Section 5

Quark Model

- Quarks have colour charge r, b, g
  Antiquarks have anti-colour charge $\bar{r}, \bar{b}, \bar{g}$

Quarks exist in colour neutral states (confinement)

Meson	Baryon	
$q\ \bar{q}$	$q\ q\ q$	"Conventional"
$r\ \bar{r}$ $g\ \bar{g}$ $b\ \bar{b}$	r g b	

Tetraquark	Pentaquark	
$q\ \bar{q}$ $q\ \bar{q}$	$\bar{q}\ q\ q$ $q\ q$	"Exotic" (exact nature not understood)

Ordinary matter consists of 1st generation (u, d) and was all we know until 1960's.

In 1960's hundreds of strongly interacting particles were being discovered - "zoo" - and a classification system was needed.

These classification systems rely on SU(n) flavour 'symmetries' of the SM.

- Approx. sym of SM at low energies
- Group hadrons of ~ mass and same J^P

- SU(2) flavour sym
  ($M_u \sim M_d$ and have ~ same strong interactions)

$$\underset{q}{2} \otimes \underset{\bar{q}}{\bar{2}} = \overset{\text{triplet}}{3} \oplus \underset{\text{singlet}}{1}$$

[originally isospin for (p, n)]

$$\left(\underset{\pi^-}{d\bar{u}},\ \underset{\pi^0}{\tfrac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})},\ \underset{\pi^+}{u\bar{d}}\right) \qquad \underset{\eta}{\left(\tfrac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\right)}$$

In 1964 strange quark postulated - Quark model with 3 quarks

- SU(3) flavour sym
  ($M_u \sim M_d \neq M_s$ but still ok)

$$3 \otimes \bar{3} = \underset{\text{octet}}{8} \oplus 1$$

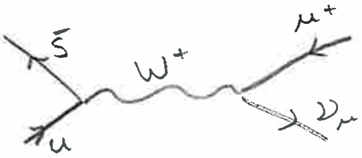
(Slide 11 + 12)

Cabibbo angle [u, d, s known quarks]

- Consider

$K^+ \to \mu^+ \nu_\mu$

$\bar{s}u$



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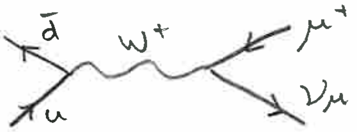
$\pi^+ \to \mu^+ \nu_\mu$

$u\bar{d}$



k^+ decay is 20x smaller than π^+ decay rate
⟹ $u \to s$ suppressed relative to $u \to d$

Cabibbo first suggested physical (mass) e-state of d quark is admixture of d and s interaction (flavour) e states
⟹ Quark mixing

$$d^I = (d\cos\theta_c + s\sin\theta_c)$$

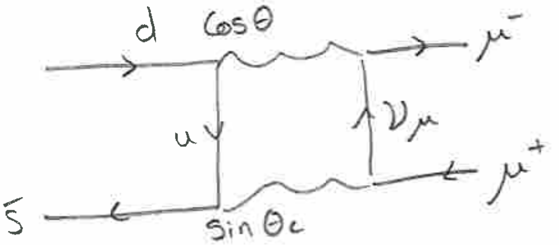
↑ Cabibbo angle

- $\sin\theta_c$ experimentally determined as 0.22

UT

- Cabibbo's solution gives rise to tree-level FCNC (with Z^0) which we do not see.
- Also $\underset{d\bar{s}}{K^0} \to \mu^+\mu^-$ has much smaller rate than expected even for a loop process

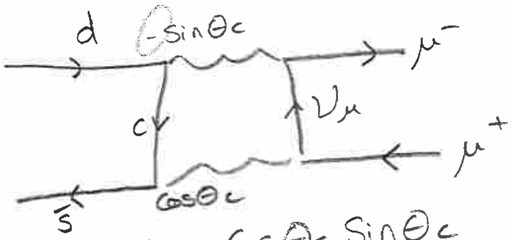
Rate << $\sin^2\theta_c \cos^2\theta_c$



FCNC via a loop
$A \propto \cos\theta_c \sin\theta_c$

Interfere

Introduce Charm quark



$A \propto -\cos\theta_c \sin\theta_c$

- Sol^n is GIM mechanism
  - If $M_u = M_c$ complete cancellation
  - $B(K^0 \to \mu^+\mu^-) \sim 7 \times 10^{-9}$

GIM mech. explains the suppression by introducing charm quark

⇒ Indirect evidence of charm quark

$$\begin{pmatrix} d \\ s \end{pmatrix}^{I} = \begin{pmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

ie. u or c quark couples to superposition of down quarks

In 1970's have quark model with 4 quarks in 2 generations and one mixing angle θ_c

(Slide 13)

1970	1974
Indirect evidence of Charm	Direct evidence of Charm
- GIM mech. to explain $K^0 \to \mu^+\mu^-$ suppression	- J obs at Brookhaven
	- ψ obs. at SLAC
	J/ψ $(c\bar{c})$

We will now see how CPV discovery led to prediction of 3rd generation...

