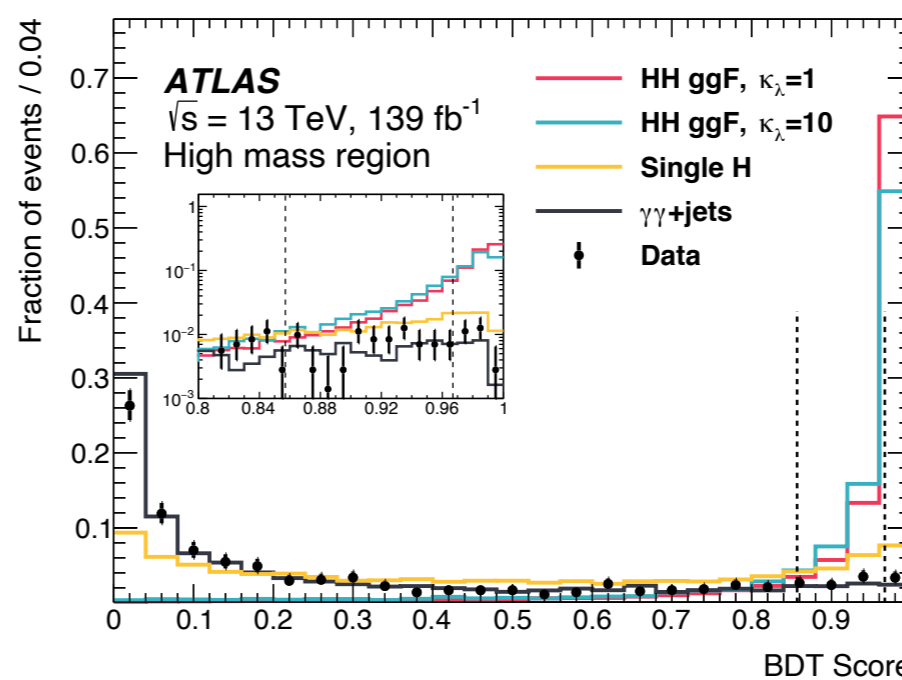
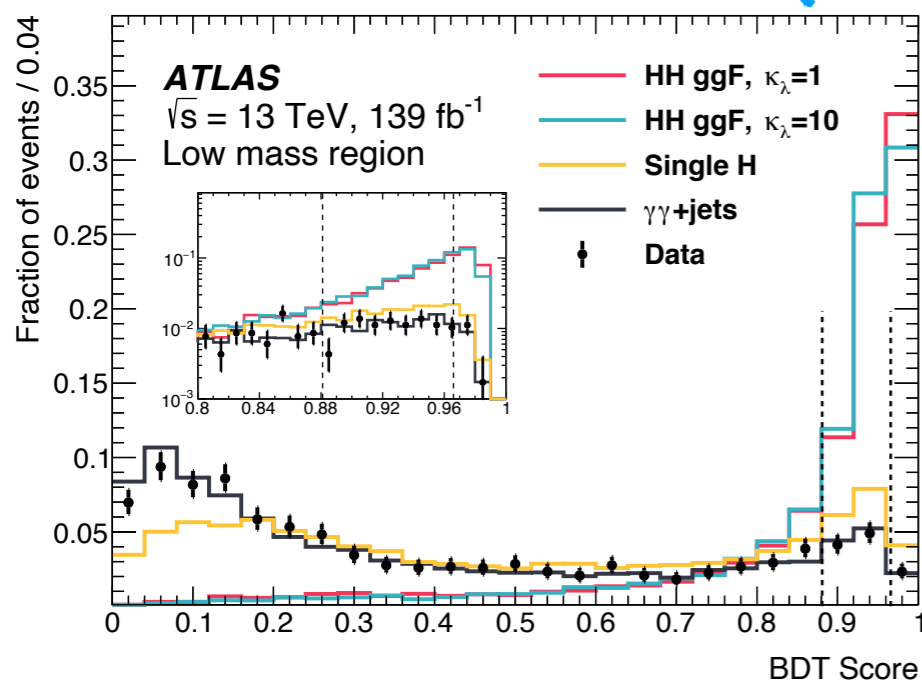
Fit $m_{\gamma\gamma}$ in 4 SR's simultaneously to extract signal

$b\bar{b}\gamma\gamma$ Analysis Strategy



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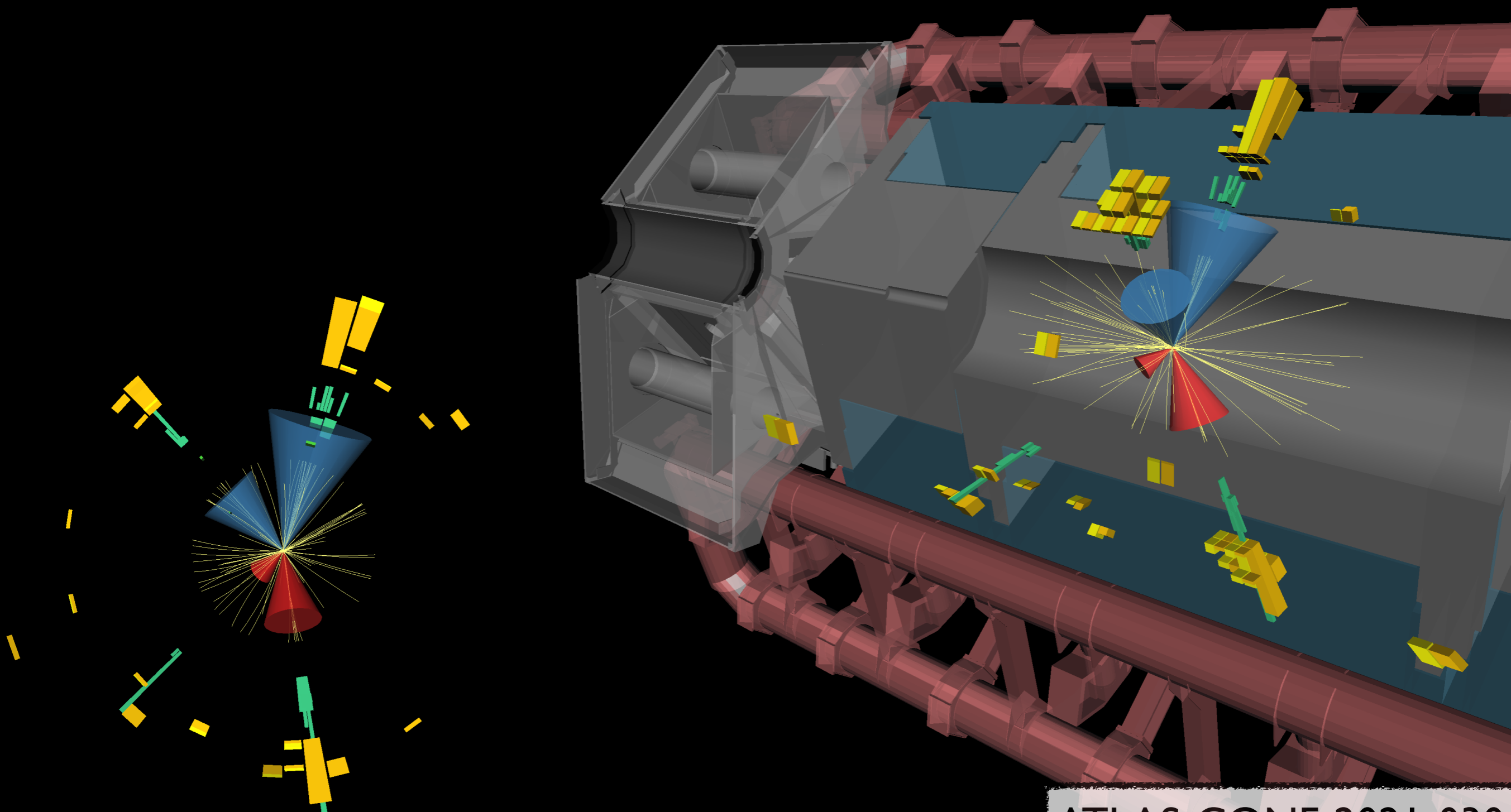
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No obvious signs of new physics

$$HH \rightarrow b\bar{b}\tau\bar{\tau}$$

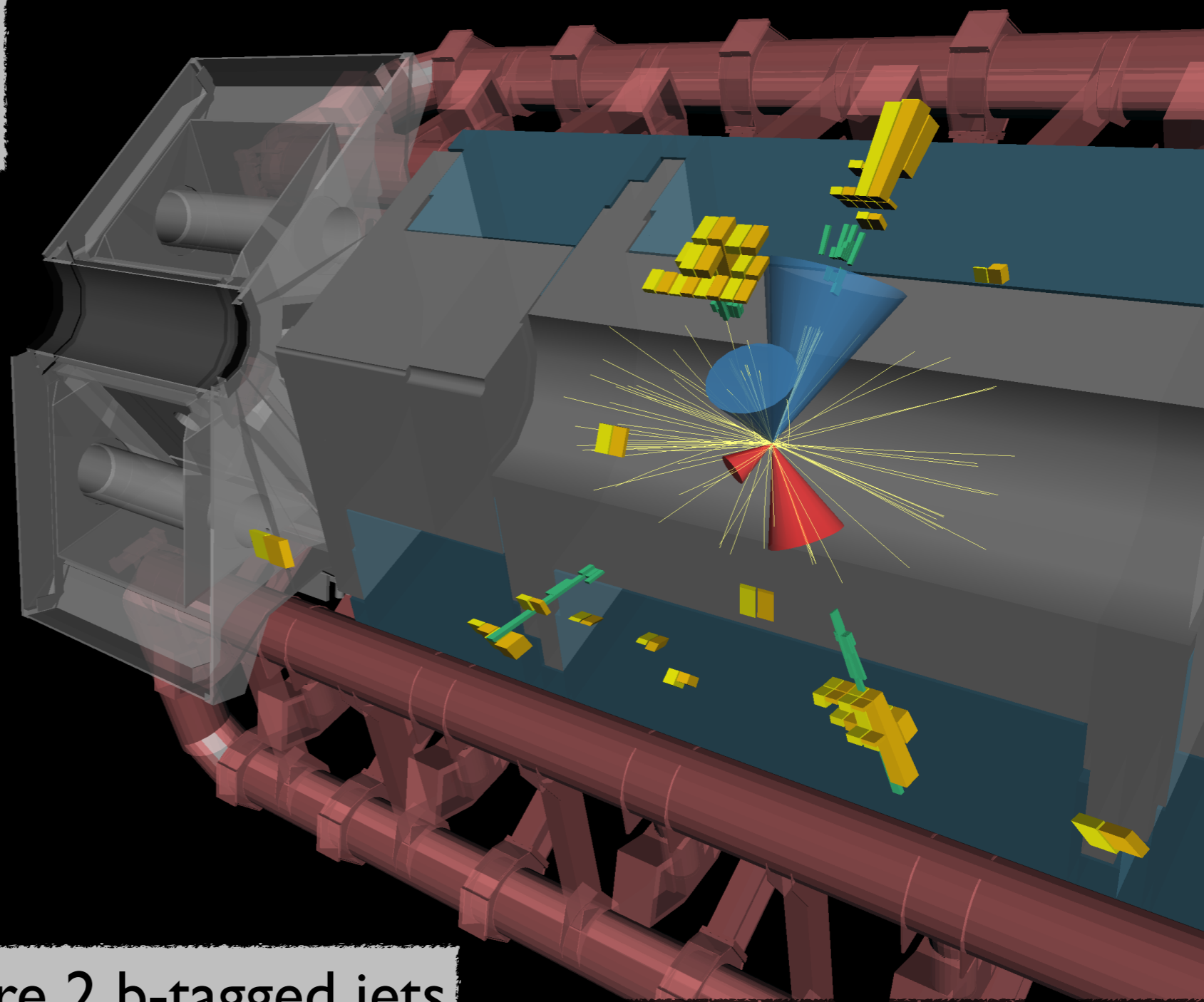
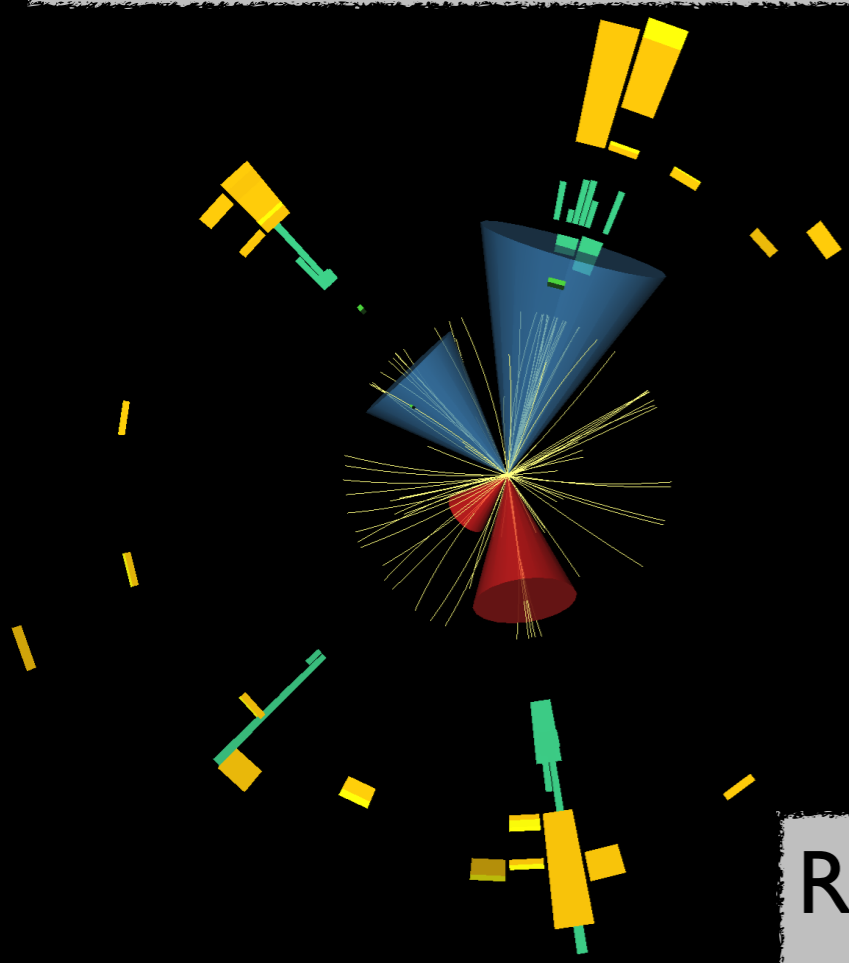


$$HH \rightarrow b\bar{b}\tau\bar{\tau}$$

Separate into $\tau_h\tau_h$ and $\tau_\ell\tau_h$ channels

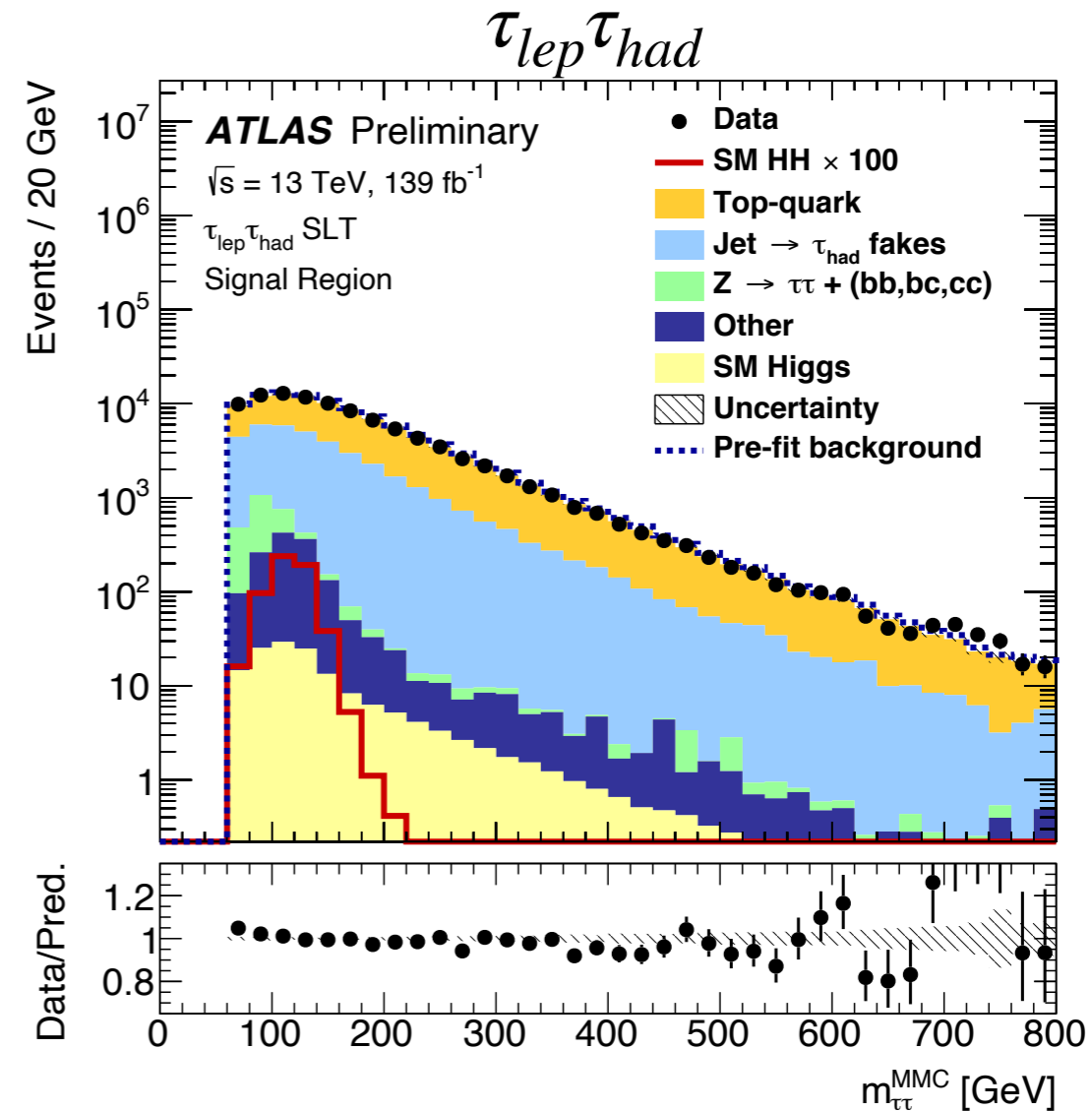
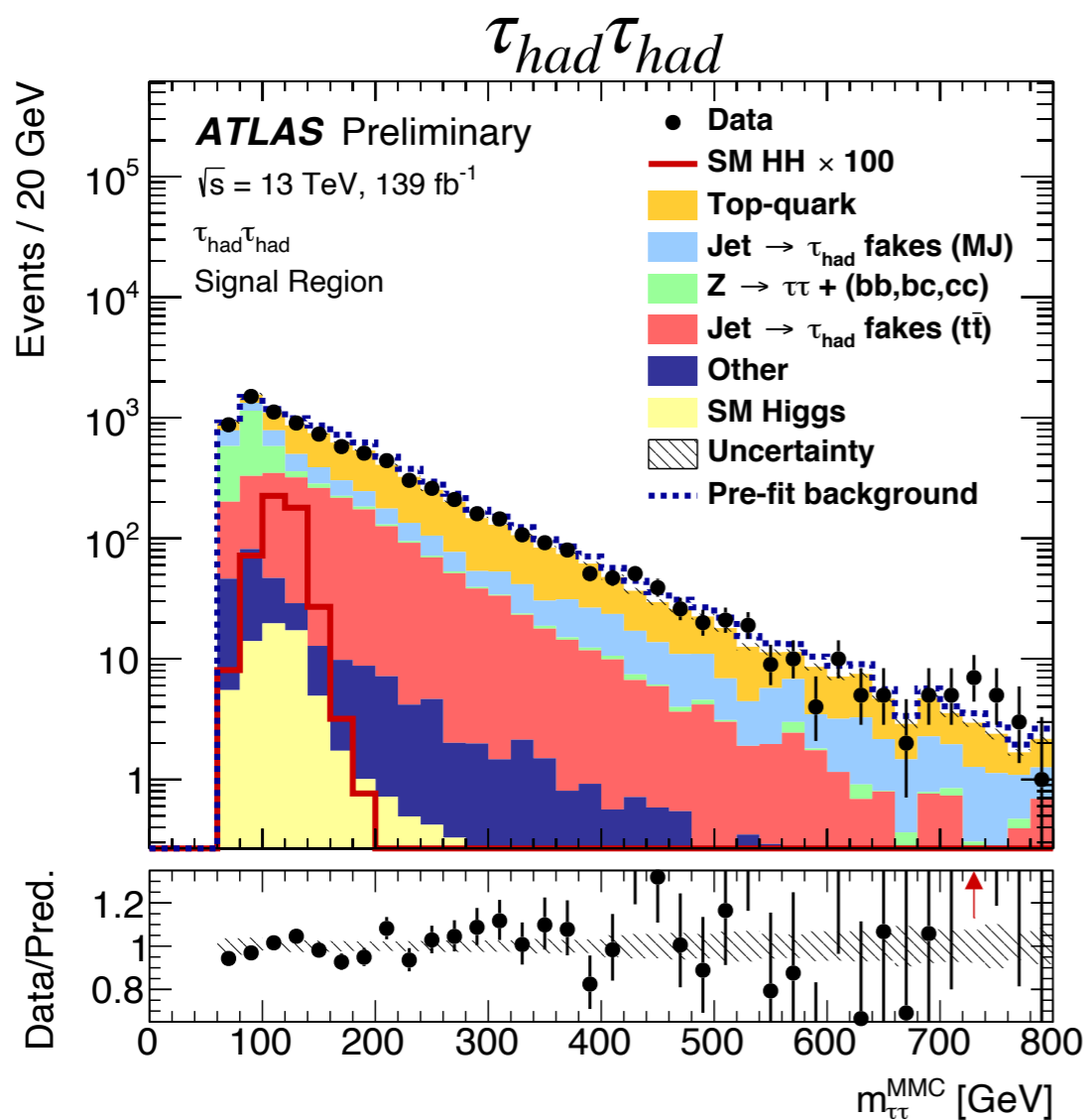
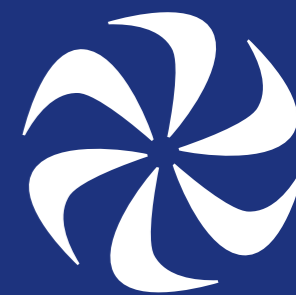
Trigger on di- τ , $\ell + \tau$,
or single ℓ

Require 1 or 2 'loose' τ :
 $m_{\tau\tau} > 60$ GeV



Require 2 b-tagged jets
($\epsilon = 77\%$)

$b\bar{b}\tau\tau$ Background Estimate

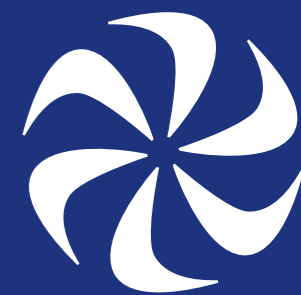


Top-quark background from MC, normalization floating in final fit

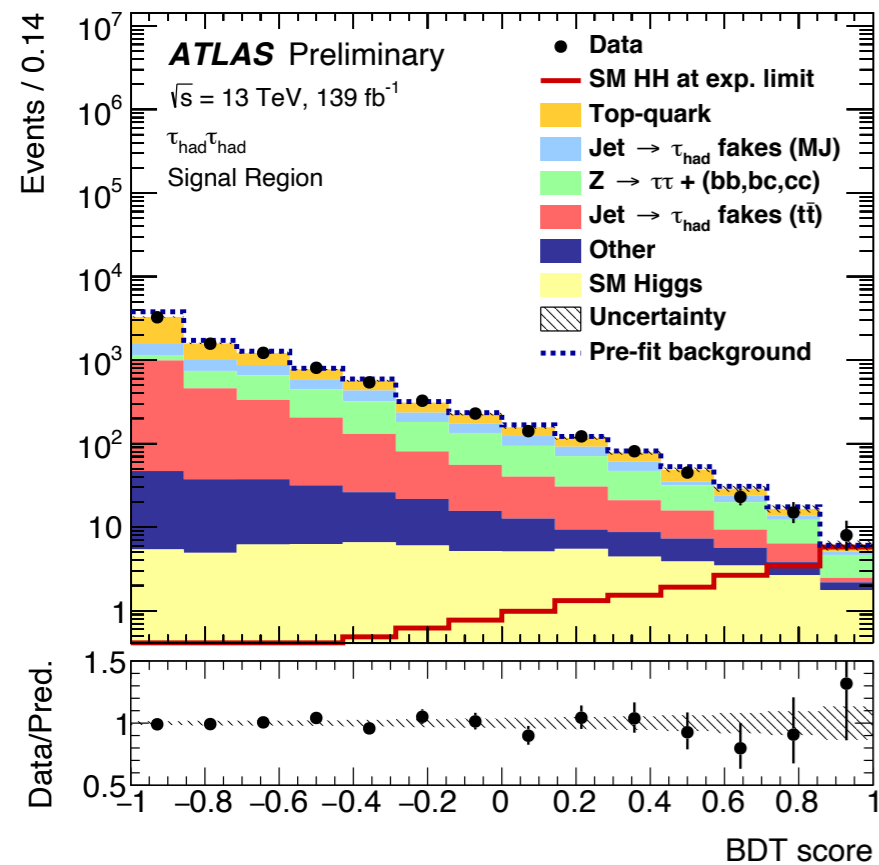
Z+jets background from MC, normalization from leptonic control region

Fake τ estimated from data

$b\bar{b}\tau\bar{\tau}$ Strategy and Results

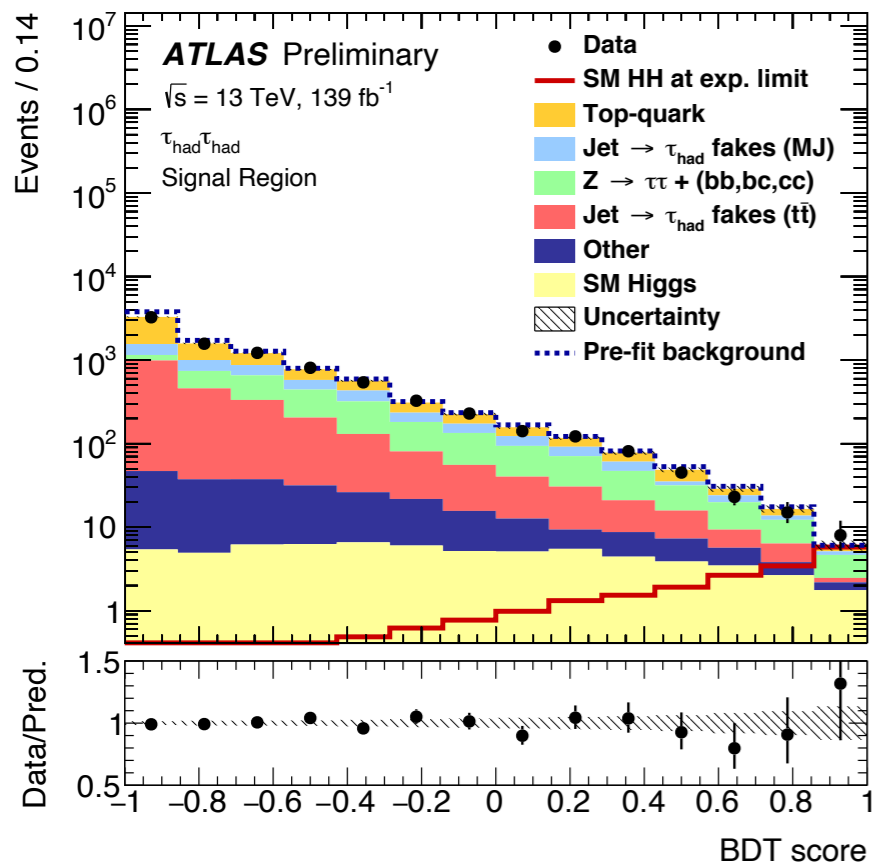
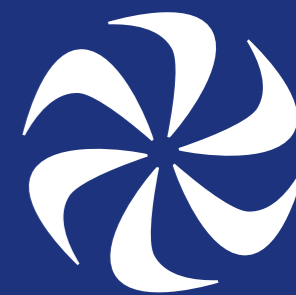


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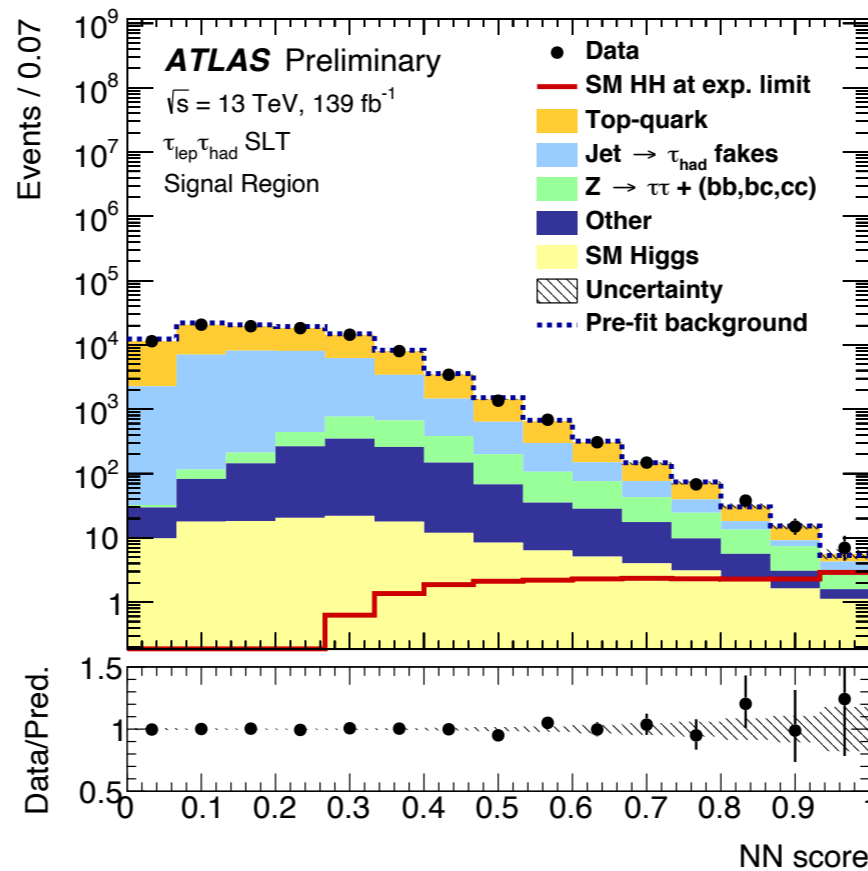


$\tau_{had}\tau_{had}$ BDT

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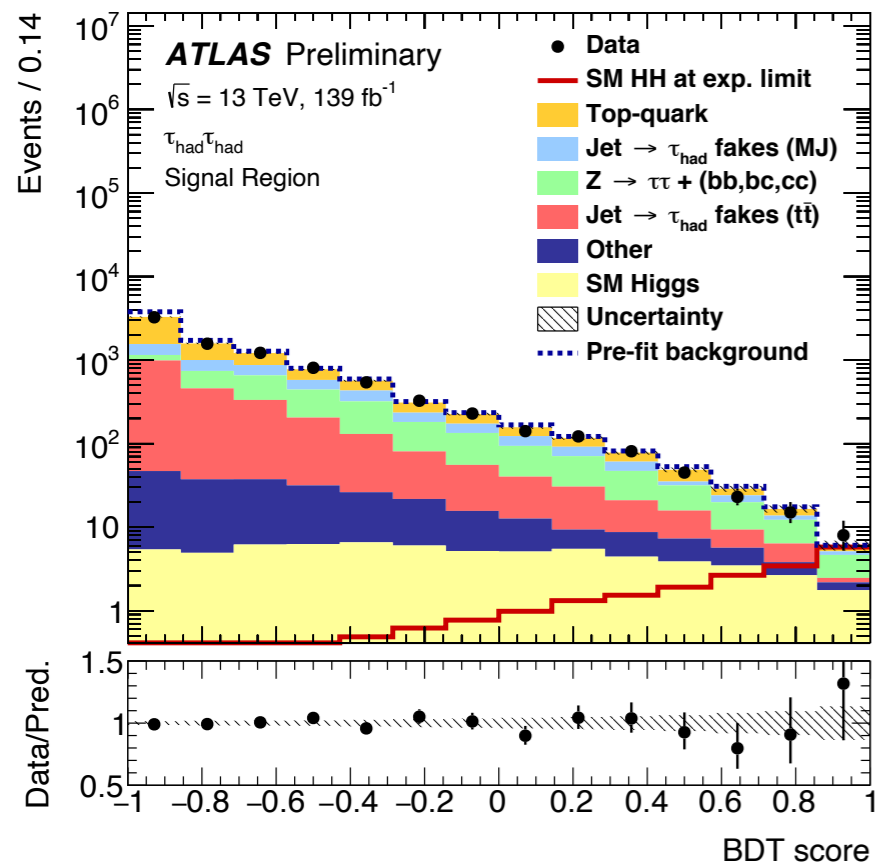


