Unitarity triangles

- V_{CKM} can be expressed in terms of mixing angles and δ CPV phase.

- However Wolfenstein parameterisation illustrates the clear flavour hierarchy
  - 4 real parameters of $O(1)$: A, λ, η, ρ

$\lambda = \sin\theta_{12}$ $\quad A\lambda^2 = \sin\theta_{23}$ $\quad A\lambda^3(\rho + i\eta) = \sin\theta_{13} e^{i\delta}$

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

(Slide 18)

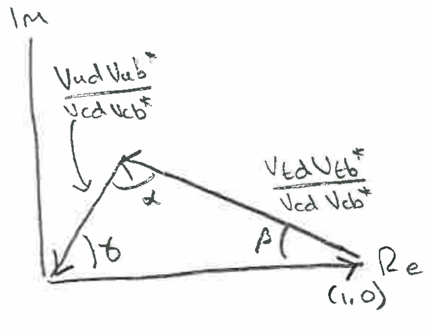
  - Note V_{ts} gains complex phase at $O(\lambda^4)$
  - $\lambda \sim 0.2$ so $\lambda^2 << \lambda$

- V_{CKM} is almost diagonal showing clear hierarchy favouring intra-generational mixing

  Same gen → Cabibbo favoured CF
  Cross 1 gen → Cabibbo suppressed CS
  Cross 2 gen → Doubly Cabibbo suppressed DCS

- Unitary property of V_{CKM} means inner product of any 2 rows/columns must be 0
- Can illustrate this in the complex plane, normalising to one side so we get 1 along real axis

- Columns 1 and 3 ⇒ $\underbrace{V_{ud}V_{ub}^*}_{\sim\lambda^3 \text{ complex } O(\lambda^3)} + \underbrace{V_{cd}V_{cb}^*}_{\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{\lambda^3 \text{ complex } O(\lambda^3)} = 0$



"B^0 unitary triangle"

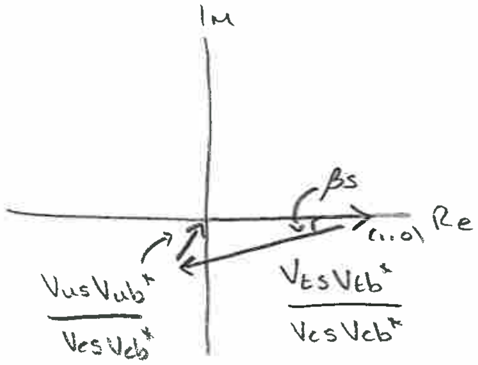
- Why is this favourable?

5/6

- Columns 2 and 3 $\Rightarrow V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$

$\sim \quad \lambda^4 \quad \lambda^2 \quad \lambda^2$

Complex $O(\lambda^3)$ Complex $O(\lambda^4)$



"B_s UT"

- These 2 triangles will come back but can form a triangle from any 2 rows/columns

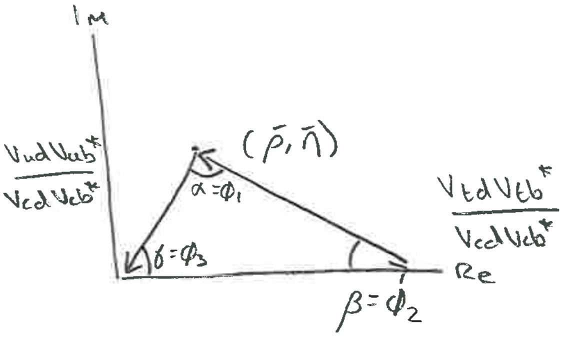
Matter-antimatter asym problem

- All triangles made from V_{CKM} have same area, $J/2$, where J is Jarskog invariant
- J is convention indep measure of CPV in quark sector of SM
- Baryogenesis is creation of an excess of baryons over antibaryons in early Universe, starting from sym. state
- CPV is one of 3 Sakharov conditions for Baryogenesis
  1. Baryon number violation
  2. C and CP violation
  3. Interactions outside thermal eqm
- Using knowledge of J from experiment we can estimate size of baryon asym allowed by SM $\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$ ← number density
- This is 10^{-17} whereas cosmological observation is 10^{-10}

$\Rightarrow$ CPV in quark sector cannot explain observed Universe!

