

Gravitational Wave

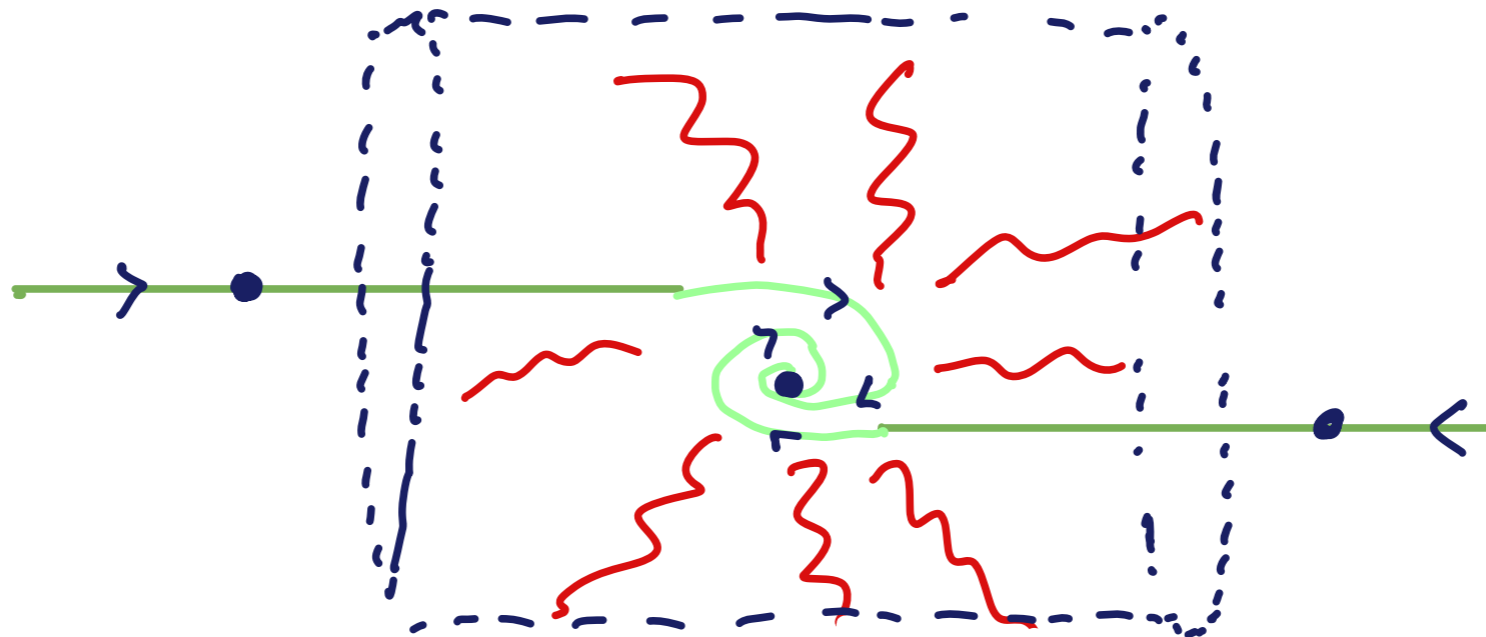
Astronomy: An HEP view

Ira Rothstein (CMU)

Lake Louise Winter Institute 2022

The HEP paradigm In Gravitational Wave Astronomy

☺ 10^{-18} Steradians
of coverage



- Black holes are a like elementary particles (no hair) but with a continuous spectrum
- Black holes binary are remarkably similar to quarkonia! In fact one can use QCD to calculate the measured signal

Outline of the talk

- 1) The experiment in context (a little history)**
- 2) The experiment: Sensitivity and Signal discrimination, event classification**
- 3) Science Objectives**
- 4) Current Catalog: What we have learned so far**
- 5) Effective Field Theory approach**
- 6) Future out look of GW as probe of “Fundamental” Physics**

1) A Brief history of Gravitational Wave Astronomy

- 1921: Gravitational Waves predicted by Einstein
- 1962: Idea of building an interferometer with hanging mirrors proposed by Gertsenshtein and Pustovoi.
- 1972: Weiss completed the invention of the interferometric gravitational wave detector by identifying all the fundamental noise sources, and conceiving ways to deal with each of them, and by showing that — at least in principle — these ways could lead to detector sensitivities good enough to detect waves from astrophysical sources.
- 1980 US NSF approves prototype detectors.
- 1989 Cal-Tech+ MIT submit a proposal for LIGO. Two detectors working in tandem. At this point all the technology had been proven with sensitivity sufficient for “possible” detection. With the plan for an advanced detector with “new” technology with a “high probability” of detection.
- 1991 US congress approves funding for LIGO with one detector in Livingston Louisiana and the other in Hannover Washington.
- 1993 VIRGO Project approved by CNRS and INFN. (Pisa Italy)
- 2005-2010 LIGO/VIRGO place “astro-physically interesting” bounds on gravitational wave sources, but did not find any waves.
- 2015 Advanced LIGO improves upon sensitivity by more than an order of magnitude.

On September 14, 2015, a detection with a large signal to noise ratio, 24. The measured waveform matched the predictions of Einstein’s general relativity for waves from two black holes spiraling together, colliding and merging.

Predictions for the Rates of Compact Binary Coalescences Observable by Ground-based Gravitational-wave Detectors

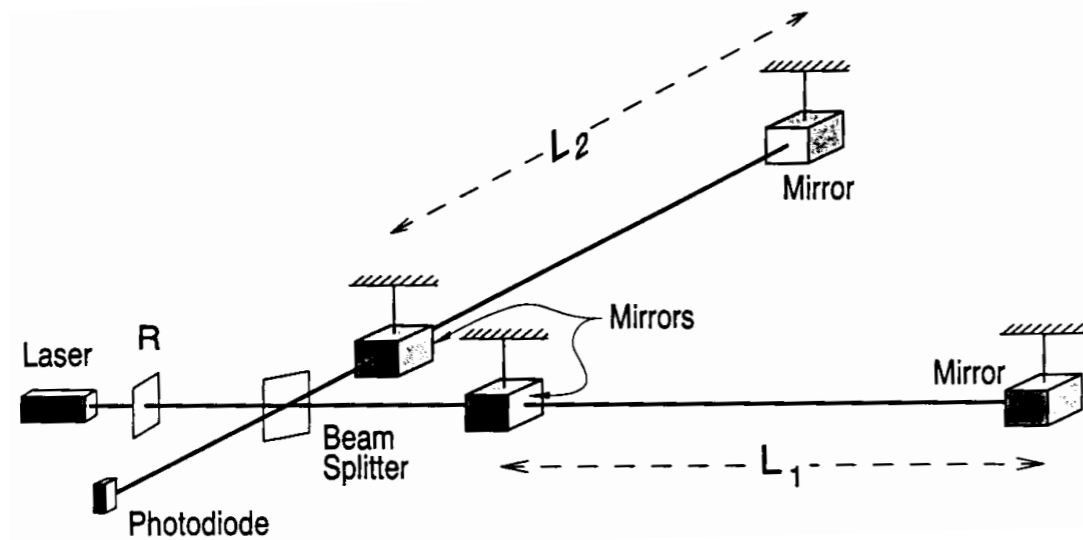
(Dated: March 25, 2010; DCC number: ligo-p0900125)

We present an up-to-date, comprehensive summary of the rates for all types of compact binary coalescence sources detectable by the Initial and Advanced versions of the ground-based gravitational-wave detectors LIGO and Virgo. Astrophysical estimates for compact-binary coalescence rates depend on a number of assumptions and unknown model parameters, and are still uncertain. The most confident among these estimates are the rate predictions for coalescing binary neutron stars which are based on extrapolations from observed binary pulsars in our Galaxy. These yield a likely coalescence rate of 100 Myr^{-1} per Milky Way Equivalent Galaxy (MWEG), although the rate could plausibly range from $1 \text{ Myr}^{-1} \text{ MWEG}^{-1}$ to $1000 \text{ Myr}^{-1} \text{ MWEG}^{-1}$ [1]. We convert coalescence rates into detection rates based on data from the LIGO S5 and Virgo VSR2 science runs and projected sensitivities for our Advanced detectors. Using the detector sensitivities derived from these data, we find a likely detection rate of 0.02 per year for Initial LIGO-Virgo interferometers, with a plausible range between 2×10^{-4} and 0.2 per year. The likely binary neutron-star detection rate for the Advanced LIGO-Virgo network increases to 40 events per year, with a range between 0.4 and 400 per year.

TABLE I: Rate statement terminology.

Abbreviation	Rate statement	Physical significance
$R_{\text{max}}, \dot{N}_{\text{max}}^a$	Upper limit	Rates should be no higher than...
$R_{\text{high}}, \dot{N}_{\text{high}}$	Plausible optimistic estimate	Rates could reasonably be as high as...
$R_{\text{re}}, \dot{N}_{\text{re}}$	Realistic estimate	Rates are likely to be...
$R_{\text{low}}, \dot{N}_{\text{low}}$	Plausible pessimistic estimate	Rates could reasonably be as low as...