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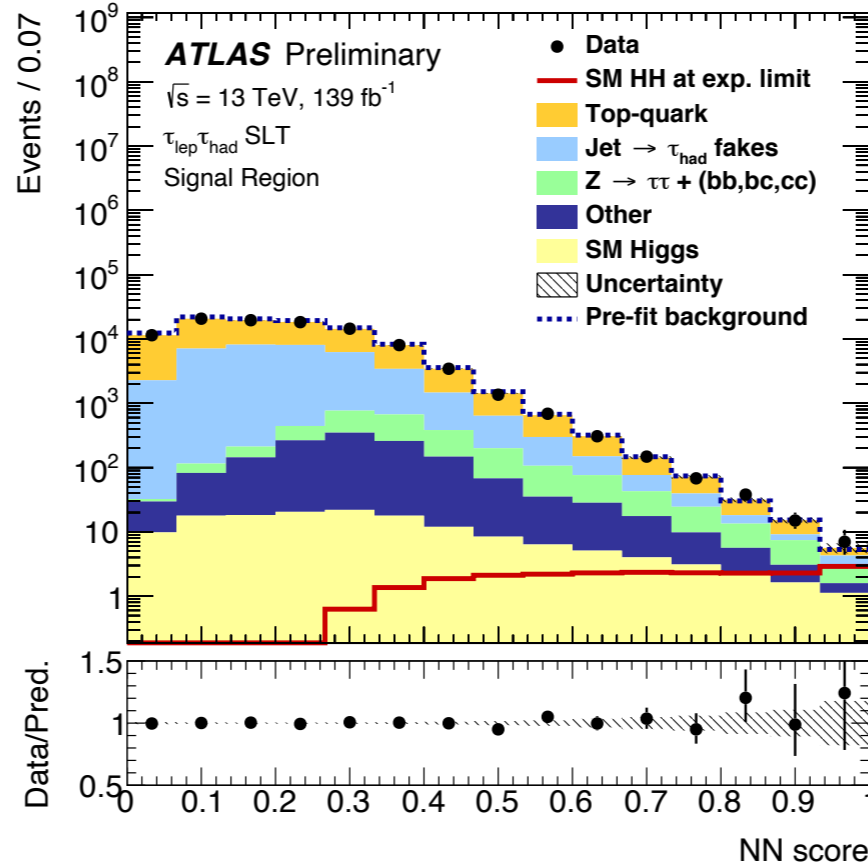


$\tau_{lep}\tau_{had}$ I-Lepton Trigger
 Neural Network

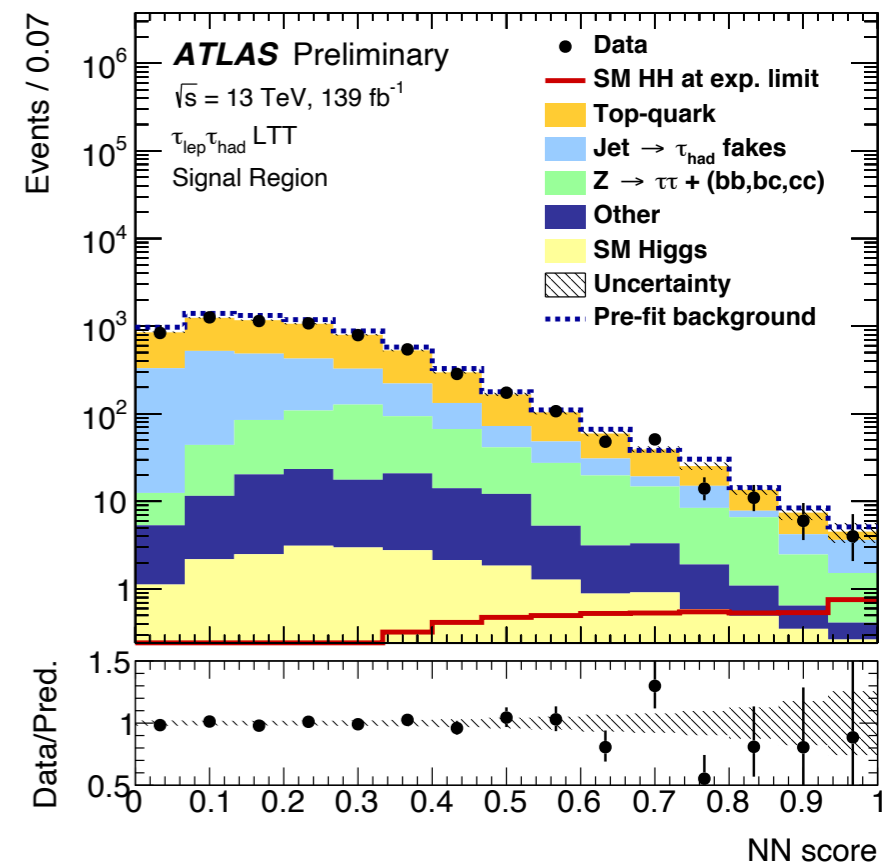
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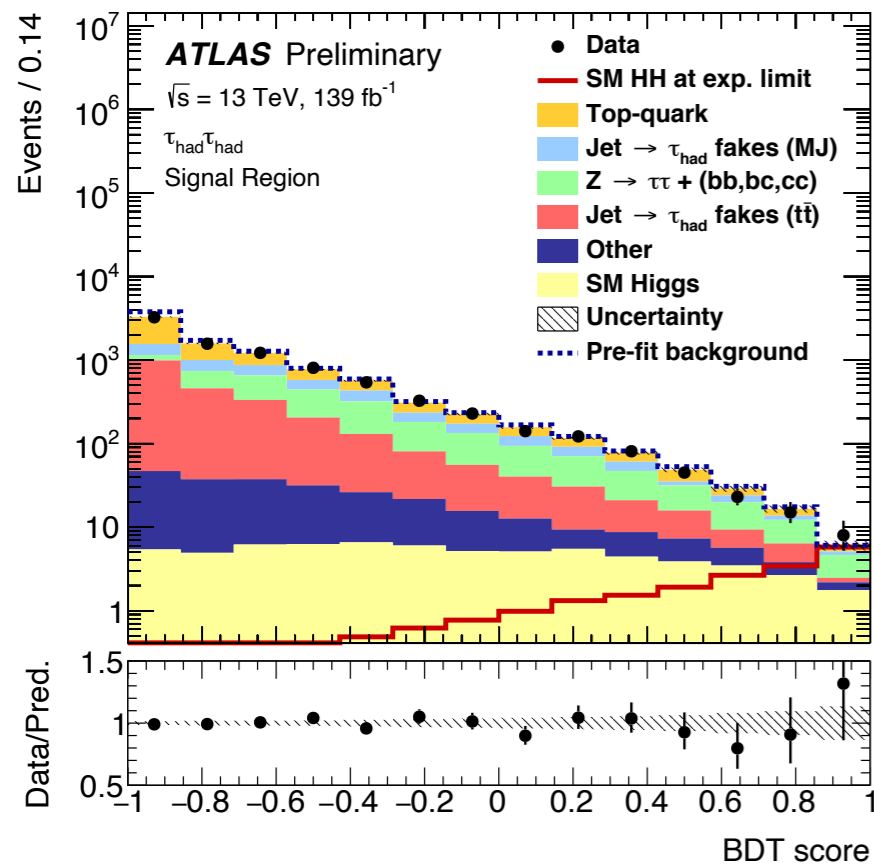


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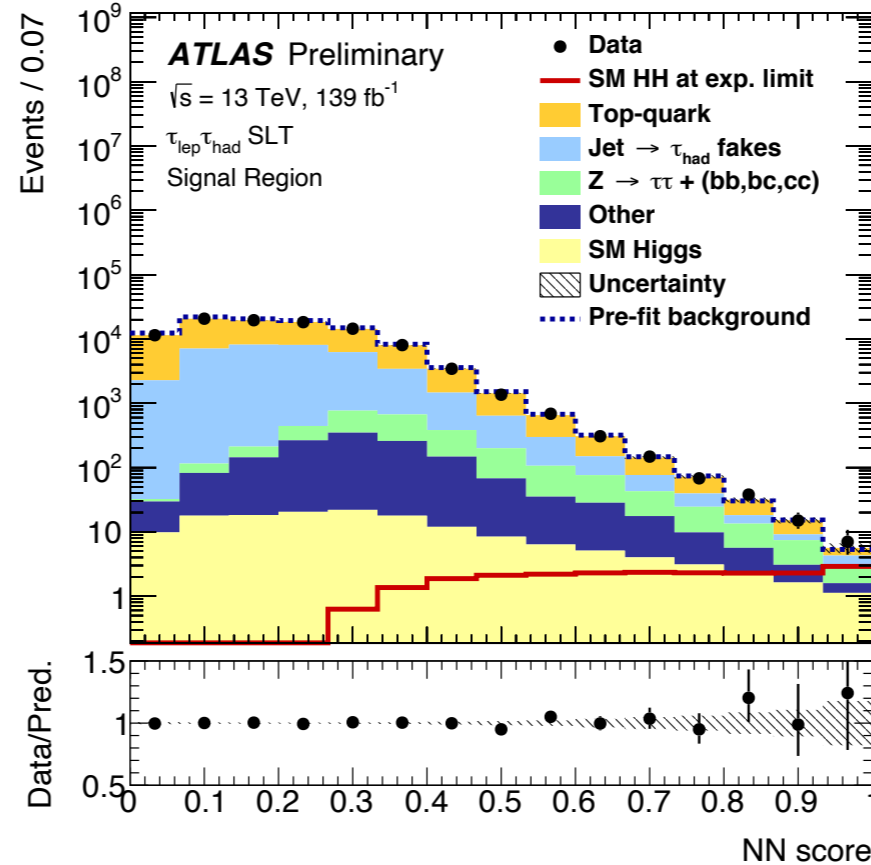


$\tau_{lep}\tau_{had}$ Lepton+Tau
 Neural Network

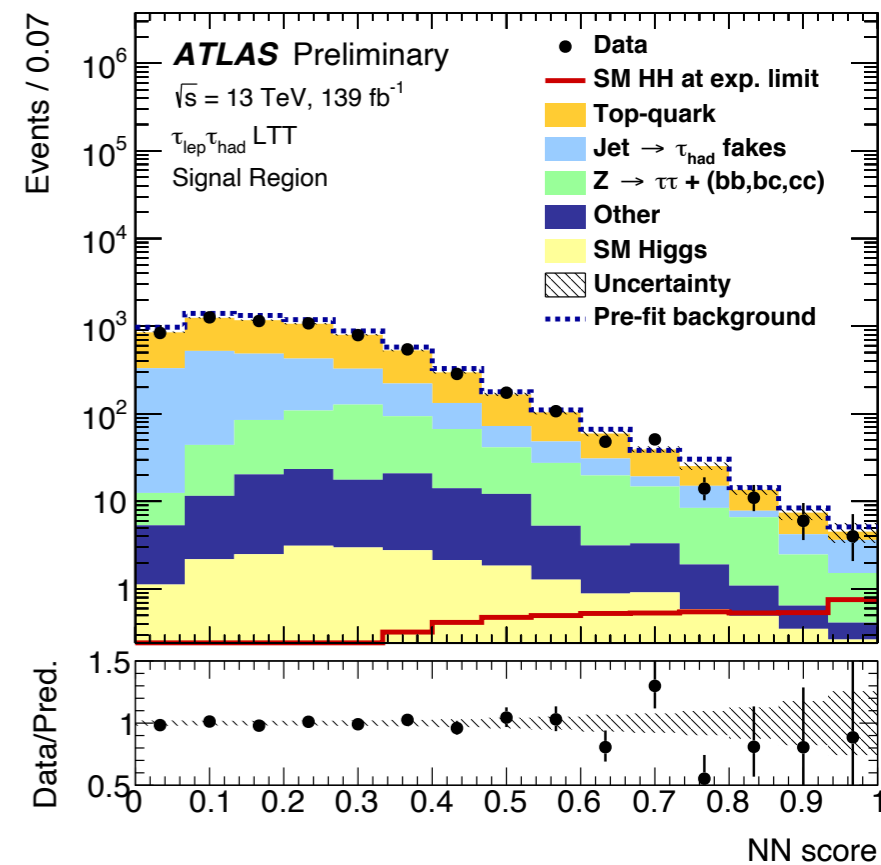
$b\bar{b}\tau\tau$ Strategy and Results



$\tau_{had}\tau_{had}$ BDT



$\tau_{lep}\tau_{had}$ I-Lepton Trigger
Neural Network



$\tau_{lep}\tau_{had}$ Lepton+Tau
Neural Network

Fits to BDT/NN shape used for final analysis

Data agrees well with background prediction

$\tau_{had}\tau_{had}$ has strongest sensitivity, but other channels also contribute

Limits on the SM

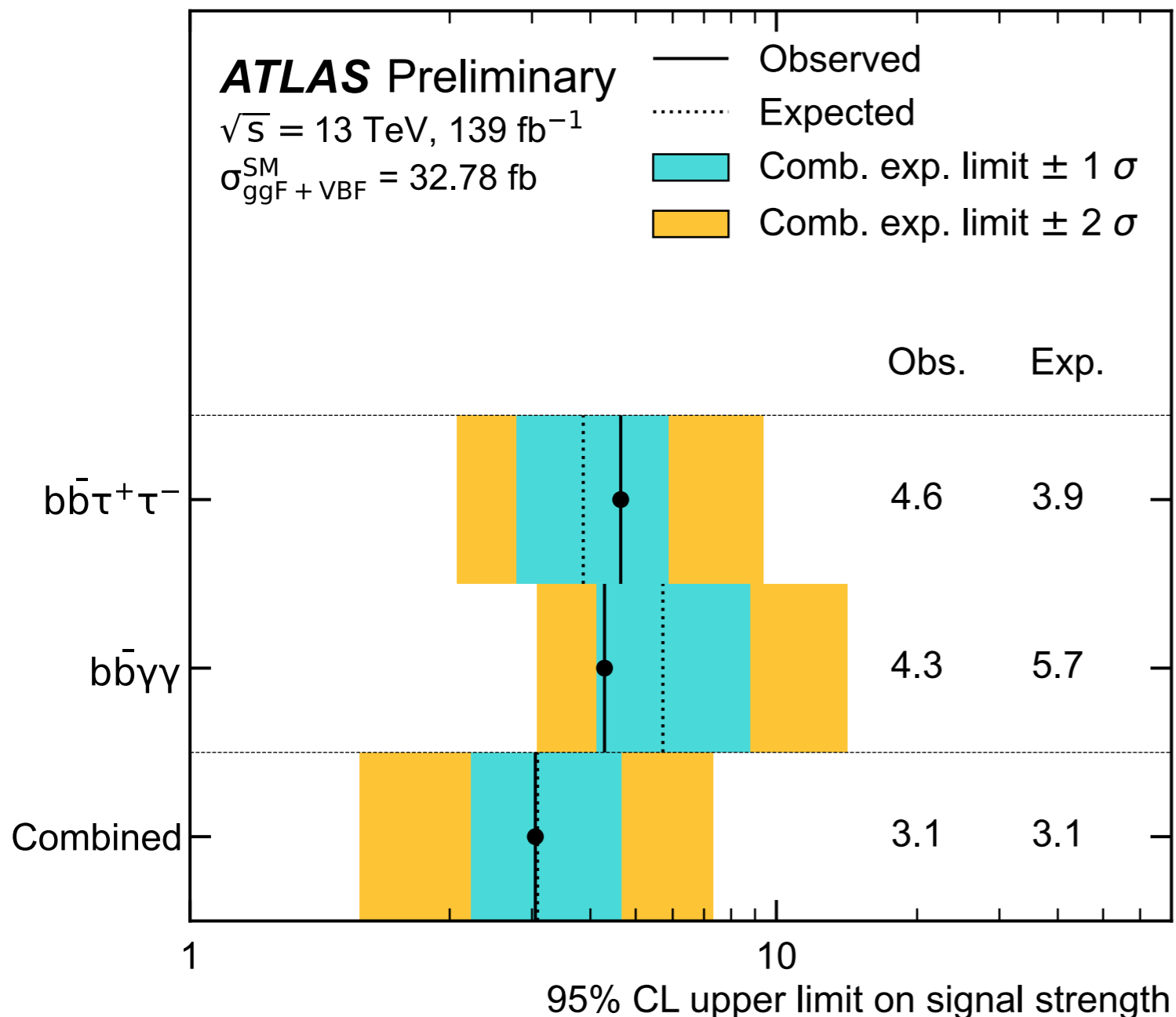
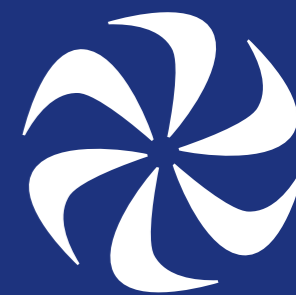


Limits on the SM



Let's put it all together:
can we see HH?

Limits on the SM



Let's put it all together:
 can we see HH?

Here, show sensitivity to SM
 signal: what factor larger
 would the signal have to be,
 for us to be sensitive?

Individual analyses set
 limits at $\sim 4.5x$ SM

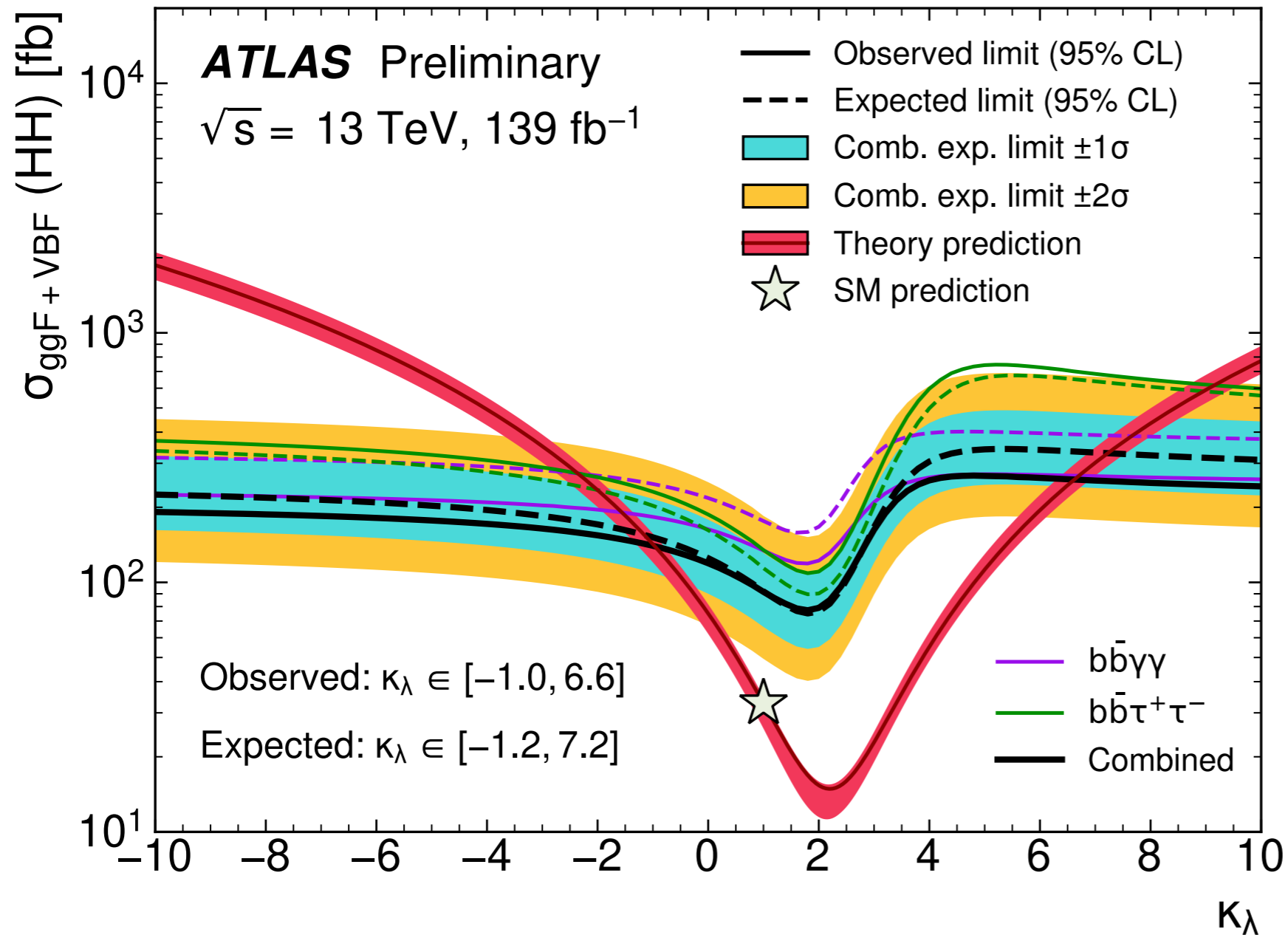
Together, set
limit at 3.1x SM

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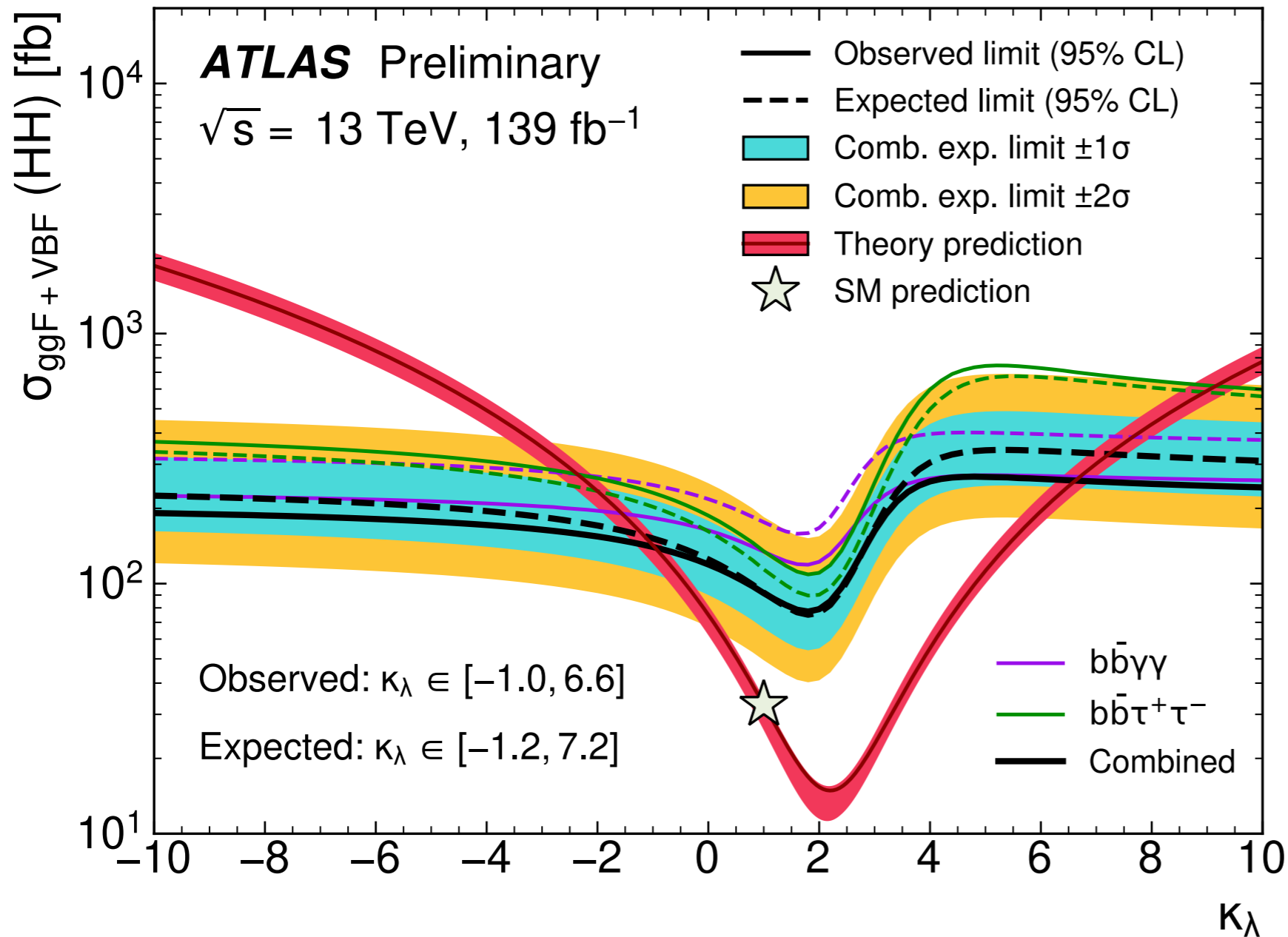
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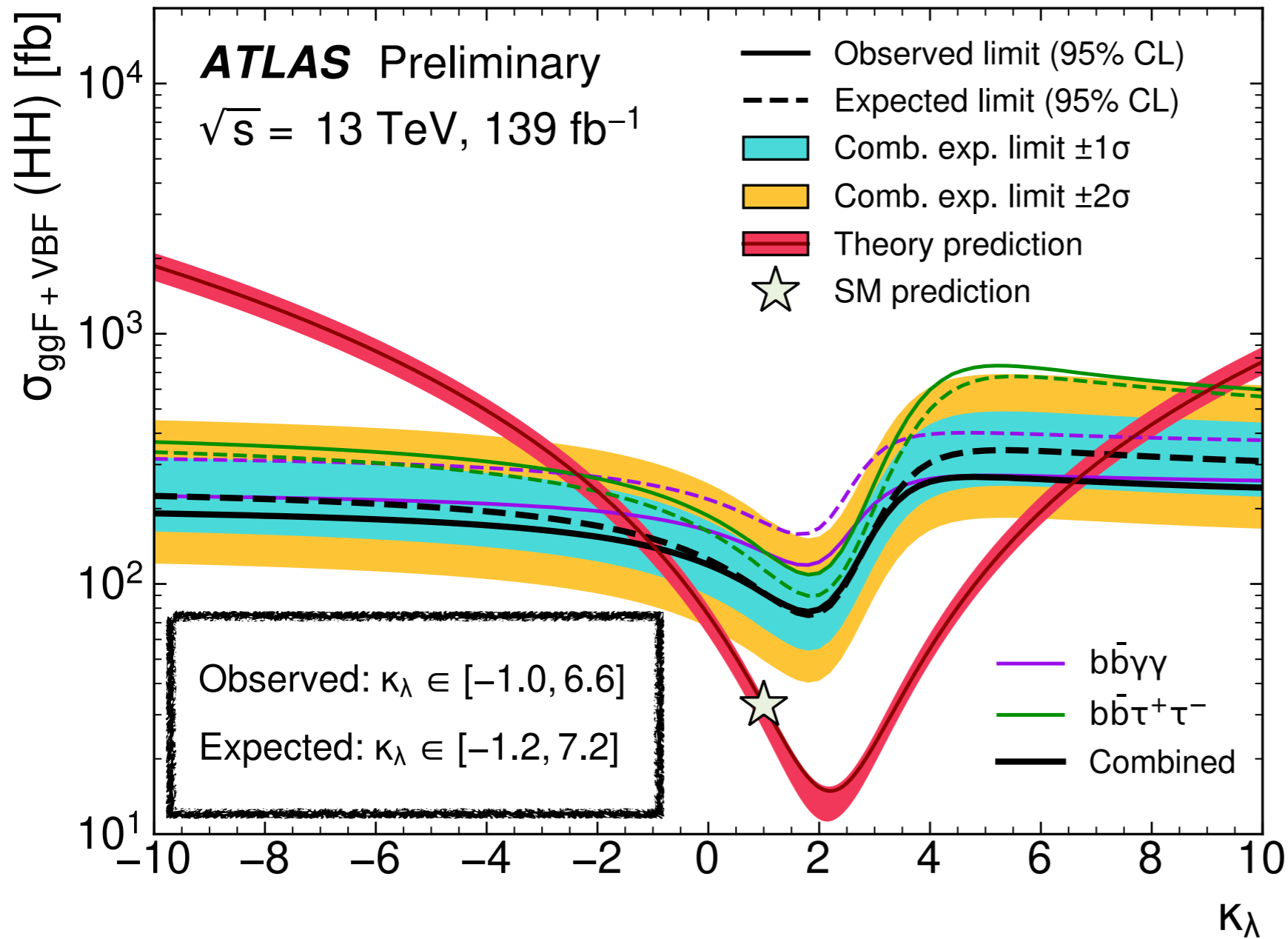


Signal σ goes up for extreme κ_λ : produce more signal

Limits also go up at extreme κ_λ : signal is growing, but is concentrated at low m_{HH} , same as backgrounds

Both analyses contribute to combination!

Measuring the Potential



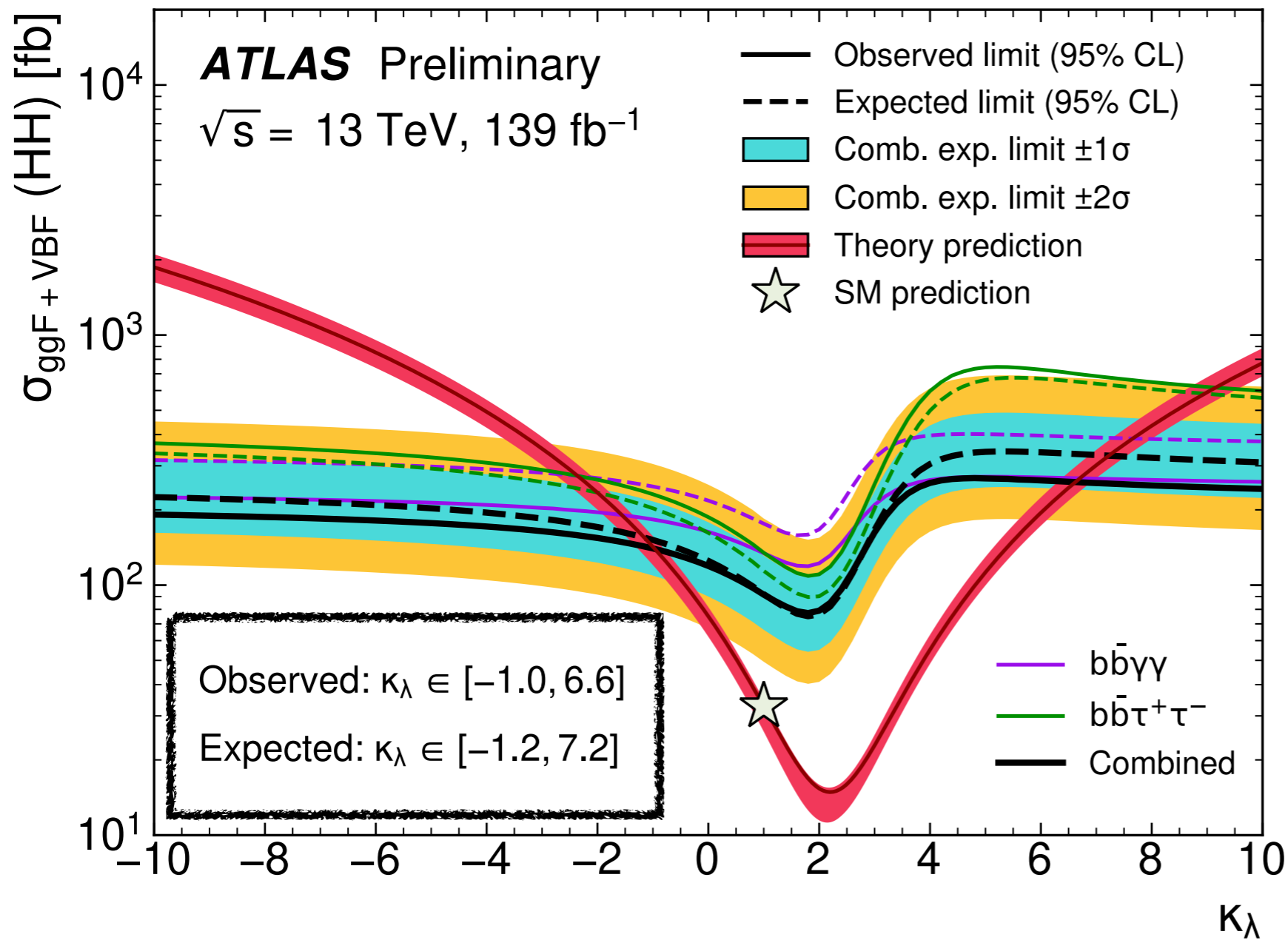
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Allowed range:
 $-1.0 < \kappa_\lambda \leq 6.6$

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Limits also go up at extreme κ_λ : signal is growing, but is concentrated at low m_{HH} , same as backgrounds

Both analyses contribute to combination!