Phases $\alpha, \beta, \gamma, \beta_s$

- lets return to B^0 UT



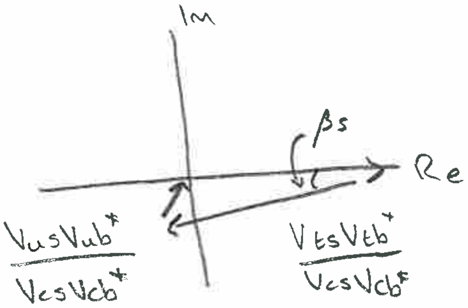
- Apex $(\bar{\rho}, \bar{\eta})$ is $(1 - \frac{\lambda^2}{2})(\rho, \eta)$ using Wolfenstein para.

$$\alpha \equiv \arg\left[- \frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right]$$

$$\beta \equiv \arg\left[- \frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right]$$

$$\gamma \equiv \arg\left[- \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right]$$

- Returning to B_s UT



$$\beta_s \equiv \arg\left[\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right]$$

- Through Wolfenstein para. write

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + O(\lambda^5)$$

  - Note that β and β_s only accessible through top transitions $\Rightarrow$ loops

- This matrix is very helpful for CKM measurements

- We will now look at different types of CPV and how to measure CKM parameters.

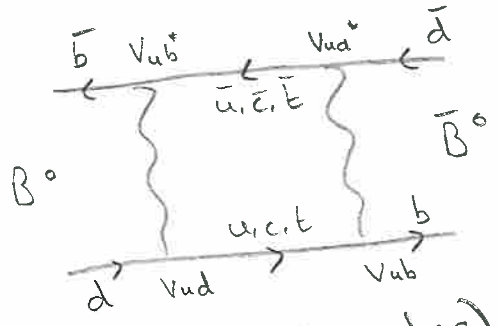
Mixing and CPV

Neutral Meson Mixing

- Flavour (weak int) estates of quarks ≠ mass (strong int) estates

$$\underset{\text{weak}}{\begin{pmatrix} d \\ s \\ b \end{pmatrix}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \underset{\text{mass}}{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}$$

- The d quark that partakes in weak int. is superposition of mass estates d, s, b and v.v.
- Particle and antiparticle ~~flavour~~ flavour estates for neutral mesons oscillate btw each other via weak int and always have equal and opposite charge, same mass and lifetimes by $\hat{C}\hat{P}\hat{T}$
- The mass estates therefore also oscillate btw their particle and antiparticle states but here there may be mass/lifetime differences (change of basis)
- In the SM neutral mesons mix btw their flavour estates via box diagrams and weak charged current



(t quark dominates)

- Mixing has been observed in all 4 neutral mesons

K^0	D^0	B^0	B_s^0
$\bar{s}d$	$c\bar{u}$	$\bar{b}d$	$\bar{b}s$

Mixing phenomenology

- Neutral meson oscillating btw its flavour estates with time is described by time-dep. Schrödinger eqn with *effective* Hamiltonian $\hat{H}$.

