Flavour Physics (of quarks)

Part 2: Mixing and CP violation

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Warwick Week Graduate Lectures

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Overview

Lecture 1: Flavour in the SM

- Flavour in the SM
- Quark Model History
- The CKM matrix

Lecture 2: Mixing and CP violation (Today)

- Neutral Meson Mixing (no CPV)
- B-meson production and experiments
- CP violation

Lecture 3: Measuring the CKM parameters

- Measuring CKM elements and phases
- Global CKM fits
- CPT and T-reversal
- Dipole moments

Lecture 4: Flavour Changing Neutral Currents

- Effective Theories
- New Physics in B mixing
- New Physics in rare $b \to s$ processes
- Lepton Flavour Violation

1. Recap

Recap

- Last time we introduced the role of flavour in the SM
- We saw how measurements of meson decays led to the predictions and subsequent discoveries of strange, charm, beauty and top decays
- We saw how various meson and baryon states are built out of the consitituent quarks
- We introduced the CKM matrix (much more on that in the next two lectures)

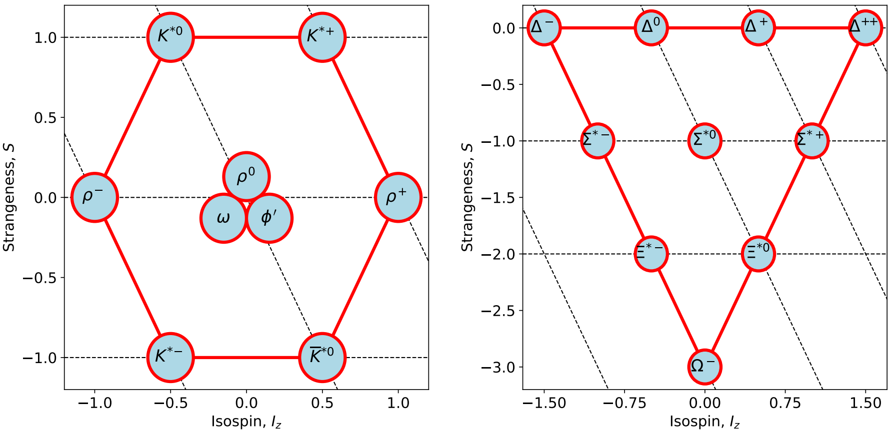
Discuss any points from the problem sheets

1. Can you explain the 2:1 ratio:

$$\sigma(p + p \to d + \pi^+) : \sigma(p + n \to d + \pi^0) = 2 : 1?$$

2. What do the spin-1 and spin-3/2 multiplets look like?

Higher resonance multiplets



Let's talk about these states, their decays and how we detect them

Recap

- Recall the CKM matrix which governs quark weak transitions

CKM exhibits a clear hierarchy

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix}$$

experimentally
determined values

Commonly represented in the Wolfenstein parametrisation

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

4 $\mathcal{O}(1)$ real parameters (A, λ, ρ, η)

Recap

- Wolfenstein parameterisation ensures that

$$\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*) \tag{1}$$

is phase convention independent and CKM matrix written in $(A, \lambda, \bar{\rho}, \bar{\eta})$ is unitary to all orders in λ

$$\bar{\rho} = \rho(1 - \lambda^2/2 + \dots) \quad \text{and} \quad \bar{\eta} = \eta(1 - \lambda^2/2 + \dots) \tag{2}$$

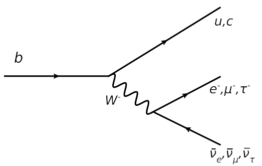
- The amount of CP violation in the SM is equivalent to asking
$\rightarrow$ how big is η relative to ρ?

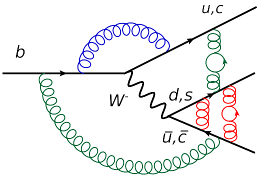
2. Weak decays of heavy hadrons

The free c- and b-quark decays

The heavy hadrons (b and c) decay via the charged weak interaction

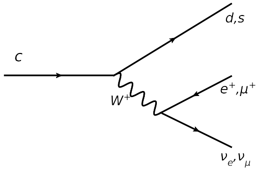
free b-quark tree-level decay [i]

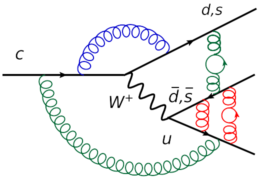




$$b \to \left\{ \begin{array}{c} c \\ u \end{array} \right. + W^- \to \left\{ \begin{array}{c} c \\ u \end{array} \right. + \left\{ \begin{array}{l} \bar{u} + d \\ \bar{c} + s \\ \bar{u} + s \\ \bar{c} + d \\ e^- + \bar{\nu}_e \\ \mu^- + \bar{\nu}_\mu \\ \tau^- + \bar{\nu}_\tau \end{array} \right.$$

free c-quark tree-level decay





$$c \to \left\{ \begin{array}{c} s \\ d \end{array} \right. + W^+ \to \left\{ \begin{array}{c} s \\ d \end{array} \right. + \left\{ \begin{array}{l} \bar{d} + u \\ \bar{d} + s \\ e^+ + \nu_e \\ \mu^+ + \nu_\mu \end{array} \right.$$

[i] Figures stolen from A. Lenz

The free c- and b-quark decays

- Final state quarks lead to sizeable QCD-corrections (gluon lines in Feynman diagrams) trigged by quark transition with $W^{\pm}$ exchange
- The basic vertex is

$$W^{+}: i\frac{g}{2\sqrt{2}}\gamma_{\mu}(1-\gamma_{5})V_{xy} \text{ and } W^{-}: i\frac{g}{2\sqrt{2}}\gamma_{\mu}(1-\gamma_{5})V_{xy}^{*} \tag{3}$$

- The couplings, V_{xy}, are the CKM elements which as we have seen are hierarchical (decays between generations are suppressed)
- There **is no tree level flavour changing neutral current (FCNC)** - can only happen at loop level
- These loop level processes are called "penguin" decays (we'll see more later) and if the tree-level process is heavily CKM suppressed they can be dominant
- In principle it is relatively straightforward for theorists to make predictions for "inclusive" decays considering only the bare quarks, *e.g.* $b \to c\bar{c}s$
- For experimentalists it is **much** easier to measure "exclusive" modes in which every final state hadron is identified, *e.g.* $\overline{B}^{0} \to D^{+}D^{-}$

HOMEWORK: Can you think about why? What are the theory / experiment trade-offs?