Allowed range:
 $-1.0 < \kappa_\lambda \leq 6.6$

Significantly improved compared to mid-Run2: $-5.0 < \kappa_\lambda \leq 12.0$

Resonant Searches



Resonant Searches



$$V(\phi) = -m^2\phi^2 + \lambda\phi^4$$

The SM's potential only choice that
is *gauge invariant, renormalizable*

Resonant Searches



$$V(\phi) = -m^2\phi^2 + \lambda\phi^4$$

The SM's potential only choice that
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$$V(\phi) = -m^2\phi^2 + \lambda\phi^4 + C\phi^6 + D\phi^8 + \dots$$

If we want modifications like these
C and *D* terms: they have to
emerge from new physics

Resonant Searches

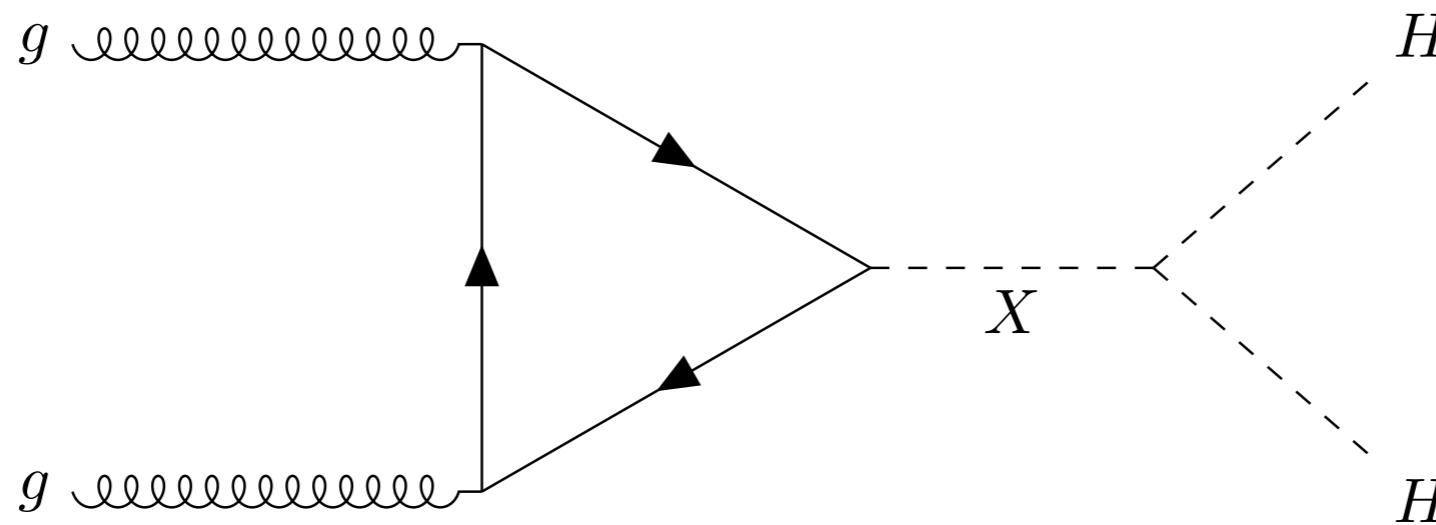


$$V(\phi) = -m^2\phi^2 + \lambda\phi^4$$

The SM's potential only choice that is *gauge invariant, renormalizable*

$$V(\phi) = -m^2\phi^2 + \lambda\phi^4 + C\phi^6 + D\phi^8 + \dots$$

If we want modifications like these C and D terms: they have to emerge from new physics

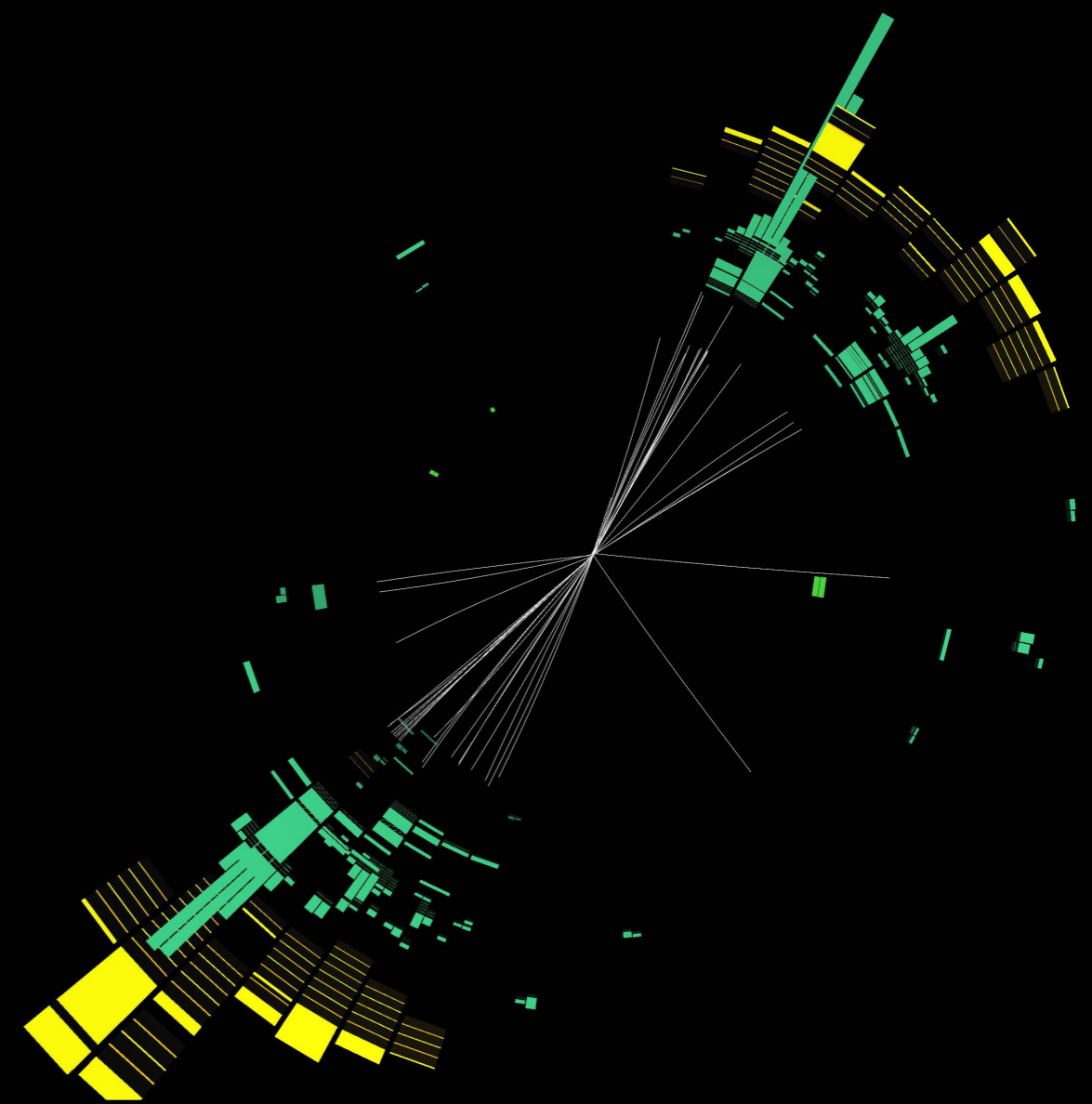
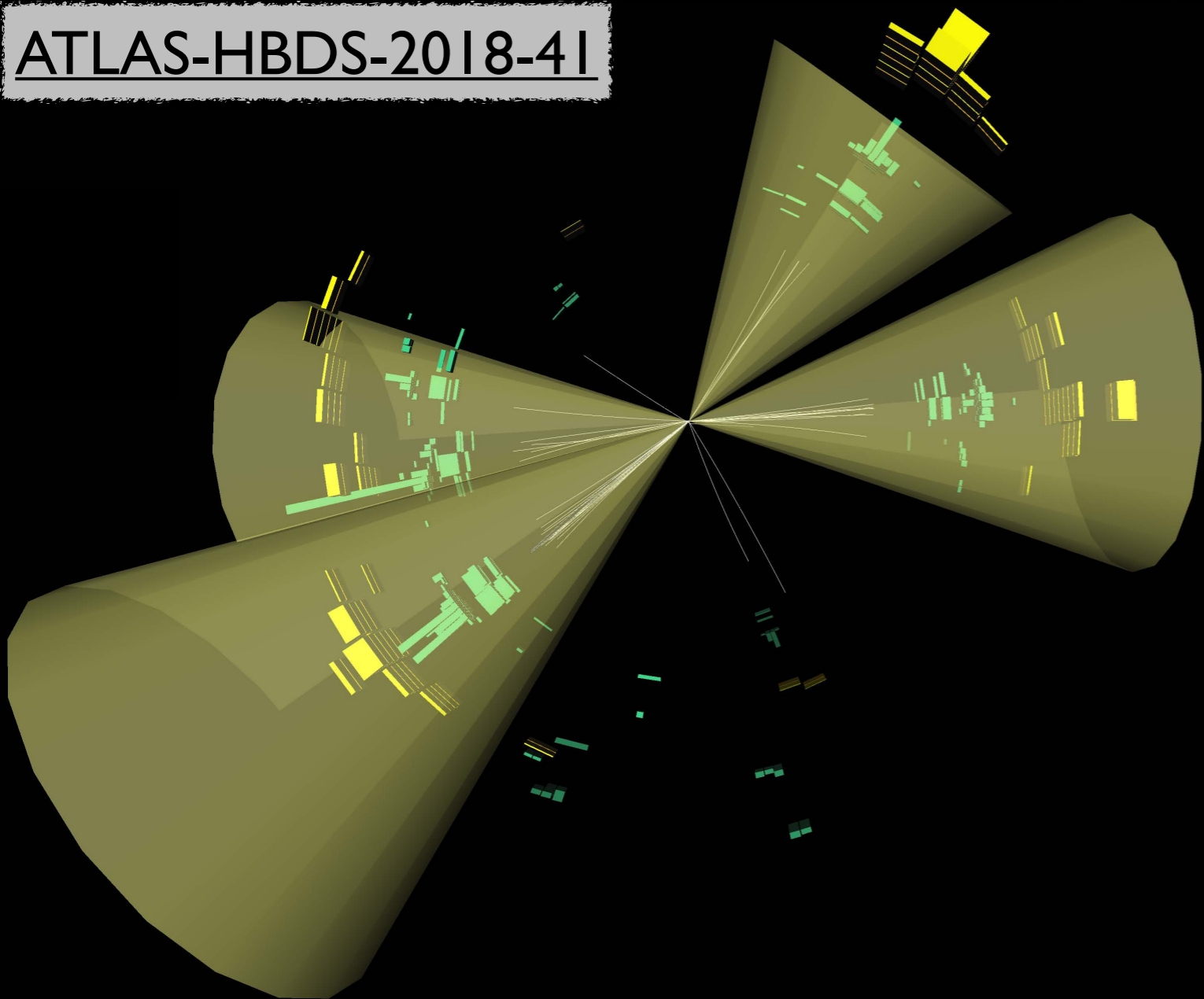


Like a new resonance!

$HH \rightarrow b\bar{b}b\bar{b}$ Resolved

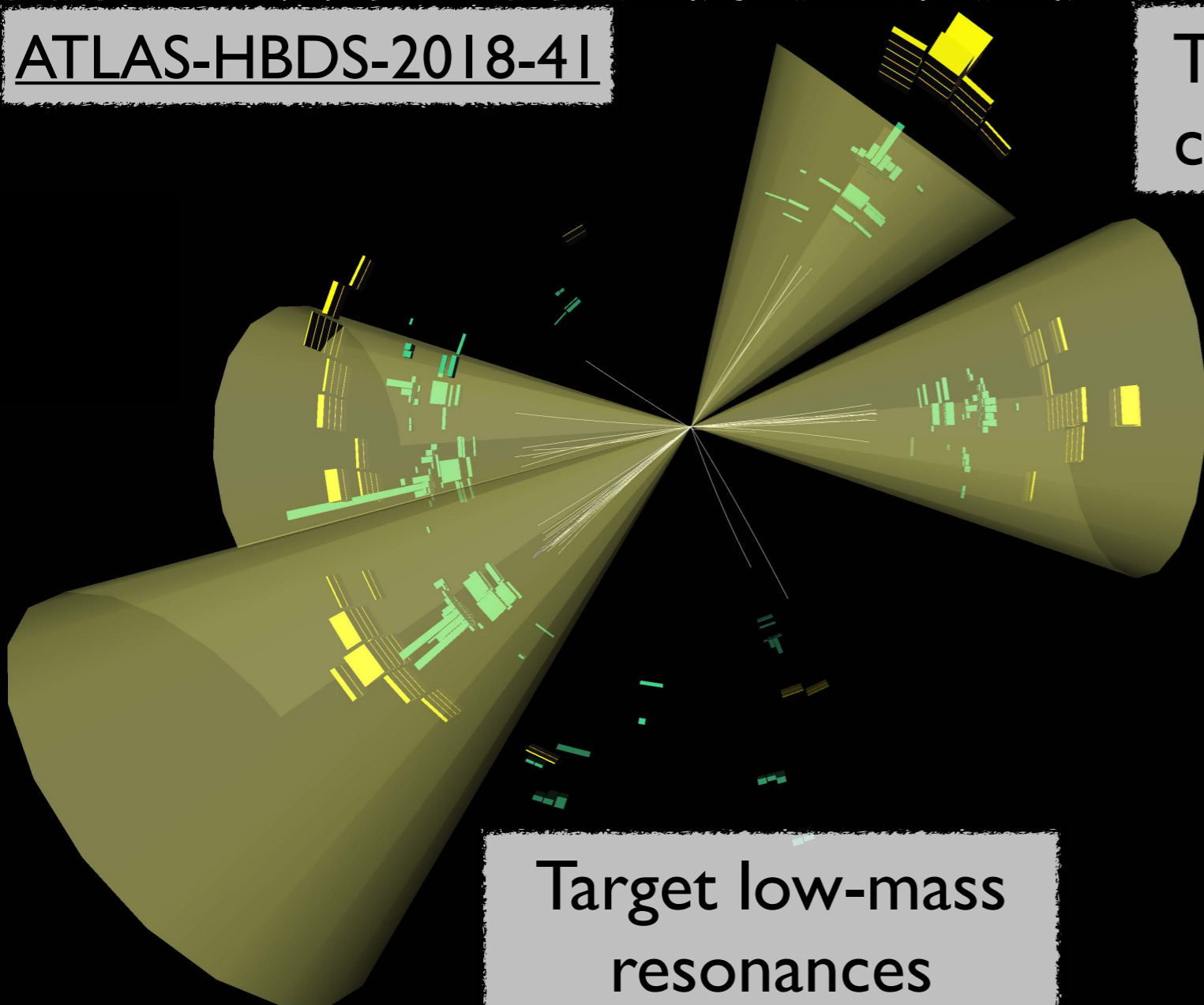
$HH \rightarrow b\bar{b}b\bar{b}$ Boosted

ATLAS-HBDS-2018-41



$HH \rightarrow b\bar{b}b\bar{b}$ Resolved

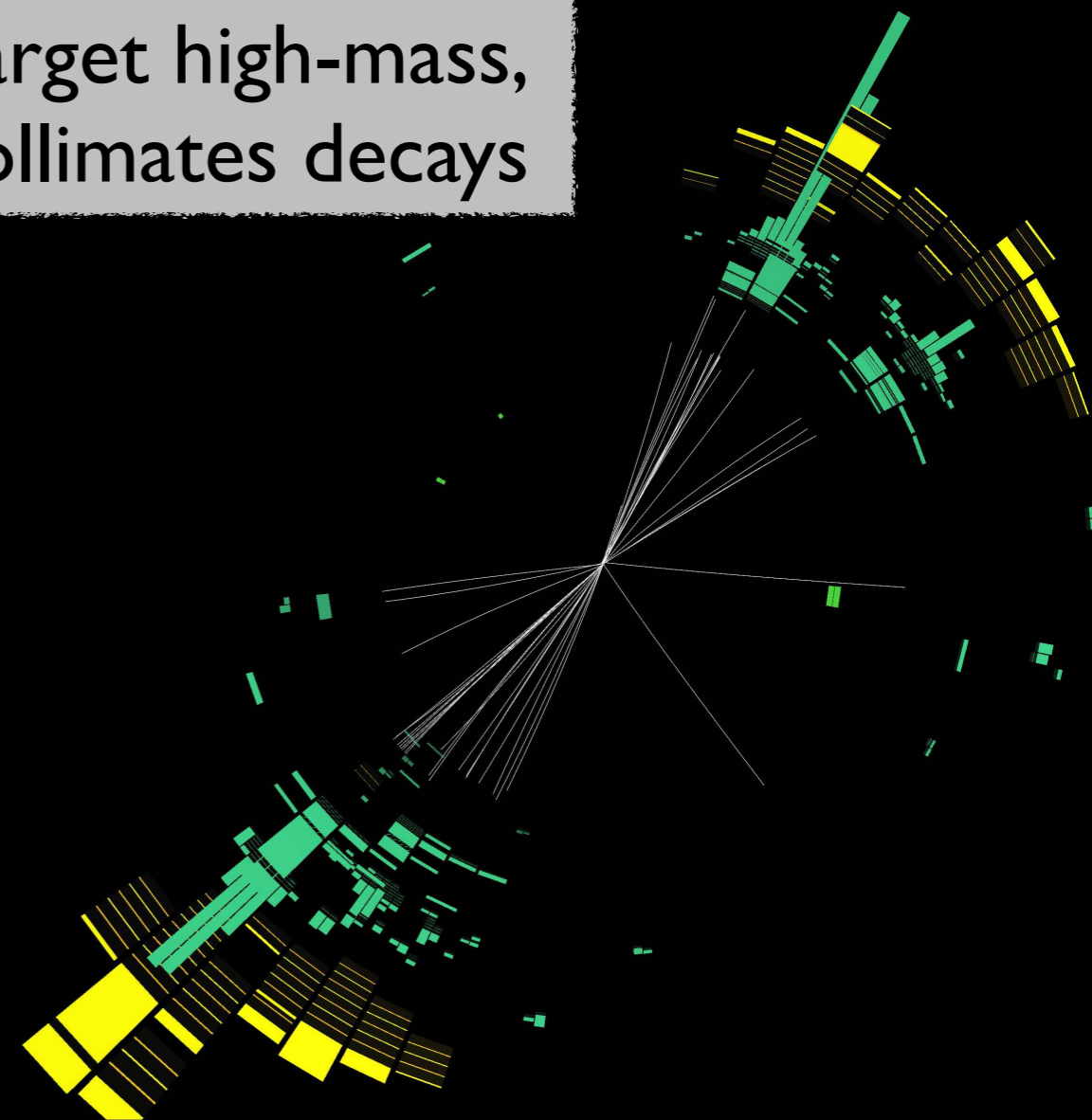
ATLAS-HBDS-2018-41



Target low-mass
resonances

$HH \rightarrow b\bar{b}b\bar{b}$ Boosted

Target high-mass,
collimates decays

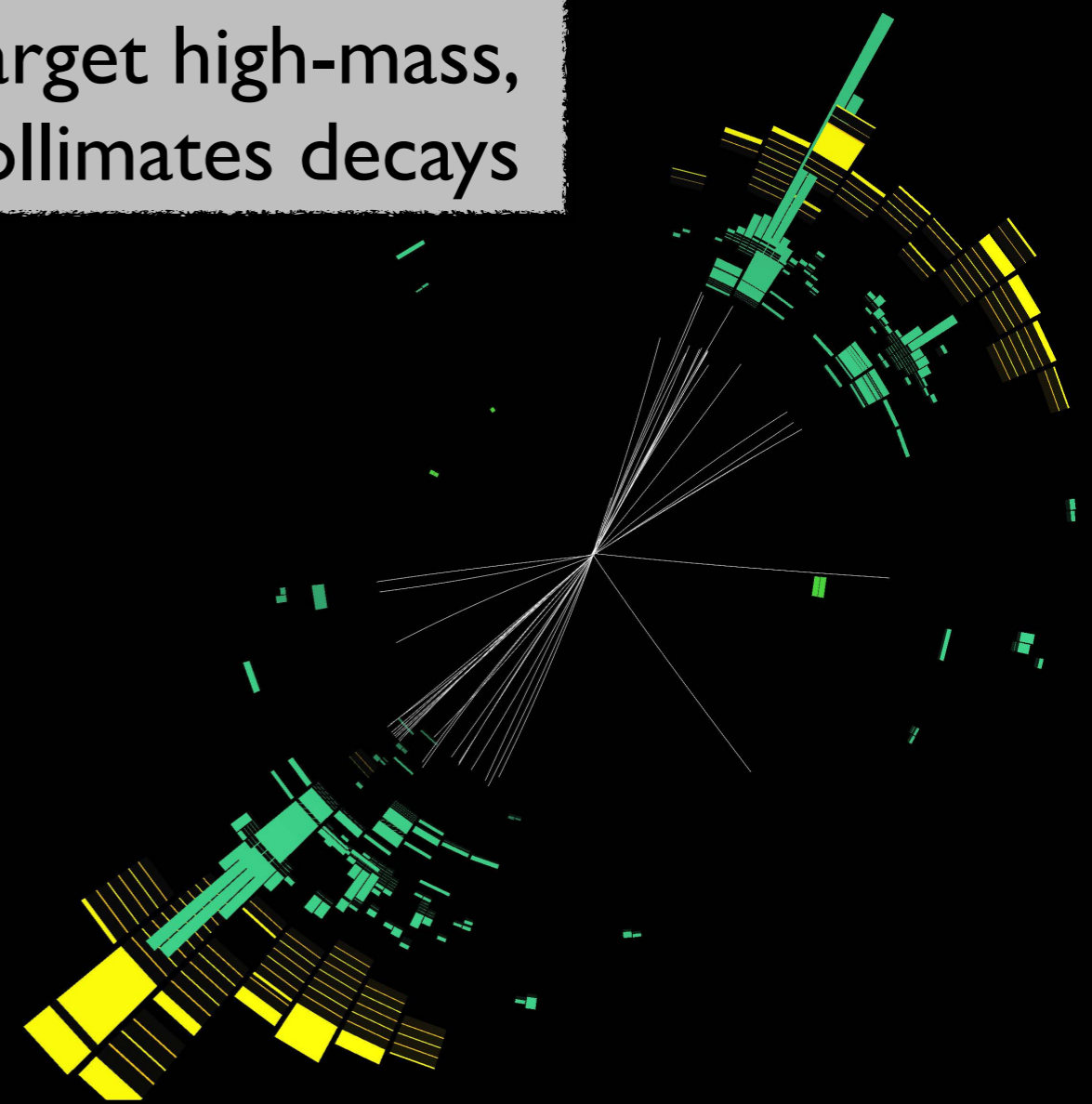
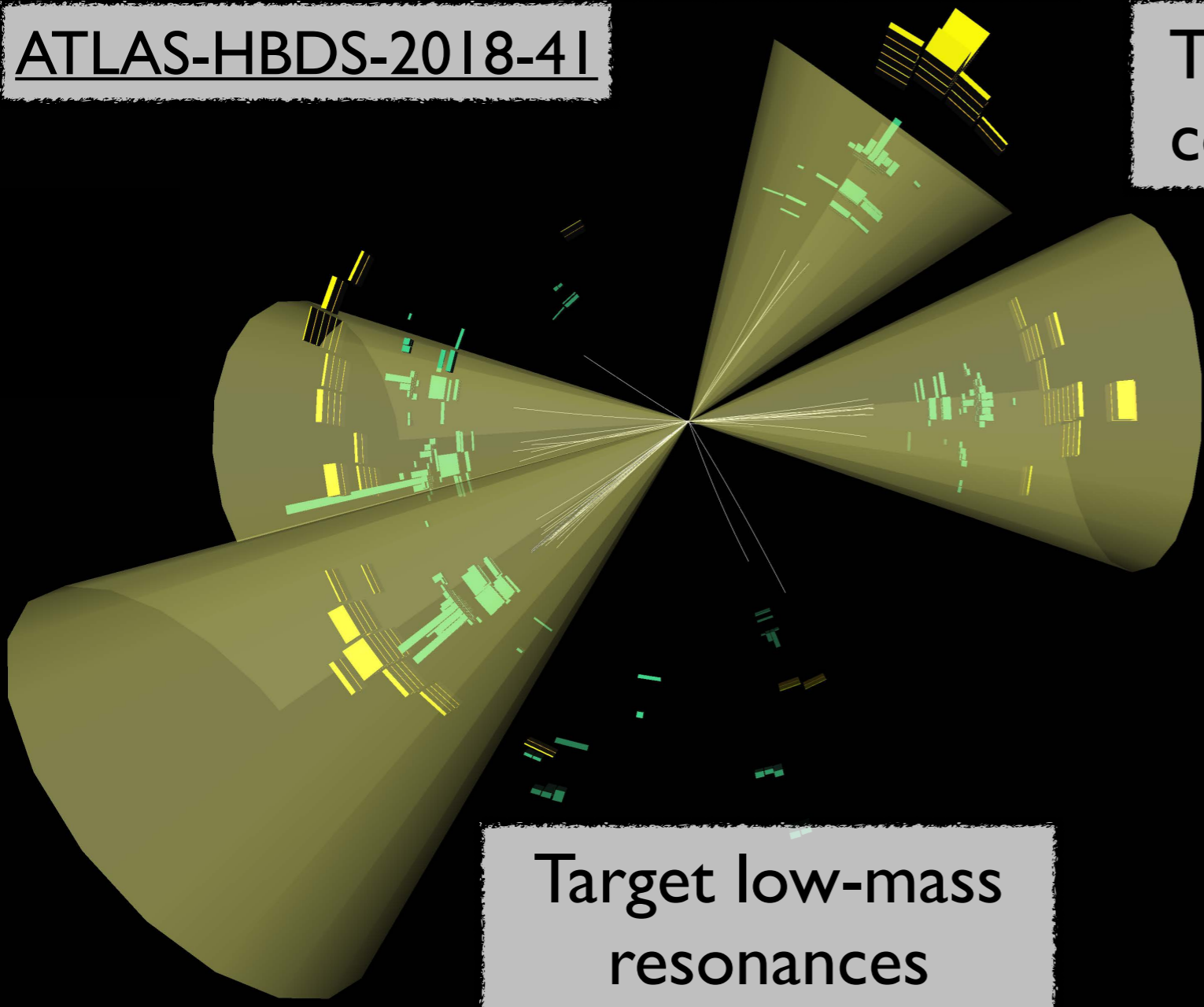


$HH \rightarrow b\bar{b}b\bar{b}$ Resolved

$HH \rightarrow b\bar{b}b\bar{b}$ Boosted

ATLAS-HBDS-2018-41

Target high-mass,
collimates decays



Target low-mass
resonances

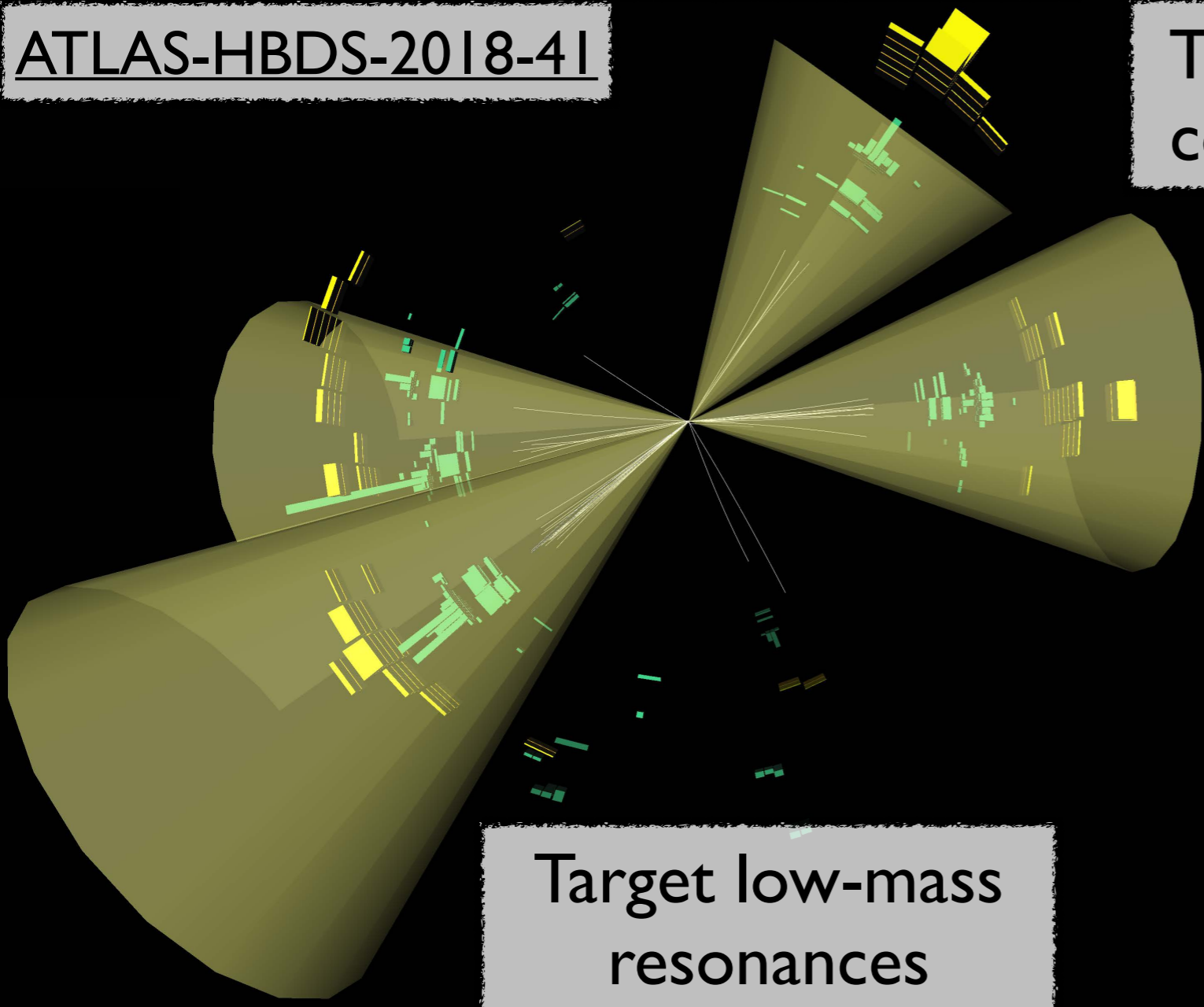
Combination of ~ 4 b-jet triggers

4 b-tagged jets
($\epsilon = 77\%$, $p_T > 40$ GeV)

Boosted Decision Tree used to pair jets

$HH \rightarrow b\bar{b}b\bar{b}$ Resolved

ATLAS-HBDS-2018-41



Target low-mass resonances

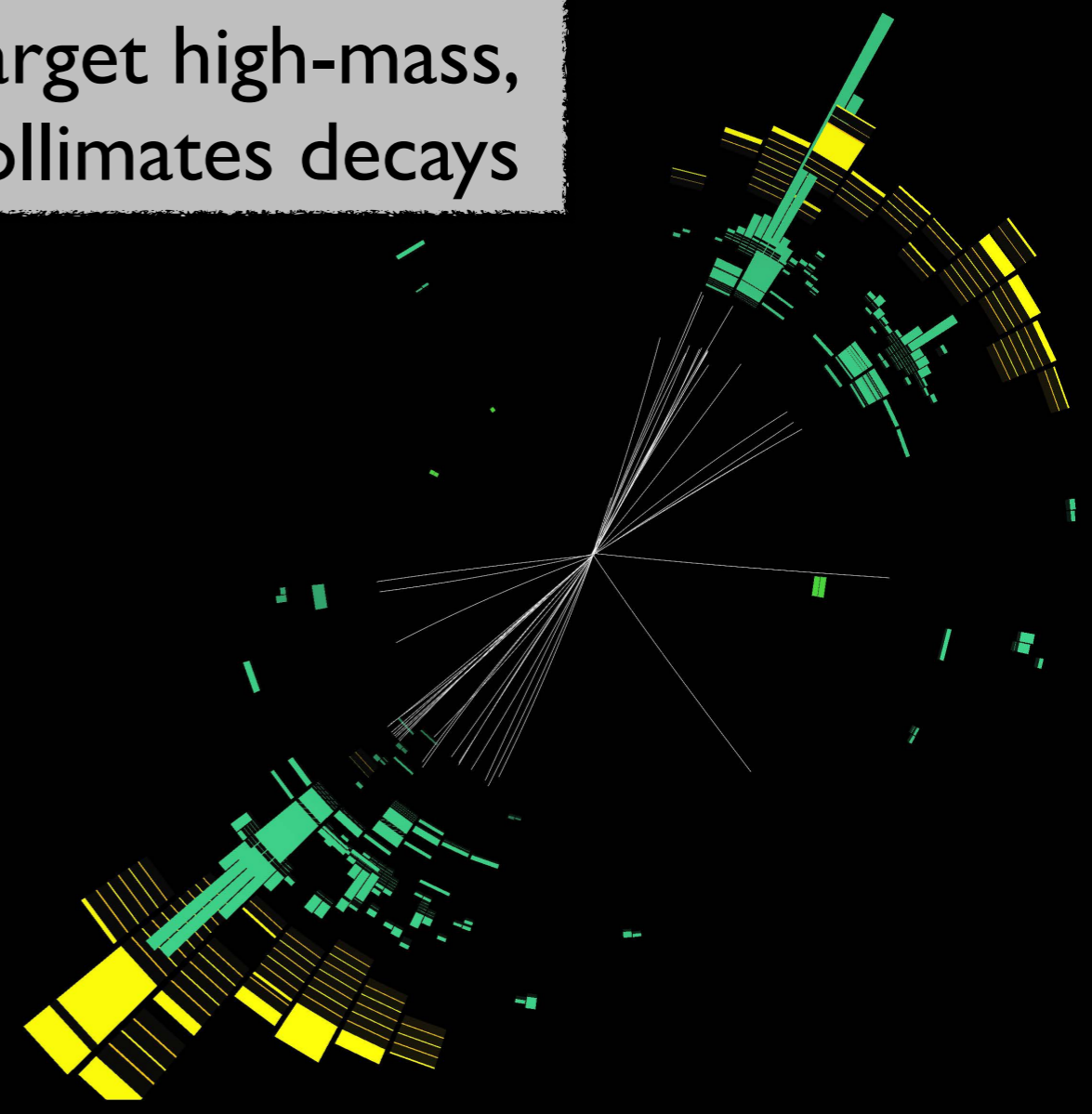
Combination of ~ 4 b-jet triggers

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($\epsilon = 77\%$, $p_T > 40$ GeV)

Boosted Decision Tree used to pair jets

$HH \rightarrow b\bar{b}b\bar{b}$ Boosted

Target high-mass, collimates decays



Large-R jet trigger ($E_T > 450$ GeV)

Two large-R jets
($R=1.0$, $p_T > 450$ (250) GeV)

2, 3, or 4 b-tags (via track-jets, $\epsilon = 77\%$)