But first a diversion to Dalitz plots...

Amplitude Analyses

- Consider a pseudoscalar D meson decaying to n final state pseudoscalars (f)

$$d\Gamma \propto \underbrace{|\,A_D(\Phi_n)|}_{\text{All decay dynamics}}{}^2 \; \overbrace{d\,\Phi_n}^{\text{Phasespace element}} \qquad (n > 2)$$

- D -> f can proceed directly or, normally, through intermediate states or "resonances"
- Amplitude analyses study relative contribution of these resonances to a decay ie. find $A_D(\Phi_n)$

- Consider D -> abc

$D \rightarrow r\,c$
$\quad \hookrightarrow ab$

$D \rightarrow a\,r$
$\quad \hookrightarrow bc$

SAME INITIAL AND FINAL STATE WITH 2 INDISTINGUISHABLE PATHS

- Similar to Young's double slit exp., these multiple, indist paths produce quantum mech. interference effects in FS phasespace due to phase differences in the amplitudes
- We observe areas of PS with

  high event density -> cons interference
  low event density -> des. interference

- If n=3 and D, a, b, c are 0^- we can show this on <u>Dalitz plot</u>

Section 4

$\frac{2}{4}$

Dalitz plot

- Visual representation of resonant sub-structure of 0^- particle decaying to 3 0^- particles

- A DP is a 2-D scatter plot with axes equal to square of invariant mass combinations of FS particles

eg. for $D_s^+ \to K^+ K^- \pi^+$ use $(p_{K^+} + p_{K^-})^2 = M_{K^+K^-}^2$

$D \to a\ b\ c$ $\quad (p_{K^-} + p_{\pi^+})^2 = M_{K^-\pi^+}^2$

- We only need 2 independent variables to fully describe this decay

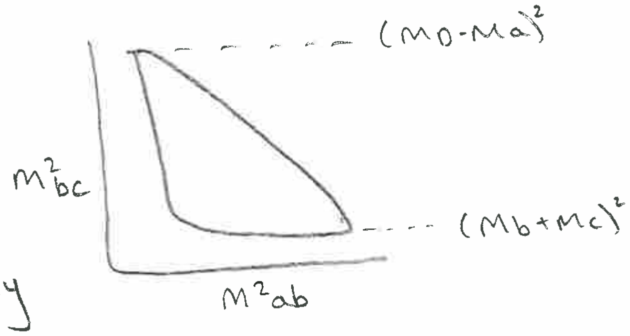
$$m^2_{ab} + m^2_{bc} + m^2_{ac} = m_D^2 + m_a^2 + m_b^2 + m_c^2$$

where $m^2_{ab} = (p_a + p_b)^2$

- The DP has a unique shape with boundaries defined by the allowed PS of the decay dictated by cons. of 4-momentum

eg.
$(m_b+m_c)^2 \leq m^2_{bc} \leq (m_D - m_a)^2$

where min and max values have b and c parallel or antiparallel respectively



$$m^2_{bc} = (p_b + p_c)^2 = \left(\begin{pmatrix} E_b \\ \underline{p}_b \end{pmatrix} + \begin{pmatrix} E_c \\ \underline{p}_c \end{pmatrix}\right)^2 = (E_b + E_c)^2 - (\underline{p}_b + \underline{p}_c)^2$$

$$= E_b^2 + E_c^2 + 2E_bE_c - (\underline{p}_b^2 + 2\underline{p}_b\underline{p}_c + \underline{p}_c^2)$$

$$= m_b^2 + m_c^2 + 2E_bE_c - 2\underline{p}_b \cdot \underline{p}_c$$

- From a DP we can infer
  - resonances present (mass)
  - Spin of resonances
  - interference (phase diff) btw them

AMPLITUDE ANALYSIS

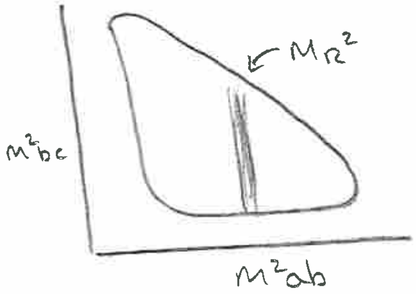
Section 4

$\frac{3}{4}$

- Consider a single amplitude $D \to R(\to ab)c$

$$A(p) = |A_1(p)| e^{i\theta_1(p)}$$

$\uparrow$ PS point

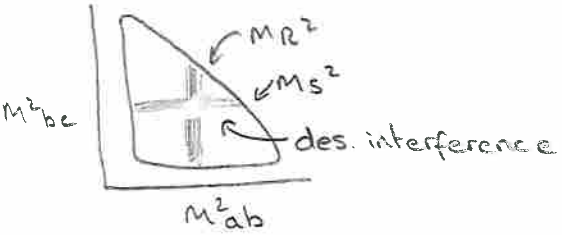


- observable is $|A_1(p)|^2$ and we get a high density band at $m^2_{ab} = m_R^2$
  - Gives mass of a resonance R but lost all phase, θ_1, info.