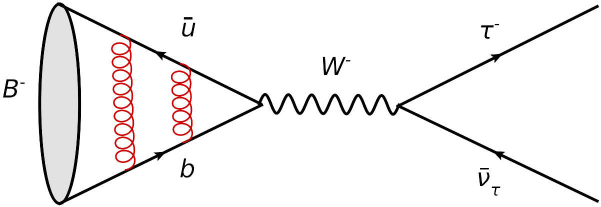
Weak decays of heavy hadrons

- In reality we don't see free quarks
- Meson decays are more (theoretically) complicated because of the (non-perturbative) strong interactions [ii]
- If we classify some common weak heavy hadron decays we can see what the phenomenological implications are
  - Leptonic decays
  - Semileptonic decays
  - Hadronic decays

[ii] It is non-perturbative because the exchange of one gluon is as important (as large) as the exchange of many

Leptonic Decays

- Only leptons in the final states
- Initial state is a hadron bound with gluons

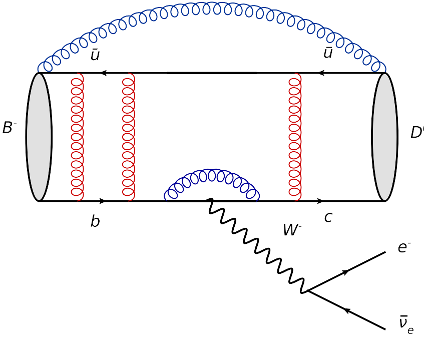


- Non-perturbative effects described by a **decay constant**, f_B, where

$$i f_{B_q} p^\mu = \langle 0 | \bar{b} \gamma^\mu \gamma_5 u | B_q(p) \rangle \tag{4}$$

- Lattice QCD can make very precise predictions of leptonic decay constants

- Leptons and hadrons in the final state (gluon lines in initial and final state)



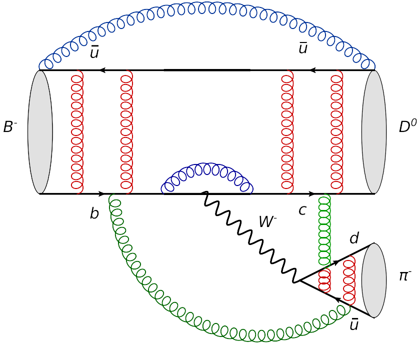
[iii]

- Non-perturbative effects described by **form factors**, $f_+(q^2)$ and $f_0(q^2)$ which depend on the momentum transfer, q^2
- Predictions can be made by either QCD sum rules or Lattice QCD (but generally in different domains of q^2 and not always in agreement)

$$\langle D^0(p_D)|\bar{c}\gamma^\mu\gamma_5 b|B^-(p_B)\rangle = f_+(q^2)\left(p_B^\mu + p_D^\mu - \frac{m_B^2 - m_D^2}{q^2}q^\mu\right) + f_0(q^2)\frac{m_B^2 - m_D^2}{q^2} \quad (5)$$

[iii] The description becomes more complex when there are $>$ 0-spin final states

- Only hadrons in the final state (gluon lines in initial and final state)



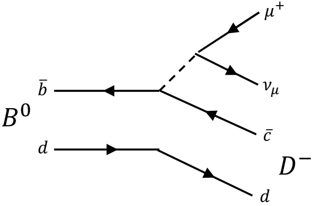
- Can only be treated with additional assumptions that allow for a *factorisation* (a **decay constant** f_π and a **form factor**, $f(q^2)$)
- Sometimes the factorisation assumption works, sometimes not (depends on mass)
- New non-perturbative objects arise called distribution amplitudes

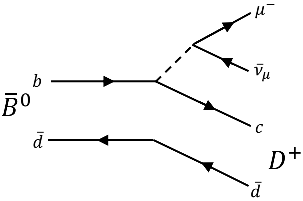
$$\begin{aligned}
&\langle D^0\pi^-|\bar{c}\gamma_\mu(1-\gamma_5)b\cdot\bar{u}\gamma^m u(1-\gamma_5)d|B^-\rangle \\
&\qquad\approx \langle D^0|\bar{c}\gamma_\mu(1-\gamma_5)b|B^-\rangle\cdot\langle\pi^-|\bar{u}\gamma^\mu(1-\gamma_5)d|0\rangle \\
&\qquad\approx f^{B\to D}(q^2)\cdot f_\pi
\end{aligned} \tag{6}$$

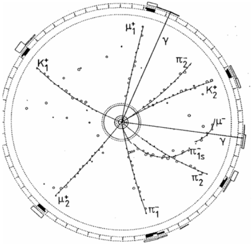
3. Neutral Meson Mixing (no CPV)

Neutral Meson Mixing

- In 1987 the ARGUS experiment coherently produced $B^0 - \overline{B}^0$ pairs and observed them decaying to same sign leptons [1]
- How is this possible?
  - Semileptonic decays "tag" the flavour of the initial state
  - *i.e.* the charge of the lepton (and hadrons from the $D^{\pm}$) tag the flavour of the b-quark in the B^0



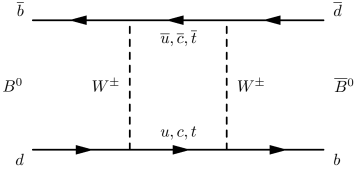


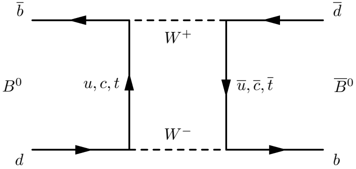


- The only explanation is that B^0–$\overline{B}^0$ can oscillate
- Rate of mixing is large $\rightarrow$ top quark must be heavy

Neutral Meson Mixing

- In the SM occurs via box diagrams involving a charged current ($W^{\pm}$) interaction
- Weak eigenstates are not the same as the physical mass eigenstates
  - The particle and antiparticle flavour states (via CPT theorem) have equal and opposite charge, identical mass and identical lifetimes
  - But the mixed states (*i.e.* the physical B_L^0 and B_H^0) can have $\Delta m, \Delta\Gamma \neq 0$