$b\bar{b}b\bar{b}$ Analysis Strategy

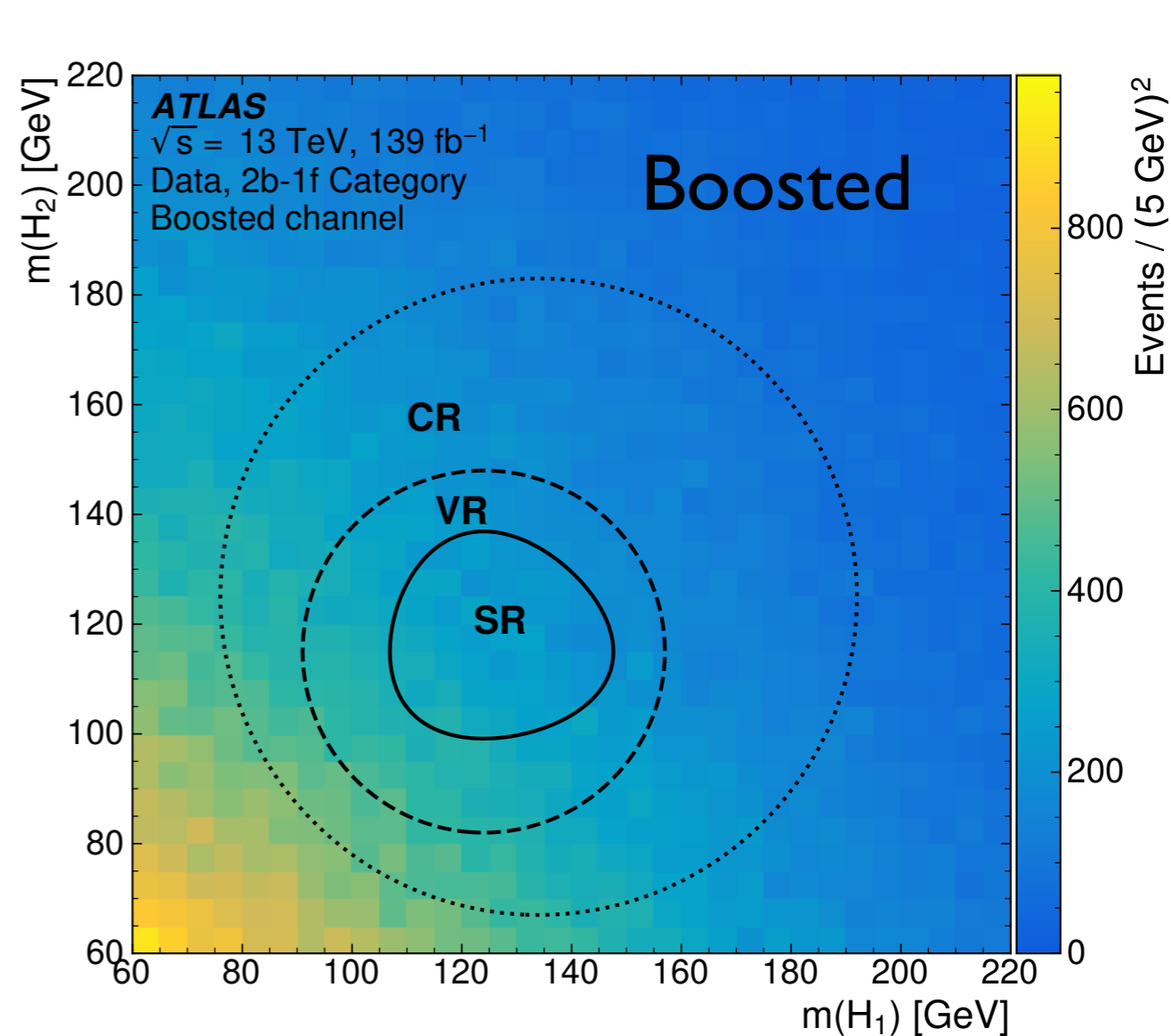
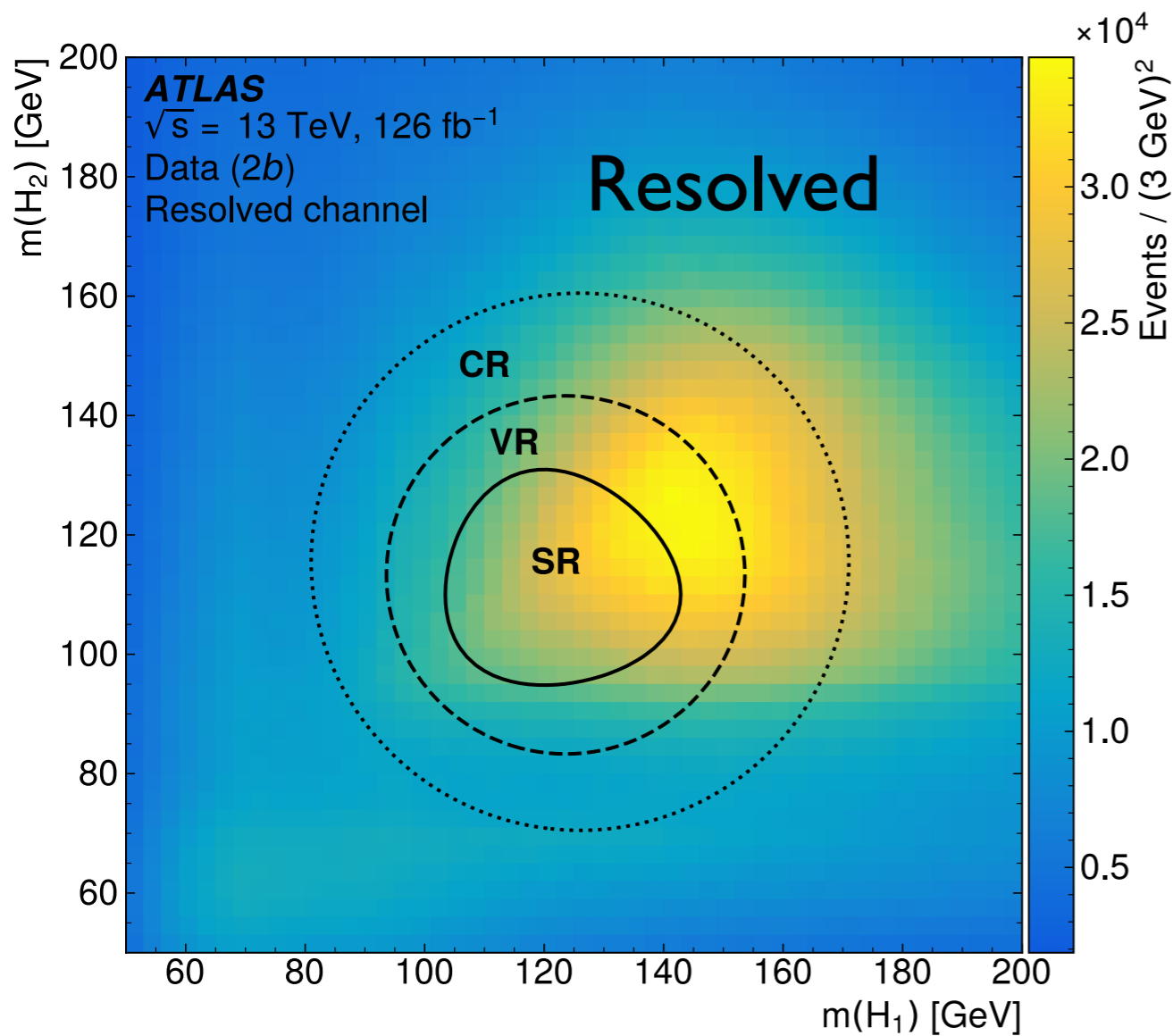
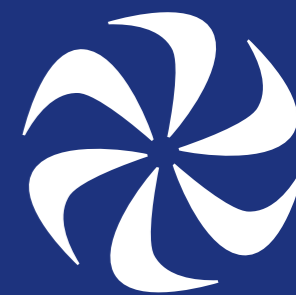


$b\bar{b}b\bar{b}$ Analysis Strategy



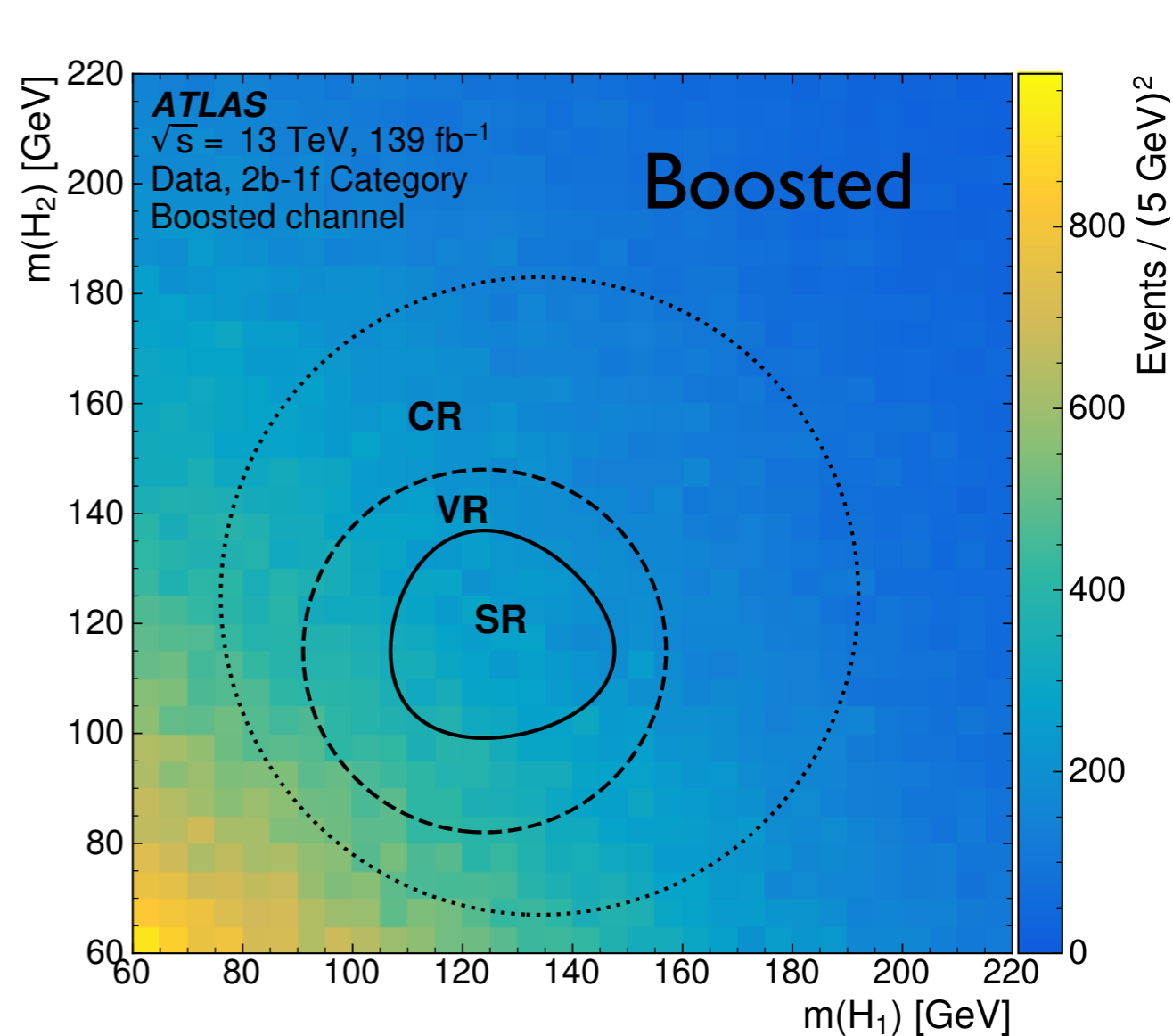
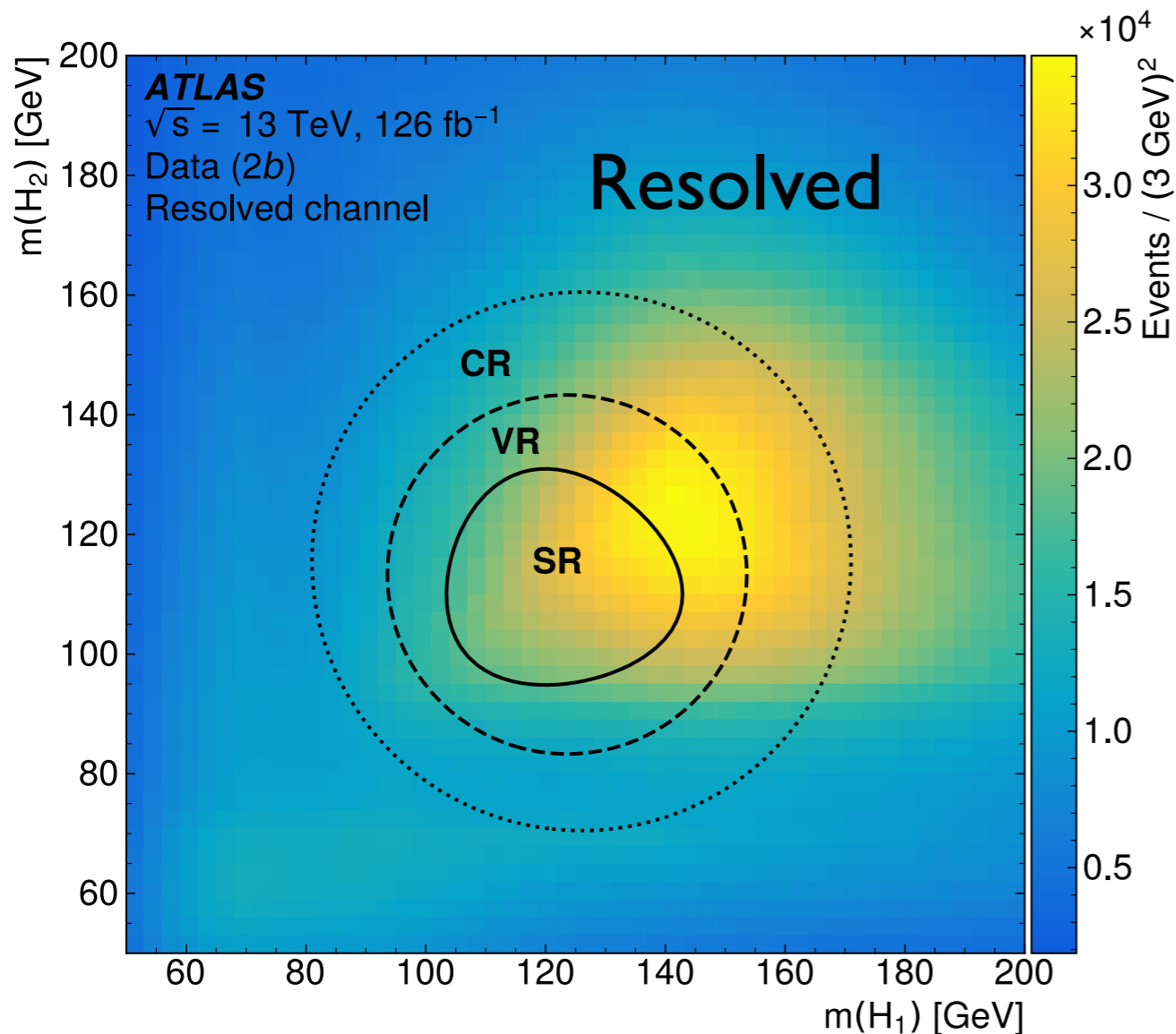
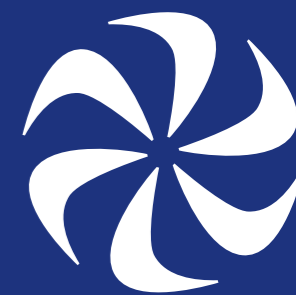
Reconstruct Higgs candidates, form “mass plane”

$b\bar{b}b\bar{b}$ Analysis Strategy



Reconstruct Higgs candidates, form “mass plane”

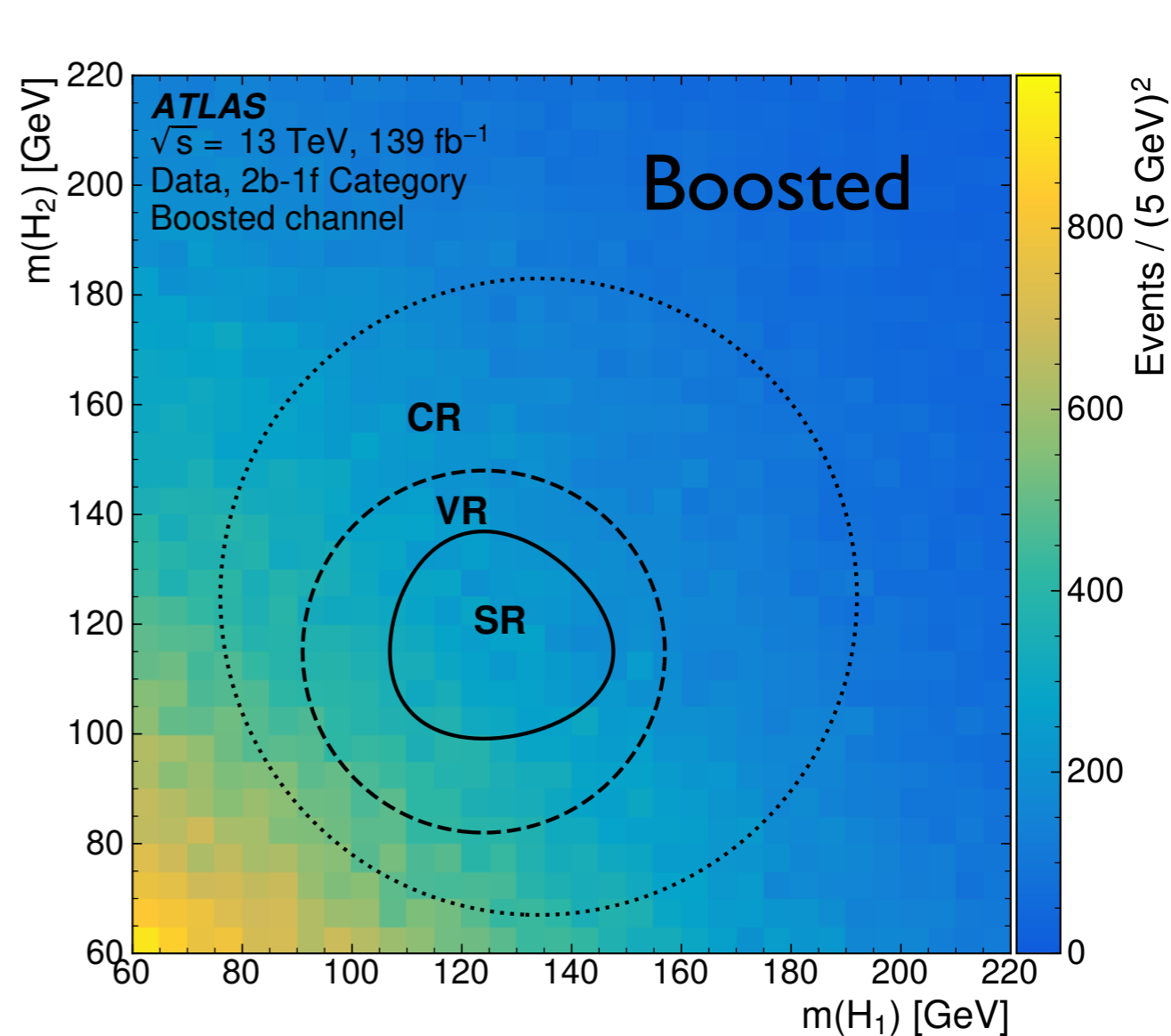
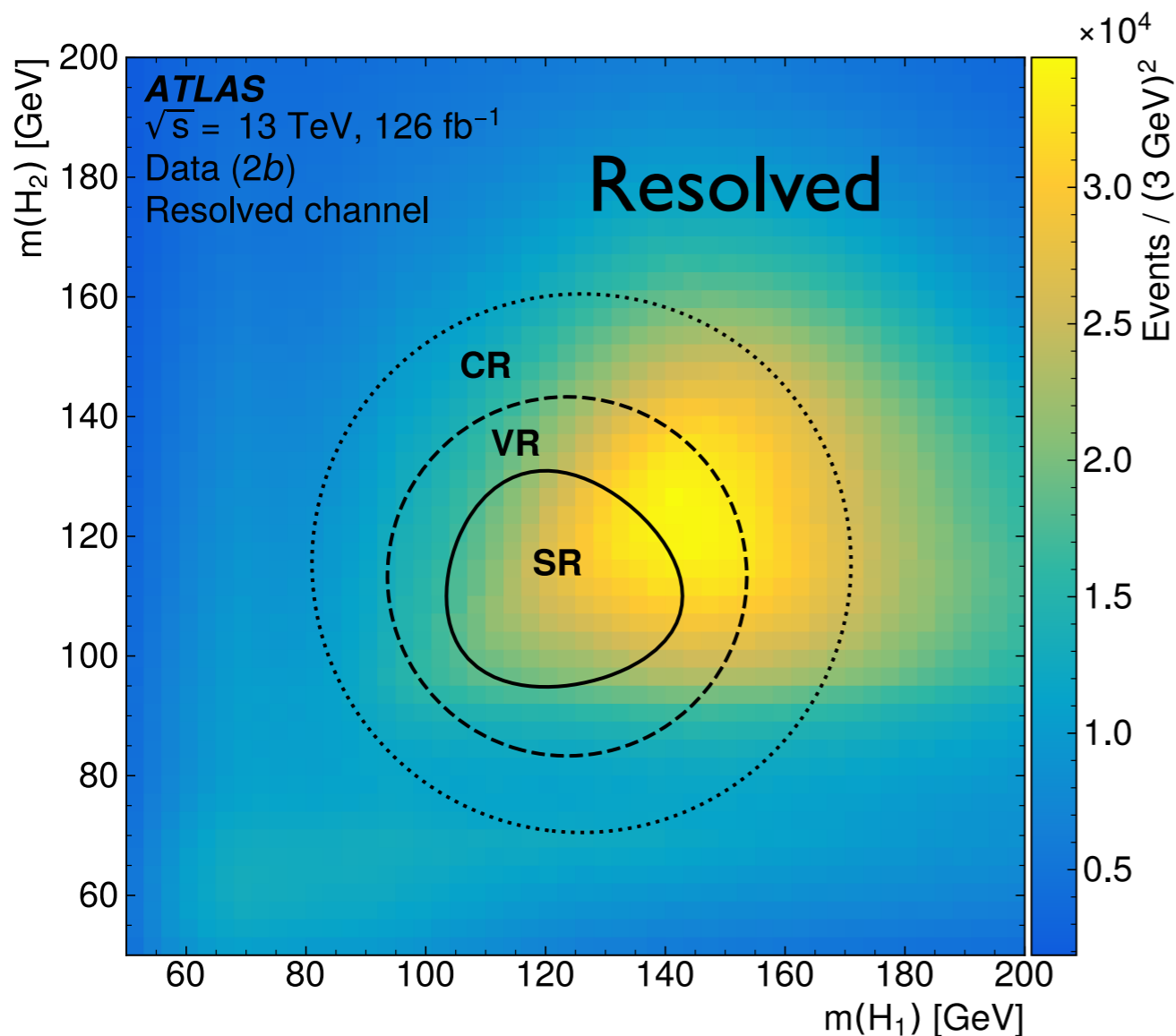
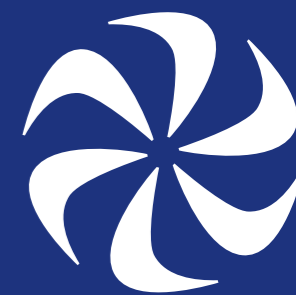
$b\bar{b}b\bar{b}$ Analysis Strategy



Reconstruct Higgs candidates, form “mass plane”

Center is signal-like; outer regions used for background estimation and validation

$b\bar{b}b\bar{b}$ Analysis Strategy



Reconstruct Higgs candidates, form “mass plane”

Center is signal-like; outer regions used for background estimation and validation

Fit m_{HH} in signal region for final analysis

$b\bar{b}b\bar{b}$ Resolved Background

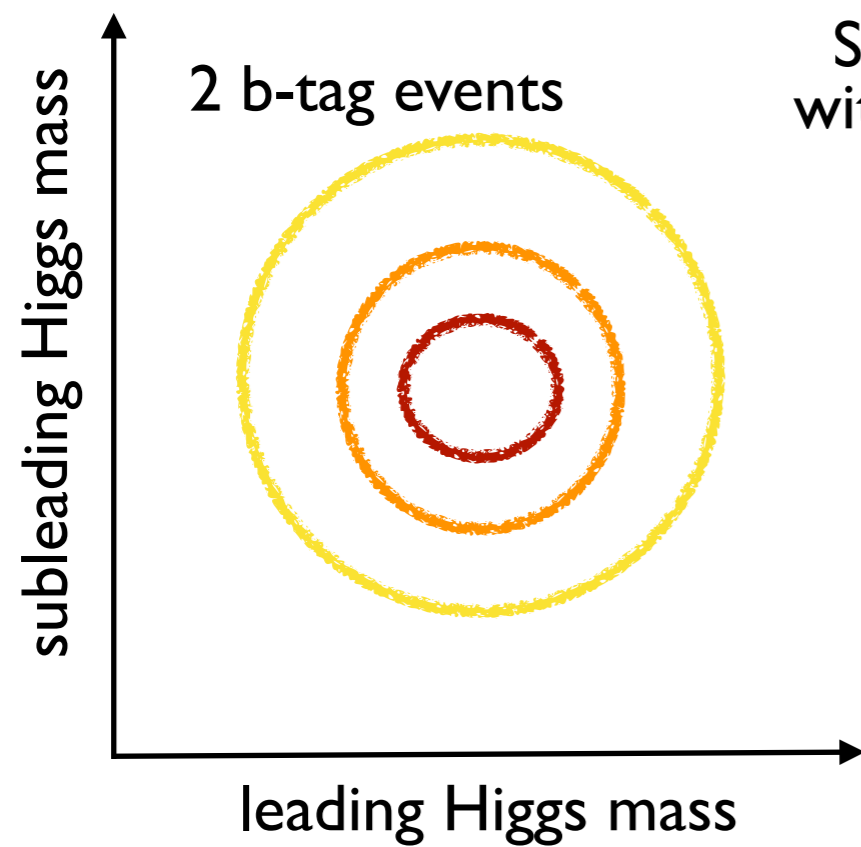


$b\bar{b}b\bar{b}$ Resolved Background

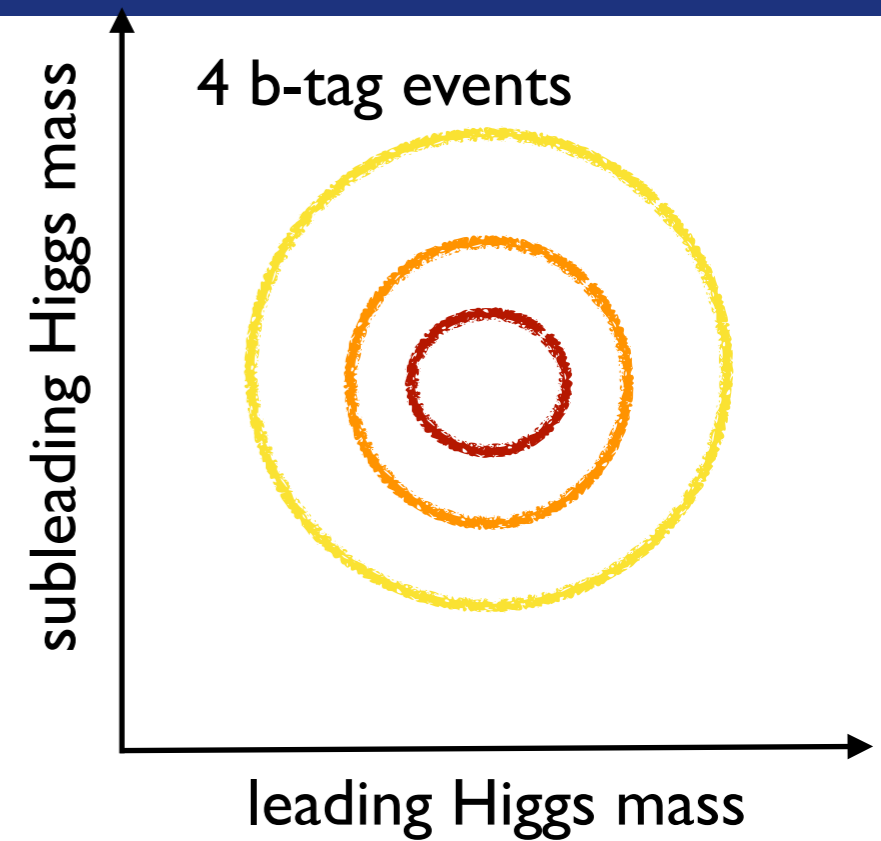


Step 0: form “mass planes”
with leading/subleading Higgs,
for 2b and 4b events

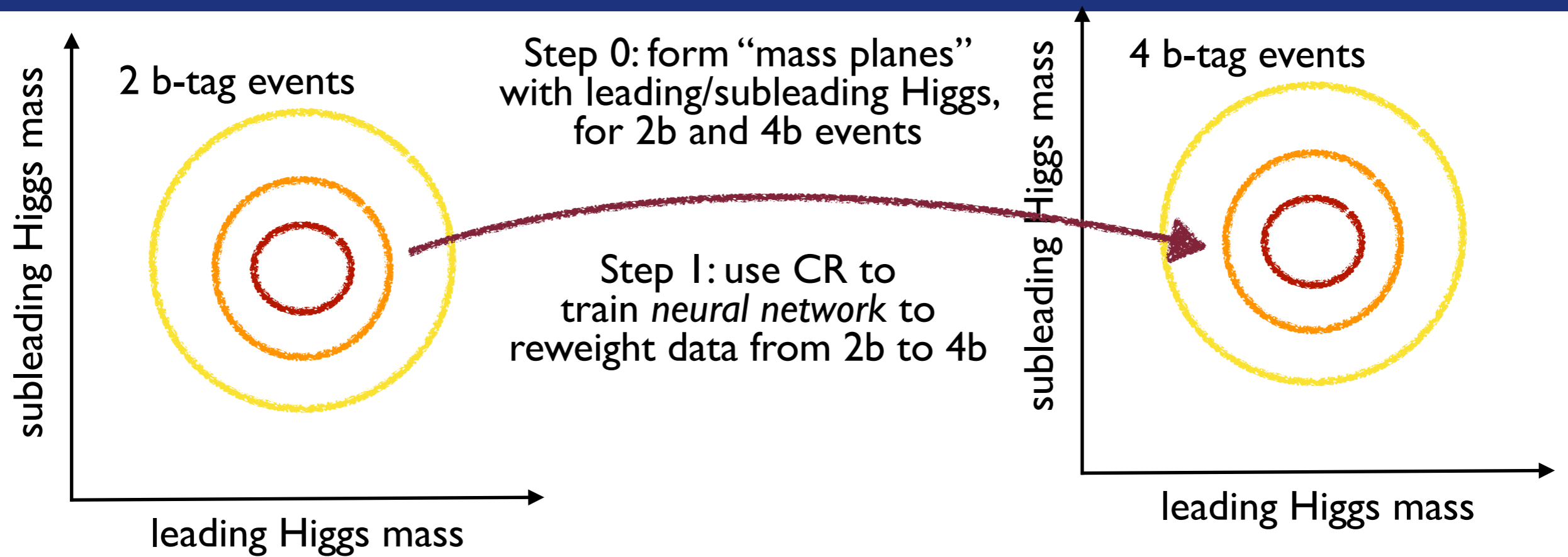
$b\bar{b}b\bar{b}$ Resolved Background



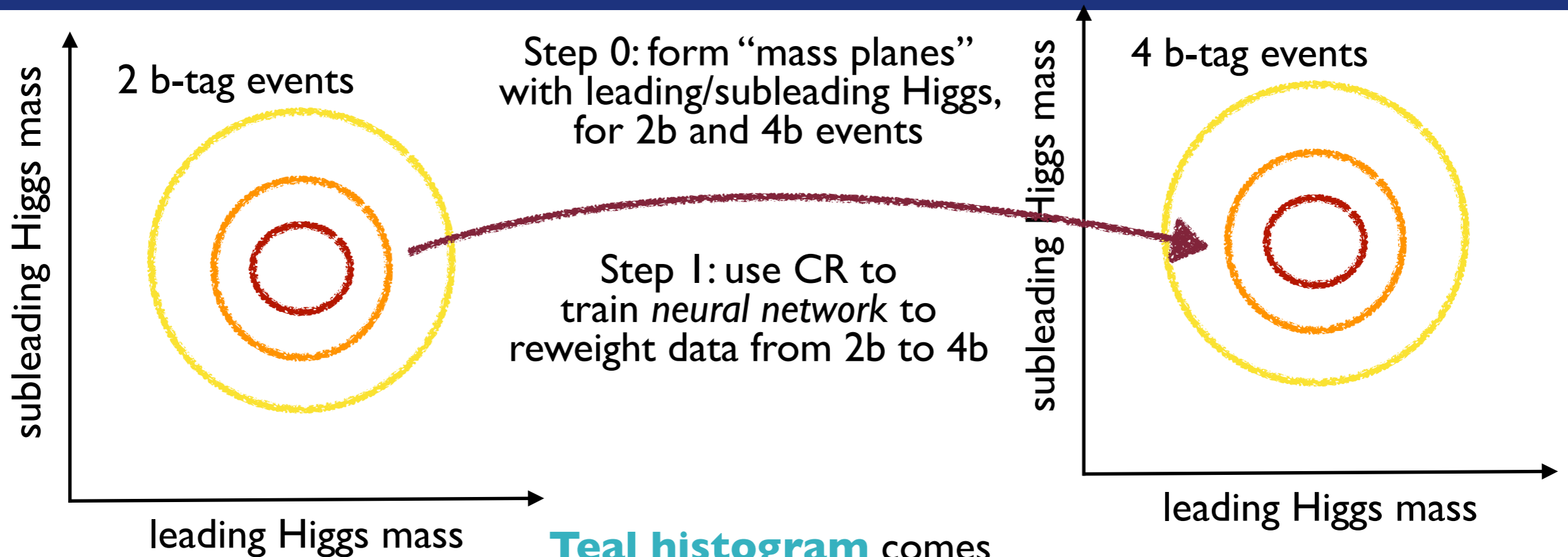
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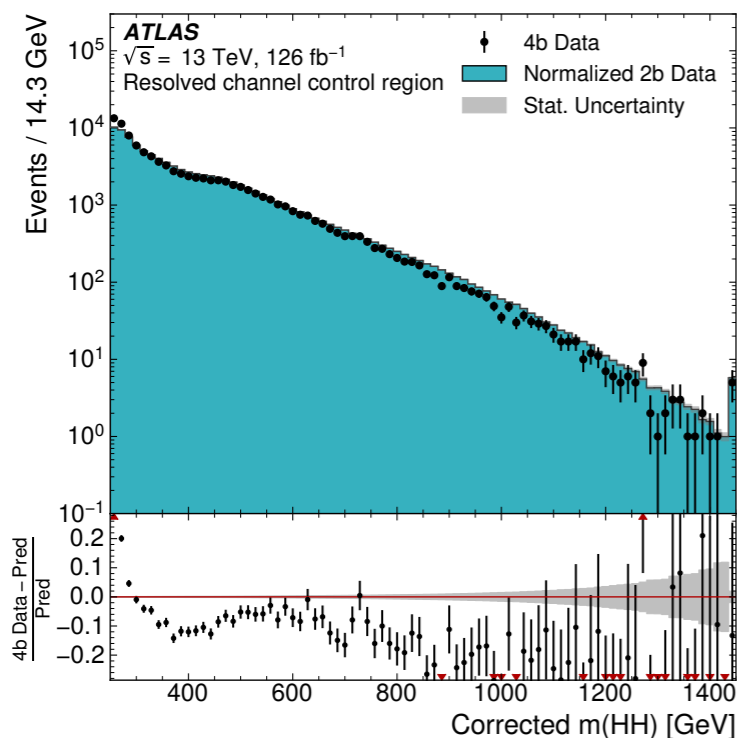
$b\bar{b}b\bar{b}$ Resolved Background



