Imperial College London

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# Quantum computing approaches for simulating parton showers in high energy collisions

Simon Williams

Lake Louise Winter Institute - 25/02/22

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[1] - A quantum walk approach to simulating parton showers, arXiv: 2109.13975

In collaboration with Sarah Malik (UCL), Michael Spannowsky (IPPP, Durham) and Khadeejah Bepari (IPPP, Durham)

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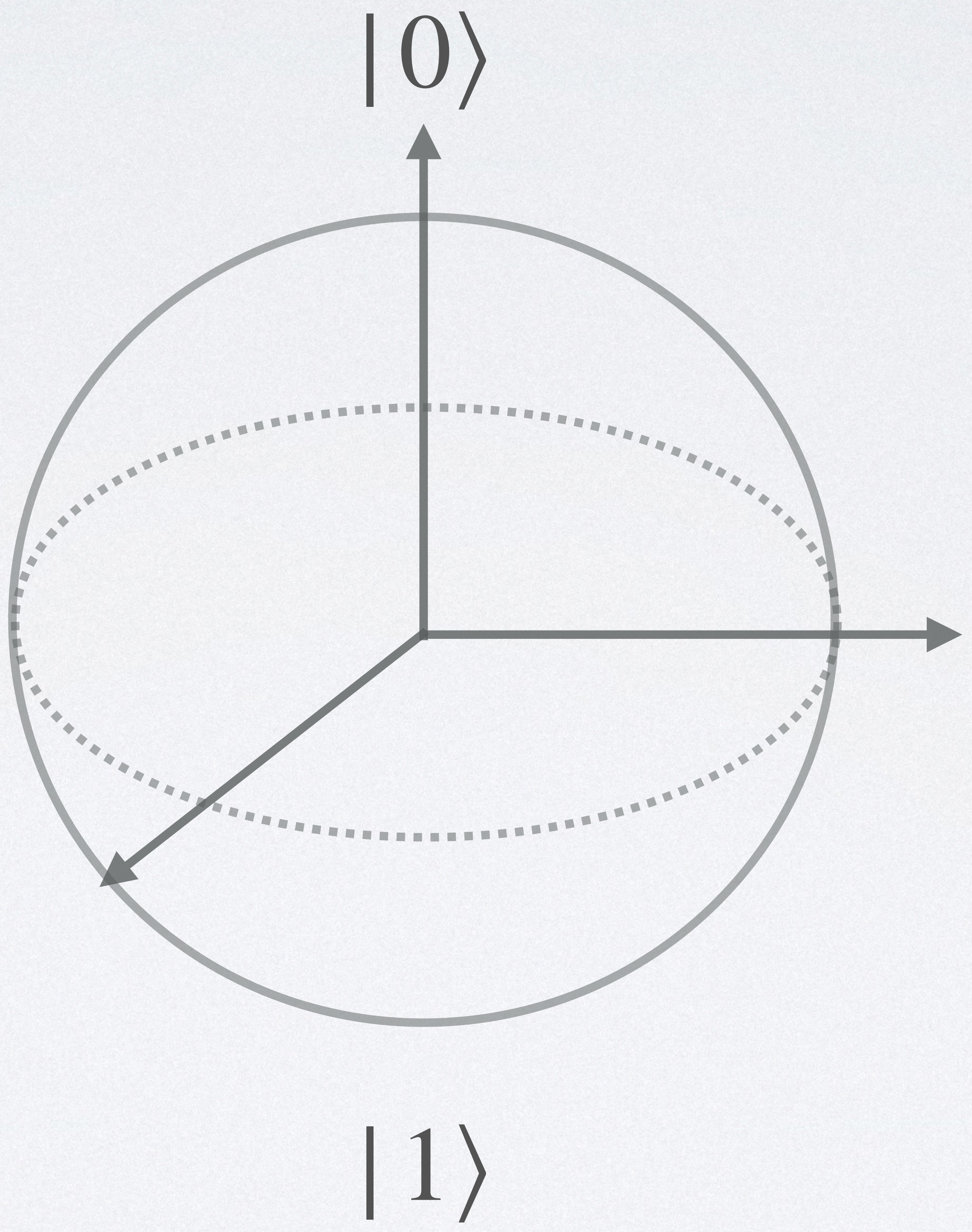

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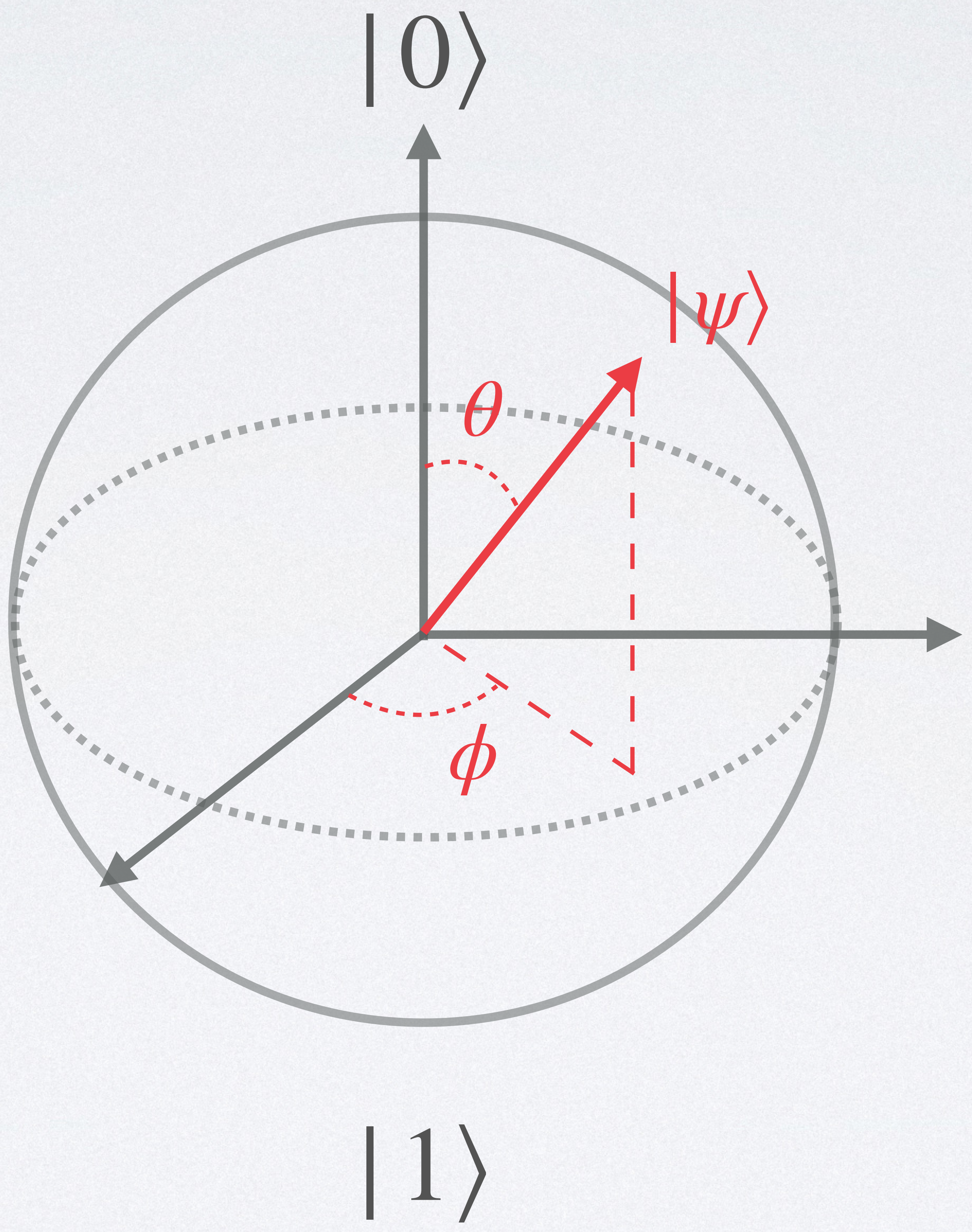

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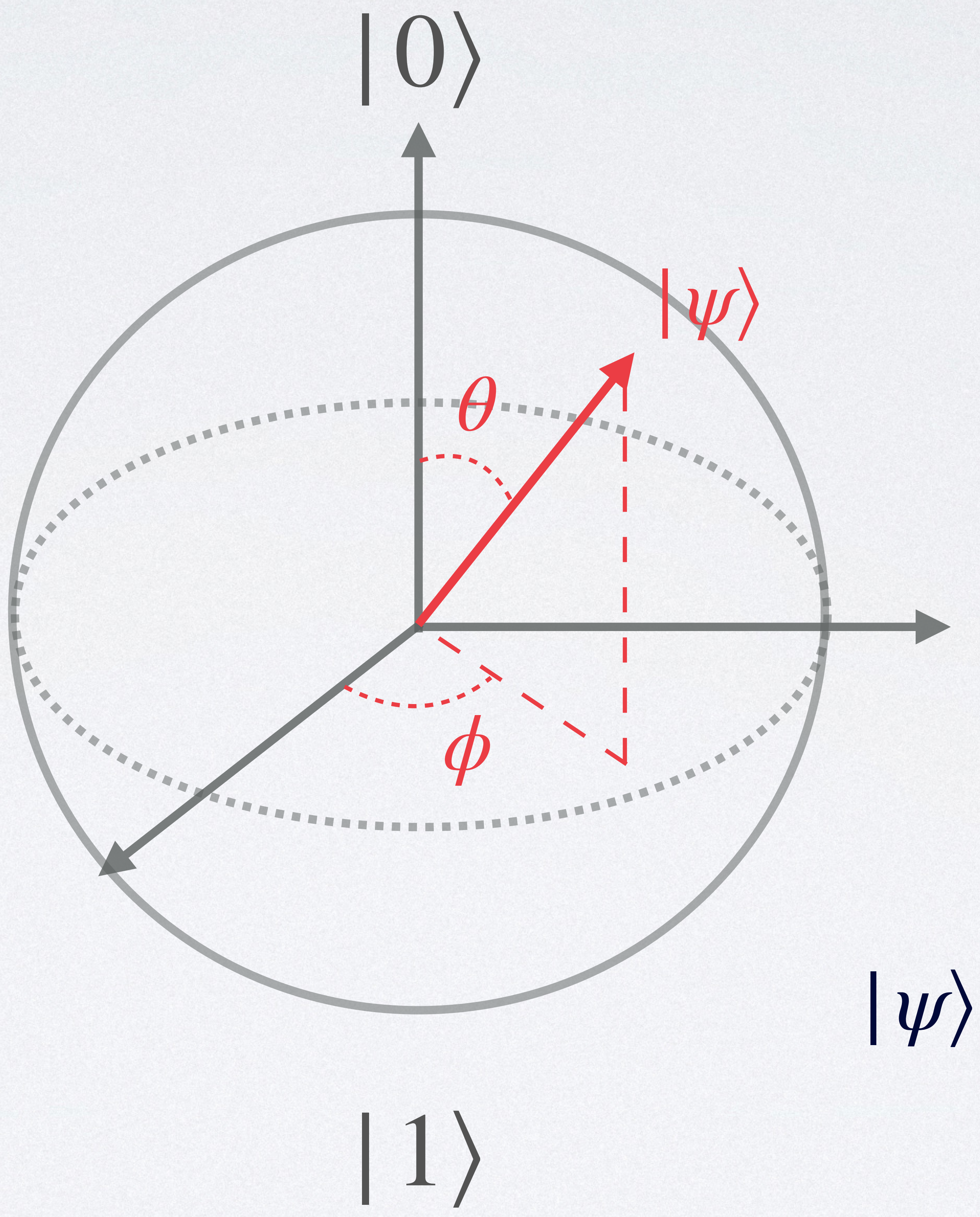


$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix}$$

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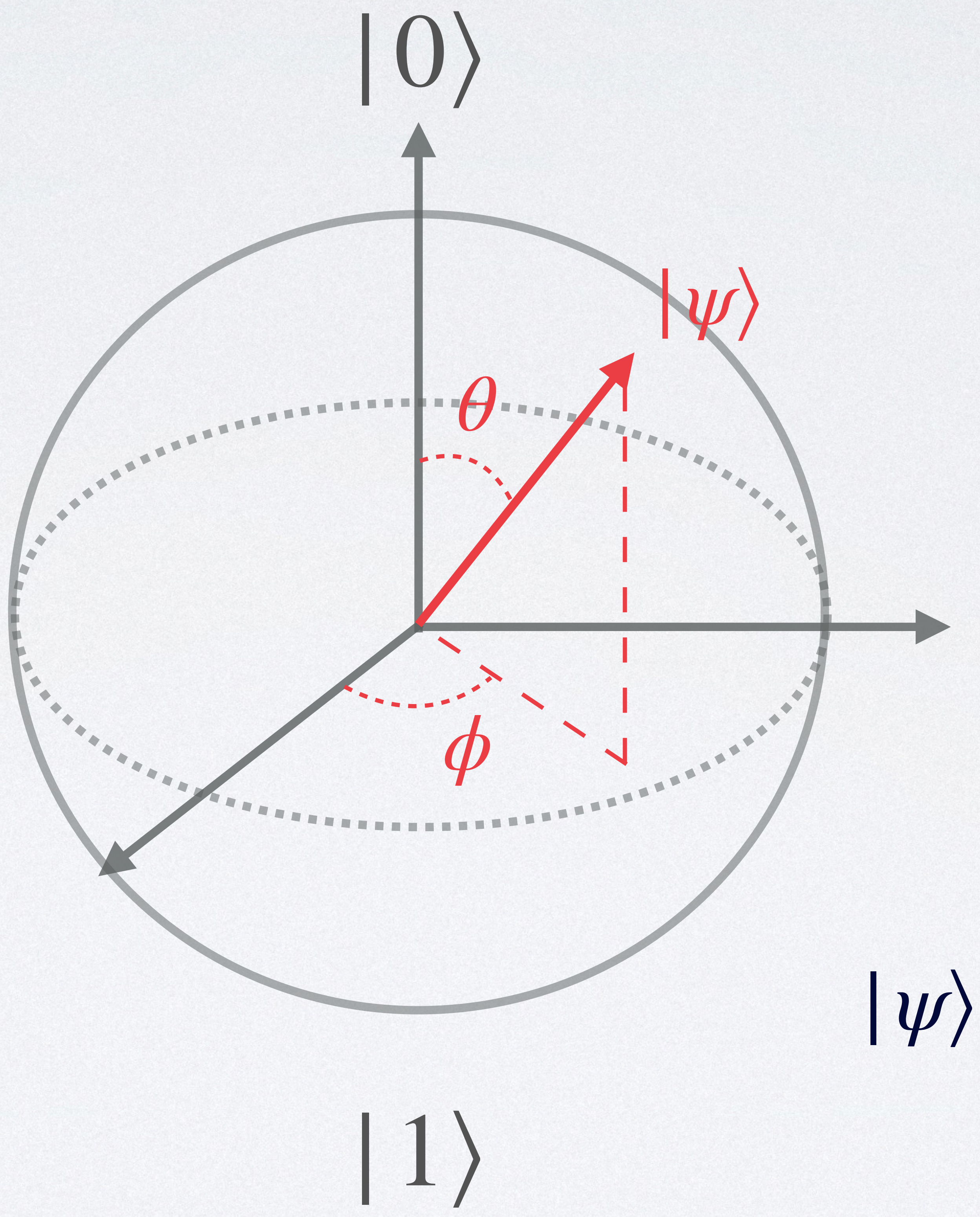


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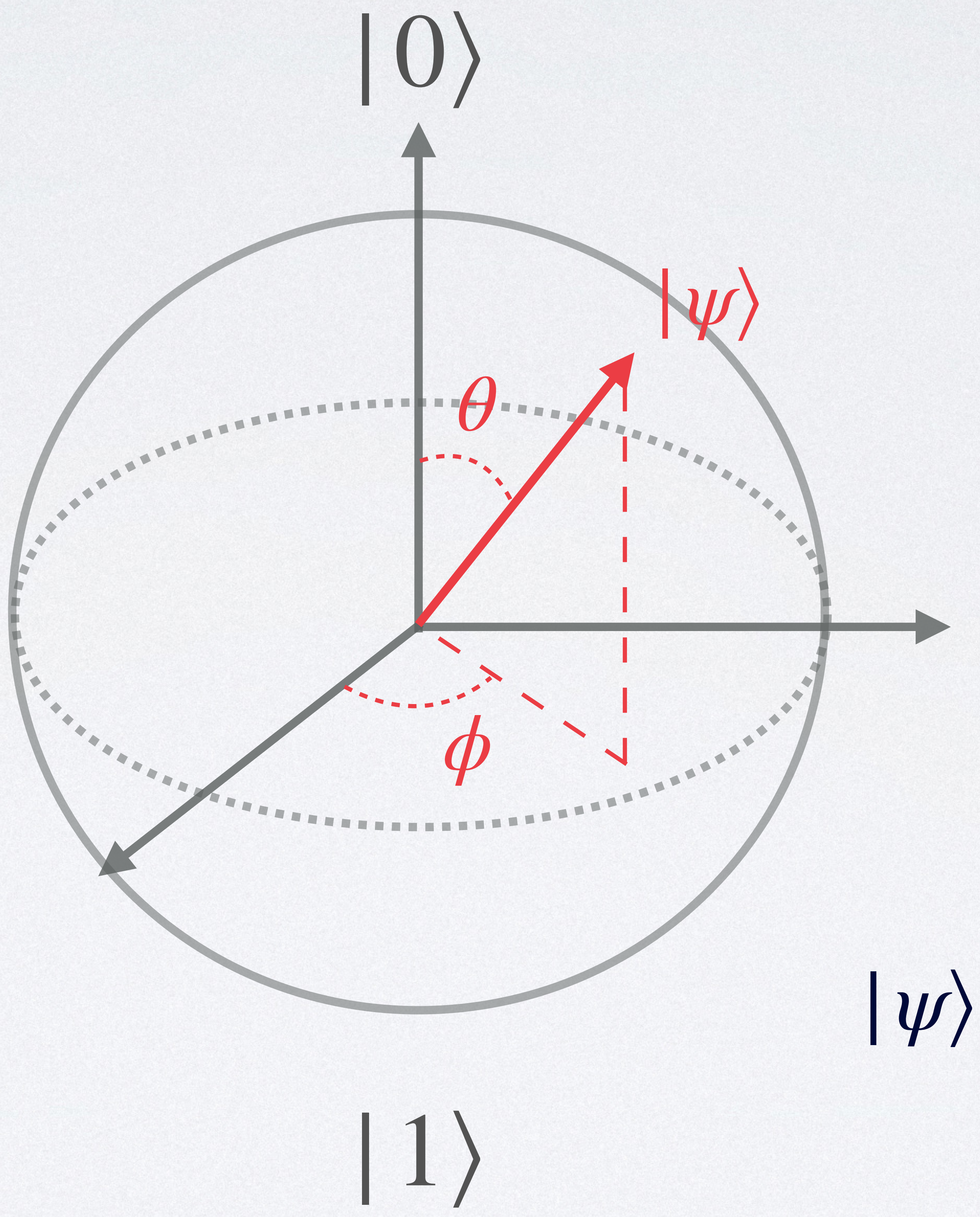


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- Extending this to a system of $N$ qubits forms a $2^N$-dimensional Hilbert Space

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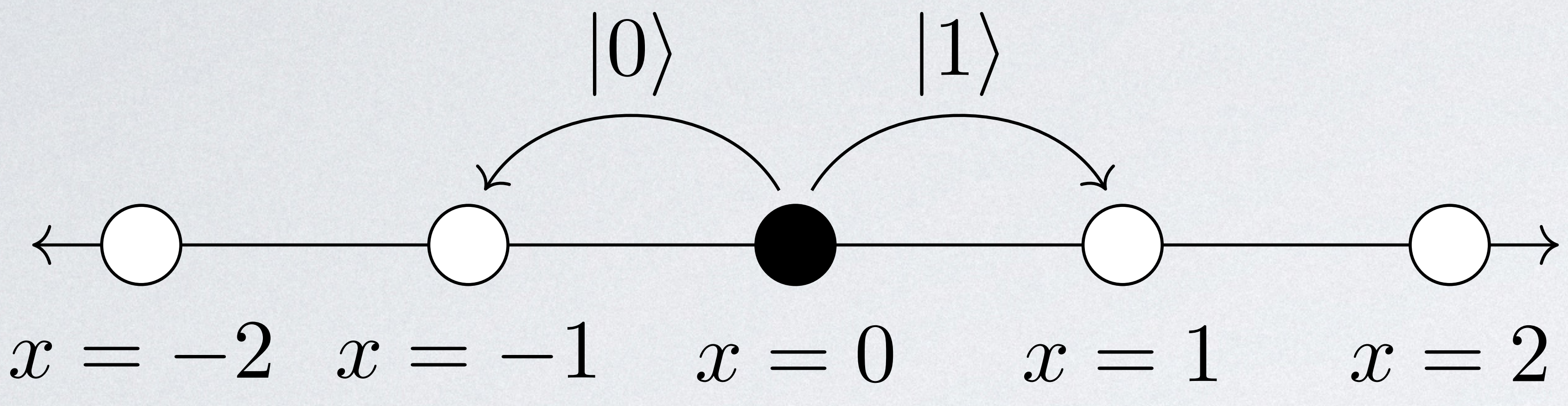