- In the SM we have four possible neutral meson states
  - K^0, D^0, B^0, B_s^0 (mixing has been observed in all four)
  - Although they all have rather different properties (as we will see in a second)

Coupled meson systems

- A single particle system evolves according to the time-dependent Schrödinger equation

$$i\frac{\partial}{\partial t}|X(t)\rangle = \mathcal{H}|X(t)\rangle = \left(M - i\frac{\Gamma}{2}\right)|X(t)\rangle \tag{7}$$

- For neutral mesons, mixing leads to a coupled system

$$i\frac{\partial}{\partial t}\begin{pmatrix}|B^0\rangle \\ |\overline{B}^0\rangle\end{pmatrix} = \mathcal{H}\begin{pmatrix}|B^0\rangle \\ |\overline{B}^0\rangle\end{pmatrix} = \left(\boldsymbol{M} - i\frac{\boldsymbol{\Gamma}}{2}\right)\begin{pmatrix}|B^0\rangle \\ |\overline{B}^0\rangle\end{pmatrix} \tag{8}$$

$$= \begin{pmatrix}M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2\end{pmatrix}\begin{pmatrix}|B^0\rangle \\ |\overline{B}^0\rangle\end{pmatrix} \tag{9}$$

- The off-diagonal terms arise because of mixing
  - Flavour eigenstates are not mass eigenstates
- Not all the parameters are independent

$$\begin{gathered}M_{11} = M_{22} \text{ and } \Gamma_{11} = \Gamma_{22} \text{ (CPT invariance)} \\ M_{21} = M_{21}^* \text{ and } \Gamma_{21} = \Gamma_{12}^* \text{ (Hermicity)}\end{gathered} \tag{10}$$

- To obtain the mass states we diagonalise the matrix
- To start with we will neglect CP-violation in mixing (approximately the case for all four neutral meson species)
- Neglecting CP-violation, the physical states are an equal mixture of the flavour states

$$|B_L^0\rangle = \frac{|B^0\rangle + |\overline{B}^0\rangle}{2}, \qquad |B_H^0\rangle = \frac{|B^0\rangle - |\overline{B}^0\rangle}{2}$$

with mass and width differences

$$\Delta\Gamma = \Gamma_H - \Gamma_L = 2|\Gamma_{12}|, \quad \Delta M = M_H - M_L = 2|M_{12}|$$

so that the physical system evolves as

$$i\frac{\partial}{\partial t}\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} = \mathcal{H}\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} = \left(\boldsymbol{M} - i\frac{\boldsymbol{\Gamma}}{2}\right)\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} \tag{11}$$

$$= \begin{pmatrix}M_L - i\Gamma_L/2 & 0 \\ 0 & M_H - i\Gamma_H/2\end{pmatrix}\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} \tag{12}$$

Time evolution

- The time evolution of the mass eigenstates (either $|{B^0}_H\rangle$ or $|{B^0}_L\rangle$ at $t=0$) is trivial

$$|{B^0}_{H,L}(t)\rangle = e^{-iM_{H,L}}e^{-i\Gamma_{H,L}}|{B^0}_{H,L}\rangle \tag{13}$$

- Time evolution of the flavour eigenstates comes from solving the Schrödinger equation, Eq. 7 (a useful homework exercise)
- For a pure flavour state $|B^0\rangle$ or $|\overline{B}^0\rangle$ at time $t=0$

$$\begin{aligned}|B^0(t)\rangle &= g_+(t)|B^0\rangle + g_-(t)|\overline{B}^0\rangle \\ |\overline{B}^0(t)\rangle &= g_+(t)|\overline{B}^0\rangle + g_-(t)|B^0\rangle\end{aligned} \tag{14}$$

where

$$\begin{aligned}g_+(t) &= e^{-iMt}e^{-\Gamma t/2}\left[\quad\cosh\left(\frac{\Delta\Gamma t}{4}\right)\cos\left(\frac{\Delta m t}{2}\right) - i\sinh\left(\frac{\Delta\Gamma t}{4}\right)\sin\left(\frac{\Delta m t}{2}\right)\right] \\ g_-(t) &= e^{-iMt}e^{-\Gamma t/2}\left[-\sinh\left(\frac{\Delta\Gamma t}{4}\right)\cos\left(\frac{\Delta m t}{2}\right) + i\cosh\left(\frac{\Delta\Gamma t}{4}\right)\sin\left(\frac{\Delta m t}{2}\right)\right]\end{aligned} \tag{15}$$

and $M = (M_L + M_H)/2$ and $\Gamma = (\Gamma_L + \Gamma_H)/2$

- We will see these equations again when we discuss CP-violation in mixing

Time evolution

- Using Eq. (15) flavour remains unchanged $(+)$ or will oscillate $(-)$ with probability

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \tag{16}$$

- With no CP violation in the mixing, the time-integrated mixing probability is

$$\frac{\int |g_{-}(t)|^2 dt}{\int |g_{-}(t)|^2 dt + \int |g_{+}(t)|^2 dt} = \frac{x^2 + y^2}{2(x^2+1)} \tag{17}$$

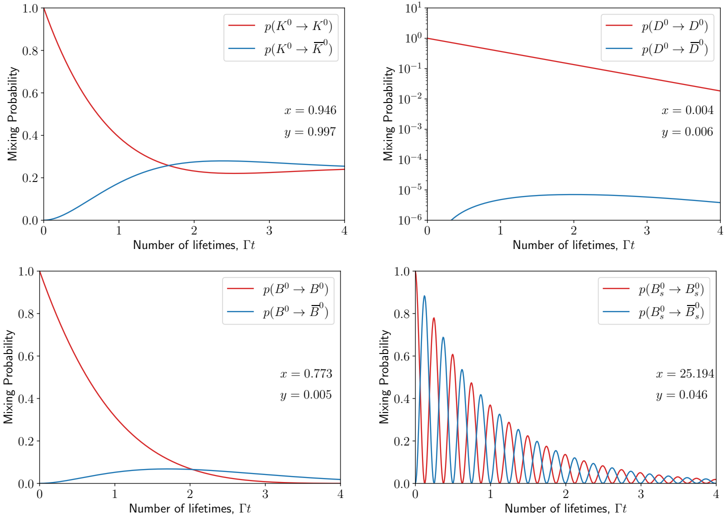
where

$$x = \frac{\Delta m}{\Gamma} \quad \text{and} \quad y = \frac{\Delta\Gamma}{2\Gamma} \tag{18}$$