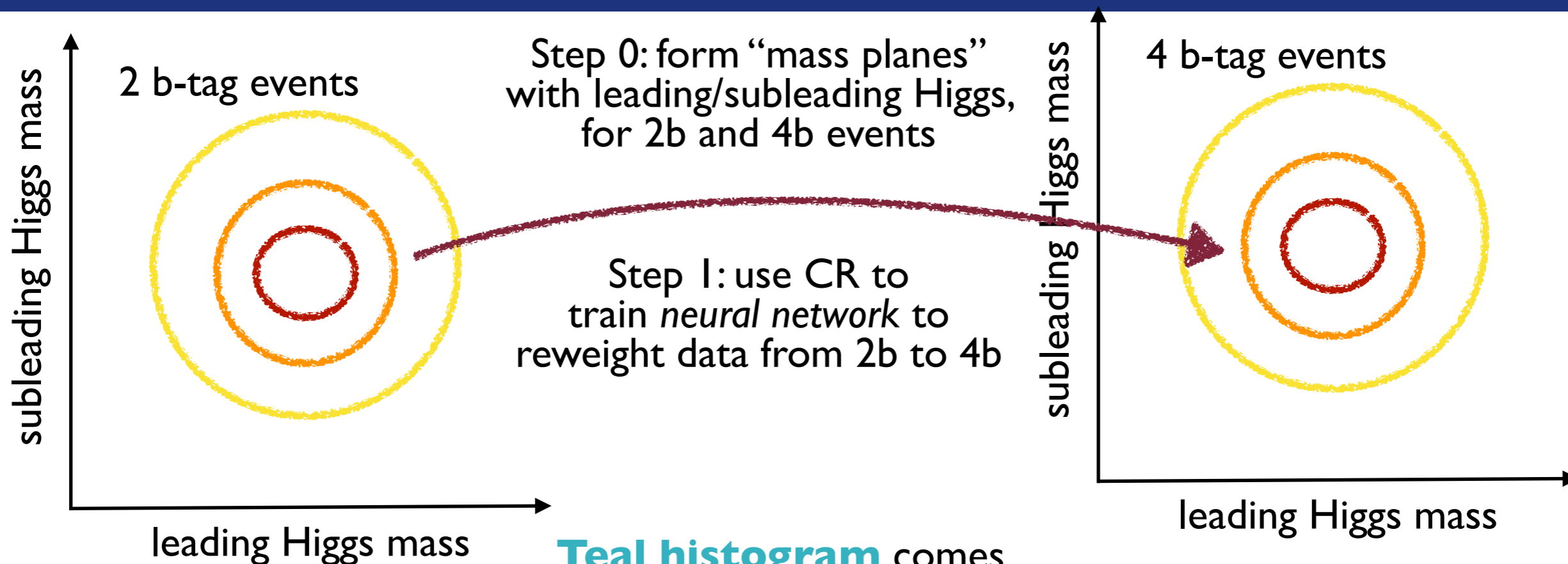
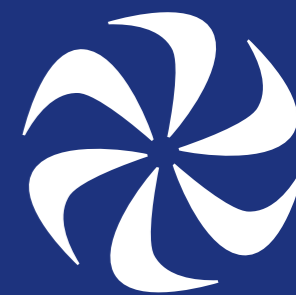
$b\bar{b}b\bar{b}$ Resolved Background



Teal histogram comes from 2b, **black points** from 4b

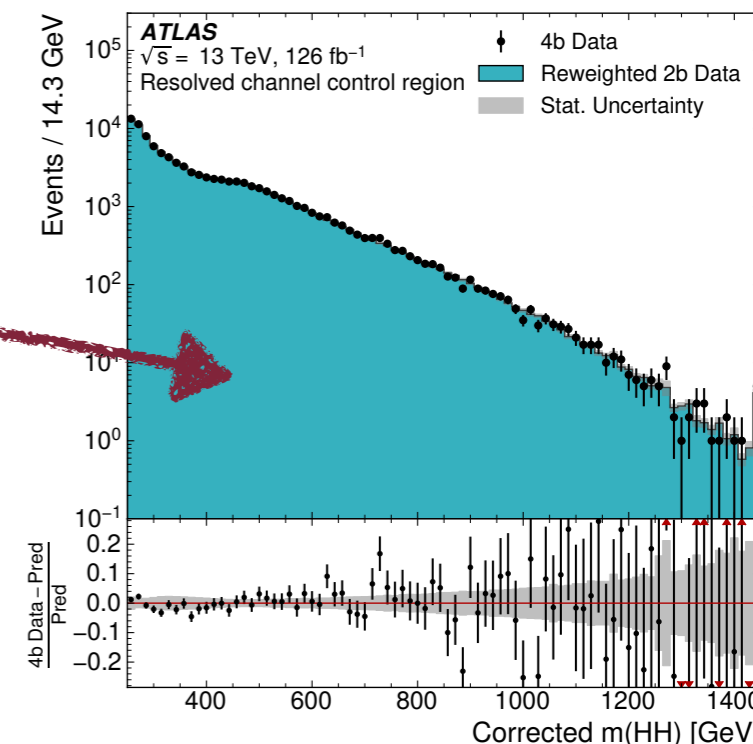
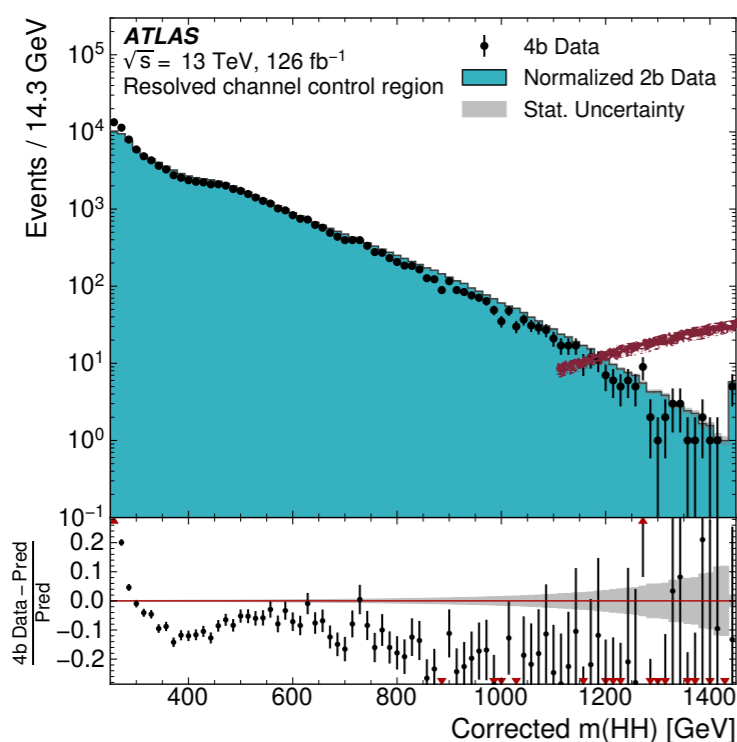


$b\bar{b}b\bar{b}$ Resolved Background

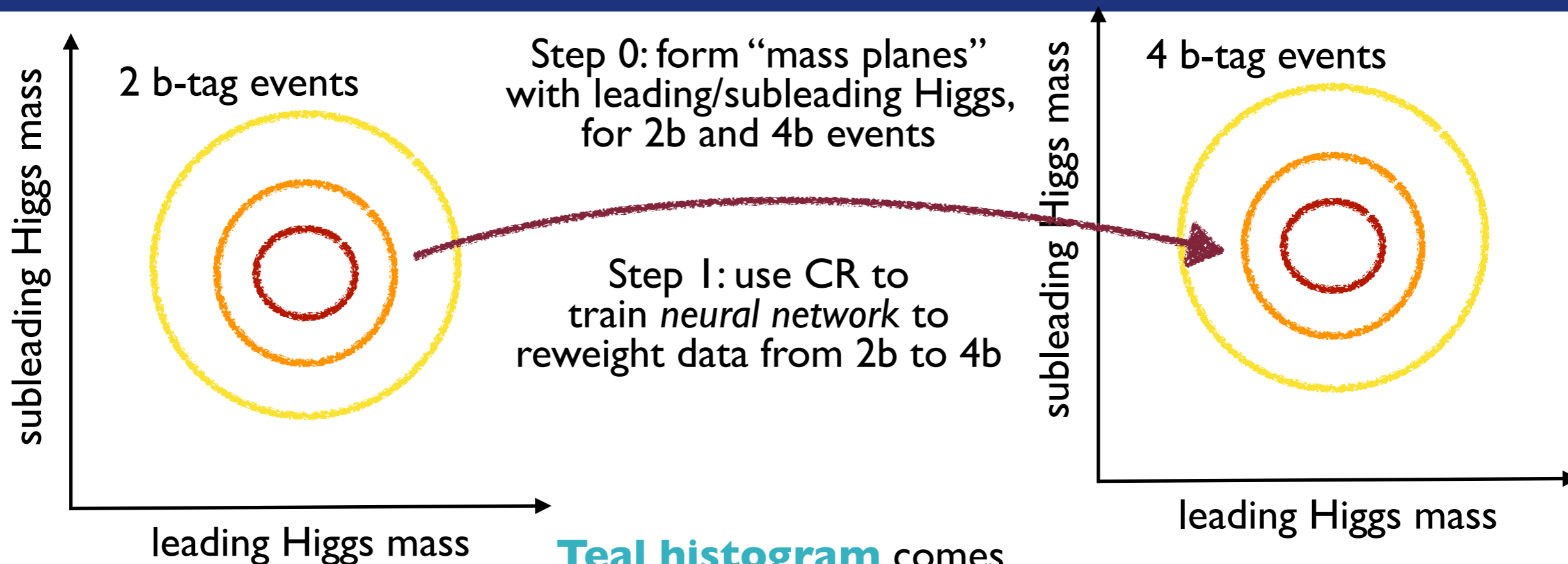
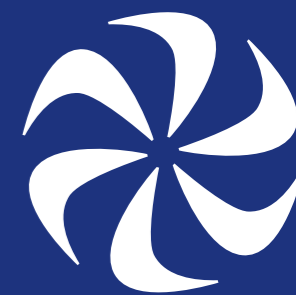


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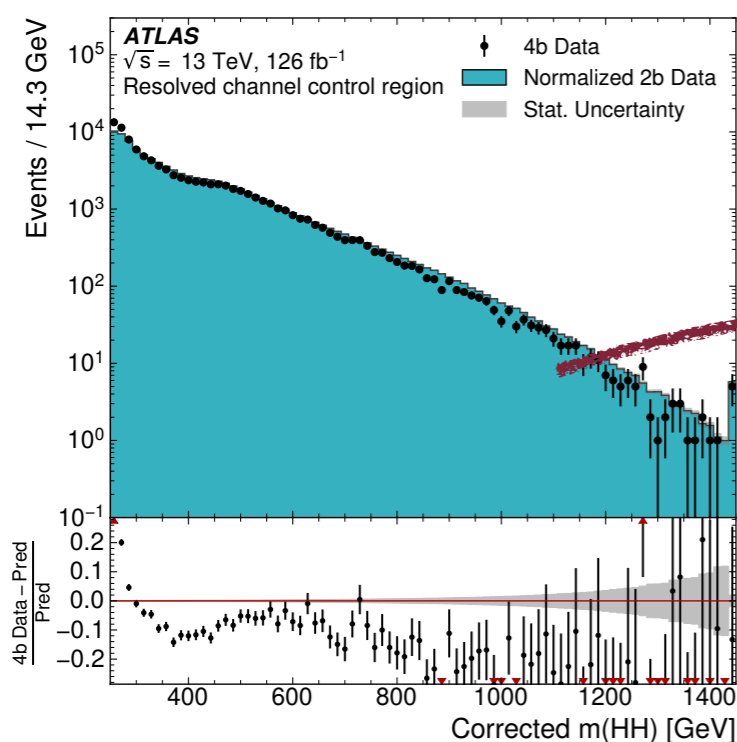
Neural network



$b\bar{b}b\bar{b}$ Resolved Background

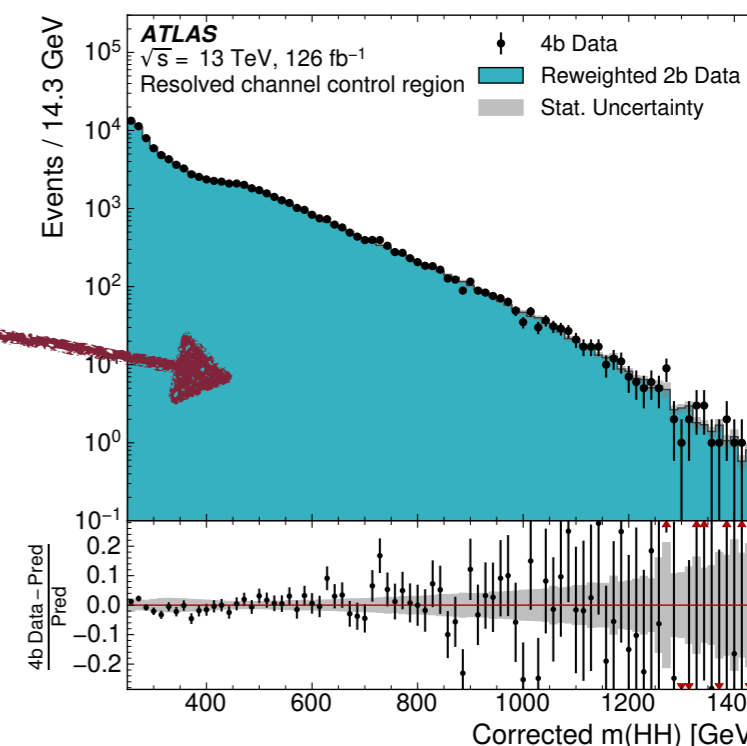


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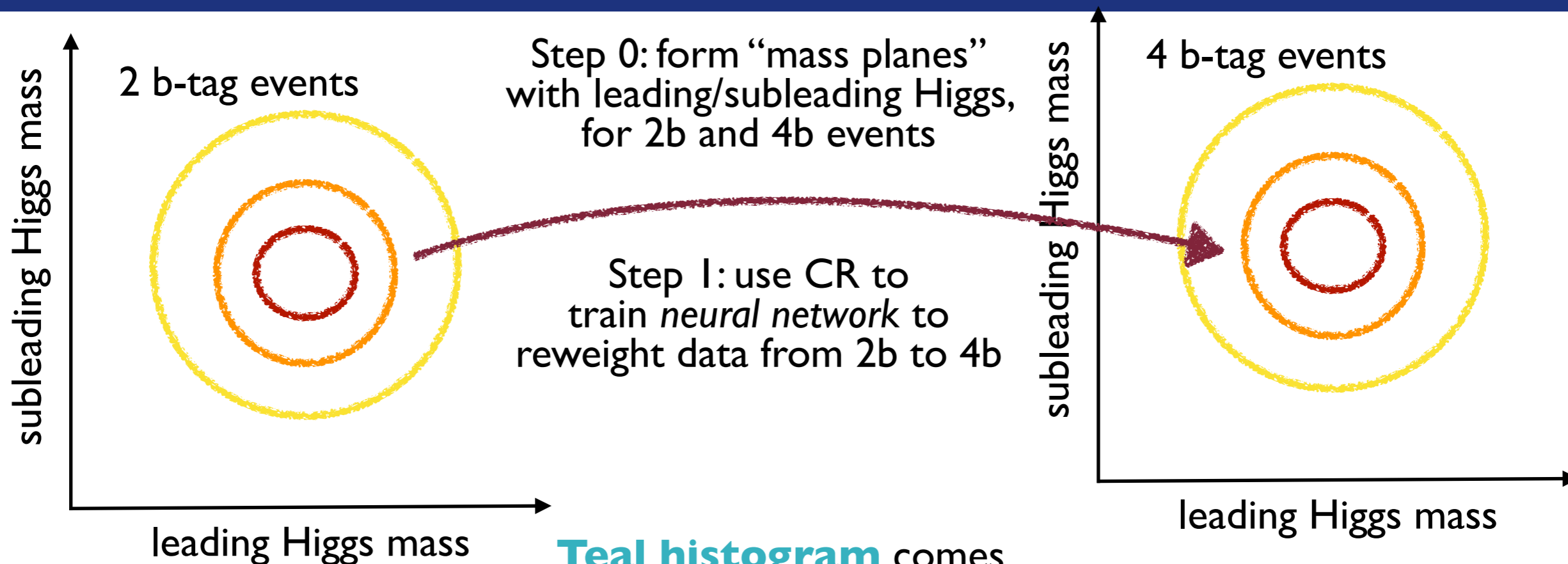
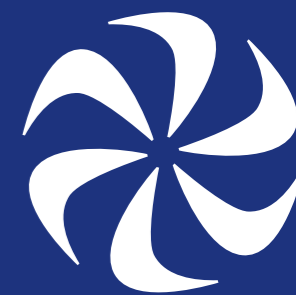


Neural network

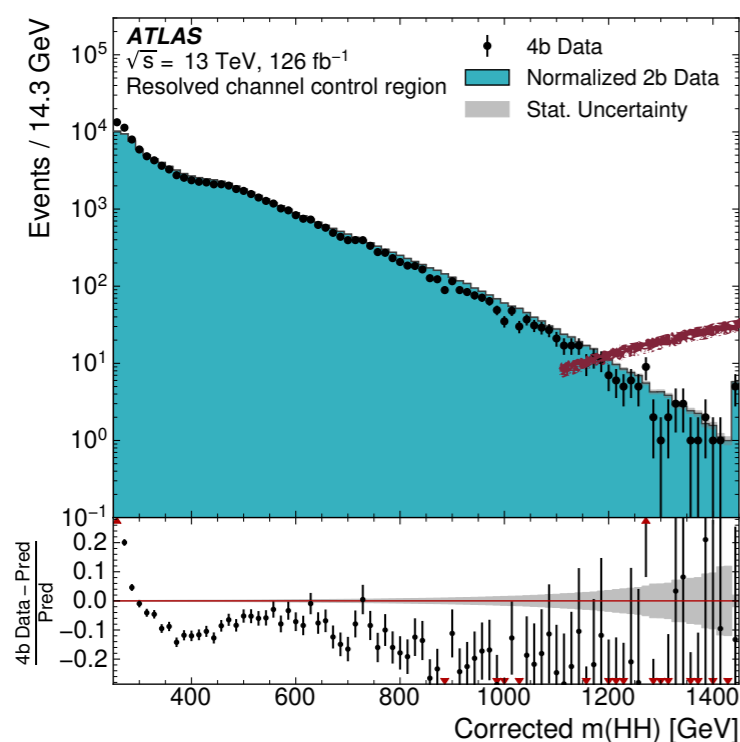
Step 2: Apply this NN to 2b SR: prediction for 4b SR



$b\bar{b}b\bar{b}$ Resolved Background



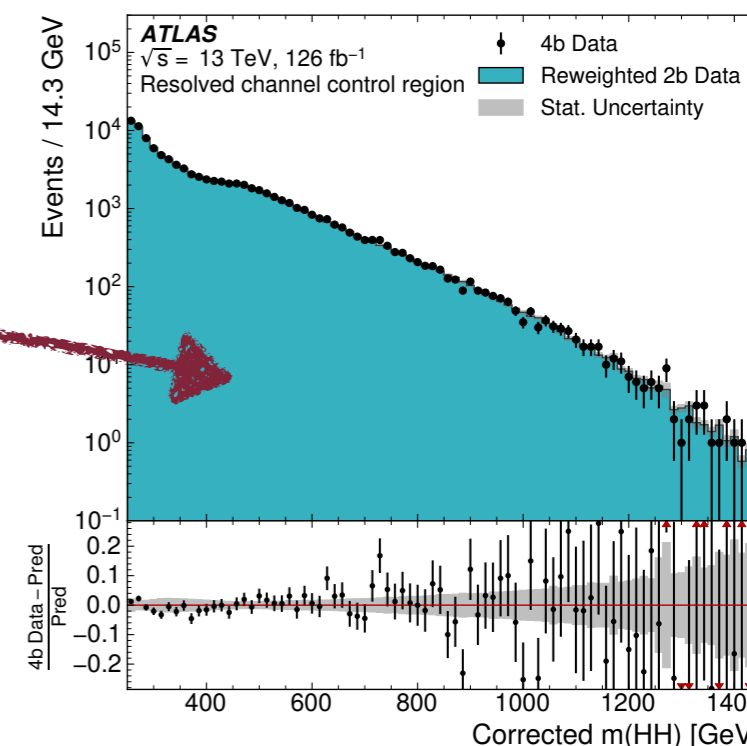
Teal histogram comes from 2b, **black points** from 4b



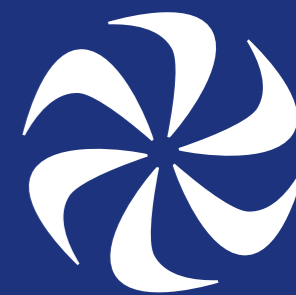
Neural network

Step 2: Apply this NN to 2b SR: prediction for 4b SR

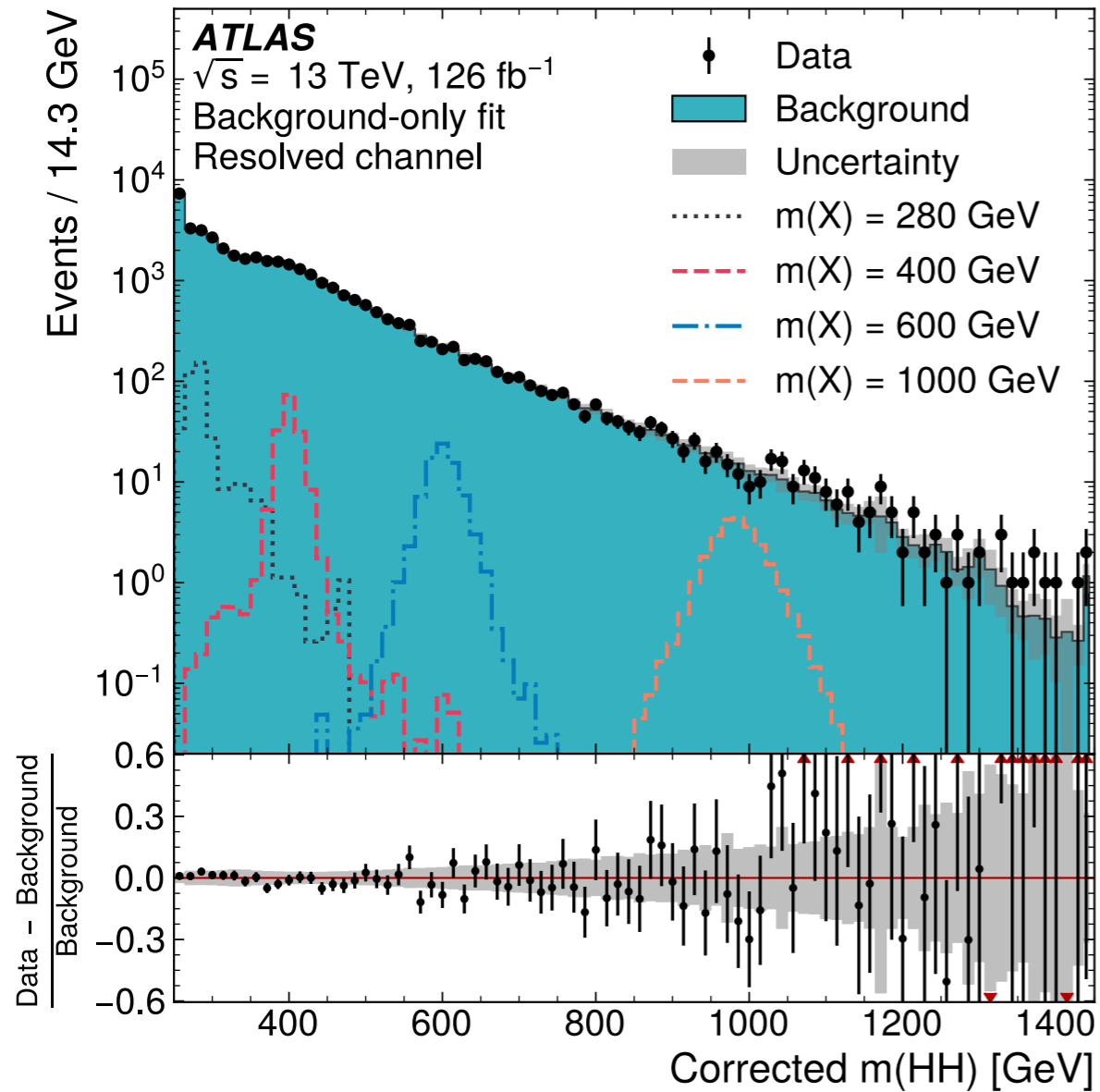
Systematics from alternate validation region



$b\bar{b}b\bar{b}$ Results

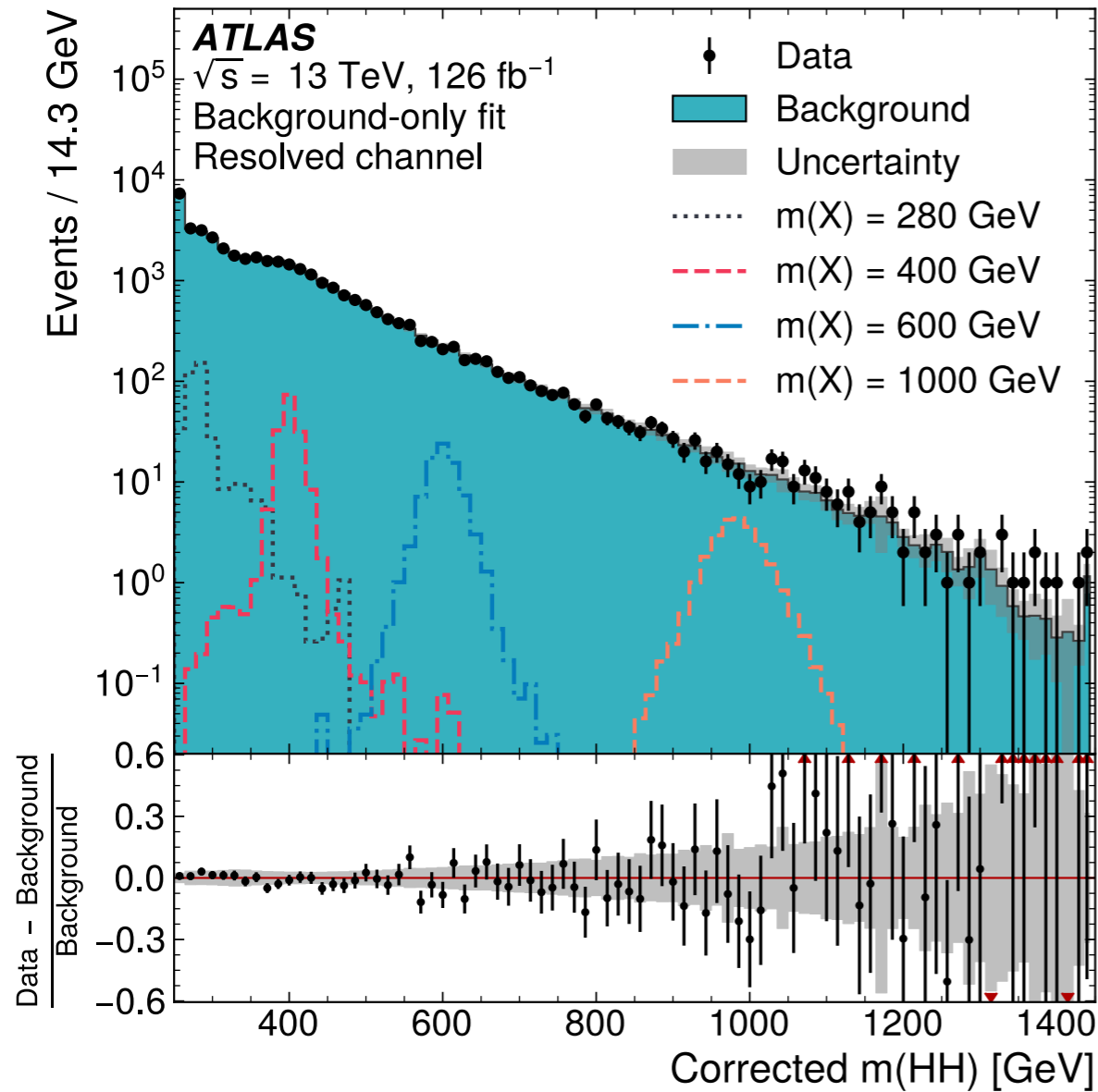


$b\bar{b}b\bar{b}$ Results

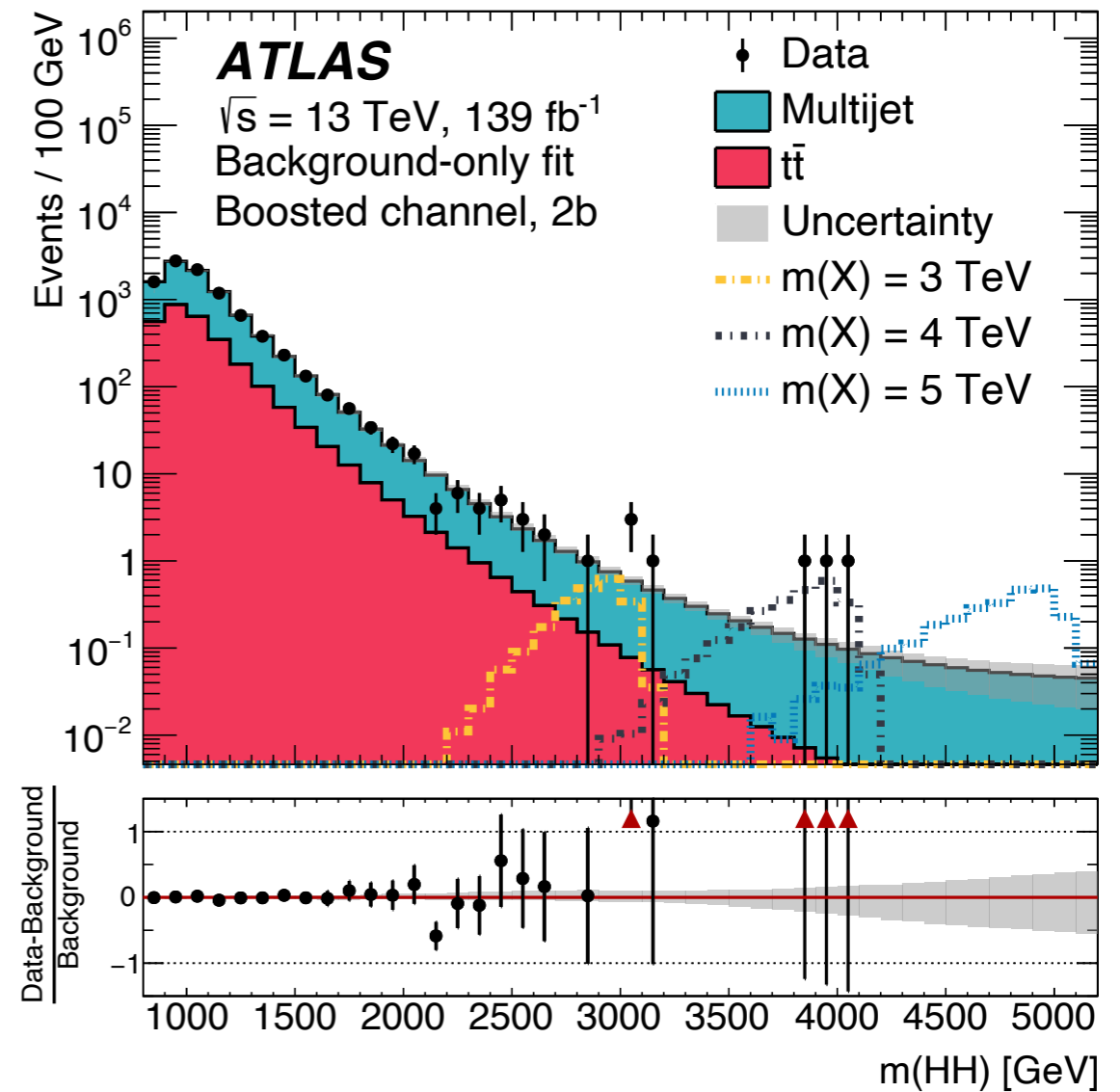


Data agrees well with background prediction

$b\bar{b}b\bar{b}$ Results



Data agrees well with background prediction



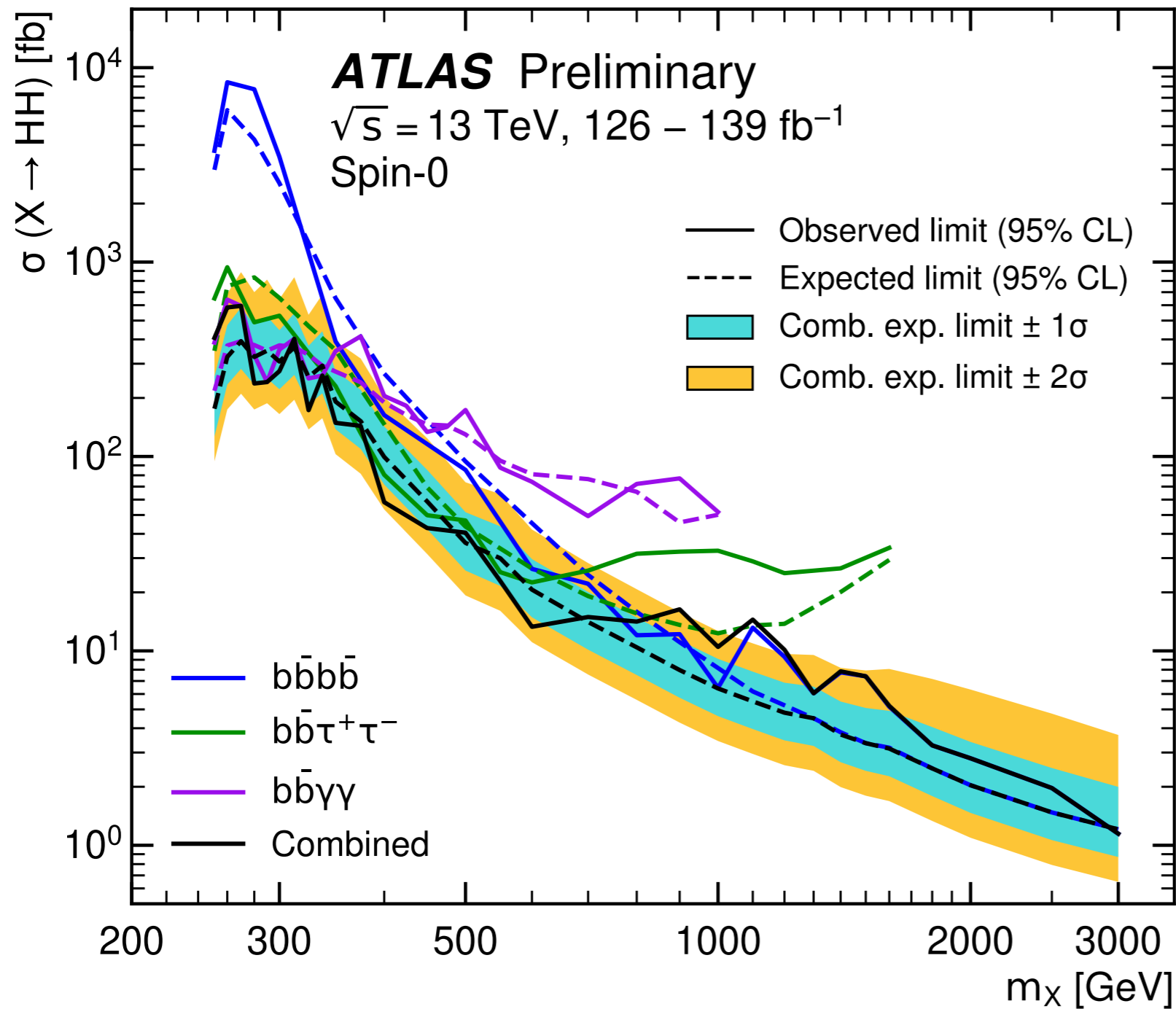
Boosted analysis is similar:
 simpler spline based reweighting