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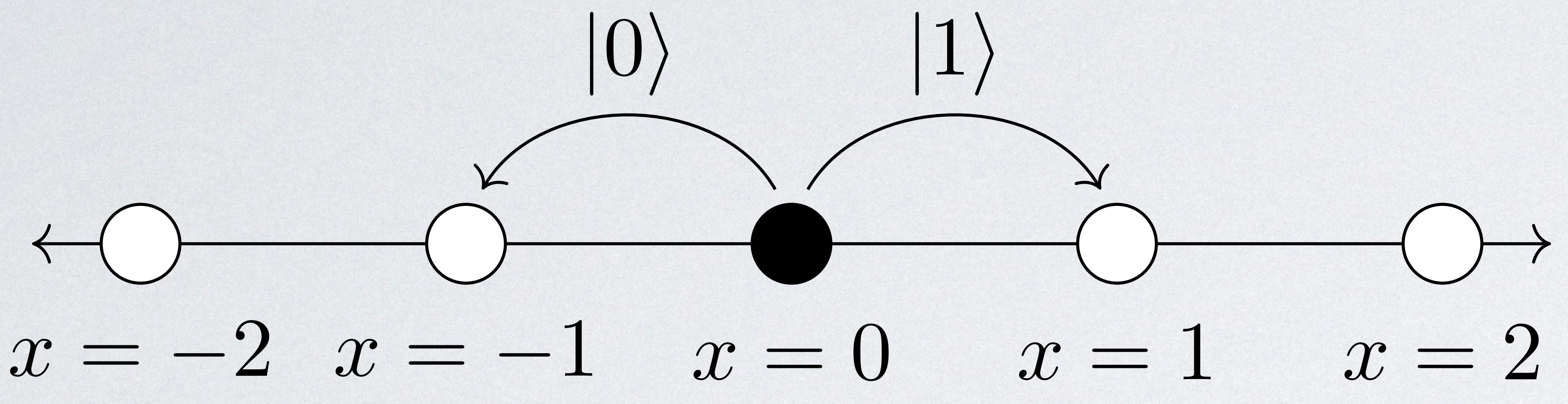


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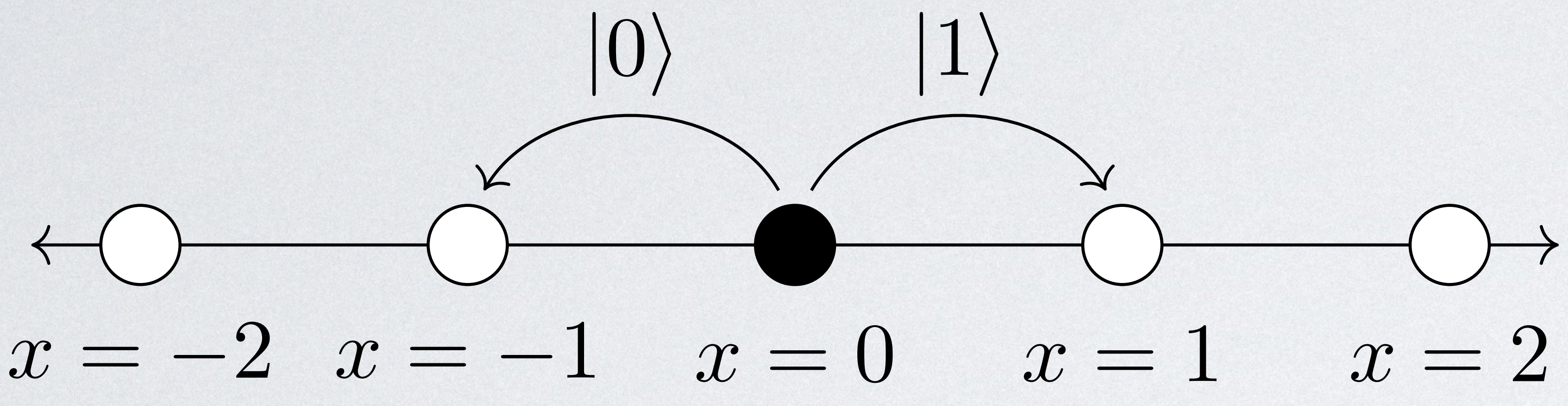


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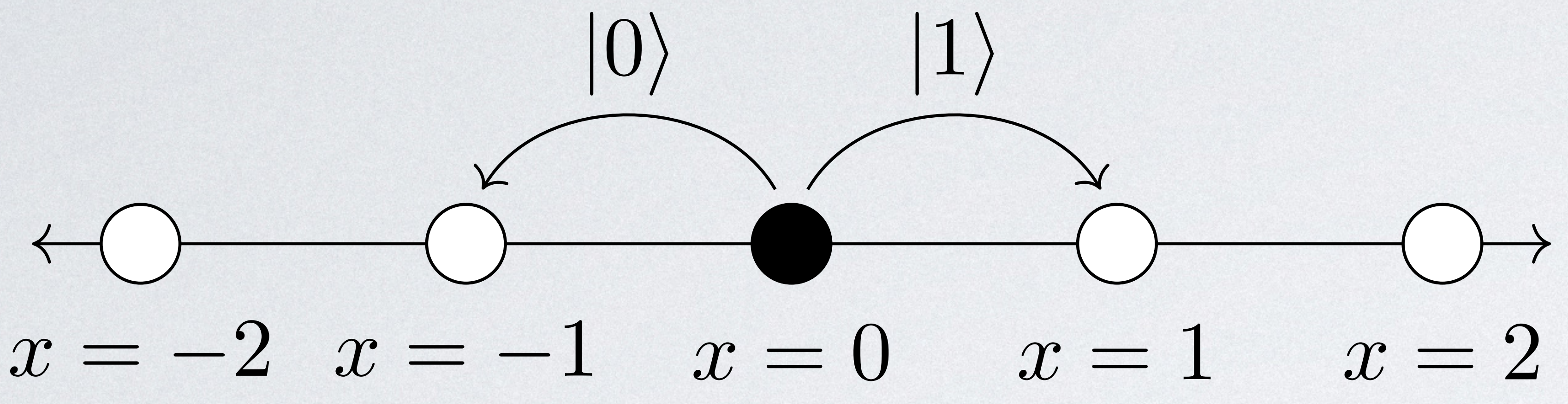


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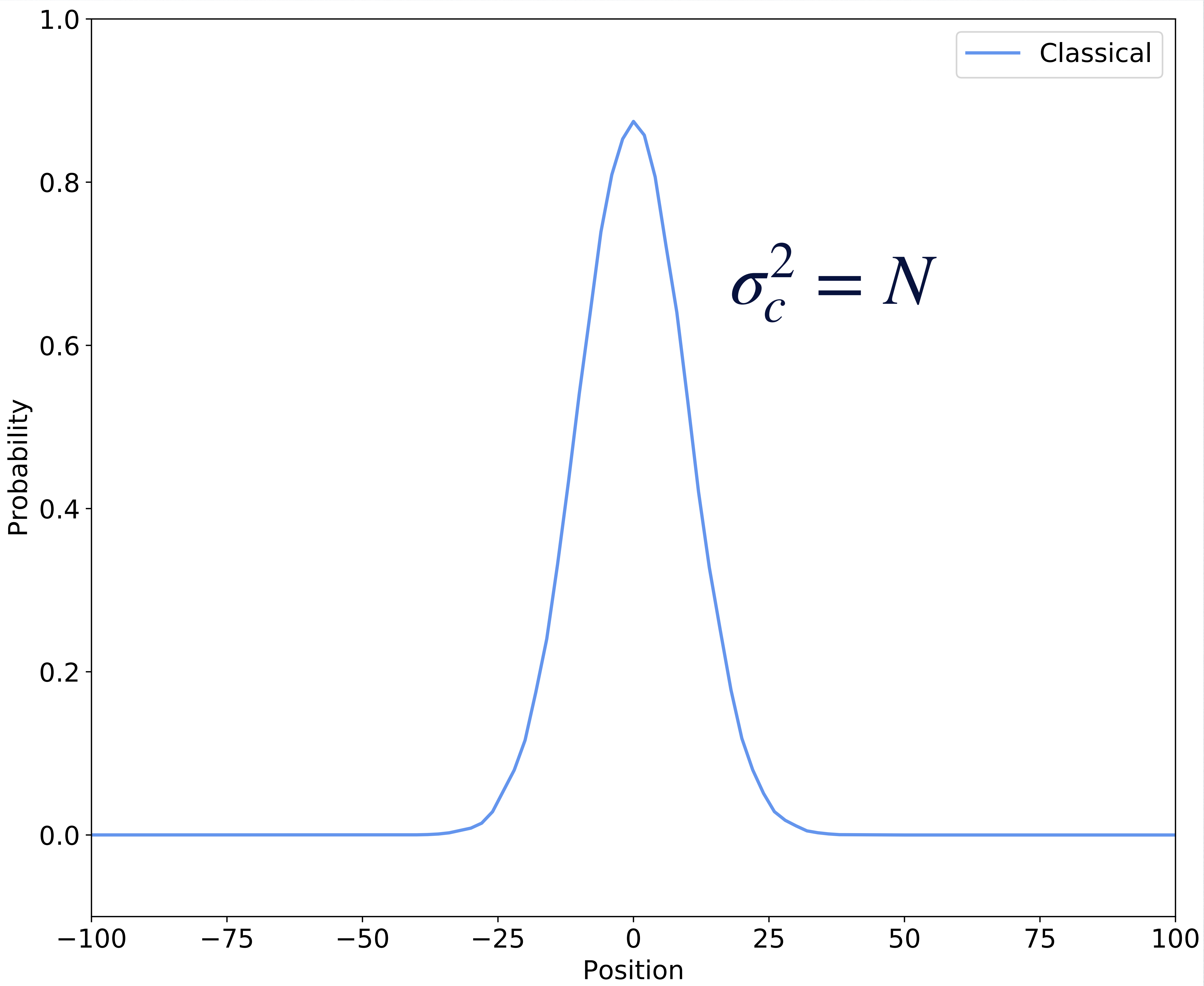

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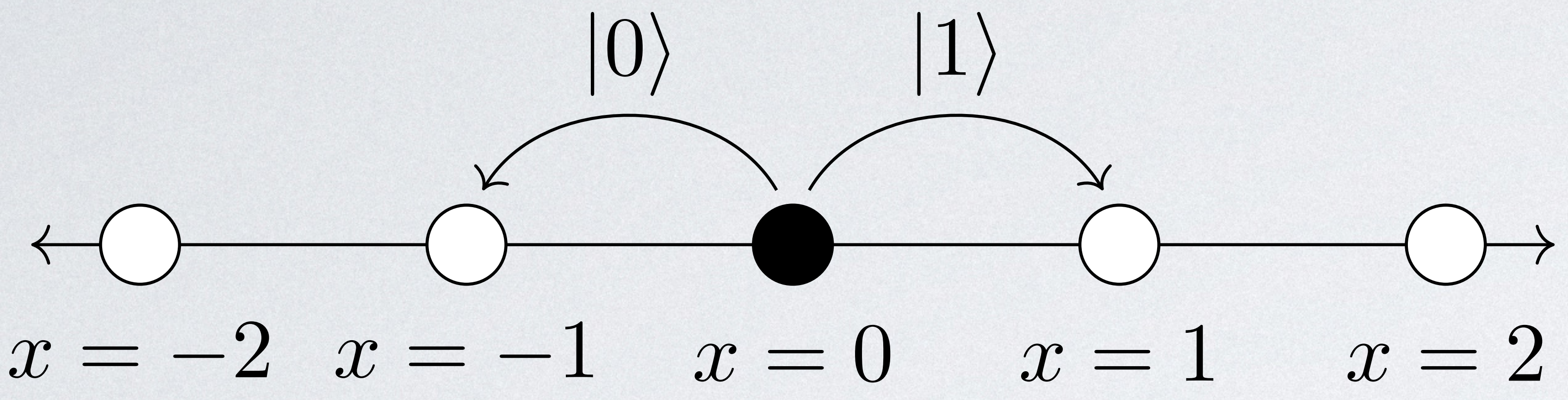


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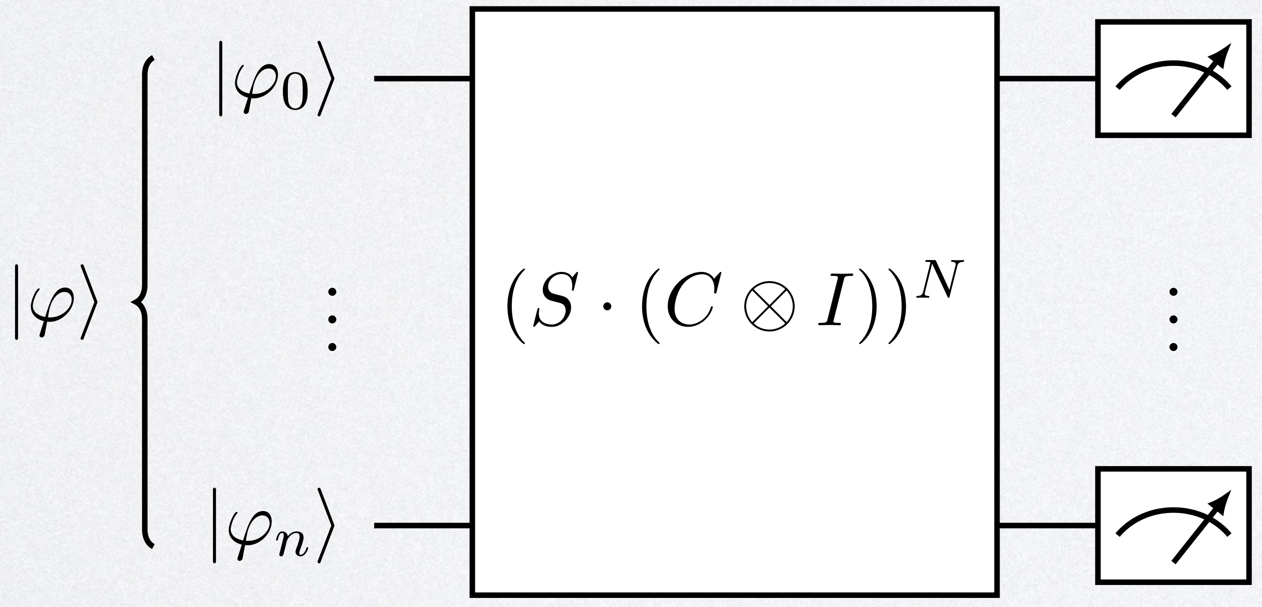


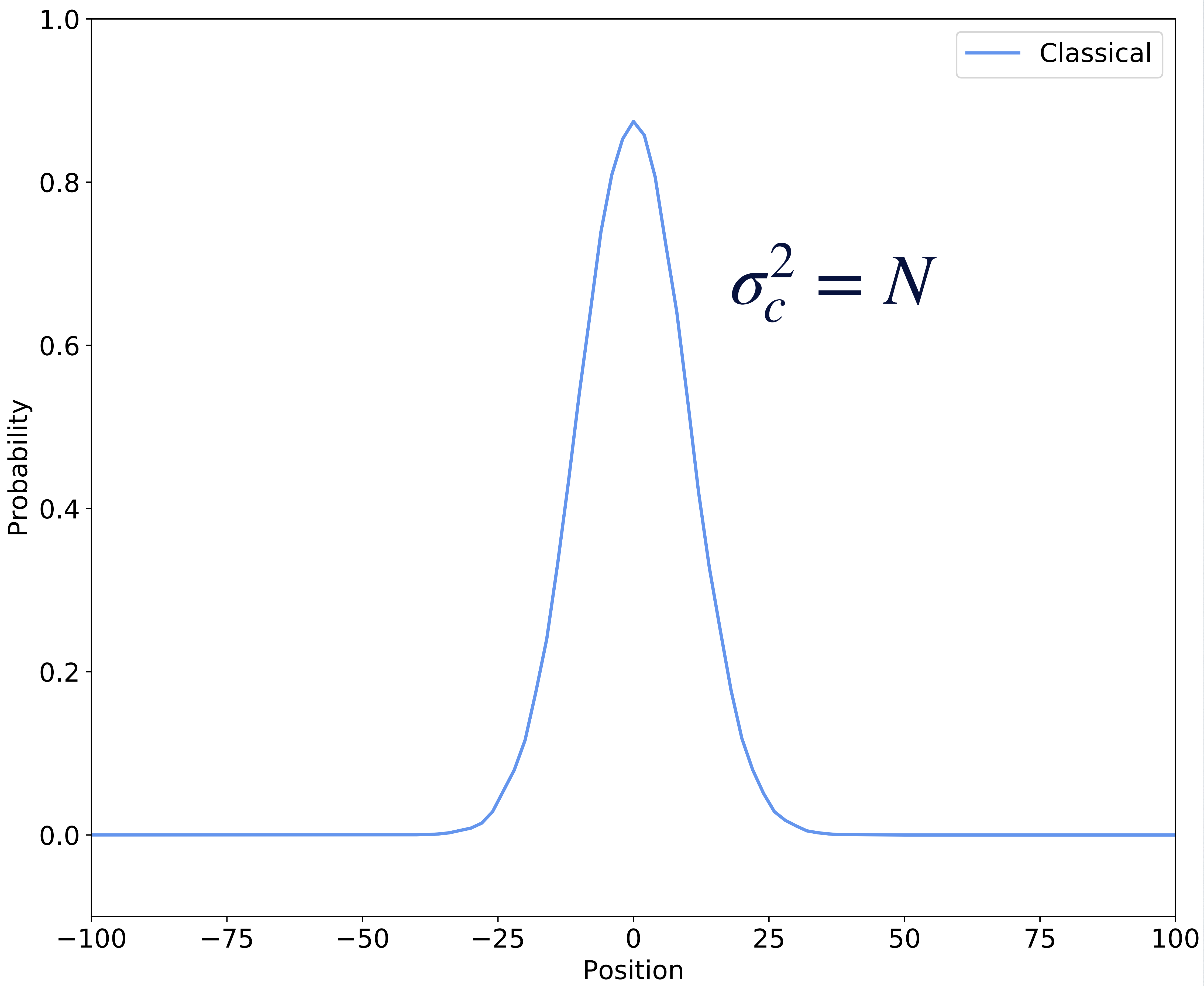

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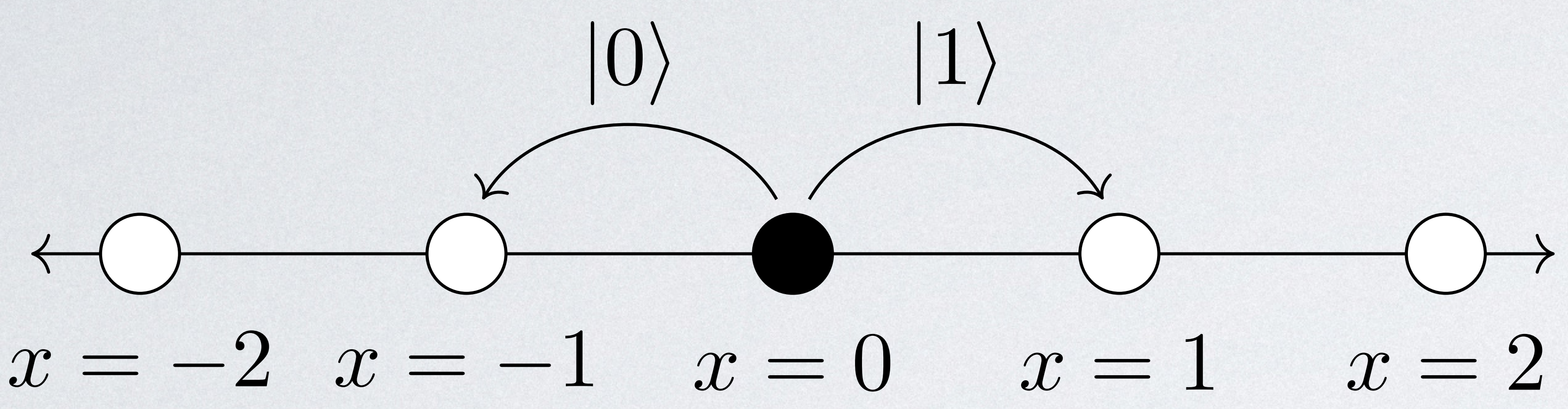