- The four different neutral meson species which mix have very different values of (x, y) and therefore very different looking time evolution properties

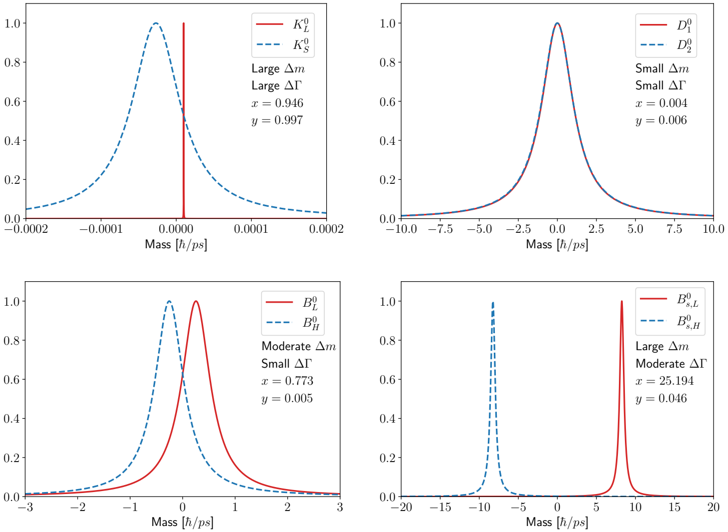
Neutral Meson Mixing

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \tag{19}$$

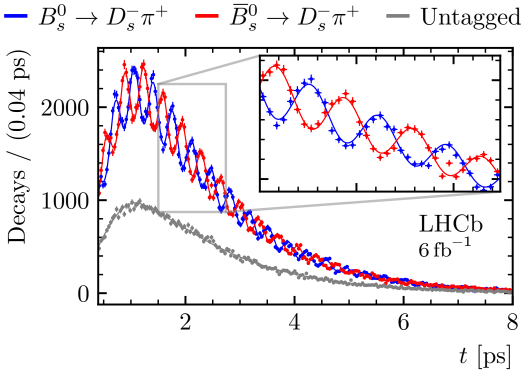


Neutral Meson Mixing

- Mass and width differences of the neutral meson mixing systems



- Very nice demonstration of the B_s^0 oscillation from the LHCb experiment [2]
- Seen in $B_s^0 \to D_s^- \pi^+$ decays
- Tag the flavour of the initial state at **production** and compare to the flavour at decay (the $D_s^- \pi^+$ final state tags the decaying flavour)
- HOMEWORK: Why is this so different from the plot on the previous slide (damped oscillation and turn on at low values)?

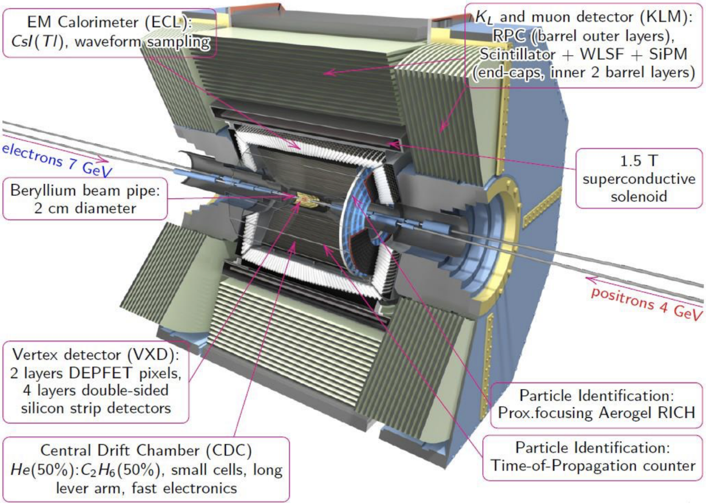


4. B-meson production and experiments

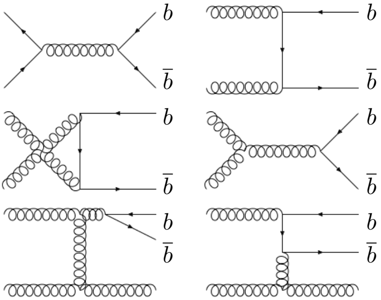
B-factories at the $\Upsilon(4S)$

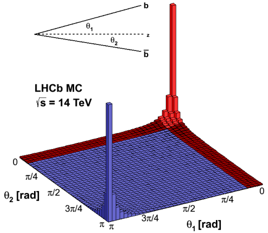
- Asymmetric e^+e^- colliders
- Produce excited $\Upsilon(4S)$ resonance (10.58 GeV) which decays strongly and produces a coherent pair of $B^0\overline{B}^0$ (50%) or B^+B^- pair (50%) moving in the lab frame
  - BaBar produced $\sim 500\text{M}\ B\overline{B}$ pairs in $\sim 530\,\text{fb}^{-1}$ of data from 9 GeV and 3.1 GeV beams at SLAC [3]
  - Belle produced $\sim 770\text{M}\ B\overline{B}$ pairs in $\sim 710\,\text{fb}^{-1}$ of data from 8 GeV and 3.5 GeV beams at KEK [4]
  - Belle-II expected to produce up to $\sim 50\text{B}\ B\overline{B}$ pairs in $\sim 50\,\text{ab}^{-1}$ of data [5]
- Very clean environments but notice that the B_s^0 is not in range of the $\Upsilon(4S)$ resonance. This requires specific running at the $\Upsilon(5S)$.
  - In comparison to LHCb, $B\overline{B}$ pairs are not produced at high boost which makes resolution of B_s^0 oscillations impossible at B-factories
- Because B mesons are produced in pairs from a known resonance you get very high flavour tagging power and very good resolution for missing energy (*i.e.* final state neutrals)
- For Belle-II to acheieve desired luminosity requires incredible squeezing of the beam (target is $8 \times 10^{35}\text{cm}^{-2}\text{s}^{-1}$ which is 40 $\times$ Belle)