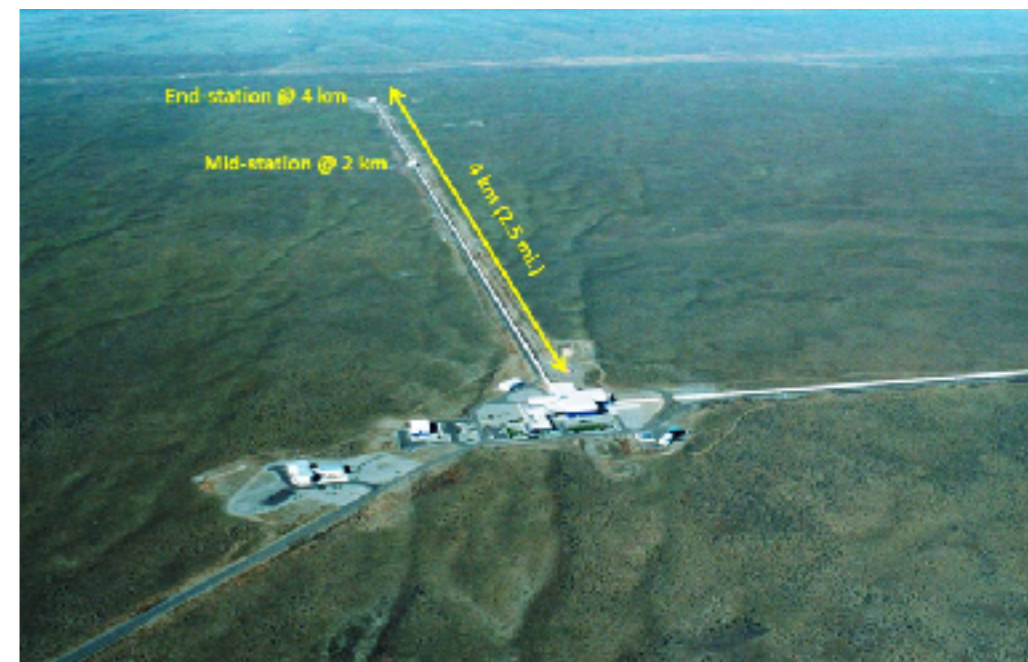
2) A little about the detectors



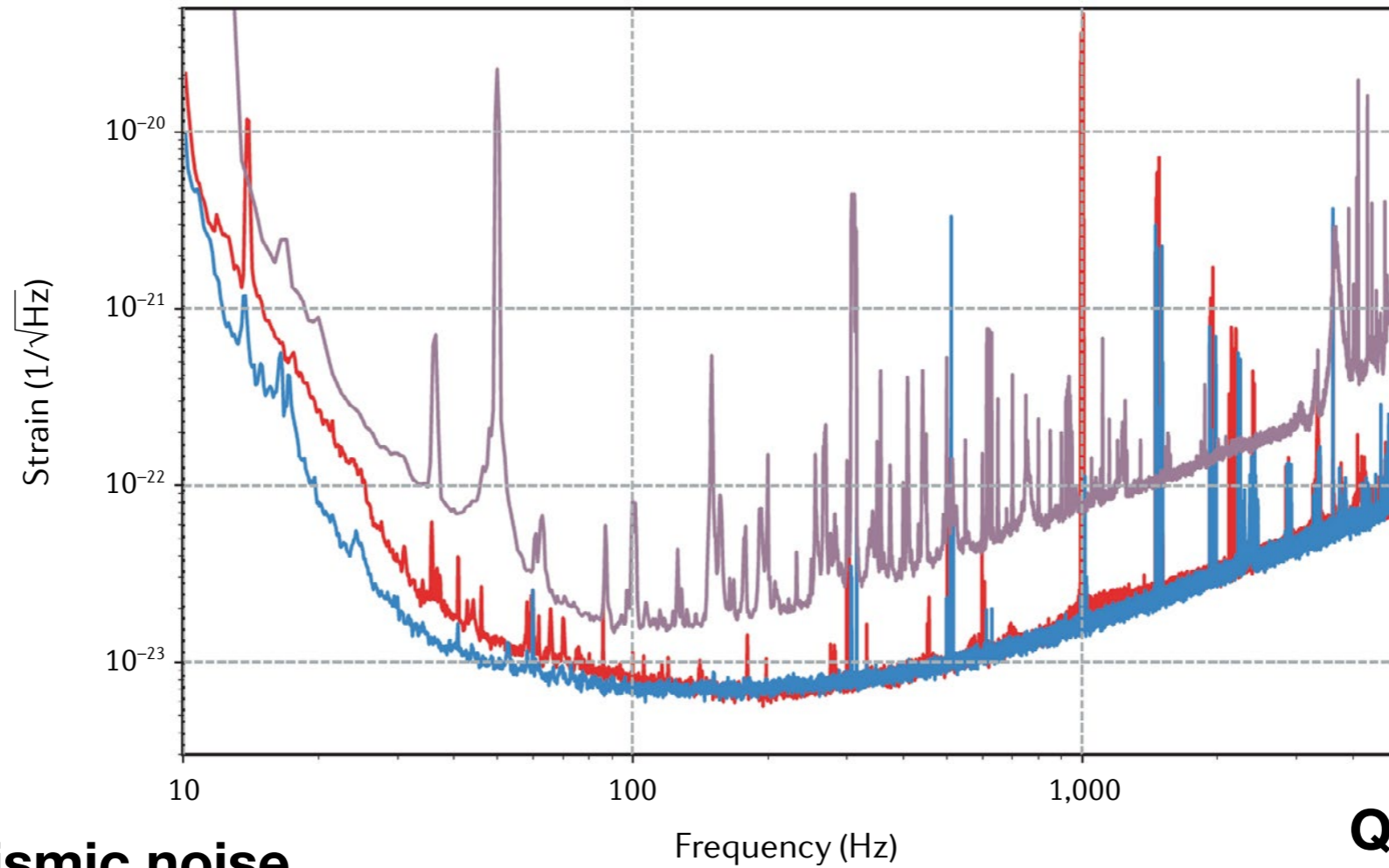
$$h = \Delta L/L \sim 10^{-21} - 10^{-22}$$

**Mirror suspension system
reduces effects of ground
motion by a factor of 10^{12} .**

Each interferometer can resolve GW's in a frequency band 1- 10^4 Hz



Strain Sensitivity



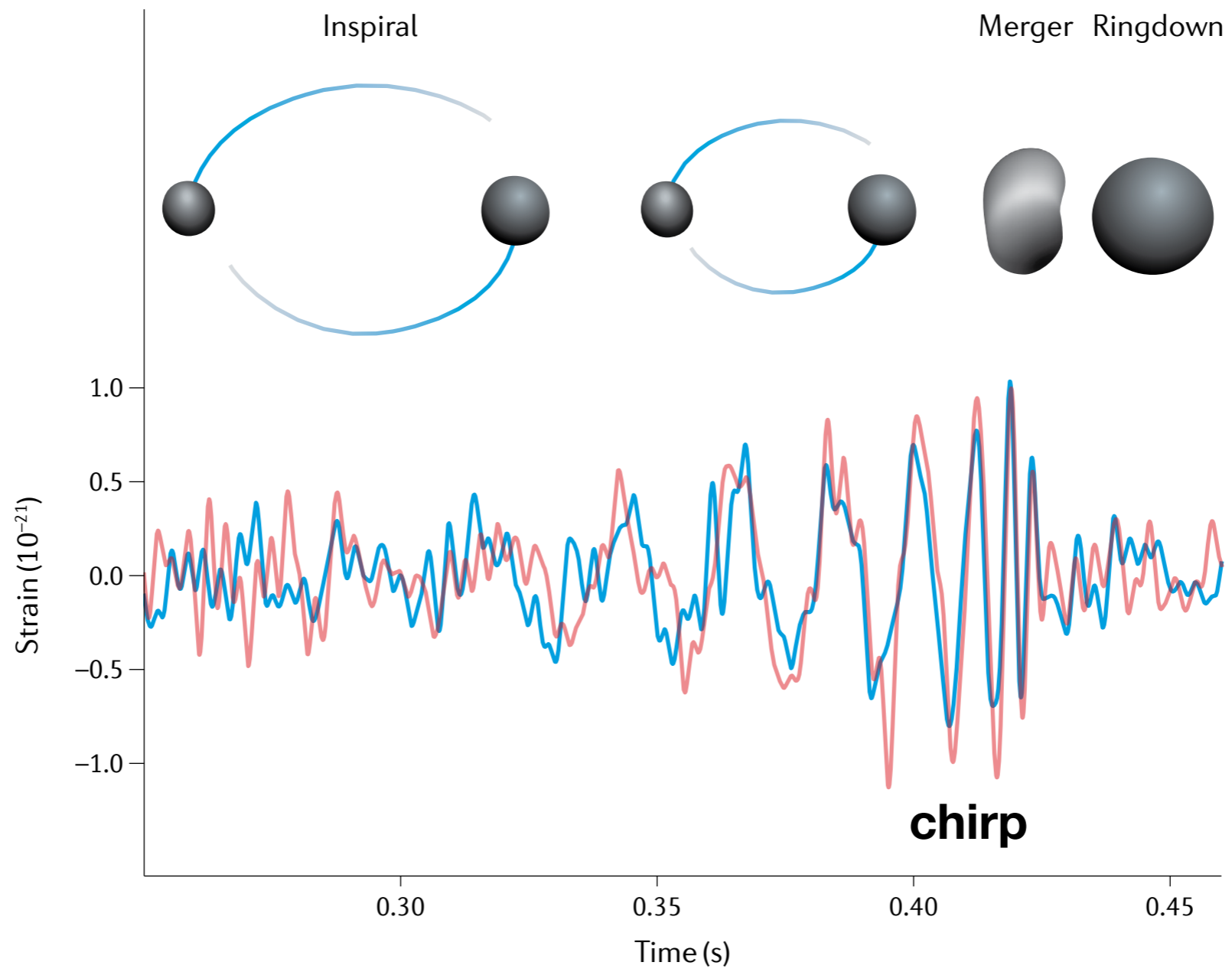
**seismic noise,
light scattering**

(this is the soft side, will be extended)

Quantum noise
(shot noise in
photodetectors, E+M
vacuum fluctuations..)

(insurmountable?)

The Signal



Classification of Events and What LIGO/VIRGO Captures in the Bucket

Frequency at last stable orbit (plunge)

$$f_{ISCO} = 2.2 \left(\frac{M_{\odot}}{m_T} \right) kHz$$

Terrestrial interferometers (TI) can only see stellar mass black holes and neutron stars

Wave Amplitude:

$$h \sim \frac{G}{r} \ddot{M}$$

Limits fiducial volume Ad-LIGO: 300 Mpc

M : Second mass moment of binary

$$N_{cycles} = 1.6 \times 10^4 \left(\frac{10Hz}{f_{min}} \right)^{5/3} \left(\frac{1.2M_{\odot}}{M_c} \right)^{5/3} .$$
$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

(“Chirp mass”)