- Now consider 2 amplitudes $D \to R(\to ab)c$ $A_1(p) = |A_1(p)| e^{i\theta_1(p)}$
  $D \to a\, S(\to bc)$ $A_2(p) = |A_2(p)| e^{i\theta_2(p)}$

- We regain phase info

$$|A(p)|^2 = |A_1(p)|^2 + |A_2(p)|^2 + 2|A_1(p)||A_2(p)| \cos(\underbrace{\theta_1(p) - \theta_2(p)}_{\Delta\theta})$$



$\Delta\theta = 0 \Rightarrow$ cons. interference

$\Delta\theta = \pi \Rightarrow$ des. interference

- Finally if R is spin-X then orbital ang mom. btw R and c, known as L, must be spin of R (X)

$$\begin{array}{ccccccc} & D & \to & R & c & & \\ J: & 0 & \to & 1 & 0 & \Rightarrow & L=1 \end{array}$$

- Orbital ang. mom. has eigenfunc^s of spherical harmonics

$Y_0^0 \propto 1$ (S wave)

$Y_1^0 \propto \cos\theta$ (P wave)

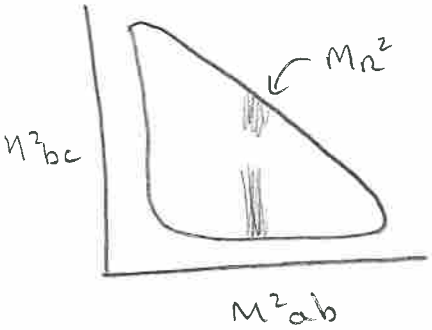
$Y_2^0 \propto \cos^2\theta$ (D wave)

[θ is angle btw a and c]

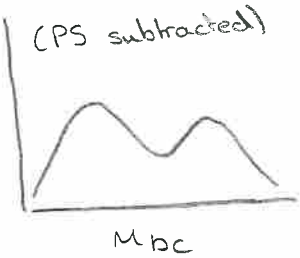
$\Rightarrow$ Spin-S resonances have $\cos^S\theta$ dependence in amplitude
$\to$ S gaps in resonance band

Reflections

BEWARE



- R is spin-1 in M^2_{bc}
- looking in 1D at M_{bc} we see 2 peaks
  - we should not conclude we have found 2 new states that decay to bc!
  - In reality we have a spin-1 resonance in ab



(Slide 14)

CP violation in SM

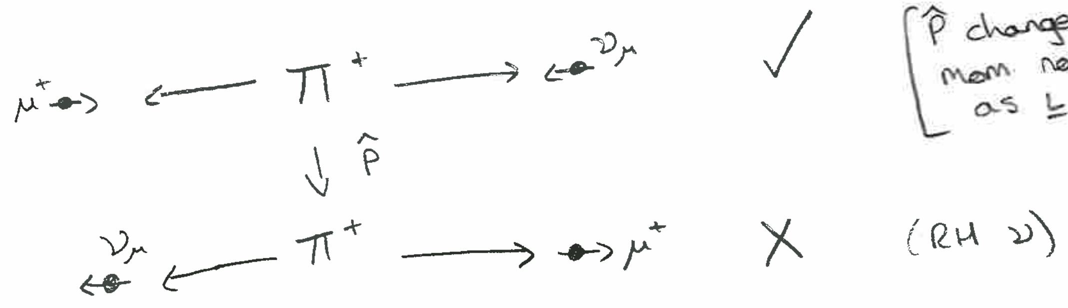
- Prior to 1956 was thought laws of physics invariant under the parity operator $\hat{P}$. Particles have well defined parity

$$\hat{P}\,\psi(\underline{r}) = \psi(-\underline{r}) = \eta\,\psi(\underline{r})$$

$$\eta = \pm 1 \quad \text{as} \quad \hat{P}(\hat{P}\psi(\underline{r})) \equiv \psi(\underline{r})$$

$$\text{eg. } \hat{P}|\pi^+\rangle = -|\pi^+\rangle \qquad J^P = 0^-$$

- Weak interaction $\pi^+ \rightarrow \mu^+ \nu_\mu$ violates $\hat{P}$ sym.