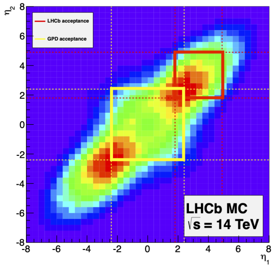
Belle-II Experiment

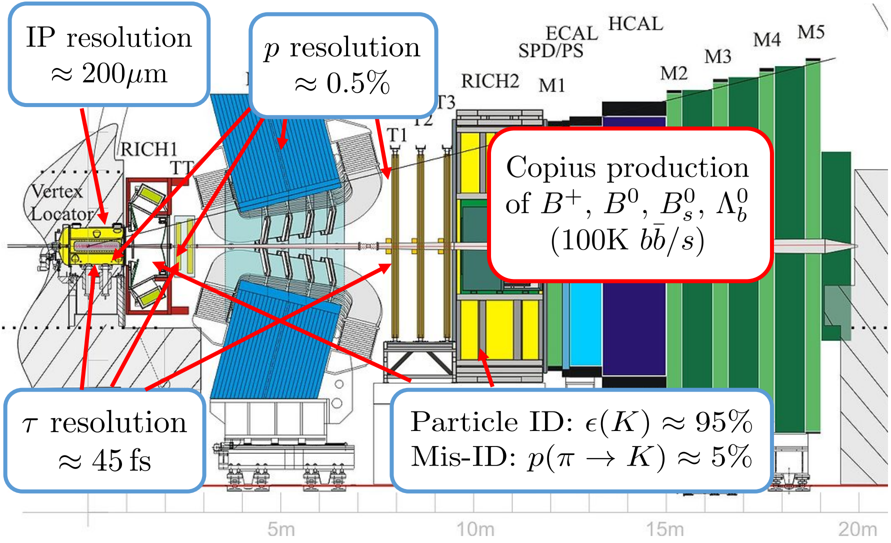


- The LHC is predominantly a gluon collider
- b-quarks are produced in pairs and predominantly in the forward region with a very large boost
  - Hence the very forward geometry of LHCb
- The very large boost and very high quality vertexing makes decay time measurements much easier
  - Can resolve the very rapid B_s^0 oscillations









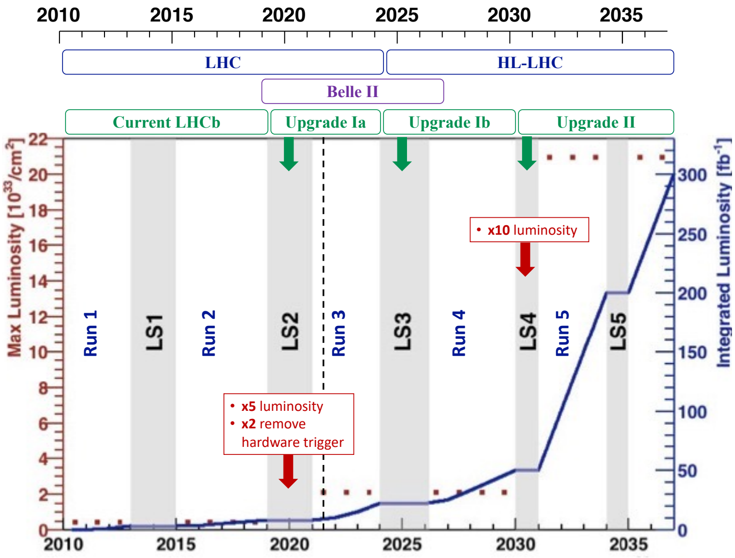
IP resolution $\approx 200\mu$m

p resolution $\approx 0.5\%$

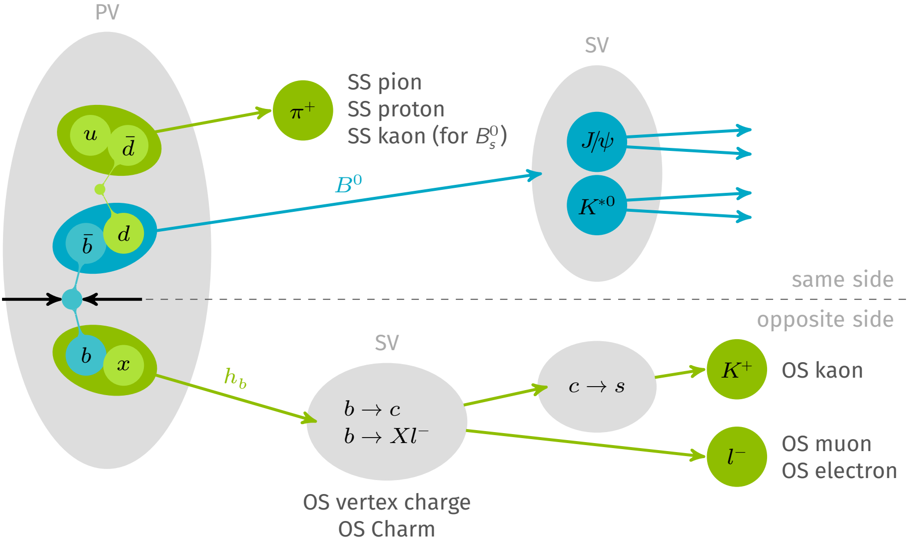
Copius production of B^+, B^0, B^0_s, Λ^0_b (100K $b\bar{b}/s$)

τ resolution $\approx 45\,$fs

Particle ID: $\epsilon(K) \approx 95\%$
Mis-ID: $p(\pi \to K) \approx 5\%$



- COVID has pushed back future schedule by one year and extended Run 3 by one year


PV
SV
SS pion
SS proton
SS kaon (for B_s^0)
B^0
J/ψ
K^{*0}
same side
opposite side
u $\bar{d}$
$\bar{b}$ d
b x
π^+
h_b
SV
$b \to c$
$b \to Xl^-$
$c \to s$
K^+
OS kaon
l^-
OS muon
OS electron
OS vertex charge
OS Charm

Dalitz plot formalism

- For a nice overview of this, take a look at Sec. 2 of [arXiv:1711.09854] [6]
- Provides a nice method and visualisation of 3-body decays, *e.g.* $B \to XYZ$
- The n-body decay rate is

$$d\Gamma = \frac{(2\pi)^4}{2M} |\mathcal{M}|^2 d\phi(p_1, p_2, \ldots, p_n) \tag{20}$$