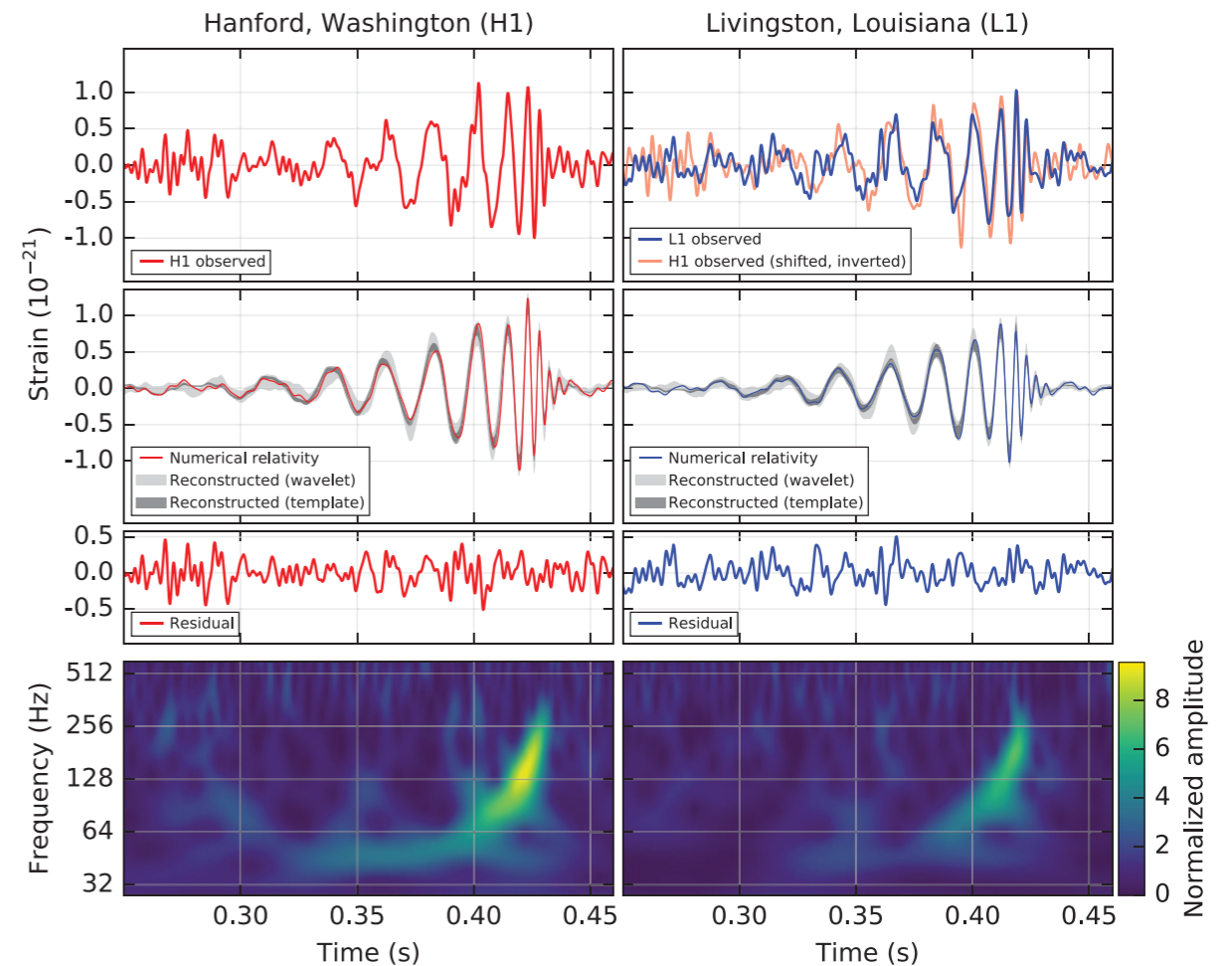
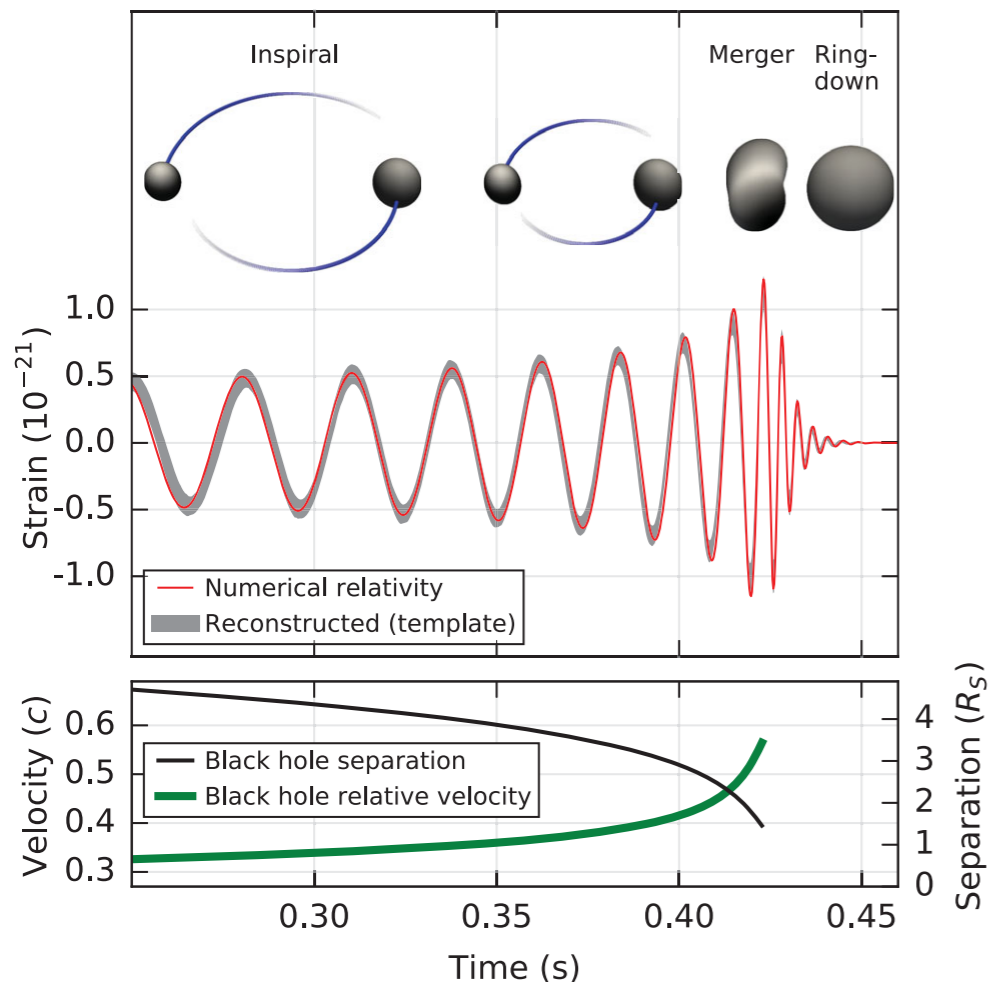
**For LIGO-VIRGO the detector can track thousands of cycles,
necessitates precise theoretical prediction**

Reconstructing Event Parameters

Leading order (“easy”): Masses, Spins, Orbital plane, distance, direction

Next to Leading order: Internal Dynamics (gravitational susceptibilities)

Rely about theoretical predictions to produce **bank of templates**
(more on this later)



Parameter Estimation via Matched Filtering

**Detector
Output**

$$s(t) = h(t) + n(t)$$

$$n(t)$$

**Stochastic
Noise**

$$\frac{1}{T} \int_0^T dt s(t)h(t) = \frac{1}{T} \int_0^T dt h(t)^2 + \frac{1}{T} \int_0^T dt n(t)h(t)$$

Theoretical prediction

Washes out

Stochastic noise dominates the signal necessitating precision theoretical templates for parameter extractions

3) SCIENCE OBJECTIVES

Black holes masses depend upon two factors:

OBSERVABLE (indirect)

BH Mass spectrum:

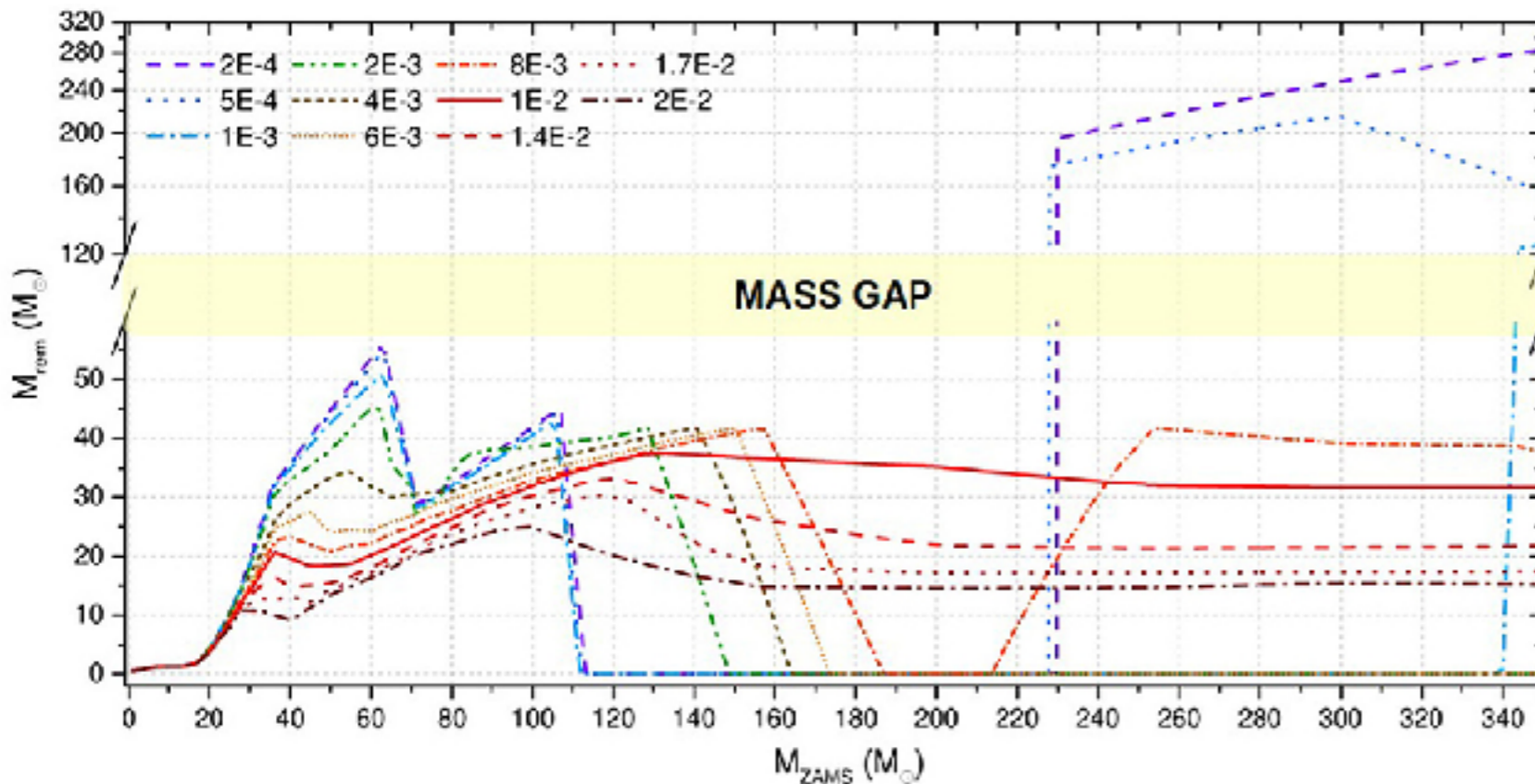
1) The metallicity (Z)
(stellar winds)

$$\dot{m} \sim Z^\beta$$

2) The progenitor mass
(mechanism of core collapse)