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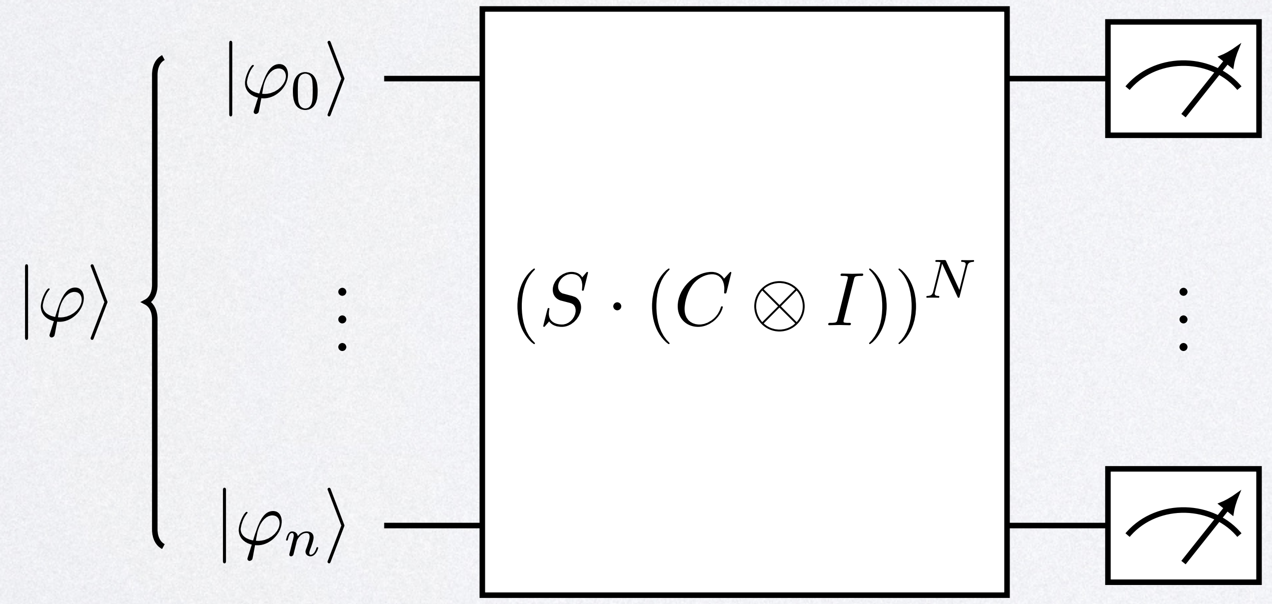


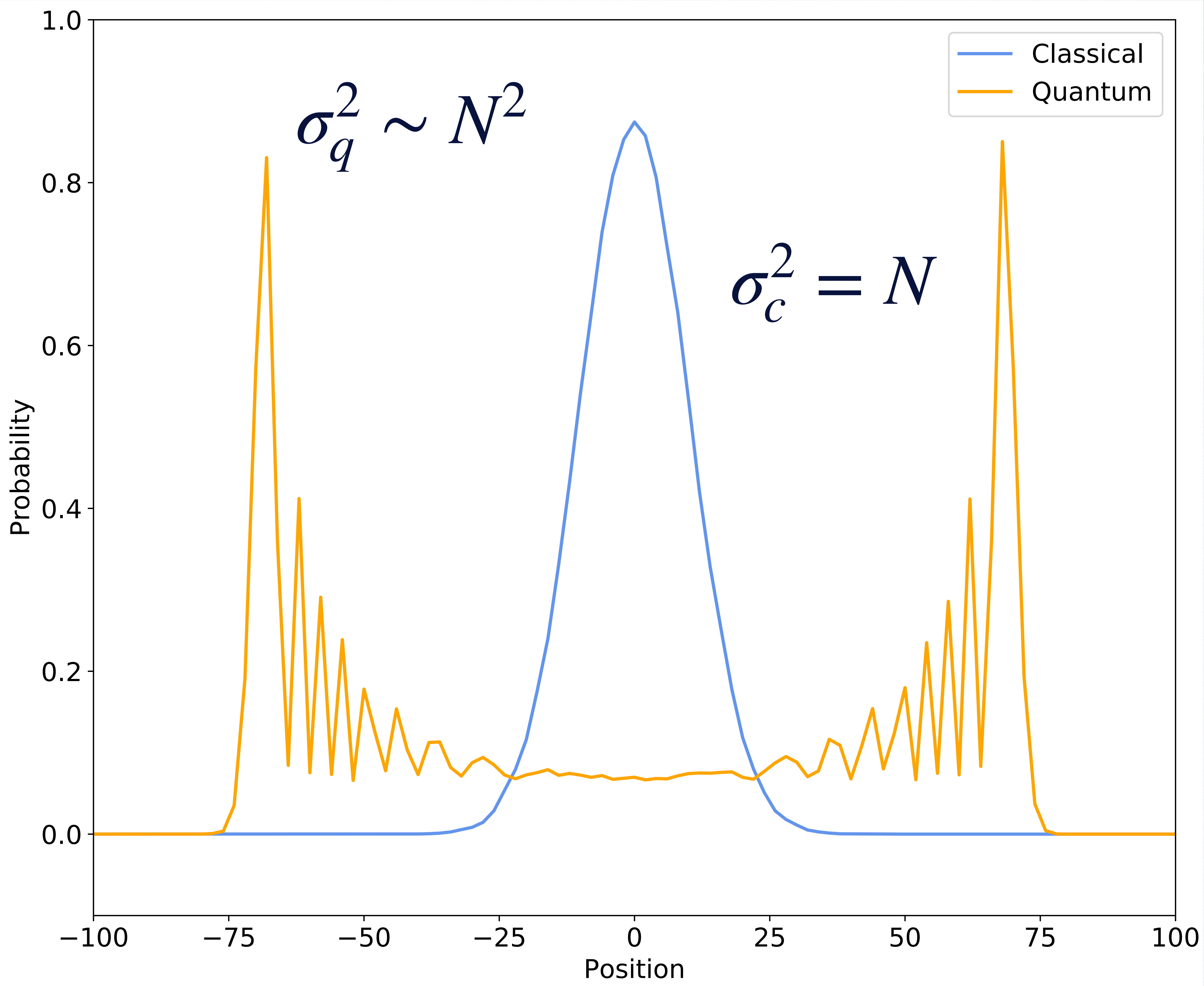

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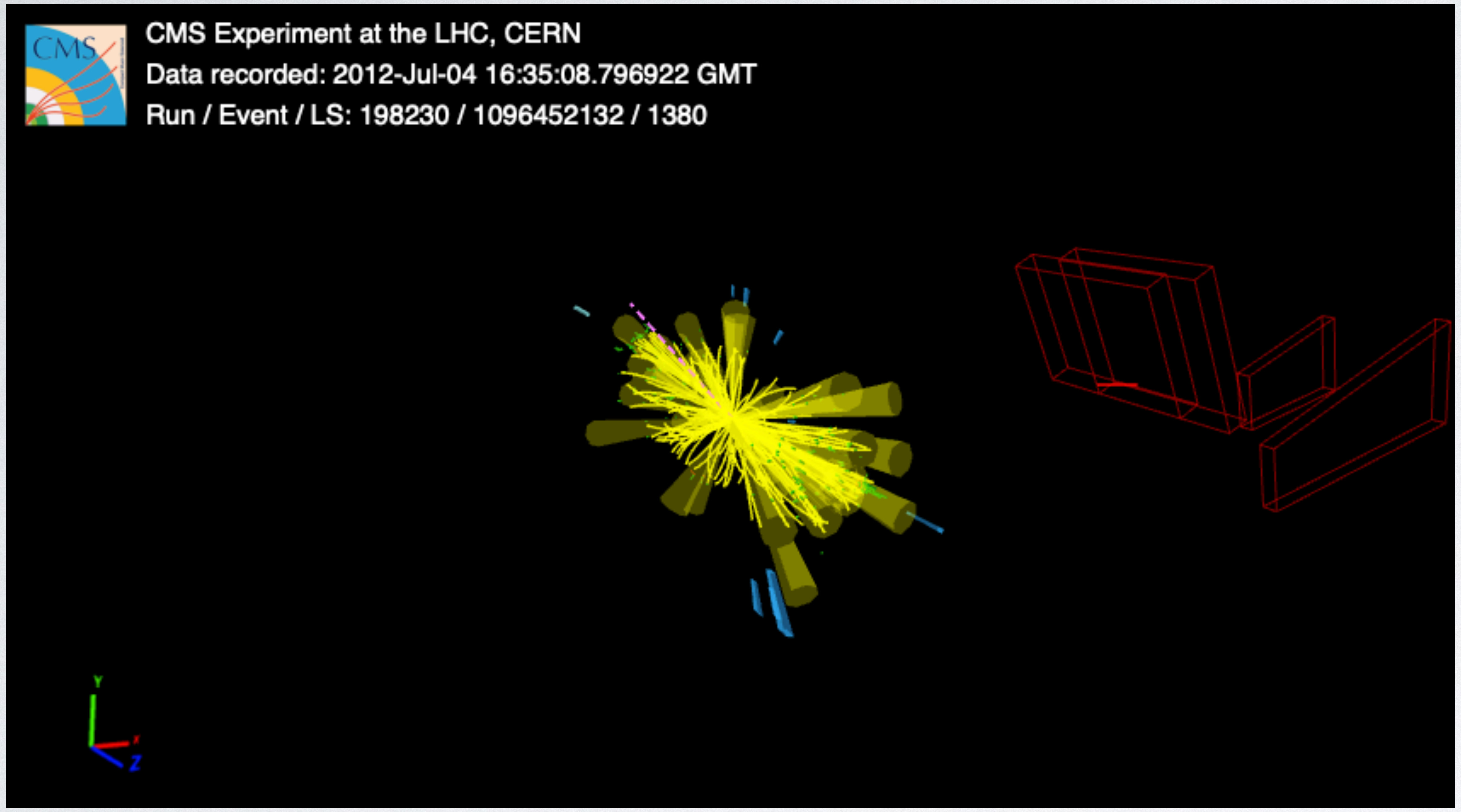


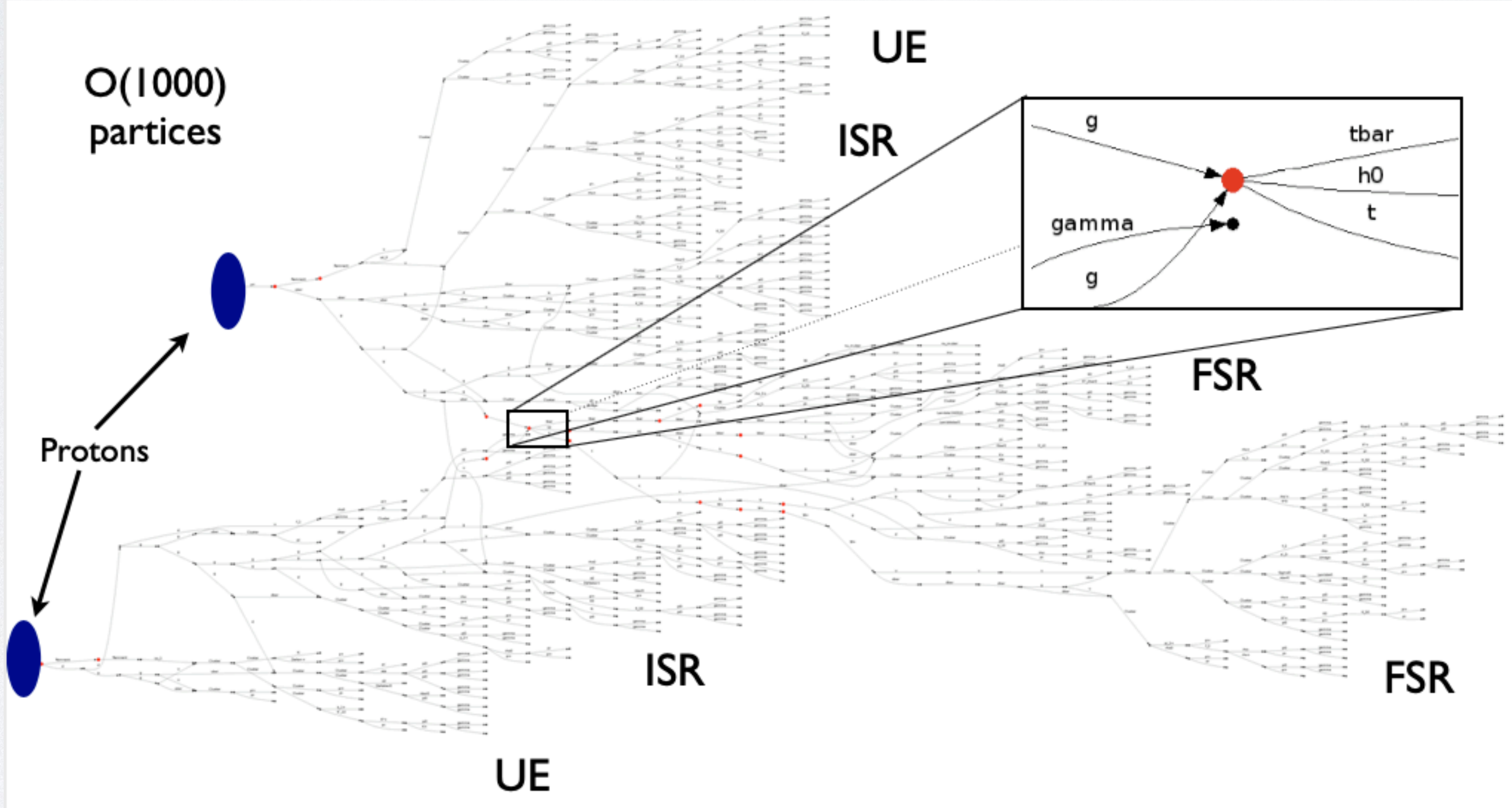

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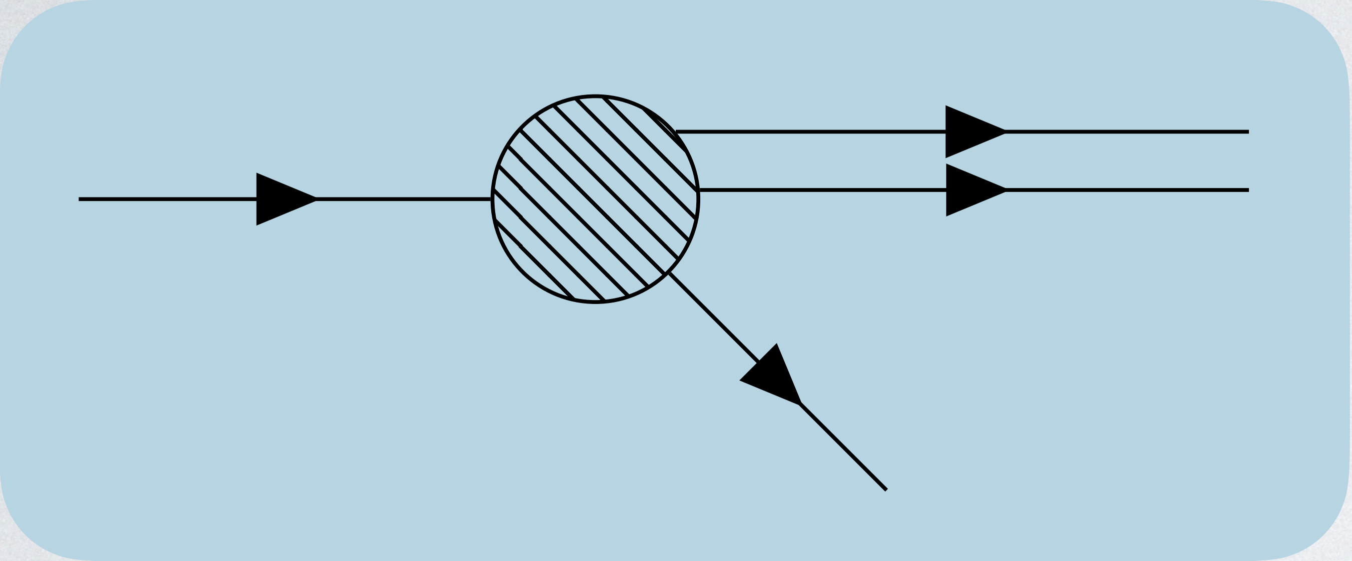
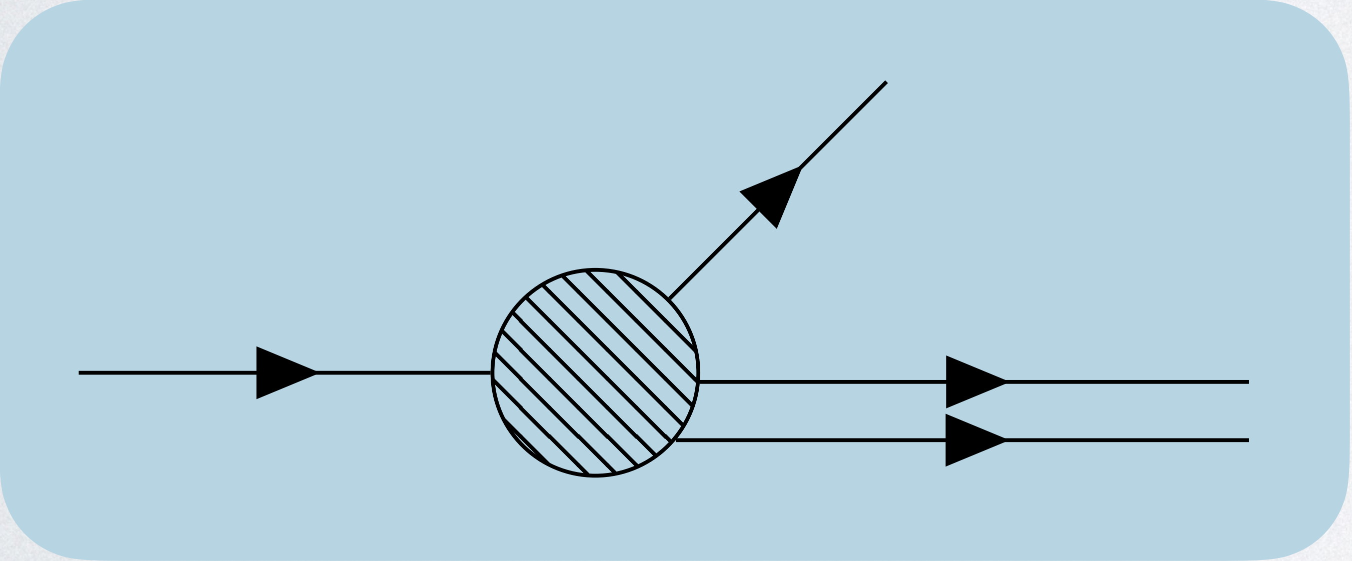
Parton Density Functions