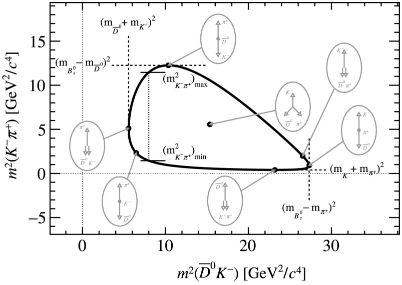
- So for a 3-body decay

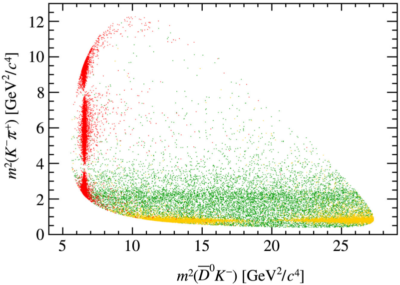
$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} \overline{|\mathcal{M}|}^2 dm_{12}^2 dm_{23}^2 \tag{21}$$

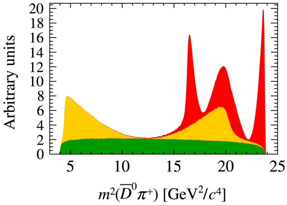
- Note how 3-body phase-space is flat in the Dalitz plot
- Resonances appear as bands in the Dalitz plot where The number of "lobes" in the Dalitz plot is related to the particle spin
  - Spin-0 "scalar" contributions have 1 lobe
  - Spin-1 "vector" contributions have 2 lobes
  - Spin-2 "tensor" contributions have 3 lobes

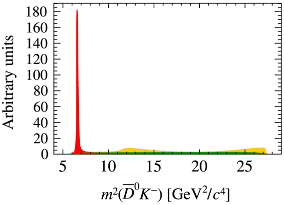
Dalitz plot formalism

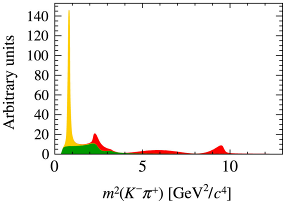
- Example shown for a $B^0 \to \overline{D}^0 K^- \pi^+$ decay











5. CP violation

Measuring CP violation

1. Need at least two interfering amplitudes
2. Need two phase differences between them
   - One CP conserving ("strong") phase difference (δ)
   - One CP violating ("weak") phase difference (ϕ)

- If there is only a single path to a final state, f, then we cannot get direct CP violation
- If there is only one path we can write the amplitudes for decay as

$$\mathcal{A}(B \to f) = A_1 e^{i(\delta_1 + \phi_1)}$$
$$\mathcal{A}(\bar{B} \to \bar{f}) = A_1 e^{i(\delta_1 - \phi_1)}$$

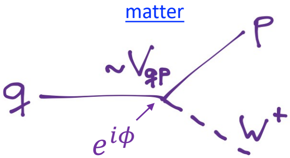
- Which gives an asymmetry of

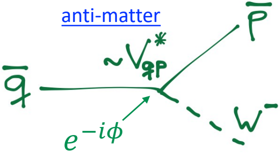
$$\mathcal{A}_{CP} = \frac{|\mathcal{A}(\overline{B} \to \bar{f})|^2 - |\mathcal{A}(B \to f)|^2}{|\mathcal{A}(\overline{B} \to \bar{f})|^2 + |\mathcal{A}(B \to f)|^2} = 0 \qquad (22)$$

- In order to observe CP-violation we need a second amplitude.
- This is often realised by having interefering tree and penguin amplitudes

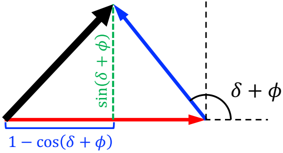
Measuring CP-violation

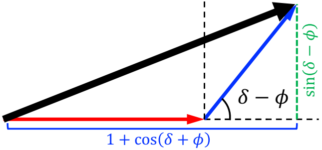
- We measure quark couplings which have a complex phase
- This is only visible when there are two amplitudes





- Below we represent two amplitudes (red and blue) with the same magnitude $= 1$
  - The strong phase difference is, $\delta = \pi/2$
  - The weak phase difference is, $\phi = \pi/4$





$$\Gamma(B \to f) = |A_1 + A_2 e^{i(\delta+\phi)}|^2 \qquad \Gamma(\bar{B} \to \bar{f}) = |A_1 + A_2 e^{i(\delta-\phi)}|^2$$

Measuring (direct) CP-violation

- Introducing the second amplitude we now have

$$\mathcal{A}(B \to f) = A_1 e^{i(\delta_1+\phi_1)} + A_2 e^{i(\delta_2+\phi_2)} \tag{23}$$

$$\mathcal{A}(\bar{B} \to \bar{f}) = A_1 e^{i(\delta_1-\phi_1)} + A_2 e^{i(\delta_2-\phi_2)} \tag{24}$$

- Which gives an asymmetry of

$$\mathcal{A}_{CP} = \frac{|\mathcal{A}(\overline{B} \to \bar{f})|^2 - |\mathcal{A}(B \to f)|^2}{|\mathcal{A}(\overline{B} \to \bar{f})|^2 + |\mathcal{A}(B \to f)|^2} \tag{25}$$

$$= \frac{4A_1A_2\sin(\delta_1-\delta_2)\sin(\phi_1-\phi_2)}{2A_1^2+2A_2^2+4A_1A_2\cos(\delta_1-\delta_2)\cos(\phi_1-\phi_2)} \tag{26}$$

$$= \boxed{\frac{2r\sin(\delta)\sin(\phi)}{1+r^2+2r\cos(\delta)\cos(\phi)}} \tag{27}$$

where $r = A_1/A_2$, $\delta = \delta_1 - \delta_2$ and $\phi = \phi_1 - \phi_2$

- This is only non-zero if the amplitudes have **different** weak **and** strong phases
- This is CP-violation in decay (often called "direct" CP violation).
  - This is the only possible route of CP violation for a charged initial state
  - For a neutral initial state there are also other ways of realising CP violation