$M > 40M_\odot$ (No SN direct to BH, leads to larger BH's)

various metallicities



zero-age main-sequence (ZAMS) mass of the progenitor star (M_{ZAMS})

$$M_{core}^{He} > 60M_\odot$$

$$T > 2m_e$$

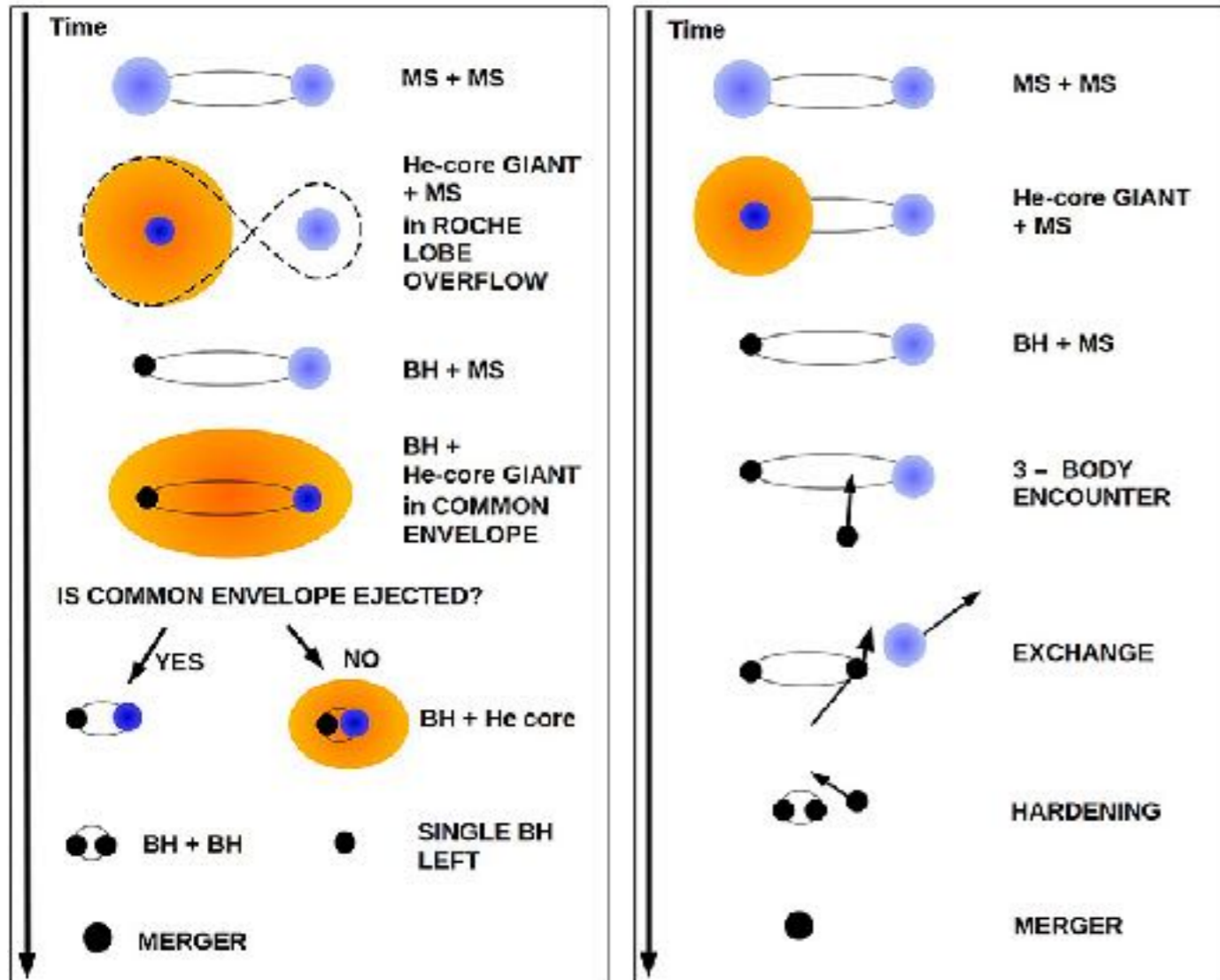
“pair instability mass gap”

(Softens equation of state leading to instability and explosion with no remnant)

OBSERVABLE

The rate of BH-BH inspirals:

This number is a function of the stellar population at a given redshift as well as the capture rate. The plunge time must be less than the age of universe. This is theoretical challenge as in isolation this is highly improbable.



Two mechanisms which can hasten decay are "common core formation" (LHS) and "dynamical formation" (RHS)

OBSERVABLE

The NS mass spectrum:

In general it is very difficult if not impossible to distinguish between a BH-NS and BH-BH merger without an **accompanying E+M signal**. NS-NS mergers are much more likely to have E+M signal.

The mass spectrum is sensitive to the Equation Of State(EOS)

$$1.9M_{\odot} < M_{NS} \leq 2.3M_{\odot}$$

At this point it is still unknown whether or not the EOS is “hadronic” or “quark-gluon”.

OBSERVABLE

Detailed shape of signal

Detailed shape of the signal contains information about the “microscopic physics”.

1) Tidal deformability depends upon EOS:

Induced quadrupole moment $Q \sim \chi \nabla \nabla h$

χ “Love Number”



2) Tests of GR in the strong field regime (testing modified gravity)

experiments⁹⁹⁻¹⁰¹. Nevertheless, there is a general consensus that GR is, at best, incomplete, representing an approximation to a more complete theory that cures some or all of its problems¹⁰². These issues include the loss of information down a black hole¹⁰³, which contradicts unitary evolution of physical states in quantum mechanics; the inevitability of spacetime singularities^{104,105}, for example, at the centre of a black hole where physical quantities such as the density and curvature of spacetime become infinitely large; a cosmological constant that is responsible for the late-time accelerated expansion of the Universe^{106,107}, whose value cannot be accounted for in the standard model of particle physics¹⁰⁸; and the lack of a viable formulation of quantum gravity, which might resolve all of these problems but has, so far, been elusive. These difficulties led to increased interest in searching for GR violations in observations in the hope that they will provide clues to an alternative theory of gravity.

**Nature 2020
`Roadmap:
Gravitational
Physics 2020-2030**

3) Dark Matter

ULDM:

**Could form halos extract mass
and angular momentum via super-radiance instability**

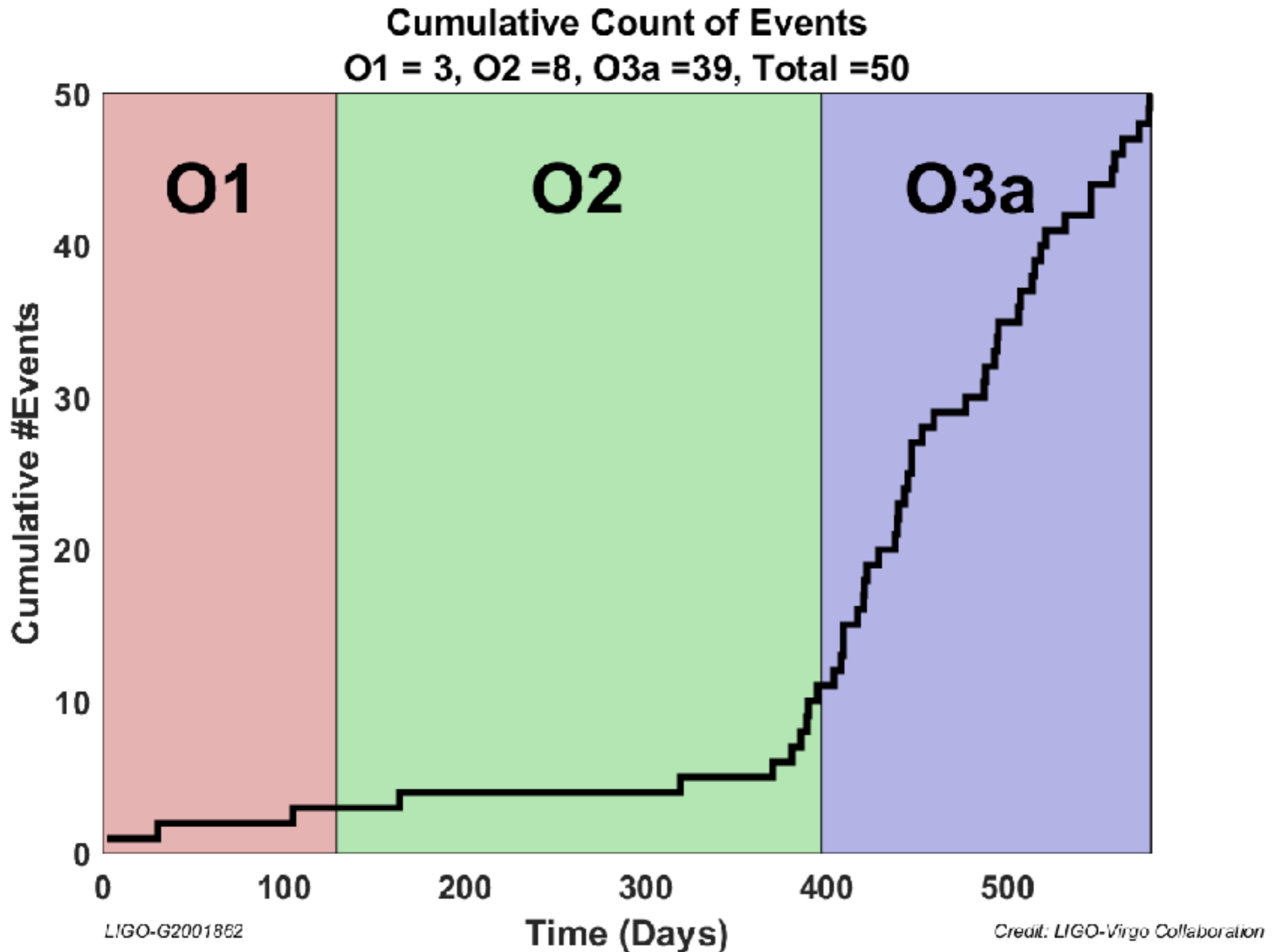
Heavy DM:

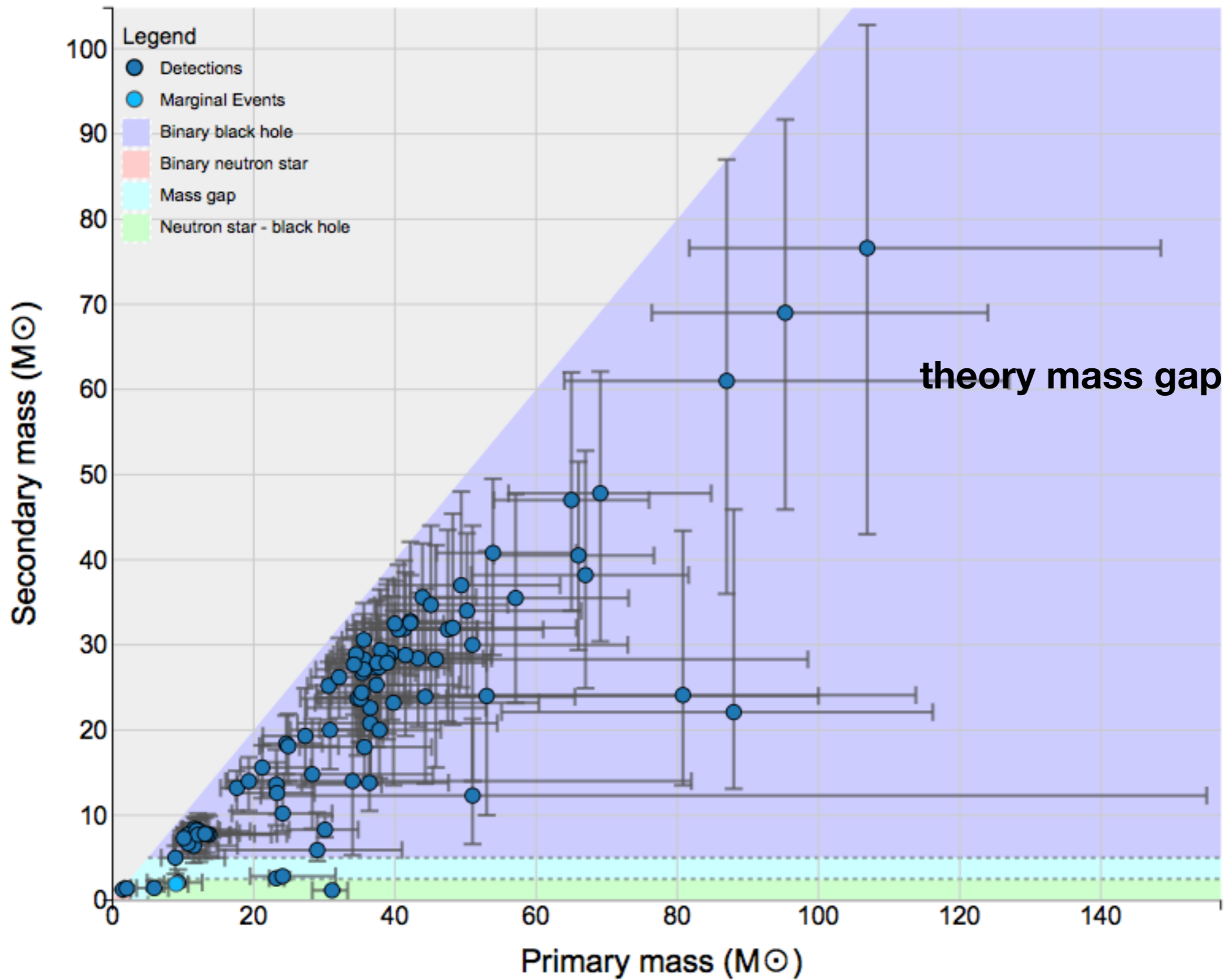
**DM stars could have distinctive EOS
(maximal parity violation?)**

4) Cosmology

**Measure the Hubble Parameter to within a few percent
with KAGRA and LIGO-India and EM observations.
Only relies on a standard siren of multi-messenger event.**

4) What have we learned so far?





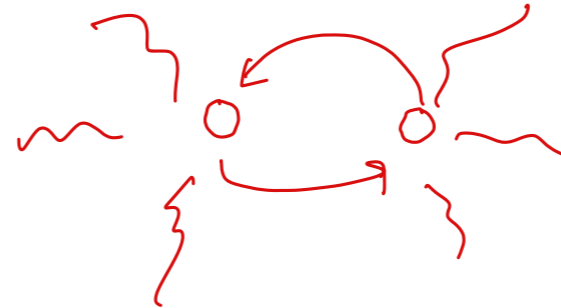
- **Behavior of black holes is entirely consistent with GR. GW travel at the speed of light, up to one part in 10^{-15} . GW are tensor (spin 2) particles.**
- **First Definitive Link between BH-NS mergers and gamma ray bursts. Birth of multi-messenger Astronomy.**
- **First conclusive proof that BH-NS mergers create heavy elements via r-process nucleosynthesis (spectroscopy)**
- **Learned a lot about BH formation via the mass spectrum and event rate. Second generation BH fill the mass gap?
Higher metallicity environments?**

5) *Effective Field Theory approach*

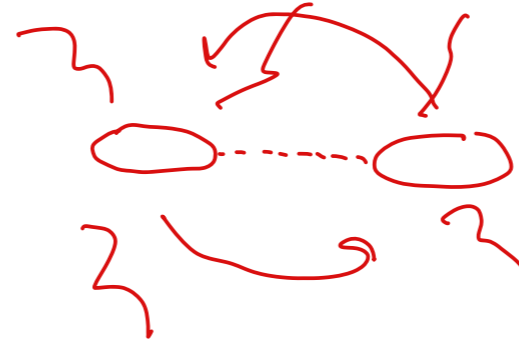
(Goldberger/
IZR 2006)

“The problem of Motion”

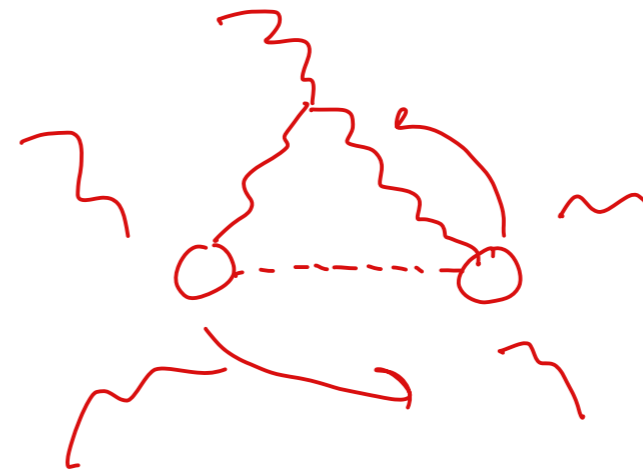
Radiation causes inspiral



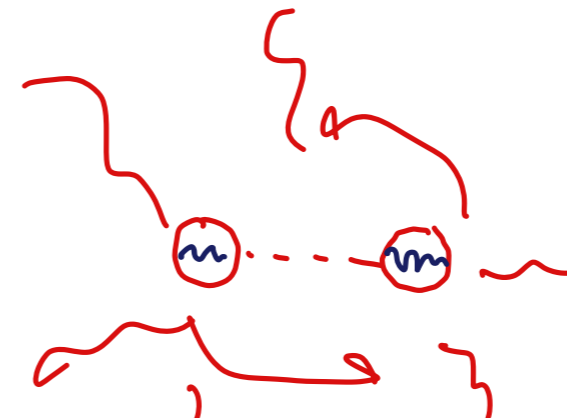
Tidal effects induce multipole moments which then effects the force between objects



Back reacts changes the radiation pattern and the inspiral rate.