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle \quad \text{with sol}^n\text{s} \quad |\Psi(t)\rangle = e^{-i\hat{H}t}|\Psi(0)\rangle$$

$$\Rightarrow i\frac{\partial}{\partial t}\begin{pmatrix}|B^0\rangle \\ |\bar{B}^0\rangle\end{pmatrix} = \Big(\underbrace{\underline{M}}_{\text{Dispersive}} - \frac{i}{2}\underbrace{\Gamma}_{\text{absorbtive}}\Big)\begin{pmatrix}|B^0\rangle \\ |\bar{B}^0\rangle\end{pmatrix} \quad \text{Coupled system}$$

$$\Rightarrow \begin{pmatrix} M_{11}-\frac{i}{2}\Gamma_{11} & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{21}-\frac{i}{2}M_{21} & M_{22}-\frac{i}{2}\Gamma_{22}\end{pmatrix}$$

- Imagine turning off mixing → only want diag. elements of $\hat{H}$
  - M is mass (energy) term and Γ gives exp. decay
    $\Rightarrow |B^0(t)\rangle = e^{-iM_{11}t-\Gamma_{11}t/2}|B^0\rangle$
  - Due to $\hat{C}\hat{P}\hat{T}$ $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$

- With mixing turned on → off-diag elements
  - for conservation of probability $\underline{M}$ and $\underline{\Gamma}$ are hermetian
    $M_{12}=M_{21}^*$, $\Gamma_{21}=\Gamma_{12}^*$
    (note $\hat{H}$ overall is not hermetian due to the states decaying)

- The mass estates are superposition of flavour estates. In the limit of no CPV in mixing

$$|B_H^0\rangle = \frac{|B^0\rangle + |\bar{B}^0\rangle}{2} \quad 4$$

$$|B_L^0\rangle = \frac{|B^0\rangle - |\bar{B}^0\rangle}{2} \quad 5$$

$$\Rightarrow |B^0(t)\rangle = |B_H^0(t)\rangle + |B_L^0(t)\rangle \quad 3$$

- By diagonalising $\hat{H}$ we can find

$$|B_H^0(t)\rangle = e^{-iM_Ht-\Gamma_Ht/2}|B_H^0(0)\rangle \quad 1$$

$$|B_L^0(t)\rangle = e^{-iM_Lt-\Gamma_Lt/2}|B_L^0(0)\rangle \quad 2$$

(mass/energy estates)

- Putting 1 and 2 into 3 and then 4 and 5 into the result

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + g_-(t)|\bar{B}^0\rangle$$

$$|\bar{B}^0(t)\rangle = g_+(t)|\bar{B}^0\rangle + g_-(t)|B^0\rangle$$

where $$g_\pm(t) = \frac{1}{2}\left[e^{-i(M_H - i\Gamma_H t/2} \pm e^{-i(M_L - i\Gamma_L t/2}\right]$$

- If we begin with a $|B^0\rangle$ flavour estate (as created in weak int), the probability to find $|\bar{B}^0\rangle$ at time t is

$$P = |\langle \bar{B}^0 | B^0(t)\rangle|^2$$

$$= |g_+(t)\underbrace{\langle \bar{B}^0 | B^0\rangle}_{=0 \text{ orthonormal}} + g_-(t)\underbrace{\langle \bar{B}^0|\bar{B}^0\rangle}_{=1}|^2$$

$$= |g_-(t)|^2$$

- Probability to remain $|\langle \overset{(-)}{B^0} | \overset{(-)}{B^0}(t)\rangle|^2 = |g_+(t)|^2$

to change $|\langle B^0 | \bar{B}^0(t)\rangle|^2 = |\langle \bar{B}^0 | B^0(t)\rangle|^2 = |g_-(t)|^2$

- Expressing $\Delta\Gamma = \Gamma_H - \Gamma_L$, $\Delta M = M_H - M_L$ and $\Gamma = \frac{(\Gamma_L + \Gamma_H)}{2}$

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right]$$

+ prob unchanged
- prob oscillate

- If no CPV in mixing time integrated mixing prob. is

$$\frac{\int |g_-(t)|^2 dt}{\int |g_-(t)|^2 dt + \int |g_+(t)|^2 dt} = \frac{x^2 + y^2}{2(x^2+1)}$$

with $x = \frac{\Delta M}{\Gamma}$ and $y = \frac{\Delta\Gamma}{2\Gamma}$ (Δm causes the mixing as hyperbolic func^{n}s don't oscillate)