- Let's extend our formalism of neutral mixing, Eqs. (14–18), to include CP violation
- Allowing for CP violation, $M_{12} \neq M_{12}^*$ and $\Gamma_{12} \neq \Gamma_{12}^*$
- The physical states can now be unequal mixtures of the weak states

$$\begin{aligned} |B_L^0\rangle &= p|B^0\rangle + q|\overline{B}^0\rangle \\ |B_H^0\rangle &= p|B^0\rangle - q|\overline{B}^0\rangle \end{aligned} \tag{28}$$

where

$$|p|^2 + |q|^2 = 1$$

- The states now have mass and width differences

$$|\Delta M| \approx 2|M_{12}|, \quad |\Delta\Gamma| \approx 2|\Gamma_{12}|\cos(\phi), \quad \phi = \arg(-M_{12}/\Gamma_{12}) \tag{29}$$

- The $g_\pm(t)$, Eq. (15), are as before but the probabilities to remain / change flavour are

Remain:
$$\boxed{\begin{aligned} |\langle B^0|B^0(t)\rangle|^2 &= |g_+(t)|^2 \\ |\langle \overline{B}^0|\overline{B}^0(t)\rangle|^2 &= |g_+(t)|^2 \end{aligned}} \tag{30}$$

Change:
$$\boxed{\begin{aligned} |\langle \overline{B}^0|B^0(t)\rangle|^2 &= \left|\frac{q}{p}\right|^2 |g_-(t)|^2 \\ |\langle B^0|\overline{B}^0(t)\rangle|^2 &= \left|\frac{p}{q}\right|^2 |g_-(t)|^2 \end{aligned}} \tag{31}$$

Classification of CP violation

- In addition to CPV in decay and CPV in mixing we must now also consider CPV in the interference between mixing and decay
- First let's consider a generalised form of a neutral meson, X^0, decaying to a final state, f
- There are four possible amplitudes to consider

$$A_f = A(X^0 \to f) = \langle f | X^0 \rangle \qquad \bar{A}_f = A(\bar{X}^0 \to f) = \langle f | \bar{X}^0 \rangle$$
$$A_{\bar{f}} = A(X^0 \to \bar{f}) = \langle \bar{f} | X^0 \rangle \qquad \bar{A}_{\bar{f}} = A(\bar{X}^0 \to \bar{f}) = \langle \bar{f} | \bar{X}^0 \rangle$$

- Define a complex parameter, λ_f (**not** the Wolfenstein parameter, λ) which encapsulates CPV in the whole process

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}, \qquad \bar{\lambda}_f = \frac{1}{\lambda_f}, \qquad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}, \qquad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}}$$

Generalised Meson Decay Formalism

The time-dependent decay rate, $\Gamma_{X^0\to f}(t) = |\langle f|X^0(t)\rangle|^2$

- contains terms for CPV in decay, mixing and the interference between the two

$$\Gamma_{X^0\to f}(t) = \boxed{|A_f|^2} \left(\boxed{|g_+(t)|^2} + \boxed{|\lambda_f|^2}\boxed{|g_-(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_f g_+^*(t) g_-(t)\right]}\right) \quad (32)$$

$$\Gamma_{X^0\to \bar{f}}(t) = \boxed{|\bar{A}_{\bar{f}}|^2}\boxed{\left|\frac{q}{p}\right|^2} \left(\boxed{|g_-(t)|^2} + \boxed{|\bar{\lambda}_{\bar{f}}|^2}\boxed{|g_+(t)|^2} + \boxed{2\mathcal{R}e\left[\bar{\lambda}_{\bar{f}} g_+(t) g_-^*(t)\right]}\right) \quad (33)$$

$$\Gamma_{\overline{X}^0\to f}(t) = \boxed{|A_f|^2}\boxed{\left|\frac{p}{q}\right|^2} \left(\boxed{|g_-(t)|^2} + \boxed{|\lambda_f|^2}\boxed{|g_+(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_f g_+(t) g_-^*(t)\right]}\right) \quad (34)$$

$$\Gamma_{\overline{X}^0\to \bar{f}}(t) = \boxed{|\bar{A}_{\bar{f}}|^2} \left(\boxed{|g_+(t)|^2} + \boxed{|\bar{\lambda}_{\bar{f}}|^2}\boxed{|g_-(t)|^2} + \boxed{2\mathcal{R}e\left[\bar{\lambda}_{\bar{f}} g_+^*(t) g_-(t)\right]}\right) \quad (35)$$

where the mixing probabilities are as before

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \quad (36)$$

$$g_+^* g_- = \frac{e^{-\Gamma t}}{2}\left[\sinh\left(\frac{\Delta\Gamma t}{2}\right) + i\sin(\Delta m t)\right] \quad (37)$$

$$g_+ g_-^* = \frac{e^{-\Gamma t}}{2}\left[\sinh\left(\frac{\Delta\Gamma t}{2}\right) - i\sin(\Delta m t)\right] \quad (38)$$

Generalised Meson Decay Formalism

The "master equations" for neutral meson decays

$$\Gamma_{X^0\to f}(t) = |A_f|^2 \qquad (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) + C_f\cos(\Delta m t) + D_f\sinh(\tfrac{1}{2}\Delta\Gamma t) - S_f\sin(\Delta m t)\Big] \tag{39}$$

$$\Gamma_{\overline{X}^0\to f}(t) = |A_f|^2\left|\frac{p}{q}\right|^2(1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) - C_f\cos(\Delta m t) + D_f\sinh(\tfrac{1}{2}\Delta\Gamma t) + S_f\sin(\Delta m t)\Big] \tag{40}$$

$$\Gamma_{X^0\to \bar{f}}(t) = |\bar{A}_{\bar{f}}|^2\left|\frac{q}{p}\right|^2(1+|\bar{\lambda}_{\bar{f}}|^2)\frac{e^{-\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) - C_{\bar{f}}\cos(\Delta m t) + D_{\bar{f}}\sinh(\tfrac{1}{2}\Delta\Gamma t) + S_{\bar{f}}\sin(\Delta m t)\Big] \tag{41}$$

$$\Gamma_{\overline{X}^0\to \bar{f}}(t) = |\bar{A}_{\bar{f}}|^2 \qquad (1+|\bar{\lambda}_{\bar{f}}|^2)\frac{e^{-\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) + C_{\bar{f}}\cos(\Delta m t) + D_{\bar{f}}\sinh(\tfrac{1}{2}\Delta\Gamma t) - S_{\bar{f}}\sin(\Delta m t)\Big] \tag{42}$$

where

$$\boxed{C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1+|\lambda_f|^2}, \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2}} \tag{43}$$