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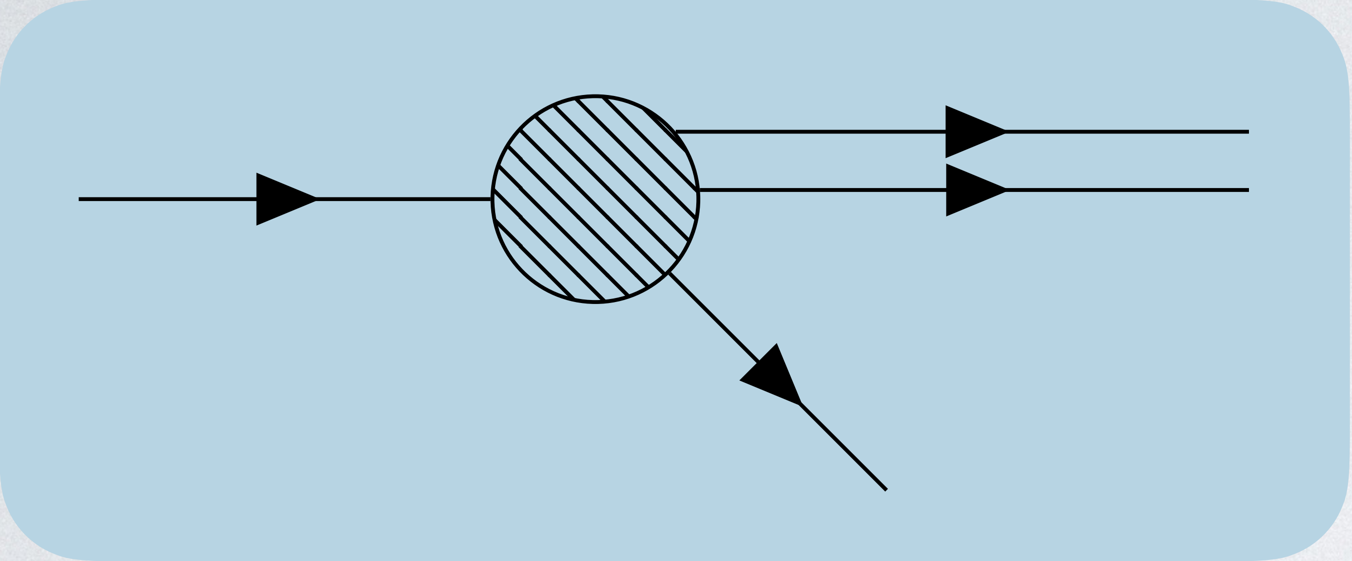
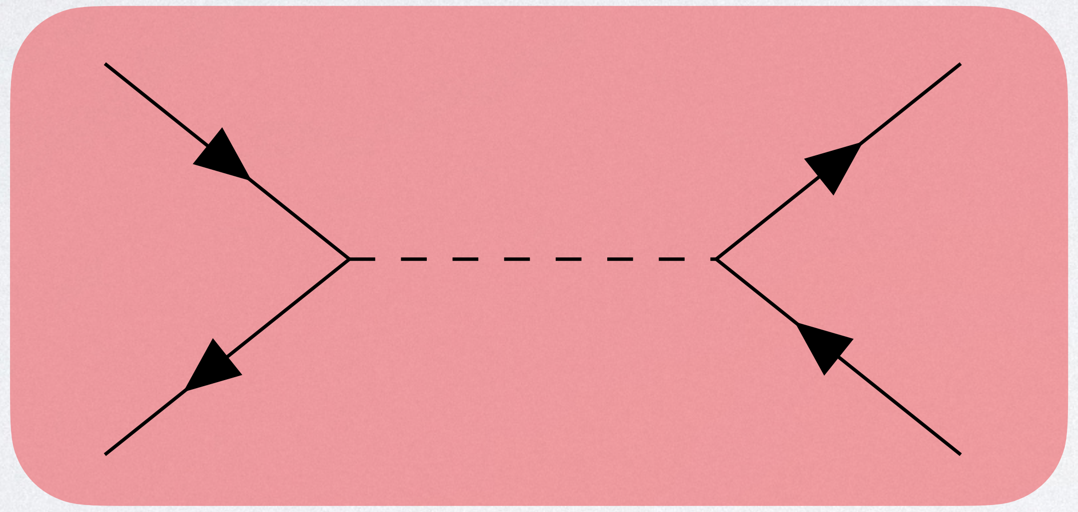


Hard Process



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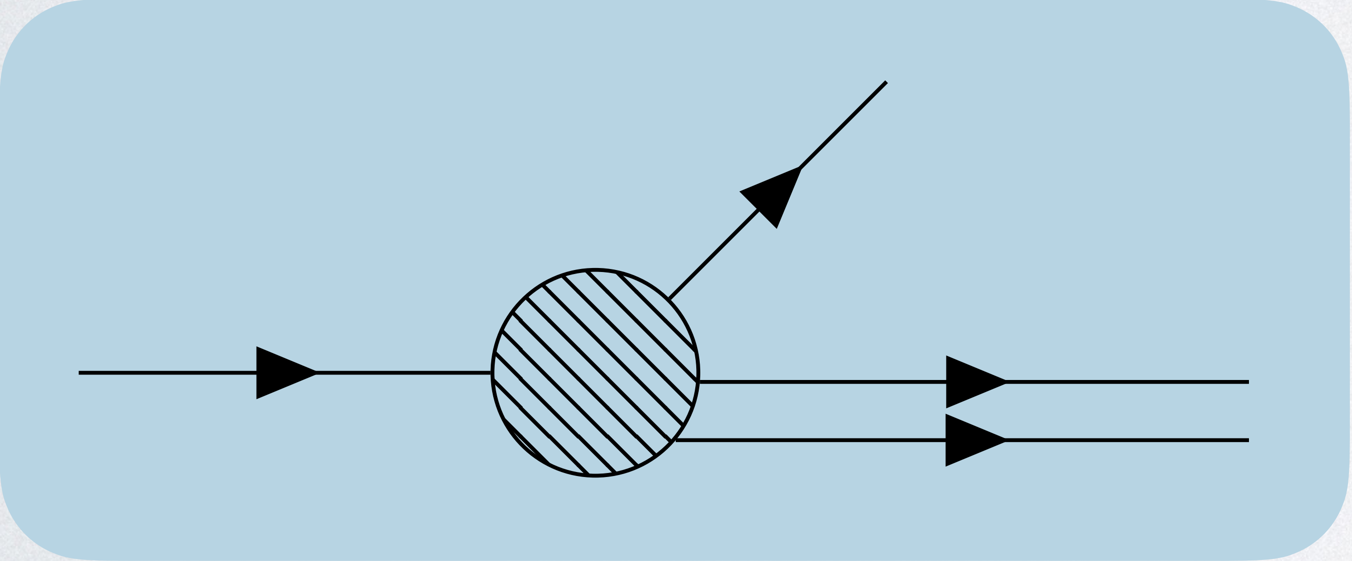

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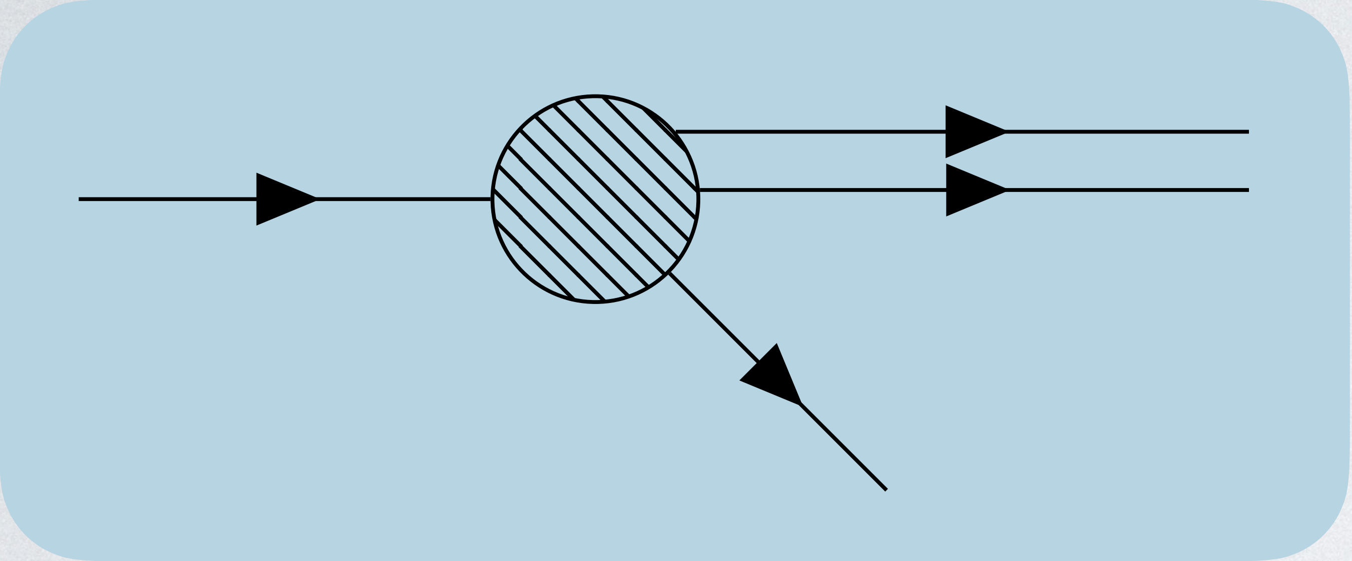

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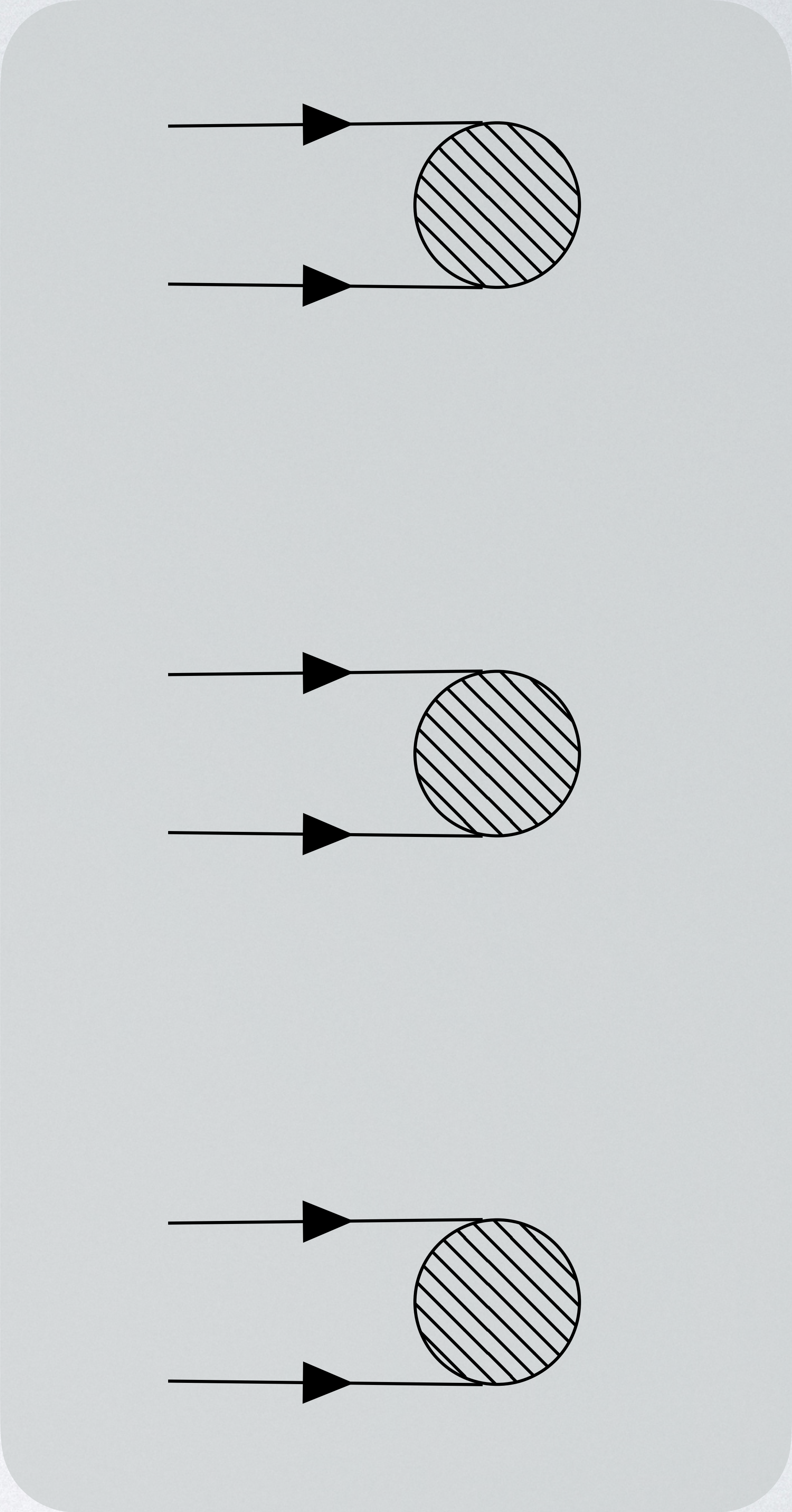

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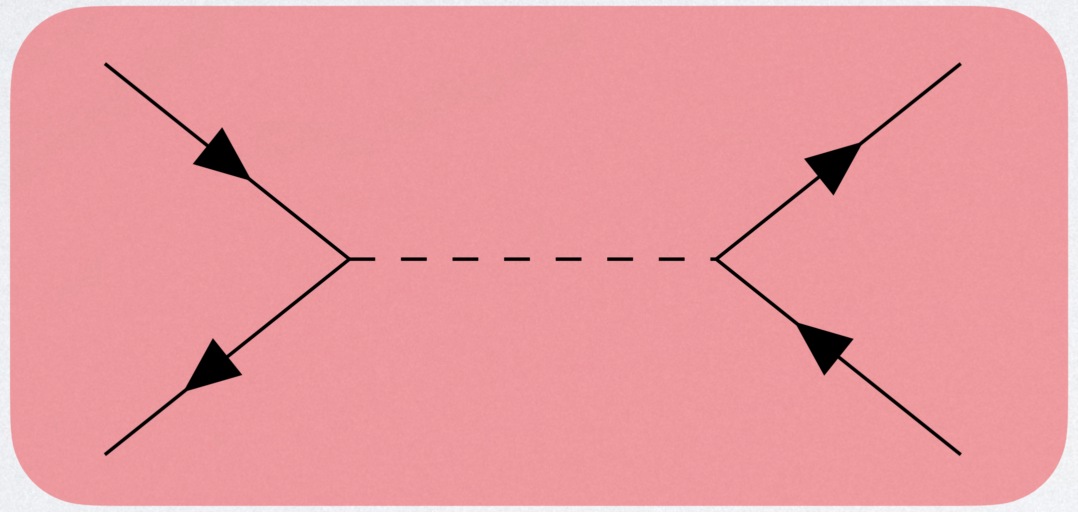

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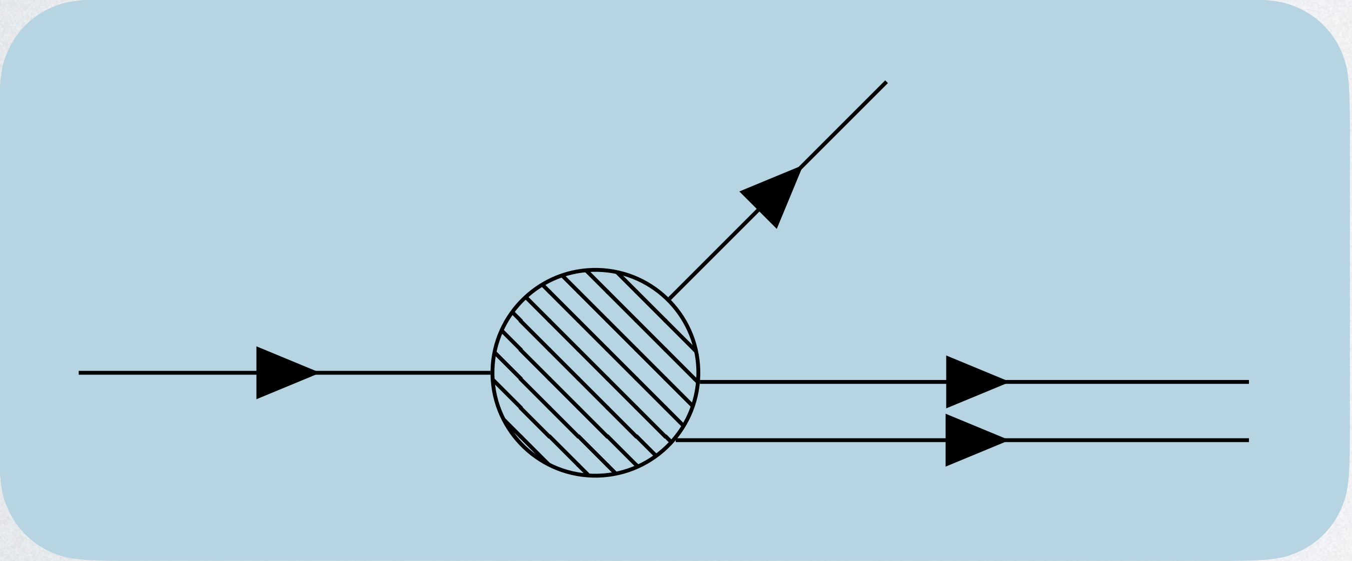

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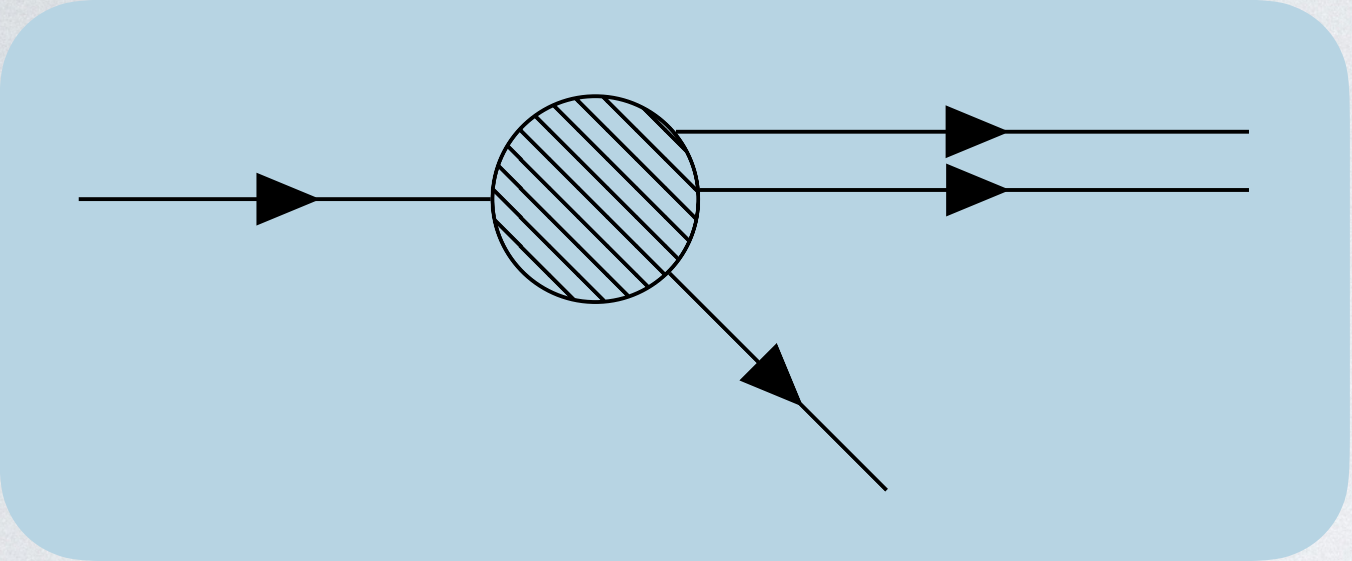

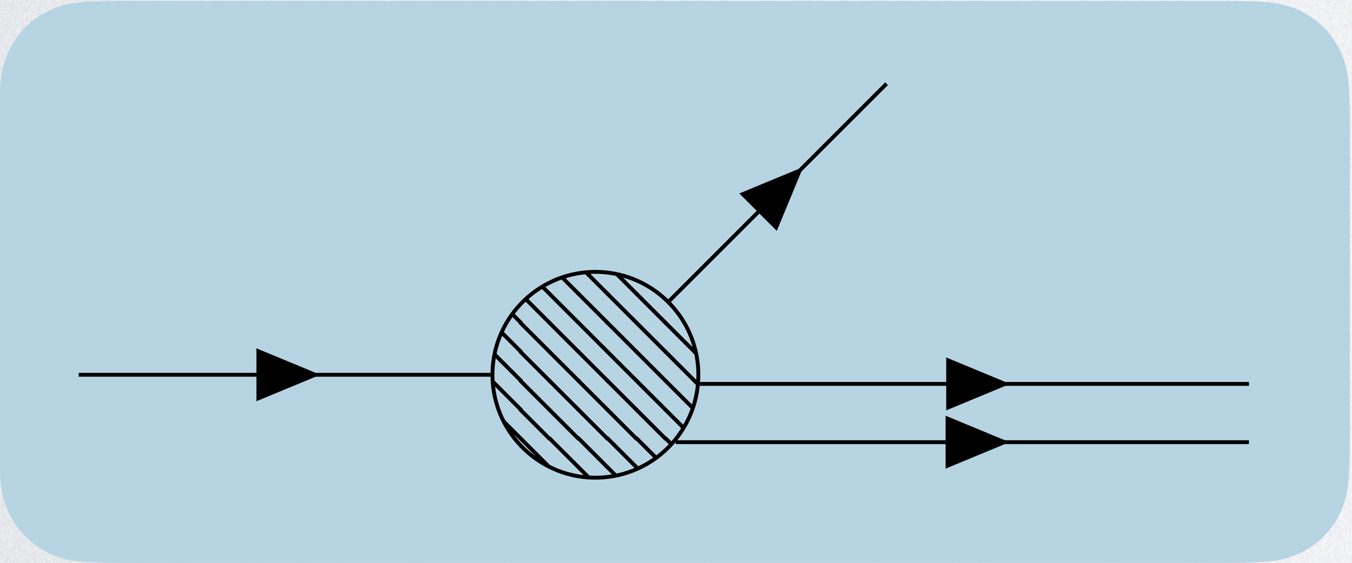

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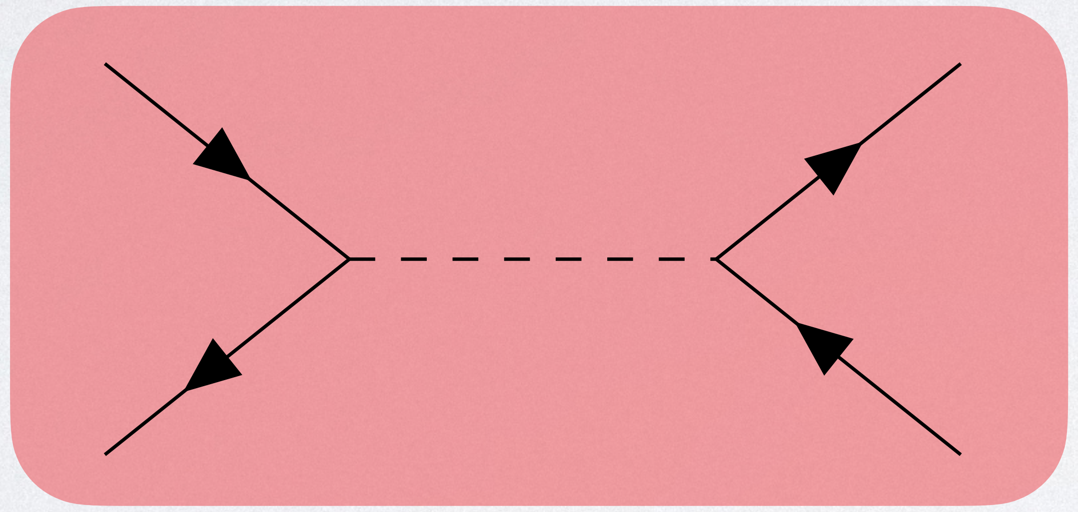