- x and y are very different for the 4 mesons (slide 19 + 20)

Types of CPV

Direct CPV

- For direct CPV / CPV in decay

$$|A(B\to f)|^2 \neq |A(\bar{B}\to \bar{f})|^2$$

- Direct CPV is the only CPV for charged initial states (no mixing)
- We need
  - ≥ 2 interfering amps.
  - CP conserving (strong) phase diff. btw them
  - CP violating (weak) phase diff. btw them.

- Consider if one single amp

$$A(B\to f) = Ae^{i(\delta-\phi)} \qquad A(\bar{B}\to\bar{f}) = Ae^{i(\underbrace{\delta}_{\text{strong}}+\underbrace{\phi}_{\text{weak}})}$$

$$A_{CP} = \frac{|A(\bar{B}\to\bar{f})|^2 - |A(B\to f)|^2}{|A(\bar{B}\to\bar{f})|^2 + |A(B\to f)|^2} = \frac{A^2 - A^2}{A^2 + A^2} = 0$$

- Consider 2 amps

$$A(B\to f) = A_1 e^{i(\delta_1+\phi_1)} + A_2 e^{i(\delta_2+\phi_2)}$$

$$A(\bar{B}\to\bar{f}) = A_1 e^{i(\delta_1-\phi_1)} + A_2 e^{i(\delta_2-\phi_2)}$$

$$A_{CP} = \frac{2r\sin\delta\sin\phi}{1+r^2+2r\cos\delta\cos\phi} \quad \text{where } r=\frac{A_1}{A_2},\ \delta=\delta_1-\delta_2,\ \phi=\phi_1-\phi_2$$

* All r, δ, ϕ must be non-zero. Can access $\phi = \gamma, \beta, \beta_s$ depending on V_{CKM} element involved

$$\Rightarrow \text{CPV if } \left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1$$

CPV in Mixing

- Allowing CPV in Mixing $|B_H^0\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$
  $|B_L^0\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$

where $|p|^2 + |q|^2 = 1$

- Now prob. to find $|\bar{B}^0\rangle$ at time t when starting with $|B^0\rangle$

$$|\langle\bar{B}^0|B^0(t)\rangle|^2 = \left|\frac{q}{p}\right|^2 |g_-(t)|^2$$
$$|\langle B^0|\bar{B}^0(t)\rangle|^2 = \left|\frac{p}{q}\right|^2 |g_-(t)|^2$$

(prob. to remain unchanged)

$$\Rightarrow \text{CPV if } \left|\frac{q}{p}\right| \neq 1$$

CPV in interference btw mixing and decay

- Consider $\overset{(-)}{B^0} \to \overset{(-)}{f}$, can proceed via

$A_f = A(B^0 \to f) = \langle f|B^0\rangle$ $\bar{A}_f = A(\bar{B} \to f) = \langle f|\bar{B}^0\rangle$

$A_{\bar{f}} = A(B^0 \to \bar{f}) = \langle \bar{f}|B^0\rangle$ $\bar{A}_{\bar{f}} = A(\bar{B} \to \bar{f}) = \langle \bar{f}|\bar{B}^0\rangle$

- We define complex parameter λ_f

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}, \quad \bar{\lambda}_f = \frac{1}{\lambda_f}, \quad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}, \quad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}}$$

- The time dep decay rate for $B^0 \to f$

$$T_{B^0\to f}(t) = |\langle f|B^0(t)\rangle|^2 = |\underbrace{g_+(t)A_f}_{\text{decay}} + \underbrace{\frac{q}{p} g_-(t)\bar{A}_f}_{\text{mix then decay}}|^2 =$$