No excess either (also in 3b and 2b SR)

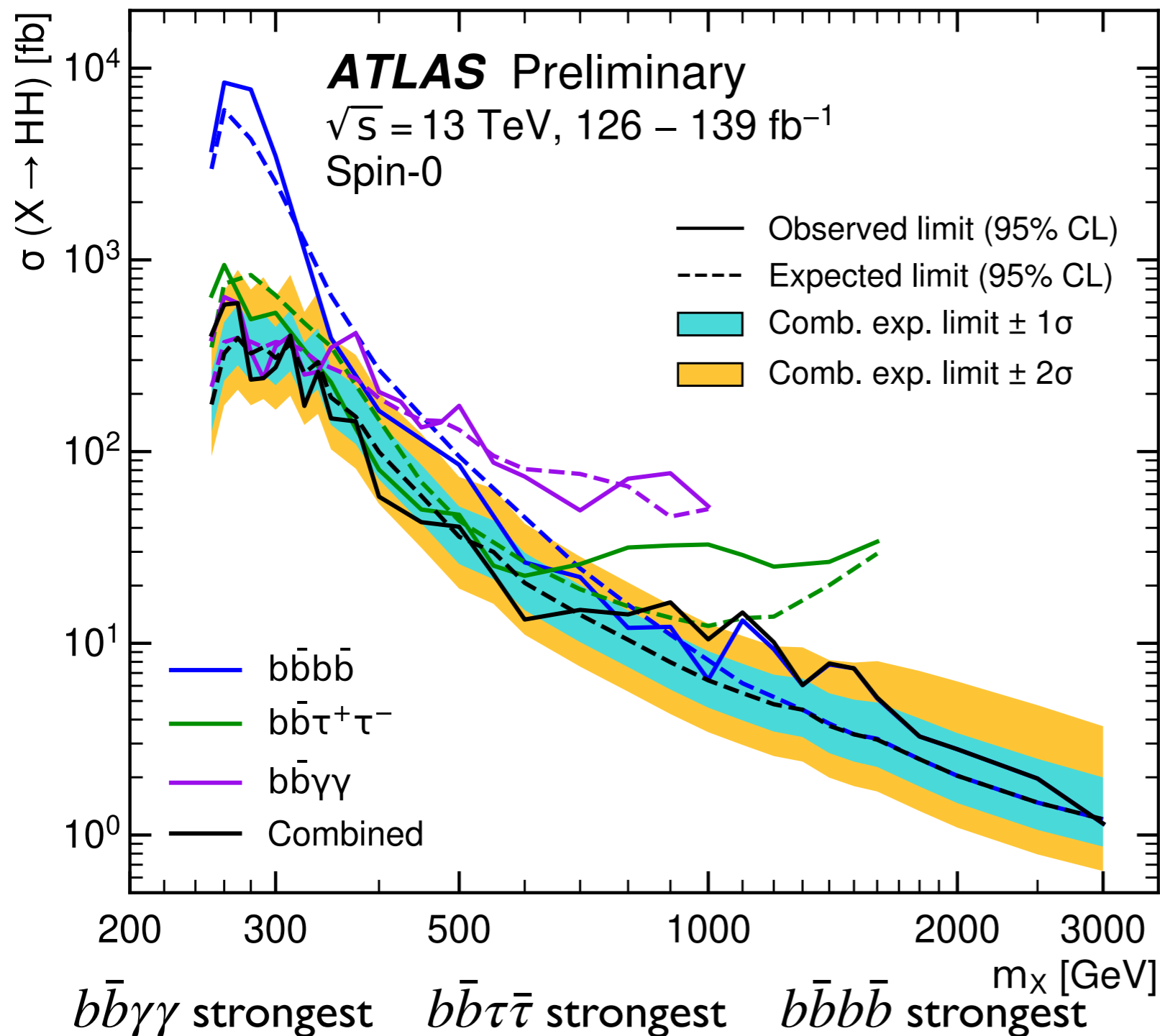
Resonant Combination



Resonant Combination



Resonant Combination



Here, show results from all three analyses

$b\bar{b}\gamma\gamma$ and $b\bar{b}\tau\tau$ have similar resonant-optimized searches

($b\bar{b}\tau\tau$ has parameterized NN for different signal mass points)

All three analyses complementary: set best limits at different ranges

Conclusions



Conclusions



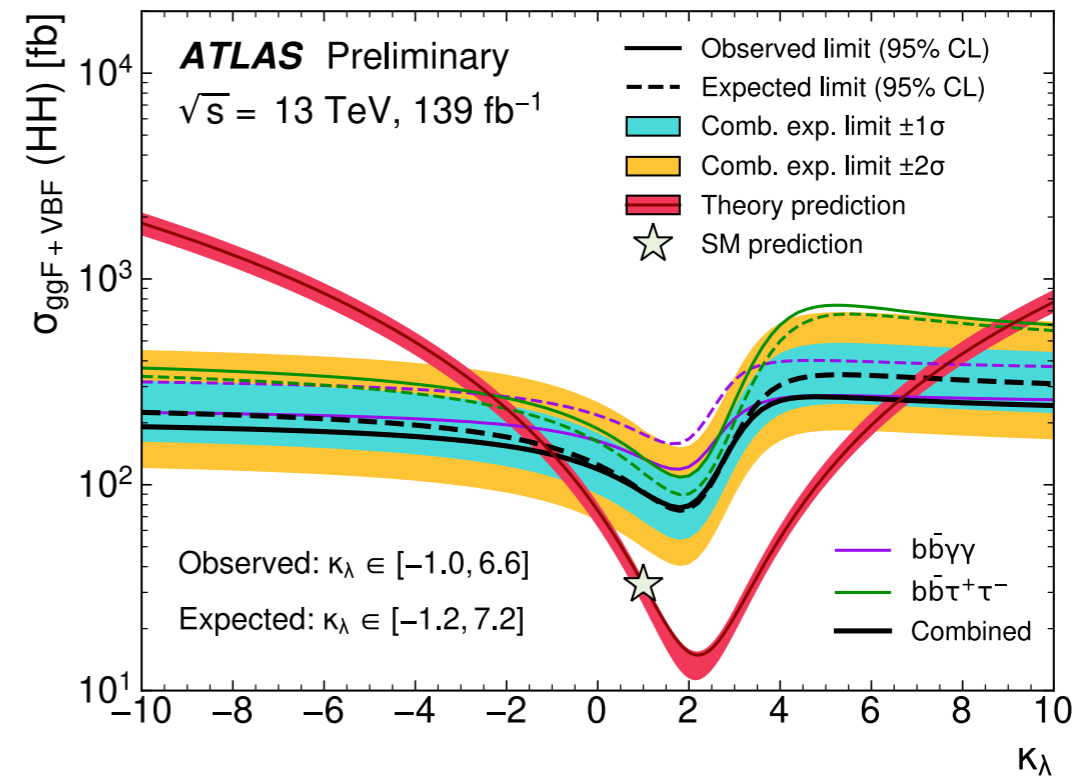
Higgs pair measurements let us directly probe the shape of the Higgs potential

Conclusions



Higgs pair measurements let us directly probe the shape of the Higgs potential

Rapidly approaching sensitivity to even the rare SM χ -sec!



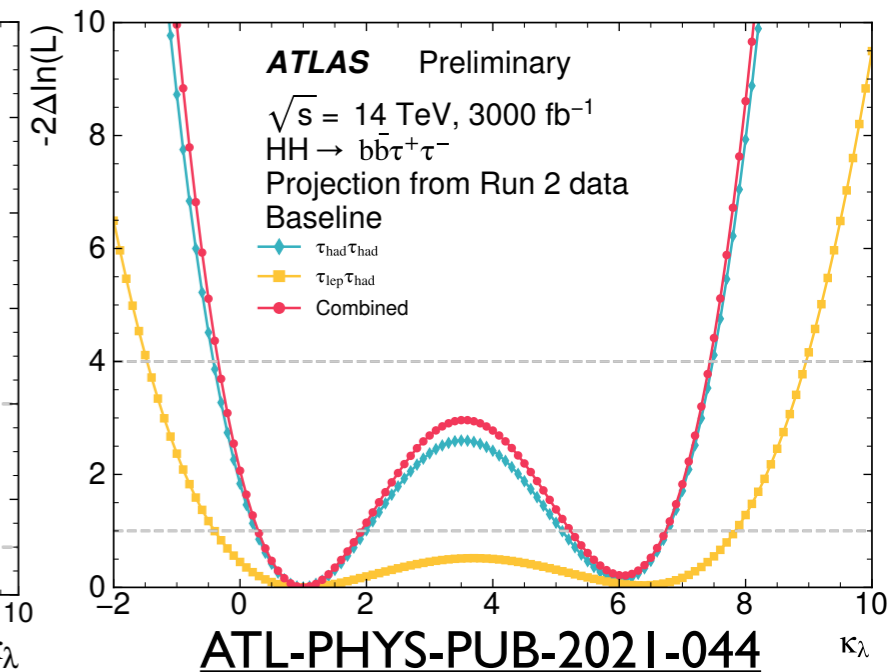
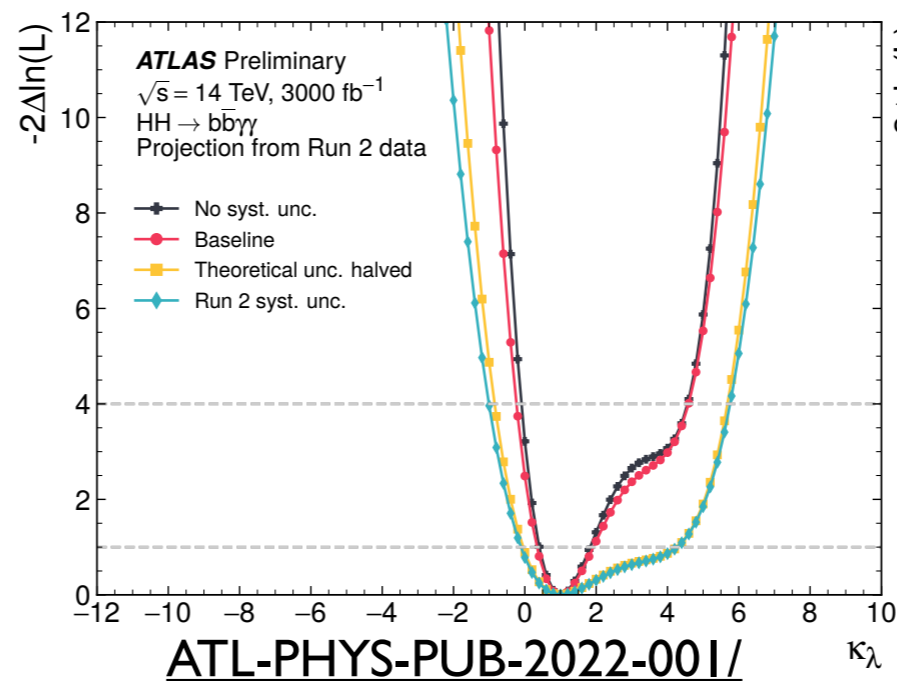
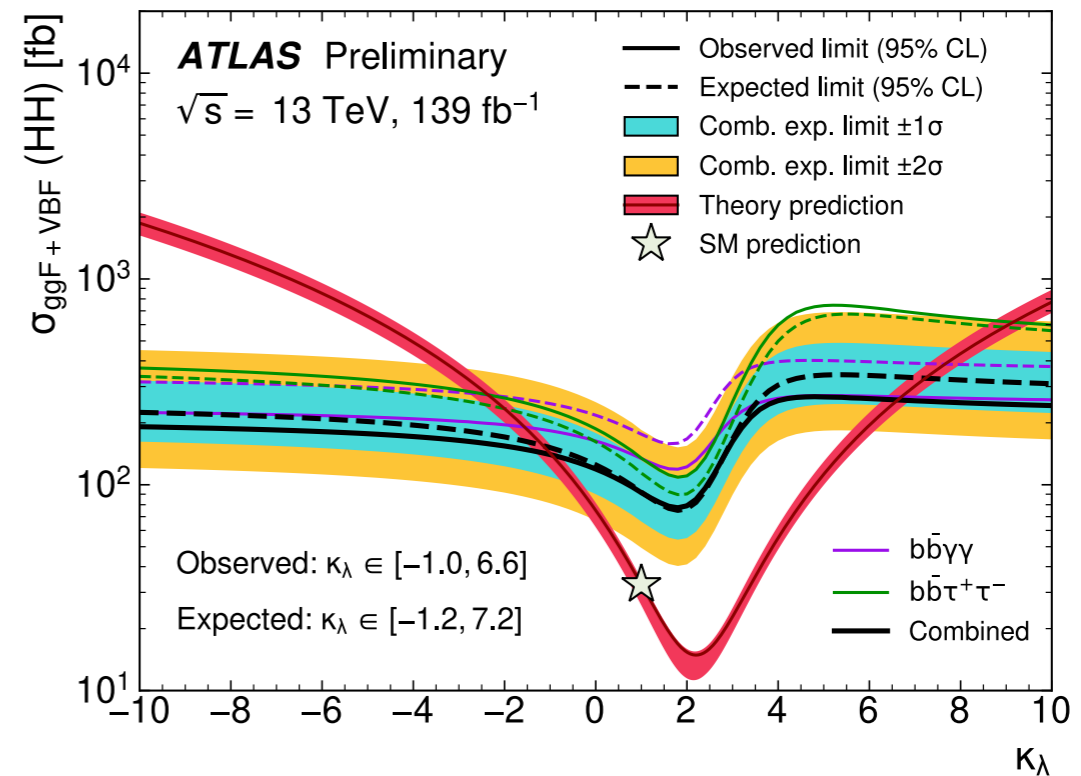
Conclusions



Higgs pair measurements let us directly probe the shape of the Higgs potential

Rapidly approaching sensitivity to even the rare SM x-sec!

Projections for HL-LHC rapidly improving as analyses are optimized: many exciting years of analysis remain!



Thank you!

More in:

[ATLAS-HDBS-2018-34](#)

[ATLAS-CONF-2021-030](#)

[ATLAS-HDBS-2018-41](#)

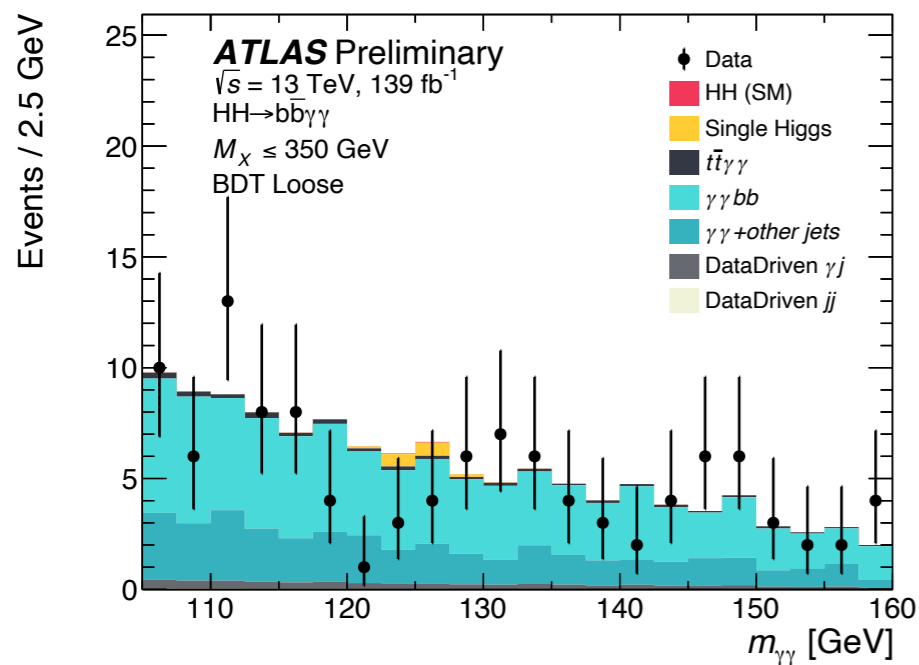
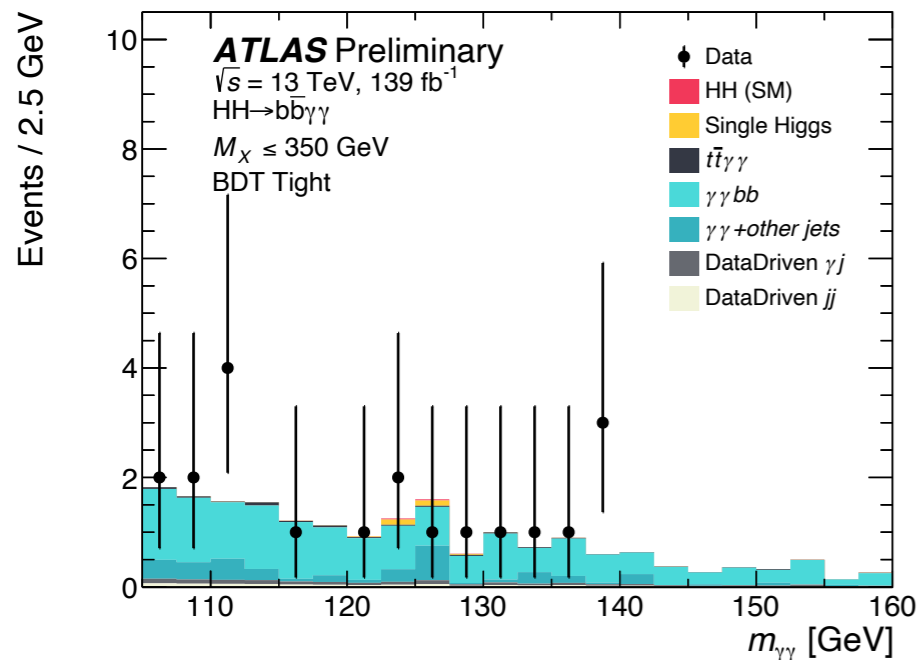
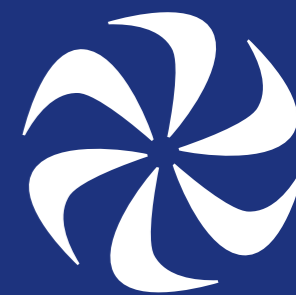
[ATLAS-CONF-2021-052](#)

Backup

$b\bar{b}\gamma\gamma$ Results

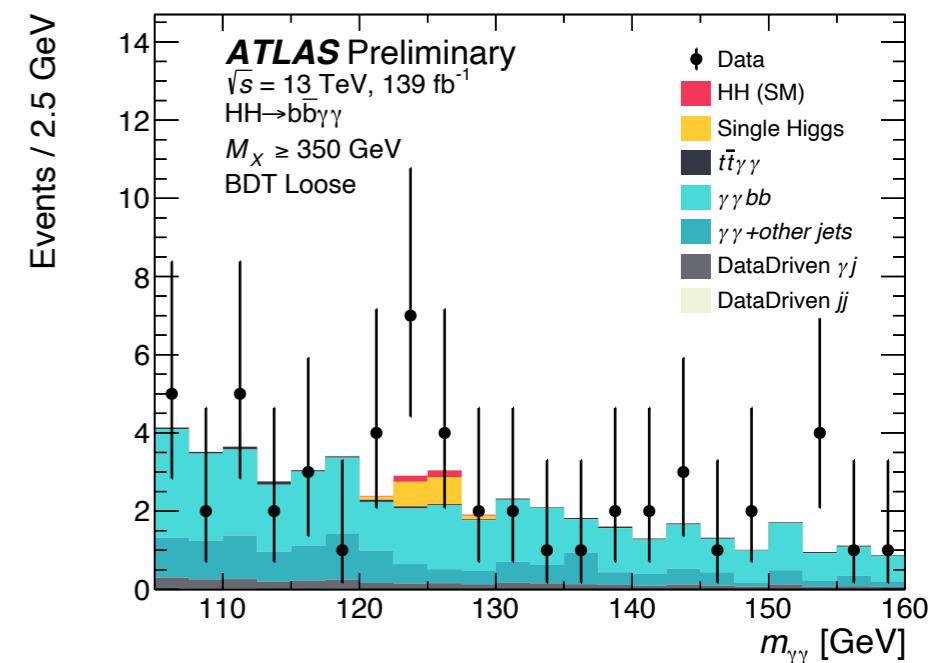
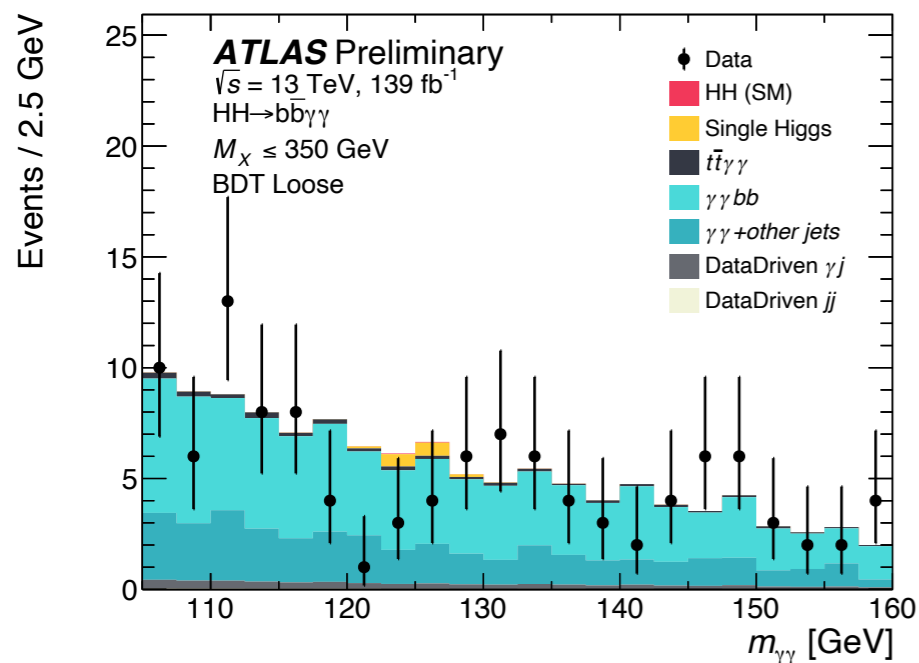
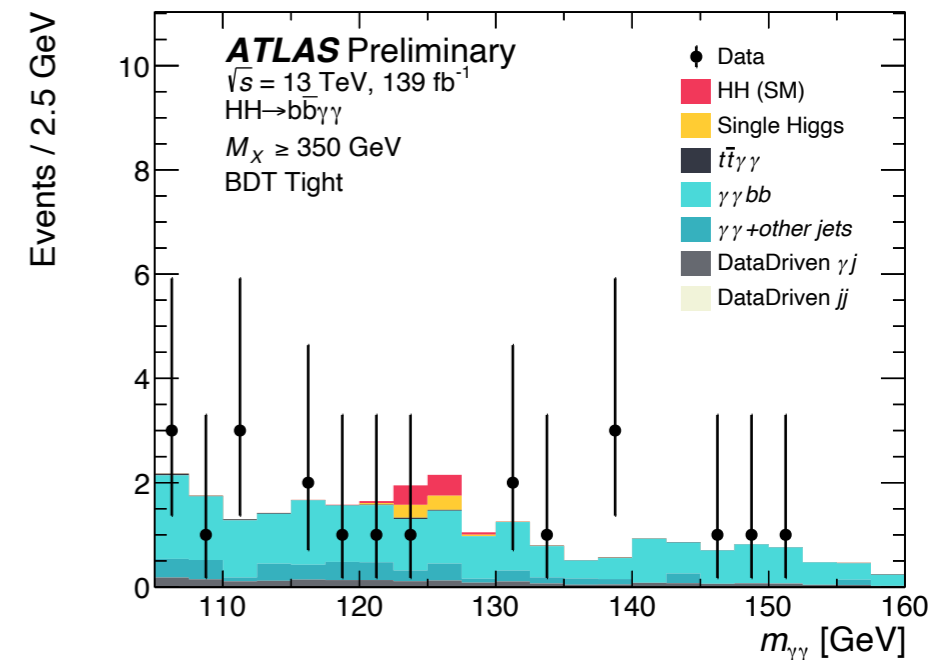
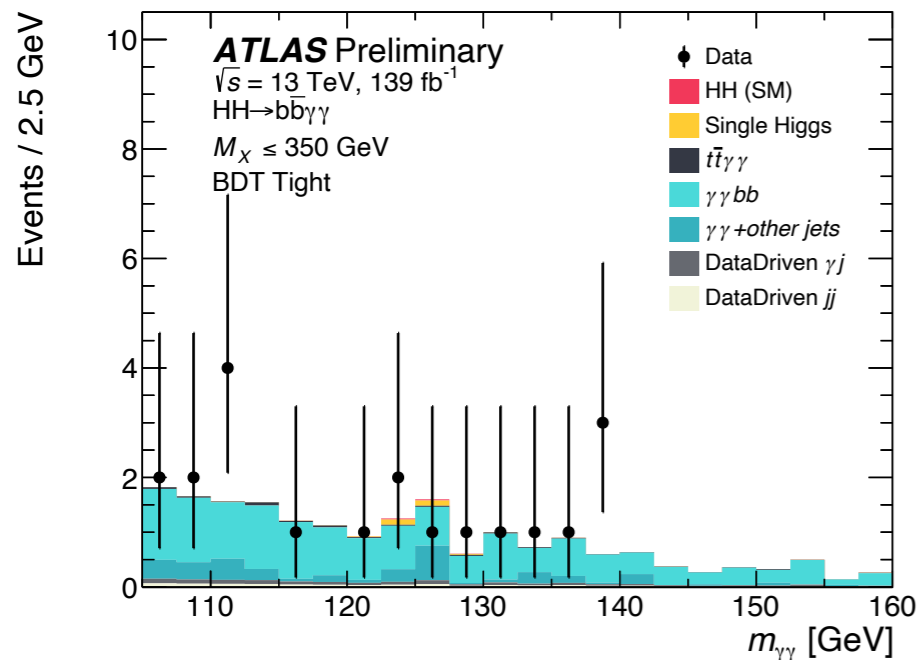
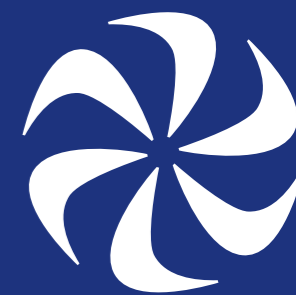


$b\bar{b}\gamma\gamma$ Results



Low mass: sensitive to K_λ

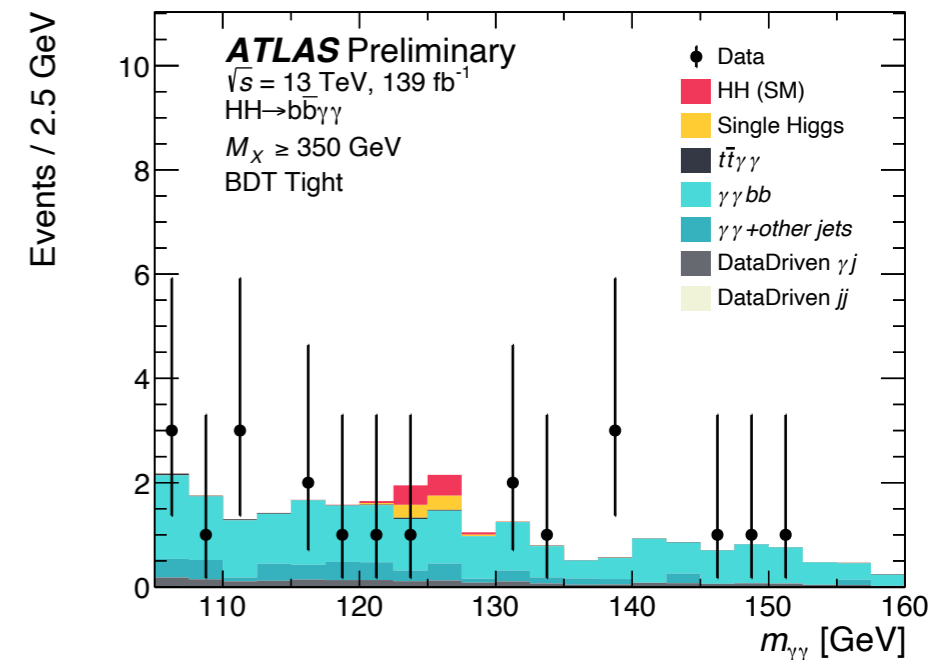
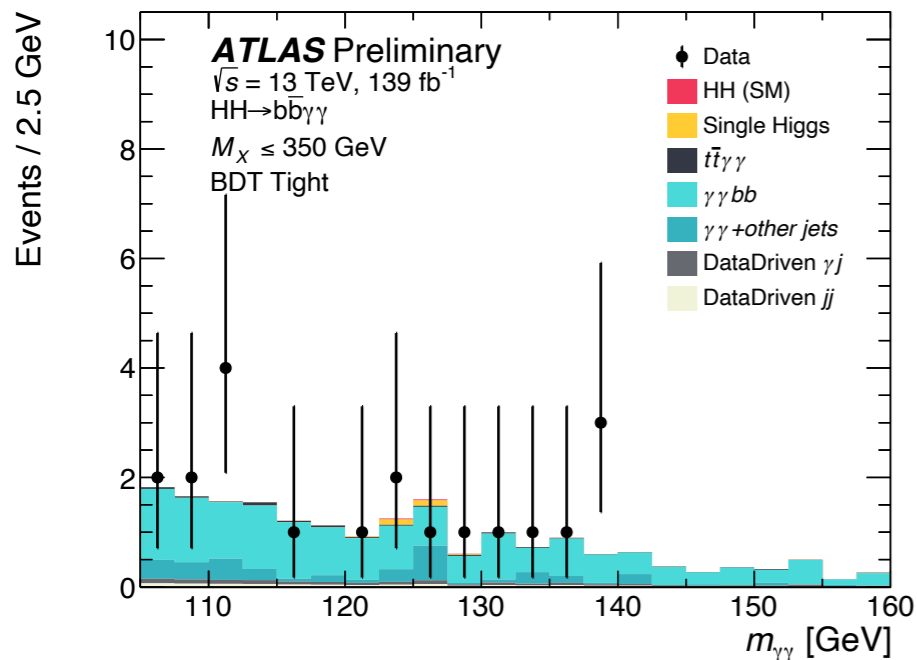
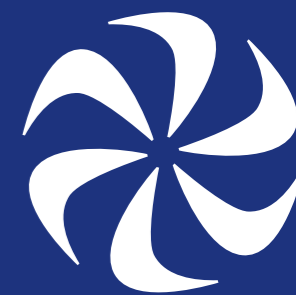
$b\bar{b}\gamma\gamma$ Results