Classification of CP violation

Can realise CP violation in three ways:

1. CP violation in decay
   - For a charged initial state this is only the type possible

$$\Gamma(X^0 \to f) \neq \Gamma(\bar{X}^0 \to \bar{f}) \Longrightarrow \boxed{\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1} \tag{44}$$

2. CP violation in mixing

$$\Gamma(X^0 \to \bar{X}^0) \neq \Gamma(\bar{X}^0 \to X^0) \Longrightarrow \boxed{\left|\frac{p}{q}\right| \neq 1} \tag{45}$$

3. CP violation in the interference between mixing and decay

$$\begin{array}{c} \Gamma([X^0 \to f] + [X^0 \to \bar{X}^0 \to f]) \\ \neq \\ \Gamma([\bar{X}^0 \to \bar{f}] + [\bar{X}^0 \to X^0 \to \bar{f}]) \end{array} \Longrightarrow \arg(\lambda_f) = \boxed{\arg\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0} \tag{46}$$

- If CPV in mixing is very small which **is the case** for the D^0, B^0 and B_s^0 systems
- Then the **time-dependent** CP **asymmetry is**

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{\frac{2C_f\cos(\Delta m t) - 2S_f\sin(\Delta m t)}{2\cosh(\frac{1}{2}\Delta\Gamma t) + 2D_f\sinh(\frac{1}{2}\Delta\Gamma t)}} \tag{47}$$

- Often we exploit final states which are themselves CP-even eigenstates, *i.e.* $f = \bar{f}$ (*e.g.* $B_s^0 \to J/\psi\phi$ and $B^0 \to J/\psi K_S^0$)
- In these cases there is one CP asymmetry (the one above), otherwise there are two
- The CP asymmetry simplifies if the transition is dominated by only one amplitude (like $B_s^0 \to J/\psi\phi$ and $B^0 \to J/\psi K_S^0$)

$$\mathcal{A}_{CP}(t) = \boxed{\frac{-\Im(\lambda_f)\sin(\Delta m t)}{\cosh(\frac{1}{2}\Delta\Gamma t) + \Re(\lambda_f)\sinh(\frac{1}{2}\Delta\Gamma t)}} \tag{48}$$

- Note that CPV can **still occur** even if both $|q/p| = 1$ and $|A(f)| = |\bar{A}_f|$, *i.e.* when $\Im(\lambda_f) \neq 0$

- In the B^0 system $\Delta\Gamma \sim 0$

$$\begin{aligned}\Gamma_{X^0\to f}(t) = |A_f|^2 \quad (1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\Big[&\cancelto{1}{\cosh(\tfrac{1}{2}\Delta\Gamma t)} + C_f\cos(\Delta m t) \\ &+ \cancelto{0}{D_f\sinh(\tfrac{1}{2}\Delta\Gamma t)} - S_f\sin(\Delta m t)\Big]\end{aligned} \tag{49}$$

$$\begin{aligned}\Gamma_{\overline{X}^0\to f}(t) = |A_f|^2\left|\frac{p}{q}\right|(1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\Big[&\cancelto{1}{\cosh(\tfrac{1}{2}\Delta\Gamma t)} - C_f\cos(\Delta m t) \\ &+ \cancelto{0}{D_f\sinh(\tfrac{1}{2}\Delta\Gamma t)} + S_f\sin(\Delta m t)\Big]\end{aligned} \tag{50}$$

- **The time-dependent CP asymmetry is**

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{C_f\cos(\Delta m t) - S_f\sin(\Delta m t)} \tag{51}$$

Specific Meson Formalism

- In the D^0 system Δm and $\Delta\Gamma$ are both small

$$\Gamma_{X^0\to f}(t) = |A_f|^2 \quad (1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\Big[1 + C_f + D_f\tfrac{1}{2}\Delta\Gamma t - S_f\Delta m t\Big] \tag{52}$$

$$\Gamma_{\overline{X}^0\to f}(t) = |A_f|^2 \left|\frac{p}{q}\right|(1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\Big[1 - C_f + D_f\tfrac{1}{2}\Delta\Gamma t + S_f\Delta m t\Big] \tag{53}$$

- **The time-dependent CP asymmetry is**

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{\frac{C_f - S_f\Delta m t}{1+\frac{1}{2}D_f\Delta\Gamma t}} \tag{54}$$

- With no tagging of flavour and no CPV in mixing we see no asymmetry (just get the sum)

$$\Gamma_{X^0\to f}(t) = |A_f|^2(1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) + D_f\sinh(\tfrac{1}{2}\Delta\Gamma t)\Big] \tag{55}$$

$$\Gamma_{\overline{X}^0\to f}(t) = |A_f|^2(1+|\lambda_f|^2)\frac{e^{-i\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) + D_f\sinh(\tfrac{1}{2}\Delta\Gamma t)\Big] \tag{56}$$

- **The time-dependent CP asymmetry is**

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{0} \tag{57}$$

	K^0	K^+	Λ^0	D^0	D^+	D_s^+	Λ_c^+	B^0	B^+	B_s^0	Λ_b^0
CP violation in mixing	✓✓	-	-	✗	-	-	-	✗	-	✗	-
CP violation in interference	✓	-	-	✗	-	-	-	✓✓	-	✓✓	-
CP violation in decay	✓	✗	✗	✓✓	✗	✗	✗	✓✓	✓✓	✓	✓

KEY:

✓✓	Strong evidence ($> 5\sigma$)
✓	Some evidence ($> 3\sigma$)
✗	Not seen
-	Not possible

6. Recap

Recap

In this lecture we have covered

- Neutral Meson Mixing (without CPV)
  - Time evolution of coupled systems
  - Differences in mixing parameters between neutral meson states
- B-meson production and experiments / techniques
  - B-factories and Belle 2
  - LHCb
  - Flavour Tagging
  - Dalitz analysis
- CP violation
  - CP violation types
  - The “master” equations for generalised meson decays

End of Lecture 2

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