- For $B^0 \to \bar{f}$

$$T_{B^0\to \bar{f}}(t) = \left|\frac{q}{p} g_-(t)\bar{A}_{\bar{f}} + g_+(t) A_{\bar{f}}\right|^2 =$$

- For $\bar{B}^0 \to f$ and $\bar{B}^0 \to \bar{f}(t)$

$$T_{\bar{B}^0\to f}(t) = \left|g_+(t)\bar{A}_f + \frac{p}{q} g_-(t) A_f\right|^2 =$$

$$T_{\bar{B}^0\to \bar{f}}(t) = \left|g_+(t)\bar{A}_{\bar{f}} + \frac{p}{q} g_-(t) A_{\bar{f}}\right|^2 =$$

Slide 21

MASTER EQNS!

$\Rightarrow$ CPV if $\arg(\lambda_f) = \arg\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$

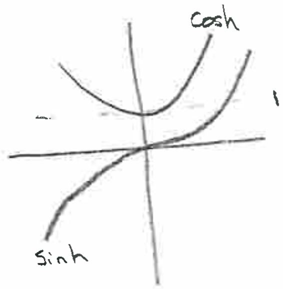
(even if $\left|\frac{p}{q}\right| = 1$ and $\left|\frac{\bar{A}_f}{A_f}\right| = 1$)

- We can use master eqns to find A_{CP} for mixing and decay but they can be simplified
  - Measurements are often made in decays to a final state common to B^0 and $\bar{B}^0$, $f = \bar{f}$ where
    $|\bar{f}\rangle = \hat{C}\hat{P}|f\rangle = \pm|f\rangle \Rightarrow f$ is CP estate
  - A_{CP} becomes $A_{CP}(t) = \dfrac{\Gamma_{B^0\to f(t)} - \Gamma_{\bar{B}^0\to f(t)}}{\Gamma_{B^0\to f(t)} + \Gamma_{\bar{B}^0\to f(t)}}$ TIME DEPENDENT ACP
  - CPV in mixing is small for D^0, B^0, B_s^0 ie. $|p| \sim |q|$

$$A_{CP}(t) \Rightarrow \frac{C_f\cos(\Delta m t) - S_f\sin(\Delta m t)}{\cosh\left(\frac{\Delta\Gamma t}{2}\right) + D_f\sinh\left(\frac{\Delta\Gamma t}{2}\right)}$$

  - Further for D^0, Δm and $\Delta\Gamma$ are small

$$A_{CP}(t) \Rightarrow \frac{C_f - S_f(\Delta m t)}{1 + \frac{1}{2}D_f\Delta\Gamma t}$$



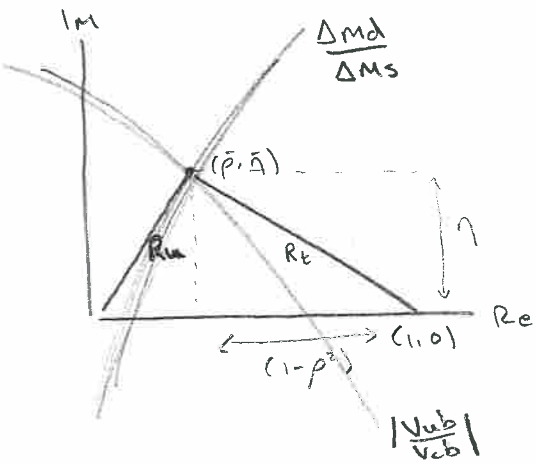
  - whereas for B^0, $\Delta\Gamma \sim 0$