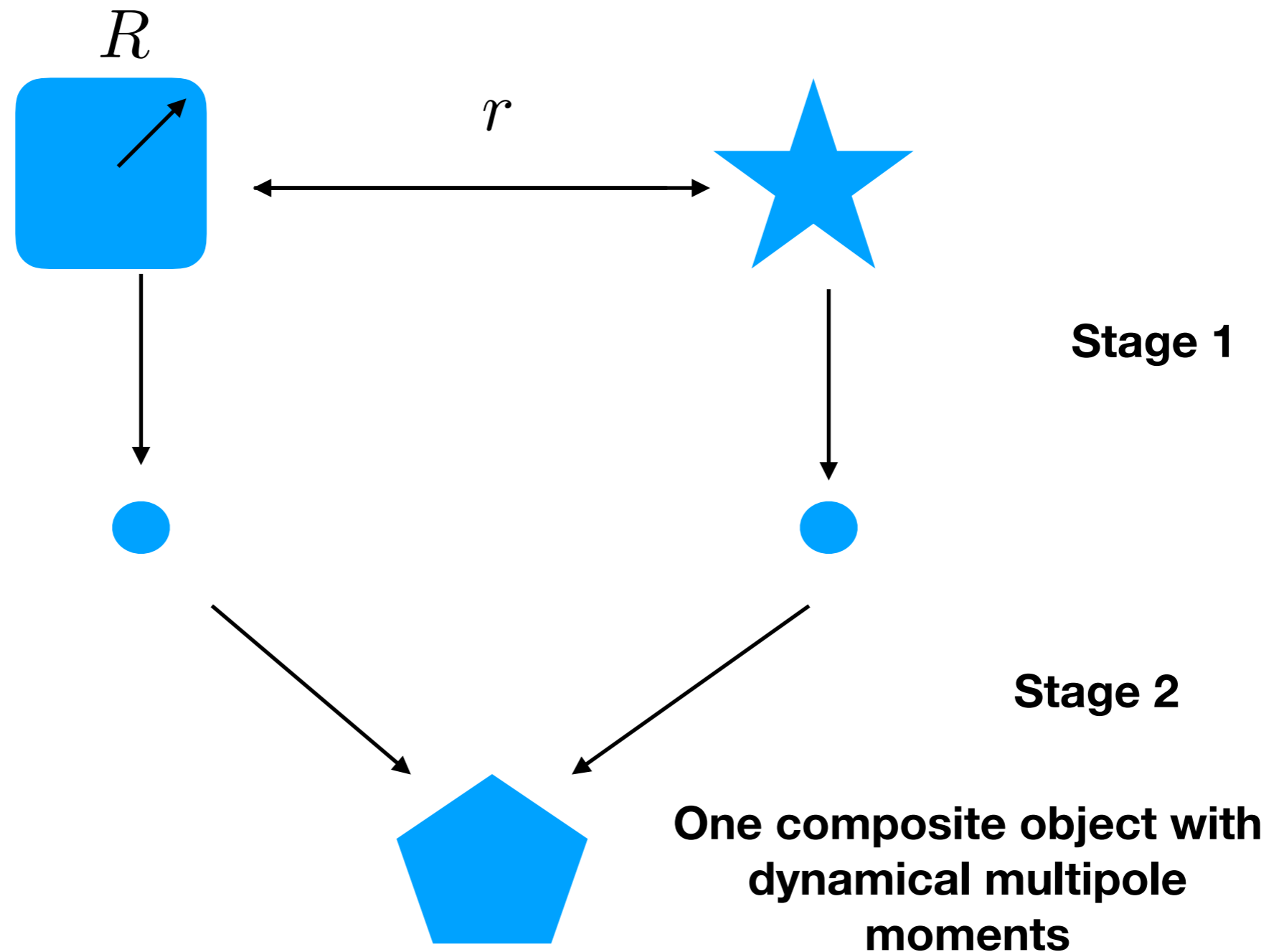


Excitation of internal modes leads to power loss and hastens the inspiral rate

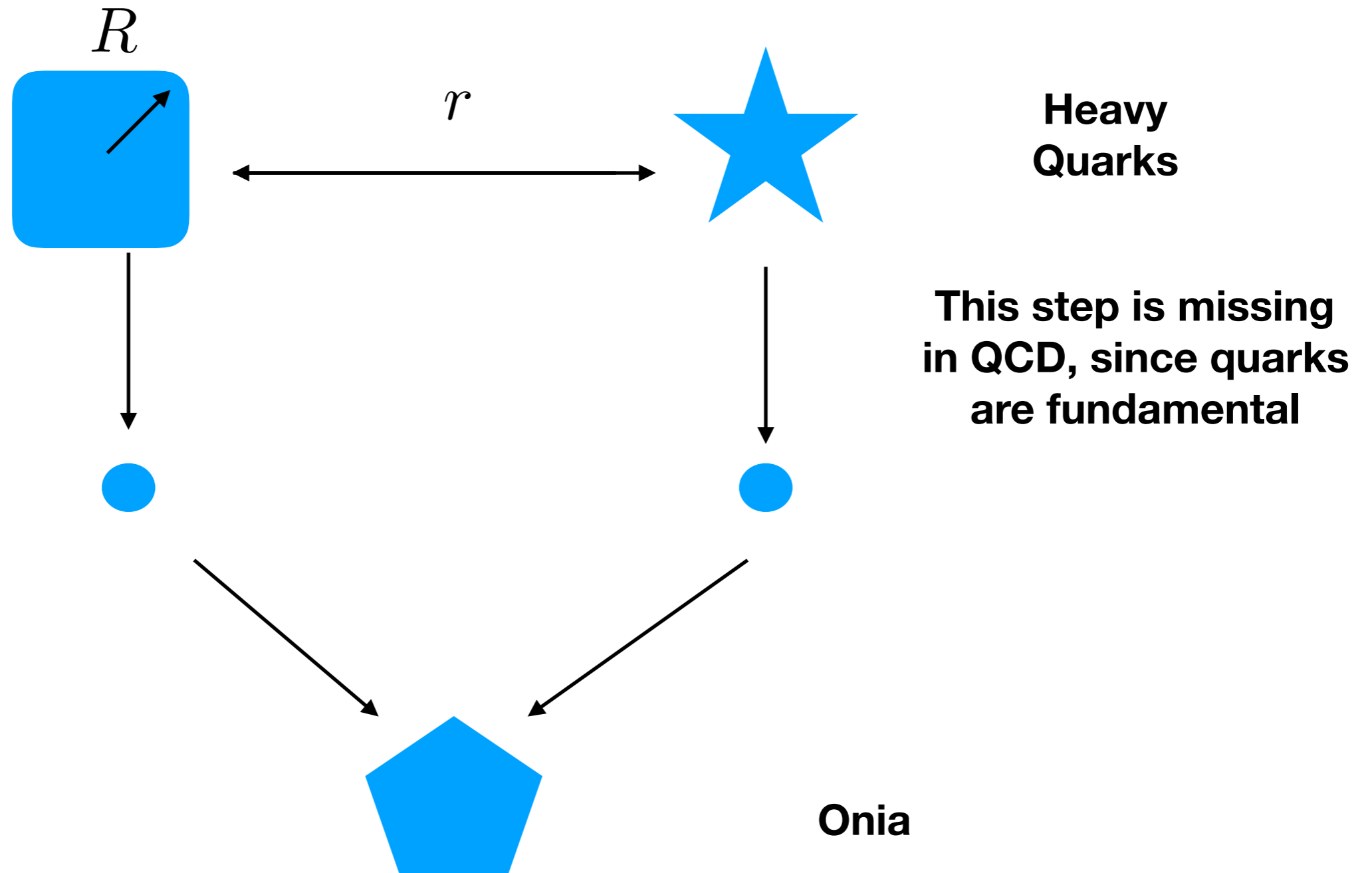


This is a multi-scale problem, perfectly suited for study using the renormalization group and **E**ffective **F**ield **T**heory

Coarse Graining Procedure



Sharp Analogy with Quarkonium



Onia

Binary

Short distances	Weak Coupling Coulomb Phase	Strong coupling
Long Distances	Confinement	Minkowski Space
Non-Linearities	Controlled by $\alpha_s \sim v$	Controlled by v^2
Quantum Effects	Controlled by $\alpha_s \sim v$	Controlled by $(M_{pl}r)^2 \sim \hbar/L$

Allows for Strong
Classical gravity

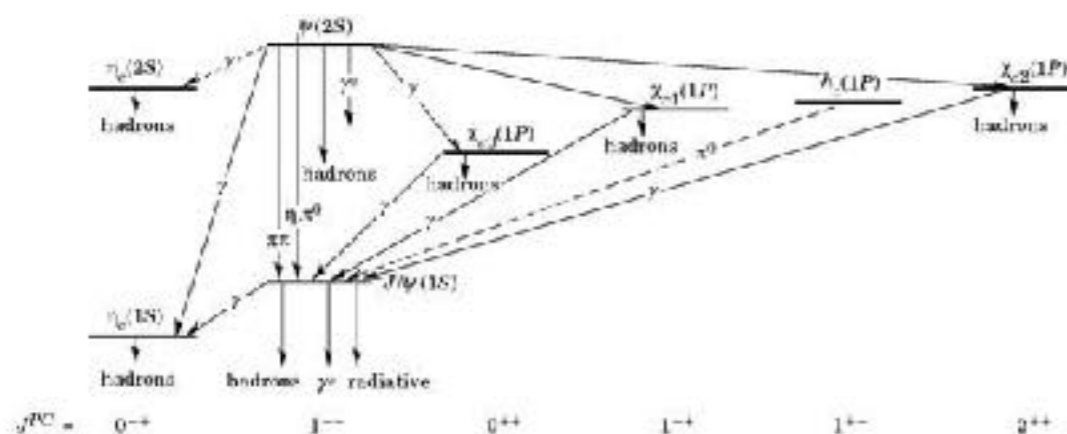
Physical Observables:

Onia

Binaries

Decay Products

Extract: Spectrum



$$m \quad \alpha_s(mv)$$

$\langle O \rangle$ Condensates

LIGO strain

$$h_{\mu\nu}(r \rightarrow \infty)$$

Phase and Amplitude contains all information

$$m_i, S_i, L_i^A$$

In the 90's a theory termed ``NRQCD'' was developed to describe quarkonia (Caswell and Lepage)

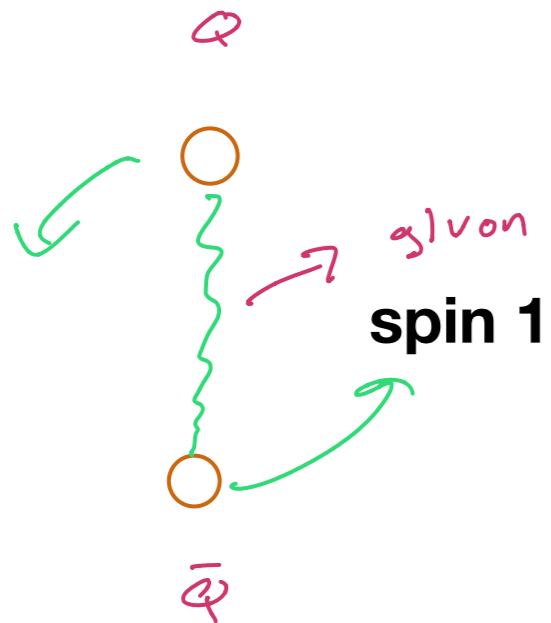
NRGR (Non-Relativistic General Relativity) (W. Goldberger/
IZR)

Key Distinctions:

- Classical Sources
- Quantum Effects are highly suppressed
- Sources are not fundamental (i.e. have internal structure)
- NRQCD is valid to arbitrarily short distances (asymptotic freedom), but breaks down at long distances (confinement). NRGR breaks down at short distances as constituents begin to overlap, but is valid to arbitrarily long distances.

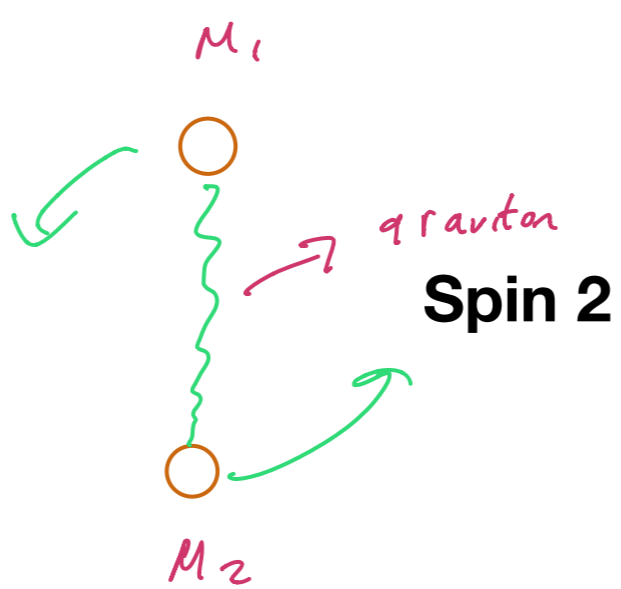
Were going to exploit this sharp analogy to utilized tools of quantum field theory to make precise predictions for the signal