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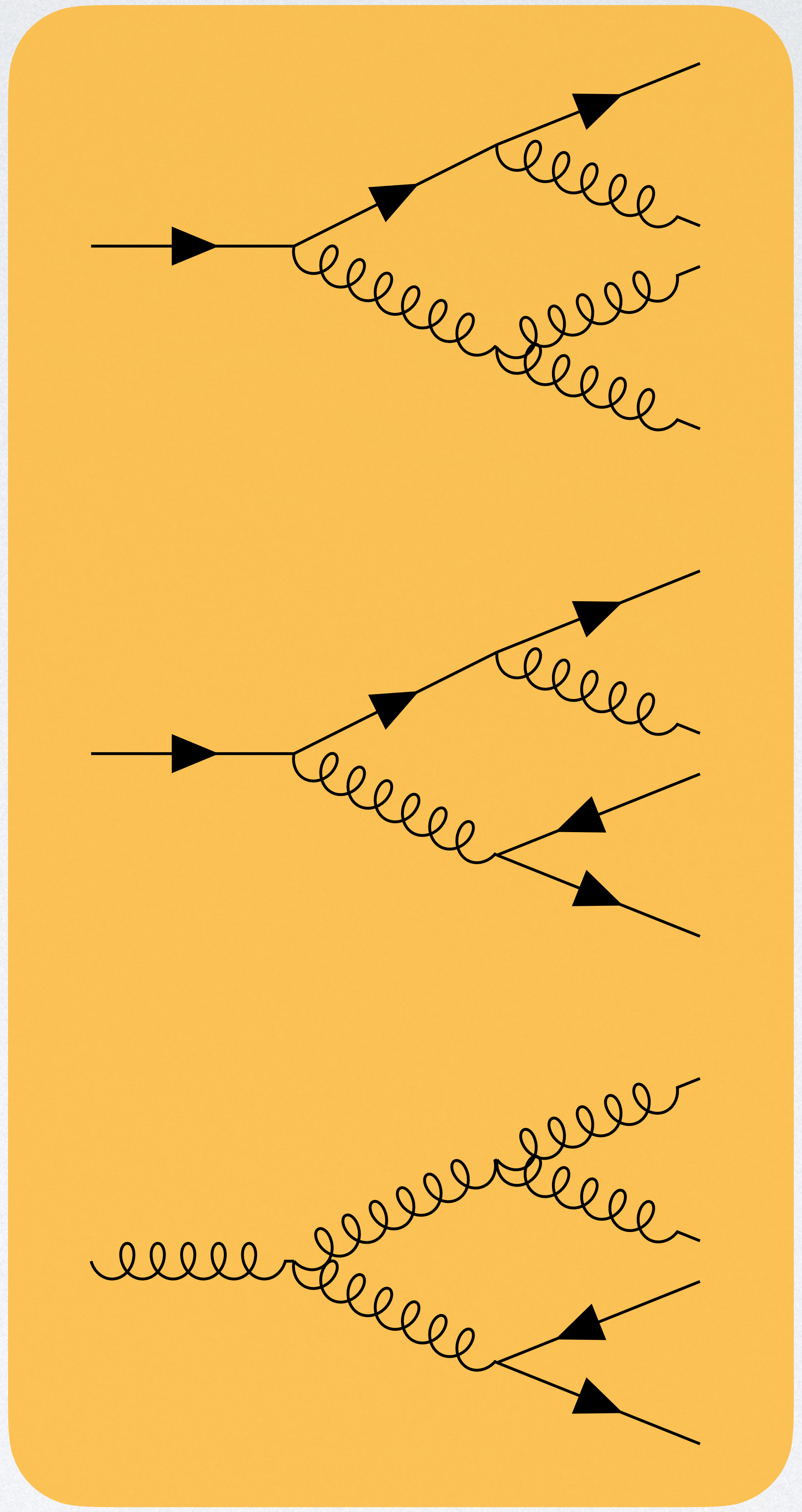
Parton Shower



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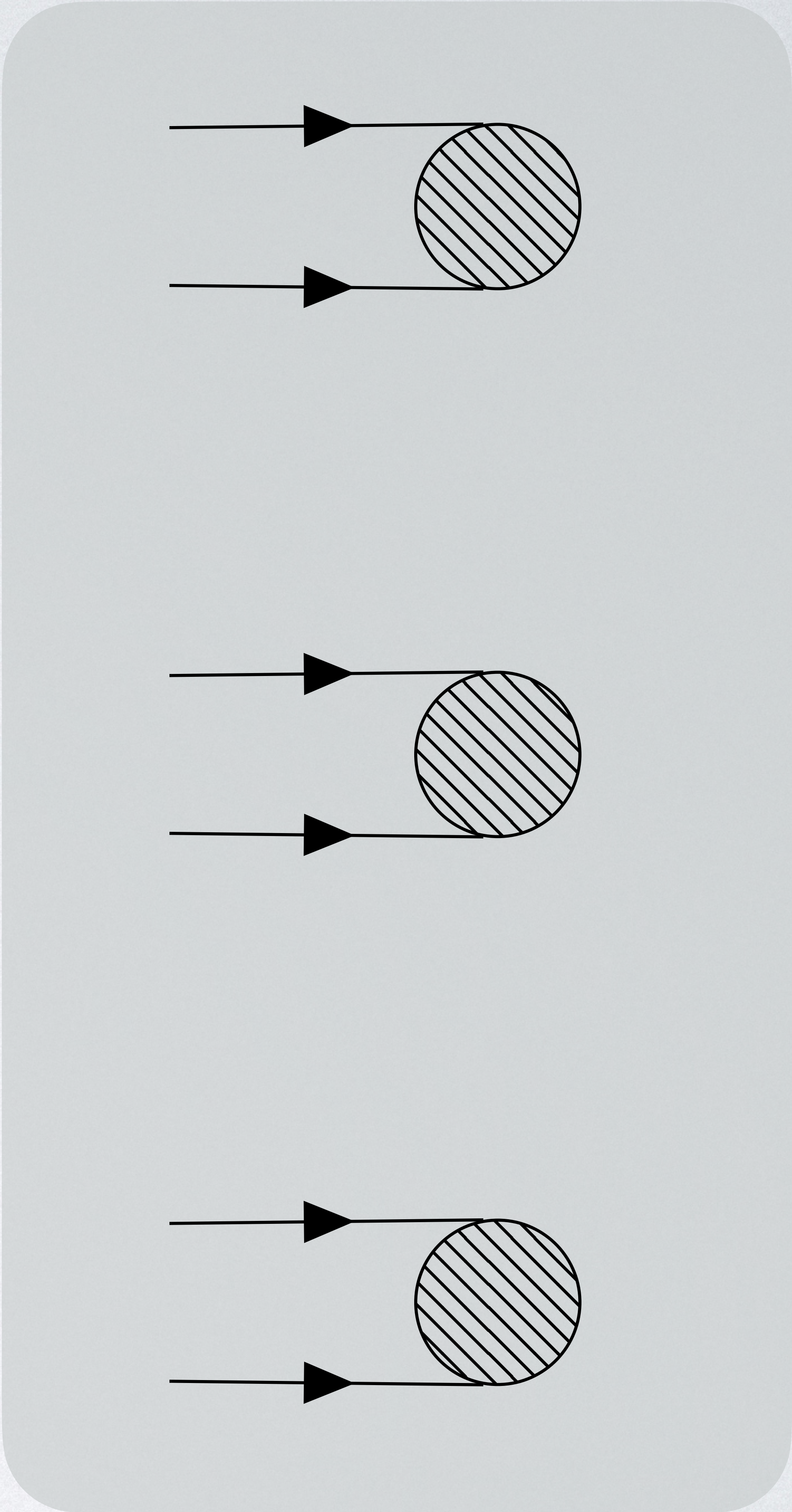

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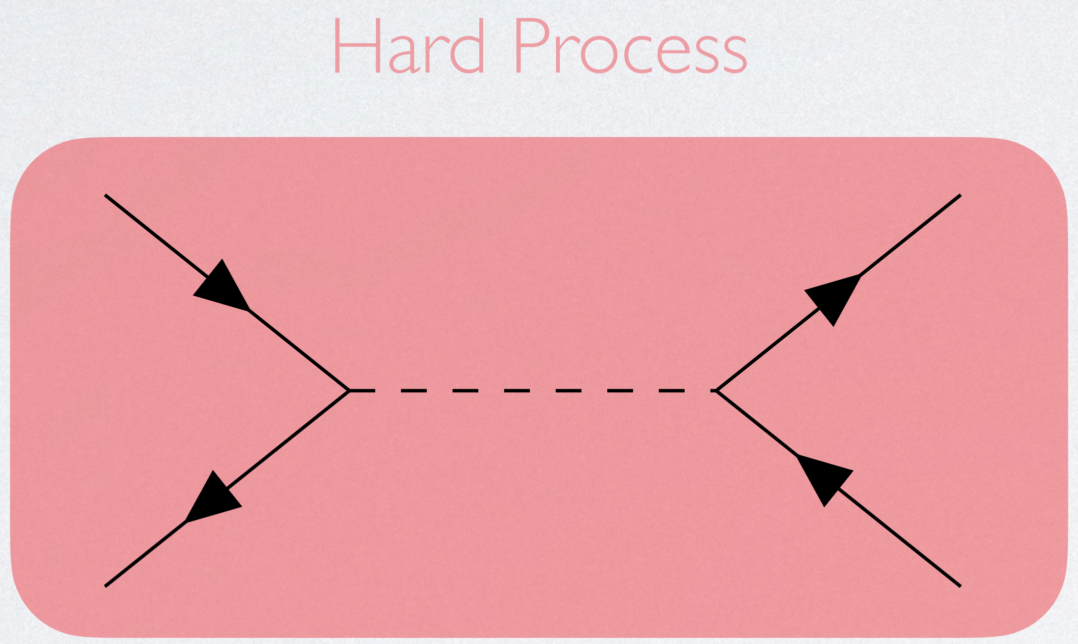


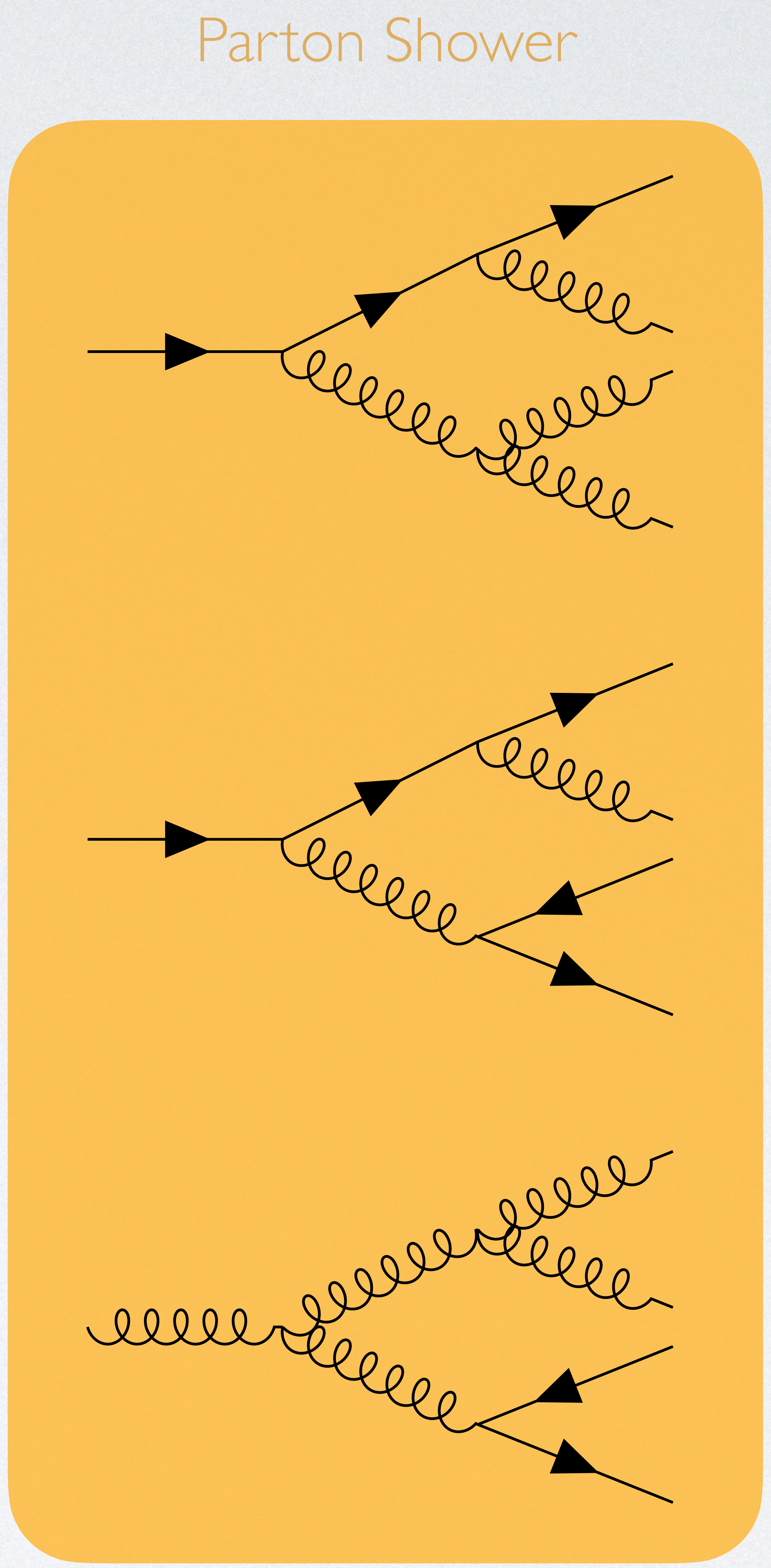


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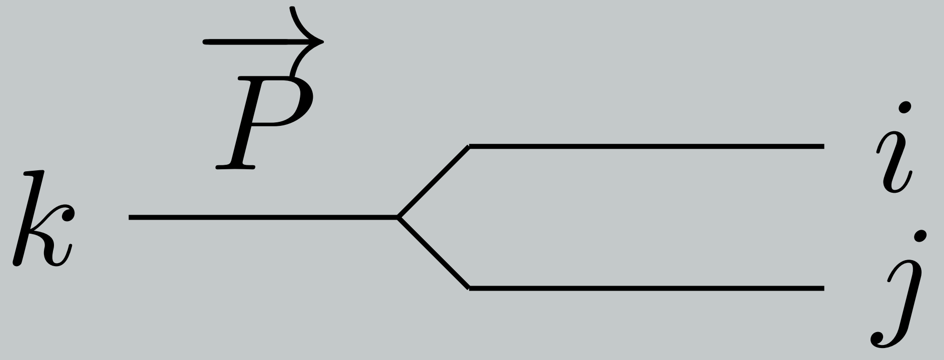


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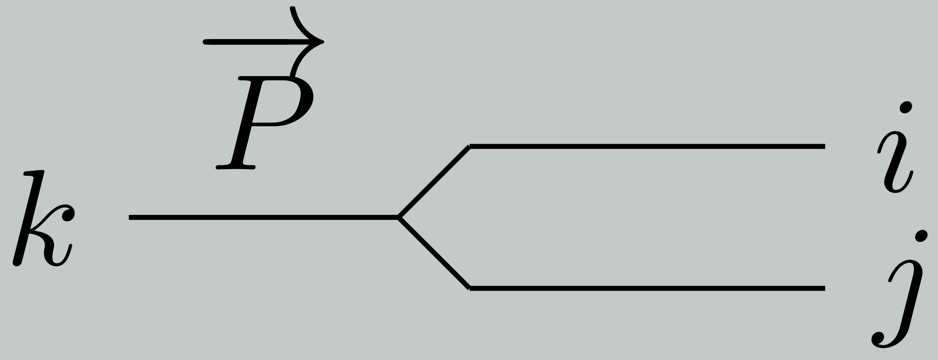


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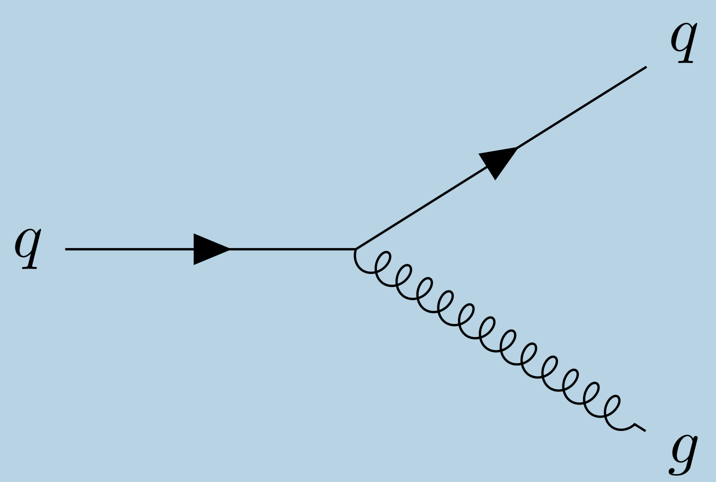


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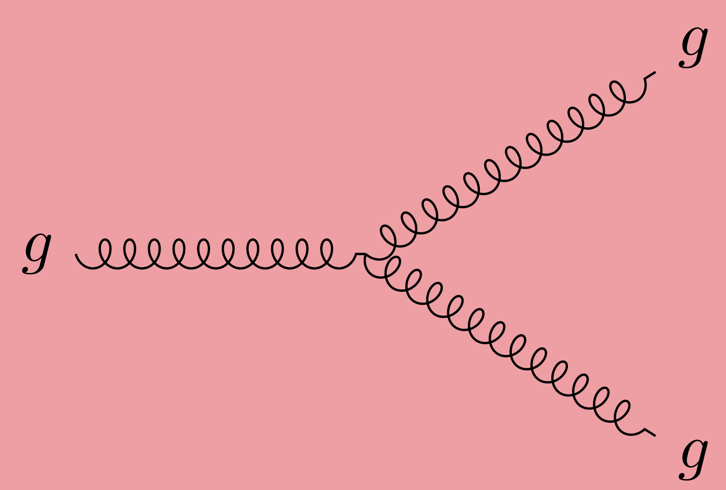


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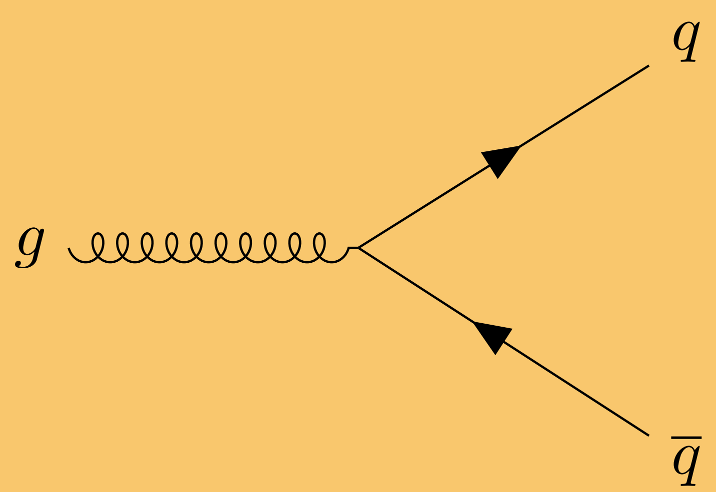