$$A_{CP}(t) \Rightarrow C_f\cos(\Delta m t) - S_f\sin(\Delta m t)$$

Measuring the CKM

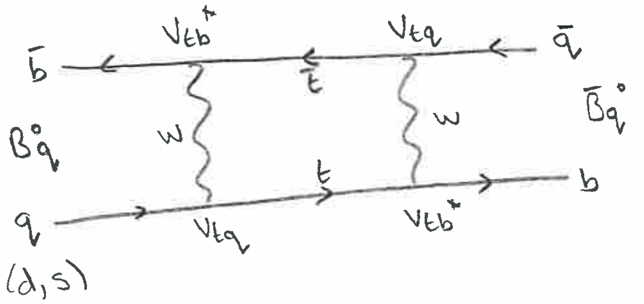
- Take the B^0 UT - there are 4 measurements that provide strongest constraints

$$|V_{ub}|,\ \Delta M,\ \sin 2\beta \ \text{and}\ \varepsilon_k$$



- R_u side constrained by $\left|\frac{V_{ub}}{V_{cb}}\right|$ as $(\bar\rho, \bar\eta) = (1 - \frac{\lambda^2}{2})(\rho, \eta)$ and $\left|\frac{V_{ub}}{V_{cb}}\right|^2 \propto (\rho^2 + \eta^2)$

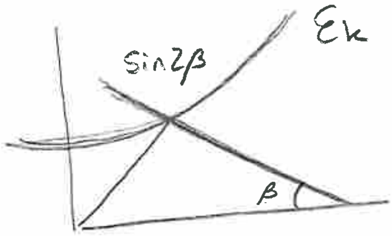
- R_t side constrained by $\frac{\Delta M_d}{\Delta M_s}$ as $\left|\frac{V_{td}}{V_{ts}}\right|^2 \propto (1-\rho)^2 + \eta^2$



- t quark contribution dominates as $M_t \gg M_c$

- Although strong constraints, on their own, these do not give CPV as η can be $0 \Rightarrow J = 0$

- Two other constraints are



- ε_k related to kaon mixing

Measuring phases

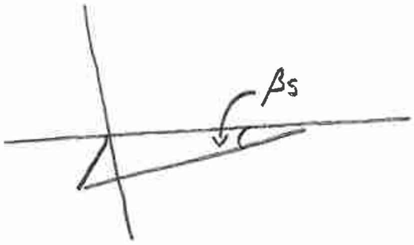
- Recall

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{-i\beta_s} & |V_{tb}| \end{pmatrix} + O(\lambda^5)$$

- Access γ through V_{ub} in interference btw $b \to u$ and $b \to c$
- Access β through V_{td} in int. btw B^0 mixing and decay
- Access β_s through V_{ts} in int btw B_s^0 mixing and decay
- Access α through int. btw different $b \to u$ transitions

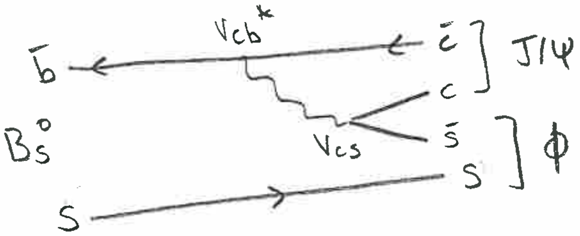
Measuring β_s

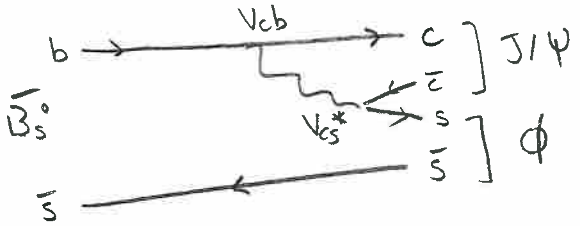
- Recall B_s UT



- β_s is small!