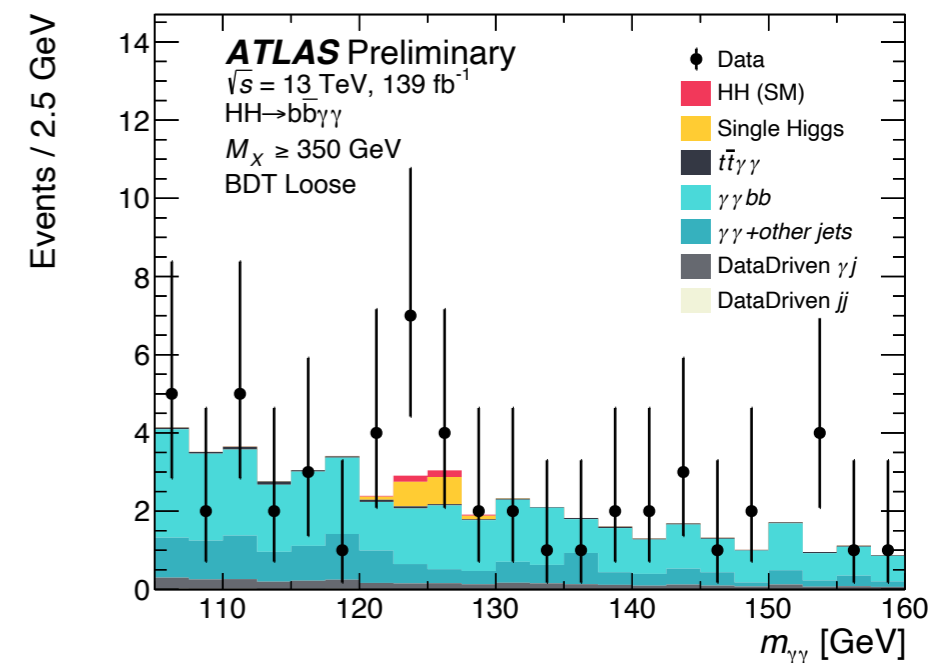
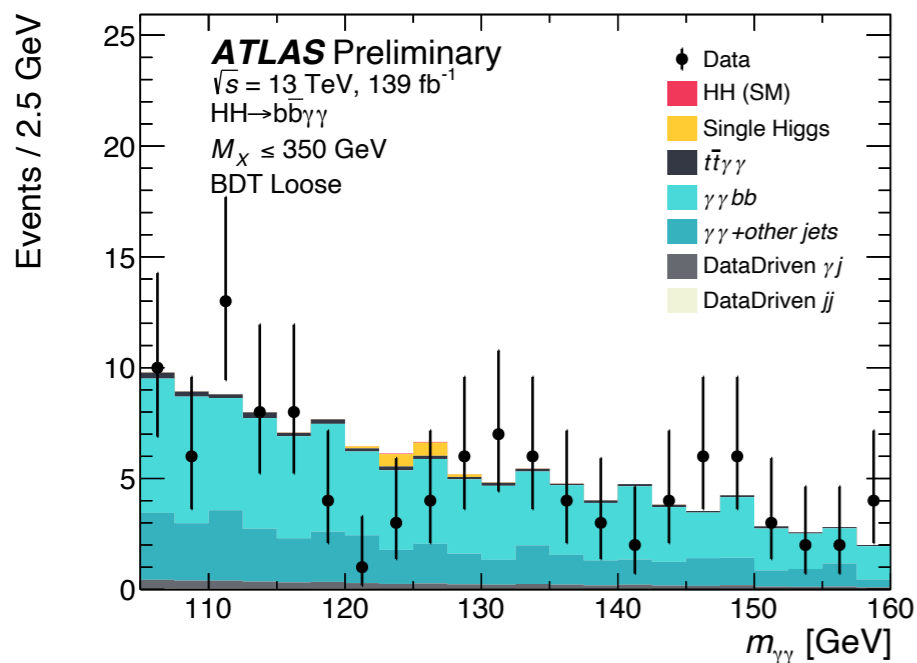
Low mass: sensitive to \mathcal{K}_λ

High mass: sensitive to SM

$b\bar{b}\gamma\gamma$ Results



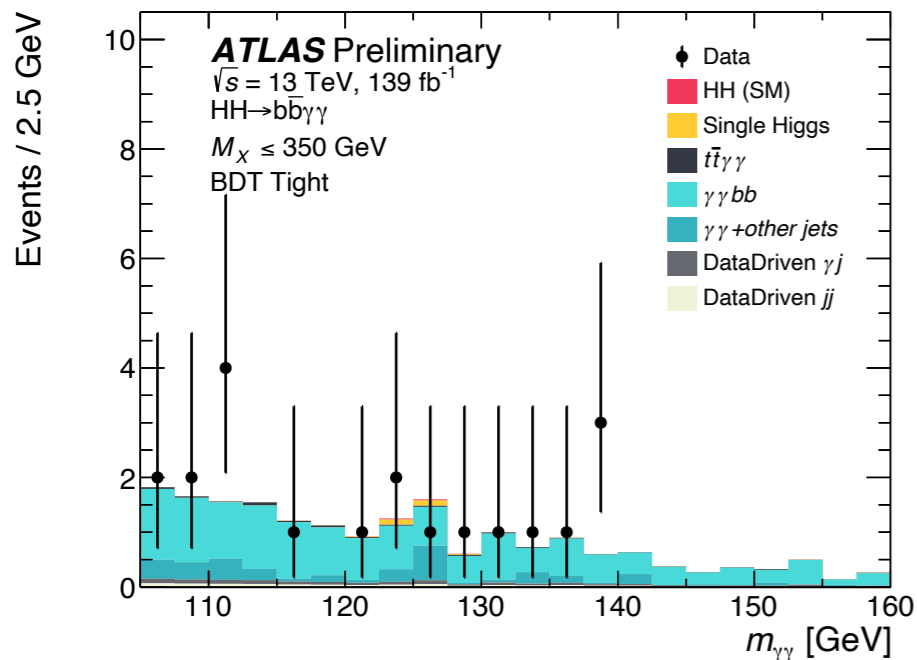
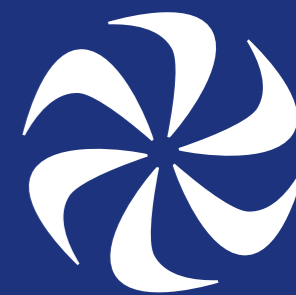
No obvious signs
of new physics!



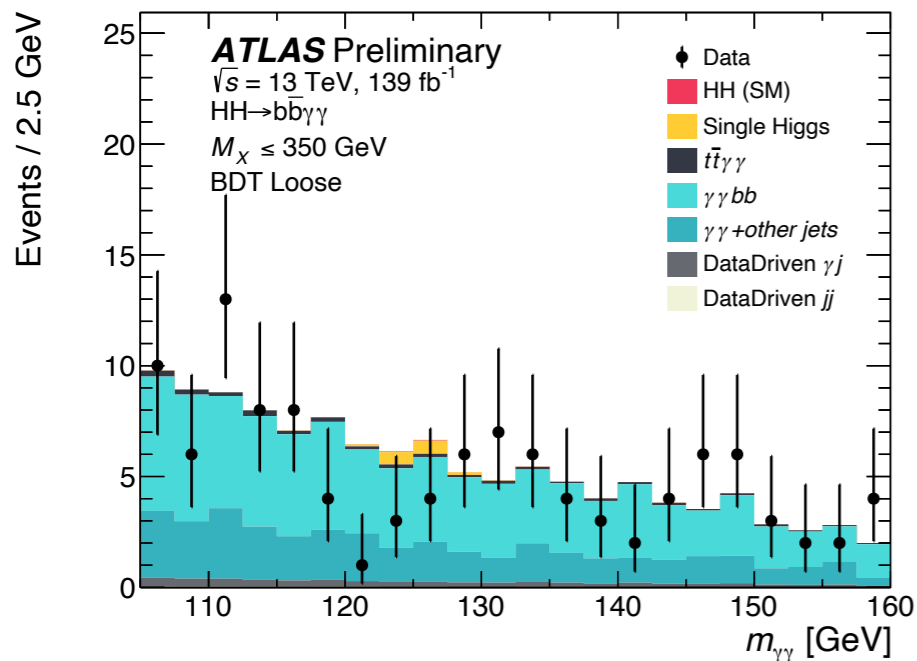
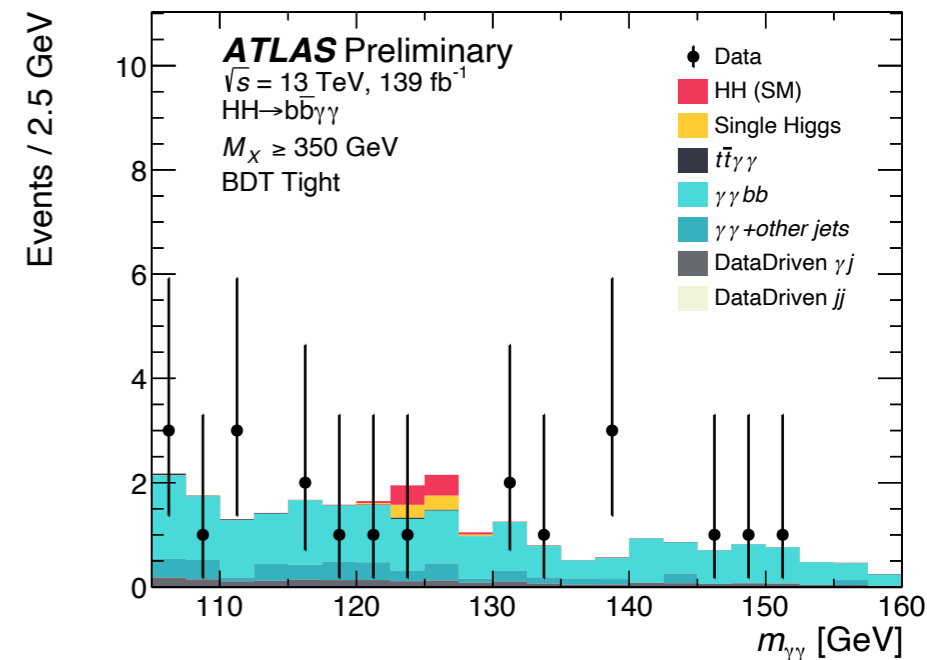
Low mass: sensitive to \mathcal{K}_λ

High mass: sensitive to SM

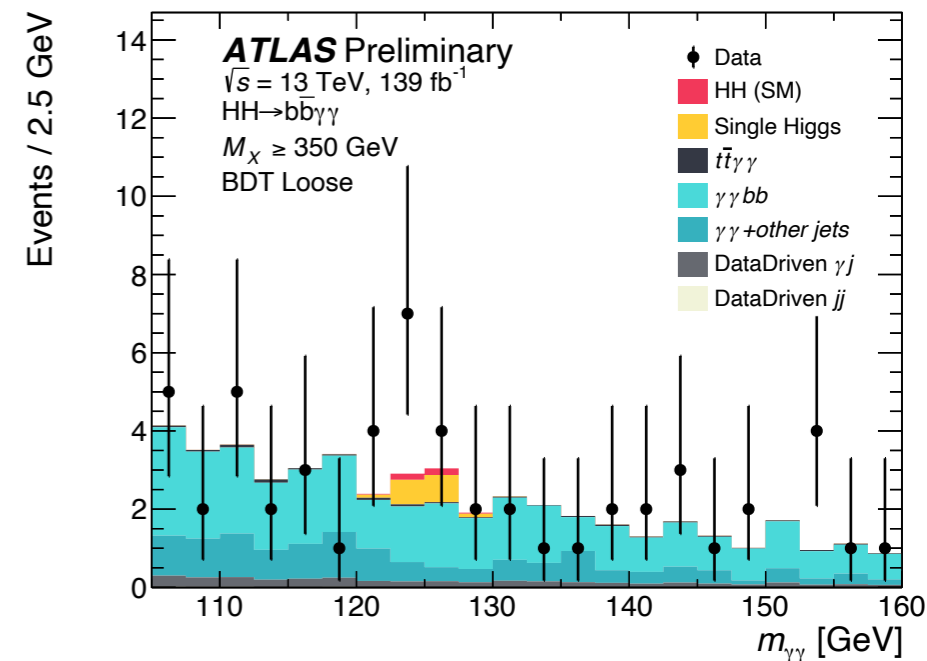
$b\bar{b}\gamma\gamma$ Results



No obvious signs
of new physics!



But some of the
best sensitivity
to HH ever...



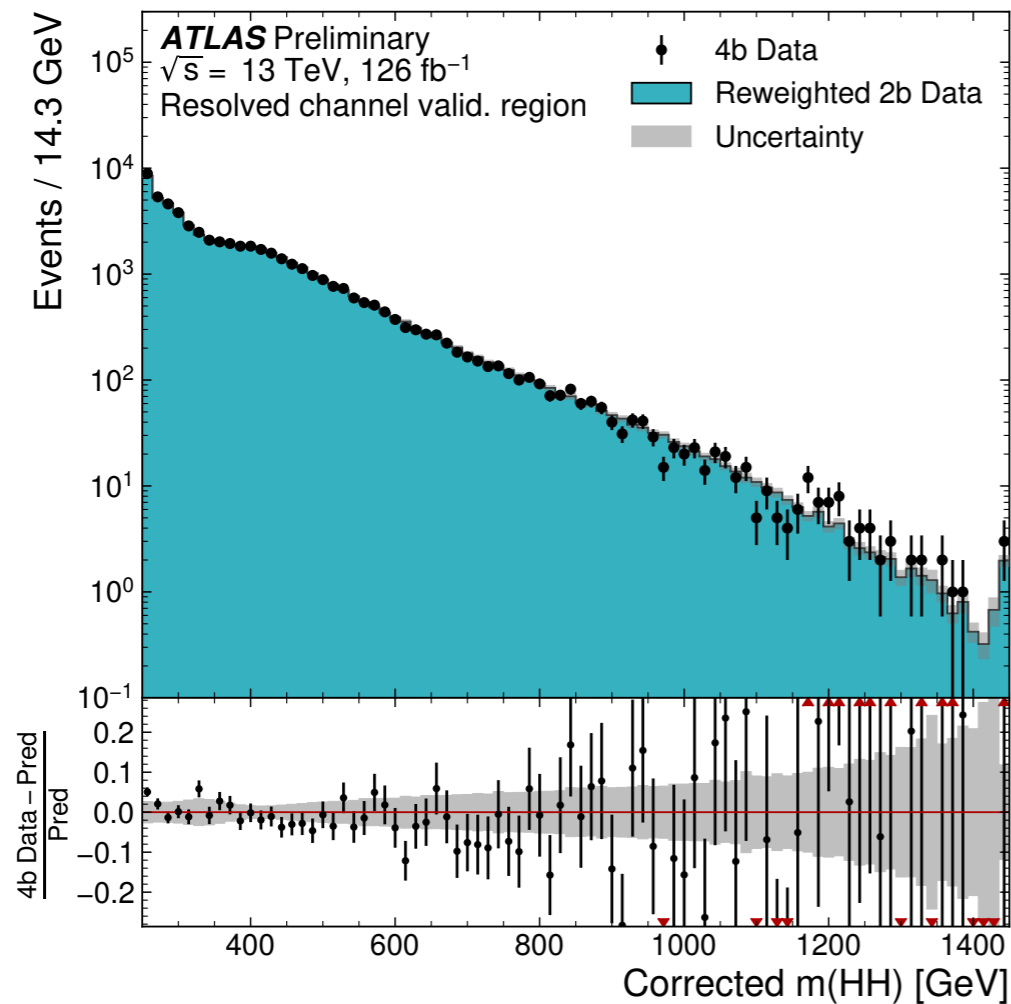
Low mass: sensitive to κ_λ

High mass: sensitive to SM

Why Neural Networks?

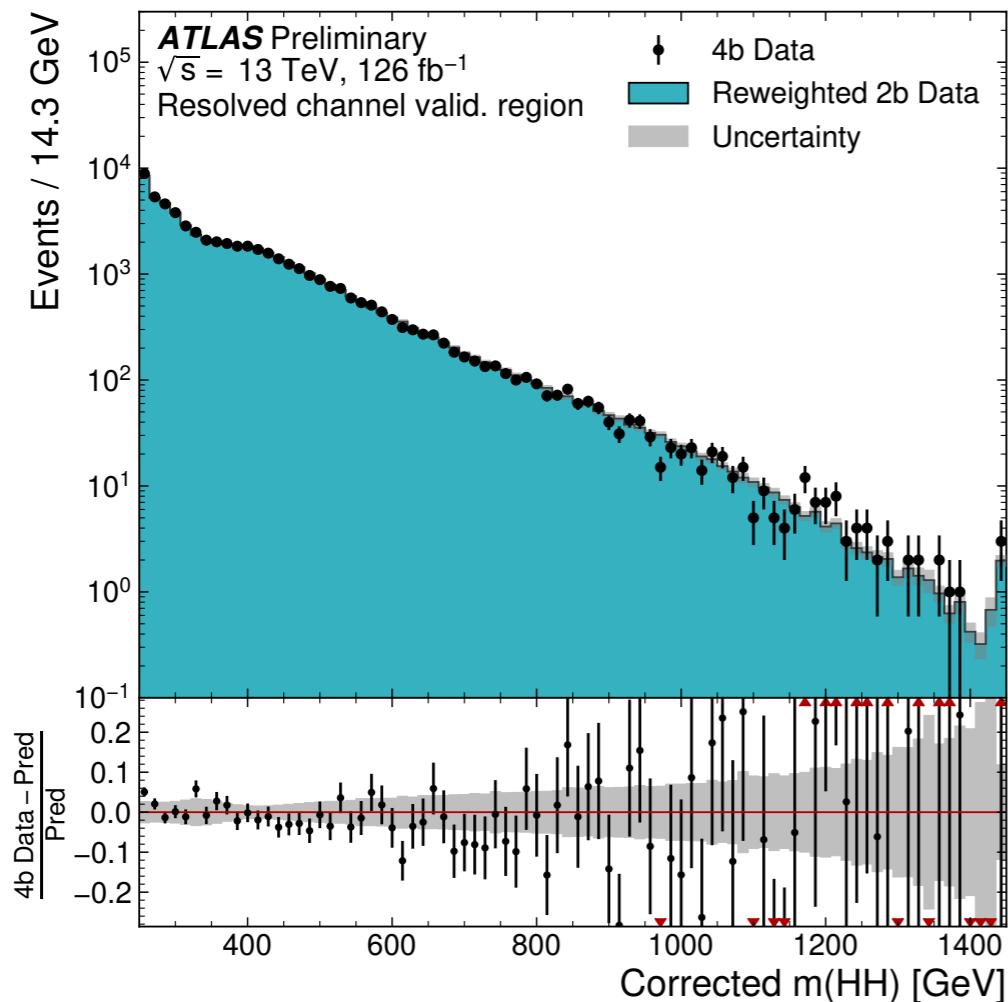


Why Neural Networks?



Here, apply NN to 2b data in VR

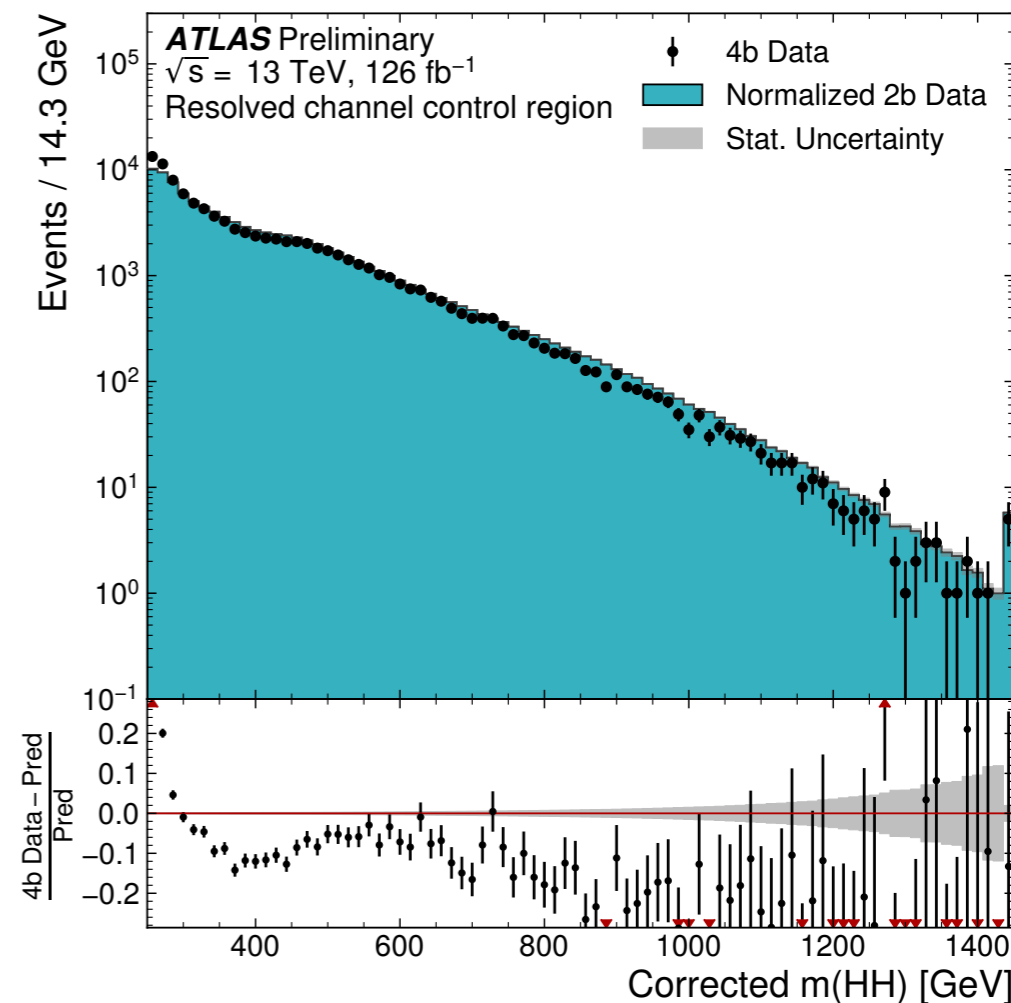
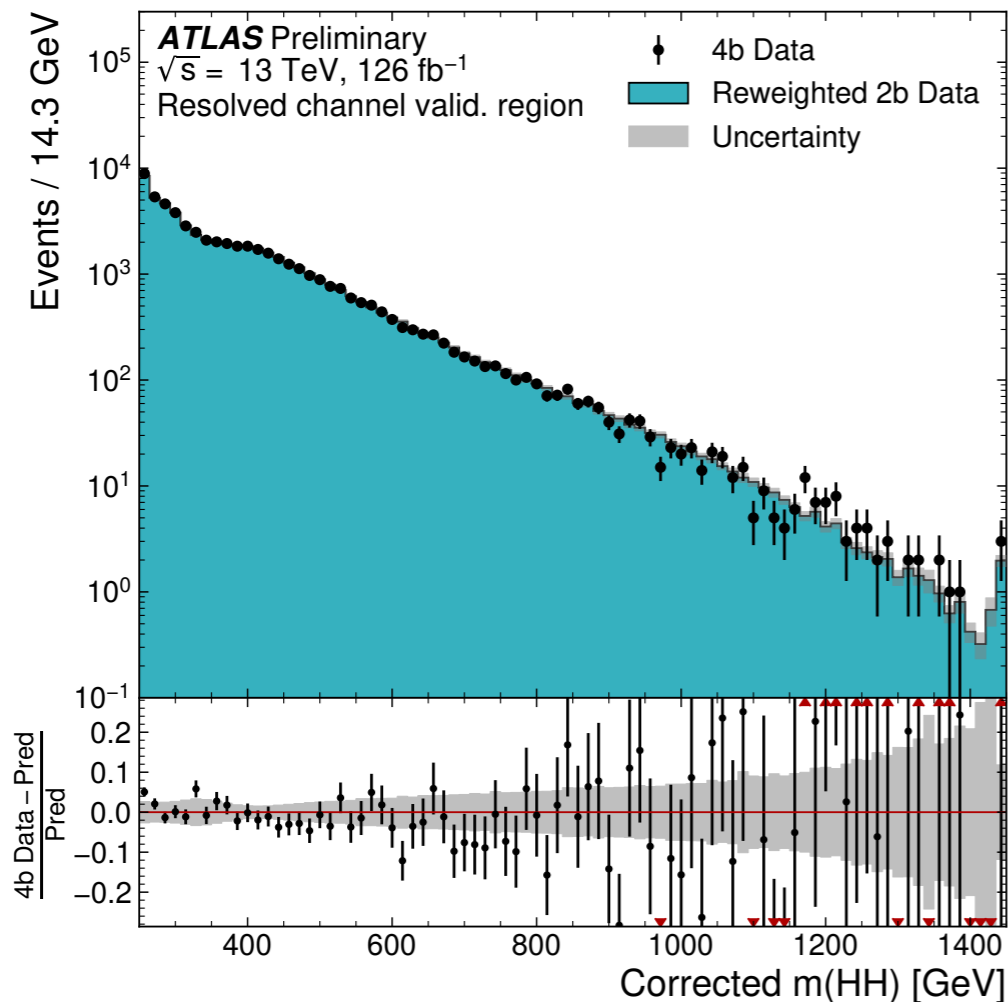
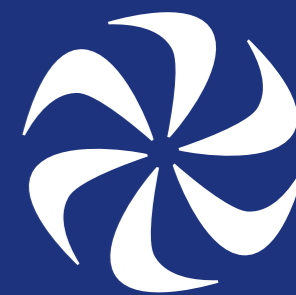
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Works well, even on data
that wasn't used in training!

Why Neural Networks?

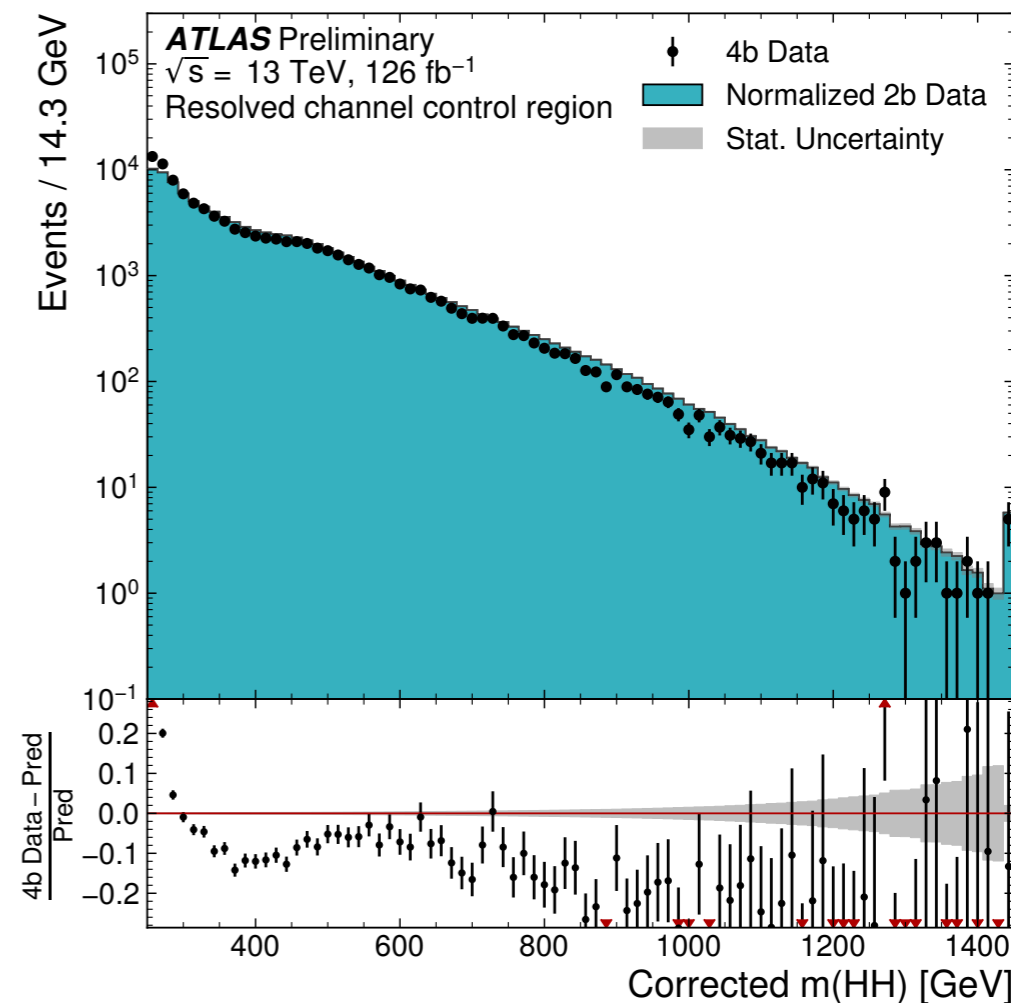
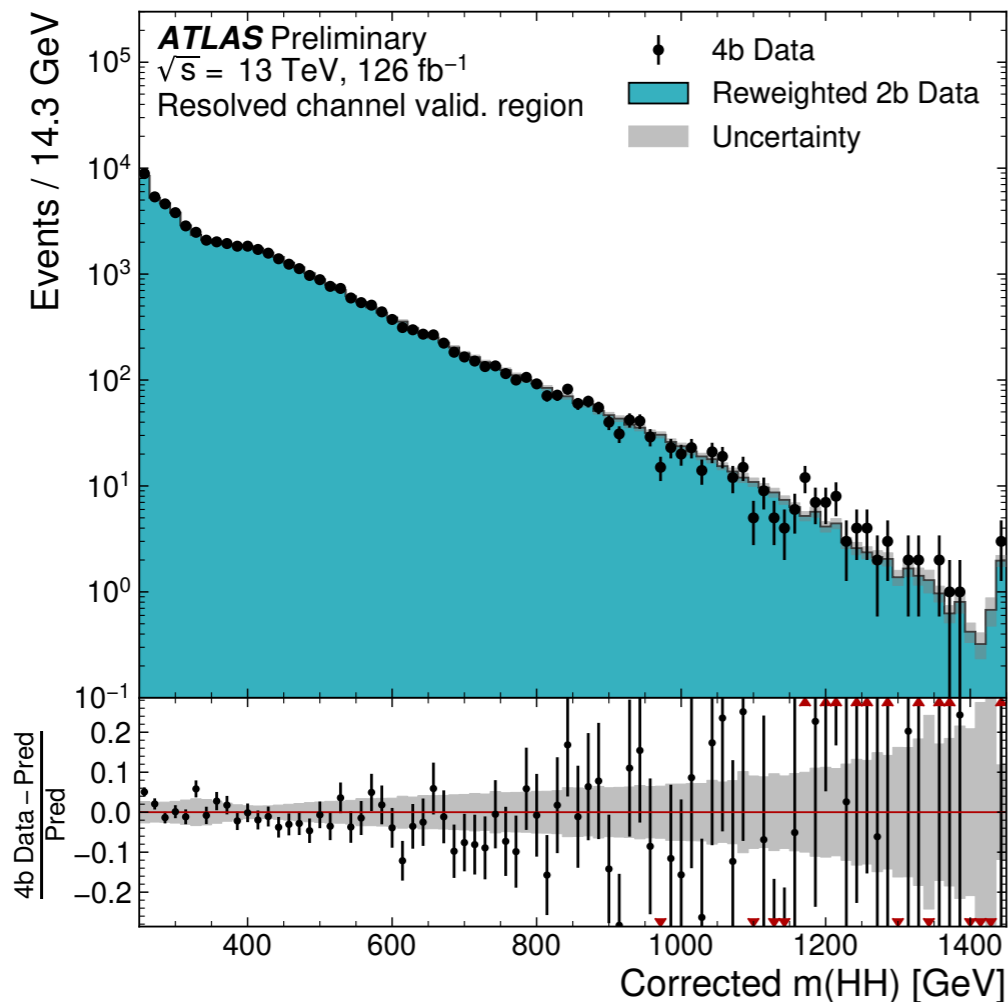
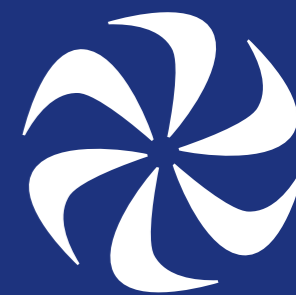


Here, apply NN to 2b data in VR

Works well, even on data that wasn't used in training!

Why does this work?

Why Neural Networks?



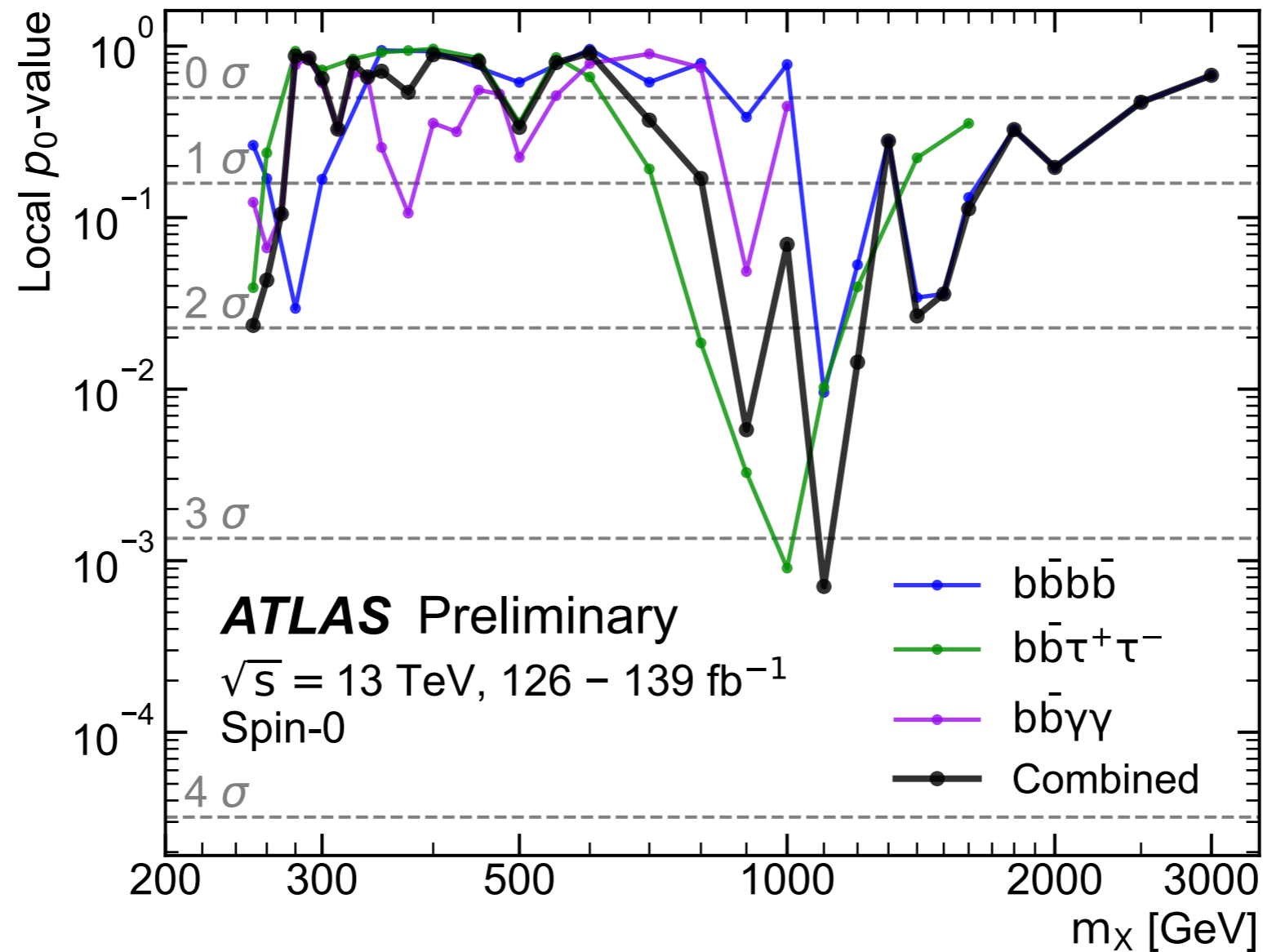
Here, apply NN to 2b data in VR

Works well, even on data that wasn't used in training!