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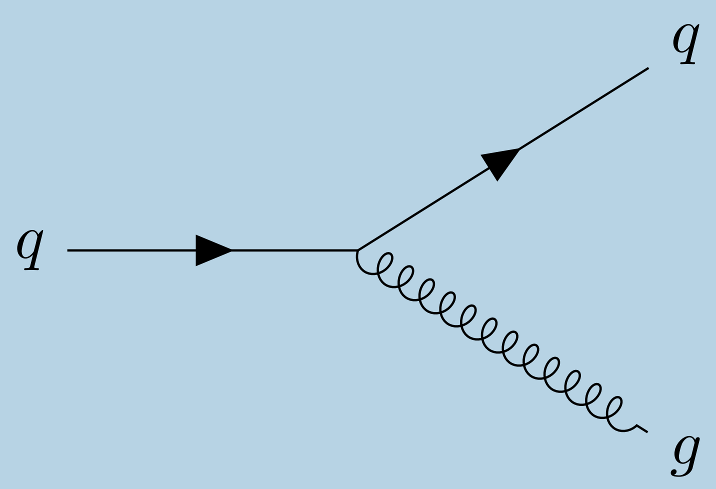


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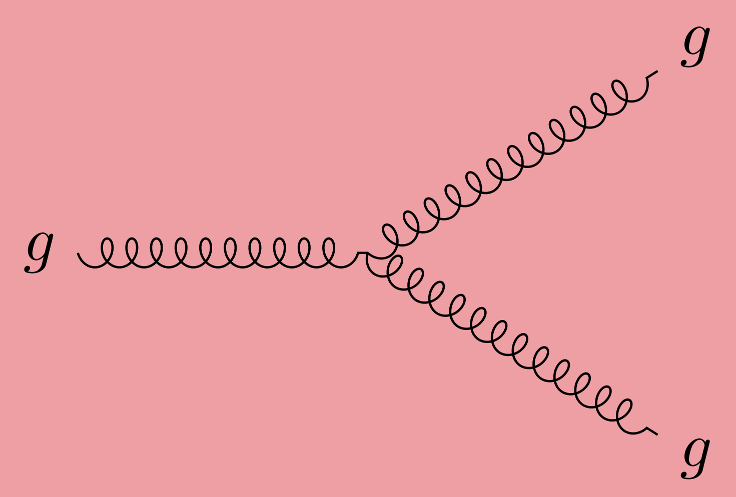


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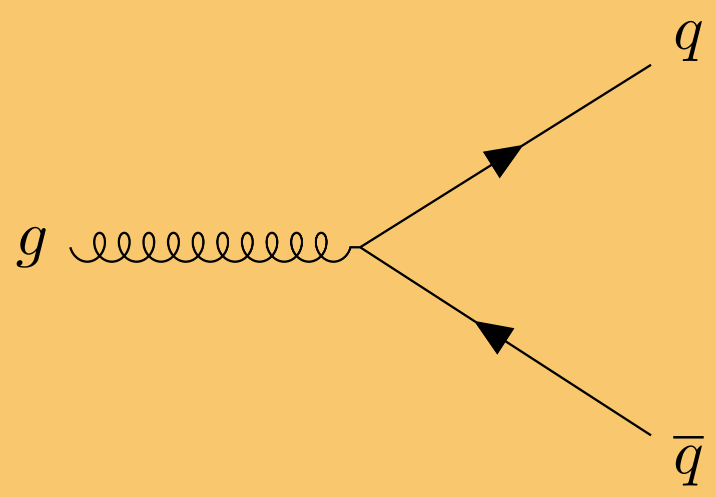


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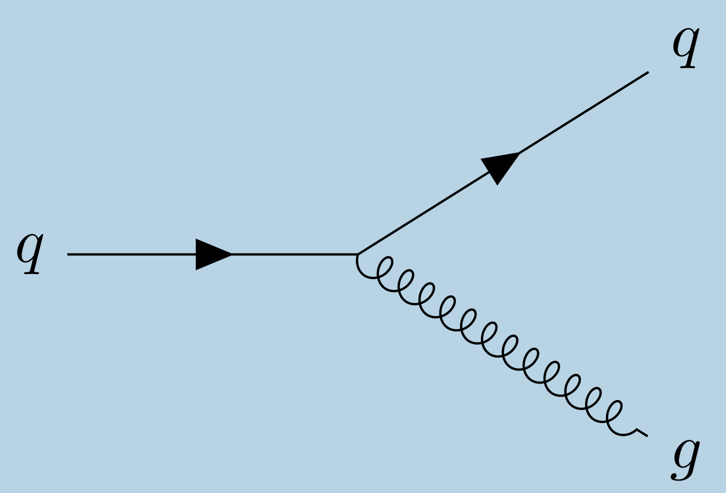


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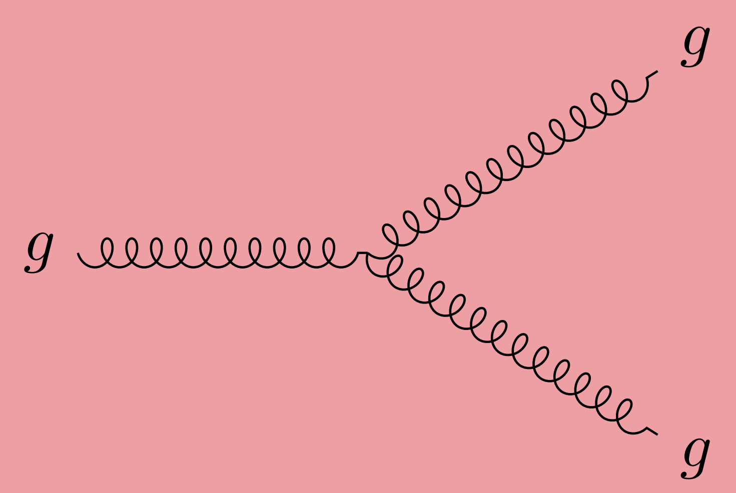


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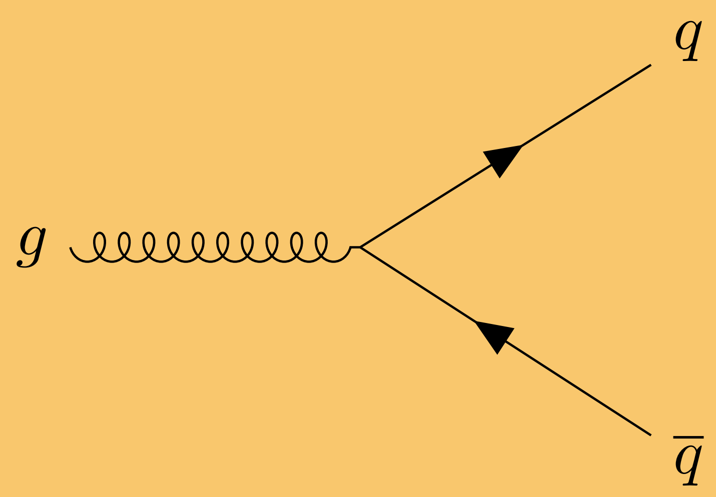


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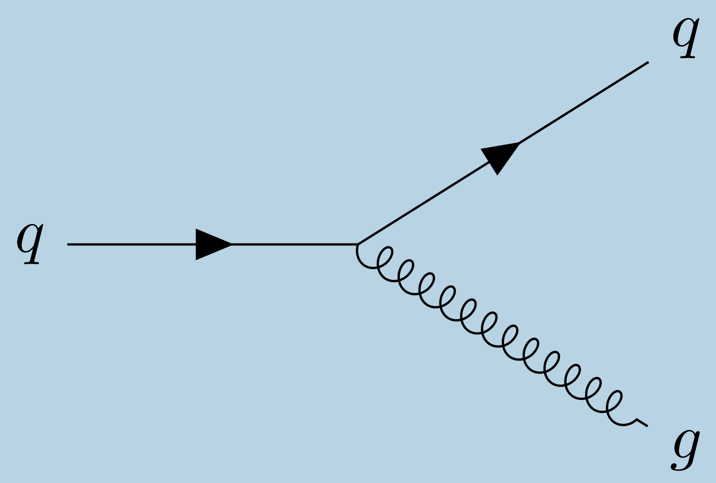


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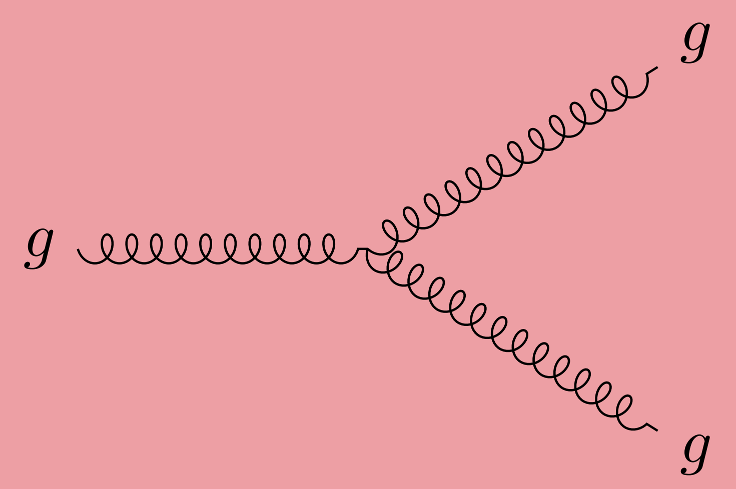


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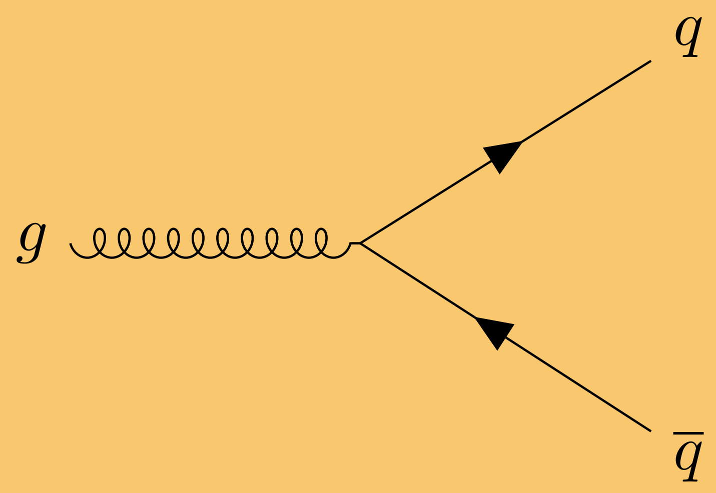


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- Combine Sudakov and splitting functions to get splitting probability for $k \to ij$ in a single shower step:

$$\text{Prob}_{k\to ij} = (1-\Delta_k)\times P_{k\to ij}(z)$$

# Quantum Walk approach to the parton shower

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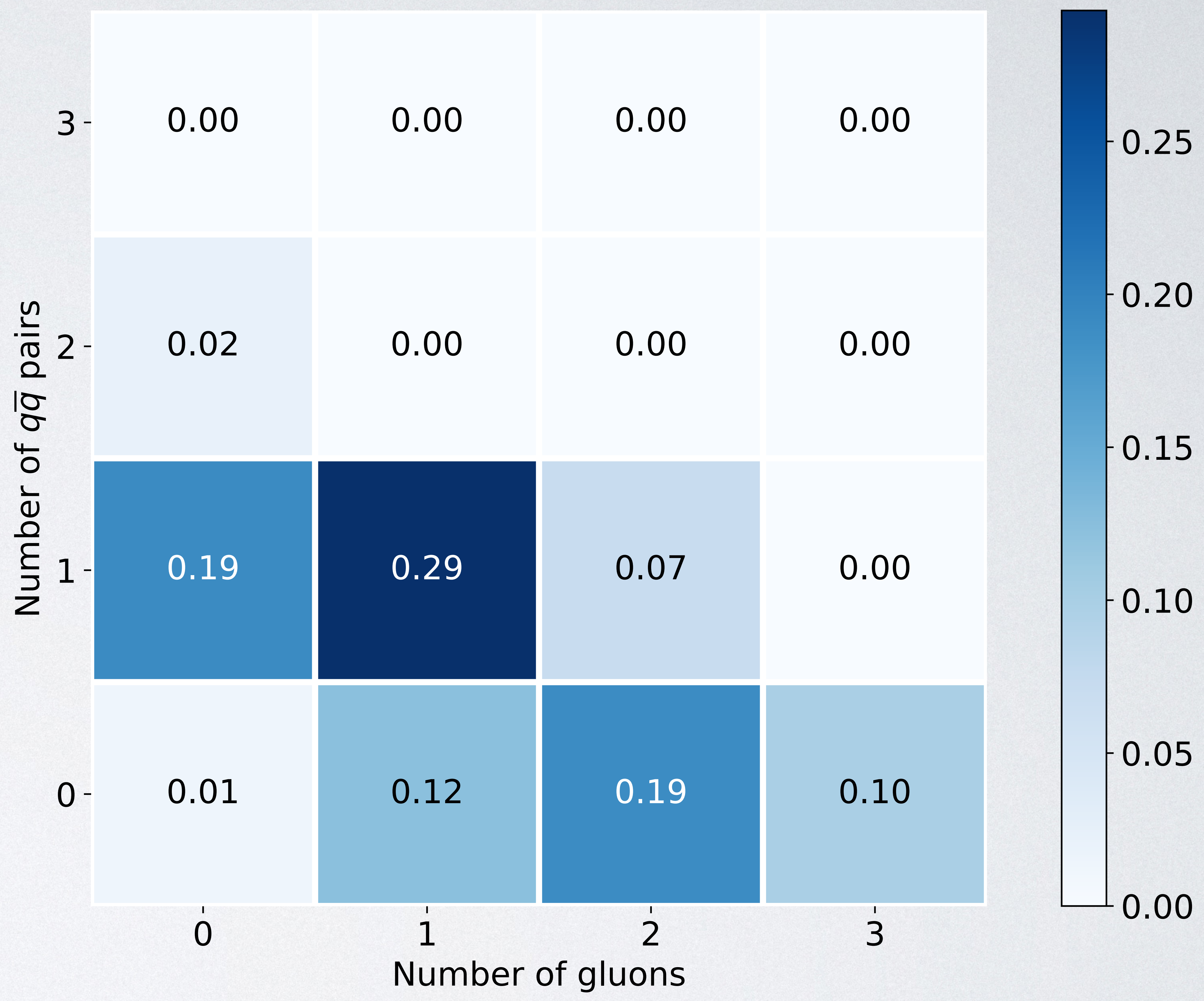
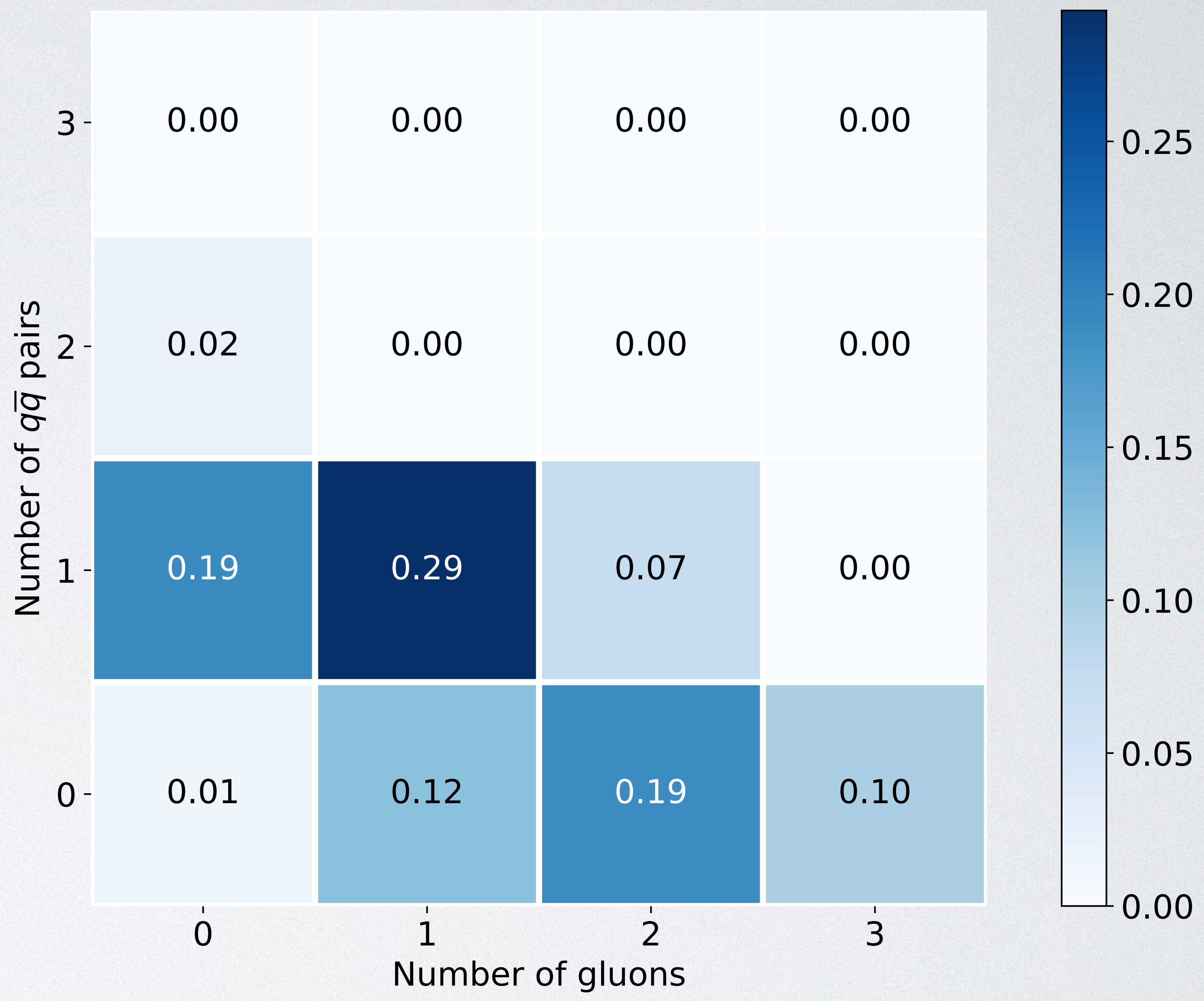


# Quantum Walk approach to the parton shower

- $\mathscr{H}_P$ : increase dimension of position space to 2D to allow for the simulation of a gluons and quarks

- $\mathscr{H}_C$ : increase dimension of coin space to accommodate for the collinear splitting probabilities