- Golden mode is $\overset{0^-}{B_s^0} \to \overset{1^{--}}{J/\psi}\ \overset{1^{--}}{\phi}$ with 3 amps
  $A_{\parallel}(\uparrow\uparrow)$ $L=2$
  $A_{\perp}(\uparrow\rightarrow)$ $L=1$
  $A_0(\uparrow\downarrow)$ $L=0$

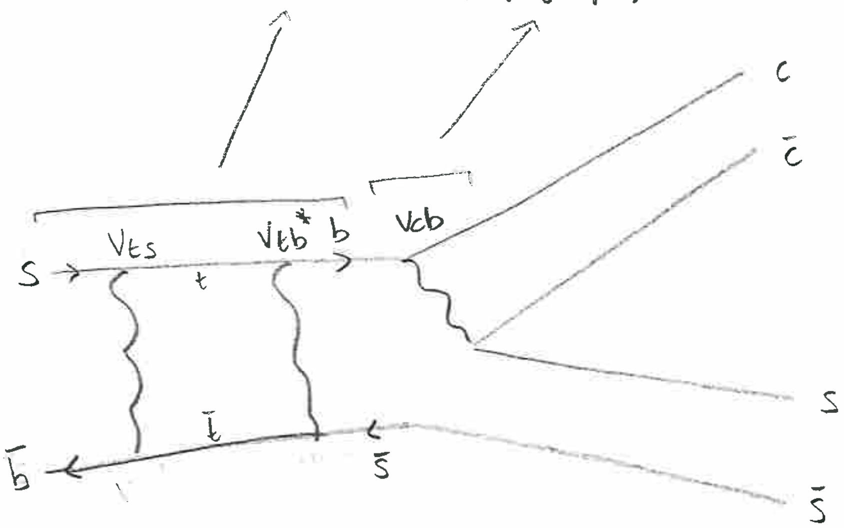




$$|A_f| \sim |\bar{A}_f| \ \therefore\ |\lambda_f| \sim 1$$

- look closer at λ_f

$$\lambda_{J/\psi\phi} = \underset{\text{mix}}{\left(\frac{q}{p}\right)_{B_s}} \underset{\text{decay}}{\left(\frac{\bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}}\right)} = \left(\frac{\cancel{V_{tb}^*} V_{ts}}{\cancel{V_{tb}} V_{ts}^*}\right)\left(\frac{\cancel{V_{cb}} \cancel{V_{cs}^*}}{\cancel{V_{cb}^*} \cancel{V_{cs}}}\right)$$



$$\lambda = \frac{-|V_{ts}|e^{-i\beta_s}}{-|V_{ts}|e^{i\beta_s}}$$

$$\lambda = e^{-2i\beta_s}$$

- Returning to A_{CP} where $|p| \sim |q|$ as for B_s^0

$$A_{CP}(t) = \frac{C_f\cos(\Delta m t) - S_f\sin(\Delta m t)}{\cosh\left(\frac{\Delta\Gamma t}{2}\right) + D_f\sinh\left(\frac{\Delta\Gamma t}{2}\right)} \xrightarrow{|\lambda_f| \sim 1} \frac{-\mathrm{Im}(\lambda_f)\sin(\Delta m t)}{\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \mathrm{Re}(\lambda_f)\sinh\left(\frac{\Delta\Gamma}{2}t\right)}$$

- Therefore

$$A_{CP}(t) = \frac{-\eta\sin(2\beta_s)\sin(\Delta m t)}{\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \eta\cos(2\beta_s)\sinh\left(\frac{\Delta\Gamma}{2}t\right)}$$

(3 amps to consider)

$$\hat{C}\hat{P}|J/\psi\phi\rangle = (-1)^L|J/\psi\phi\rangle \Rightarrow \eta = (-1)^L$$

- β is measured similarly using $B^0 \to J/\Psi\, K^0_{S(L)}$ with following differences
  - Sensitive to V_{td} $(e^{-i\beta})$ not V_{ts} $(-e^{-i\beta_s})$
  - Actual decay product is K^0 so have to account for kaon mixing in λ_f
  - Only one amplitude so $C=0$
  - $\Delta\Gamma \sim 0$

$$\Rightarrow A_{CP}(t) = C_f \cos(\Delta m t) - S_f \sin(\Delta m t)$$