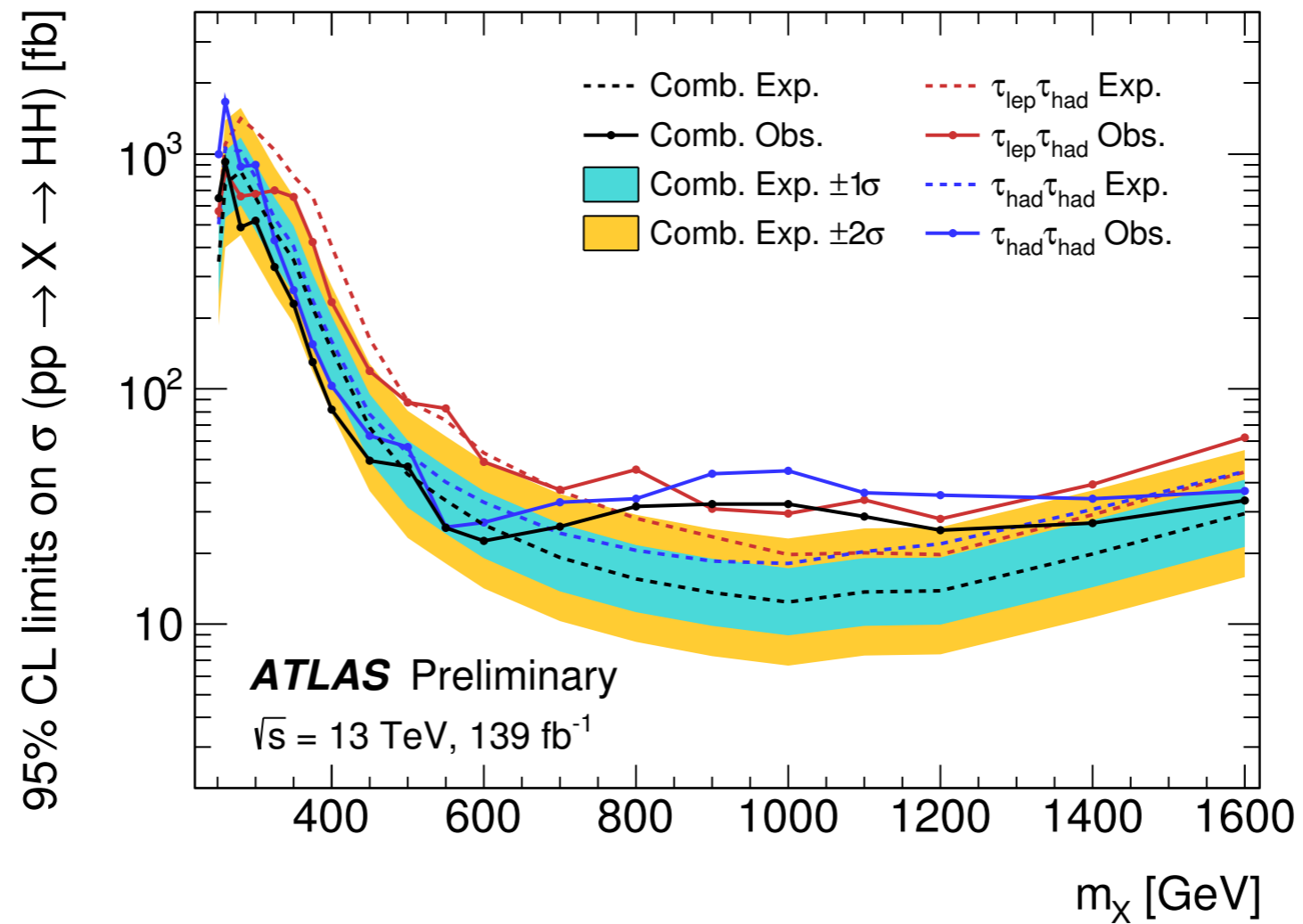
Why does this work?

NN's learn a density ratio of two classes: normally this ratio is used to isolate a single class, but can be used to reweight classes

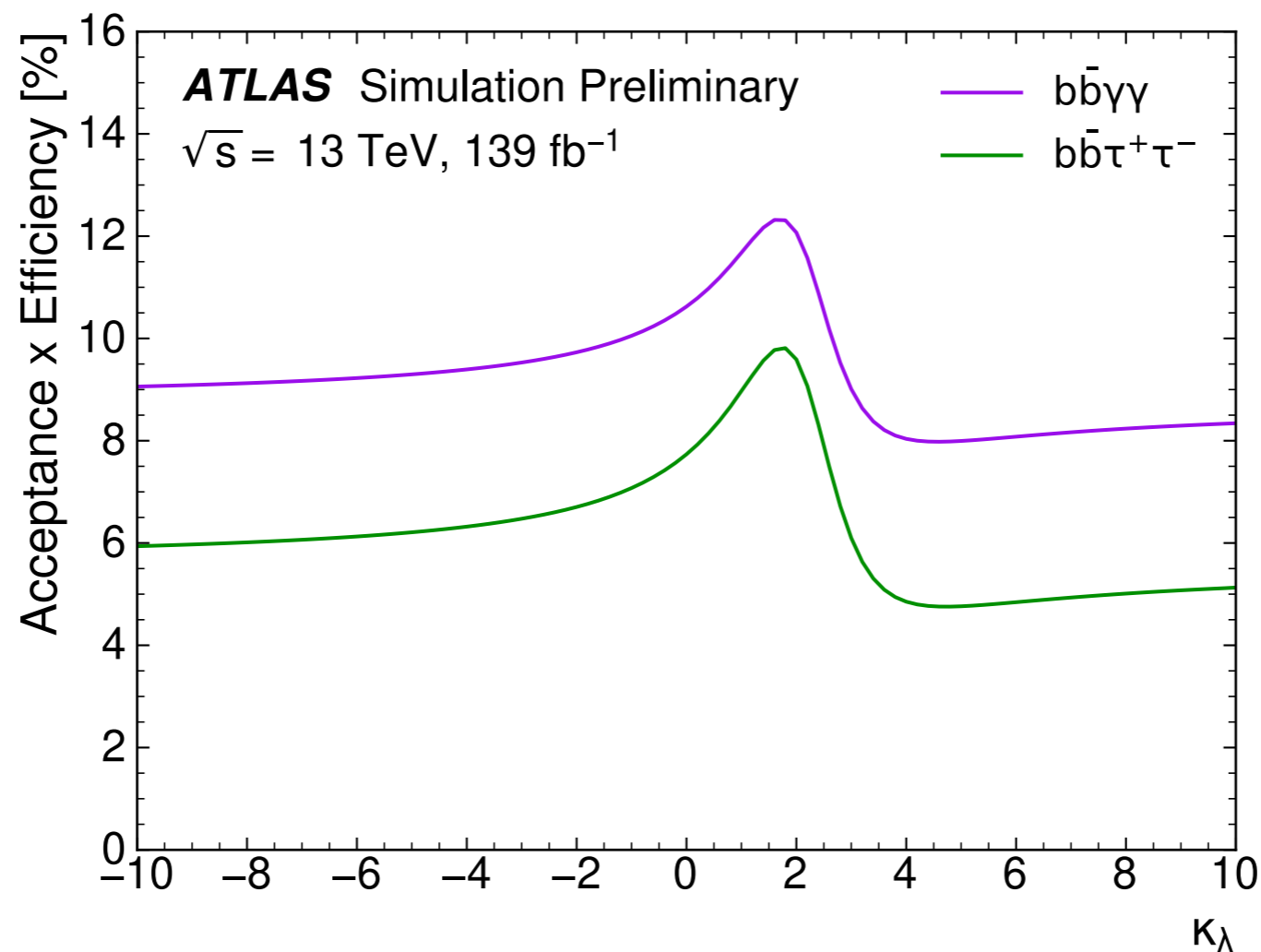
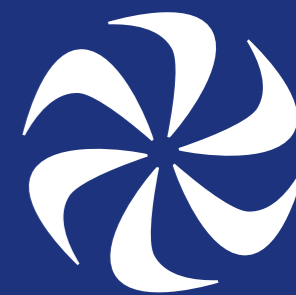
Resonant p-value



$b\bar{b}\tau\bar{\tau}$ Resonant Limits



Non-resonant Acc x Eff

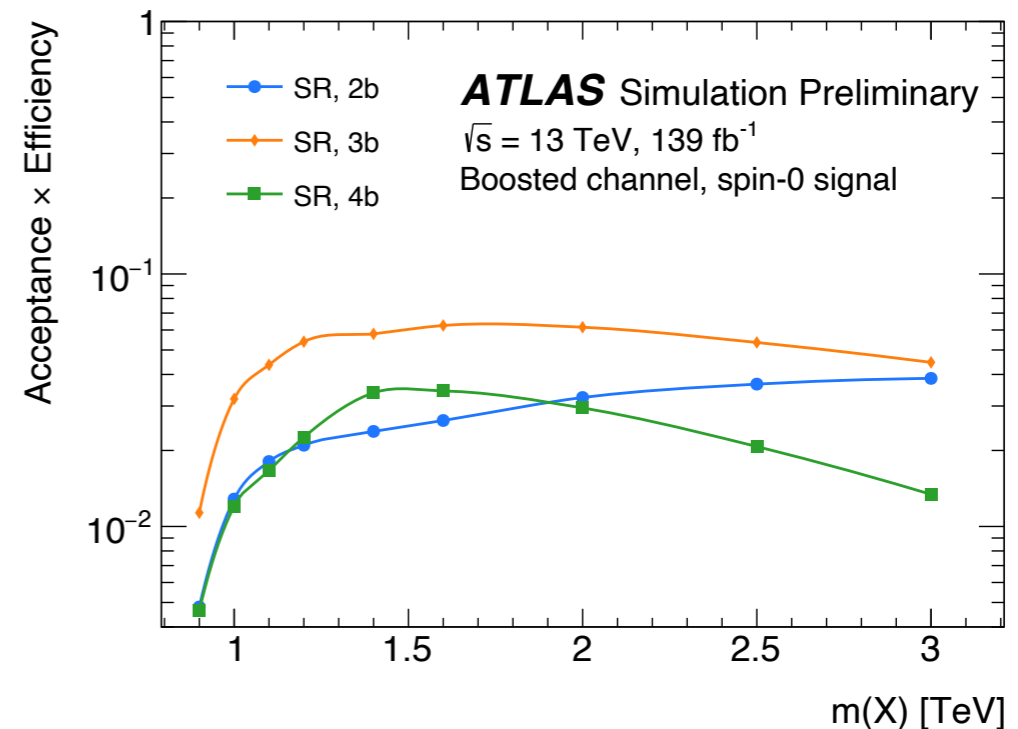
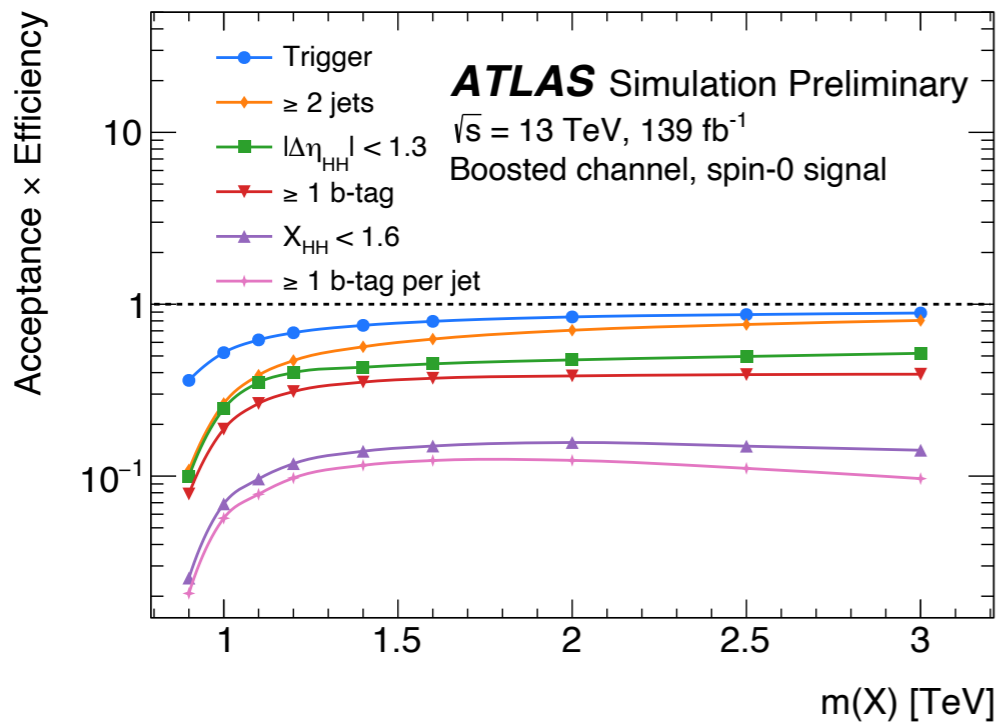
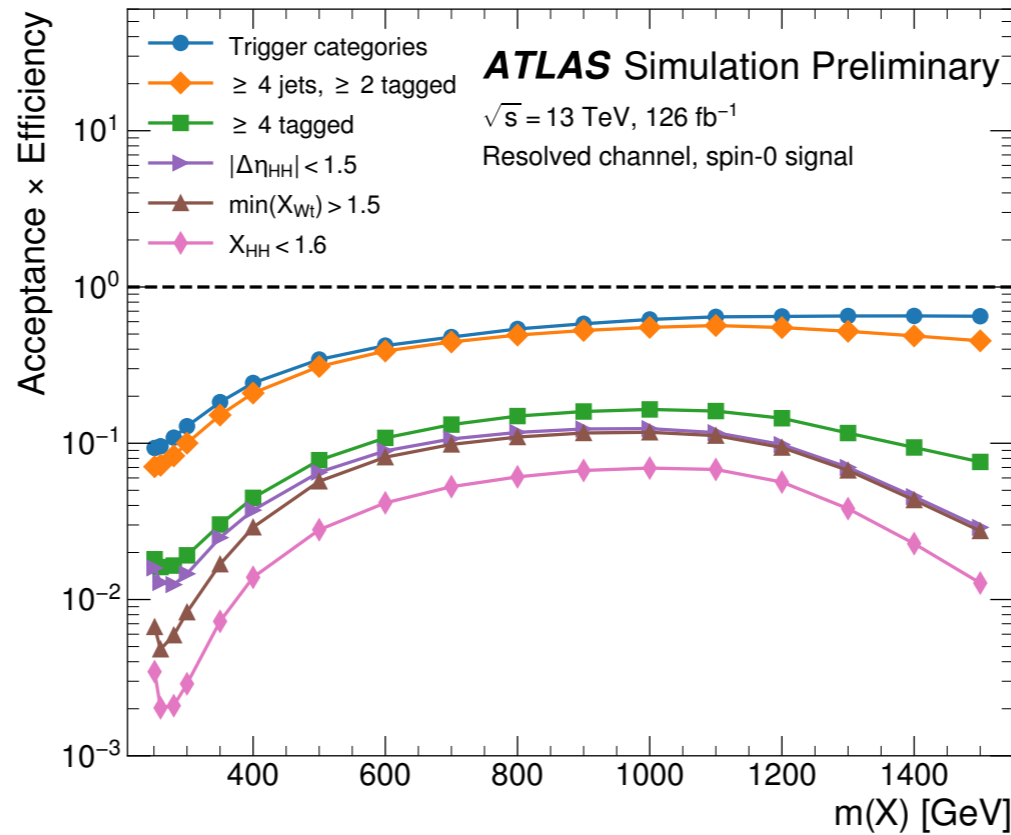


Variables for MVAs

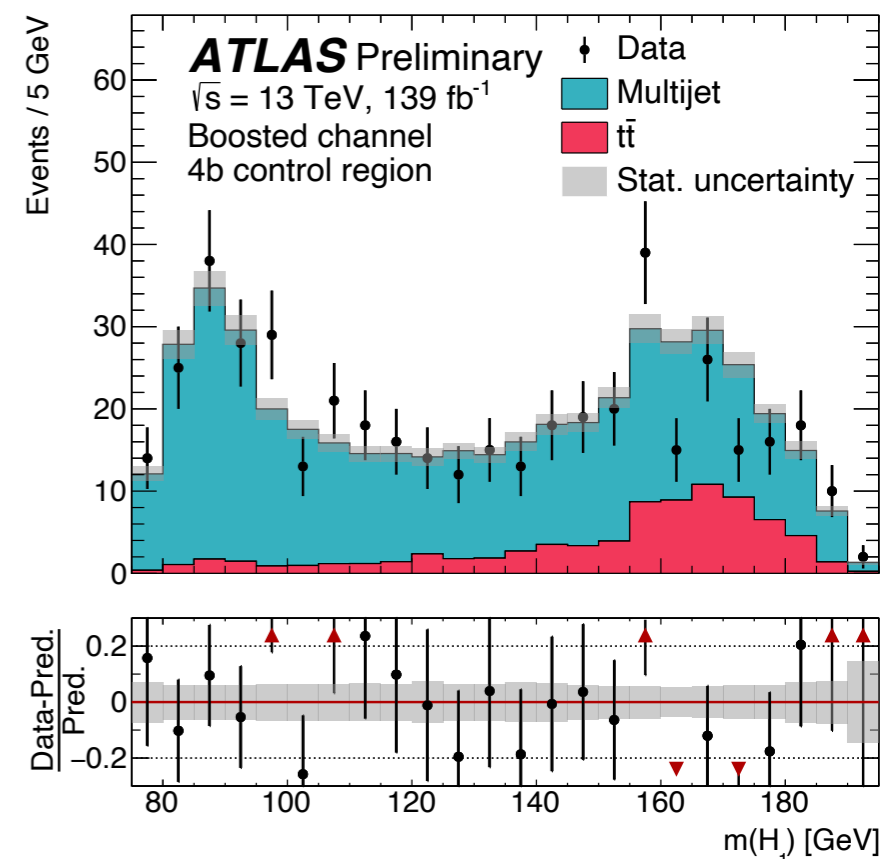
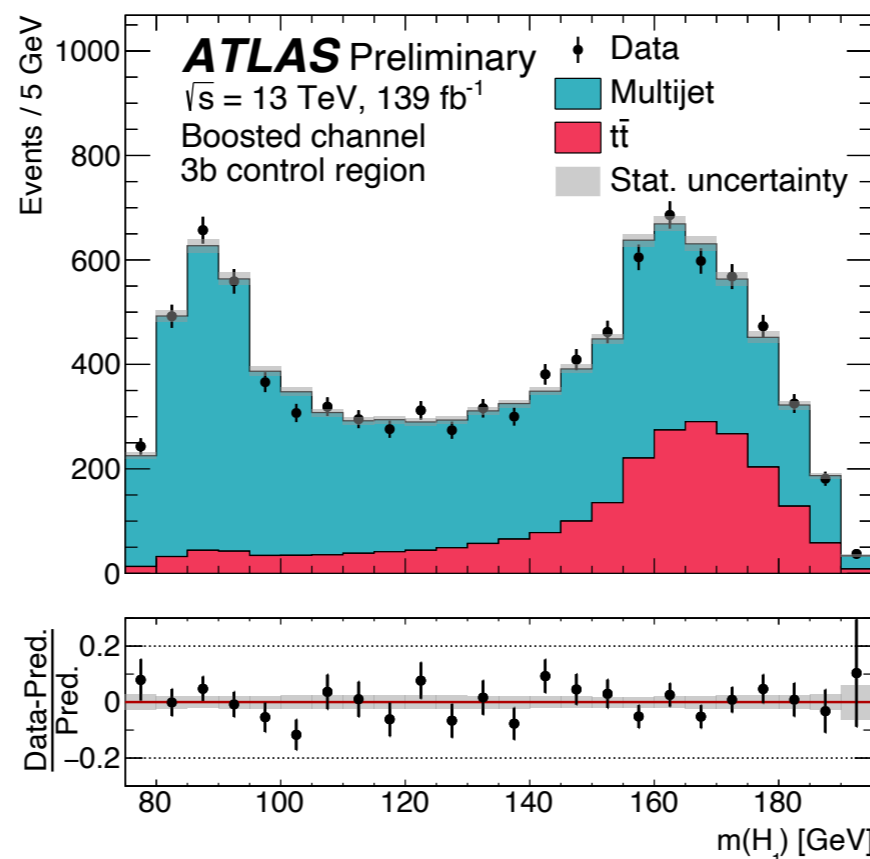
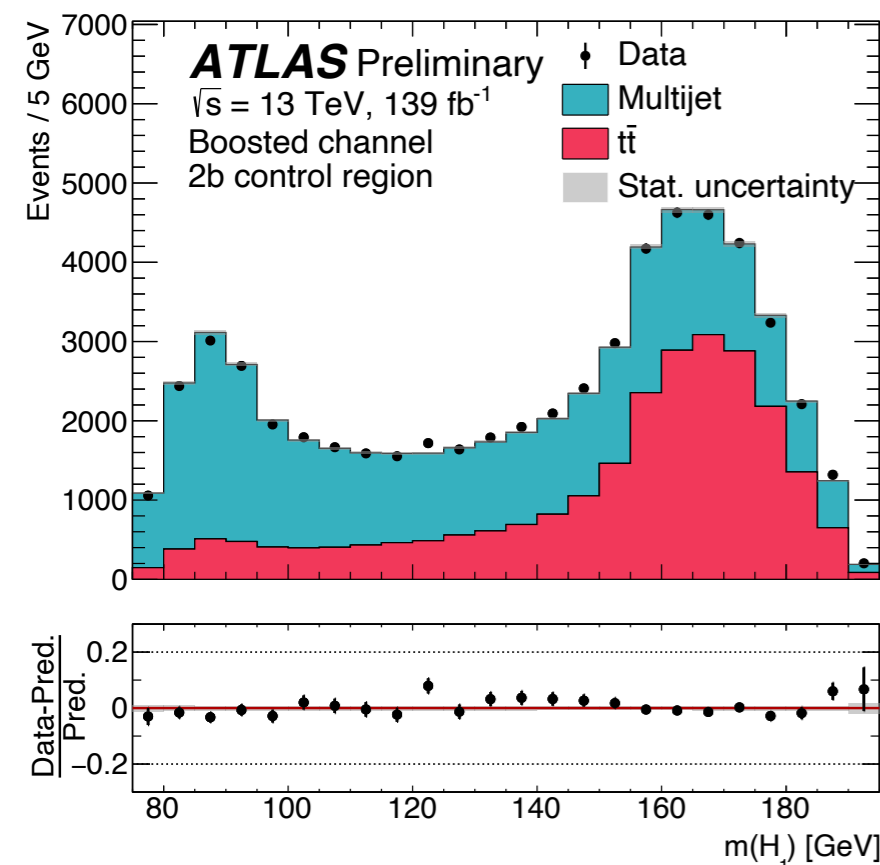
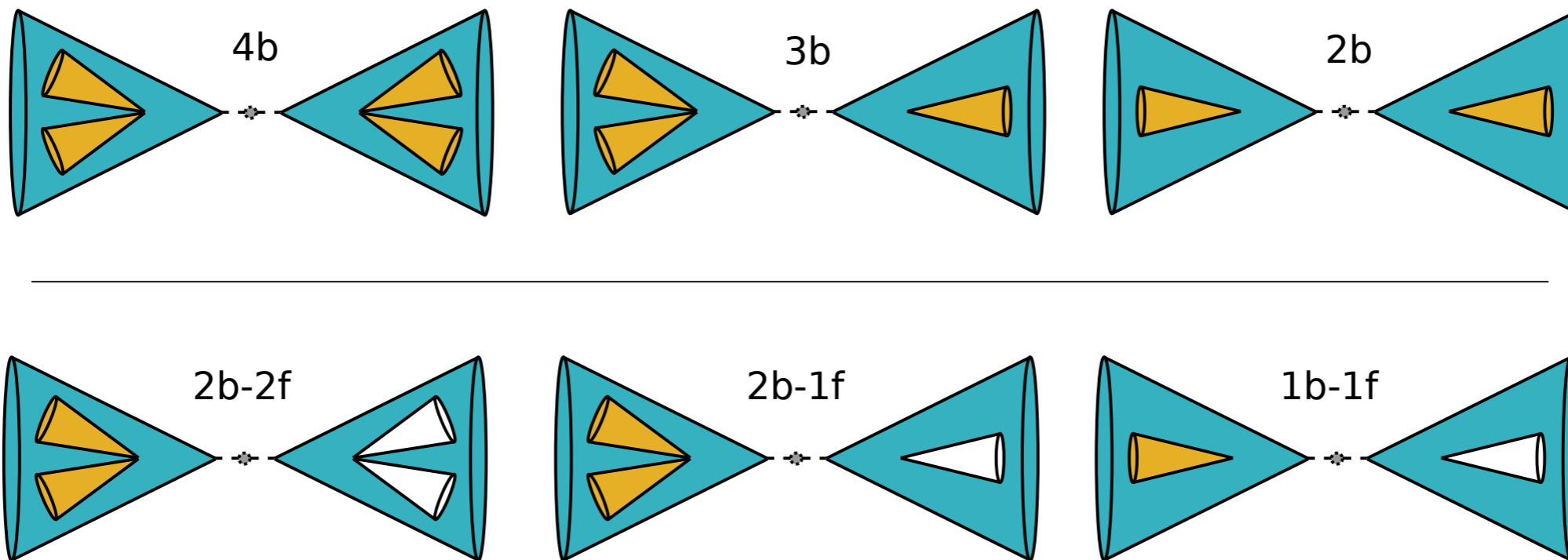
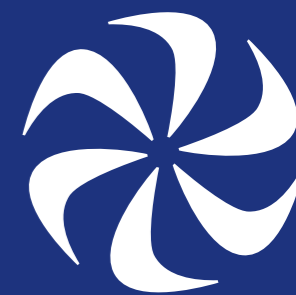


- For $b\bar{b}\gamma\gamma$: photon kinematics, b-jet kinematics, bb-system kinematics, missing energy, total energy, “top-ness”
- For $b\bar{b}\tau\bar{\tau}$: m_{HH} , m_{bb} , $m_{\tau\tau}$, $DR(b,b)$, $DR(\tau,\tau)$, $DPt(l\text{ep},\tau)$, MET, $DPhi(l\text{ep}\tau, bb)$...
- For $b\bar{b}b\bar{b}$:
 1. $\log(p_T)$ of the selected jet with the 2nd-highest p_T ,
 2. $\log(p_T)$ of the selected jet with the 4th-highest p_T ,
 3. $\log(\Delta R)$ between the two selected jets with the smallest ΔR ,
 4. $\log(\Delta R)$ between the other two selected jets,
 5. the average $|\eta|$ of selected jets,
 6. $\log(p_T)$ of the HH system,
 7. ΔR between the two H candidates,
 8. $\Delta\phi$ between the jets making up H_1 ,
 9. $\Delta\phi$ between the jets making up H_2 ,
 10. $\log(\min(X_{Wt}))$, and
 11. the number of jets in the event with $p_T > 40$ GeV and $|\eta| < 2.5$, including jets that are not selected.

Acceptance x Eff $b\bar{b}b\bar{b}$



Boosted Backgrounds



From Limits, to Discovery



From Limits, to Discovery



- **More data**

From Limits, to Discovery



- **More data**
- **Background estimation**

From Limits, to Discovery



- **More data**
- **Background estimation**
- **Jet reconstruction**

From Limits, to Discovery



- **More data**
- **Background estimation**
- **Jet reconstruction**
- **Jet triggering**

From Limits, to Discovery



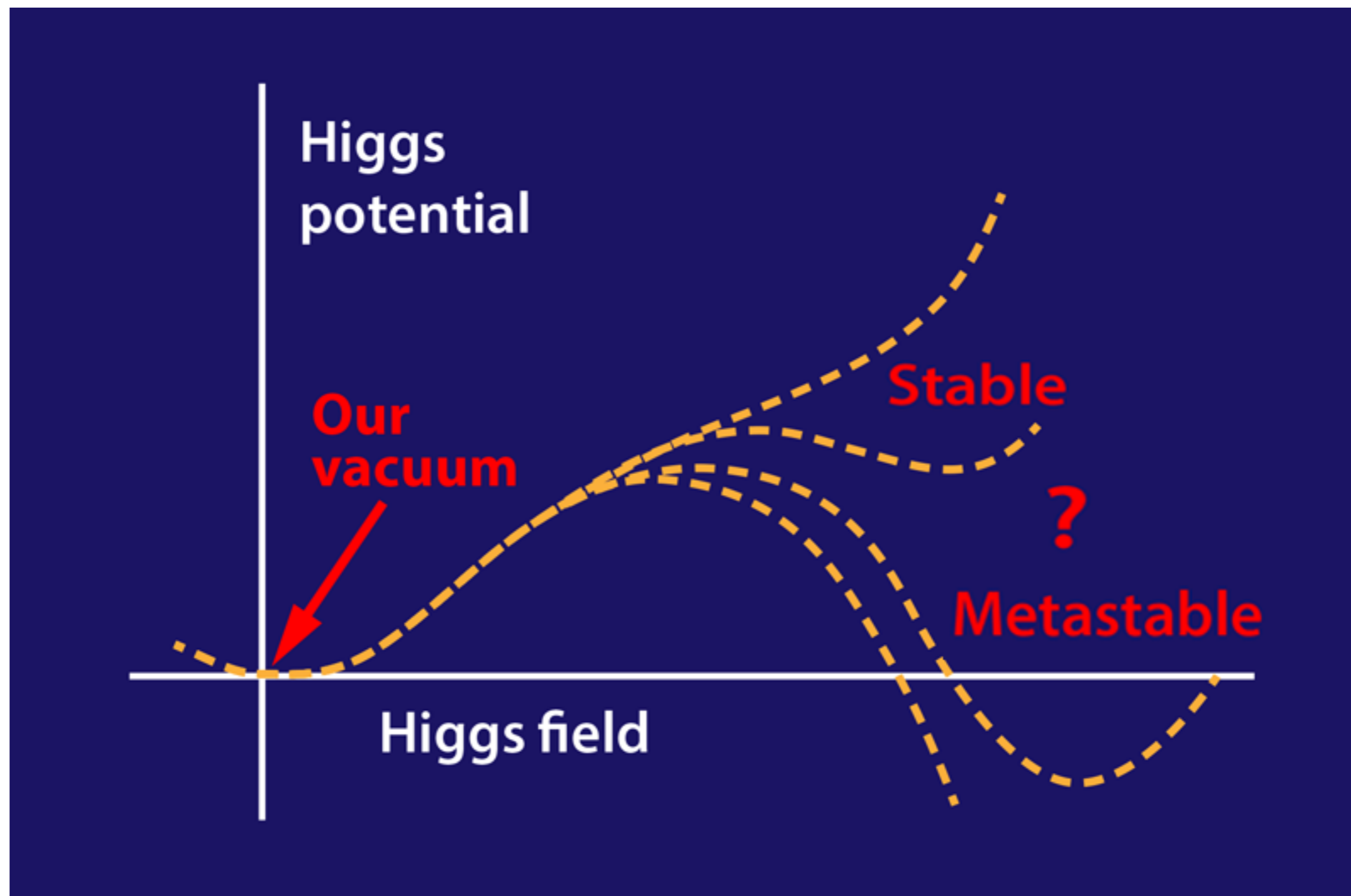
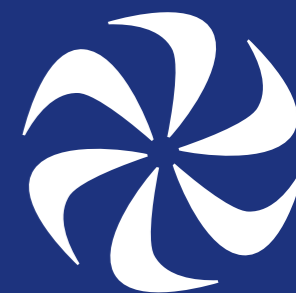
- **More data**
- **Background estimation**
- **Jet reconstruction**
- **Jet triggering**
- A common theme to these problems: how to use **more information**

From Limits, to Discovery



- **More data**
- **Background estimation**
- **Jet reconstruction**
- **Jet triggering**
- A common theme to these problems: how to use **more information**
 - And a common solution to many: **machine learning**

Universe Stability



A. Kusenko

Interference



$$\sigma \propto \left| \left(\left| \right|^2 - \left(\right) + \left| \right|^2 \right) + h.c.)$$

Interference



$$\sigma \propto \left| \left(\begin{array}{c} g \text{ } \text{-----} \\ \text{ } \nearrow \text{ } \kappa_t \\ \text{ } \searrow \text{ } \kappa_\lambda \\ g \text{ } \text{-----} \\ \text{ } \leftarrow \text{ } t/b \end{array} \right)^2 - \left(\begin{array}{c} g \text{ } \text{-----} \\ \text{ } \rightarrow \text{ } \kappa_t \\ \text{ } \rightarrow \text{ } \kappa_t \\ g \text{ } \text{-----} \\ \text{ } \leftarrow \text{ } t/b \end{array} \right)^2 + h.c. \right|$$