# Quantum Walk approach to the parton shower

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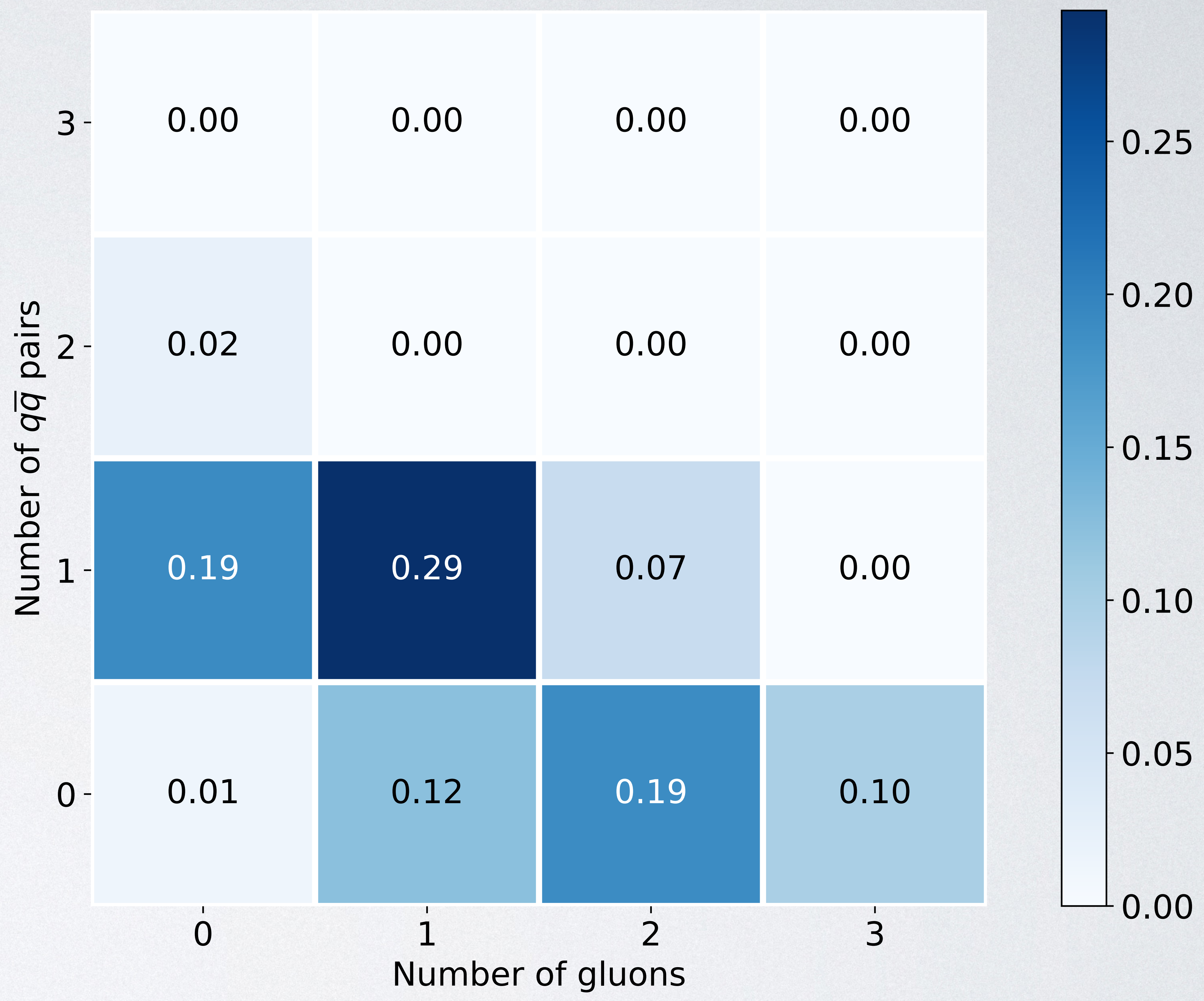

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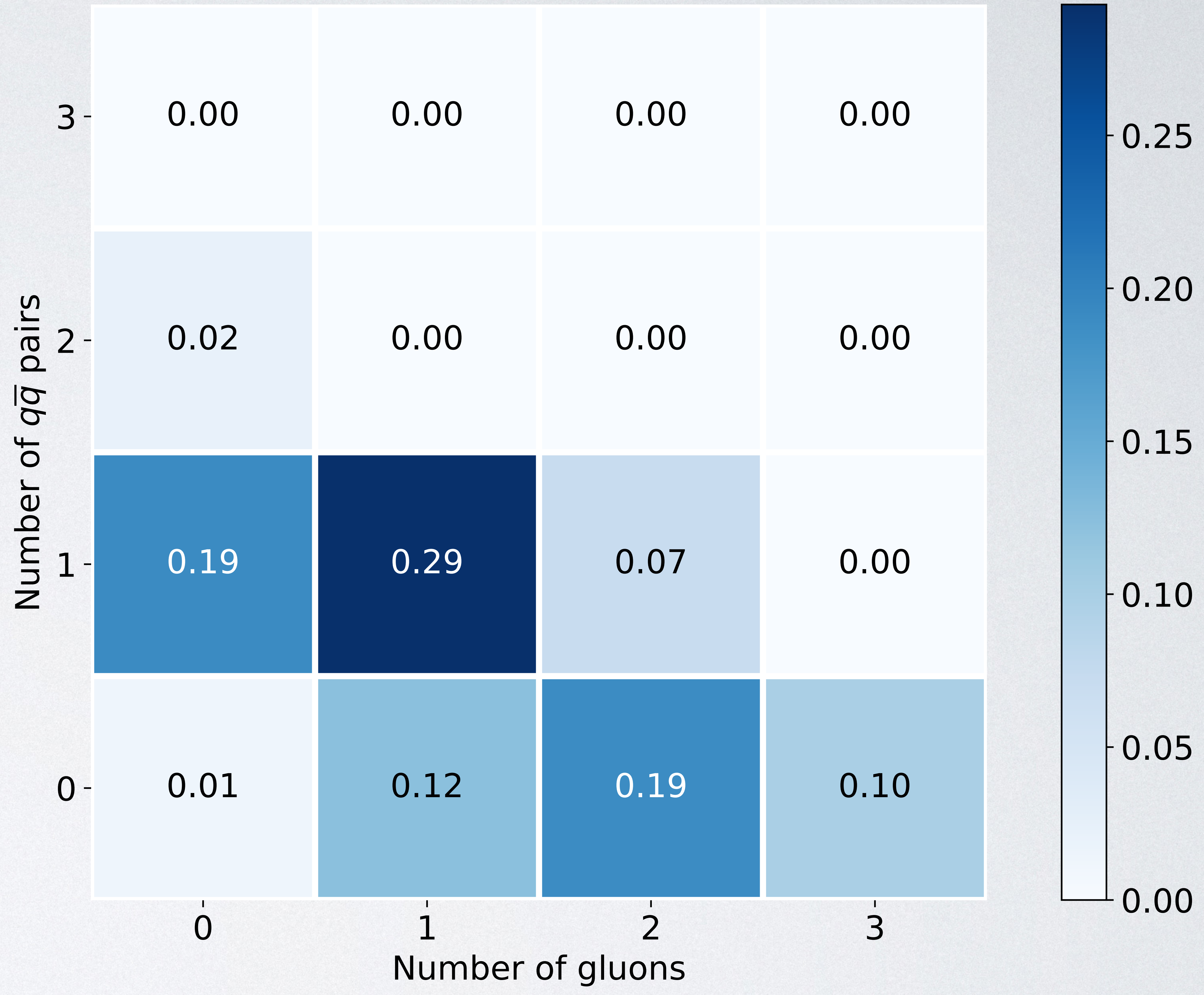

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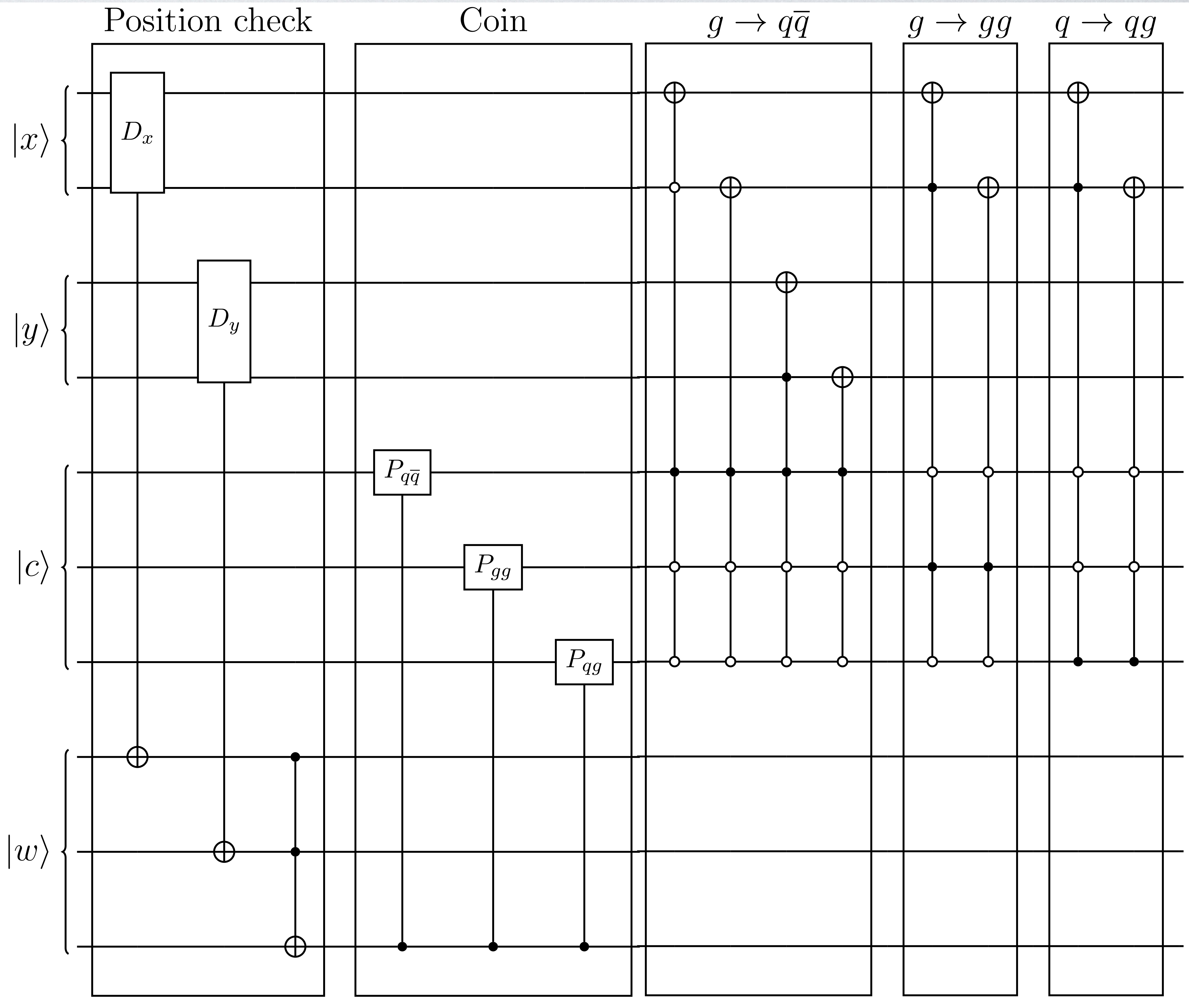
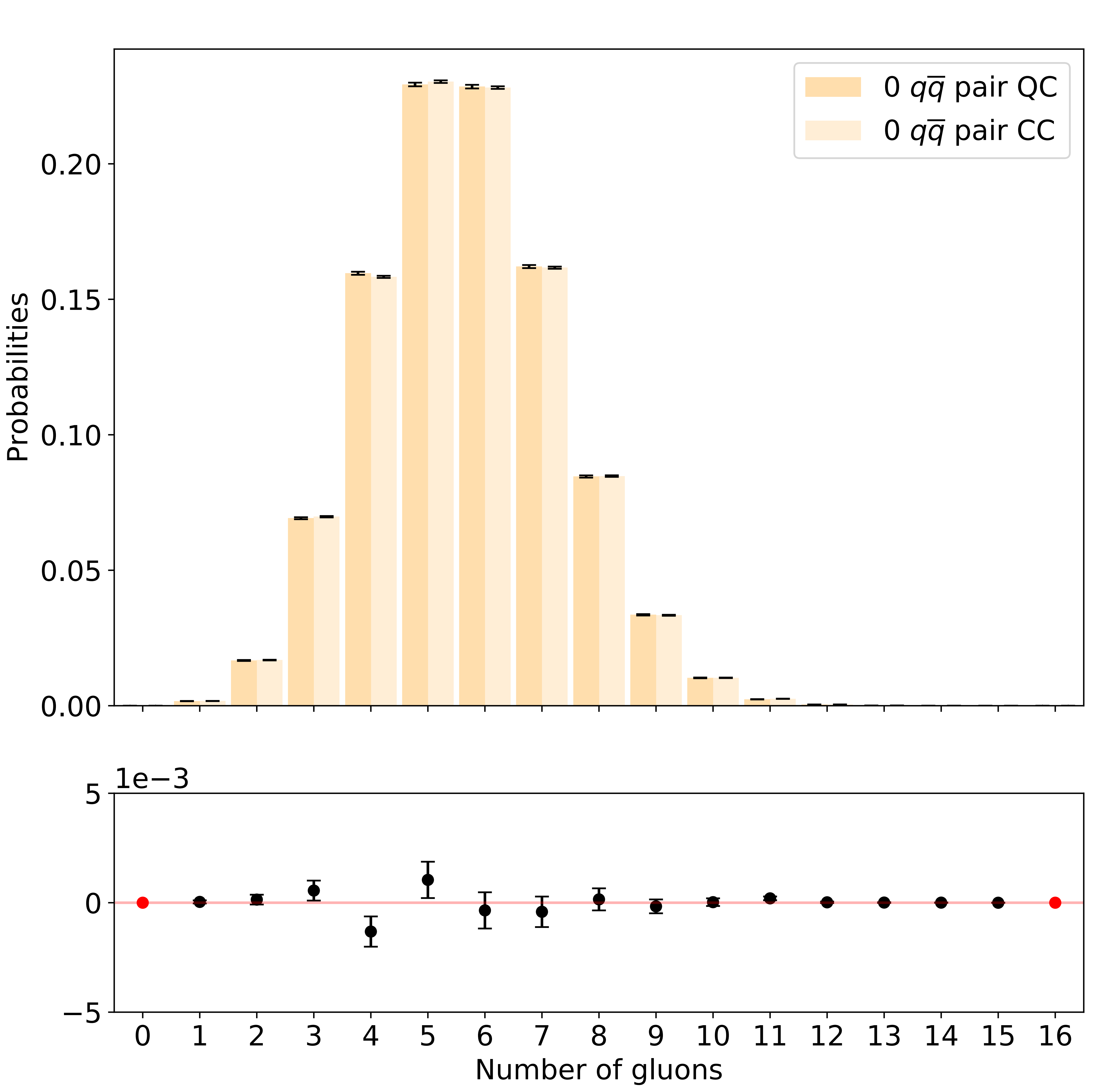


# Quantum Walk approach to the parton shower

| | Previous algorithm | QW |
|---|---|---|
| Qubits | 31 | 16 |
| Steps | 2 | 31 |
| Scaling, $n_q$ | $\frac{3N(N+1)}{2}^*$ | $2\log_2(N+1)+6$ |

*Scaling of a single register, not full circuit!

Previous - Phys. Rev. D 103, 076020 (2021)




arXiv: 2109.13975

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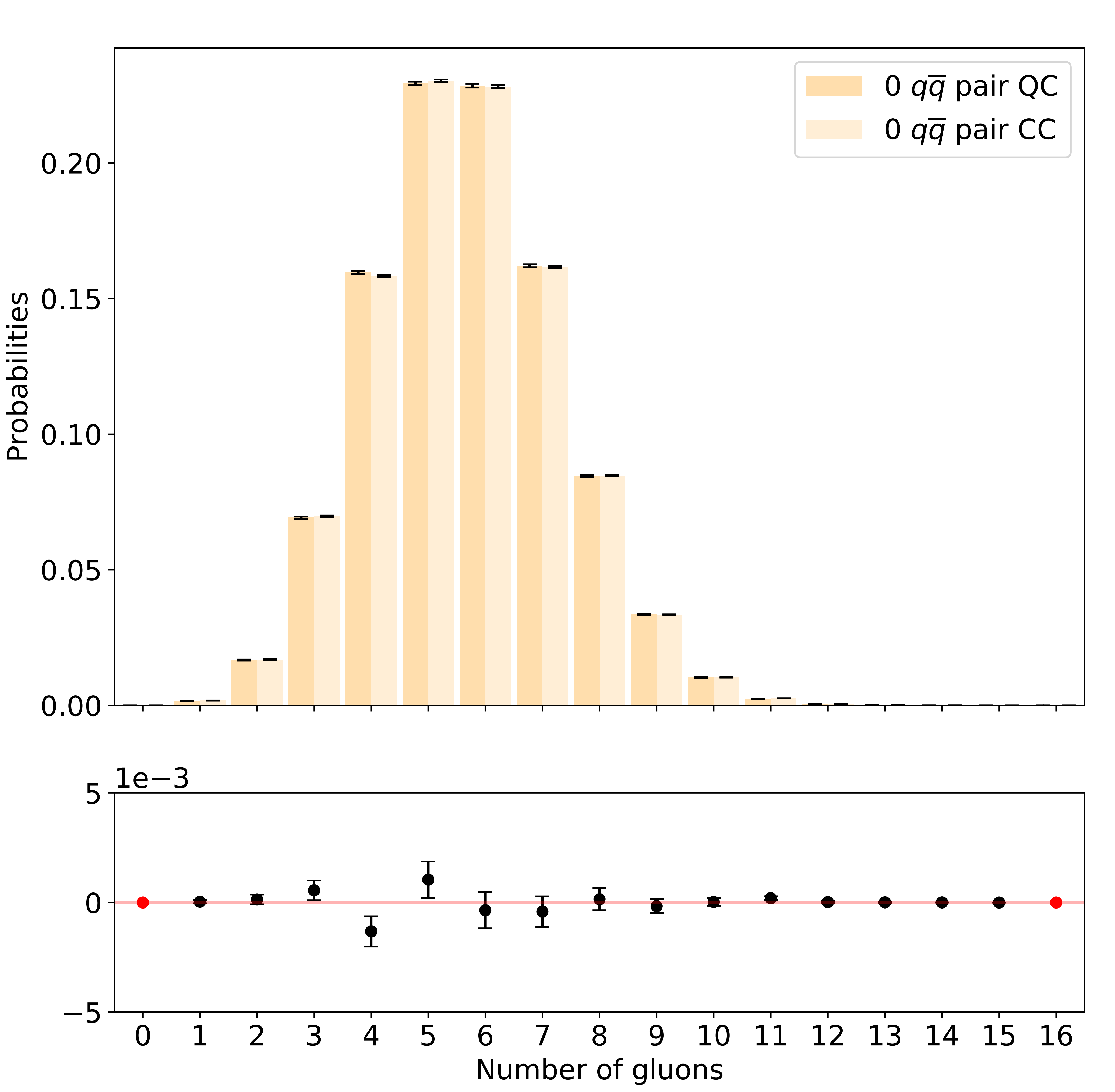


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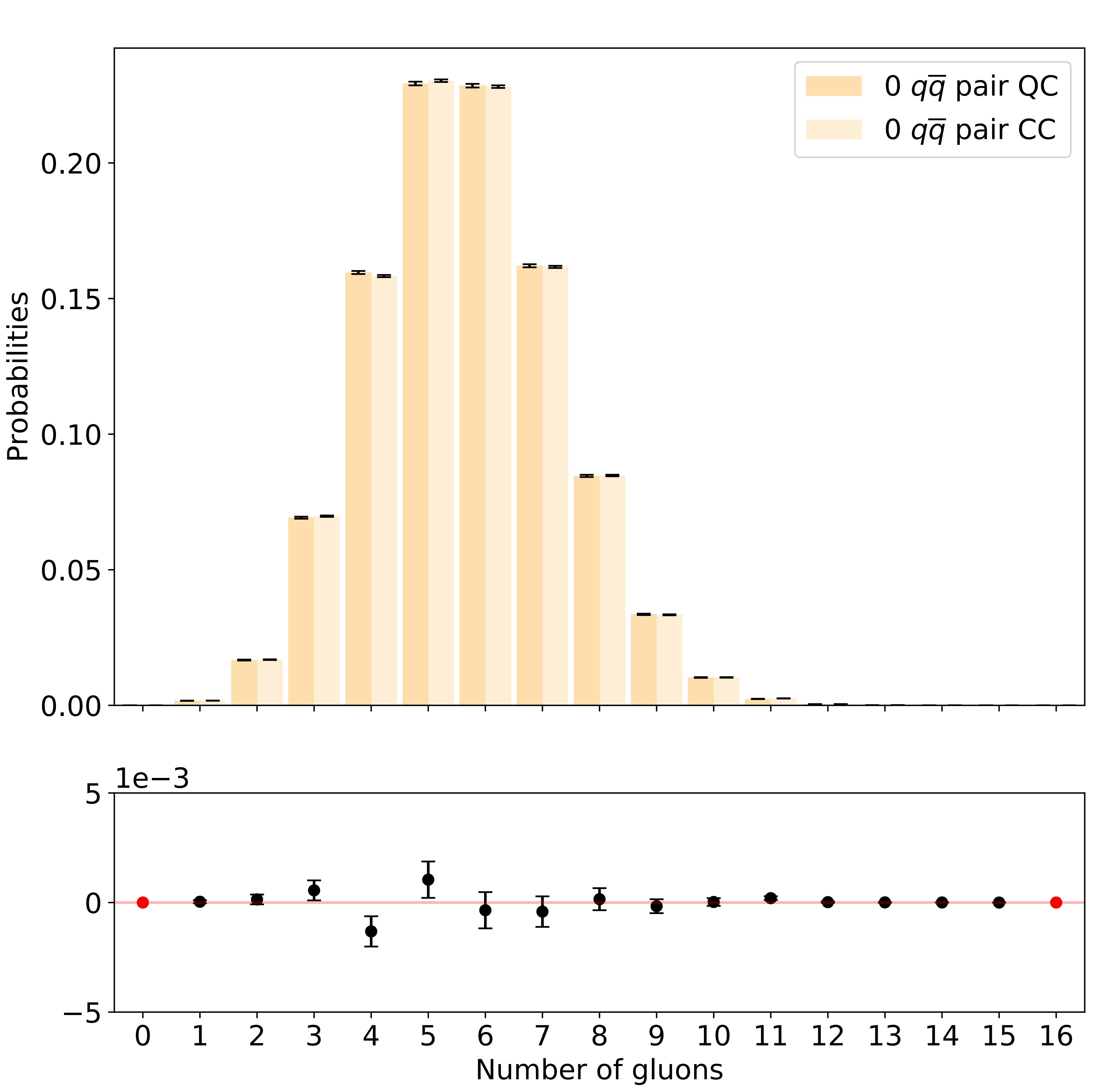