QUARKONIUM



Coulomb
 $V(r) \sim 1/r$

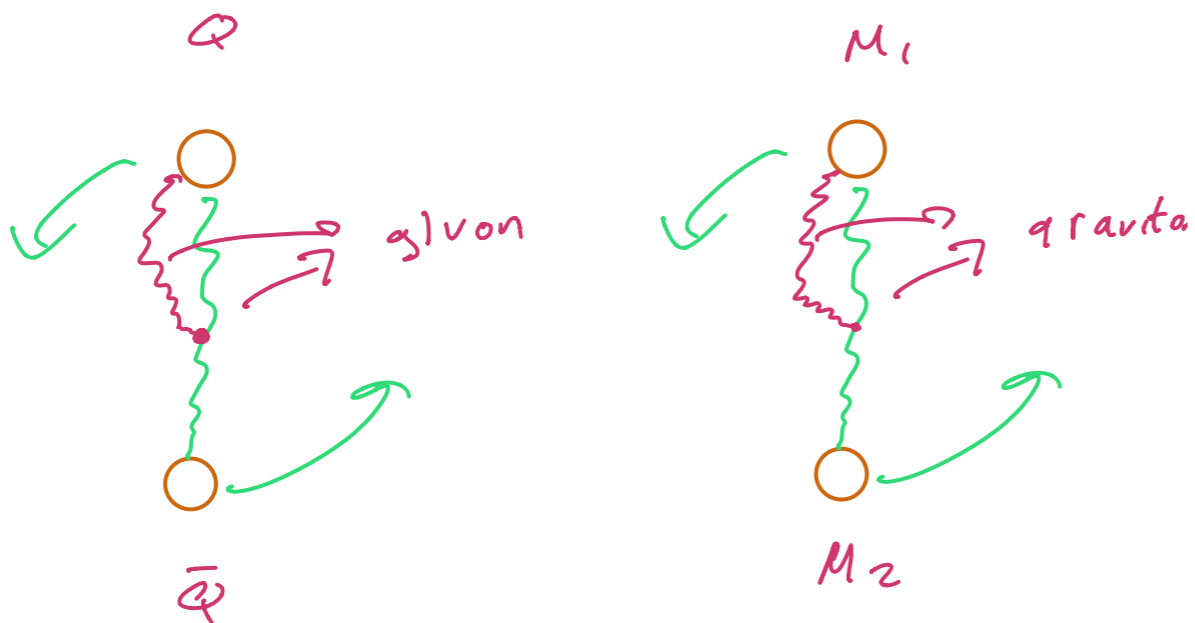
BINARY



Newton
 $V(r) \sim 1/r$

**Sharp Analogy
between the binaries
and bound states of
heavy quarks**

BOTH THEORIES SHARE NON-LINEARITIES



**However, non-
linearities are quite
distinct**

The analogy does not stop there. It turns out that the quantum mechanical scattering amplitudes from one theory can be determined directly from the other!!!, both described by gauge theories that are intimately related

$$A_{GR} \sim A_{QCD}^2$$

KLT/BCJ double copy relations for scattering amplitudes

graviton

gluon



Closed String
Amplitude



$t \rightarrow$



$t \rightarrow$

Product of two open
string amplitudes

More on this later

How do we mathematically perform this coarse graining procedure?

Matching

Full Theory (i.e. exact)

$$L(h_{\mu\nu}, T_{\mu\nu}^{matter})$$

(collection of point particles $i=1,2$)

Coarse Grained (effective) Theory

$$L_{EFT}(h_{\mu\nu}, x_i)$$

$$S = \sum_i \int m_i d\tau$$

How do we include finite size effects?

Take a clue from Electro-Dynamics:

Finite size leads to polarizability

$$S = \sum_i \int (\chi_i^E \vec{E}^2 + \chi_i^B \vec{B}^2) d\tau$$

Gravitational Case is literally analogous

$$S = \sum_i \int (\chi_i^E \vec{E}^2 + \chi_i^B \vec{B}^2) d\tau \quad \vec{E}, \vec{B} \sim \nabla \nabla h$$

To calculate **Love numbers** we put the system in a background field and calculate the response, **for a BH it is ZERO!!!!**

BH really are like fundamental particles

This seems like a remarkable fine tuning, much like the CC or Weak scale. However, recently it has been shown **that BH's have an enhanced near horizon symmetry (Penco et. al.)**

LIGO has the ability to test this remarkable prediction. How does the Love number contribute to the signal?

Next stage of coarse graining

So far we have not made any approximations except a multipole expansion.
Assumed we only probe distance large compared to the size of the object.

$$S = \sum_i \int (\chi_i^E \vec{E}^2 + \chi_i^B \vec{B}^2) d\tau + O(R^n / r^n)$$

To make further analytic progress we will
consider the early stages of the inspiral

$$v/c \ll 1$$

$$g_{\mu\nu} = \eta_{\mu\nu} + O(v^2)$$

Allows us to expand around flat space. (“**Post-Newtonian expansion**”)

Notation: nPN $\sim v^{(2n)}$

It can be shown that the Love number does not come in until 5PN! (“effacement theorem”) (D’amour). **This means we need to make all of our prediction at this level to be able to extract the Love number. What needs to be calculated?**

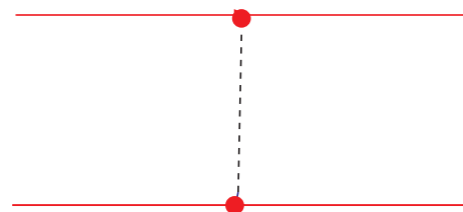
Next stage of matching is done in an expansion in v/c . Integrate out all Fourier components of the gravitational field with Fourier components $k > 1/r$.

$$h = h_{pot} + h_{rad}$$

$$(k > 1/r) \quad (k < 1/r)$$

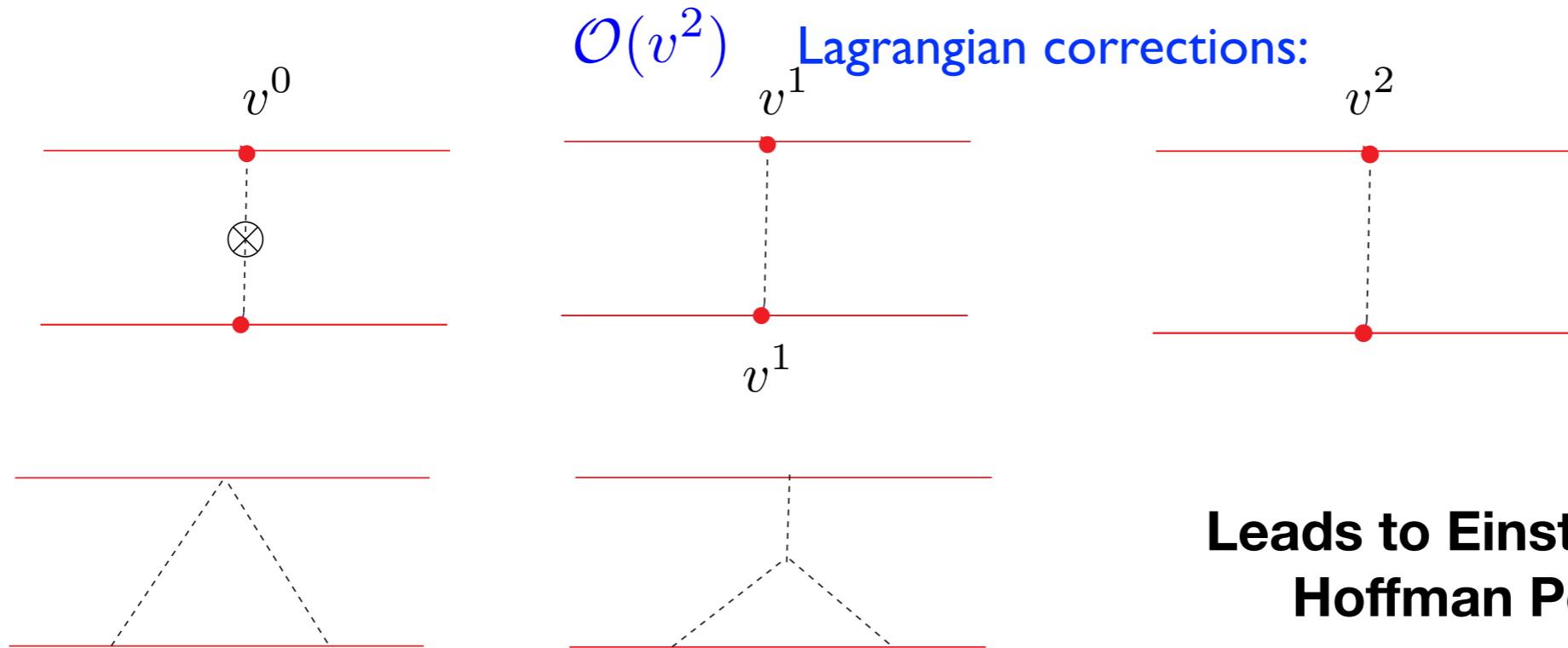
$$L = \sum_i \left(\frac{1}{2} m_i v_i^2 + \dots \right) (1 + F(h_{rad})) + \sum V_i(r) (1 + G(rad))$$

Calculate $V_i(r)$ in a systematic expansion in v



$$= \frac{GM_1 M_2}{r}$$

Newton



Leads to Einstein-Infeld-Hoffman Potential

$$L_{EIH} = \frac{1}{8} \sum_a m_a v_a^4 + \frac{G_N m_1 m_2}{2|\mathbf{x}_1 - \mathbf{x}_2|} \left[3(v_1^2 + v_2^2) - 7(\mathbf{v}_1 \cdot \mathbf{v}_2) - \frac{(\mathbf{v}_1 \cdot \mathbf{x}_{12})(\mathbf{v}_2 \cdot \mathbf{x}_{12})}{|\mathbf{x}_1 - \mathbf{x}_2|^2} \right] - \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{2|\mathbf{x}_1 - \mathbf{x}_2|^2}$$

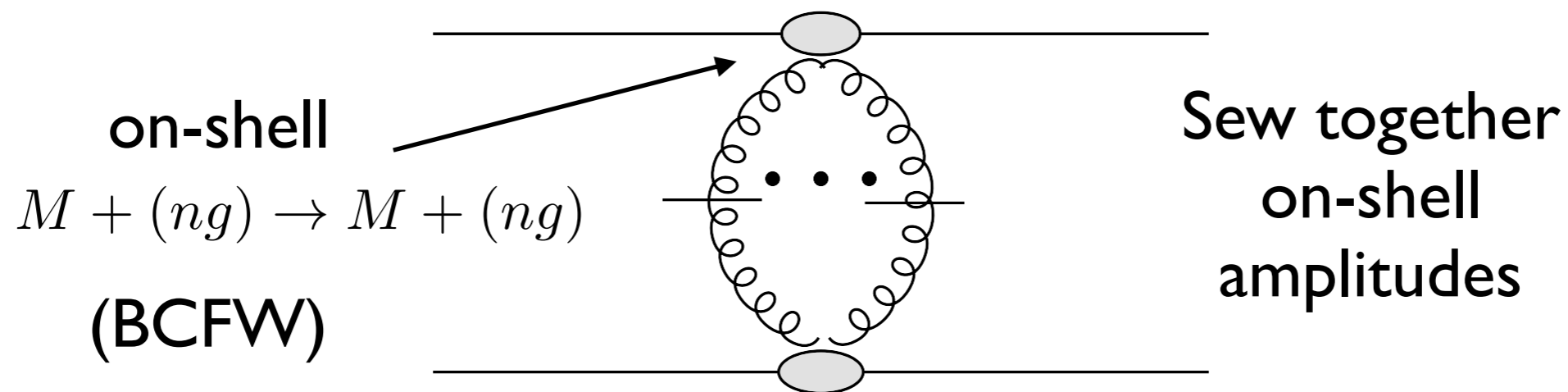
These potentials have been calculated to 5PN using these techniques (Foffa and Strani, Blanchet et. al.)