[$\hat{P}$ changes sign of mom. not spin vector as $\underline{L} = \underline{r} \times \underline{p}$]

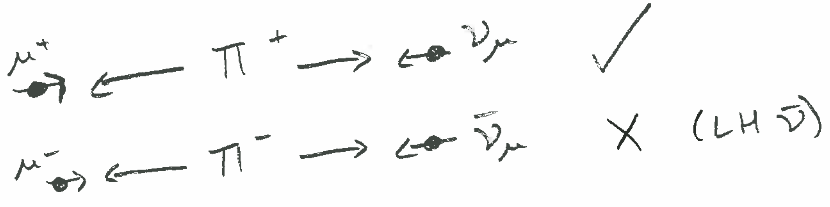
- Charge conj. operator $\hat{C}$ replaces particles with antiparticles

$$\hat{C}|\pi^+\rangle = |\pi^-\rangle \qquad \text{NOT AN E-STATE OF } \hat{C}$$

- Some particles are their own antiparticle ($\hat{C}$ e-state)

$$\hat{C}|\pi^0\rangle = +|\pi^0\rangle \qquad J^{PC} = 0^{-+}$$

- $\hat{C}$ sym is also violated in weak decays



- With 2 gen. physics invariant under combined $\hat{C}\hat{P}$
- Last operator is $\hat{T}$

$$\hat{T}\psi(\underline{r},t) = \psi(\underline{r},-t)$$

- $\hat{C}\hat{P}\hat{T}$ combination is sym of any Lorentz inv. gauge field theory

- Note parity conservation means both the sym under $\hat{P}$ and the conserved quantity (same for $\hat{C}$). This is different to $\hat{T}$ sym and conservation of energy but its the same thing.

- QED and QCD preserve P and C (respect $\hat{C}$ and $\hat{P}$ sym) but weak int does not

CPV in kaons

- Violation of $\hat{C}\hat{P}$ seen in kaons first
- Physical neutral kaons are admixtures of flavour estates

$$|K_1\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}} \qquad |K_2\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}$$

$$\hat{P}|K^0\rangle = -|K^0\rangle \qquad \hat{C}|K^0\rangle = |\bar{K}^0\rangle \qquad \hat{C}\hat{P}|K^0\rangle = -|\bar{K}^0\rangle$$

$$\Rightarrow \quad \hat{C}\hat{P}|K_{1,2}\rangle = \pm|K_{1,2}\rangle \quad \hat{C}\hat{P} \text{ e-states!}$$

- $|K_1\rangle$ and $|K_2\rangle$ have decay modes that conserve CP

$$\overset{0^-}{K_1} \to \overset{0^-}{\pi^+}\ \overset{0^-}{\pi^-} \quad l=0 \qquad K_2 \to \pi^+\pi^-\pi^0$$

CP : even   even   CP: odd   odd

$$\left[\begin{array}{l} P = (-)\cdot(-)\cdot(-1)^L \\ C = 1 \end{array}\right]$$

- $|K_1\rangle$ is shorter lived as more PS $|K_1\rangle \equiv |K_S\rangle$ $|K_2\rangle \equiv |K_L\rangle$

- To conserve CP $K_L^0 \to \pi^+\pi^-$ should be forbidden...
  - Beam of $|K^0\rangle = \frac{|K_S\rangle + |K_L\rangle}{\sqrt{2}}$ from weak decay
  - should only see decays to 3 pions far from production point   (Slide 15+16)
  - We can show obs. of CPV is indirect evidence of 3rd quark generation

CKM matrix

- V_{CKM} is 3×3 complex matrix $= 18$ dof
  - $V^\dagger V = I$ unitarity implies n^2 constraints
    - (n) for diag elements $= 1$
    - $(n^2 - n)$ for off-diag elements $= 0$
  - Leaves 9 dof
    - 3 are mixing angles $\theta_{12} (= \theta_c), \theta_{13}, \theta_{23}$
    - 6 are left as possible complex phases
- We can absorb 5/6 phases into quark fields along with rephasing of V_{CKM}

  eg. $u_L \to e^{i\phi_u} u_L$, $d_L \to e^{i\phi_d} d_L$, $V_{ud} \to e^{i\phi_u} V_{ud} e^{-i\phi_d}$

  $\mathcal{L}_{SM}$ terms are left invariant (see $\mathcal{L}_{cc}$) with this rephasing
- Left with one complex phase, δ, to give CPV in SM

	N	2	3
Mixing angles	$N(N-1)/2$	1	3
Complex phases	$(N-1)(N-2)/2$	0	1
		↑ no CPV	↑ CPV

1973	1977	1995
CPV explained by CKM giving indirect evidence for 3rd gen	$\Upsilon(b\bar{b})$ obs. at Fermilab	t obs. at CDF and D0.

(Slide 17).

- 1995 until all 6 quarks directly observed!