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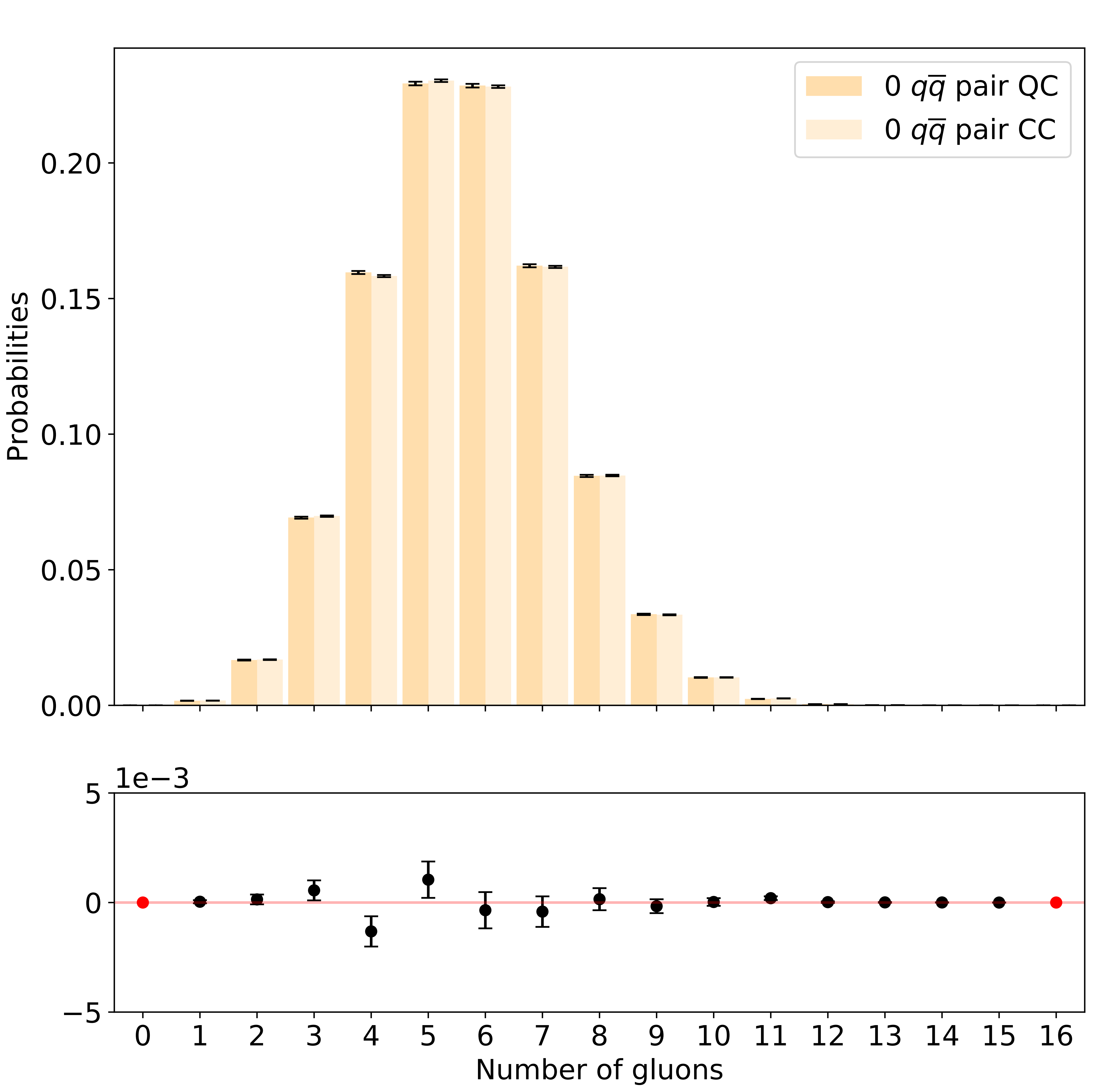


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# Summary and Looking to the Future

- Present a dedicated quantum algorithm for the simulation of parton showers in high energy collisions:

    - All shower histories calculated in full superposition constructing a final wavefunction containing all possible histories. Measurement projects out a physical quantity.

    - Reframing in the Quantum Walk framework vastly improves the efficiency of the quantum parton shower algorithm and offers a potential quadratic speed up compared to MCMC sampling

- **Looking to the future:** the introduction of kinematics to the algorithm will be a large step forward in the realism of the algorithm, with the potential of comparison to real data


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# Thank you

# Quantum Walk approach to the parton shower - A Simple Shower

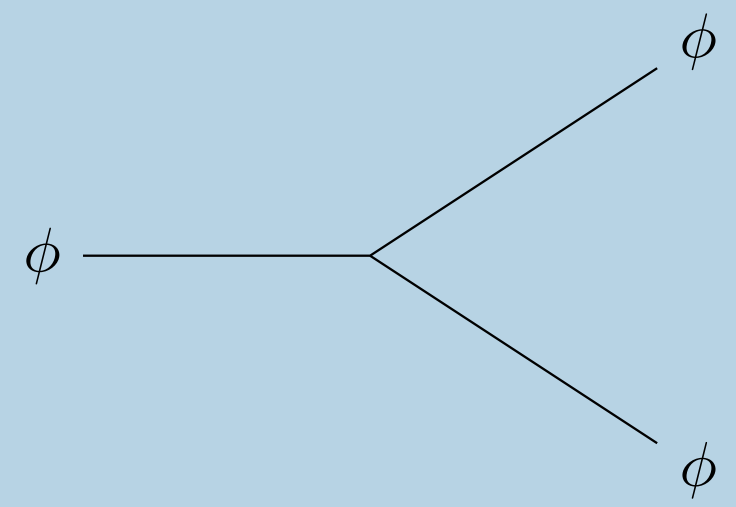
- Consider a simple shower with a single particle type $\phi$

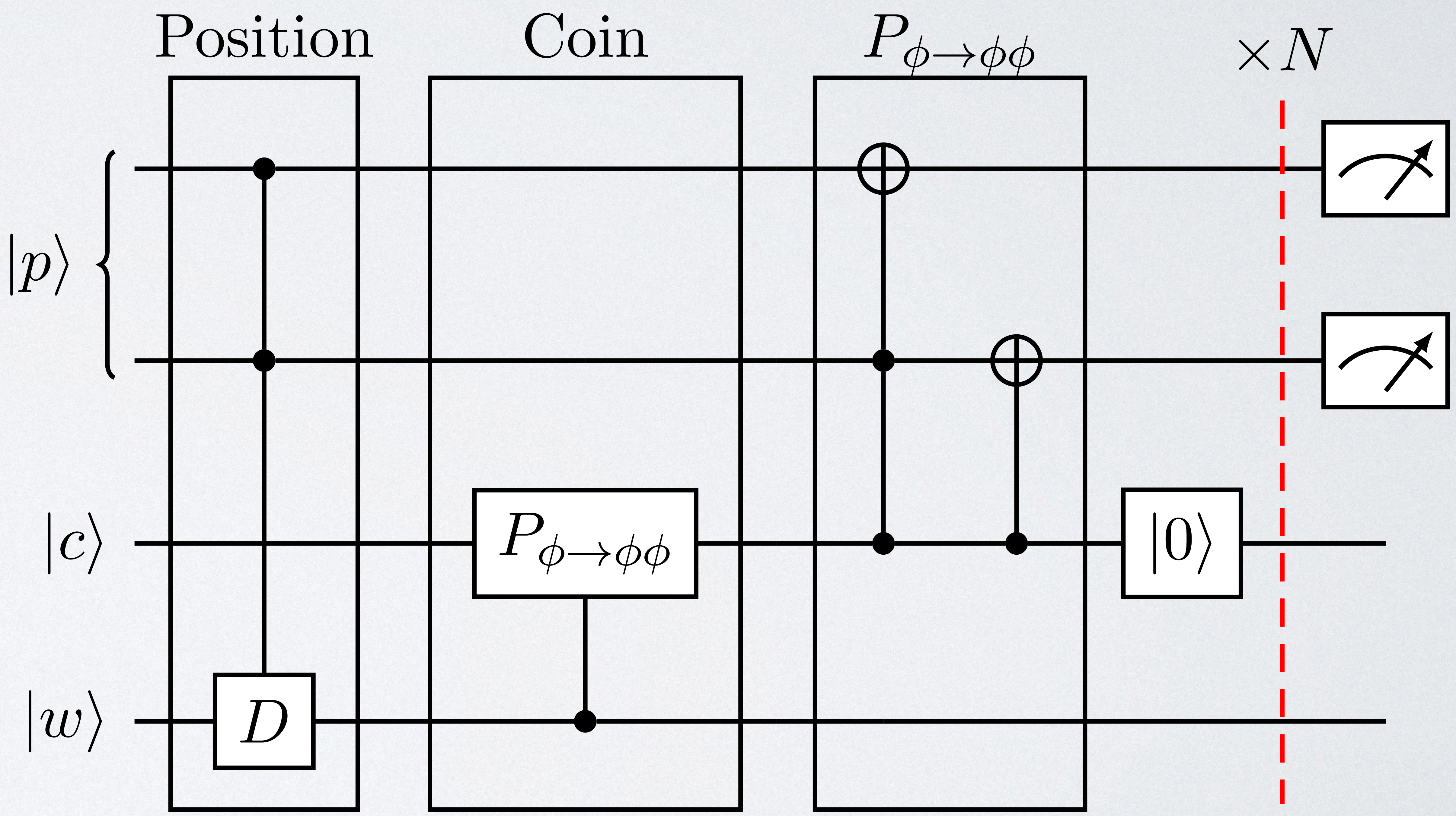
- $\mathscr{H}_c$ : Here we alter the coin operation to reflect the splitting probability $P_{\phi\to\phi\phi}$

- $\mathscr{H}_p$ : The walker position space now reflects the number of $\phi$ particles present in the shower

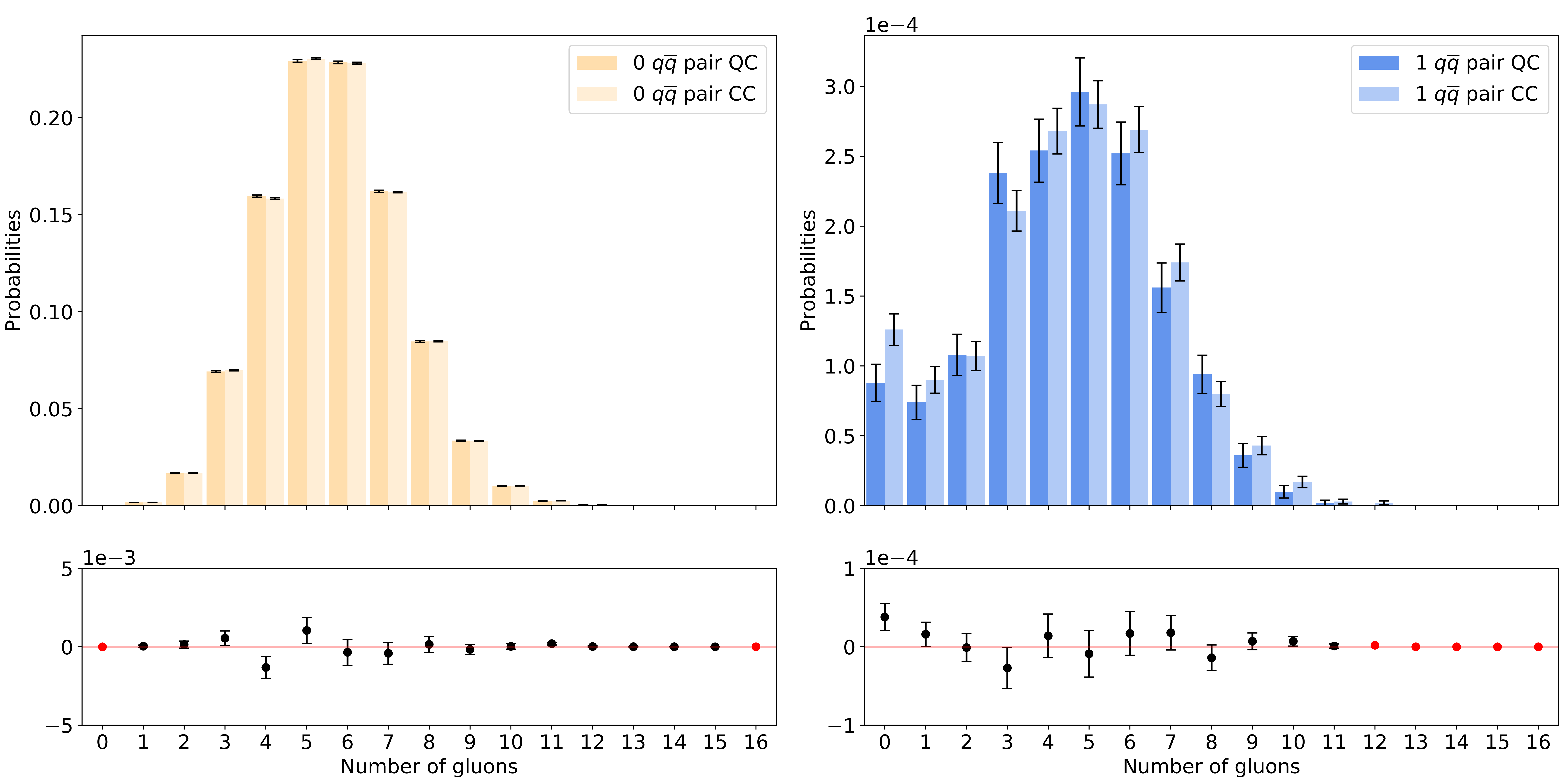
- The shift operation only increases the position of the walker, as only $\phi \to \phi\phi$ splittings

$$\phi \to \phi\phi : P_{\phi\to\phi\phi}$$



# Quantum Walk approach to the parton shower - Results

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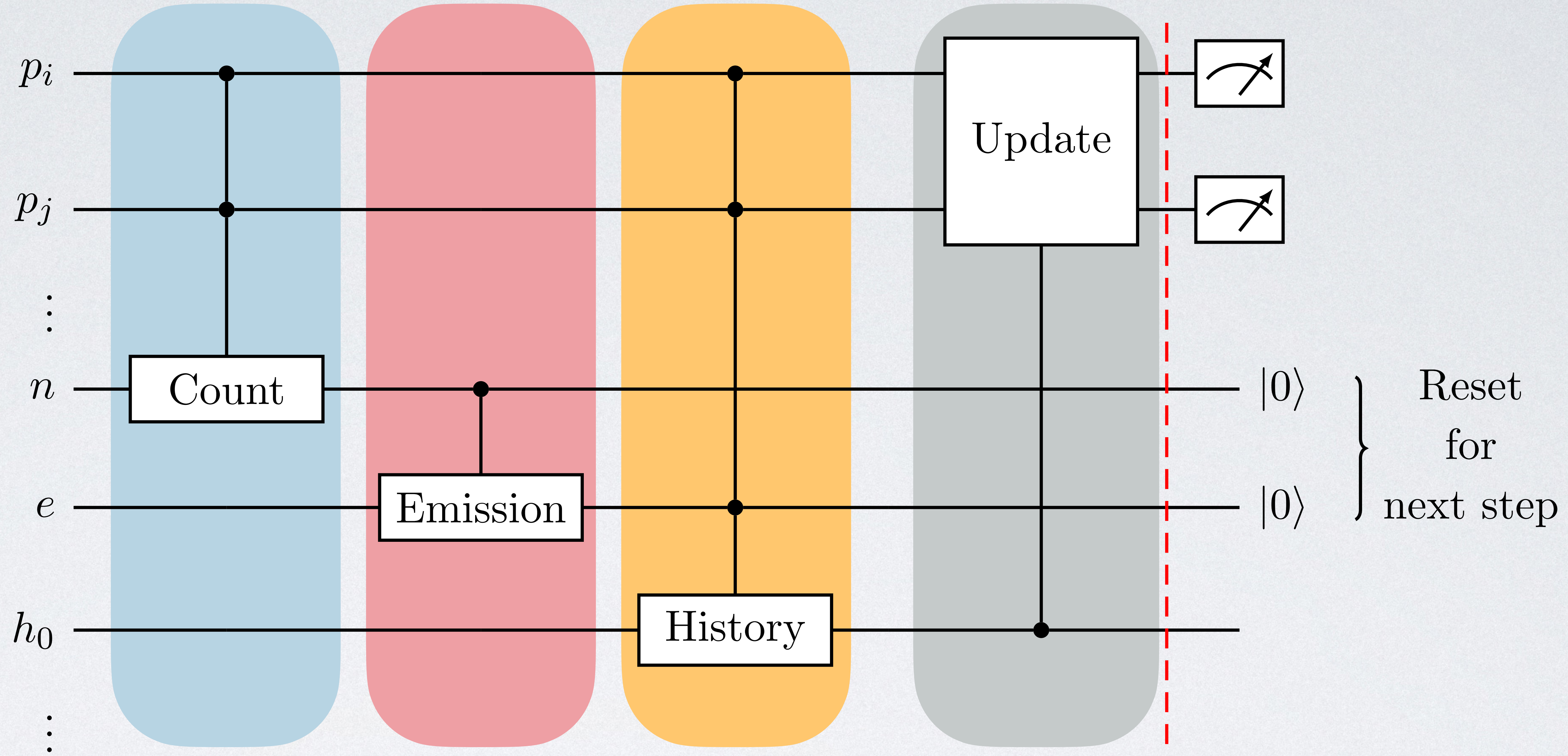
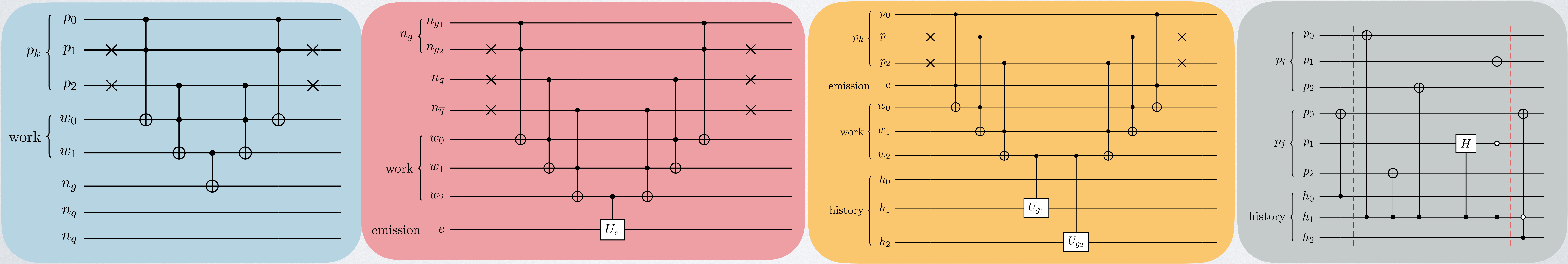
# Markov Chain parton shower implementation

Previous algorithm:




Builds on Phys. Rev. Lett. 126, 062001 (2021)



# Measurement

- Measurement of an arbitrary qubit system, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, is represented by the projection onto the $|0\rangle$ and $|1\rangle$ state, defining the projection operators $P_0 = |0\rangle\langle 0|$ and $P_1 = |1\rangle\langle 1|$.

- The probability of measuring the $|0\rangle$ state:

$$\text{Prob}(|0\rangle) = \text{Tr}(P_0|\psi\rangle\langle\psi|) = \langle\psi|P_0|\psi\rangle = |\alpha|^2$$

- Qubits are measured in this Projection-Valued Measurement regime and so the final state of the qubit is altered by the measurement. If the qubit is measured in the $|0\rangle$ state, then the final qubit state is:

$$|\psi\rangle \leftarrow \frac{P_0|\psi\rangle}{\sqrt{\langle\psi|P_0|\psi\rangle}} = |0\rangle$$

# Looking to the Future of Quantum Computers

- We are on the brink of a 'quantum revolution' - IBM on track to exceed 1000 qubits by 2023

- Quantum Walks have long been conjectured to give a quadratic speed up in the mixing time of Markov Chains

- Quadratic speed up has been proven for several quantum MCMC algorithms