Number of Feynman diagrams grows factorially with PN order

Using Modern Scattering Amplitude Methods To Reduce Work Load

(D. Neill/IZR)

Factorial growth in number of Feynman diagrams (> 100 at 4PN) a consequence of carrying around gauge dependent junk



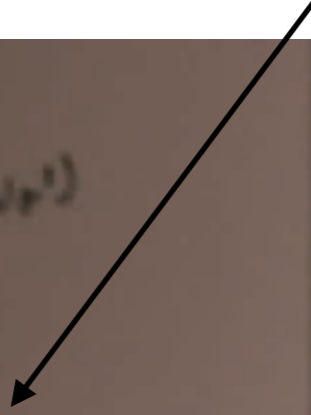
(Cheung, IZR and Solon)

Using Amplitude methods we were able to Calculate the potential to all orders in velocity at $O(G^2)$

Recently extended to $O(G^4)$ by Bern et. al and Porto et. al

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi} (1 - 2\sigma^2), \quad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi} \left[\frac{3}{4} (1 - 5\sigma^2) - \frac{4\nu\sigma (1 - 2\sigma^2)}{\gamma\xi} - \frac{\nu^2 (1 - \xi) (1 - 2\sigma^2)^2}{2\gamma^3 \xi^2} \right],$$
$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi} \left[\frac{1}{12} (3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3) - \frac{4\nu (3 + 12\sigma^2 - 4\sigma^4) \operatorname{arcsinh} \sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right. \\ \left. - \frac{3\nu\gamma (1 - 2\sigma^2) (1 - 5\sigma^2)}{2(1 + \gamma)(1 + \sigma)} - \frac{3\nu\sigma (7 - 20\sigma^2)}{2\gamma\xi} - \frac{\nu^2 (3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2) (1 - 2\sigma^2)}{4\gamma^3 \xi^2} \right. \\ \left. + \frac{2\nu^3 (3 - 4\xi)\sigma (1 - 2\sigma^2)^2}{\gamma^4 \xi^3} + \frac{\nu^4 (1 - 2\xi) (1 - 2\sigma^2)^3}{2\gamma^6 \xi^4} \right],$$

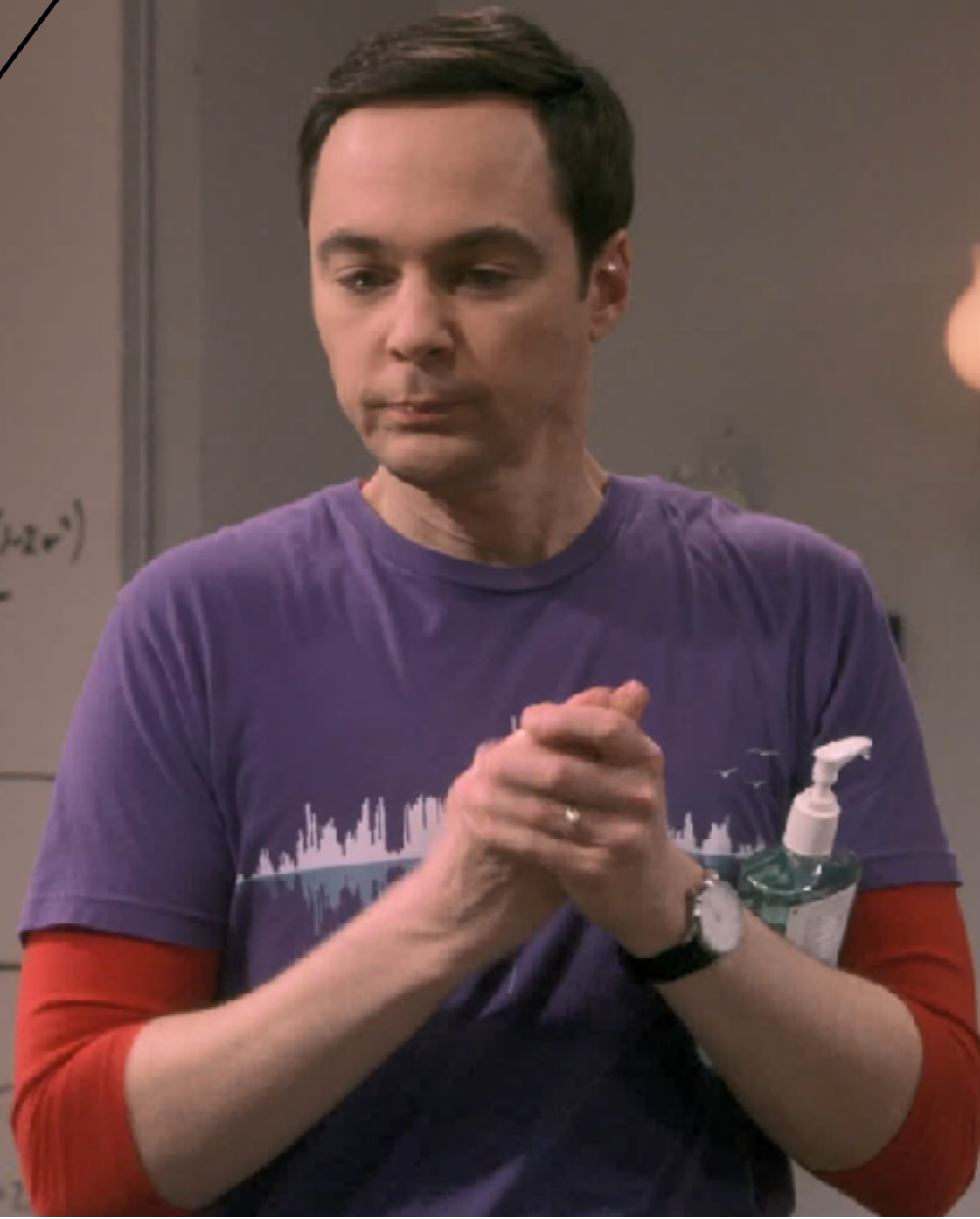
Minus sign error



Handwritten mathematical equations and a diagram on a whiteboard:

$$\frac{d}{dt} \left(\frac{v^2}{r^2} \right)$$
$$\left[\frac{d}{dt} (v^2) \cdot r^2 - v^2 \cdot \frac{d}{dt} (r^2) \right]$$
$$\frac{2v \cdot \frac{dv}{dt} \cdot r^2 - v^2 \cdot 2r \cdot \frac{dr}{dt}}{r^4}$$
$$\frac{2v \cdot \frac{dv}{dt} \cdot r^2 - 2v^2 \cdot r \cdot \frac{dr}{dt}}{2r^4}$$
$$\frac{v \cdot \frac{dv}{dt} \cdot r^2 - v^2 \cdot r \cdot \frac{dr}{dt}}{r^4}$$
$$\frac{v \cdot \frac{dv}{dt} \cdot r^2 - v^2 \cdot r \cdot \frac{dr}{dt}}{r^4}$$

Diagram showing a path with nodes 1, 2, 3, 4, 5, 6.

$$\frac{v^2}{r^2} \cdot \frac{d}{dt} \left(\frac{v^2}{r^2} \right)$$
$$\frac{v^2}{r^2} \cdot \frac{d}{dt} \left(\frac{v^2}{r^2} \right)$$
$$\frac{v^2}{r^2} \cdot \frac{d}{dt} \left(\frac{v^2}{r^2} \right)$$


But we are not quite done: We have calculated the potential which yield the equations of motion but we **still need to calculate the radiation!**

$$L = \sum_i \left(\frac{1}{2} m_i v_i^2 + \dots \right) (1 + F(h_{rad})) + \sum V_i(r) (1 + G(rad))$$

$$L_{1-body} = Q_E(t) \cdot E + Q_B(t) \cdot B + \dots$$

**Leading order
power loss:**



"TAIL EFFECT"

Cut for time

-Dissipation (4PN) : leading order finite size effect for BHs'

-Radiation reaction forces

-Gravitational Lamb Shift (classical)

-Re-Summing Logs (non-trivial classical RG flow)

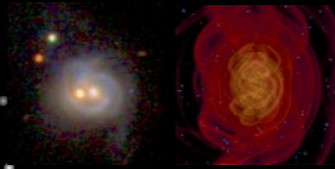
-Spin

The future of GW Astronomy

Sources



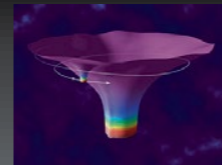
Big Bang



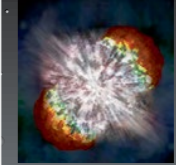
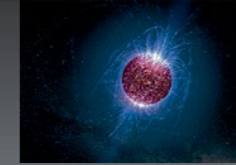
(Super-)massive black hole inspiral and merger



Compact binary inspiral and merger



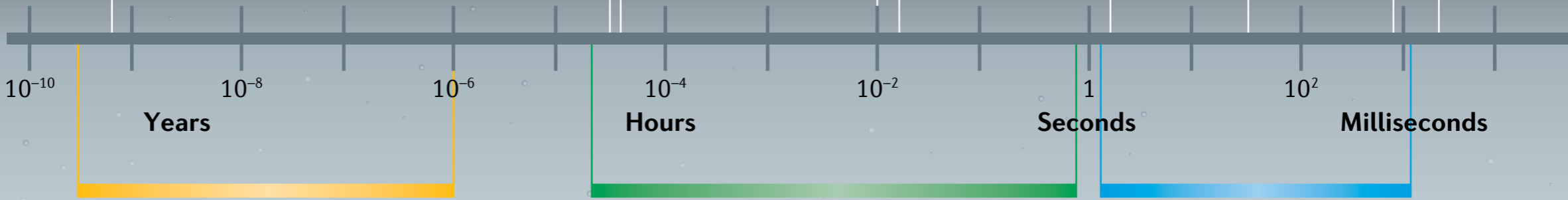
Extreme-mass-ratio inspirals



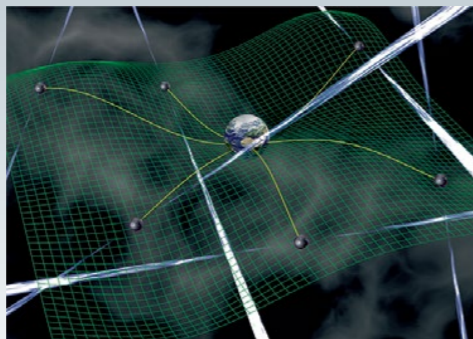
Pulsars, supernovae

Wave period

Wave frequency



Radio pulsar timing arrays



Space-based interferometers



Terrestrial interferometers



Detectors

The future is bright for this field, we will certainly learn a lot about astrophysics, and dense hadronic matter. But whether or not we learning anything about BSM physics.....??